Analysis of some features for prognosis of Alzheimer onset: Probability theory & Information theory

Luca, Alexandra, Ingrid MMIV-ML group meeting, 13 January 2022

Mild Cognitive Impairment

Mild Cognitive Impairment





Alzheimer Disease















₫ AGE

් RAVLT

© ANARTERR

ී GDTOTAL

© TRABSCOR

© CATANIMSC

© TRAASCOR

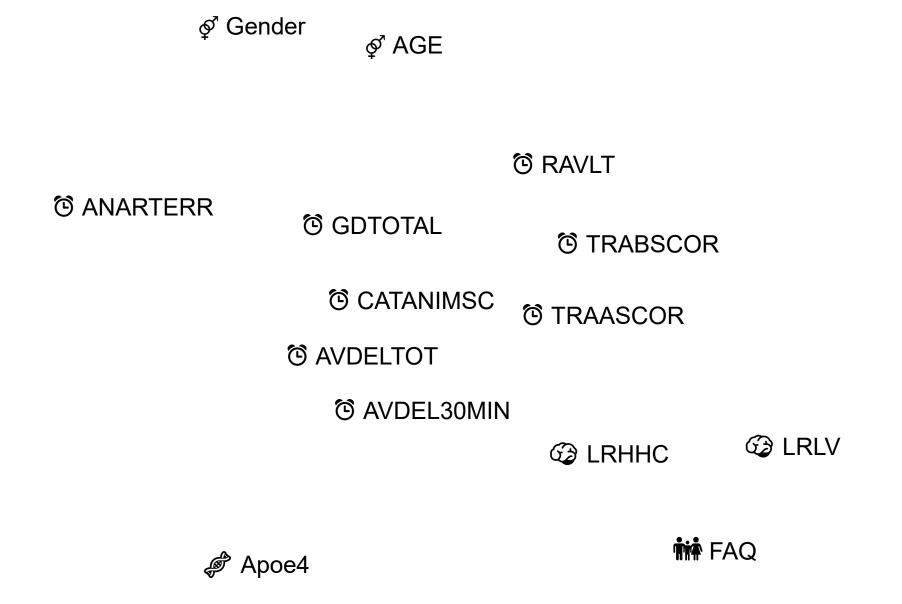
© AVDELTOT

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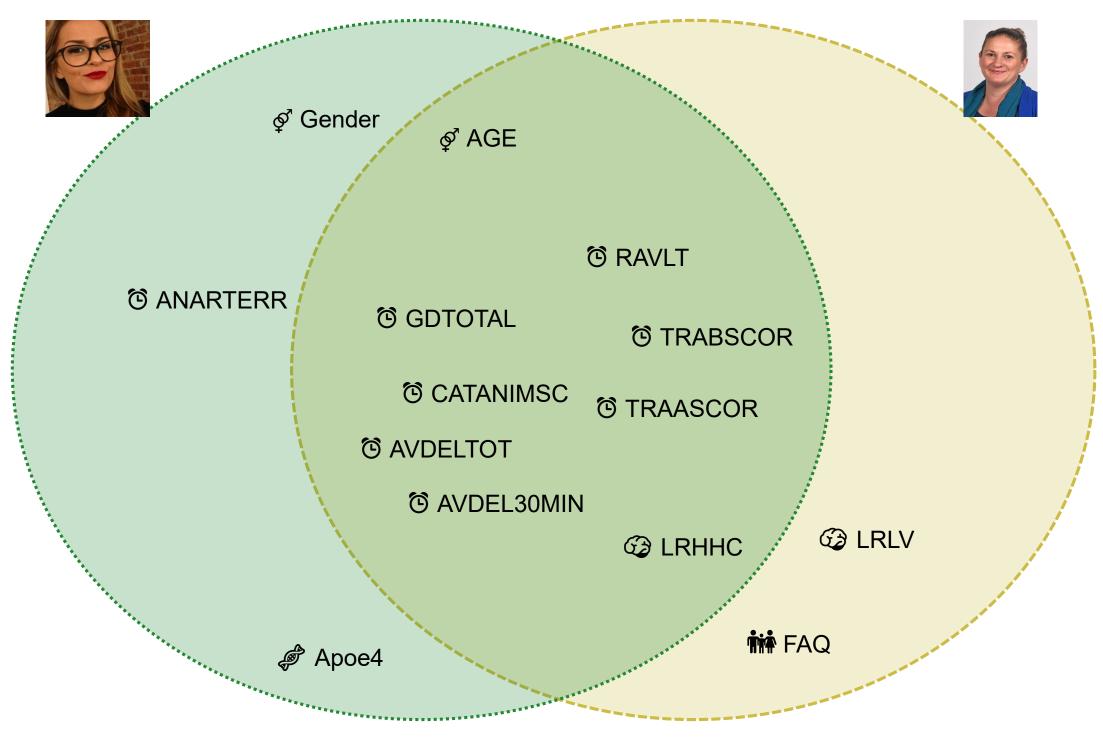
3 LRLV

Apoe4

†i FAQ



How 'good' are these features at prognosing the later onset of Alzheimer?



How 'good' are these features at prognosing the later onset of Alzheimer?



O ANAR

Functional Activities Questionnaire

Administration

Ask informant to rate patient's ability using the following scoring system:

- Dependent = 3
- Requires assistance = 2
- Has difficulty but does by self = 1
- Normal = 0
- Never did [the activity] but could do now = 0
- Never did and would have difficulty now = 1

Writing checks, paying bills, balancing checkbook				
Assembling tax records, business affairs, or papers				
Shopping alone for clothes, household necessities, or groceries				
Playing a game of skill, working on a hobby				
Heating water, making a cup of coffee, turning off stove after use				
Preparing a balanced meal				
Keeping track of current events				
Paying attention to, understanding, discussing TV, book, magazine				
Remembering appointments, family occasions, holidays, medications				
Traveling out of neighborhood, driving, arranging to take buses				
TOTAL SCORE:				

Evaluation

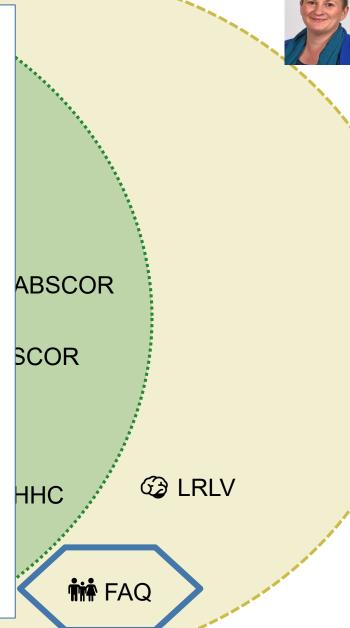
Sum scores (range 0-30). Cutpoint of 9 (dependent in 3 or more activities) is recommended to indicate impaired function and possible cognitive impairment.

Pfeffer RI et al. Measurement of functional activities in older adults in the community. J Gerontol 1982; 37(3):323-329. Reprinted with permission of The Gerontological Society of America, 1030 15th Street NW, Suite 250, Washington, DC 20005 via Copyright Clearance Center. Inc.

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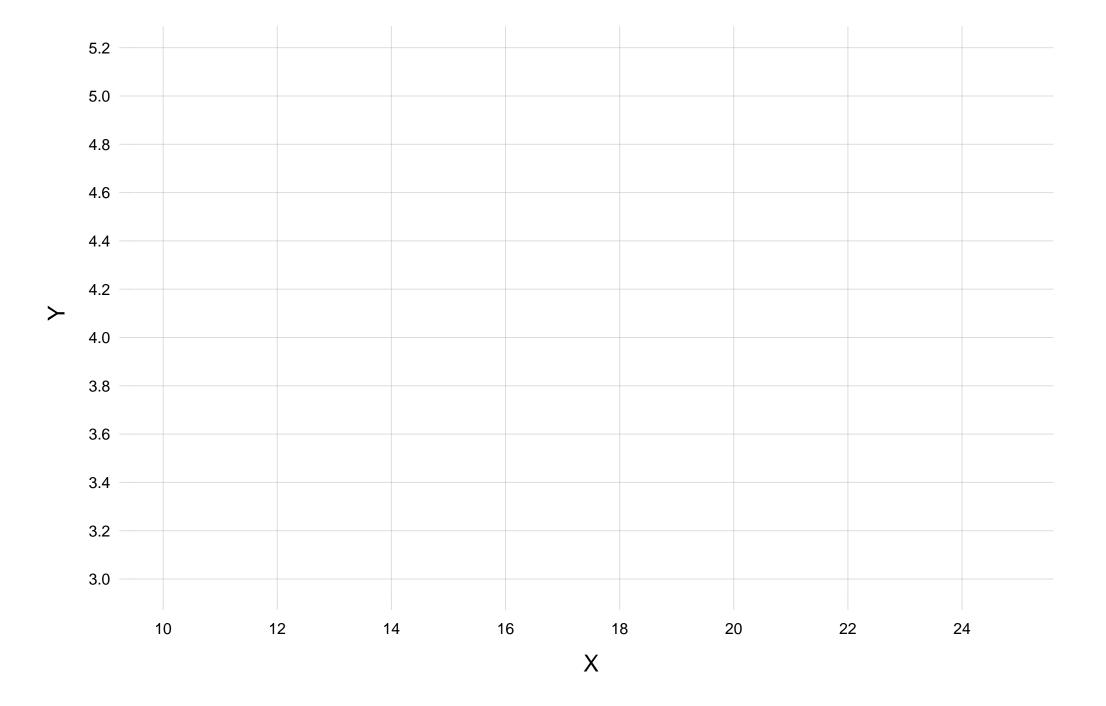


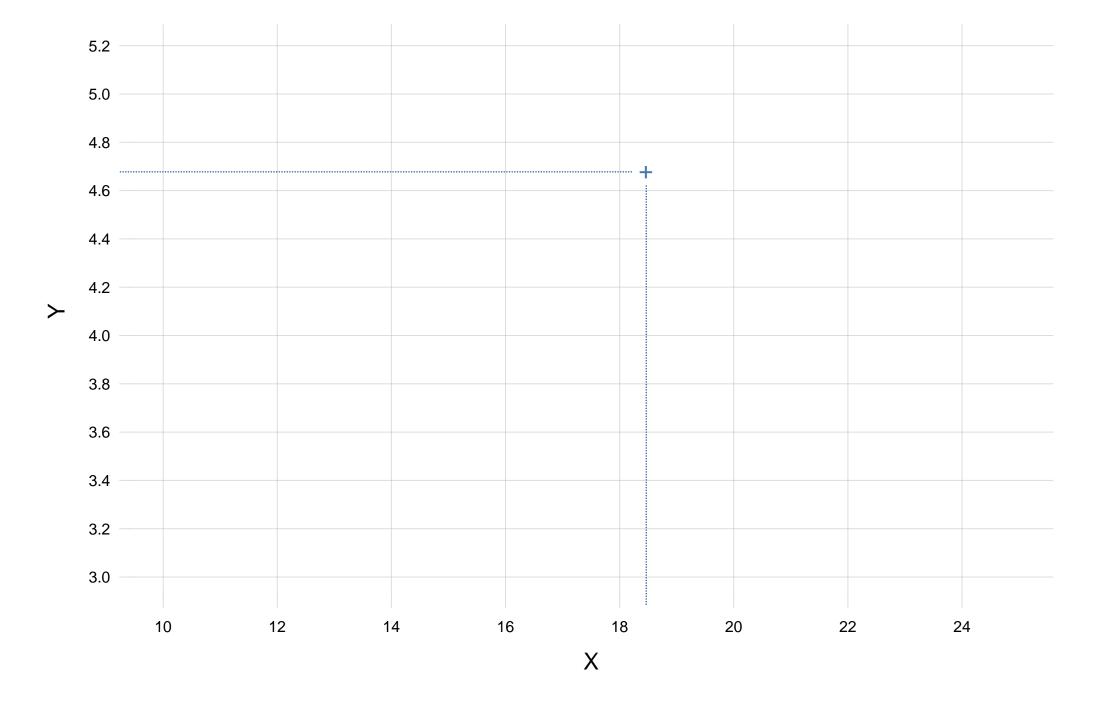
Ingrid's study: 12+1 variates, 678 datapoints

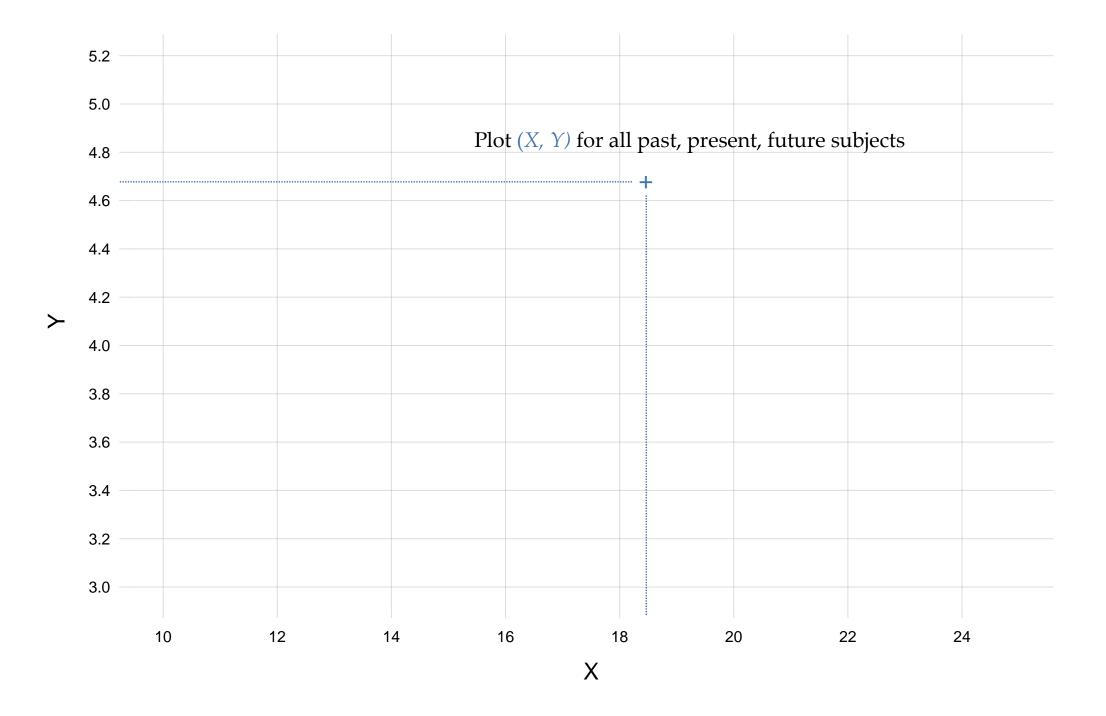
Alexandra's study: 11+1 variates, 708 datapoints (43 missing values)

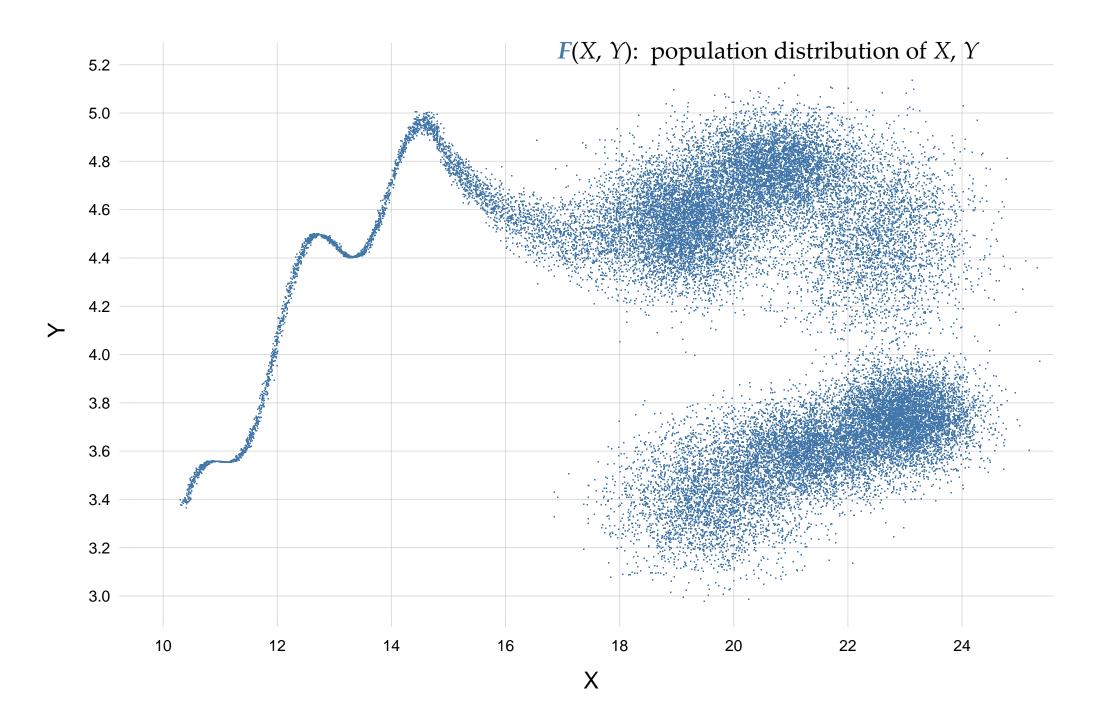
Computation time: ~65 h/study (3 parallel sessions to assess numeric convergence)

