### Mild Cognitive Impairment

Mild Cognitive Impairment (stable)

Alzheimer Disease















**©** RAVLT

් ANARTERR

**ී** GDTOTAL

**©** TRABSCOR

© CATANIMSC

**TRAASCOR** 

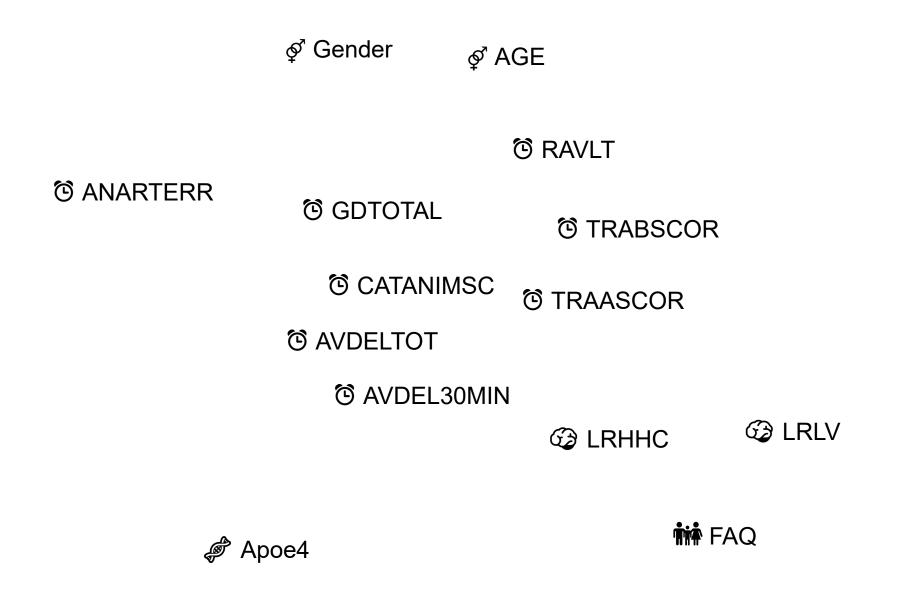
© AVDELTOT

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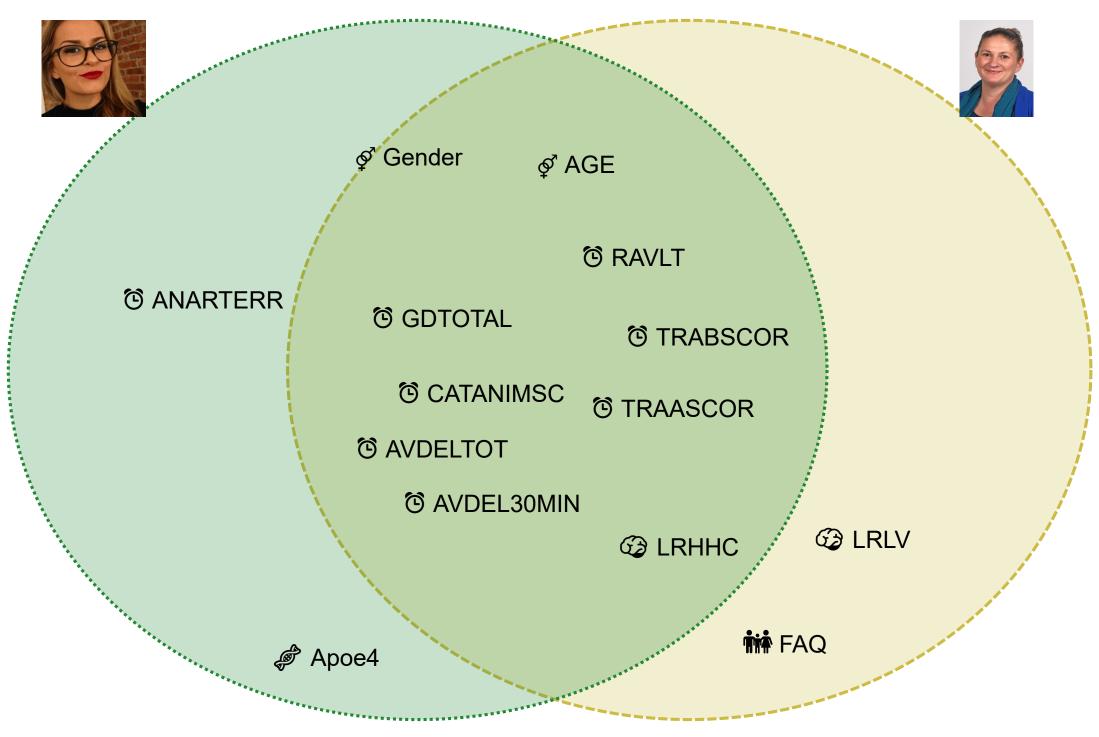
3 LRLV

Apoe4

**†i** FAQ



How 'good' are these features at prognosing the later onset of Alzheimer?



How 'good' are these features at prognosing the later onset of Alzheimer?



O ANAR

### **Functional Activities Questionnaire**

#### Administration

Ask informant to rate patient's ability using the following scoring system:

- Dependent = 3
- Requires assistance = 2
- Has difficulty but does by self = 1
- Normal = 0
- Never did [the activity] but could do now = 0
- Never did and would have difficulty now = 1

Writing checks, paying bills, balancing checkbook	
Assembling tax records, business affairs, or papers	
Shopping alone for clothes, household necessities, or groceries	
Playing a game of skill, working on a hobby	
Heating water, making a cup of coffee, turning off stove after use	
Preparing a balanced meal	
Keeping track of current events	
Paying attention to, understanding, discussing TV, book, magazine	
Remembering appointments, family occasions, holidays, medications	
Traveling out of neighborhood, driving, arranging to take buses	
TOTAL SCORE:	

#### **Evaluation**

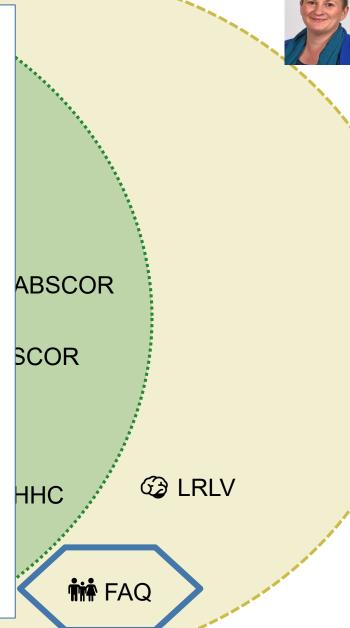
Sum scores (range 0-30). Cutpoint of 9 (dependent in 3 or more activities) is recommended to indicate impaired function and possible cognitive impairment.

Pfeffer RI et al. Measurement of functional activities in older adults in the community. J Gerontol 1982; 37(3):323-329. Reprinted with permission of The Gerontological Society of America, 1030 15<sup>th</sup> Street NW, Suite 250, Washington, DC 20005 via Copyright Clearance Center. Inc.

These guidelines/tools are informational only. They are not intended or designed as a substitute for the reasonable exercise of independent clinical judgment by practitioners considering each patient s needs on an individual basis. Guideline recommendations apply to populations of patients. Clinical judgment is necessary to design treatment plans for individual patients. For more information, visit our Web site at www.aviviahealth.com. To contact our Chief Medical Officer, please call 1-888-4AVIVIA (1-888-428-4842).

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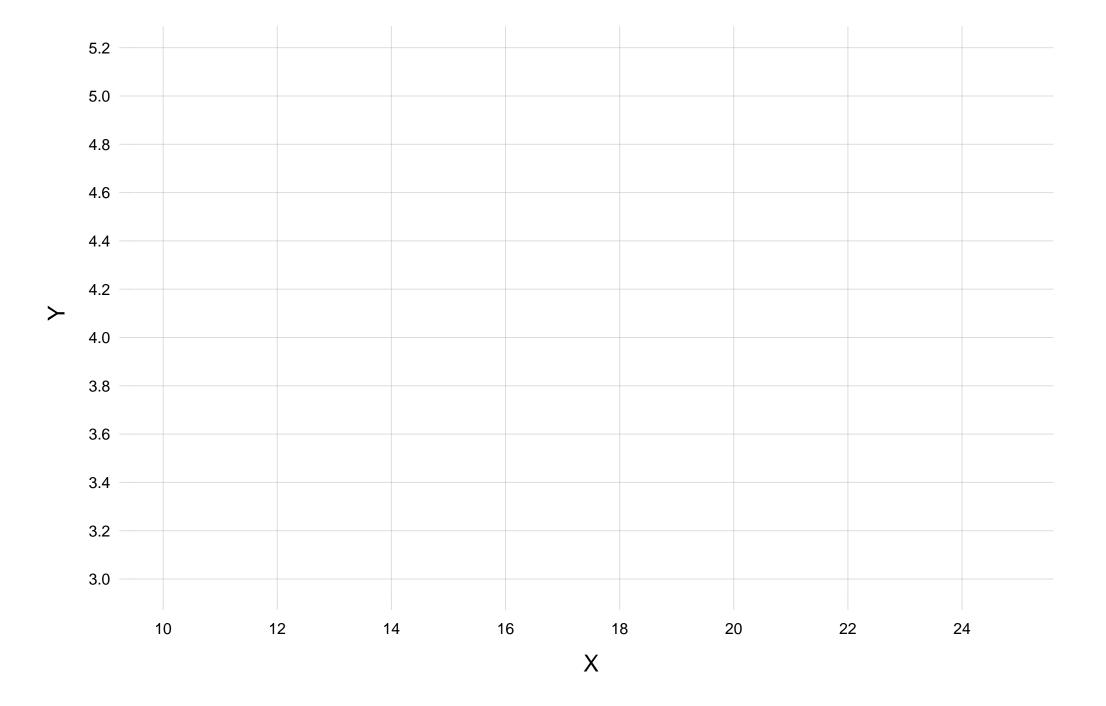


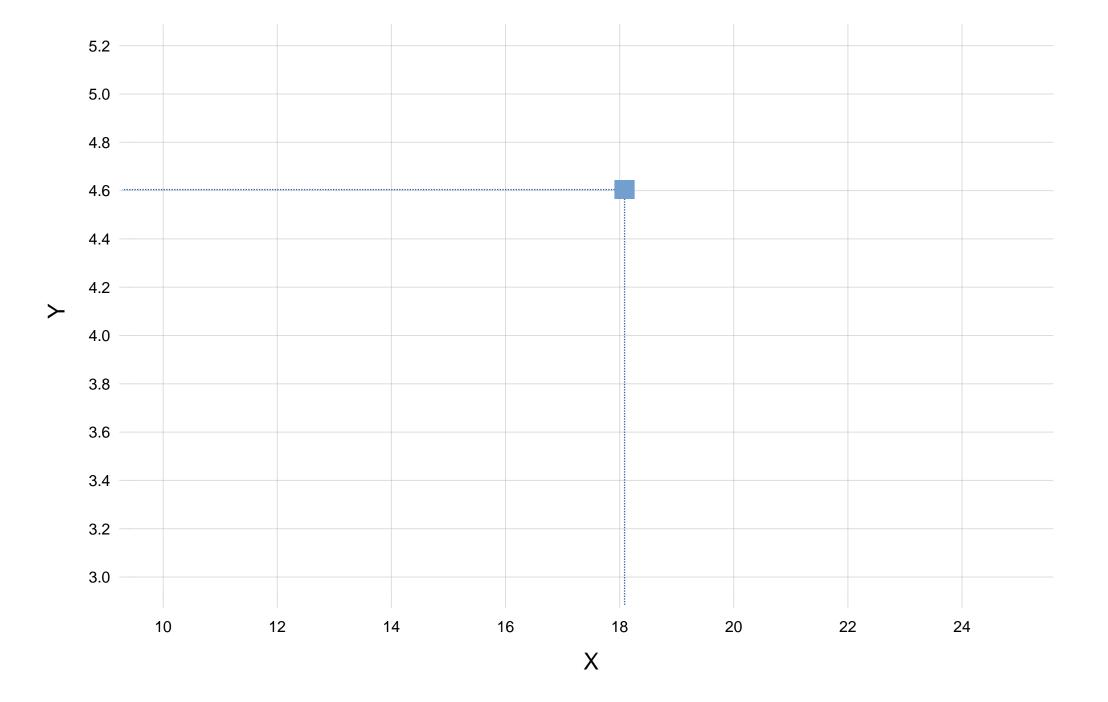
**Ingrid's study**: 12+1 variates, 678 datapoints

