Mild Cognitive Impairment

Mild Cognitive Impairment (stable)

Alzheimer Disease















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් ANARTERR

ී GDTOTAL

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TRAASCOR

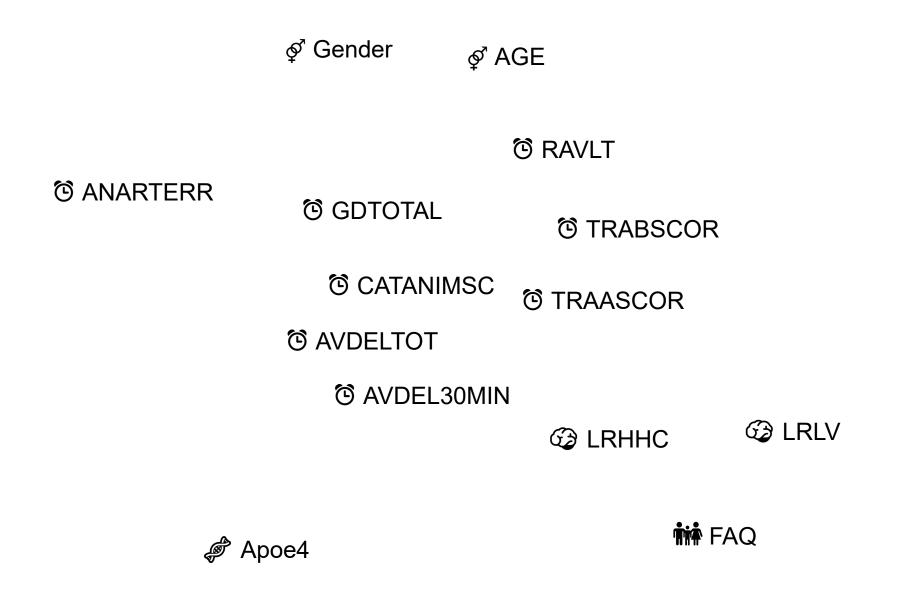
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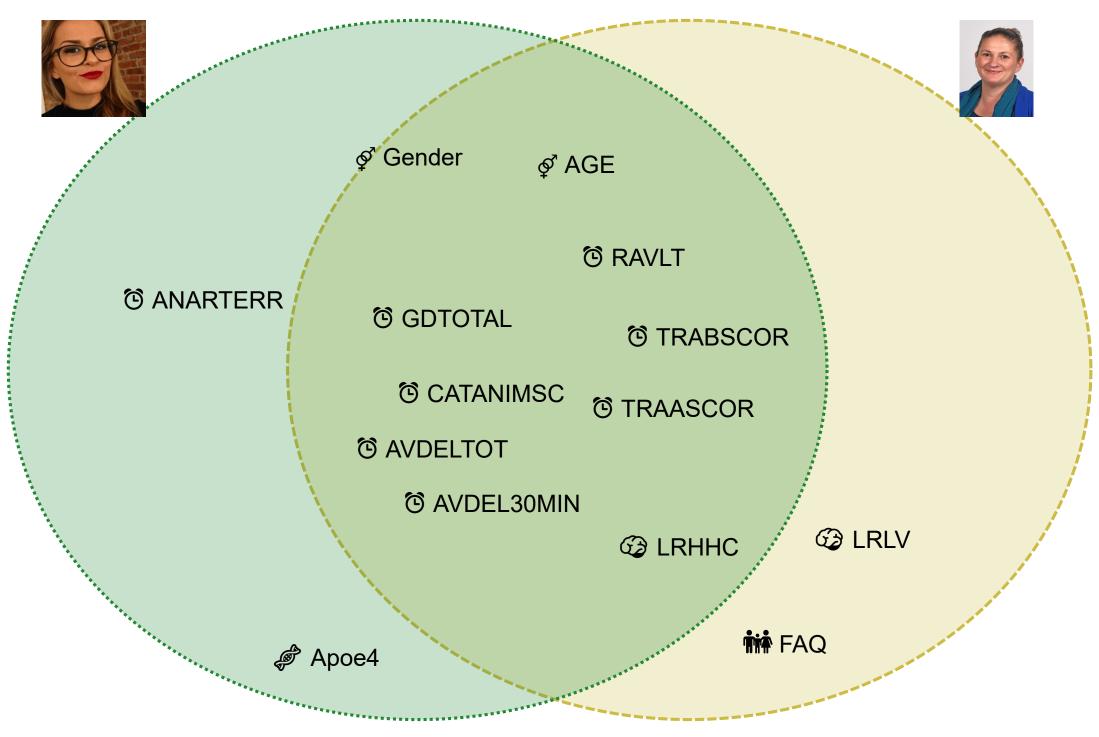
3 LRLV

Apoe4

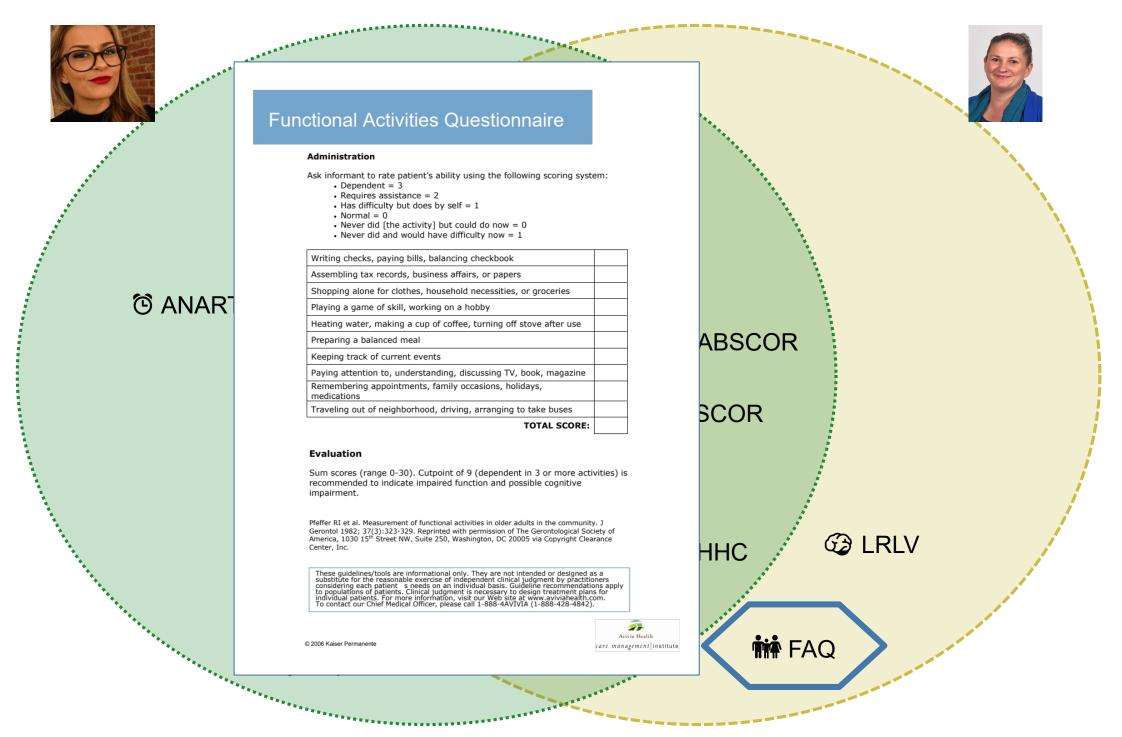
†i FAQ



How 'good' are these features at prognosing the later onset of Alzheimer?