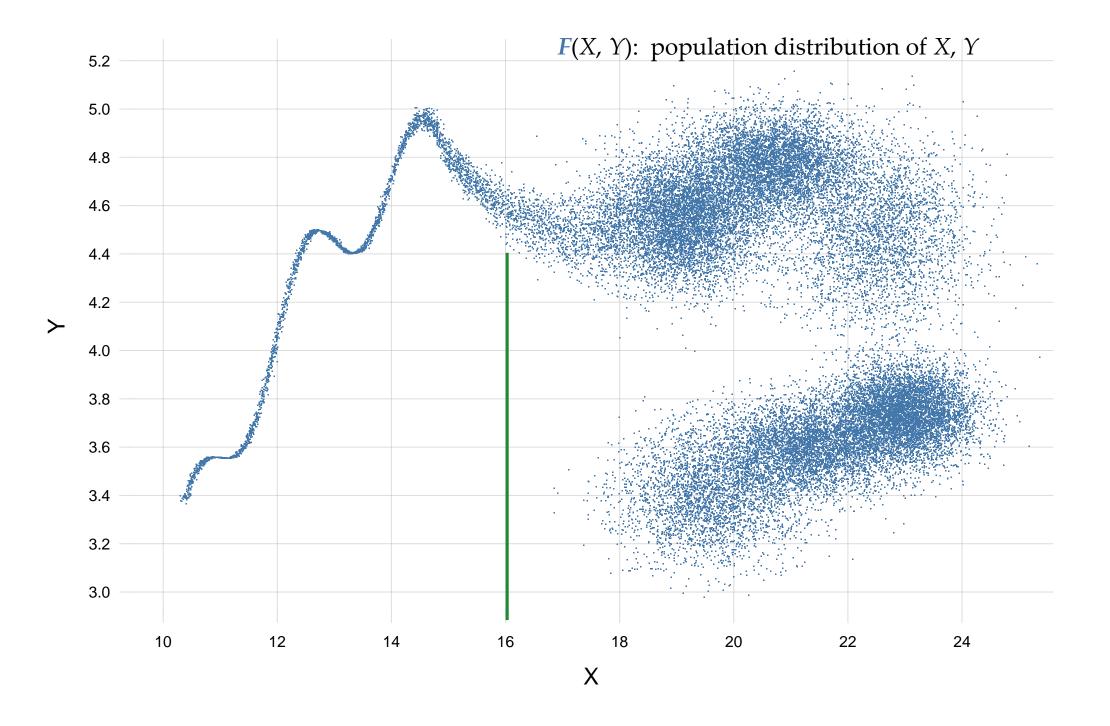
HPC UNINETT | sigma2





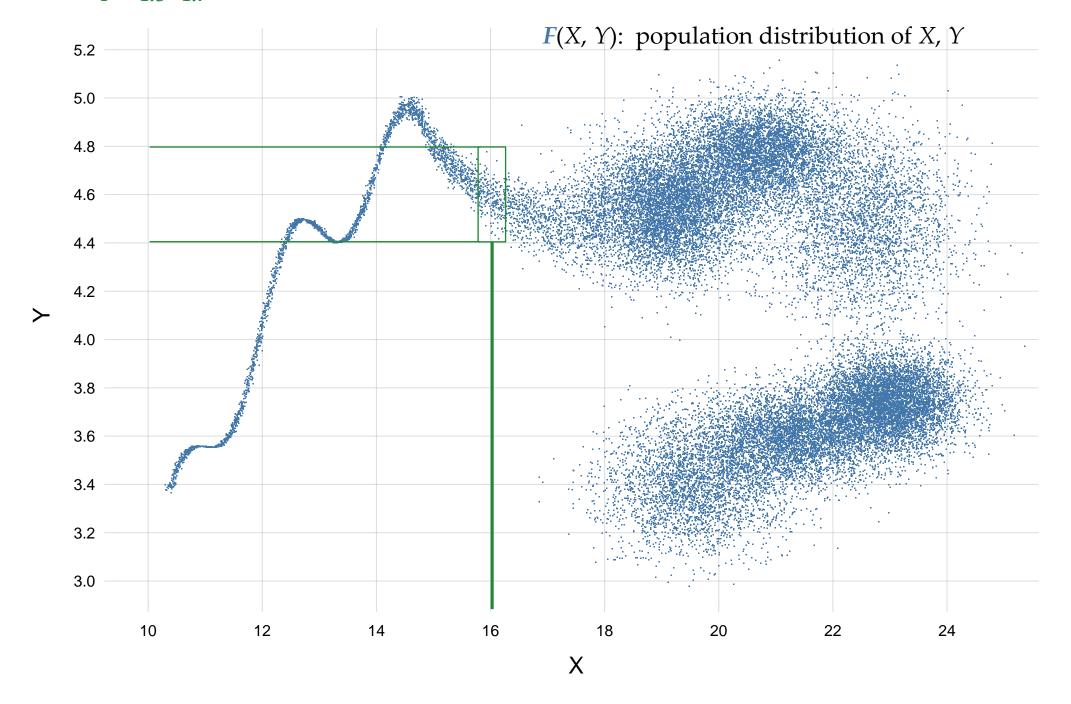






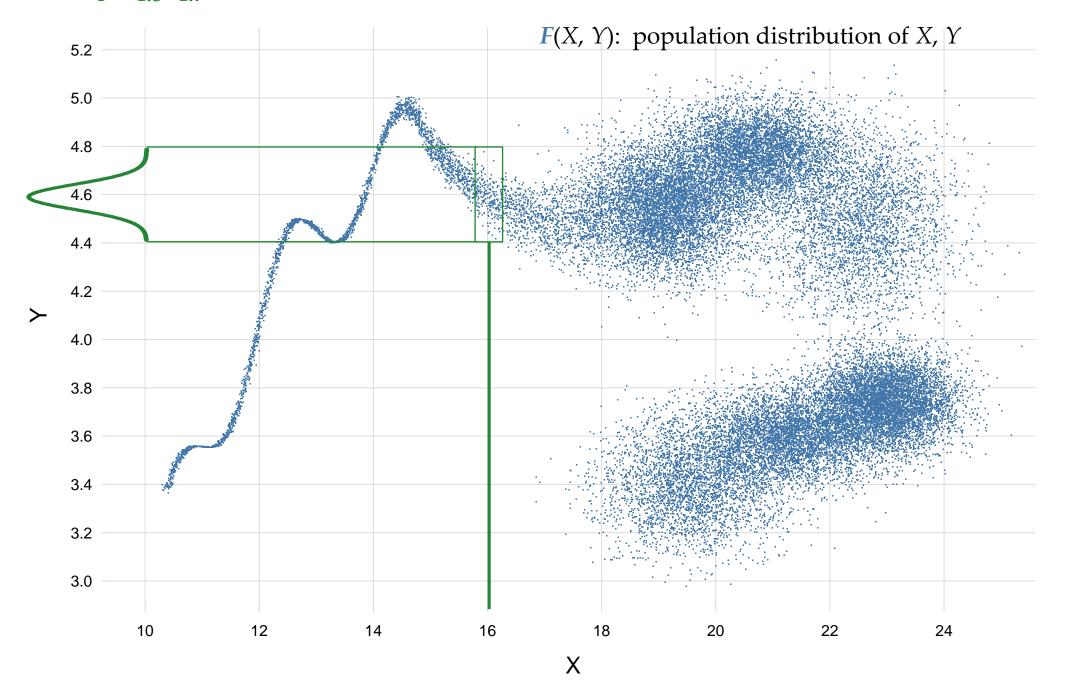
New patient: X = 16

$$\Rightarrow Y \approx 4.5-4.7$$

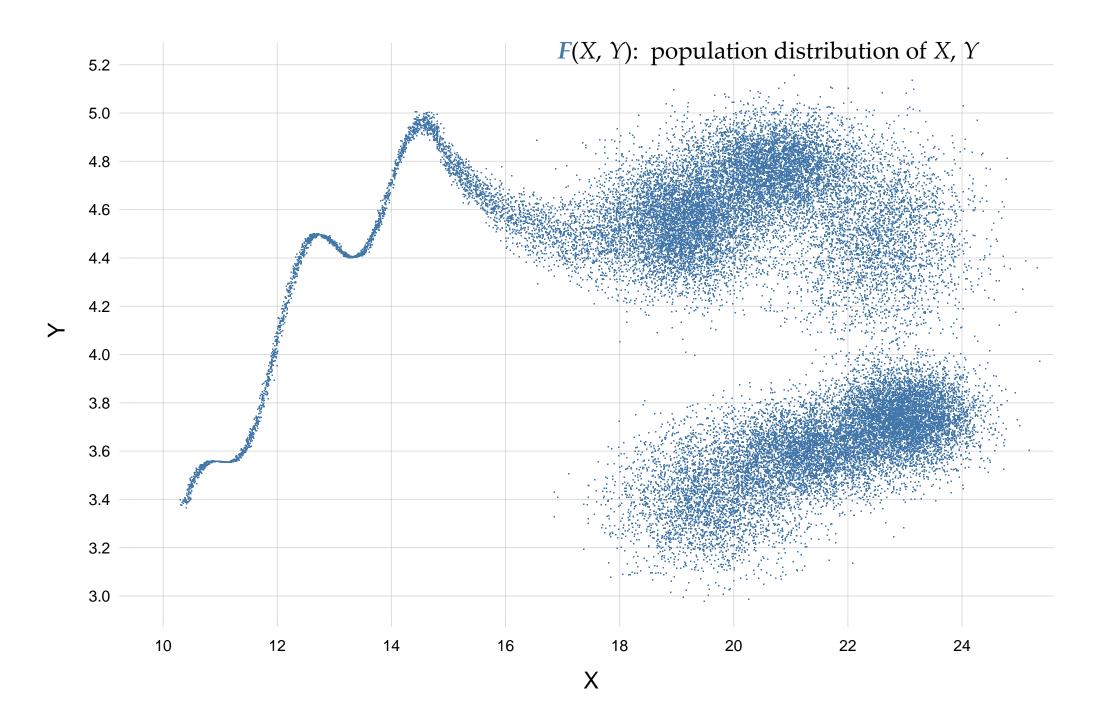


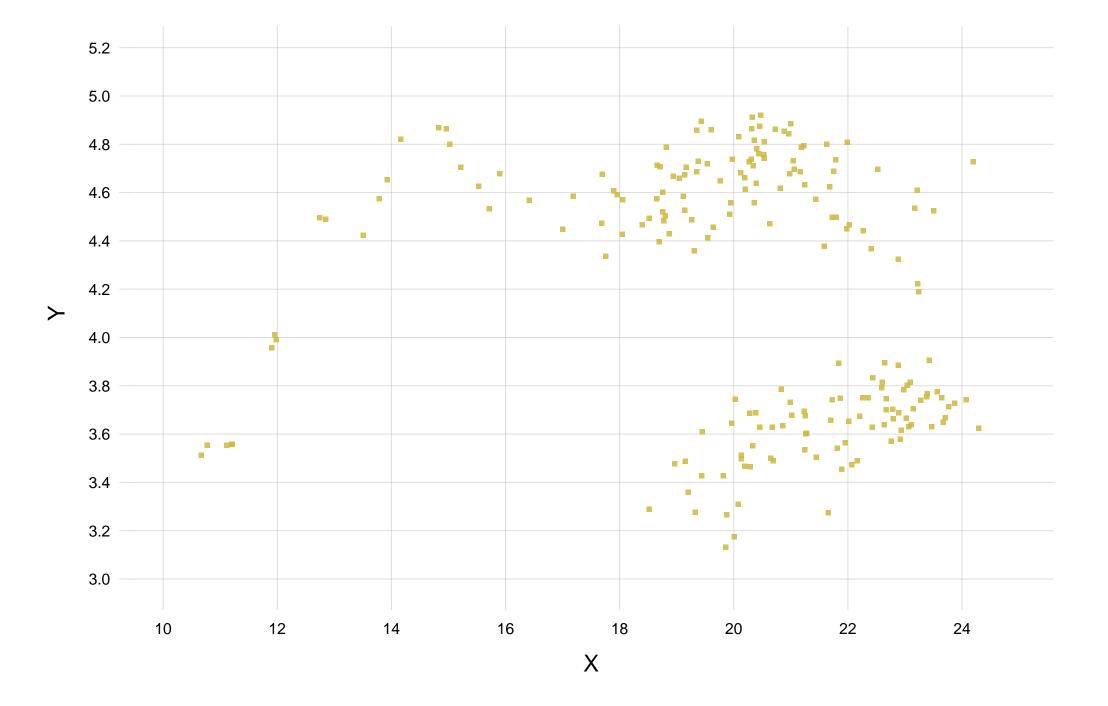
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$$P(y \mid x) = F(y \mid x)$$





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$$P(y \mid x) = \int F(y \mid x) p(F \mid data) dF$$

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$$p(F \mid data) \propto F(y_1, x_1) \times F(y_2, x_2) \times F(y_3, x_3) \times ...$$

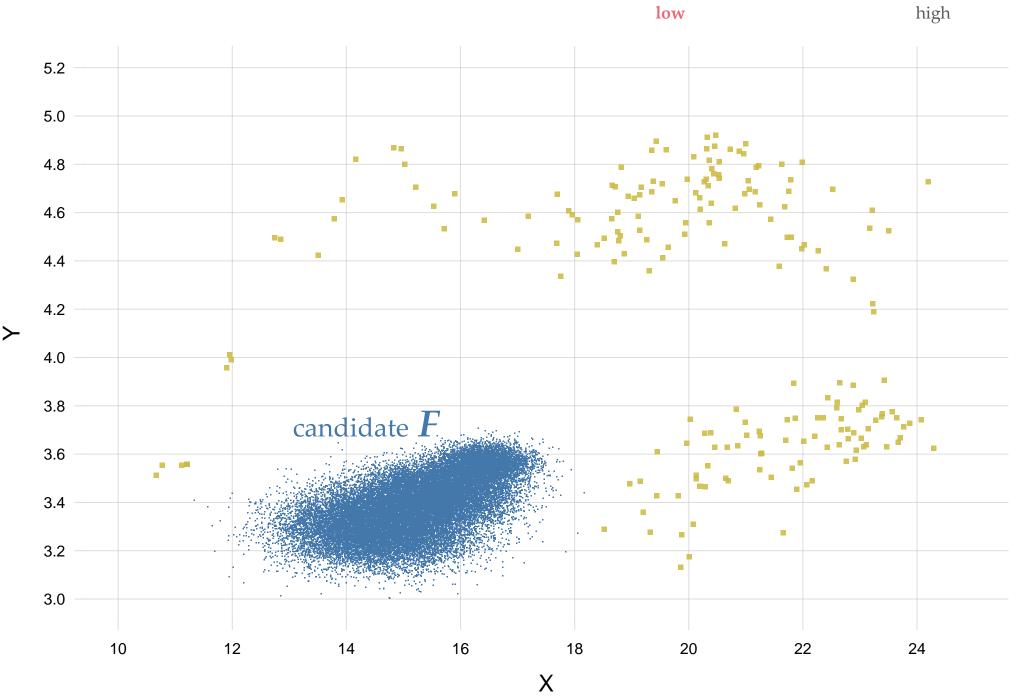
how well the 'candidate' distribution fits the data