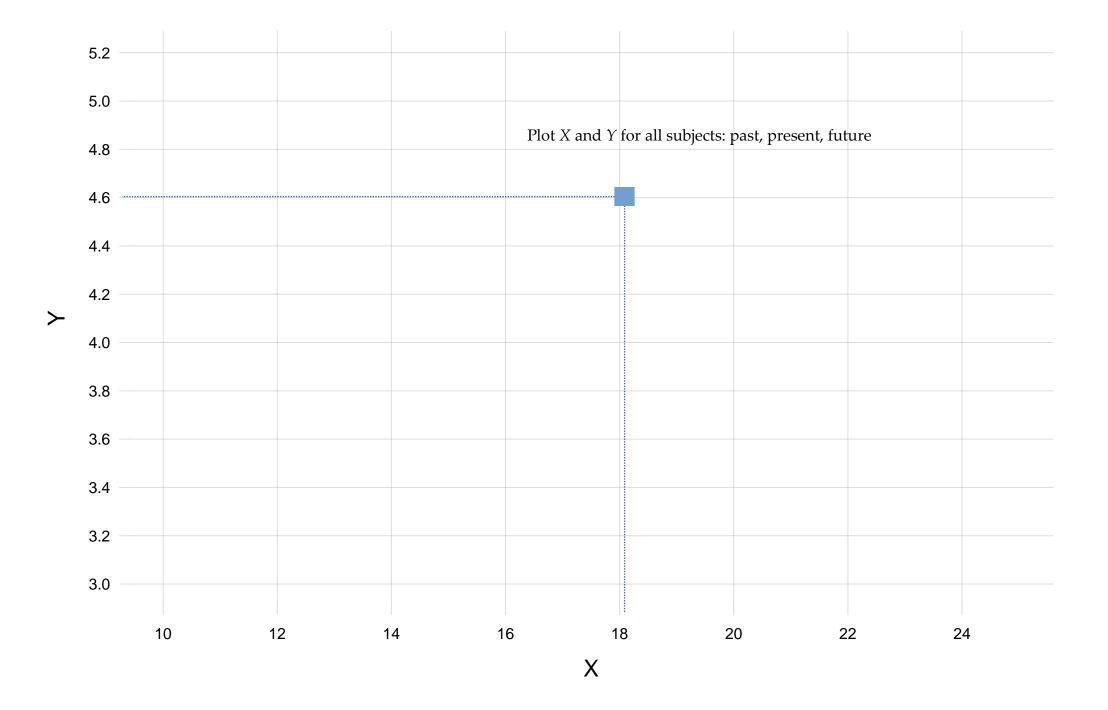
**Alexandra's study**: 11+1 variates, 708 datapoints (43 missing values)

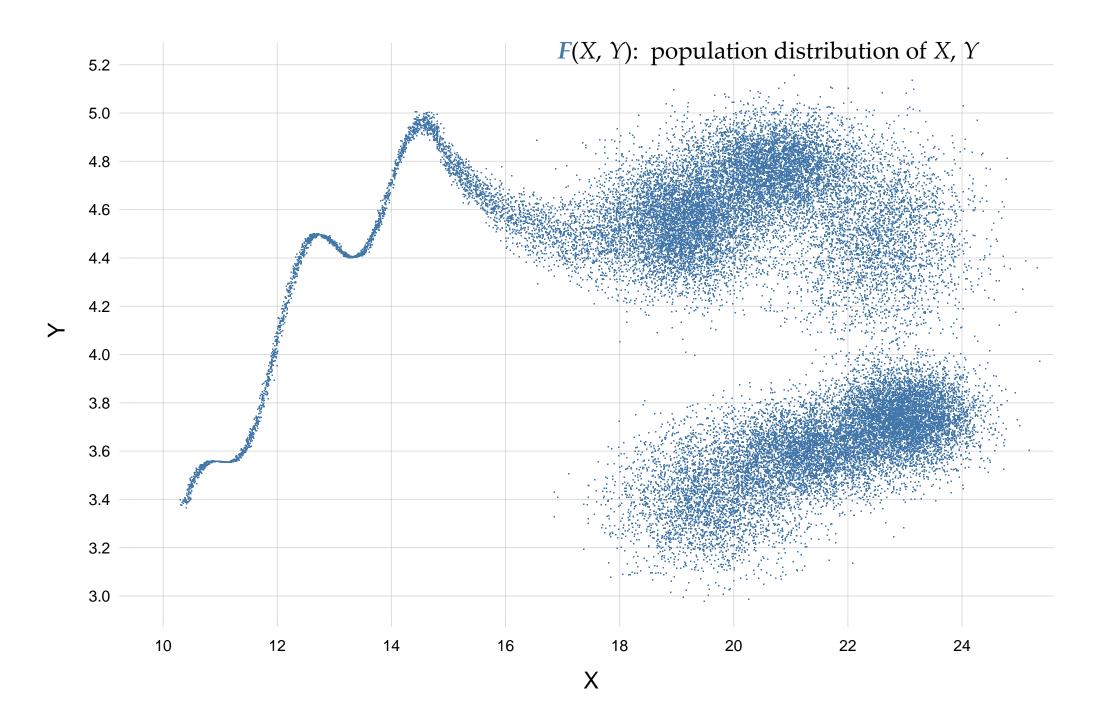
Computation time: ~65 h/study (3 parallel sessions to assess numeric convergence)

HPC UNINETT | sigma2



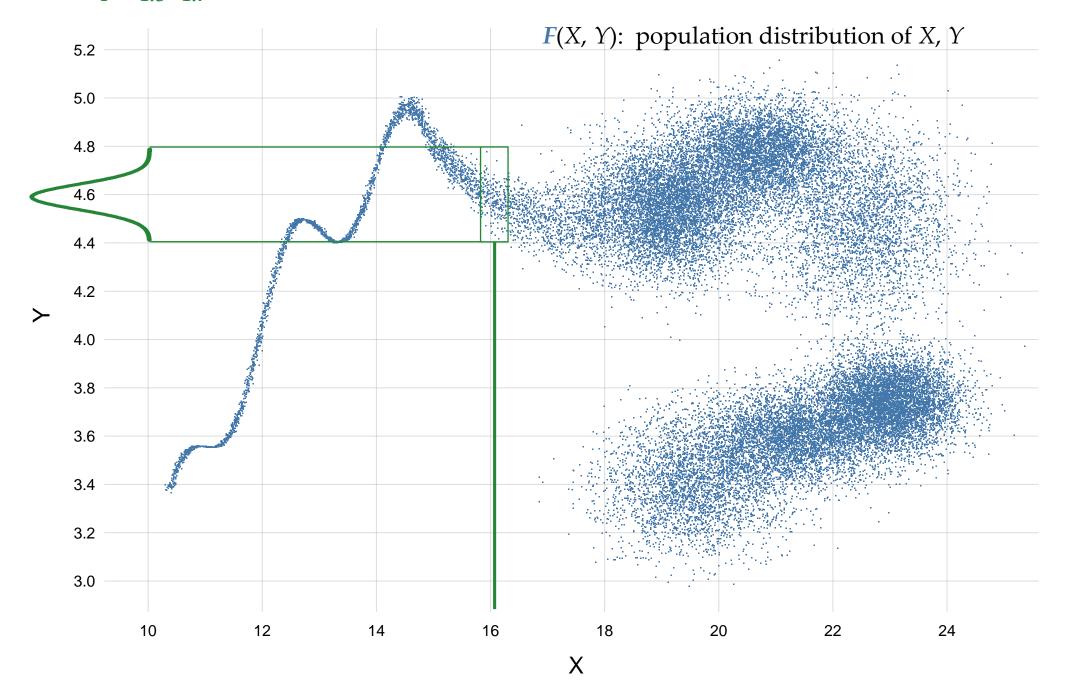




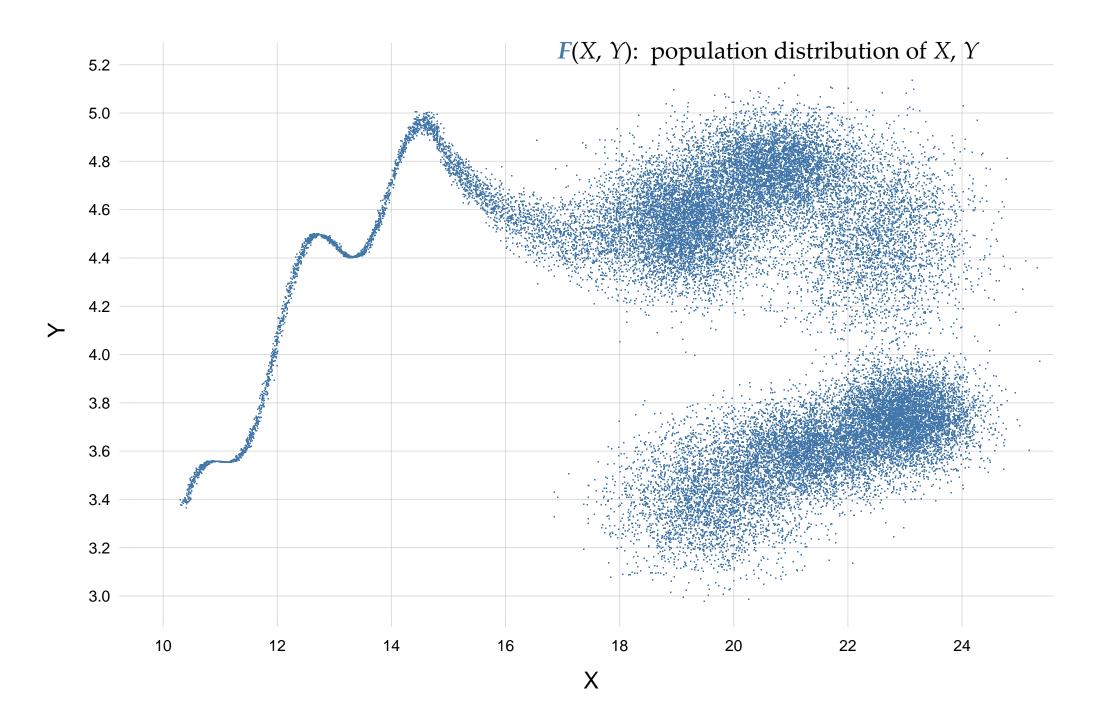


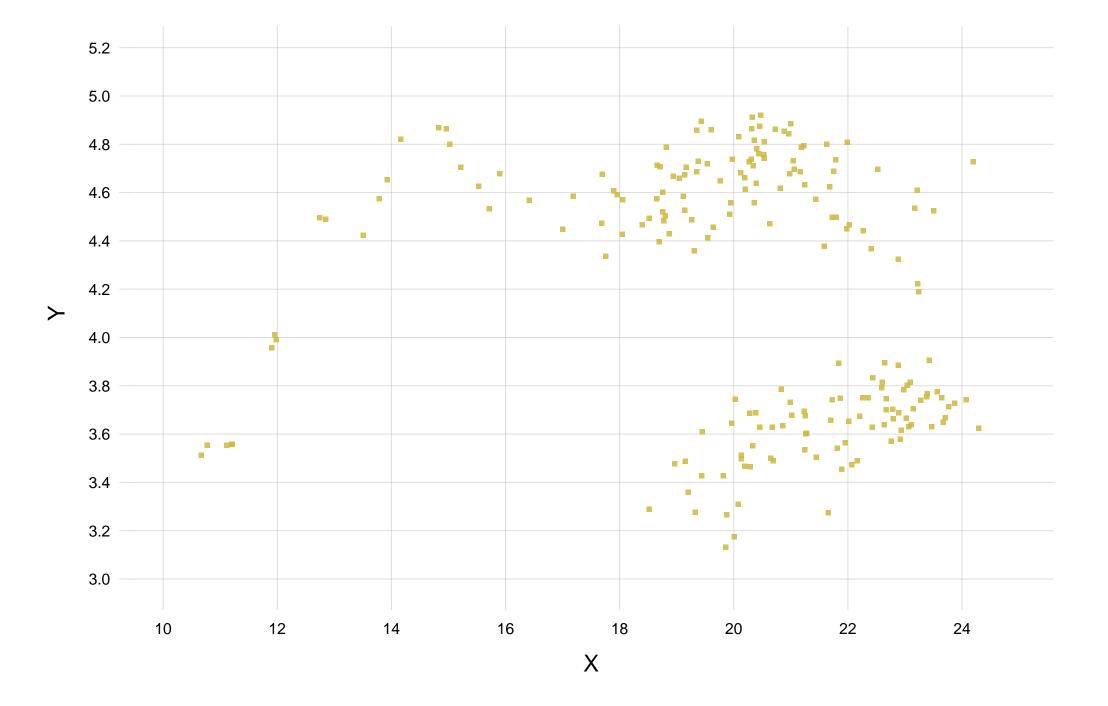
New patient: X = 16

 $\Rightarrow Y \approx 4.5-4.7$ 



$$P(y \mid x) = F(y \mid x)$$





$$P(y \mid x) = F(y \mid x)$$

$$P(y \mid x) = \int F(y \mid x) p(F \mid data) dF$$

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$$p(F \mid data) \propto F(y_1, x_1) \times F(y_2, x_2) \times F(y_3, x_3) \times ...$$

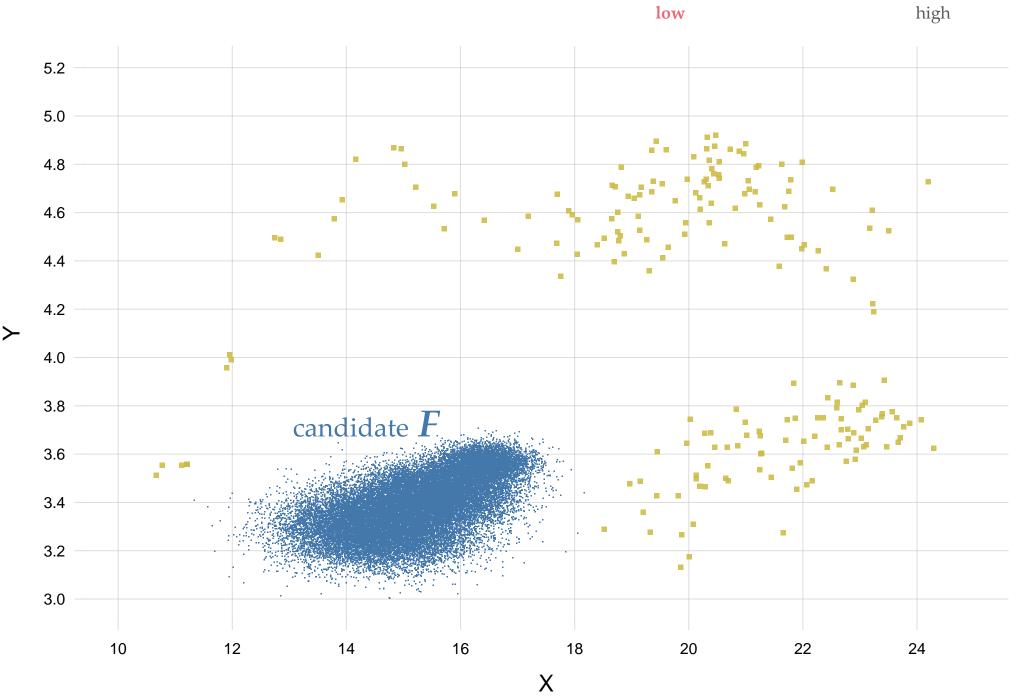
how well the 'candidate' distribution fits the data