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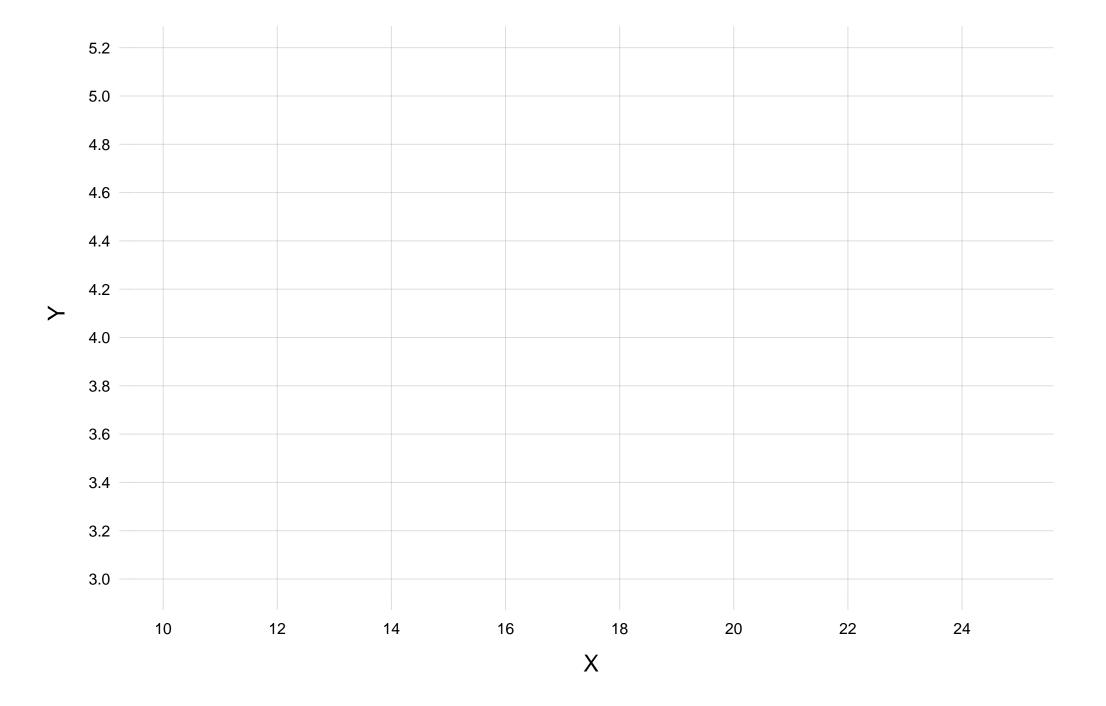


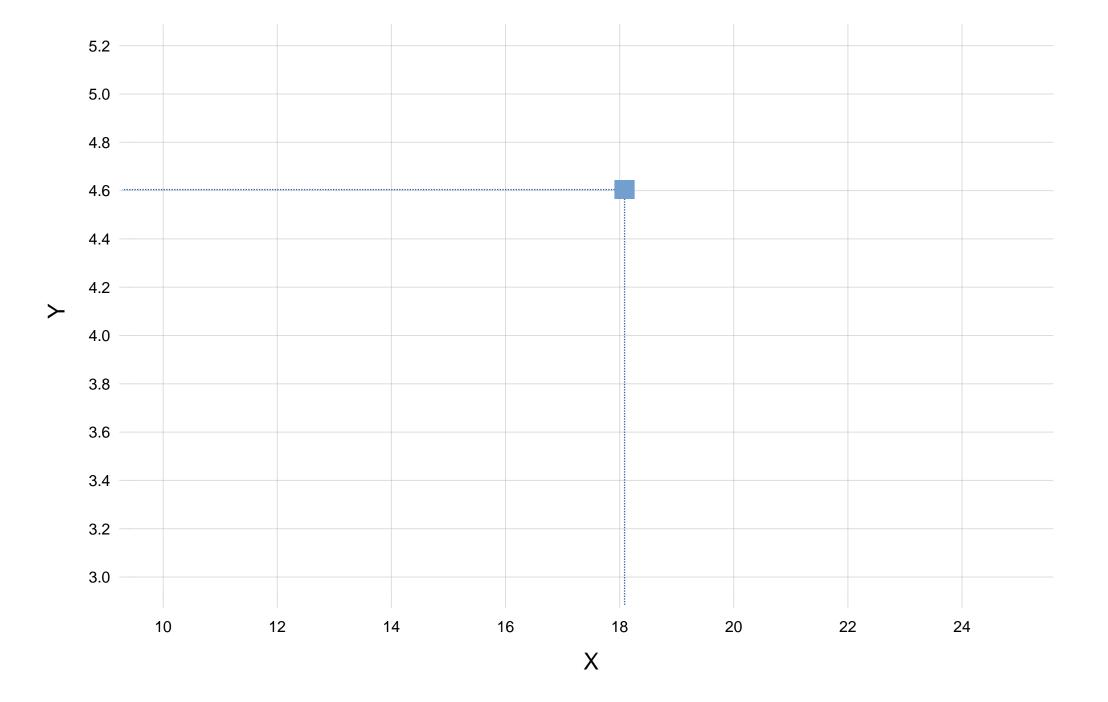
Ingrid's study: 12 + 1 variates, 678 datapoints

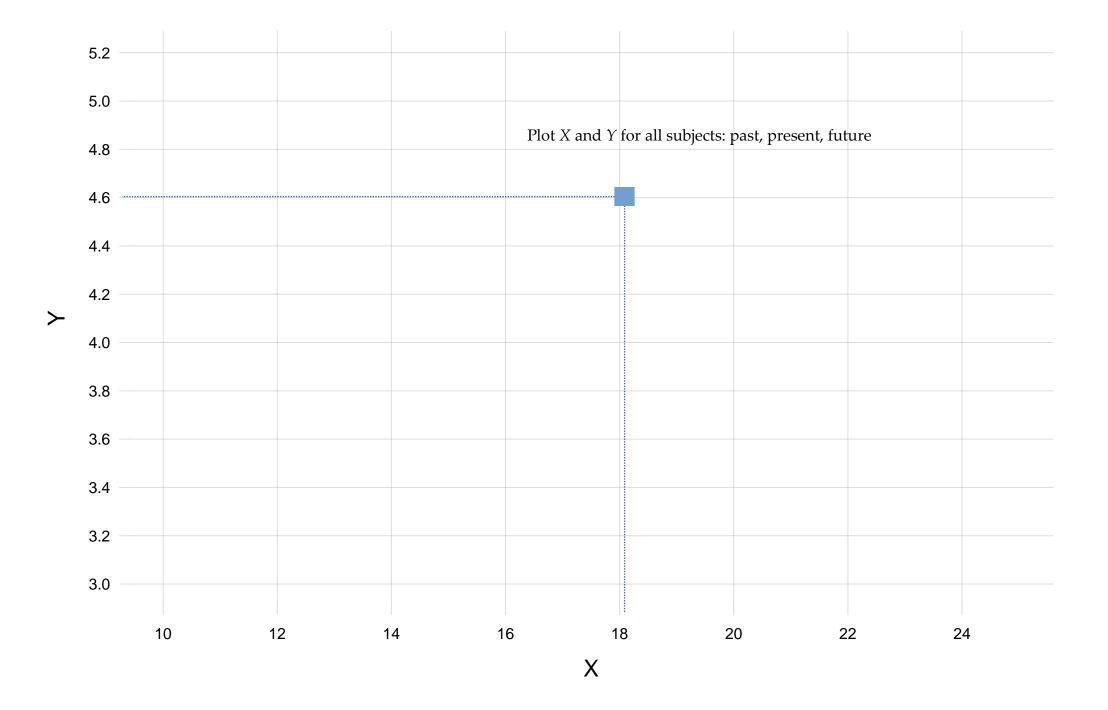
Alexandra's study: 11 + 1 variates, 708 datapoints (43 missing values)