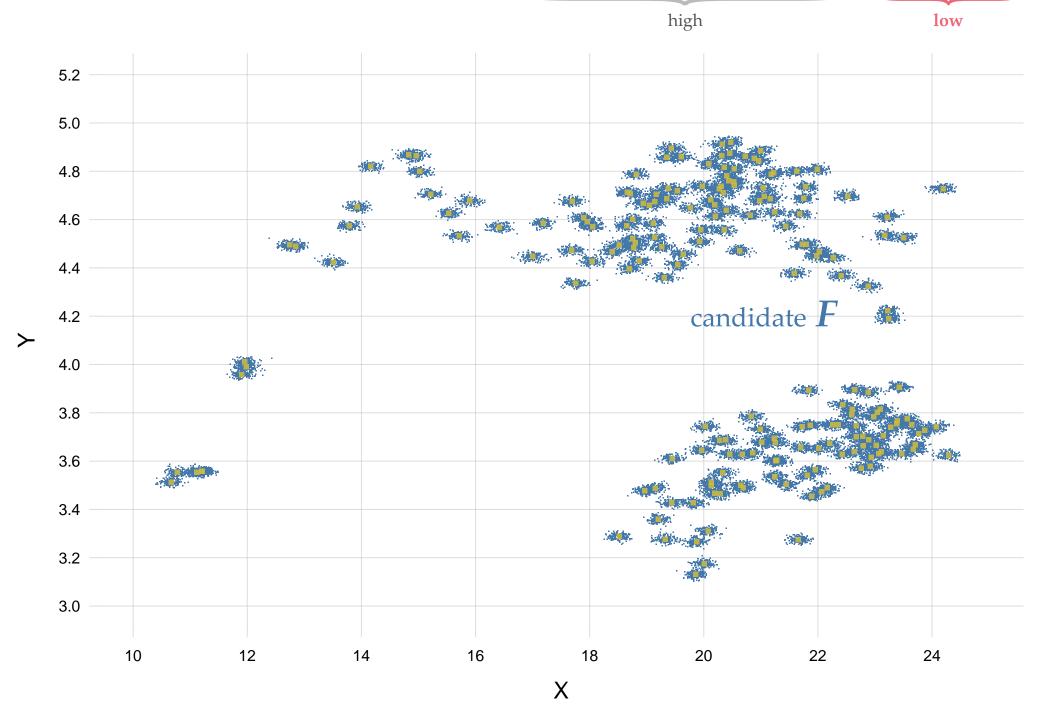
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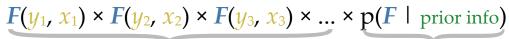
$$p(F \mid data) \propto F(y_1, x_1) \times F(y_2, x_2) \times F(y_3, x_3) \times ... \times p(F \mid prior info)$$

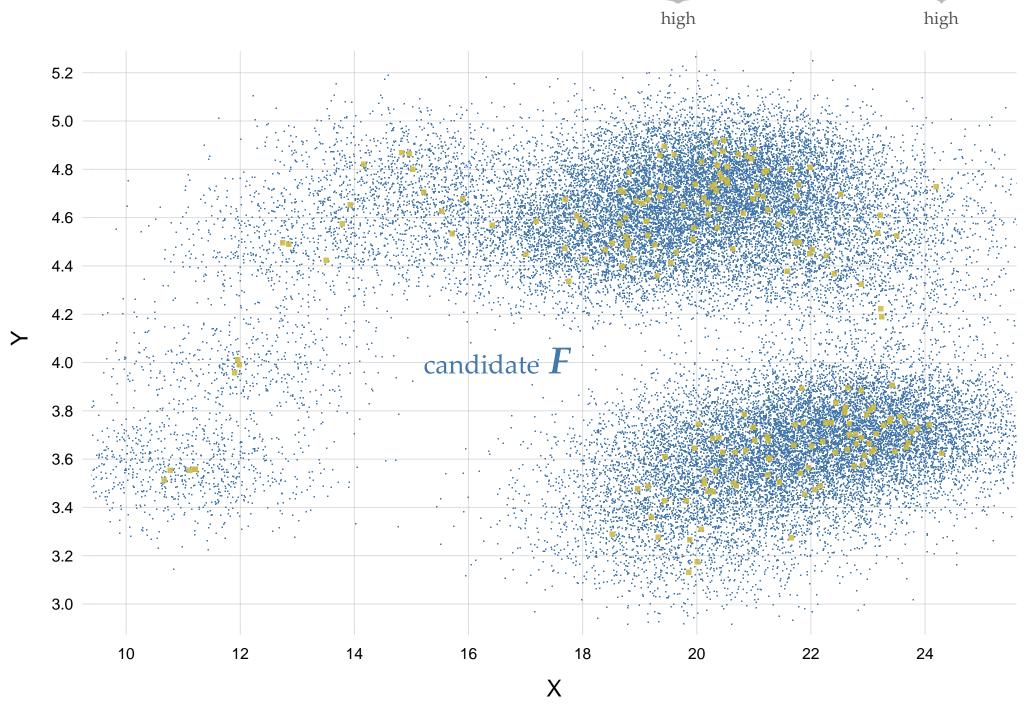
how well the 'candidate' distribution fits the data

extra-data knowledge







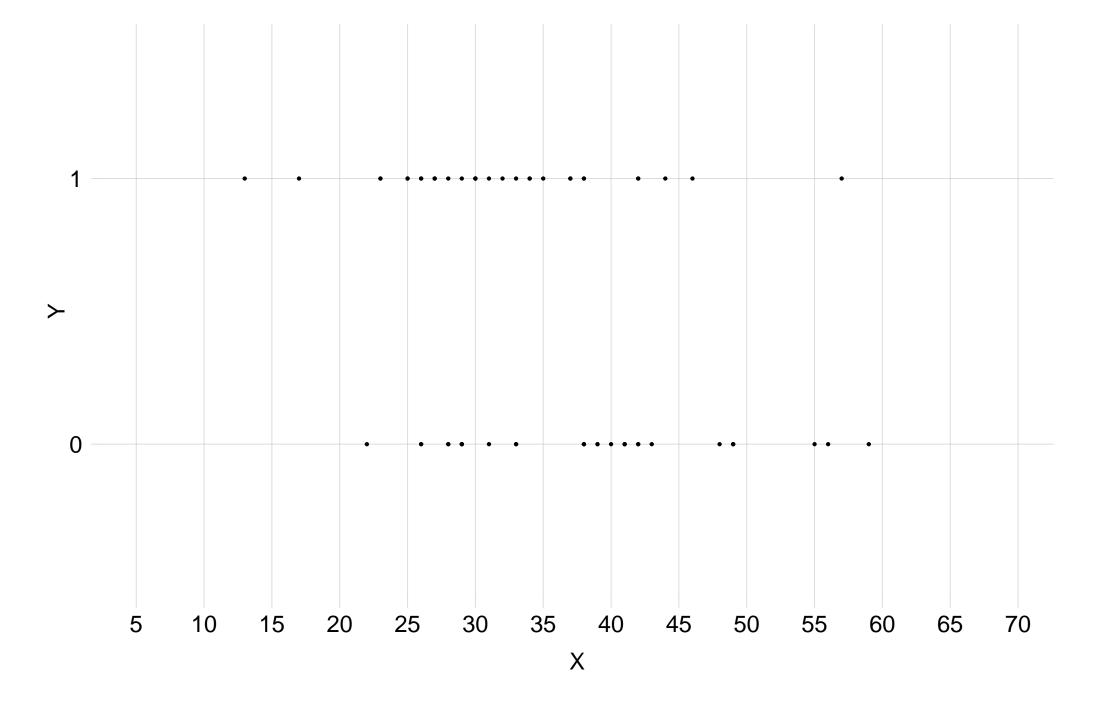


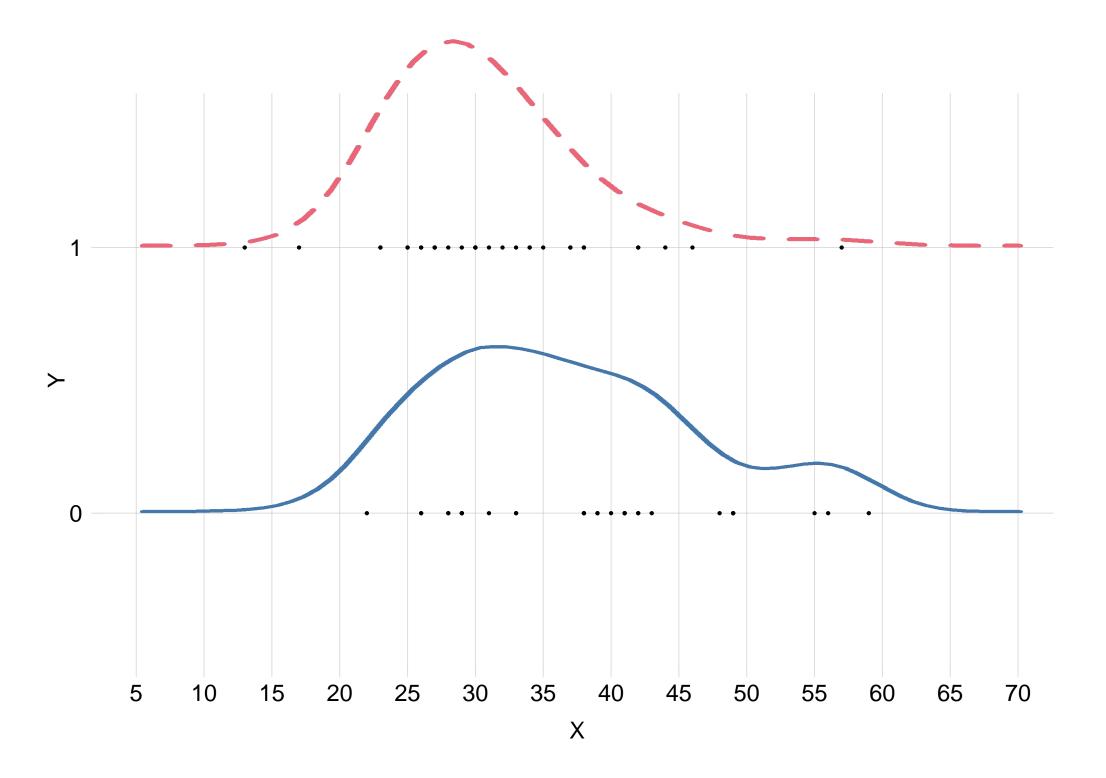
$intuition \rightarrow mathematics$

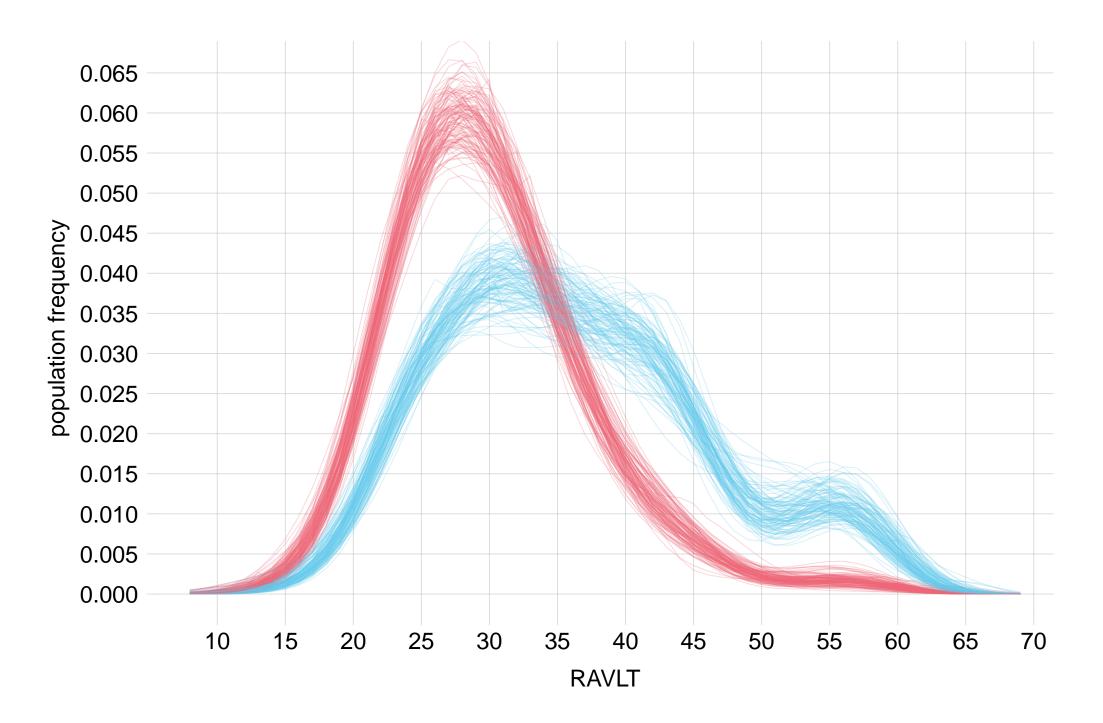
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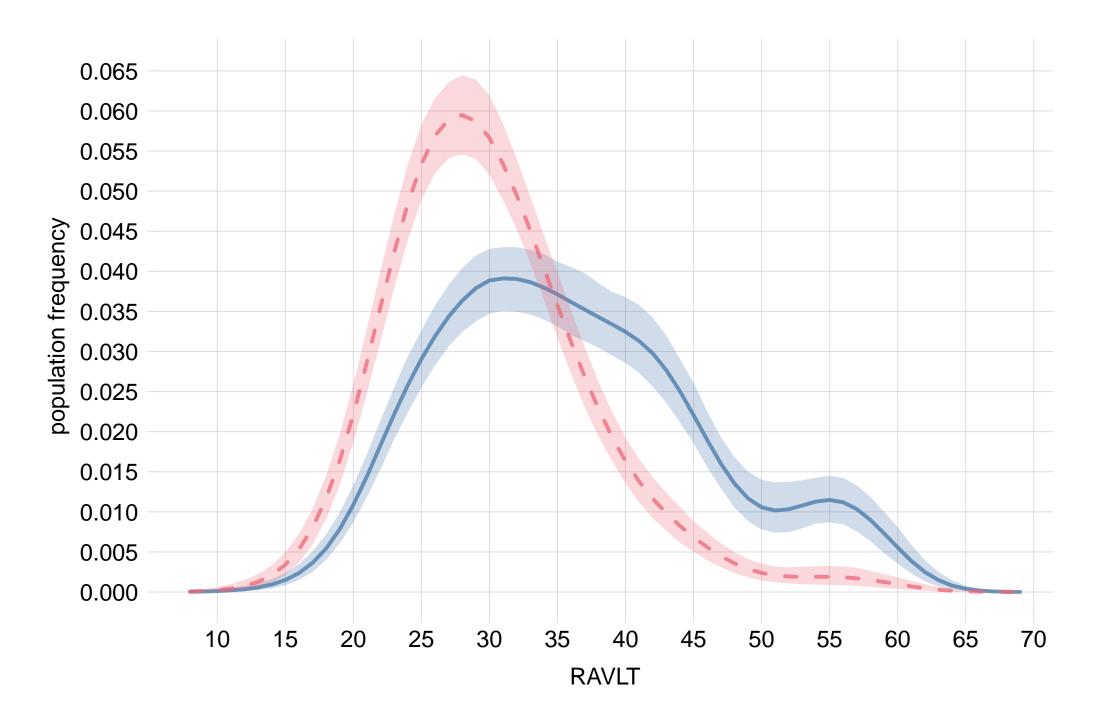
$first\ principles \rightarrow mathematics \rightarrow intuition$

('Bayesian')