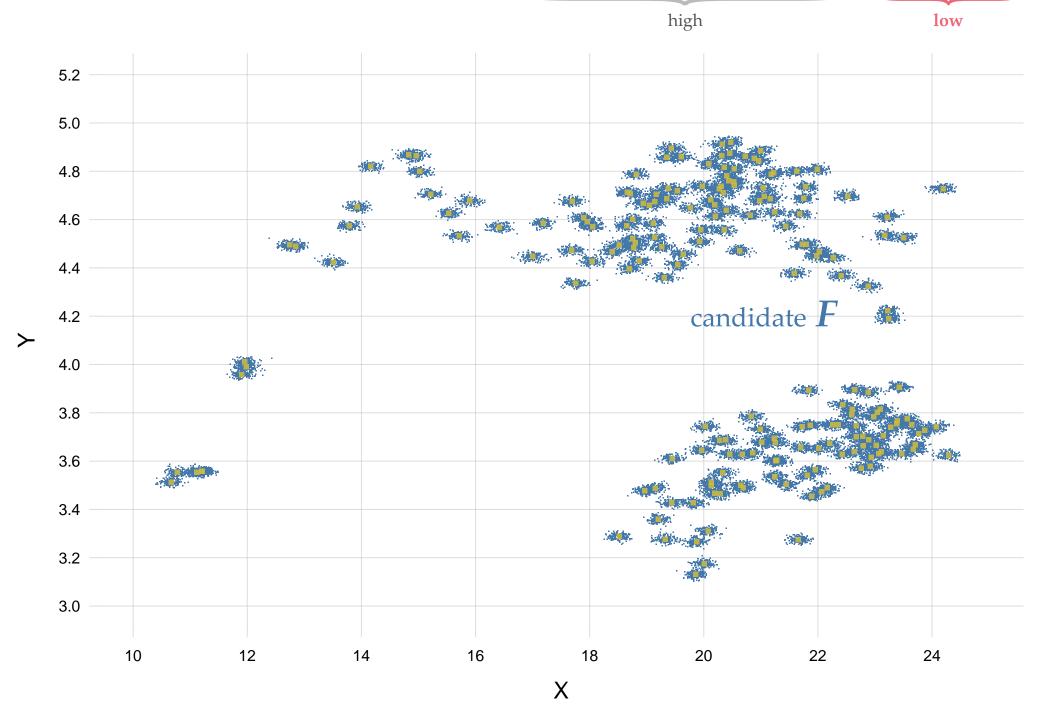
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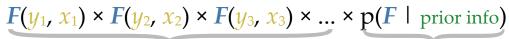
$$p(F \mid data) \propto F(y_1, x_1) \times F(y_2, x_2) \times F(y_3, x_3) \times ... \times p(F \mid prior info)$$

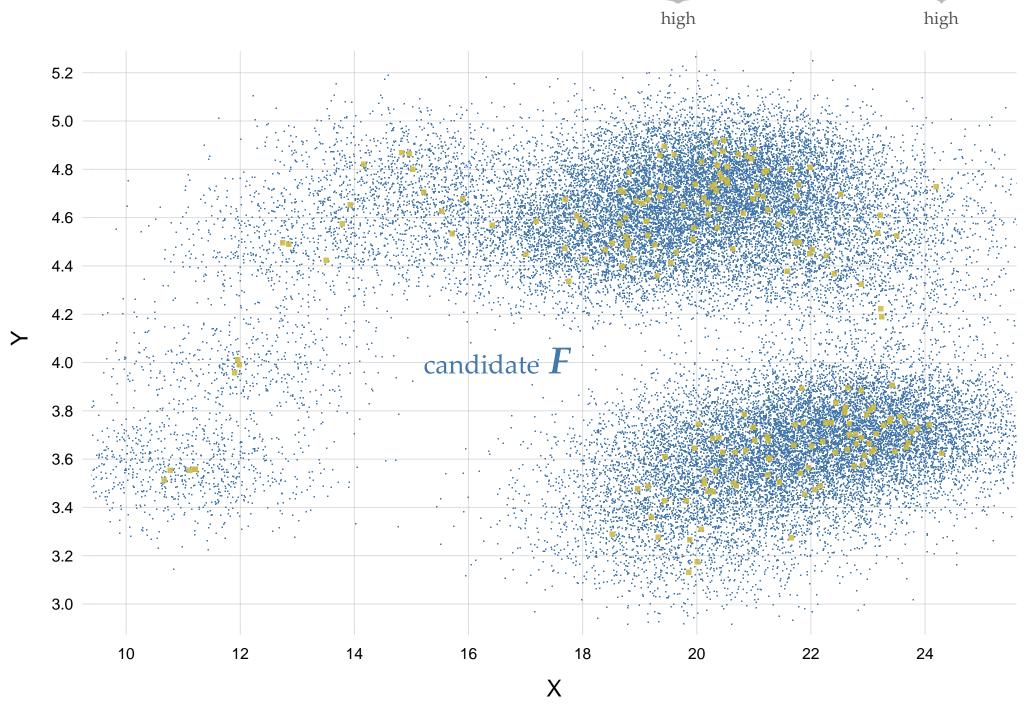
how well the 'candidate' distribution fits the data

extra-data knowledge







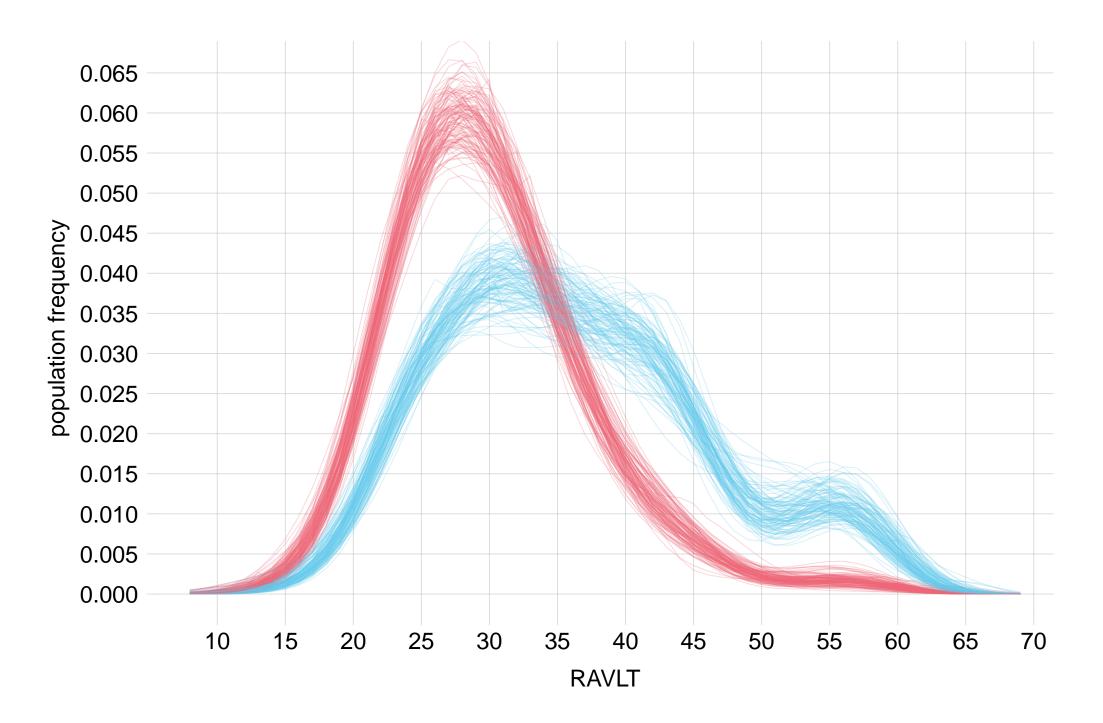


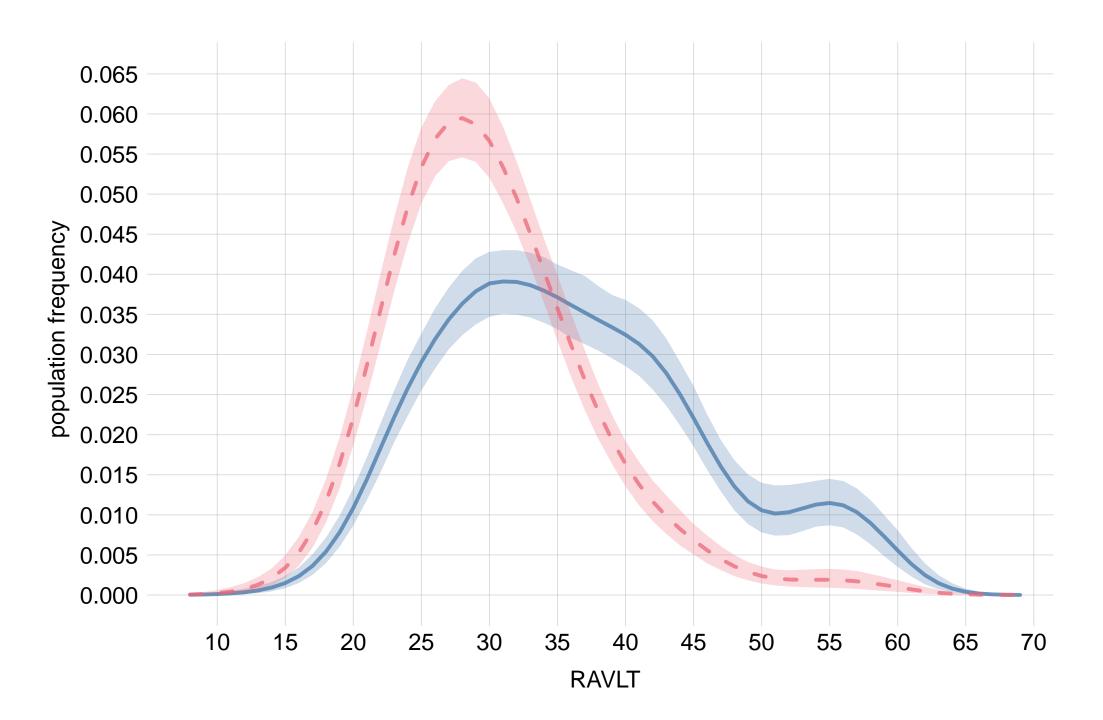
### $intuition \rightarrow mathematics$

## intuition — mathematics

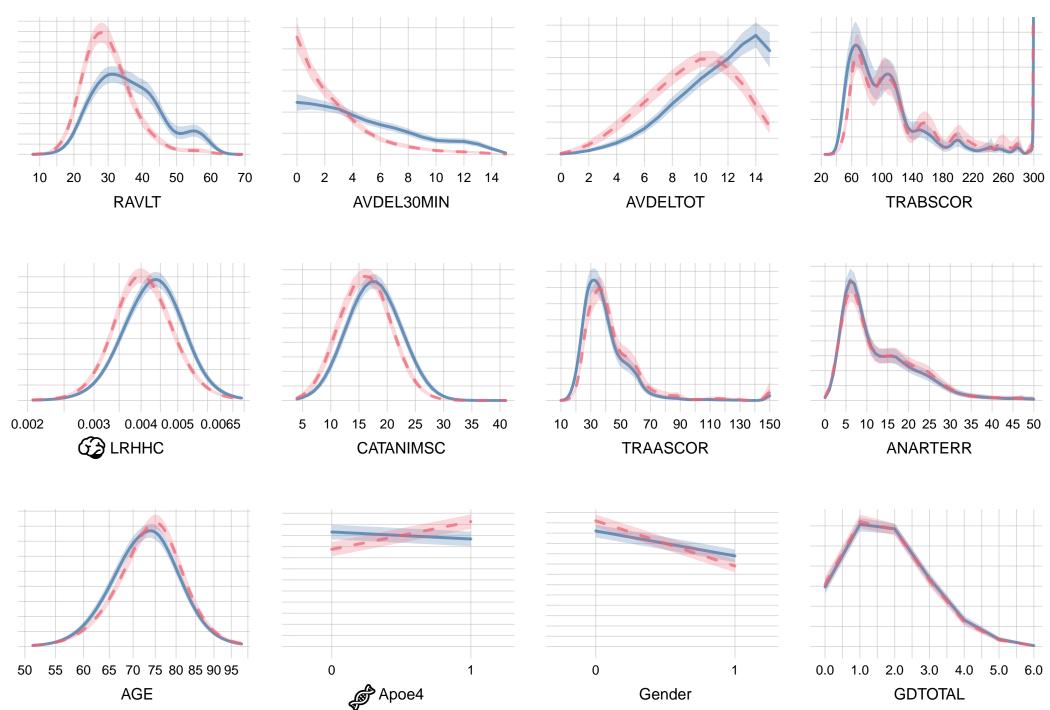
# $first\ principles \rightarrow mathematics \rightarrow intuition$