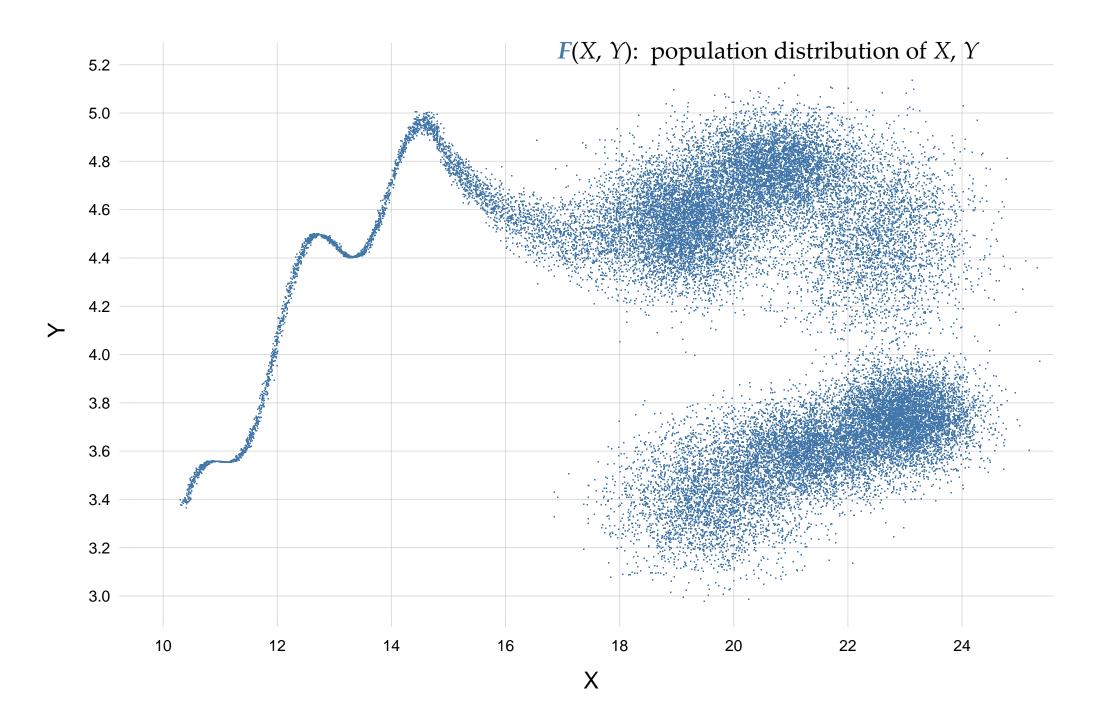
Computation time: ~65 h/study (3 parallel sessions to assess numeric convergence)