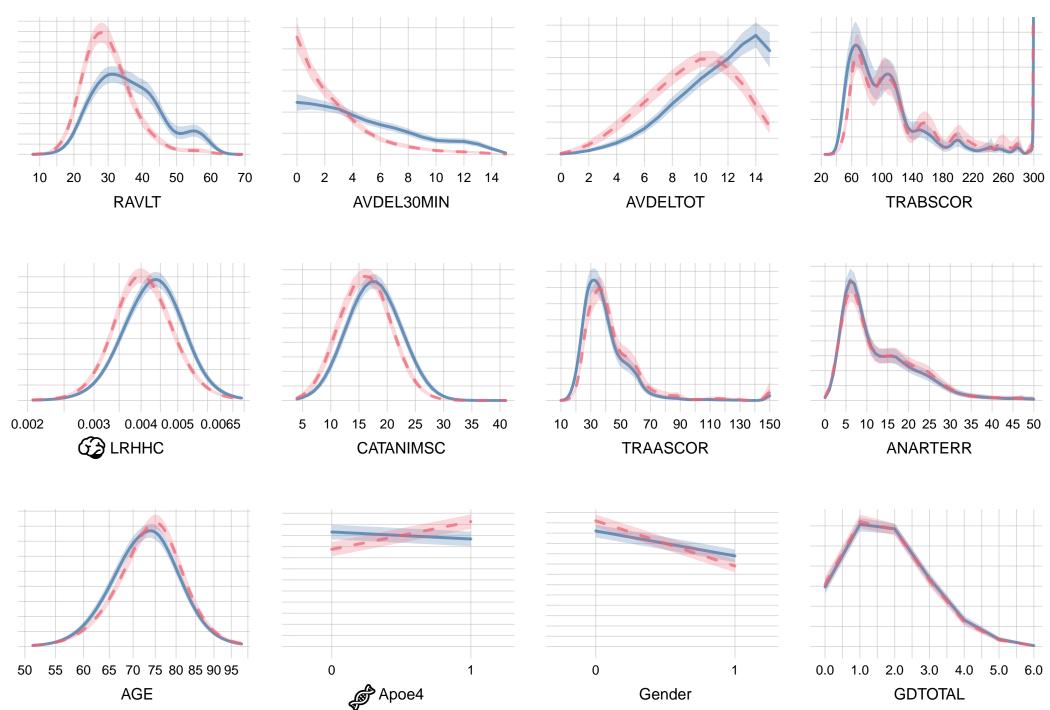




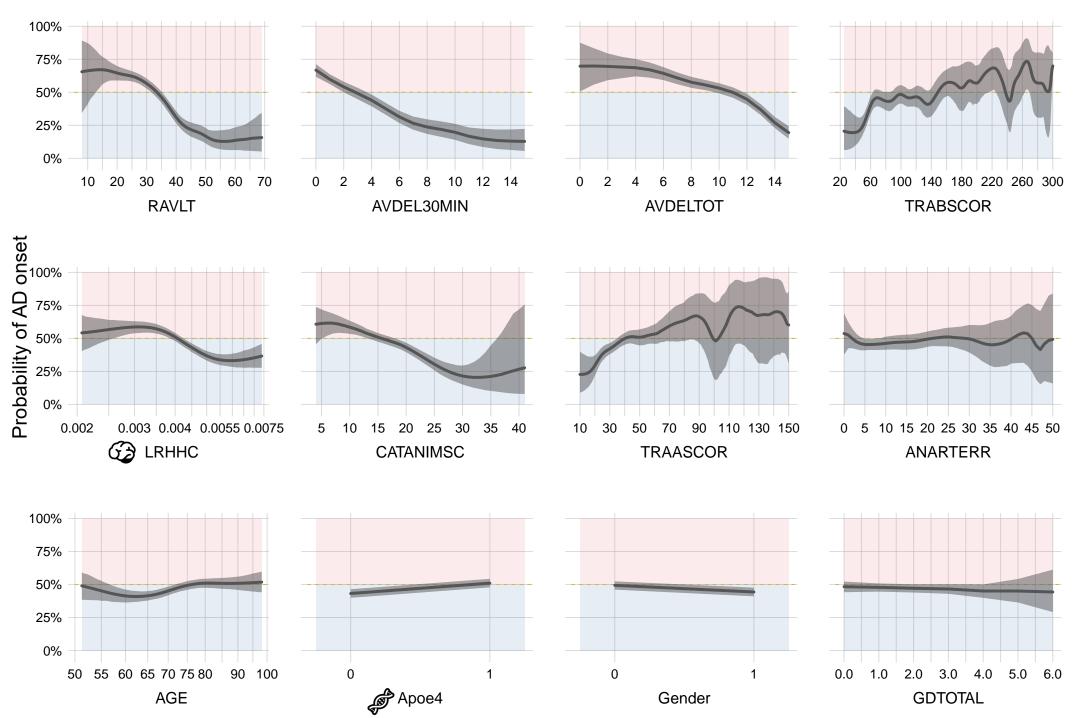




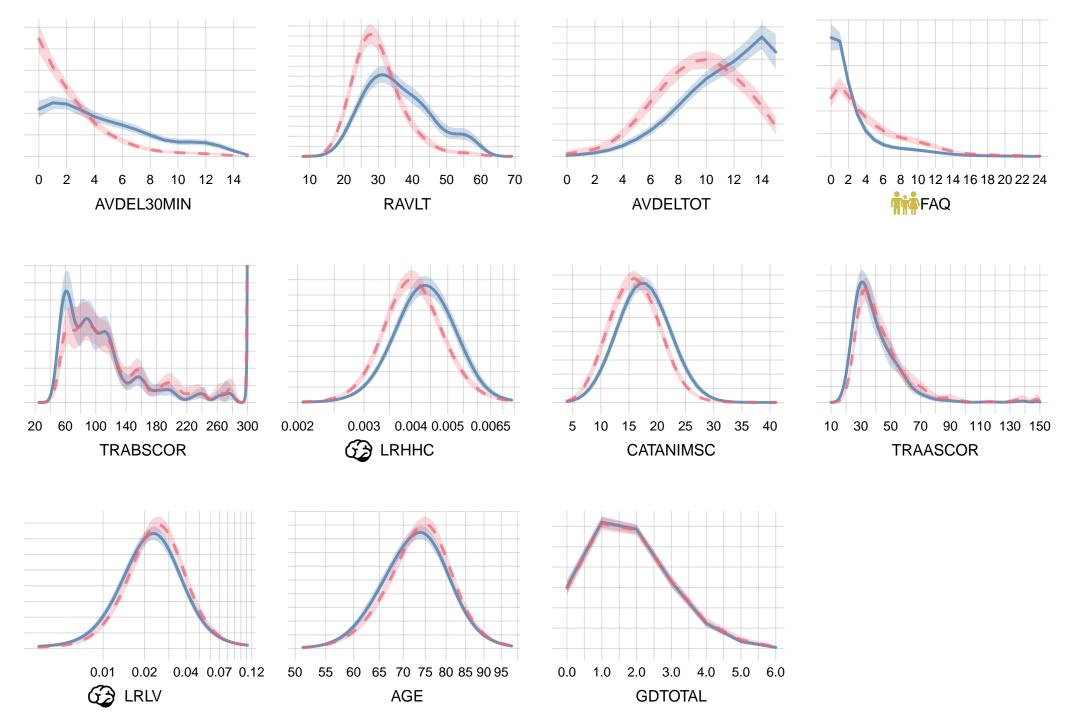




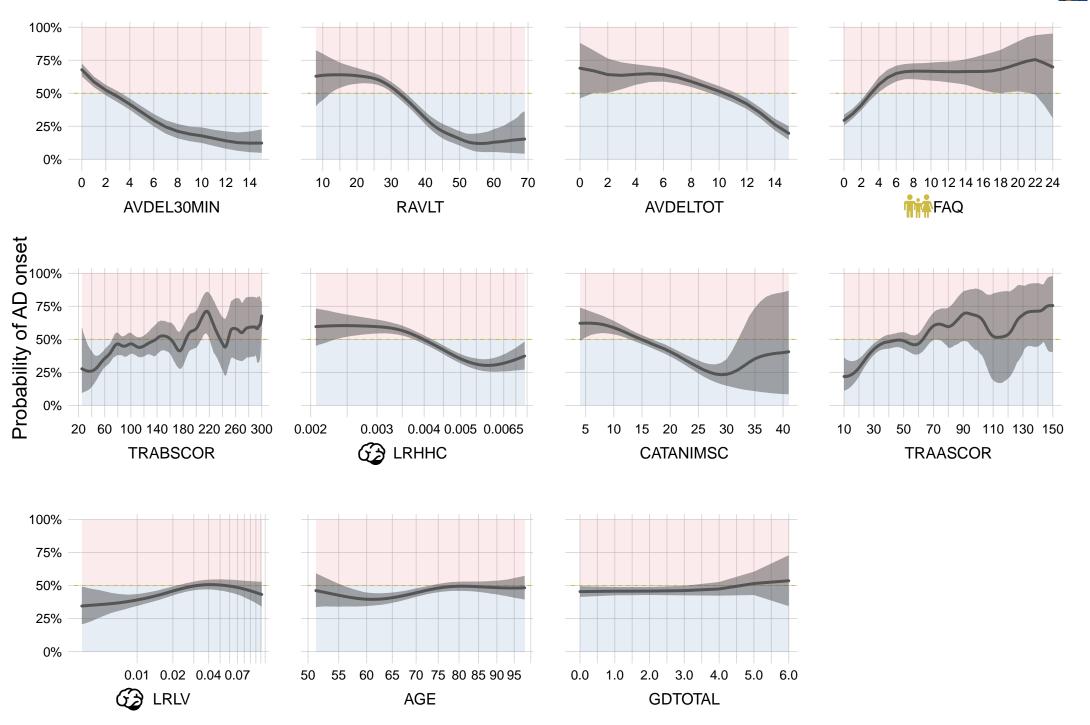












Interesting characteristics of F(Y, X) in Ingrid's & Alexandra's studies:

• Several high-density regions in the 12D space

• Some features seem more robust if used in a 'discriminative' way: $P(Y \mid X)$, others in a 'generative' way: $P(X \mid Y)$

$$P(Y|X_d, X_g) \propto P(X_g|Y) P(Y|X_d)$$



"Language is a product of, and reflects, thinking.

Sloppy usage reflects sloppy thinking, a kind of thinking incompatible with good scientific habits of mind"

(D. J. Helfand)

Prediction problem:

guess the six digits of the winning lottery ticket ??????

Clue A:

Clue B: **///?/?**

Clue C: ???///

What is the 'importance' or 'predictive power' of each clue?

Scenario 1: we can use **only one** clue

Clue **A**: **////**??

Clue **B**: **///?/?**

Clue **C**: ???**/**//

increasing importance

Best: **A** or **B** (each gives 1/81 winning chance)

Worst: **C** (gives 1/729 winning chance)

Scenario 2: we can use **all** clues

Clue **A**: **////??**

Clue **B**: **///?/?**

Clue **C**: ???///

→ We fully know the winning number!

Scenario 2: what happens if we **discard** clues?

Clue **A**: **////??**

Clue **B**: **///?/?**

Scenario 2: what happens if we discard clues?

Clue **A**: **////**??

Clue **B**: **///?/?**

Clue **C**: ???**///**

• Discard A: still 100% win \Rightarrow A has 'importance = 0'

Clue **B**: **///?/?**

- Discard A: still 100% win \Rightarrow A has 'importance = 0'
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Clue **B**: **///?/?**

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- Discard **B**: still 100% win \Rightarrow **B** has 'importance = **0**'
 - Discard **A** and **B**: 1/9 winning chance

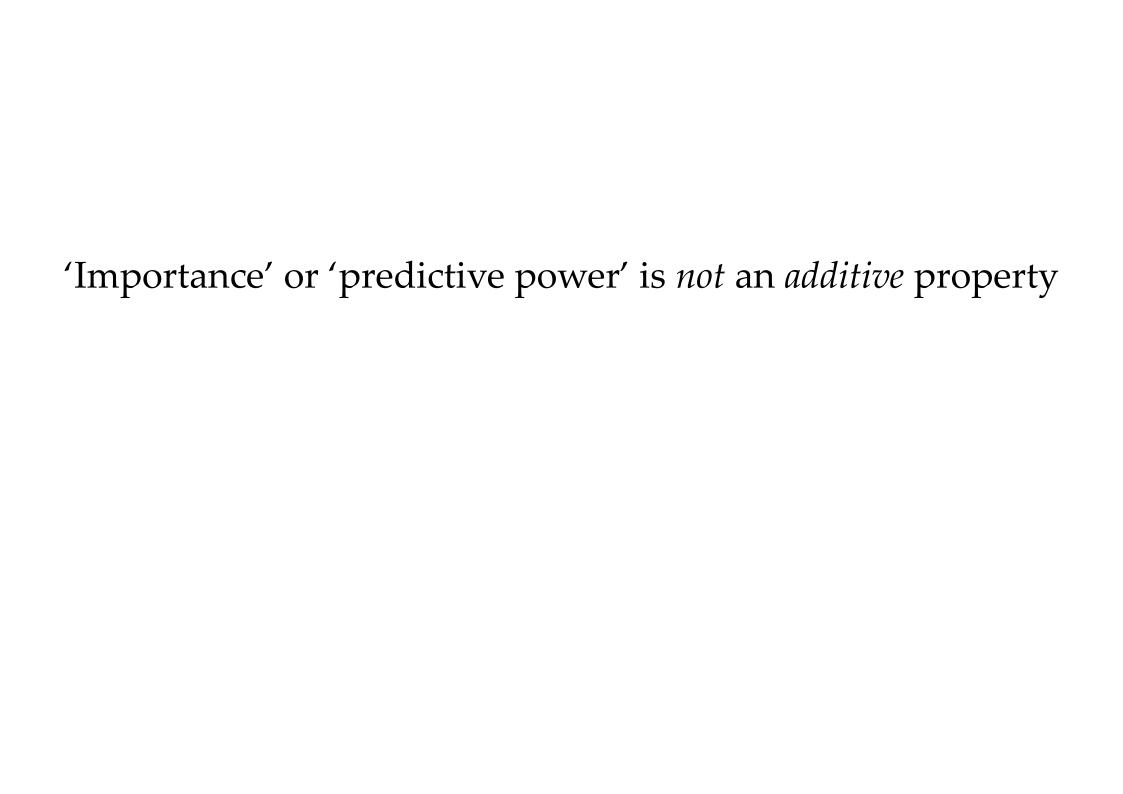
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 - \Rightarrow A and B together have 'importance > 0'

Clue **B**: **///?/?**

- Discard A: still 100% win \Rightarrow A has 'importance = 0'
- Discard **B**: still 100% win \Rightarrow **B** has 'importance = **0**'
 - Discard **A** and **B**: 1/9 winning chance
 - \Rightarrow A and B together have 'importance>0'

$$'0 + 0 \neq 0'$$



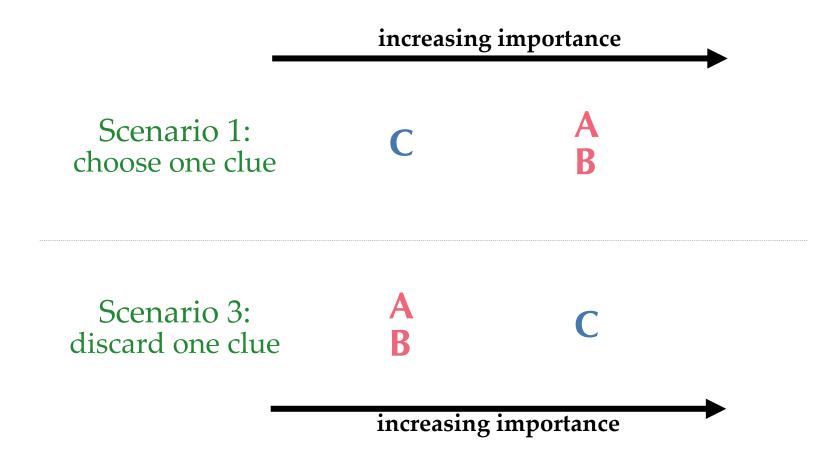
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Clue **C**: ???**/**//

increasing importance

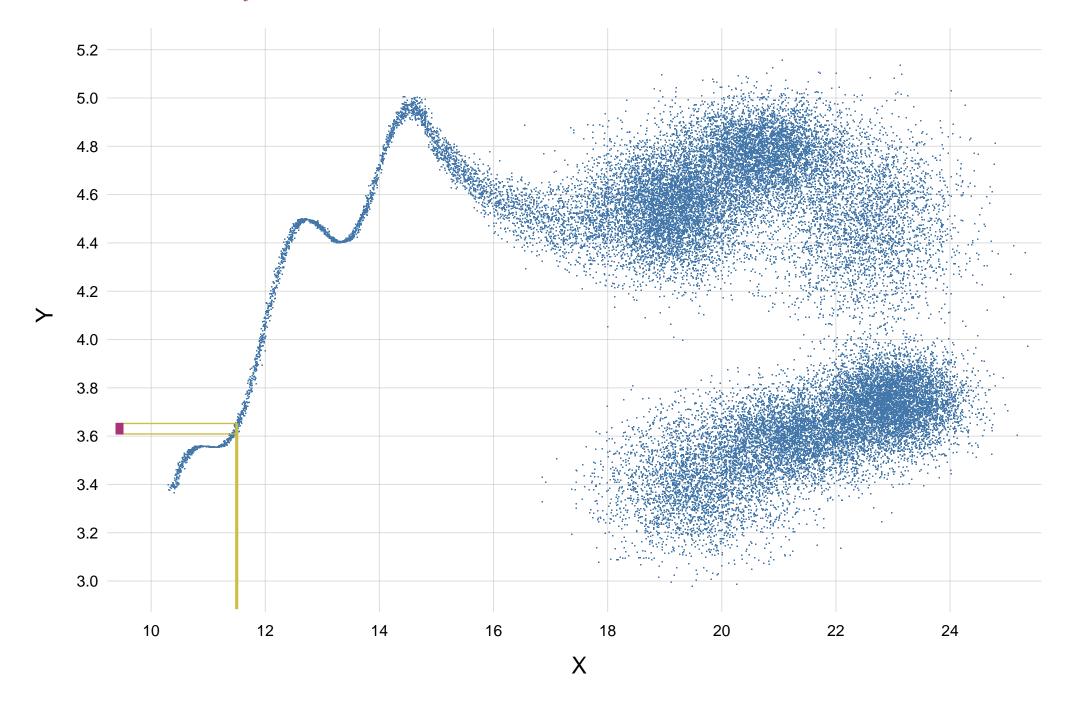
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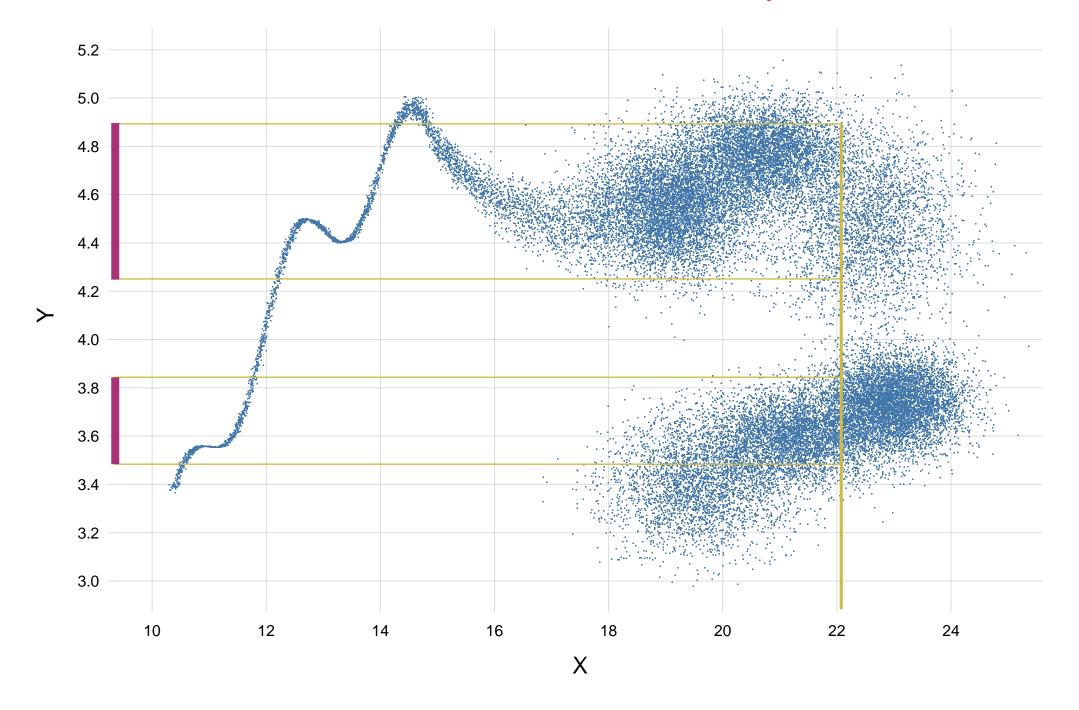
 \rightarrow If we have to discard one clue, it's most important that we keep \mathbf{C}