('Bayesian')

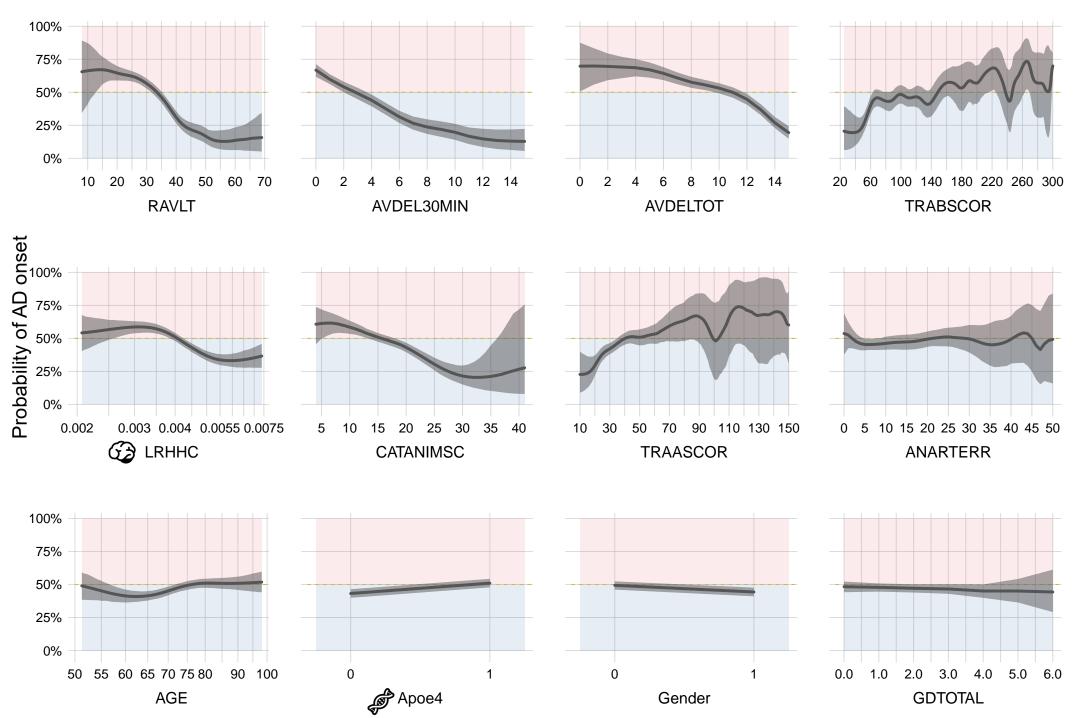




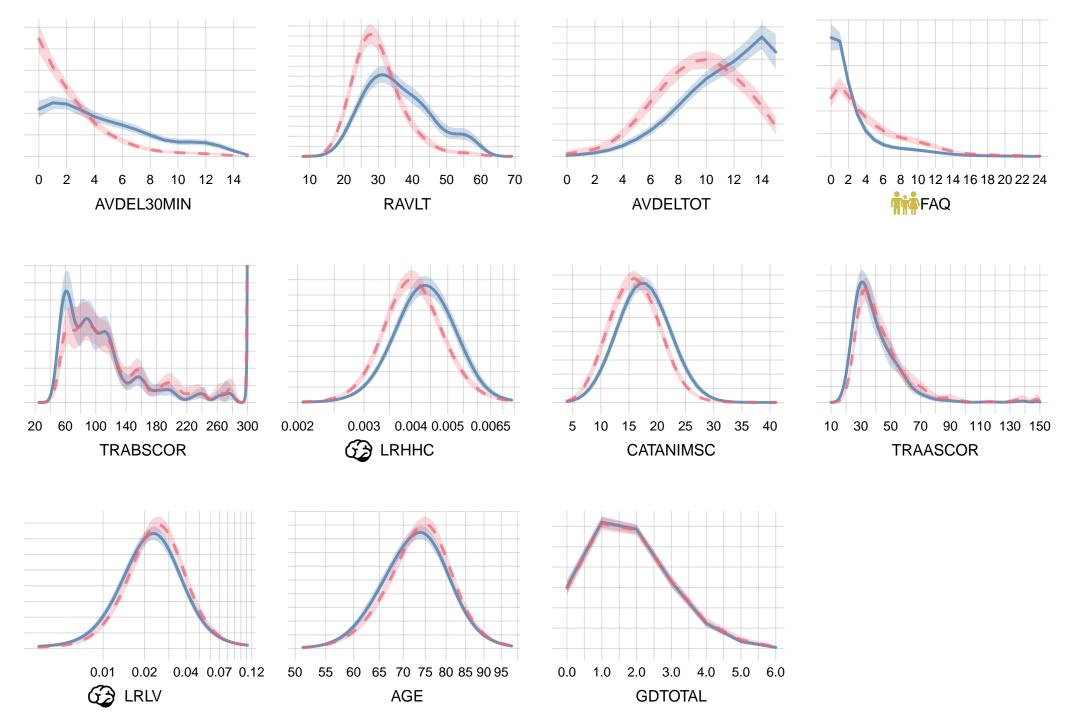




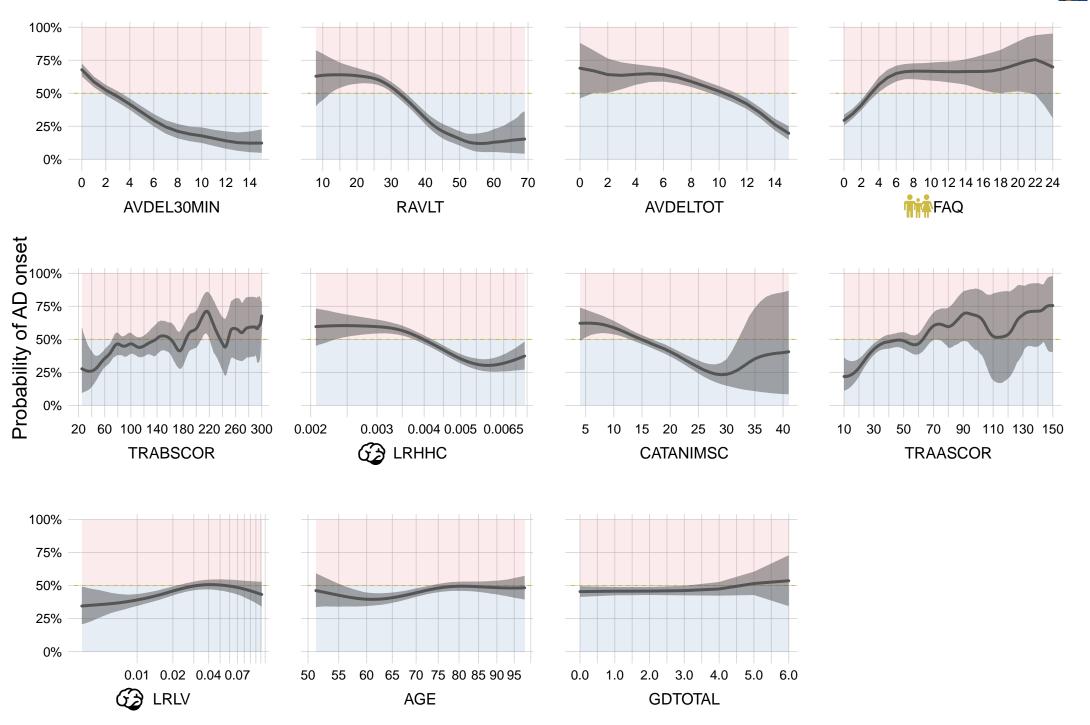


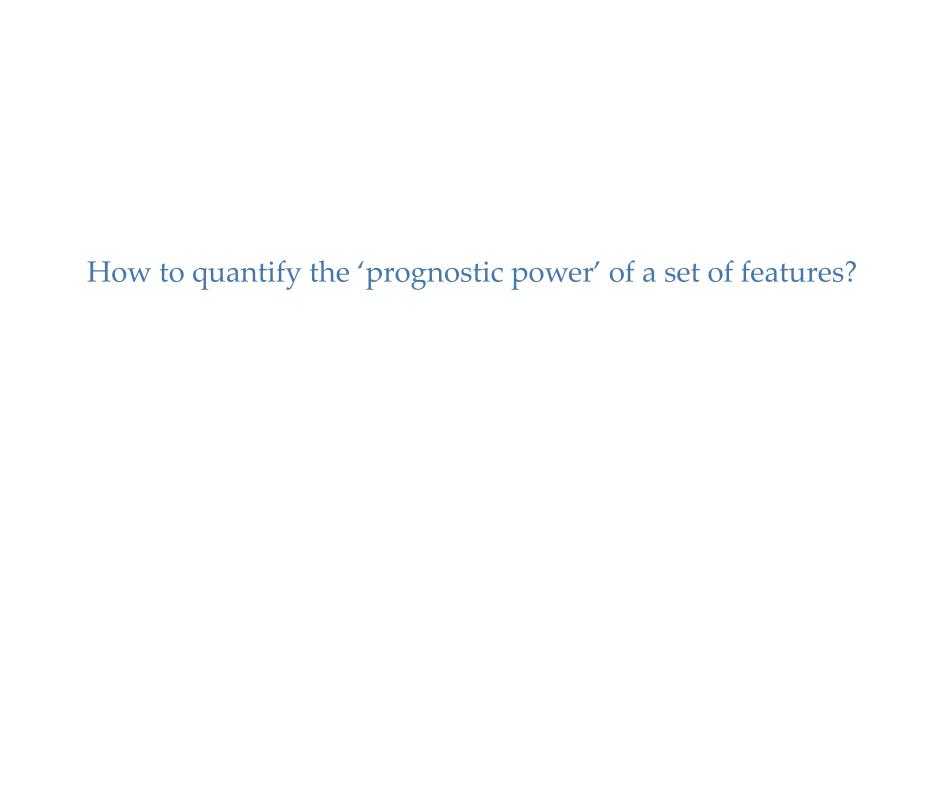












# Prediction problem:

guess the six digits of the winning lottery ticket ??????

Clue A:

Clue B: **///?/?** 

Clue C: ???///

What is the 'importance' or 'predictive power' of each clue?

# Scenario 1: we can use **only one** clue

Clue **A**: **////**??

Clue **B**: **///?/?** 

Clue **C**: ???**/**//

increasing importance

Best: **A** or **B** (each gives 1/81 winning chance)

Worst: **C** (gives 1/729 winning chance)

### Scenario 2: we can use **all** clues

Clue **A**: **////??** 

Clue **B**: **///?/?** 

Clue **C**: ???///

→ We fully know the winning number!

Scenario 2: what happens if we **discard** clues?

Clue **A**: **////??** 

Clue **B**: **///?/?** 

Scenario 2: what happens if we discard clues?

Clue **A**: **////?**?

Clue **B**: **///?/?** 

Clue **C**: ???**///** 

• Discard A: still 100% win  $\Rightarrow$  A has 'importance = 0'

Clue **B**: **///?/?** 

- Discard A: still 100% win  $\Rightarrow$  A has 'importance = 0'
- Discard **B**: still 100% win  $\Rightarrow$  **B** has 'importance = **0**'

Clue **B**: **///?/?** 

- Discard A: still 100% win  $\Rightarrow$  A has 'importance = 0'
- Discard **B**: still 100% win  $\Rightarrow$  **B** has 'importance = **0**'
  - Discard **A** and **B**: 1/9 winning chance

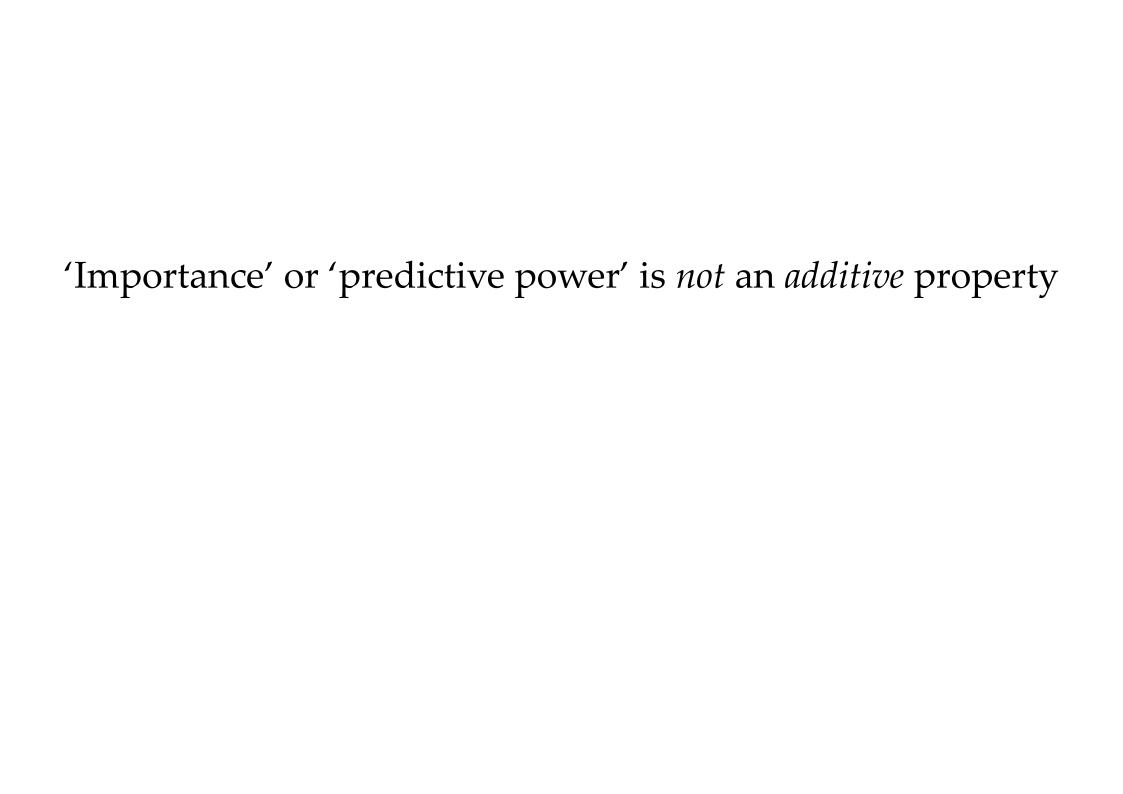
Clue **B**: **///?/?** 

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  - Discard **A** and **B**: 1/9 winning chance
  - $\Rightarrow$  A and B together have 'importance > 0'