HPC UNINETT | sigma2



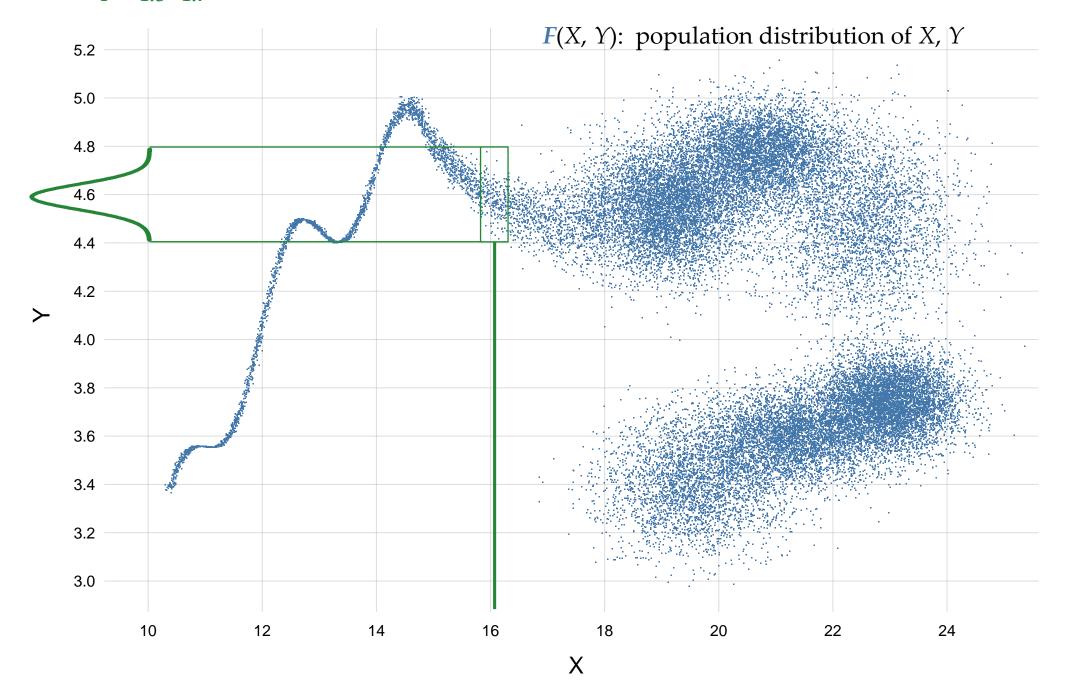




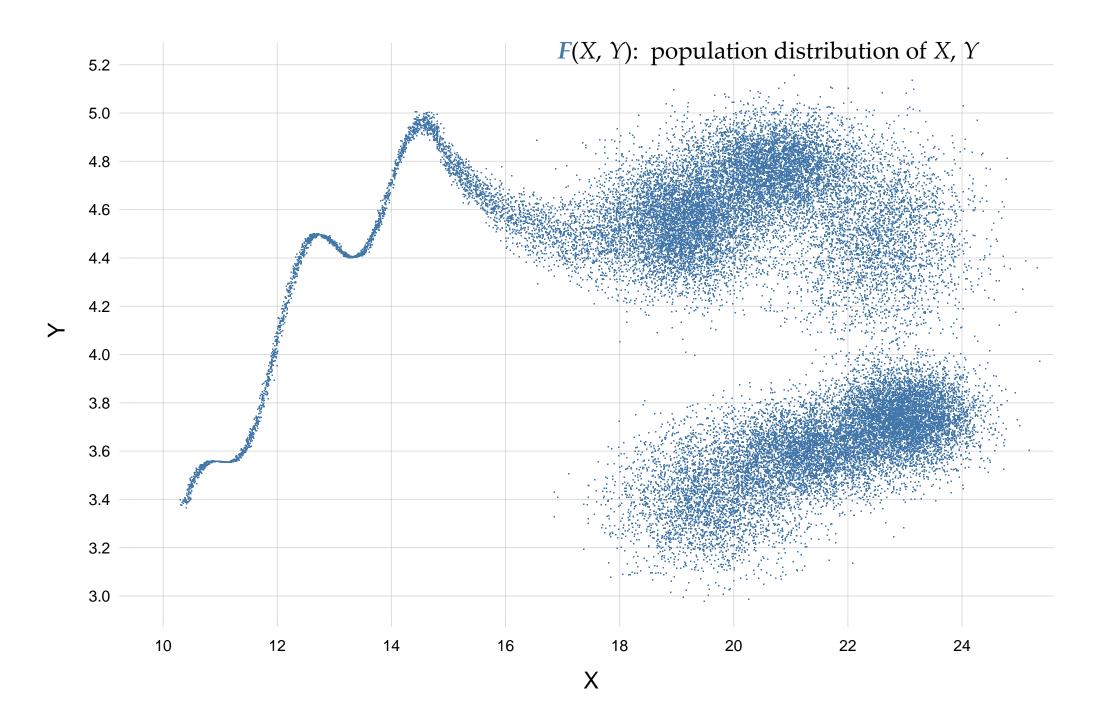


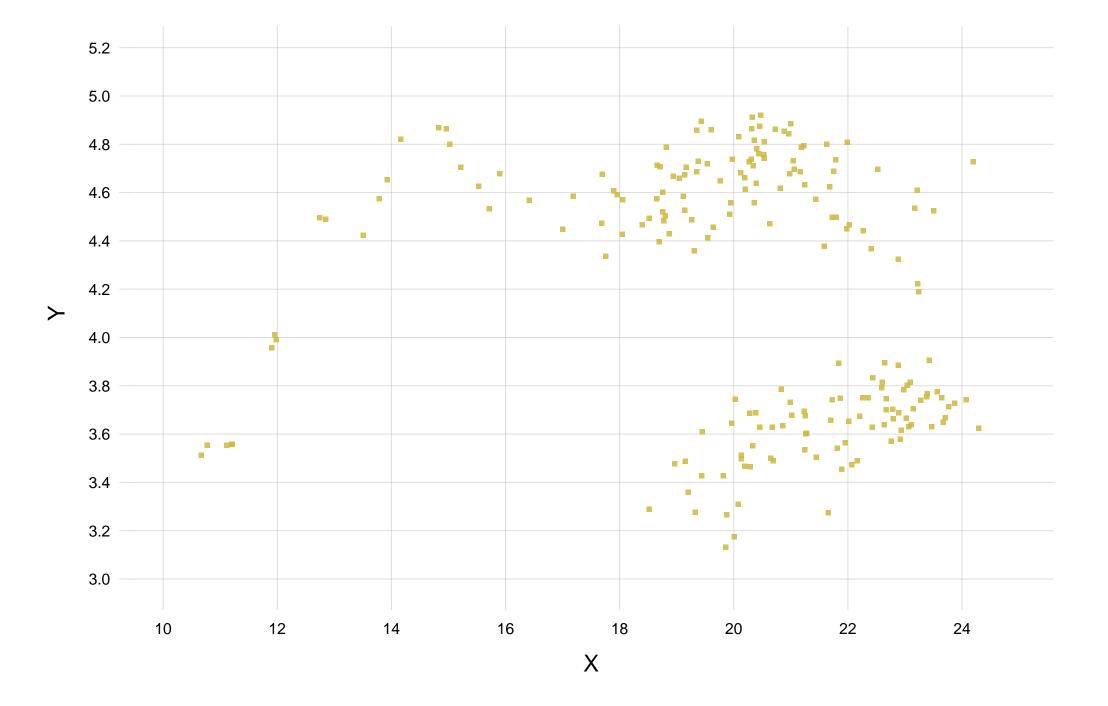
New patient: X = 16

 $\Rightarrow Y \approx 4.5-4.7$



$$P(y \mid x) = F(y \mid x)$$





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$$P(y \mid x) = \int F(y \mid x) p(F \mid data) dF$$

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$$p(F \mid data) \propto F(y_1, x_1) \times F(y_2, x_2) \times F(y_3, x_3) \times ...$$

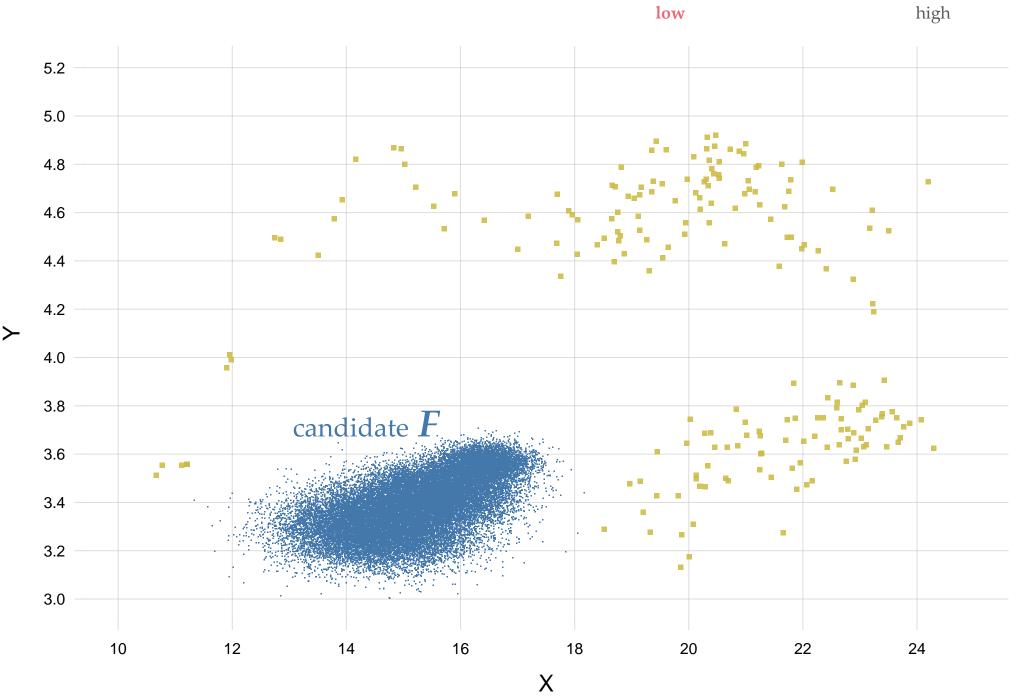
how well the 'candidate' distribution fits the data

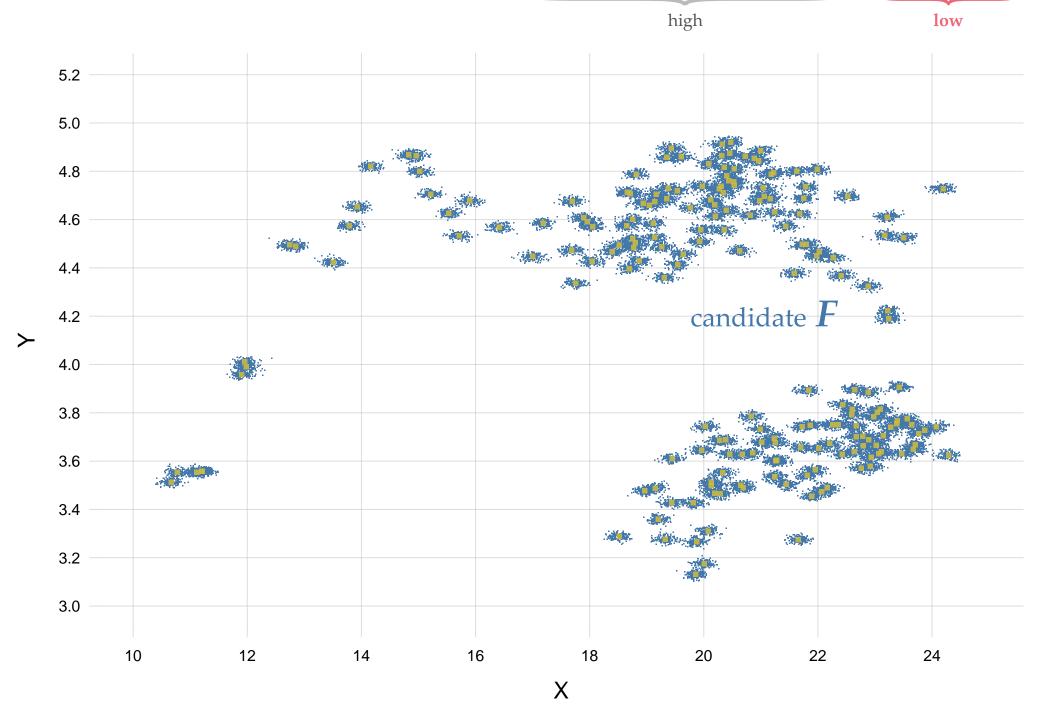
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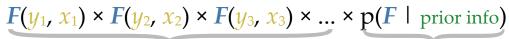
$$p(F \mid data) \propto F(y_1, x_1) \times F(y_2, x_2) \times F(y_3, x_3) \times ... \times p(F \mid prior info)$$