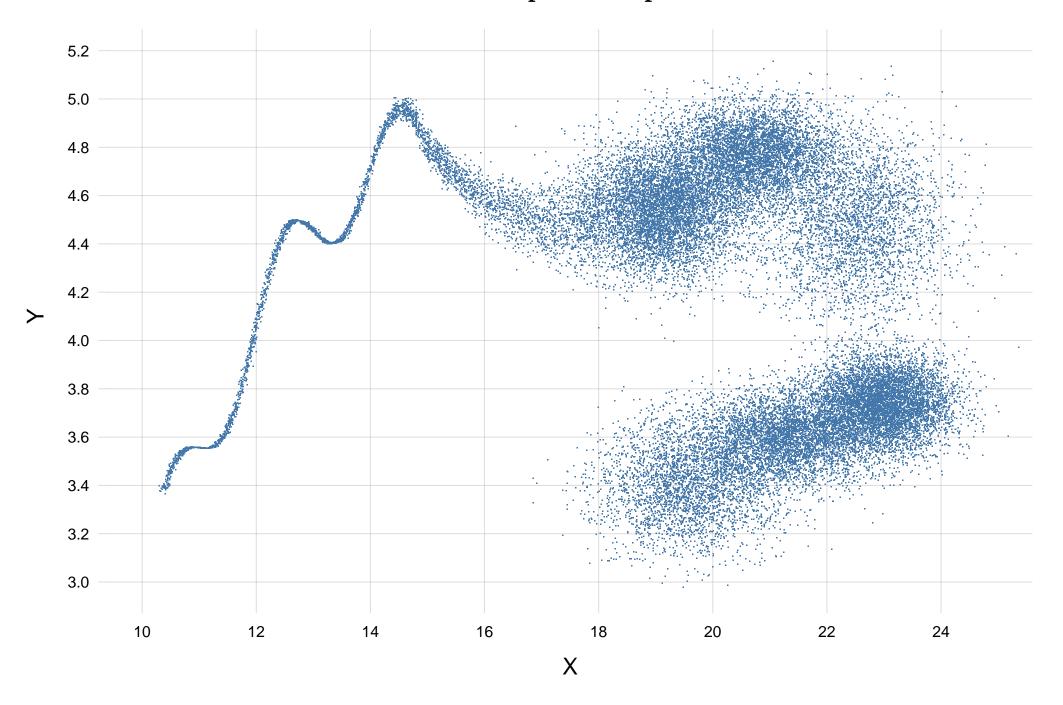
'Importance' or 'predictive power' of *X* is *context-dependent* (which other features are we considering?)

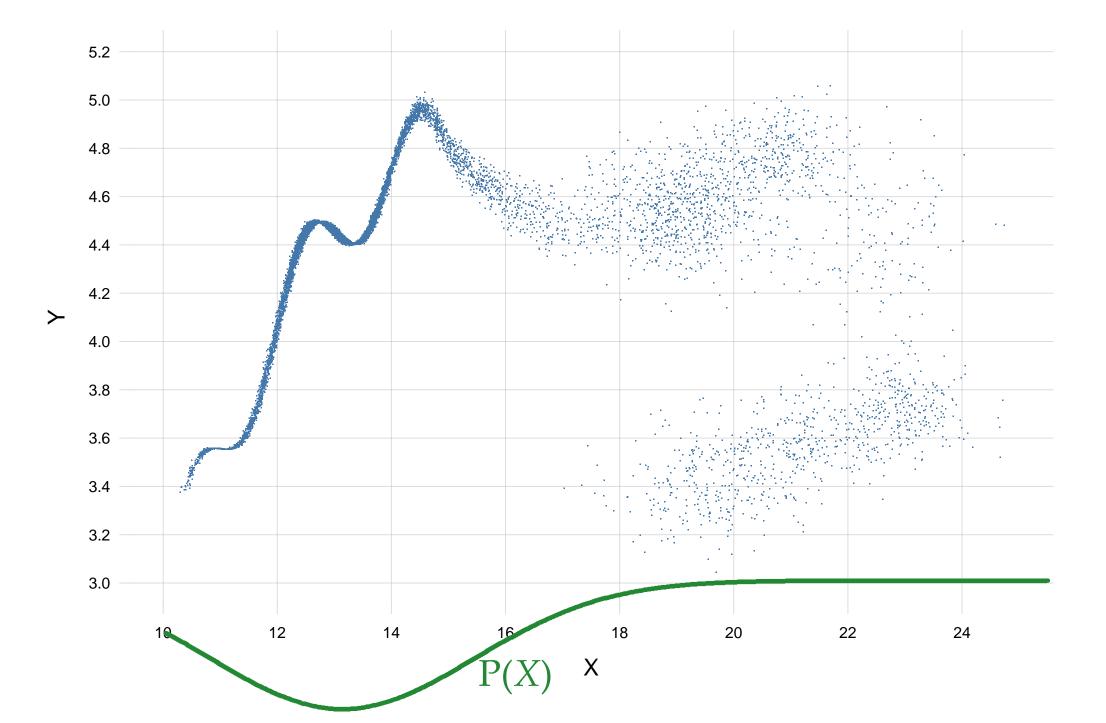
 $x = 11.5 \implies y \approx 3.60 - 3.65$

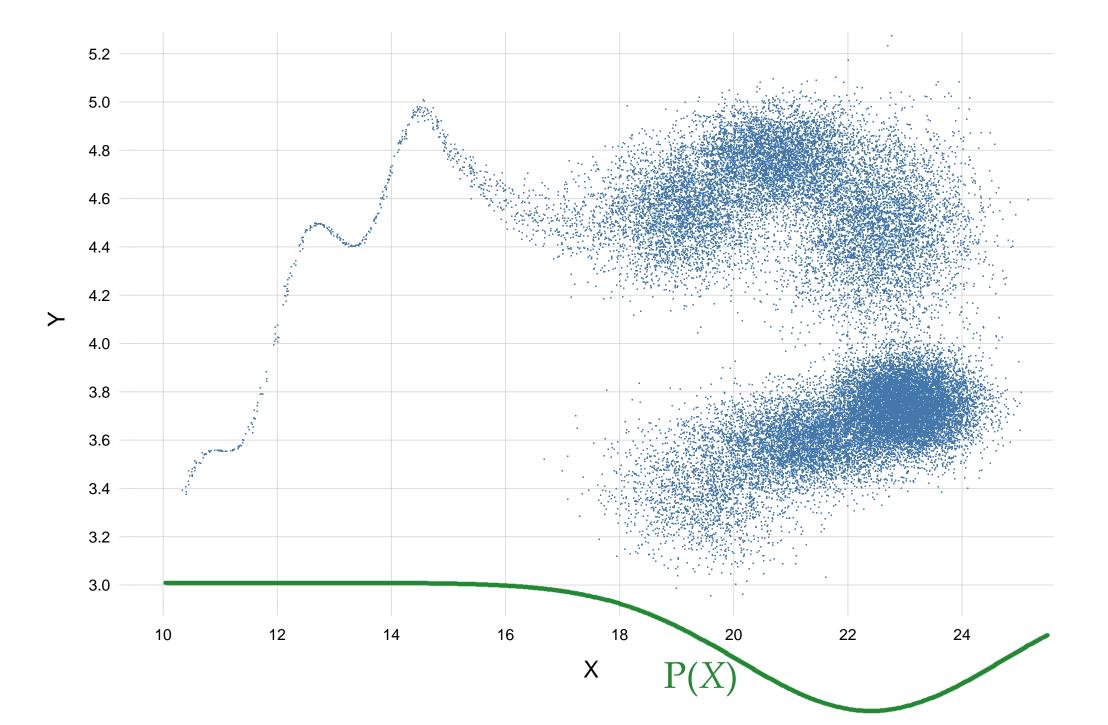




What is the 'overall predictive power' of X?







The 'importance' or 'predictive power' of X depends on $\mathrm{P}(X)$

The 'importance' or 'predictive power' of X depends on P(X)

$$P(Y|X) P(X) = P(X|Y) P(Y)$$

$$P(Y|X) P(X) \stackrel{!}{=} P(X|Y) P^*(Y)$$

$$\stackrel{?}{P^*}(Y|X)\stackrel{?}{P^*}(X) = \stackrel{?}{P^*}(X|Y) P^*(Y)$$

Information Theory

The Bell System Technical Journal

Vol. XXVII July, 1948 No. 3

A Mathematical Theory of Communication

By C. E. SHANNON

Introduction

THE recent development of various methods of modulation such as PCM and PPM which exchange bandwidth for signal-to-noise ratio has intensified the interest in a general theory of communication. A basis for such a theory is contained in the important papers of Nyquist¹ and Hartley² on this subject. In the present paper we will extend the theory to include a number of new factors, in particular the effect of noise in the channel, and the savings possible due to the statistical structure of the original message and due to the nature of the final destination of the information.

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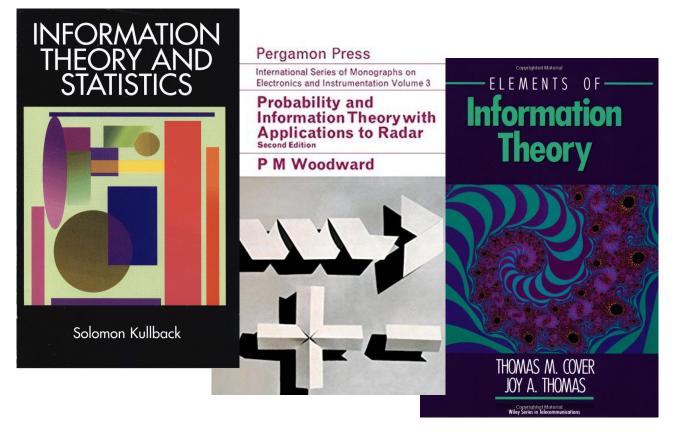
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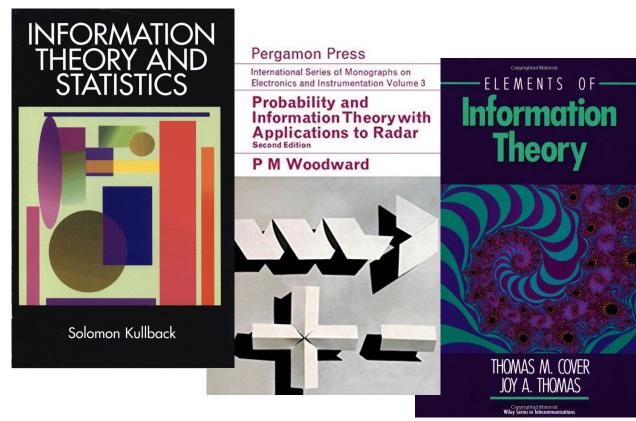
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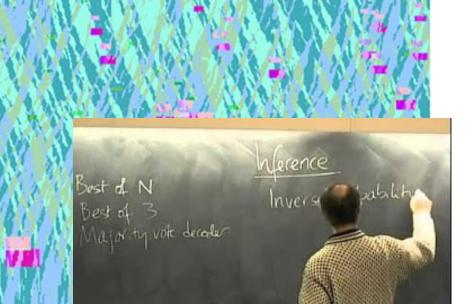
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David J.C. MacKay

Information Theory, Inference, and Learning Algorithms https://www.inference.org.uk/itila/book.html https://youtube.com/playlist?list=PLruBu5BI5n4aFpG32iMbdWoRVAA-Vcso6



Cambridge University Press, 2003

'predictive power' of X for Y := Mutual information between <math>Y and X (mean transinformation content)

$$I(X;Y) := \int p(y|x) p(x) \log \left| \frac{p(y|x)}{p(y)} \right| dy dx$$

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$$I(X;Y) := \int p(y|x) p(x) \log \left[\frac{p(y|x)}{p(y)} \right] dy dx$$

$$I(Y; X_1, X_2) \geq I(Y; X_1)$$

$$I(Y; X_1, X_2) \geq I(Y; X_2)$$

but $I(Y; X_1, X_2) \neq I(Y; X_1) + I(Y; X_2)$



Edition 1.0 2008-03

INTERNATIONAL STANDARD

NORME INTERNATIONALE

Quantities and units -

Part 13: Information science and technology

Grandeurs et unités -

Partie 13: Science et technologies de l'information





Edition 1.0 2008-03

INTERNATIONAL STANDARD

INFORMATION SCIENCE AND TECHNOLOGY QUANTITIES							
Item No.	Name	Symbol	Definition	Remarks			
13-24 (<i>902</i>)	information content fr quantité (f) d'information	I(x)	$I(x) = \operatorname{lb} \frac{1}{p(x)} \operatorname{Sh} = \operatorname{lg} \frac{1}{p(x)} \operatorname{Hart} =$ $\operatorname{ln} \frac{1}{p(x)} \operatorname{nat}$ where $p(x)$ is the probability of event x	See ISO/IEC 2382-16, item 16.03.02. See also IEC 60027-3.			
13-25 (<i>903</i>)	entropy fr entropie (f)	H	$H(X) = \sum_{i=1}^{n} p(x_i)I(x_i)$ for the set $X = \{x_1,, x_n\}$ where $p(x_i)$ is the probability and $I(x_i)$ is the information content of event x_i	See ISO/IEC 2382-16, item 16.03.03.			
13-30 (<i>908</i>)	joint information content fr quantité (f) d'information conjointe	I(x, y)	$I(x, y) = \operatorname{lb} \frac{1}{p(x, y)} \operatorname{Sh} = \operatorname{lg} \frac{1}{p(x, y)} \operatorname{Hart} =$ $\operatorname{ln} \frac{1}{p(x, y)} \operatorname{nat}$ where $p(x, y)$ is the joint probability of events x and y				
13-35 (<i>912</i>)	transinformation content fr transinformation (f)	T(x,y)	T(x,y) = I(x) + I(y) - I(x,y) where $I(x)$ and $I(y)$ are the information contents (13-24) of events x and y , respectively, and $I(x,y)$ is their joint information content (13-30)	See ISO/IEC 2382-16, item 16.04.07.			
13-36 (<i>913</i>)	mean transinformation content fr transinformation (f) moyenne	T	$T(X,Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} p(x_i, y_j) T(x_i, y_j)$ for the sets $X = \{x_1,, x_n\}, Y = \{y_1,, y_m\},$ where $p(x_i, y_j)$ is the joint probability of events x_i and y_j , and $T(x_i, y_j)$ is their transinformation content (item 13-35)	See ISO/IEC 2382-16, item 16.04.08.			