Clue **B**: **///?/?** 

- Discard A: still 100% win  $\Rightarrow$  A has 'importance = 0'
- Discard **B**: still 100% win  $\Rightarrow$  **B** has 'importance = **0**'
  - Discard **A** and **B**: 1/9 winning chance
  - $\Rightarrow$  A and B together have 'importance>0'

$$'0 + 0 \neq 0'$$



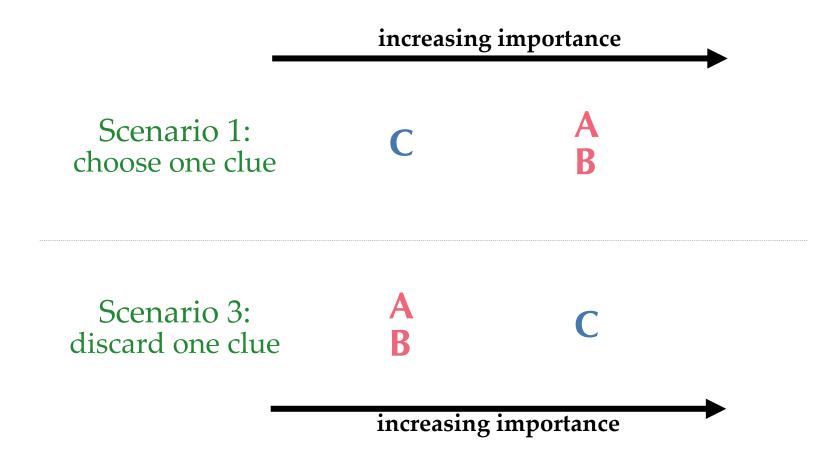
Clue **B**: **///?/?** 

Clue **C**: ???**/**//

increasing importance

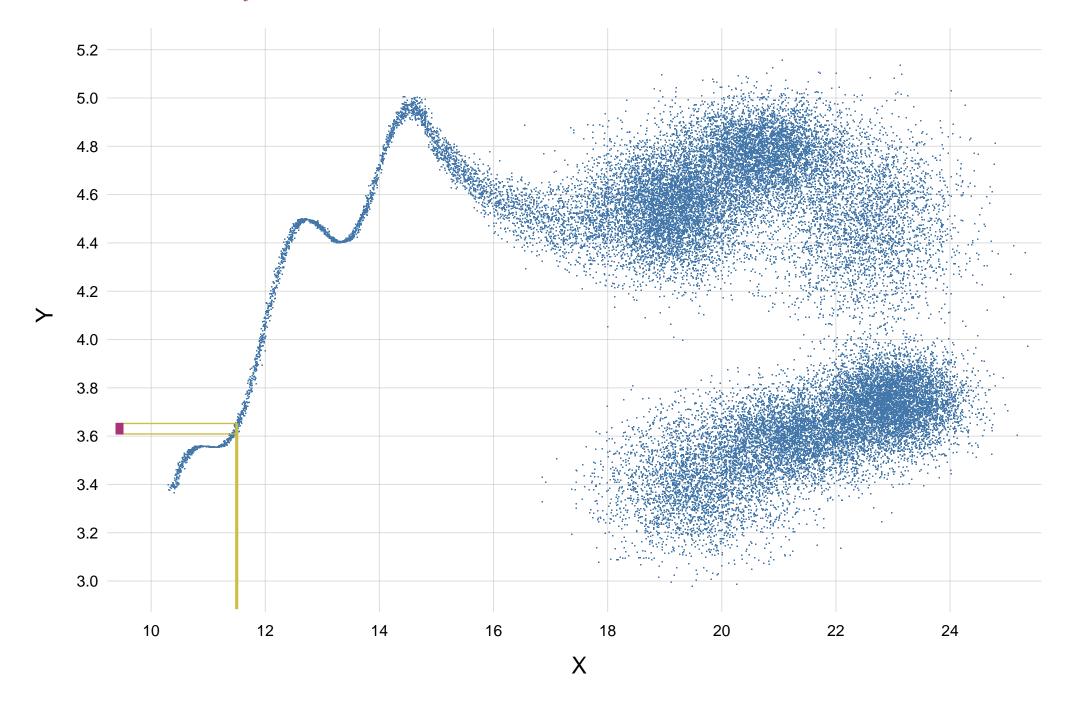
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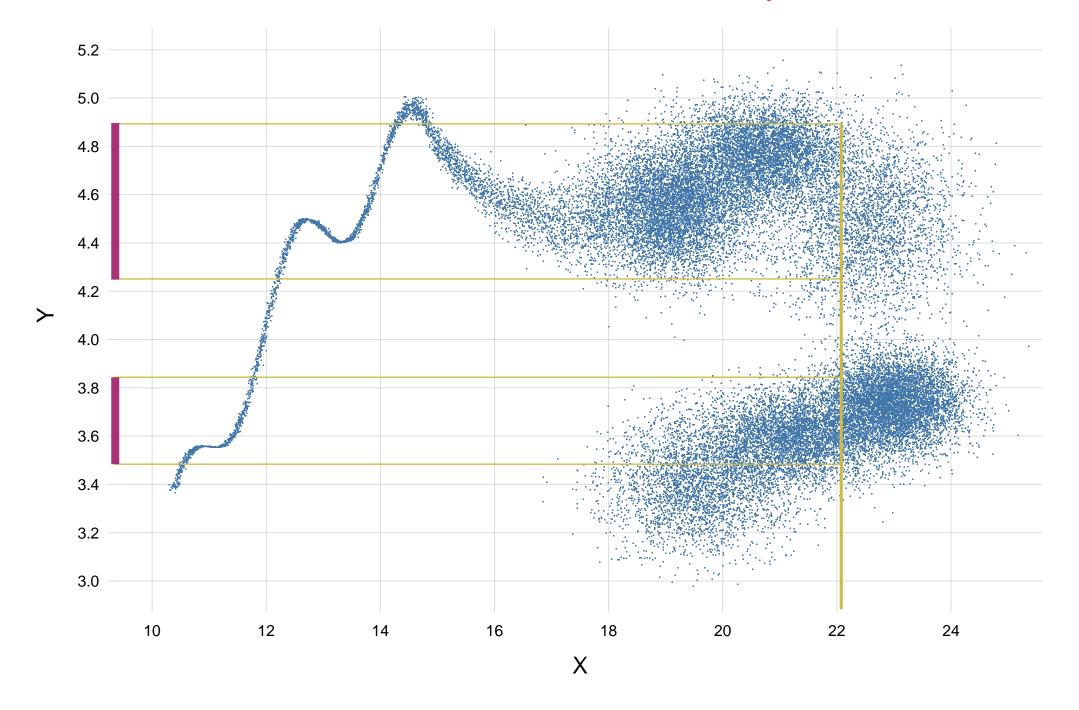
 $\rightarrow$  If we have to discard one clue, it's most important that we keep  $\mathbf{C}$ 



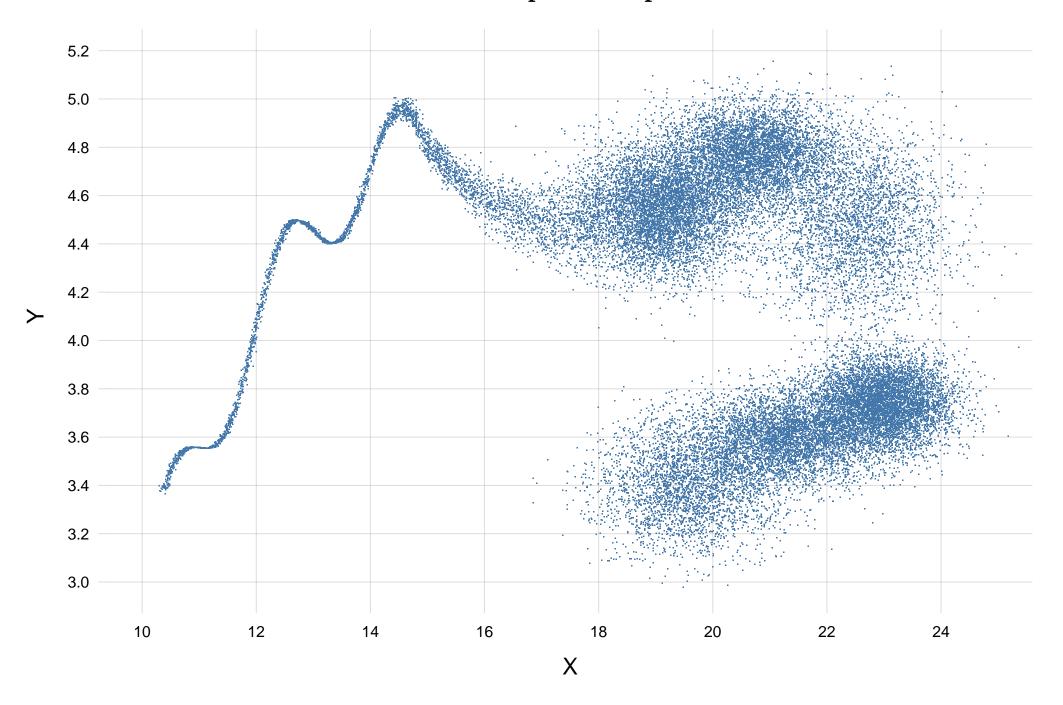
'Importance' or 'predictive power' of *X* is *context-dependent* (which other features are we considering?)

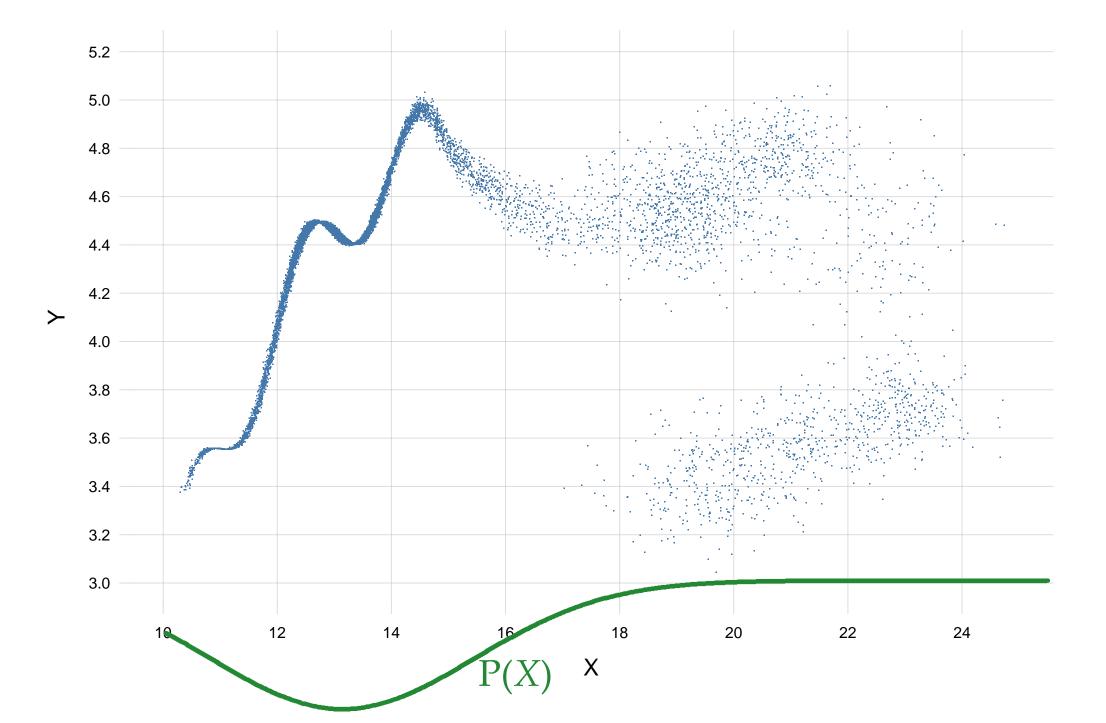
 $x = 11.5 \implies y \approx 3.60 - 3.65$ 