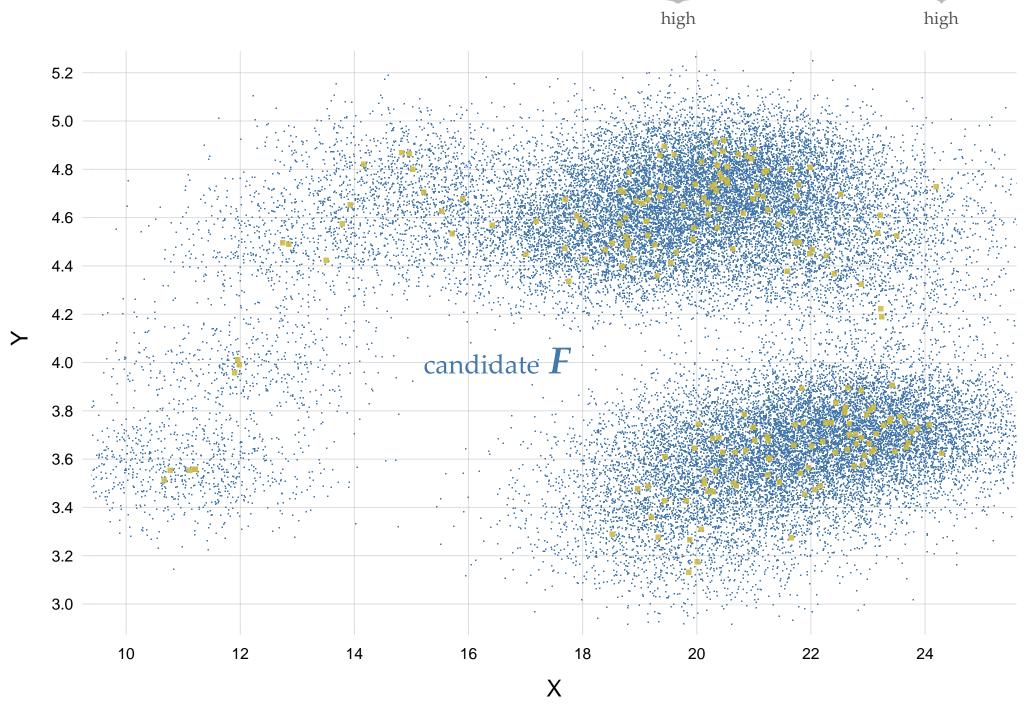
how well the 'candidate' distribution fits the data