UNITS INFORMATION SCIENCE AND TECHNOLOGY							
Item No.	Name	Symbol	Definition	Conversion factors and remarks			
13-24.a	shannon	Sh	value of the quantity when the argument is equal to 2	1 Sh ≈ 0,693 nat ≈ 0,301 Hart			
13-24.b	hartley	Hart	value of the quantity when the argument is equal to 10	1 Hart ≈ 3,322 Sh ≈ 2,303 nat			
13-24.c	natural unit of information	nat	value of the quantity when the argument is equal to e	1 nat ≈ 1,433 Sh ≈ 0,434 Hart			
13-25.a	shannon	Sh					
13-25.b	hartley	Hart					
13-25.c	natural unit of information	nat					
13-30.a	shannon	Sh					
13-30.b	hartley	Hart					
13-30.c	natural unit of information	nat					
13-35.a	shannon	Sh					
13-35.b	hartley	Hart					
13-35.c	natural unit of information	nat					
13-36.a	shannon	Sh		In practice, the unit "shannon per character" is generally used, and			
13-36.b	hartley	Hart		sometimes the units "hartley per character" and "natural unit per character".			
13-36.c	natural unit of information	nat					

$$0 \text{ Sh} \leq I(Y;X) \leq 1 \text{ Sh}$$

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X and *Y* are independent Using *X* is no better than flipping a coin

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$$I(Y; X) = 0.22 \text{ Sh}$$

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In N=100 new prognoses:

- we are **completely certain** about 22
- we are **completely uncertain** about 100-22 = 78
- \rightarrow approx 22+78/2 = 61 correct prognoses (TP+TN)

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$$\frac{1}{2} + \frac{1}{2}\sqrt{1 - (1 - 0.22)^{\frac{4}{3}}} \approx 77\%$$
 ± 0.8 $\sqrt{N}\%$ correct prognoses

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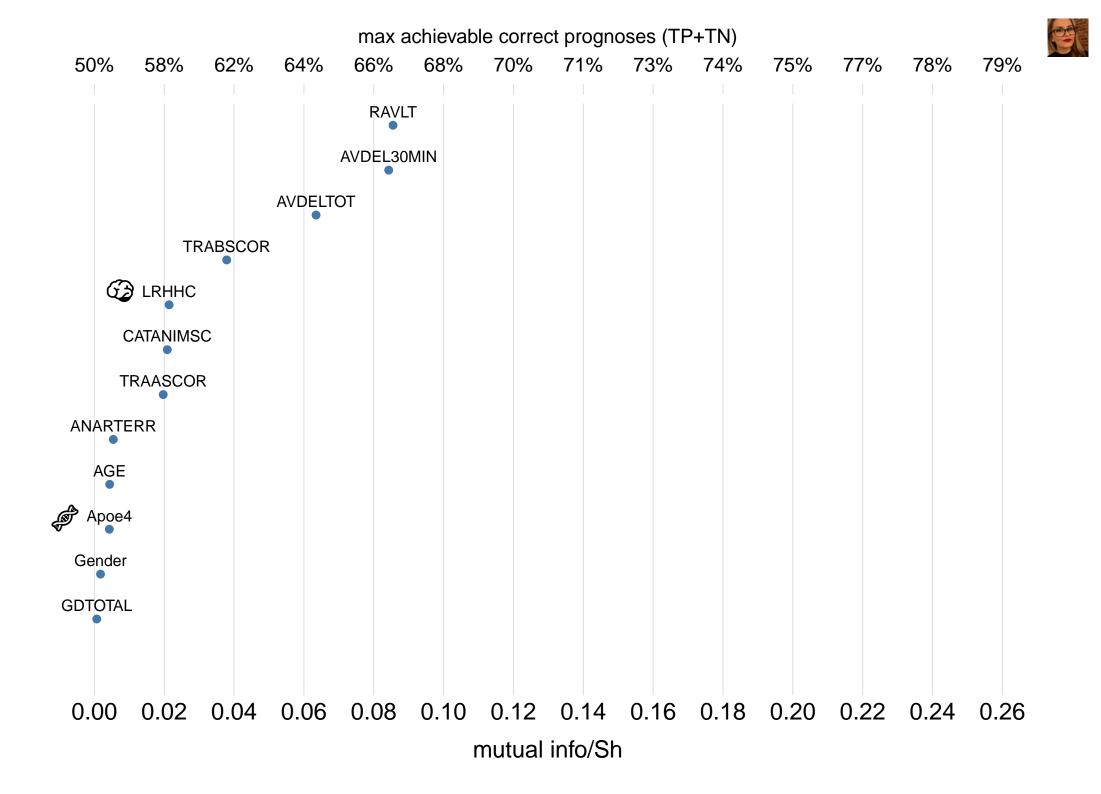
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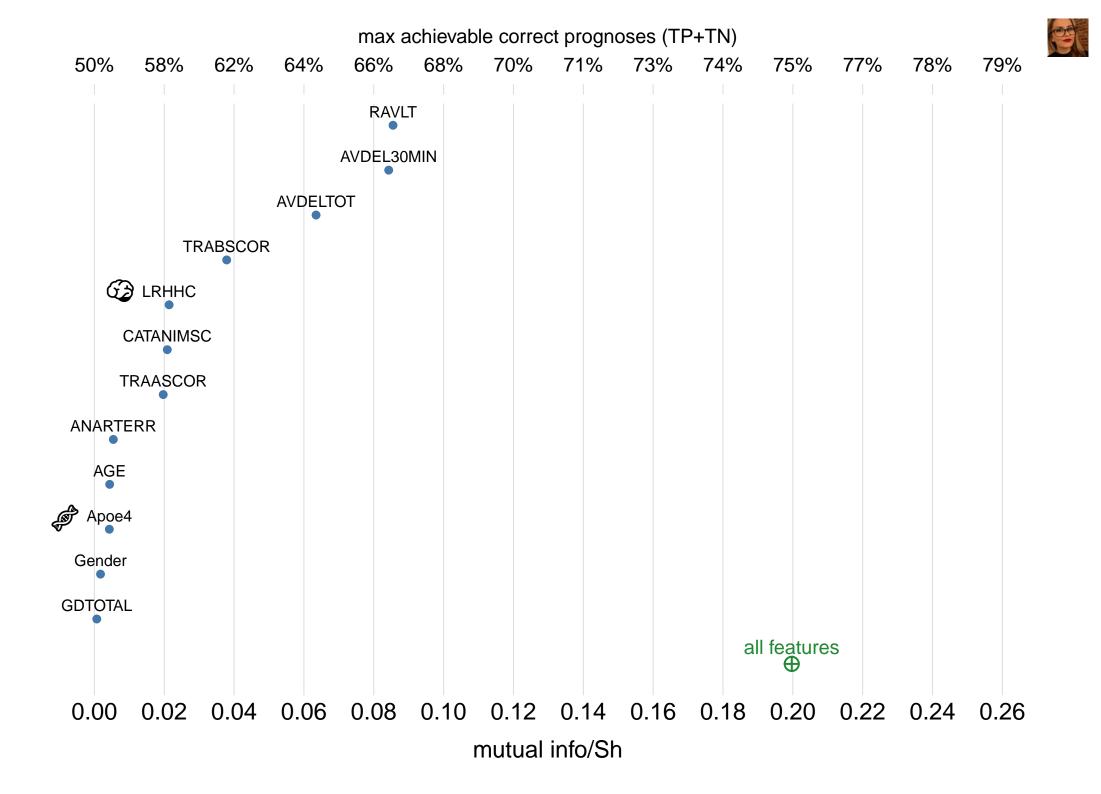
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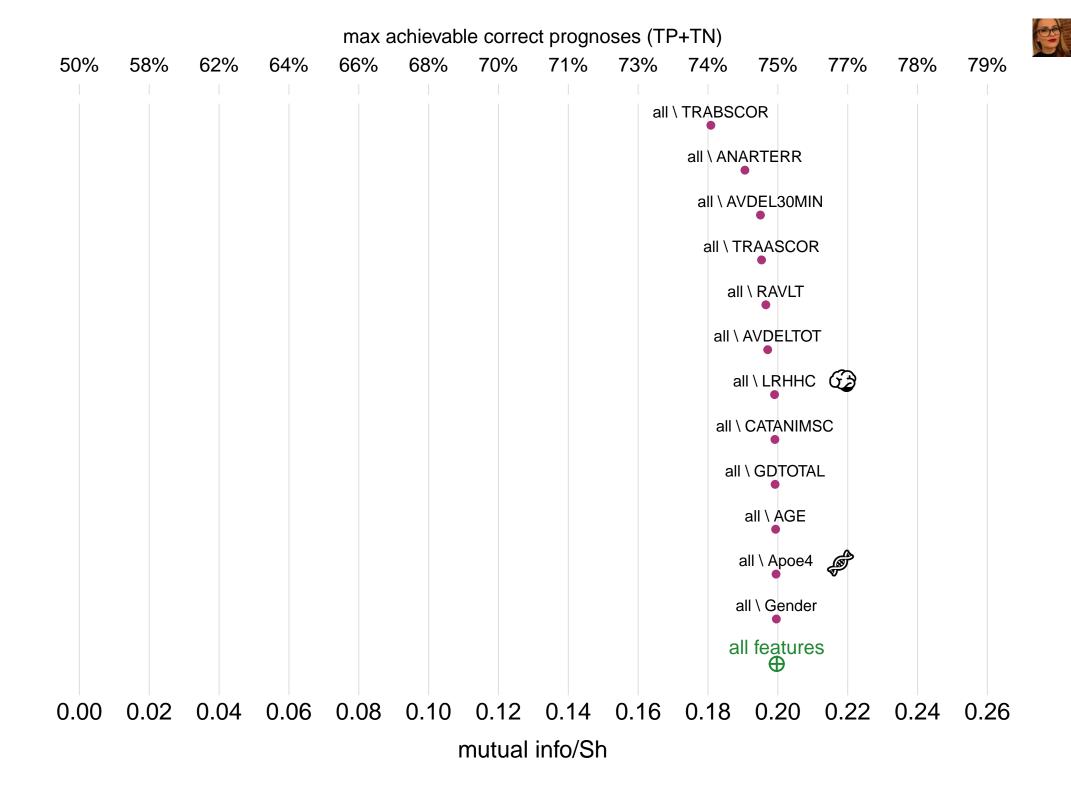
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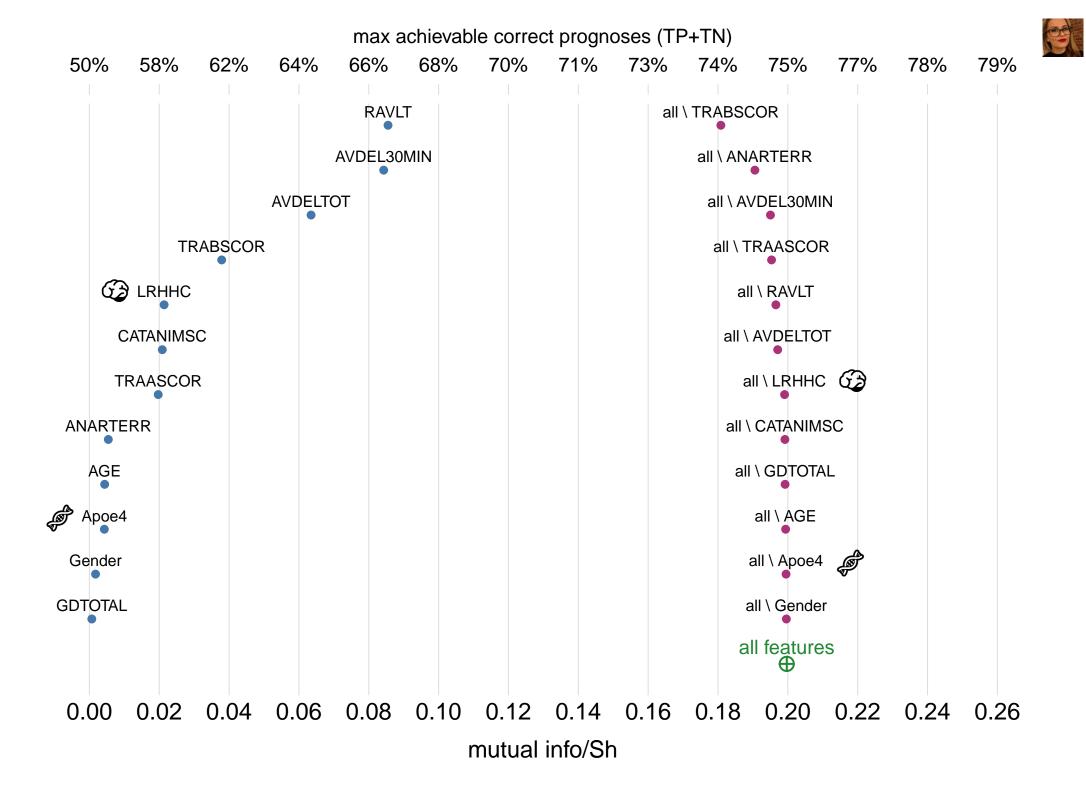
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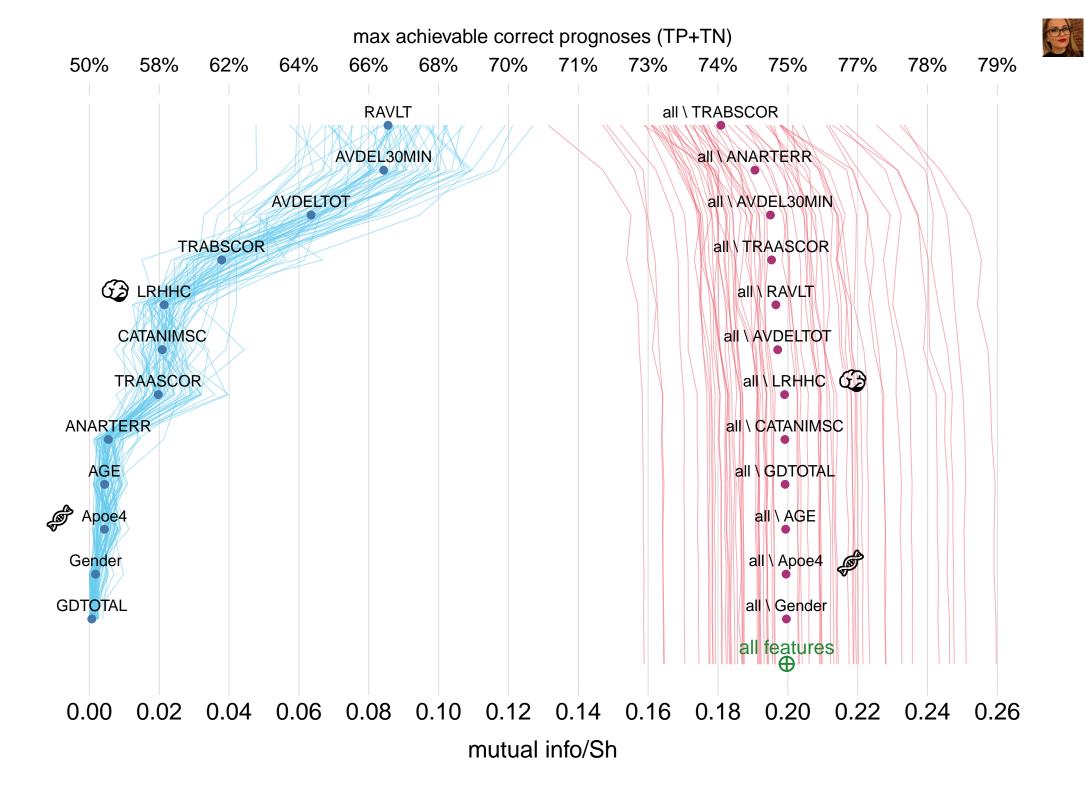
Maximum accuracy attainable by *any* algorithm which uses only feature set *X*