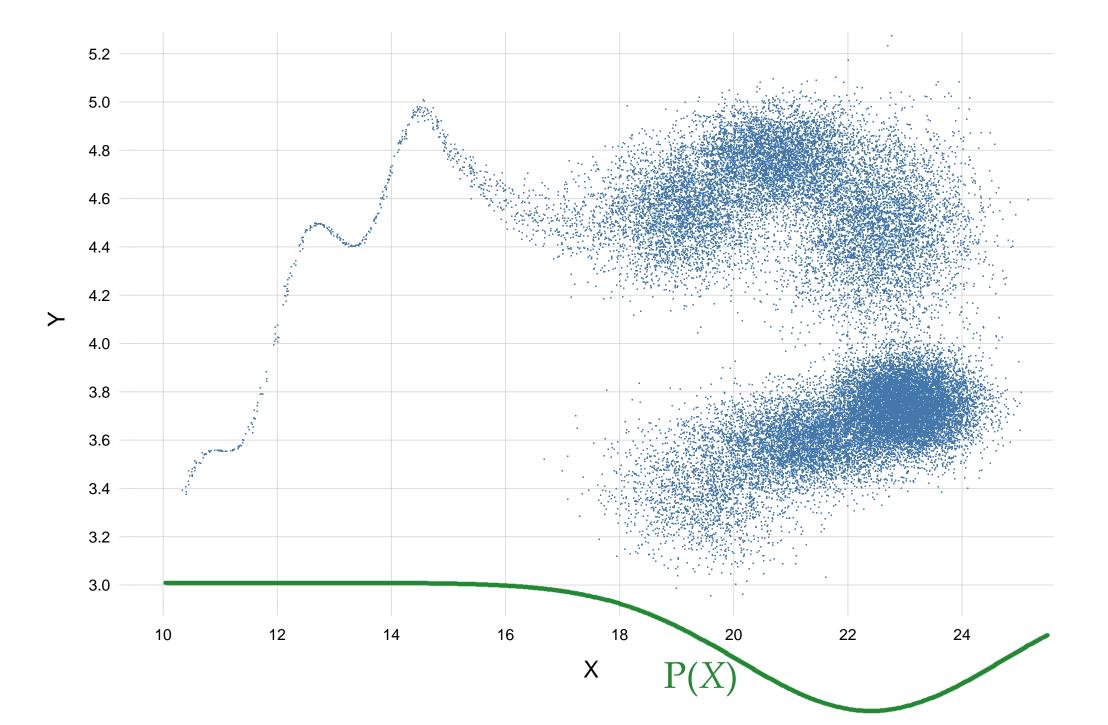




## What is the 'overall predictive power' of X?







The 'importance' or 'predictive power' of X depends on  $\mathrm{P}(X)$ 

The 'importance' or 'predictive power' of X depends on P(X)

## **Information Theory**

### The Bell System Technical Journal

Vol. XXVII July, 1948 No. 3

#### A Mathematical Theory of Communication

By C. E. SHANNON

#### Introduction

THE recent development of various methods of modulation such as PCM and PPM which exchange bandwidth for signal-to-noise ratio has intensified the interest in a general theory of communication. A basis for such a theory is contained in the important papers of Nyquist¹ and Hartley² on this subject. In the present paper we will extend the theory to include a number of new factors, in particular the effect of noise in the channel, and the savings possible due to the statistical structure of the original message and due to the nature of the final destination of the information.

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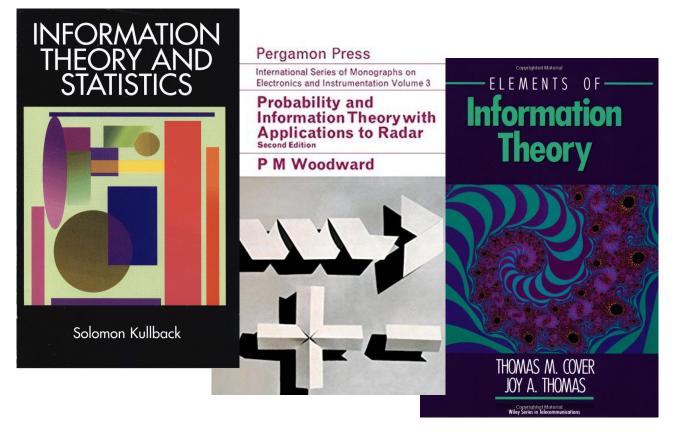
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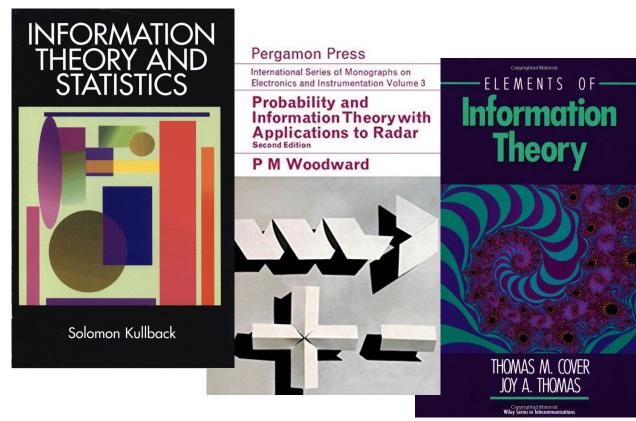
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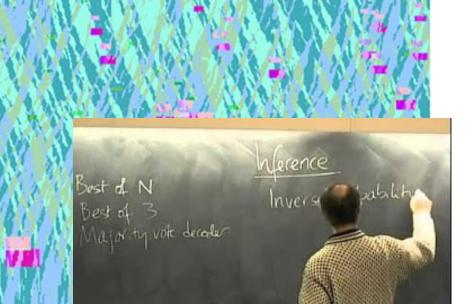
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David J.C. MacKay

# Information Theory, Inference, and Learning Algorithms https://www.inference.org.uk/itila/book.html https://youtube.com/playlist?list=PLruBu5BI5n4aFpG32iMbdWoRVAA-Vcso6



Cambridge University Press, 2003

'predictive power' of X for Y := Mutual information between <math>Y and X (mean transinformation content)

$$I(X;Y) := \int p(y|x) p(x) \log \left[ \frac{p(y|x)}{p(y)} \right] dy dx$$

$$I(Y; X_1, X_2) \geq I(Y; X_1)$$

$$I(Y; X_1, X_2) \geq I(Y; X_2)$$

but 
$$I(Y; X_1, X_2) \neq I(Y; X_1) + I(Y; X_2)$$



Edition 1.0 2008-03

# INTERNATIONAL STANDARD

# NORME INTERNATIONALE

Quantities and units -

Part 13: Information science and technology

Grandeurs et unités -

Partie 13: Science et technologies de l'information





Edition 1.0 2008-03

# INTERNATIONAL STANDARD

INFORMATION SCIENCE AND TECHNOLOGY QUANTITIES							
Item No.	Name	Symbol	Definition	Remarks			
13-24 ( <i>902</i> )	information content fr quantité (f) d'information	I(x)	$I(x) = \operatorname{lb} \frac{1}{p(x)} \operatorname{Sh} = \operatorname{lg} \frac{1}{p(x)} \operatorname{Hart} =$ $\operatorname{ln} \frac{1}{p(x)} \operatorname{nat}$ where $p(x)$ is the probability of event $x$	See ISO/IEC 2382-16, item 16.03.02. See also IEC 60027-3.			
13-25 ( <i>903</i> )	entropy fr entropie (f)	H	$H(X) = \sum_{i=1}^{n} p(x_i)I(x_i)$ for the set $X = \{x_1,, x_n\}$ where $p(x_i)$ is the probability and $I(x_i)$ is the information content of event $x_i$	See ISO/IEC 2382-16, item 16.03.03.			
13-30 ( <i>908</i> )	joint information content fr quantité (f) d'information conjointe	I(x, y)	$I(x, y) = \operatorname{lb} \frac{1}{p(x, y)} \operatorname{Sh} = \operatorname{lg} \frac{1}{p(x, y)} \operatorname{Hart} =$ $\operatorname{ln} \frac{1}{p(x, y)} \operatorname{nat}$ where $p(x, y)$ is the joint probability of events $x$ and $y$				
13-35 ( <i>912</i> )	transinformation content fr transinformation (f)	T(x,y)	T(x,y) = I(x) + I(y) - I(x,y) where $I(x)$ and $I(y)$ are the information contents (13-24) of events $x$ and $y$ , respectively, and $I(x,y)$ is their joint information content (13-30)	See ISO/IEC 2382-16, item 16.04.07.			
13-36 ( <i>913</i> )	mean transinformation content  fr transinformation (f) moyenne	T	$T(X,Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} p(x_i, y_j) T(x_i, y_j)$ for the sets $X = \{x_1,, x_n\}, Y = \{y_1,, y_m\},$ where $p(x_i, y_j)$ is the joint probability of events $x_i$ and $y_j$ , and $T(x_i, y_j)$ is their transinformation content (item 13-35)	See ISO/IEC 2382-16, item 16.04.08.			

UNITS INFORMATION SCIENCE AND TECHNOLOGY							
Item No.	Name	Symbol	Definition	Conversion factors and remarks			
13-24.a	shannon	Sh	value of the quantity when the argument is equal to 2	1 Sh ≈ 0,693 nat ≈ 0,301 Hart			
13-24.b	hartley	Hart	value of the quantity when the argument is equal to 10	1 Hart ≈ 3,322 Sh ≈ 2,303 nat			
13-24.c	natural unit of information	nat	value of the quantity when the argument is equal to e	1 nat ≈ 1,433 Sh ≈ 0,434 Hart			
13-25.a	shannon	Sh					
13-25.b	hartley	Hart					
13-25.c	natural unit of information	nat					
13-30.a	shannon	Sh					
13-30.b	hartley	Hart					
13-30.c	natural unit of information	nat					
13-35.a	shannon	Sh					
13-35.b	hartley	Hart					
13-35.c	natural unit of information	nat					
13-36.a	shannon	Sh		In practice, the unit "shannon per character" is generally used, and			
13-36.b	hartley	Hart		sometimes the units "hartley per character" and "natural unit per character".			
13-36.c	natural unit of information	nat					

$$0 \text{ Sh} \leq I(Y;X) \leq 1 \text{ Sh}$$

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In N=100 new prognoses:

- we are **completely certain** about 22
- we are **completely uncertain** about 100-22 = 78
- $\rightarrow$  approx 22+78/2 = 61 correct prognoses (TP+TN)

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$$\frac{1}{2} + \frac{1}{2}\sqrt{1 - (1 - 0.22)^{\frac{4}{3}}} \approx 77\%$$
 ± 0.8 $\sqrt{N}\%$  correct prognoses