extra-data knowledge







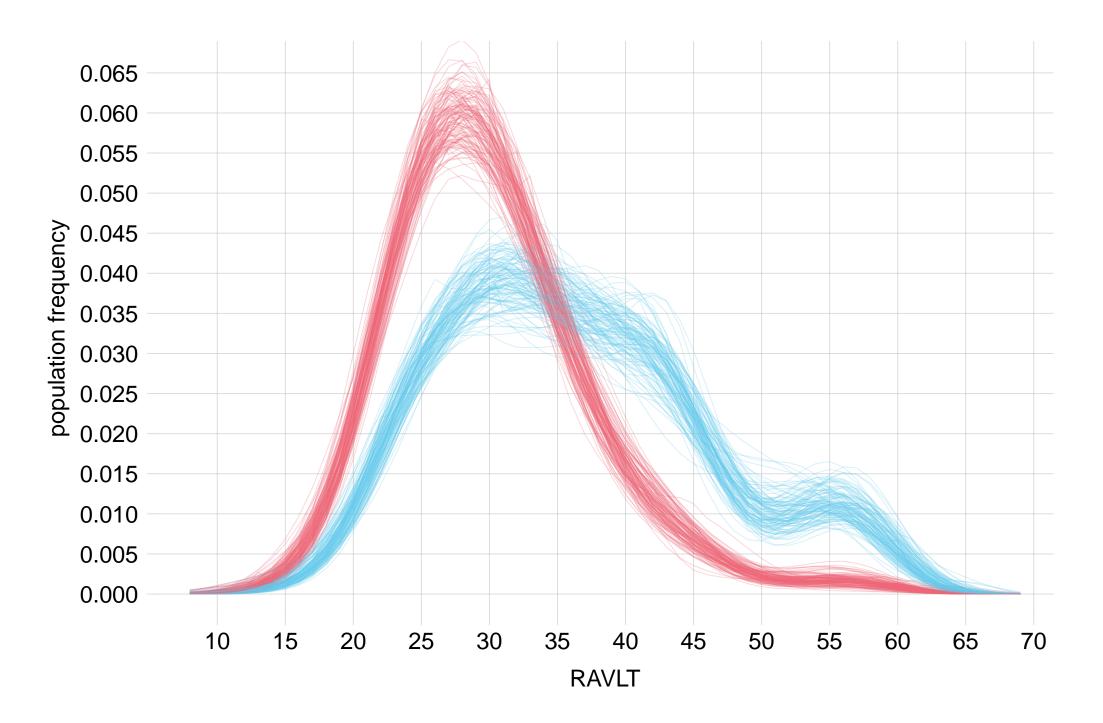


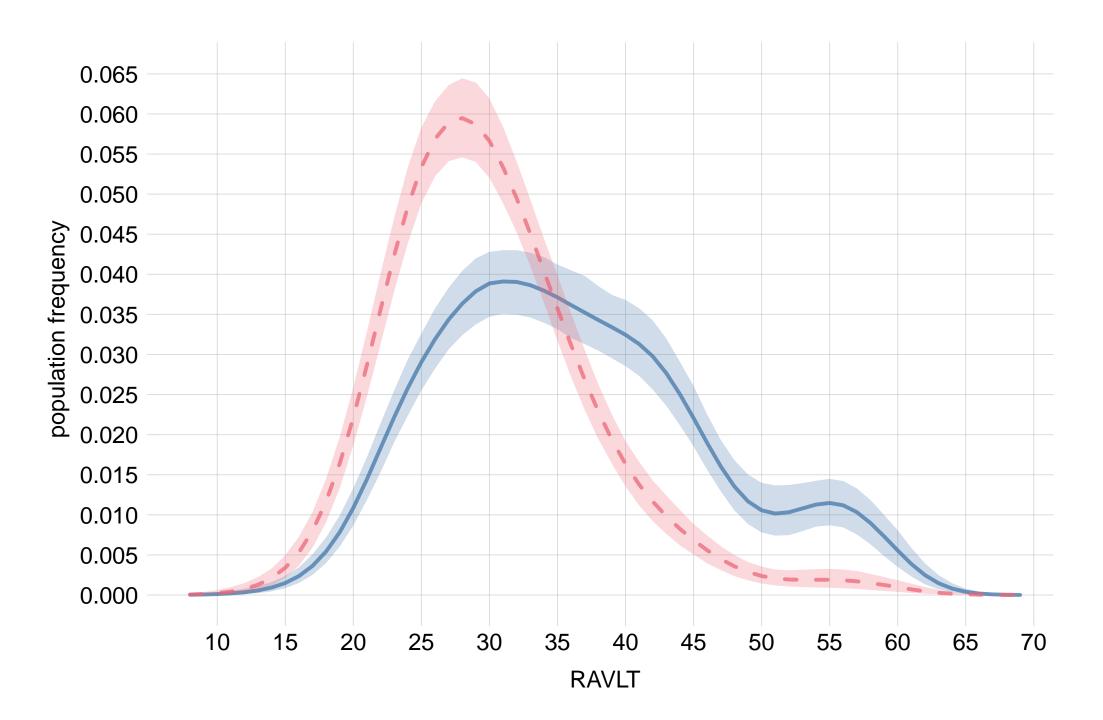
$intuition \rightarrow mathematics$

intuition — mathematics

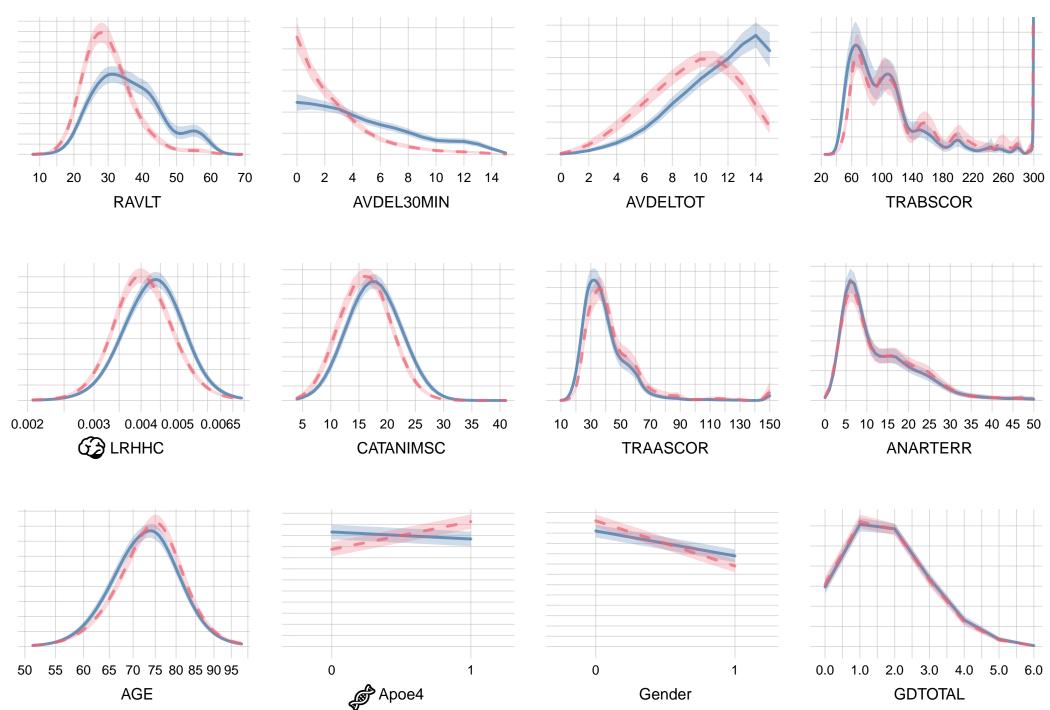
$first\ principles \rightarrow mathematics \rightarrow intuition$

('Bayesian')