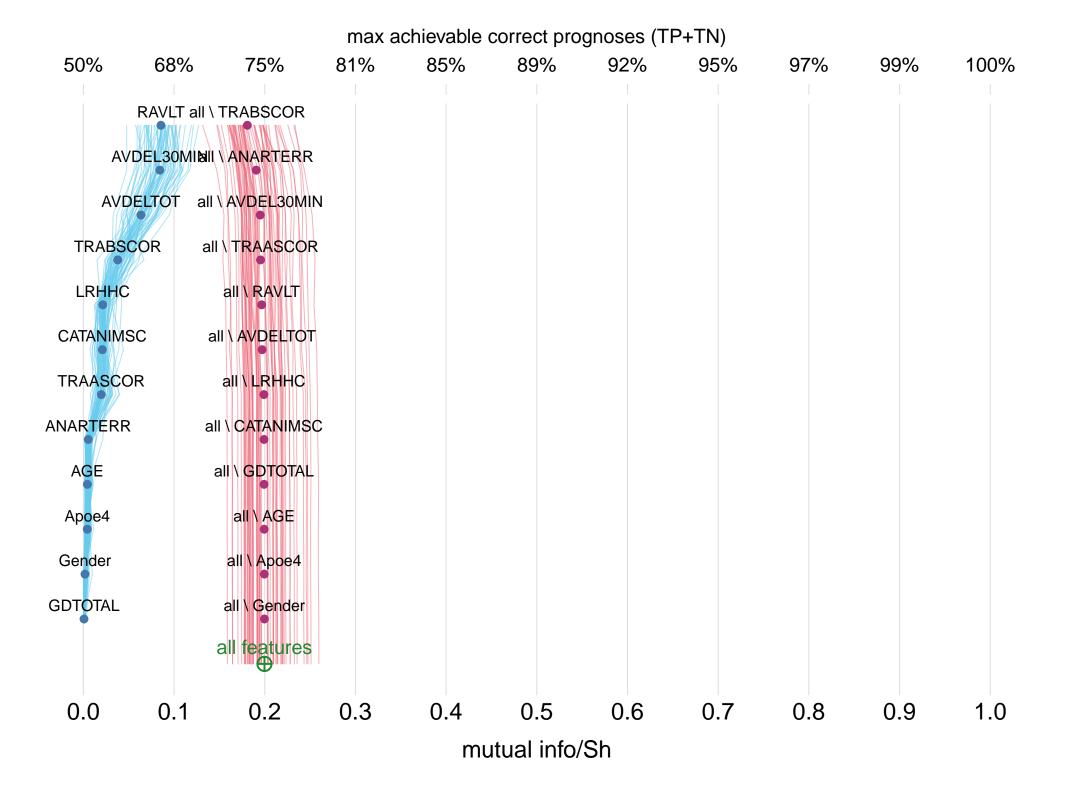


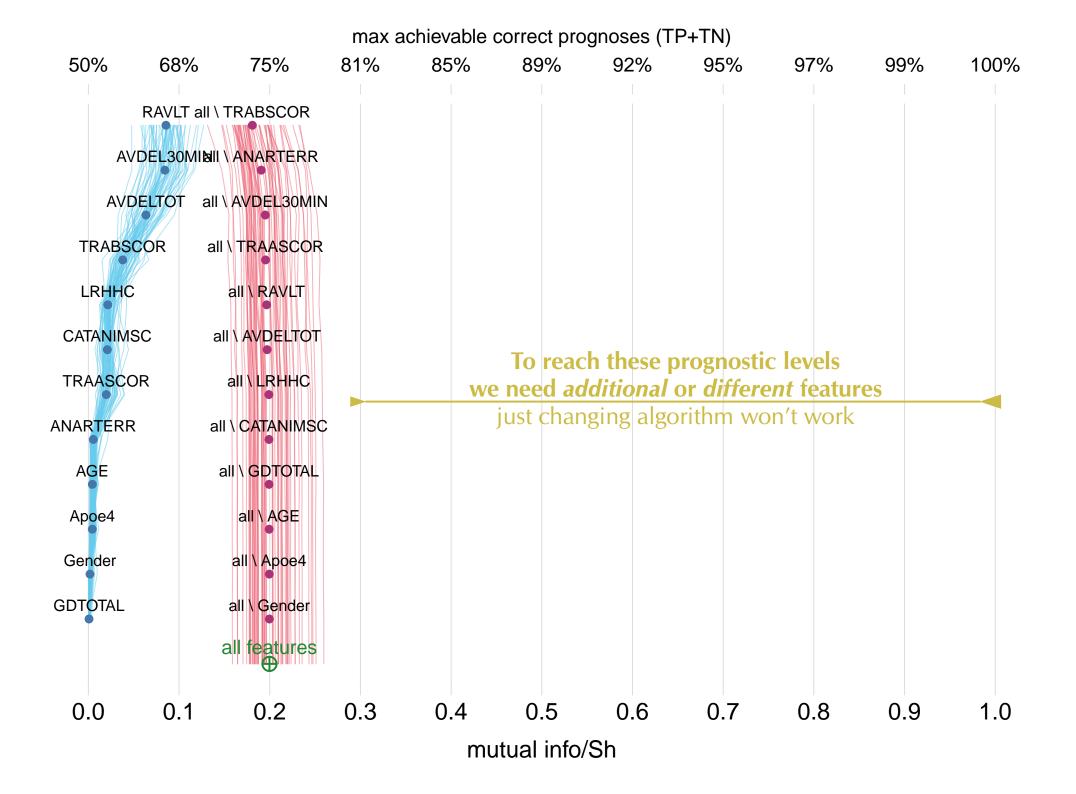


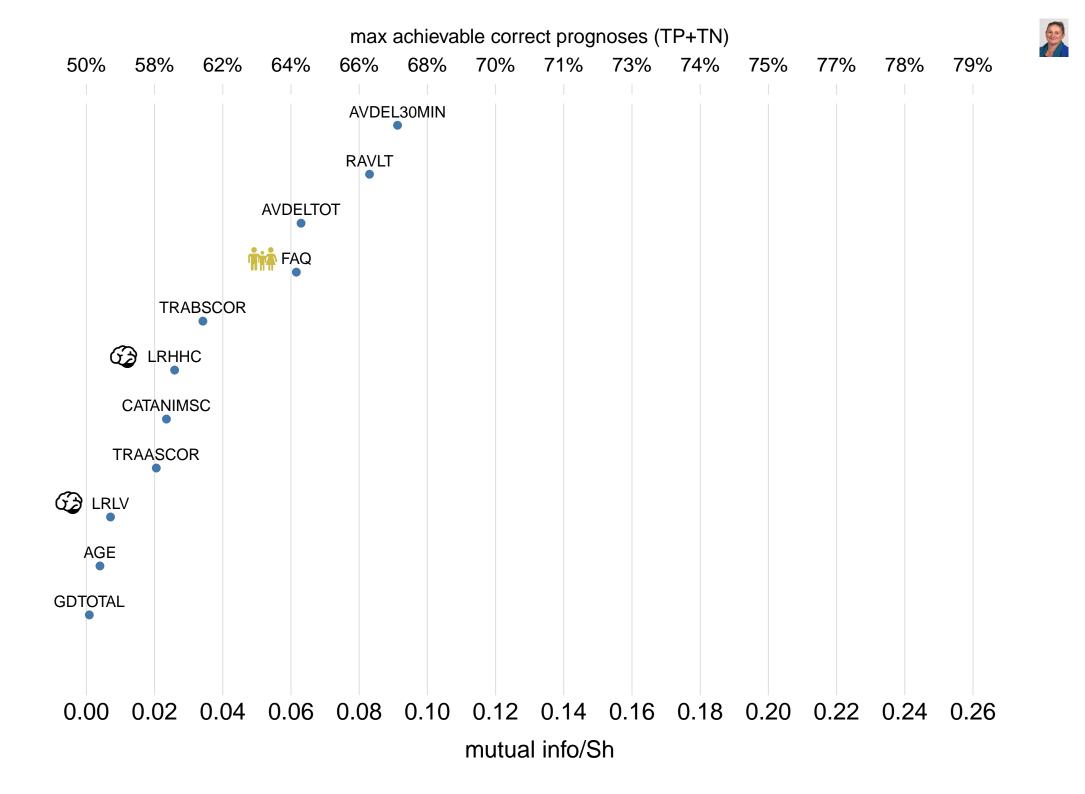


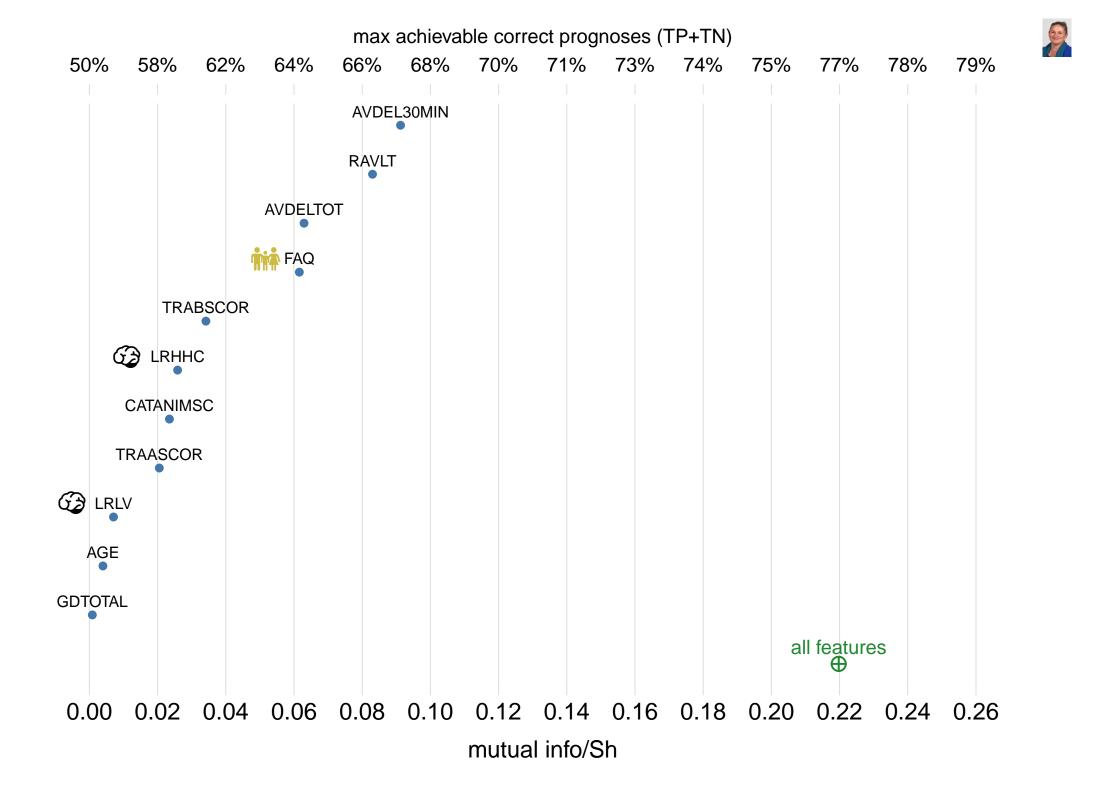


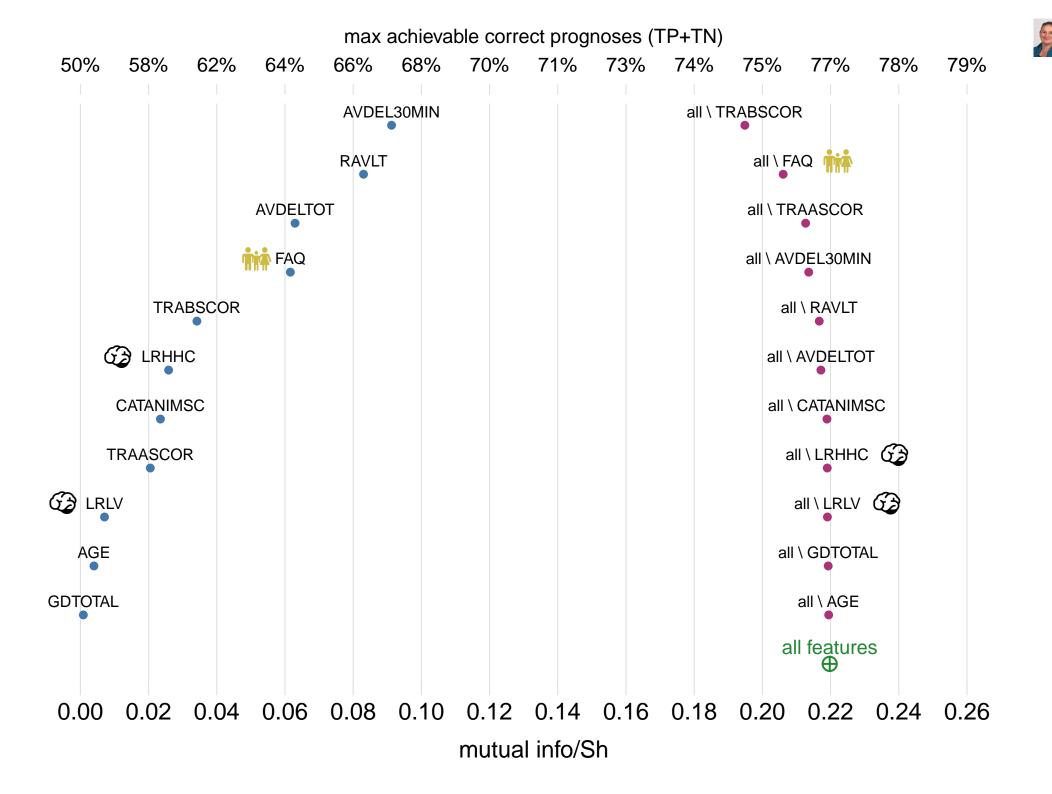


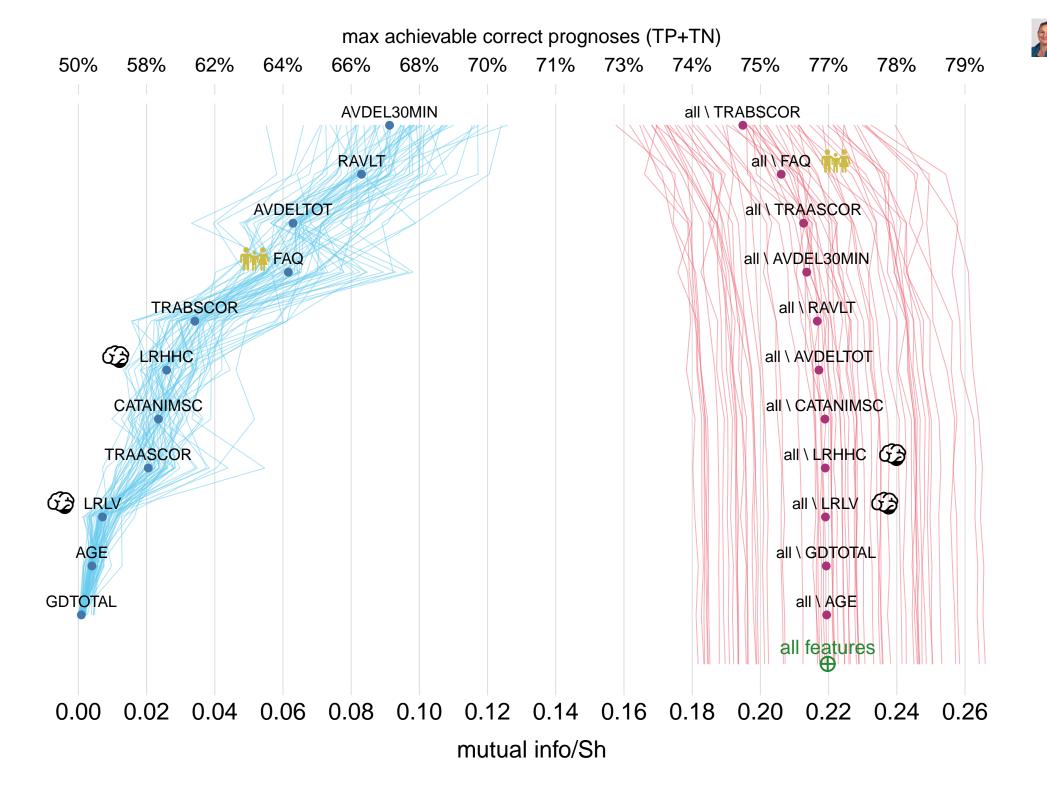






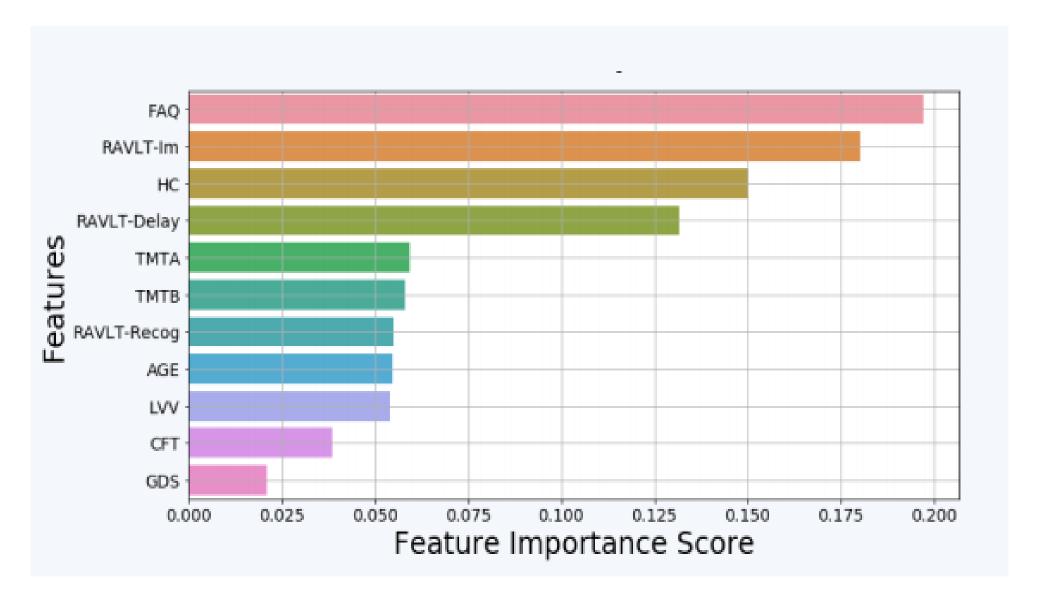