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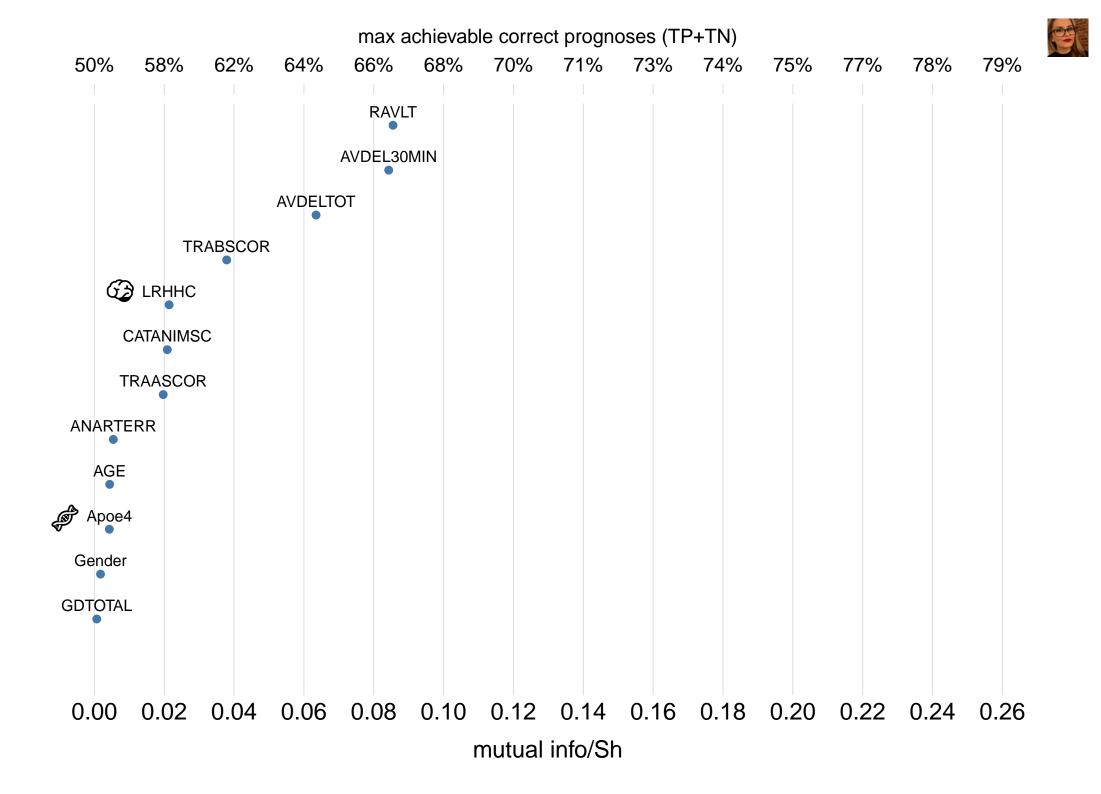
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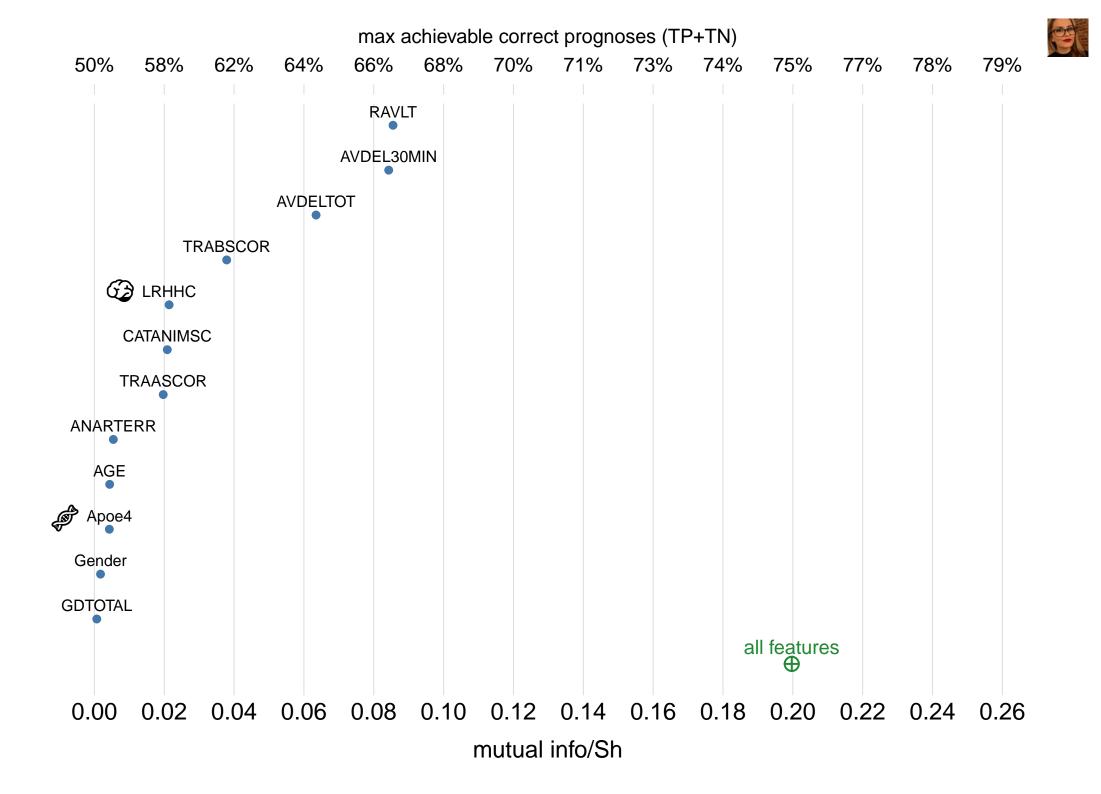
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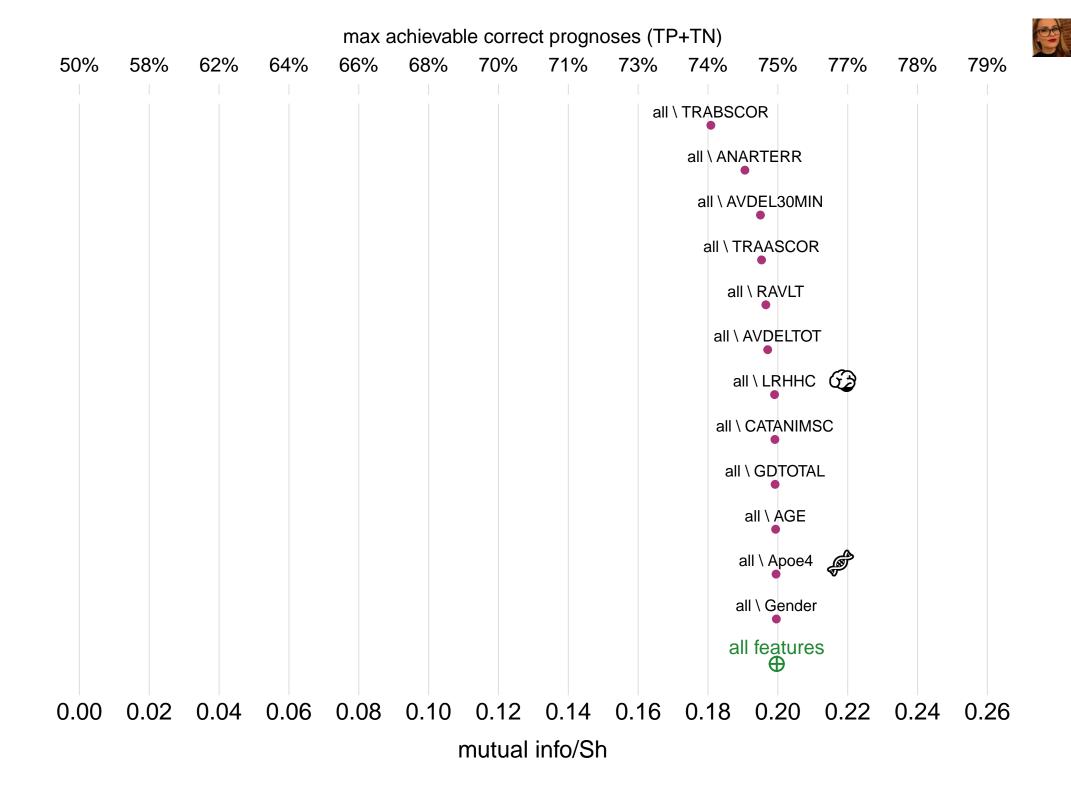
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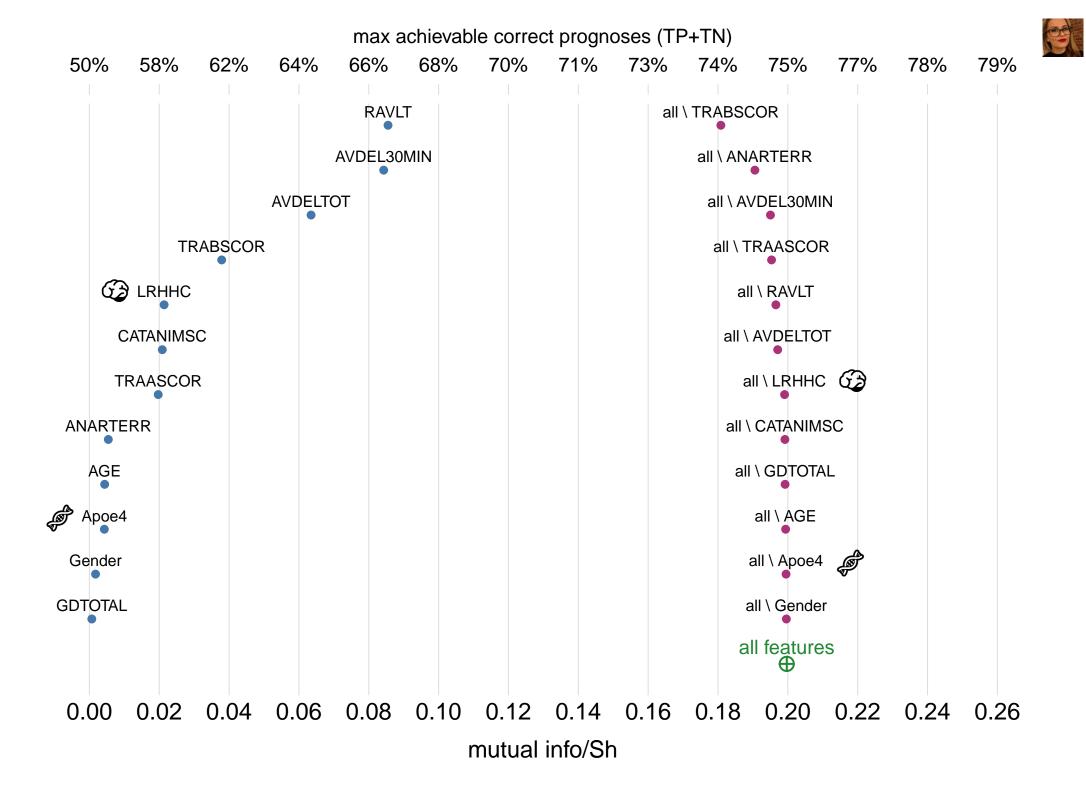
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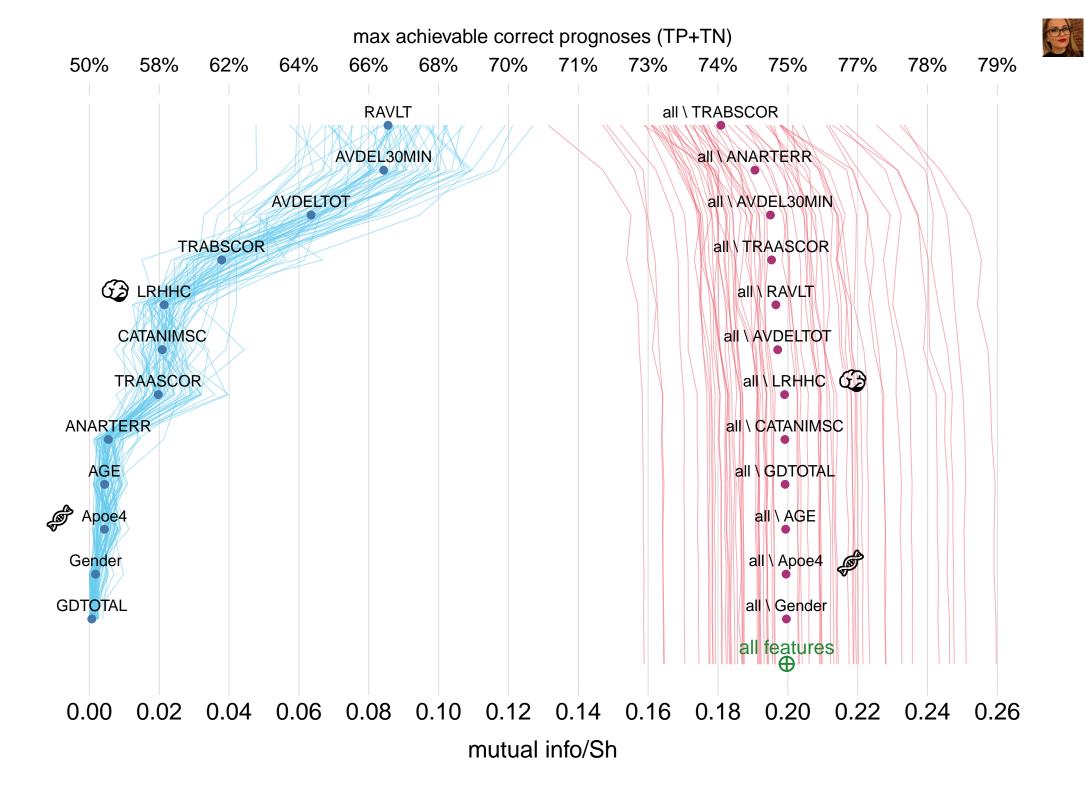
Maximum accuracy attainable by *any* algorithm which uses only feature set *X* 