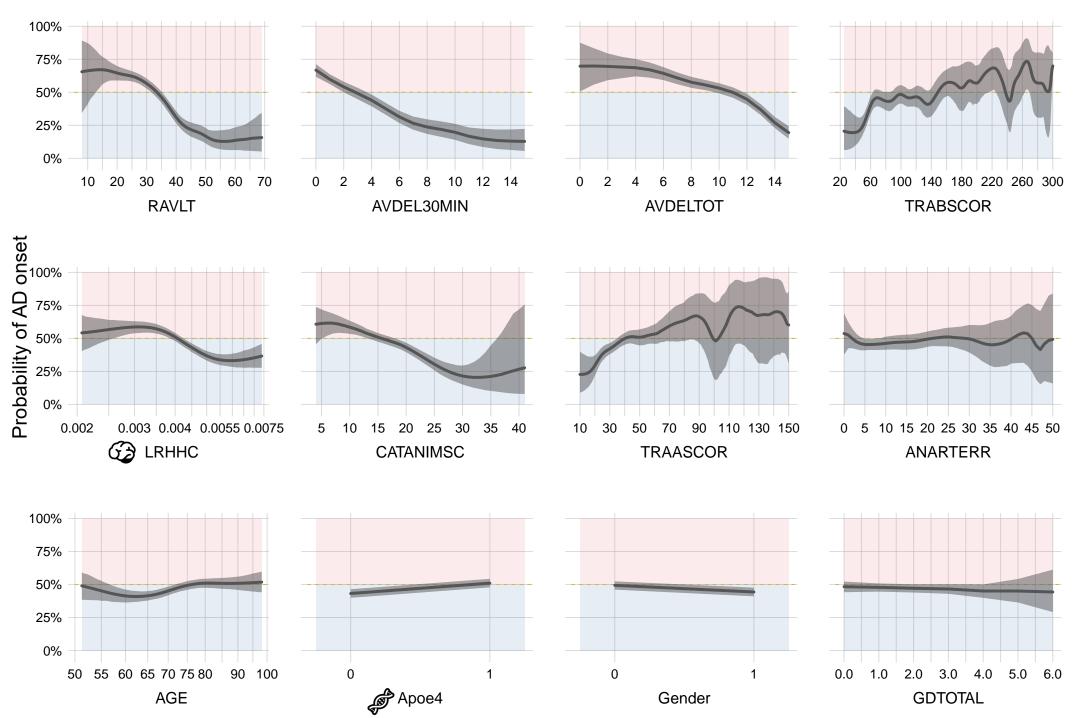




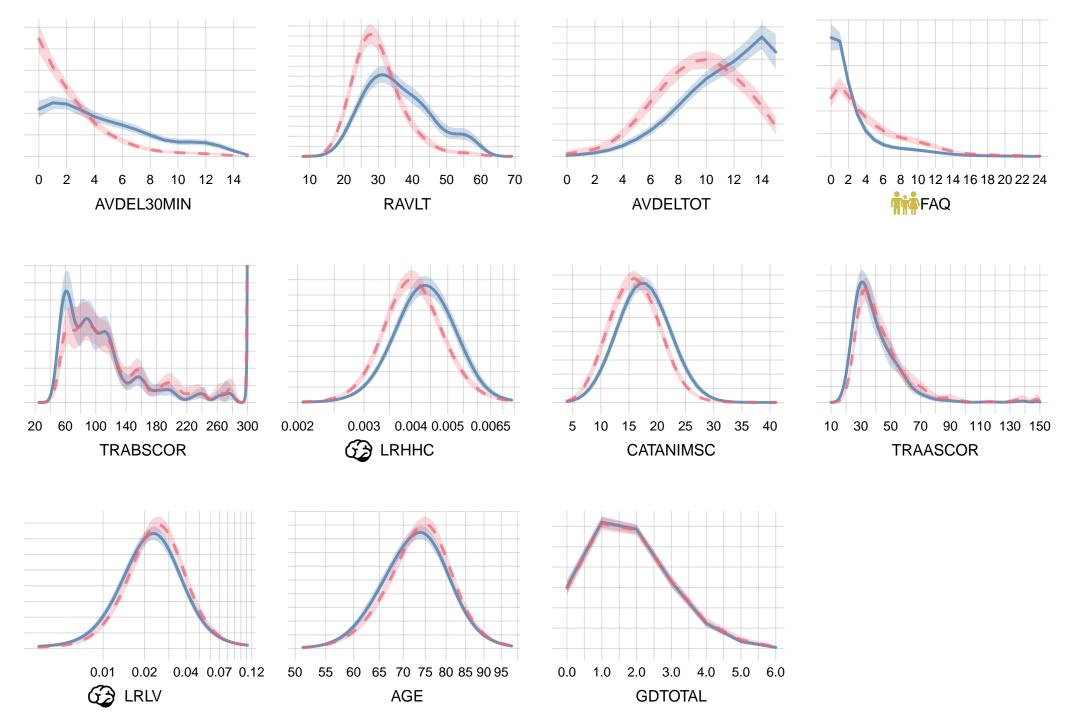




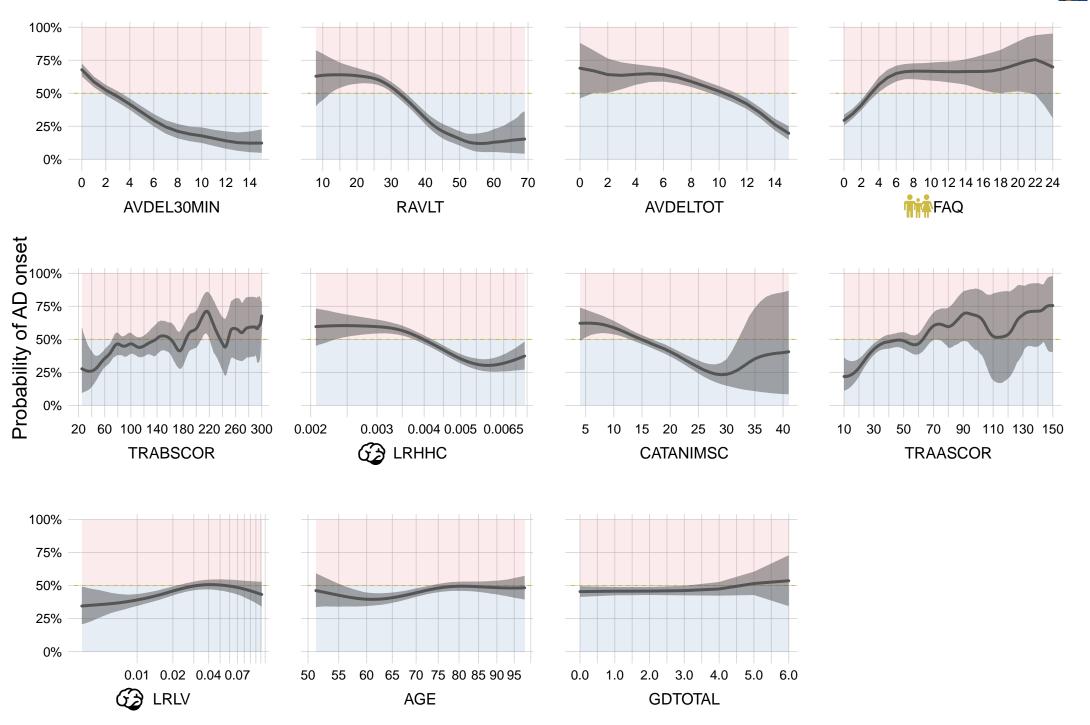


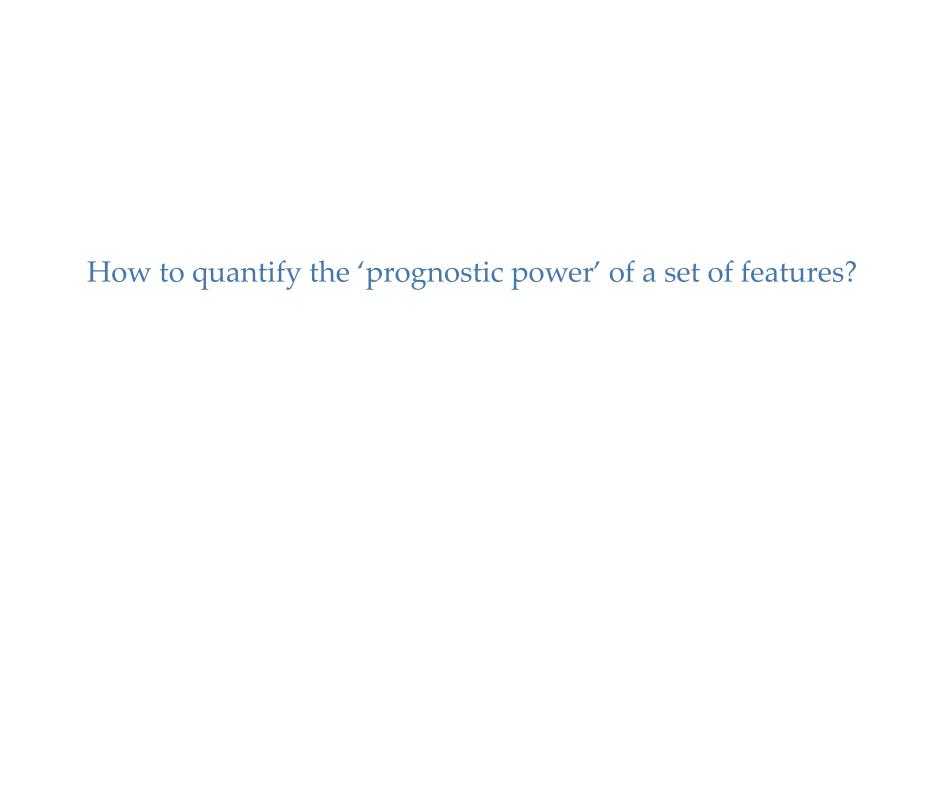












Prediction problem:

guess the six digits of the winning lottery ticket ??????

Clue A:

Clue B: **///?/?**

Clue C: ???///

What is the 'importance' or 'predictive power' of each clue?

Scenario 1: we can use **only one** clue

Clue **A**: **////**??

Clue **B**: **///?/?**

Clue **C**: ???**/**//

increasing importance

Best: **A** or **B** (each gives 1/81 winning chance)

Worst: **C** (gives 1/729 winning chance)

Scenario 2: we can use **all** clues

Clue **A**: **////??**

Clue **B**: **///?/?**

Clue **C**: ???///

→ We fully know the winning number!

Scenario 2: what happens if we **discard** clues?

Clue **A**: **////??**

Clue **B**: **///?/?**

Scenario 2: what happens if we discard clues?