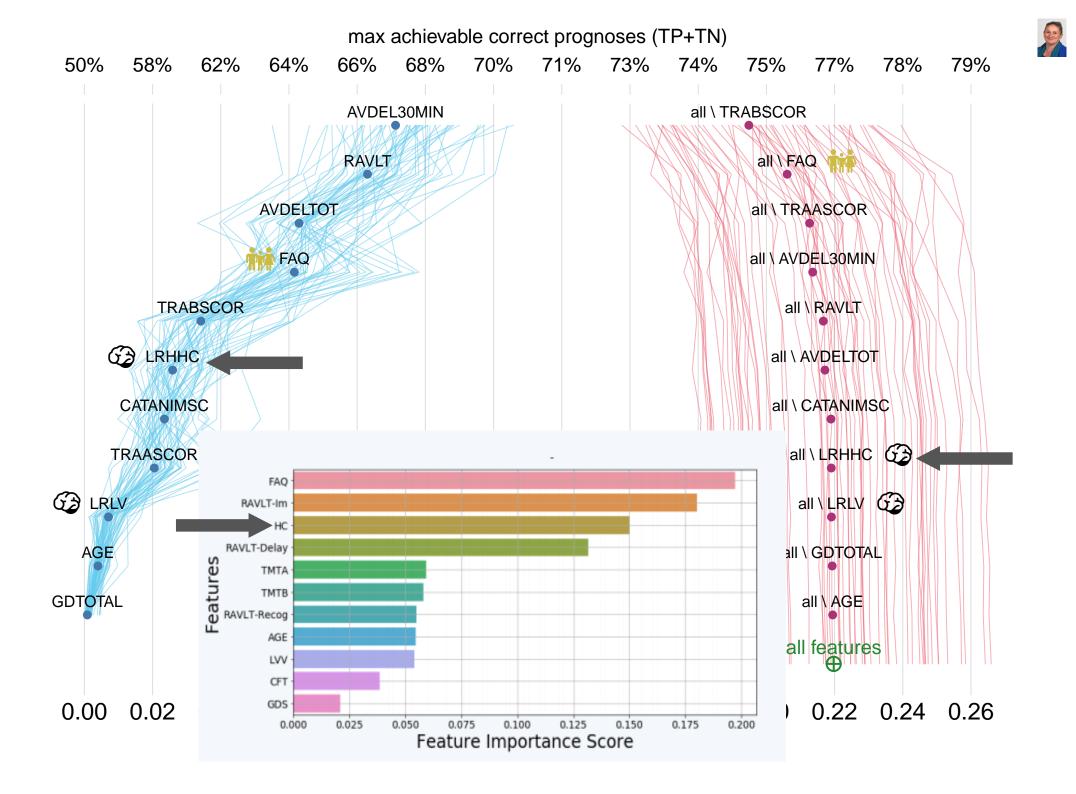


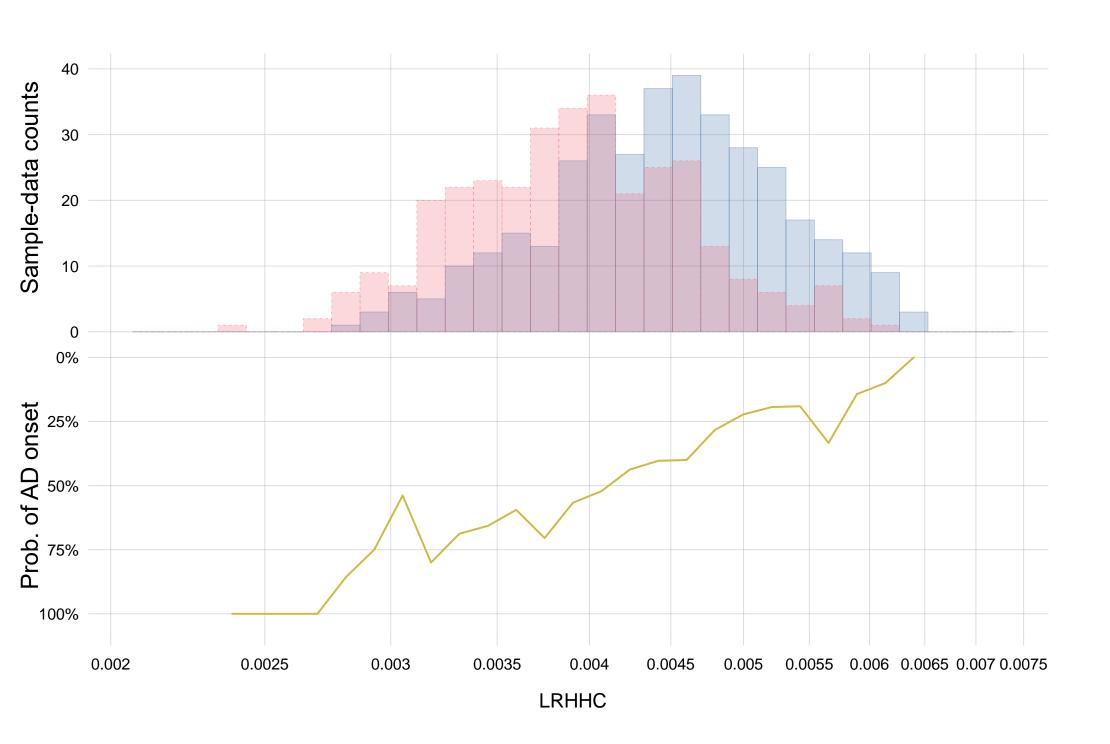


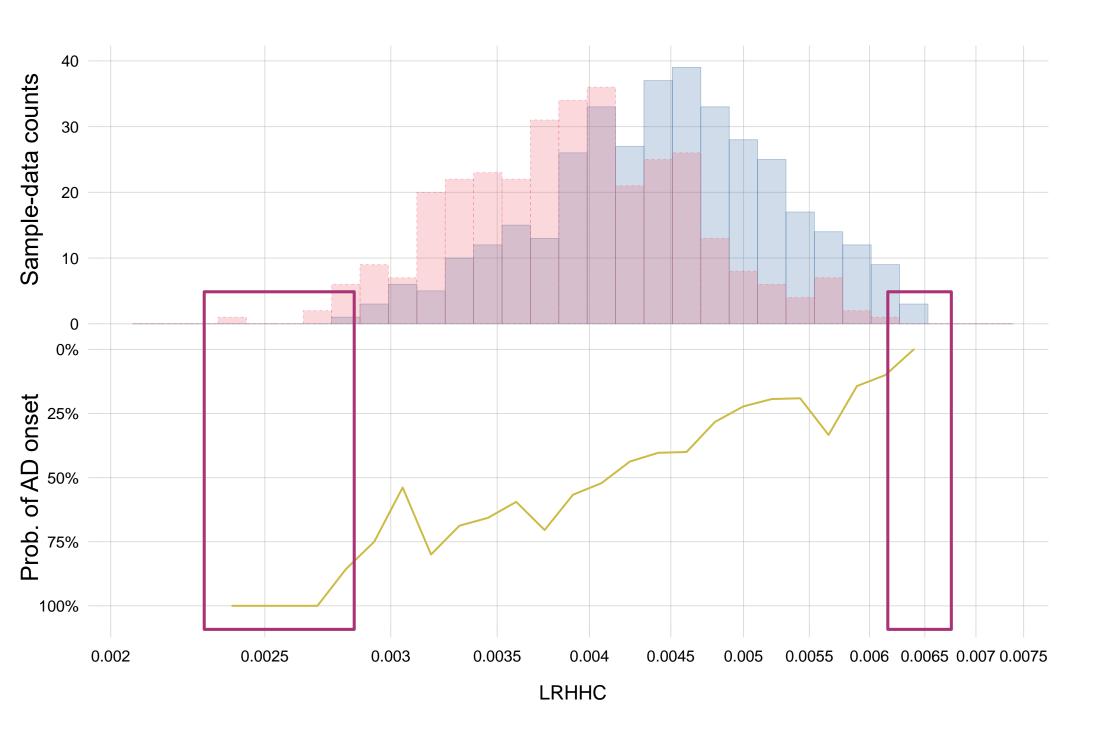


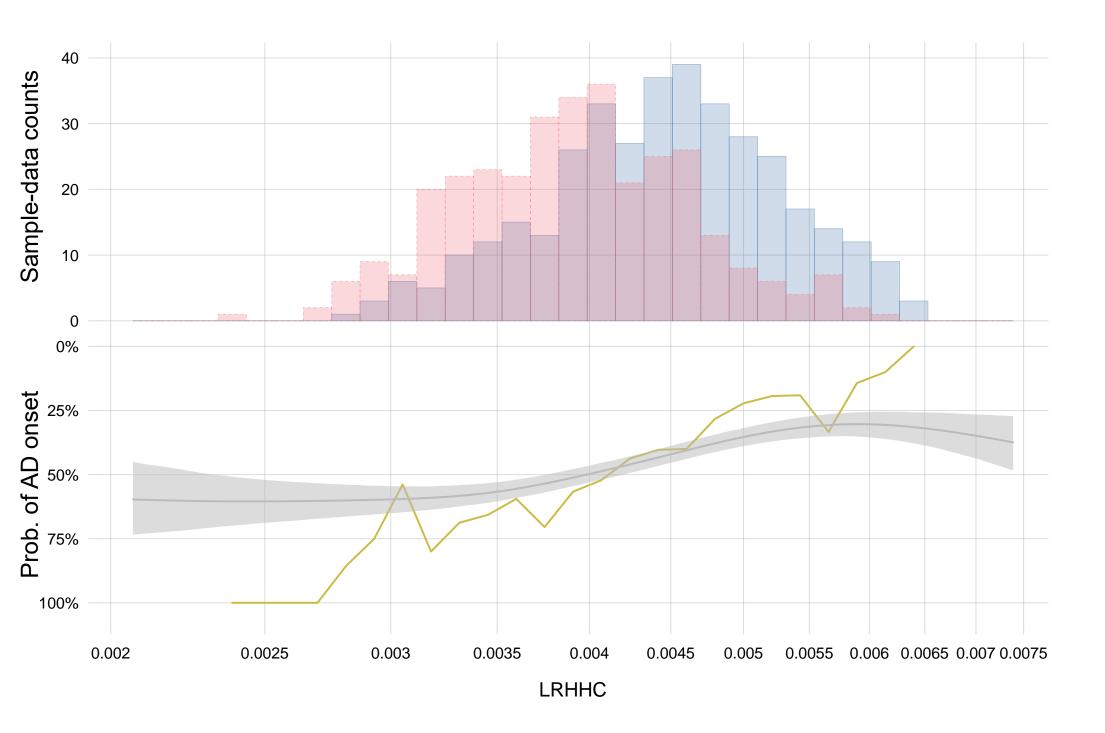
Alexandra's results



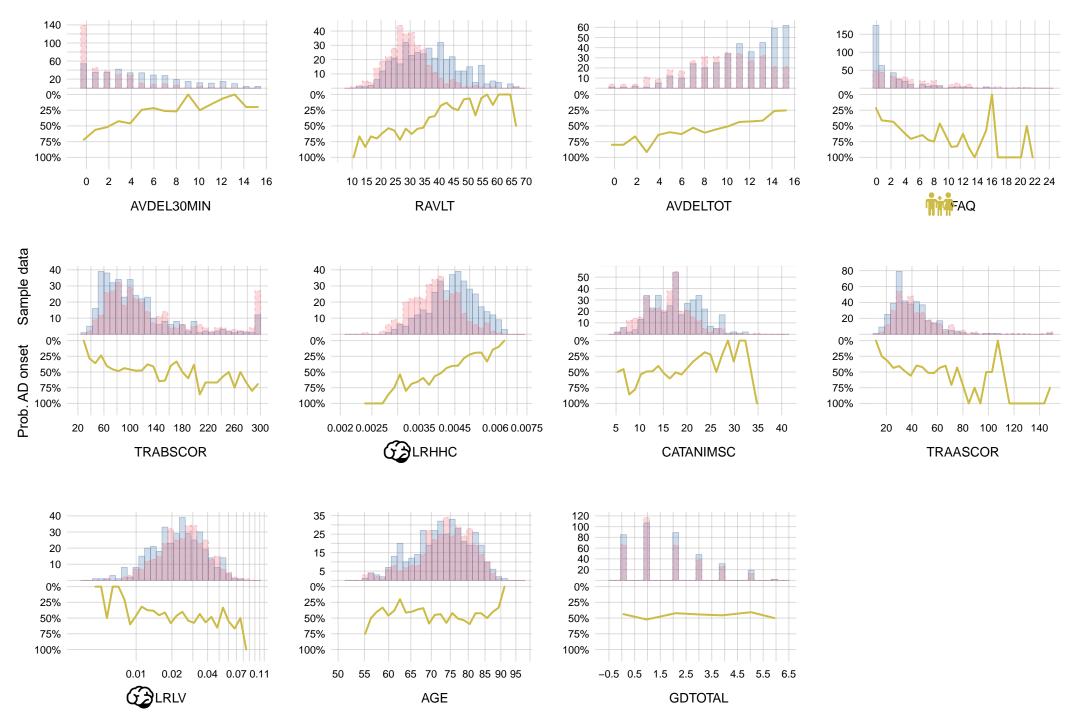




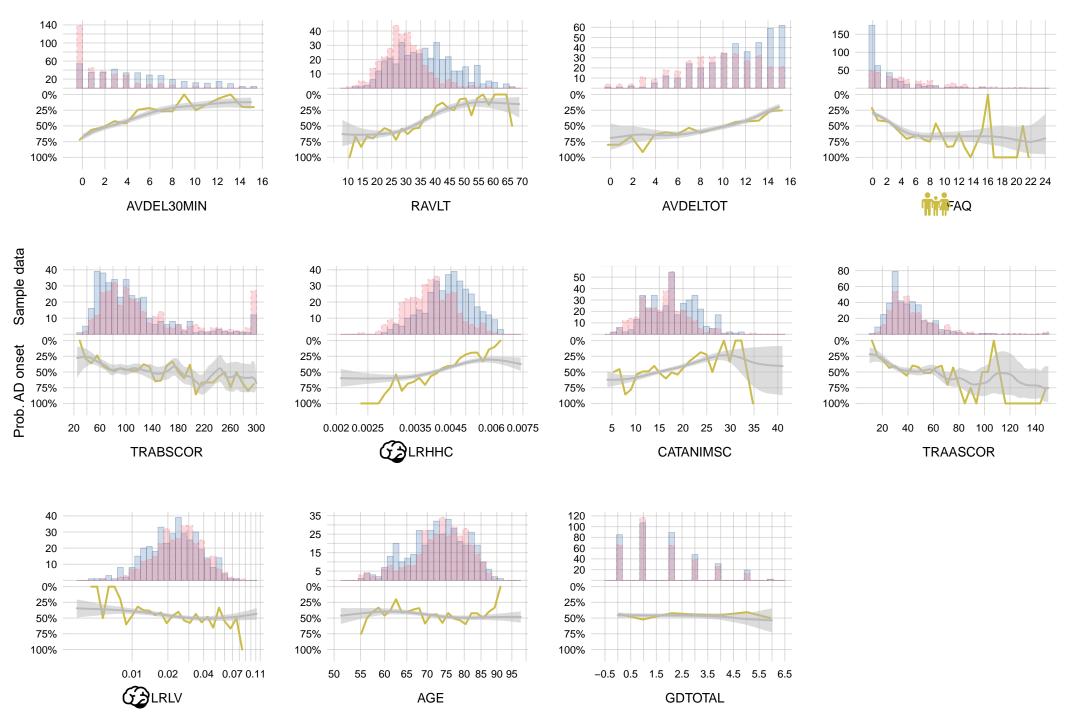












Thank you!