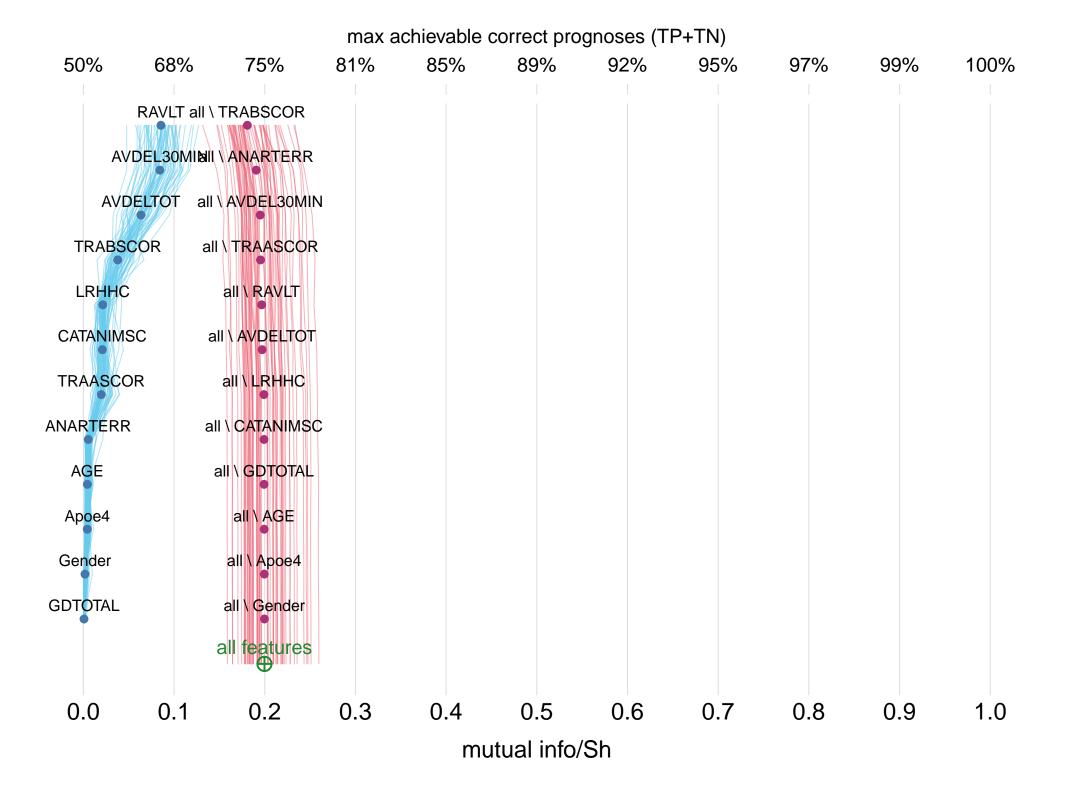


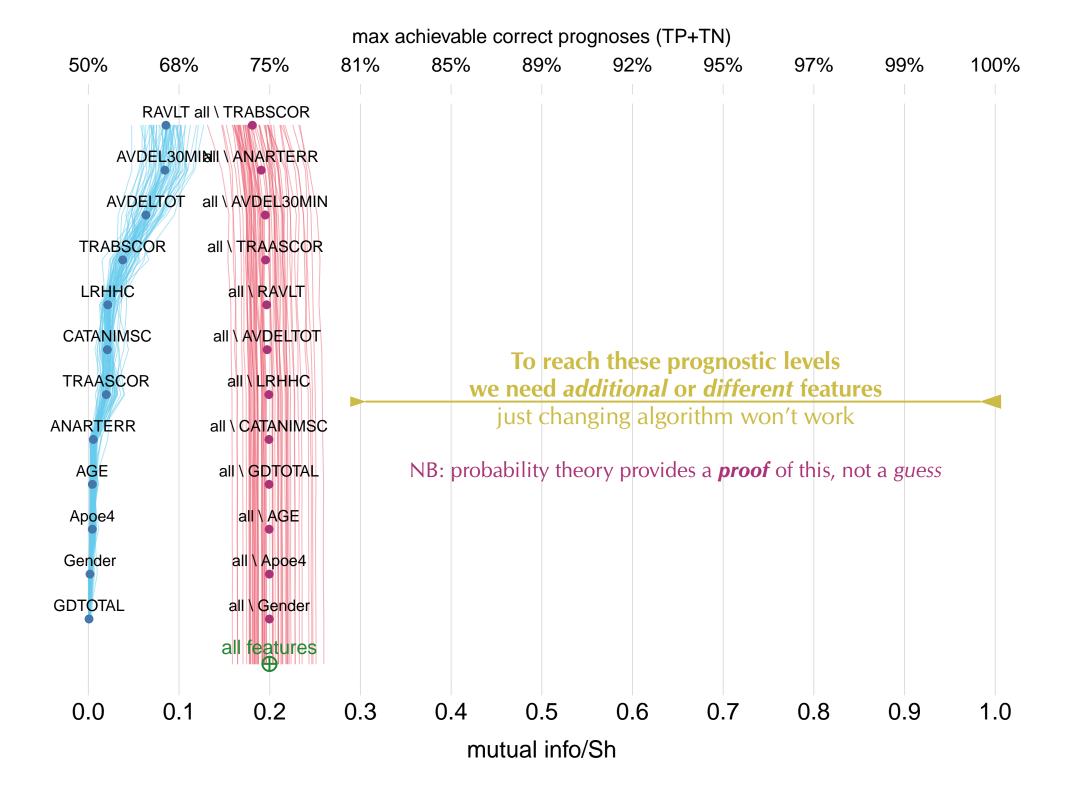


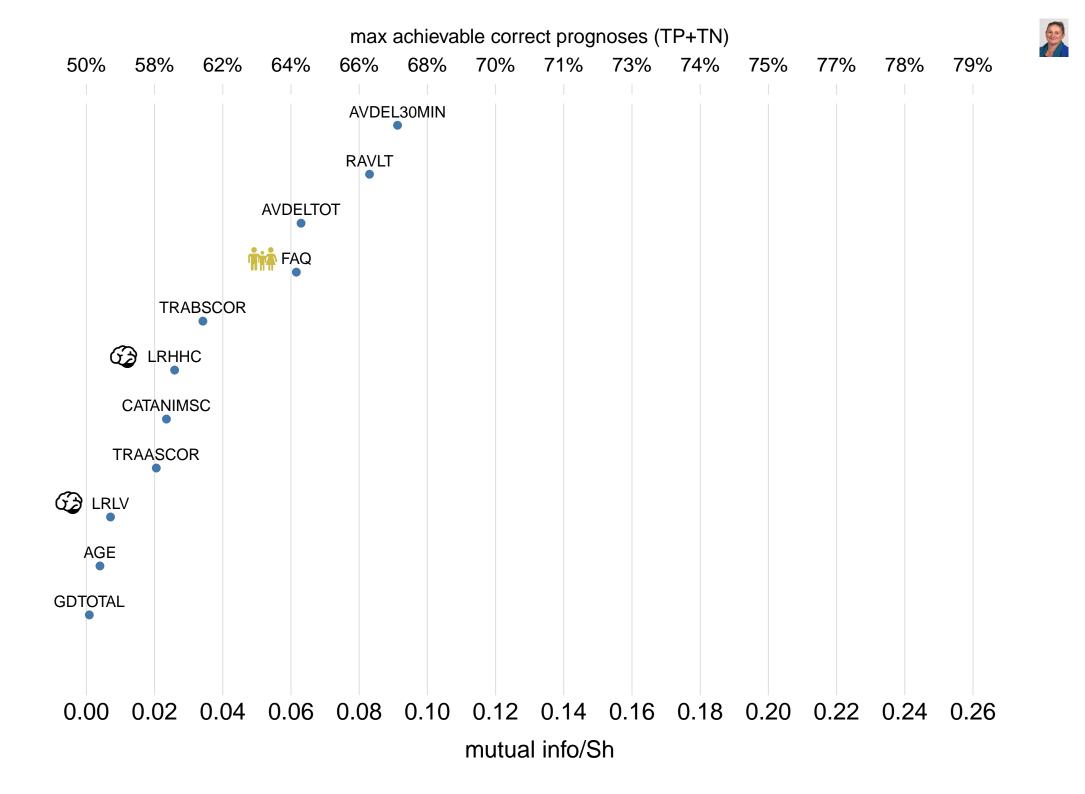


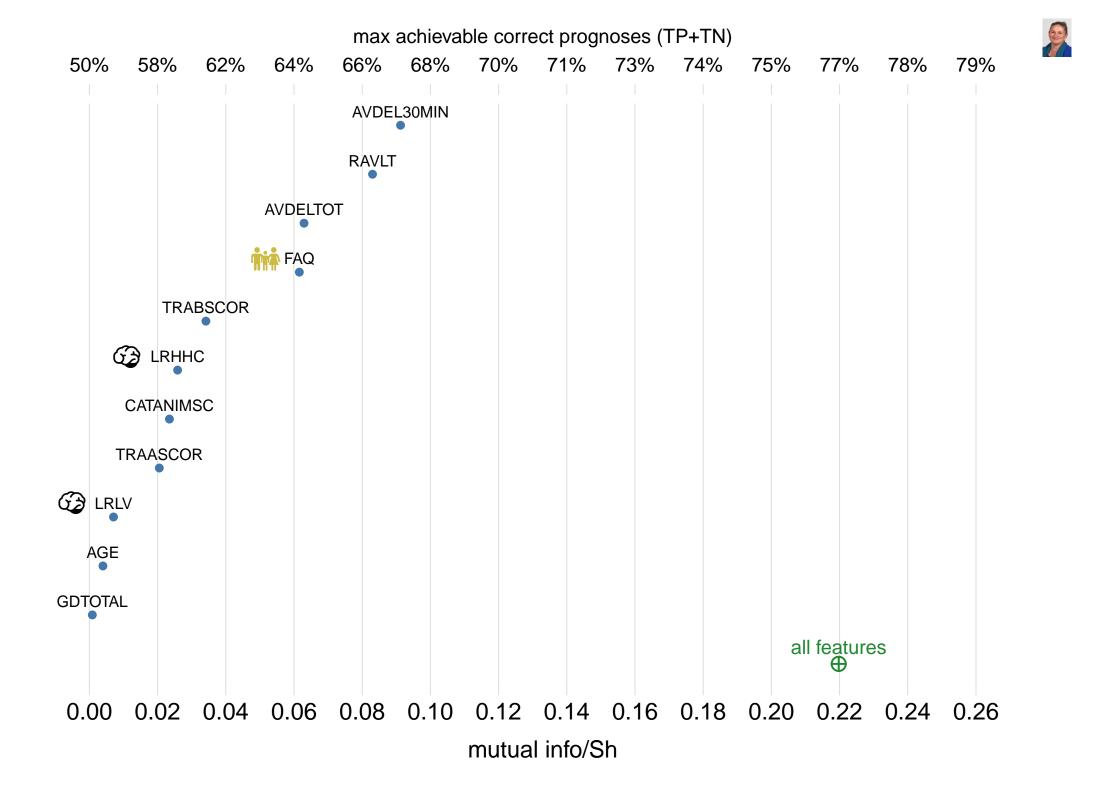


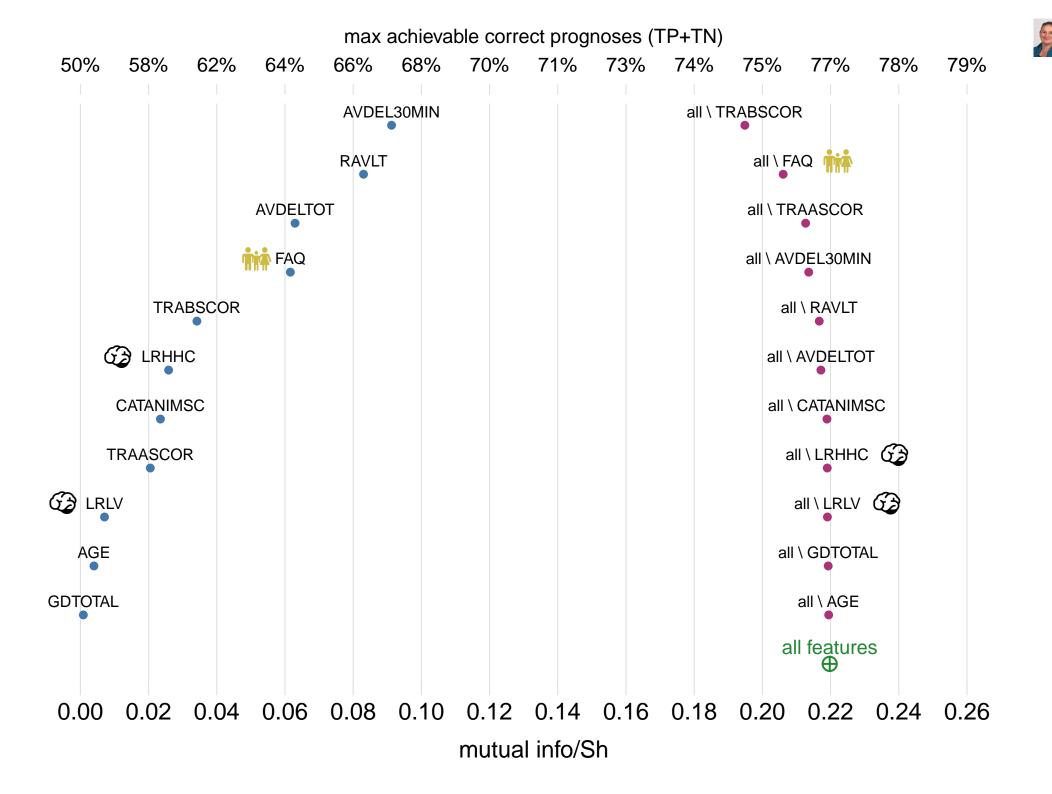


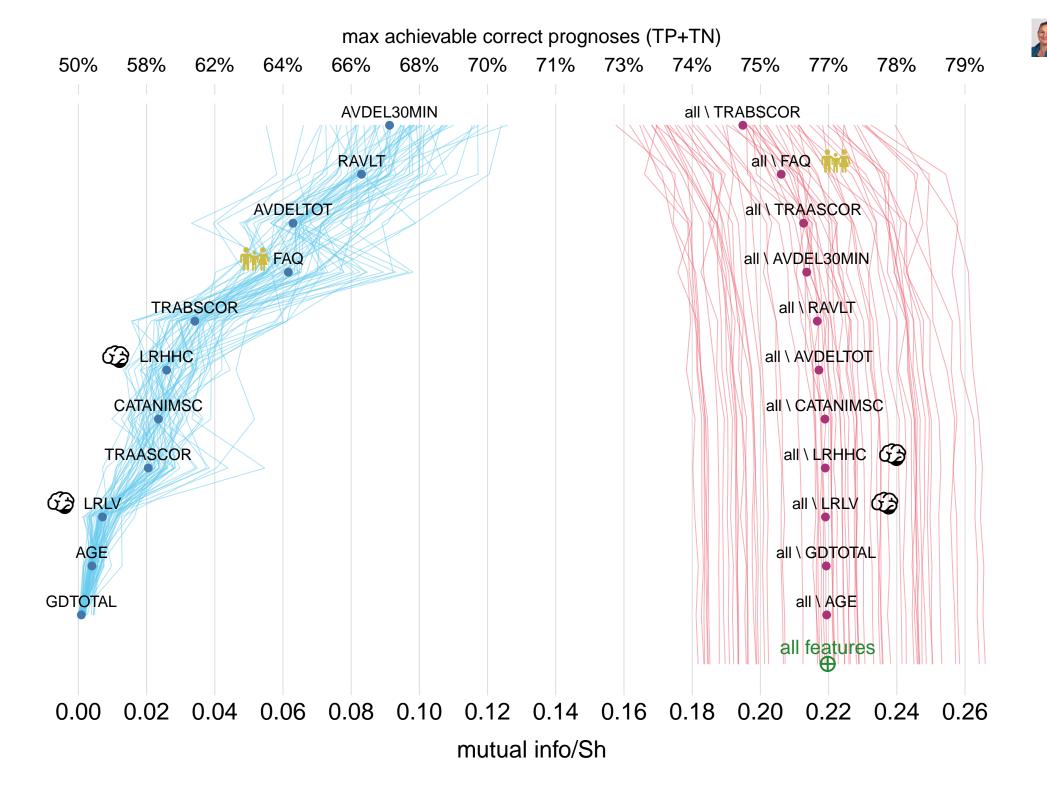














## Alexandra's results