Clue **A**: **////?**?

Clue **B**: **///?/?**

Clue **C**: ???**///**

• Discard A: still 100% win \Rightarrow A has 'importance = 0'

Clue **B**: **///?/?**

- Discard A: still 100% win \Rightarrow A has 'importance = 0'
- Discard **B**: still 100% win \Rightarrow **B** has 'importance = **0**'

Clue **B**: **///?/?**

- Discard A: still 100% win \Rightarrow A has 'importance = 0'
- Discard **B**: still 100% win \Rightarrow **B** has 'importance = **0**'
 - Discard **A** and **B**: 1/9 winning chance

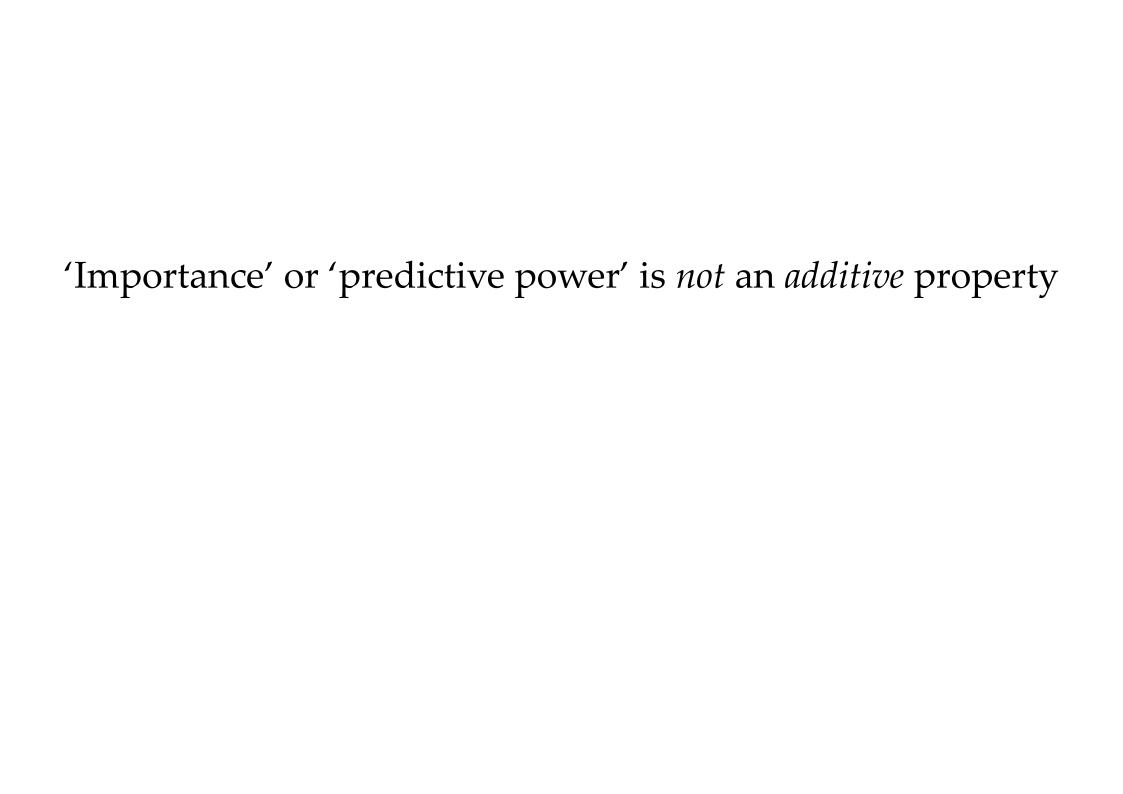
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 - \Rightarrow A and B together have 'importance > 0'

Clue **B**: **///?/?**

- Discard A: still 100% win \Rightarrow A has 'importance = 0'
- Discard **B**: still 100% win \Rightarrow **B** has 'importance = **0**'
 - Discard **A** and **B**: 1/9 winning chance
 - \Rightarrow A and B together have 'importance>0'

$$'0 + 0 \neq 0'$$



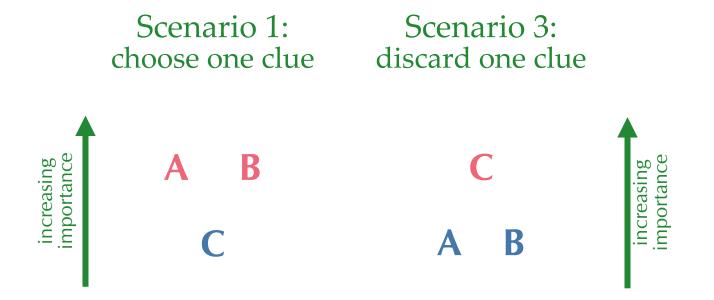
Clue **B**: **///?/?**

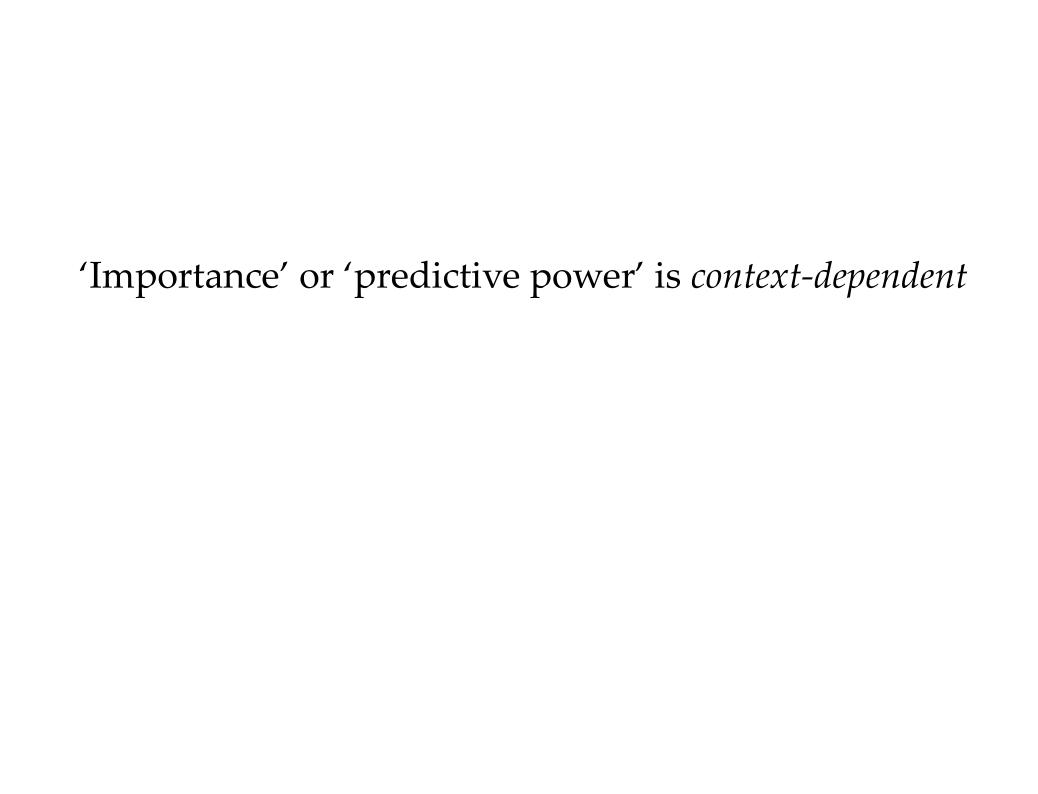
Clue **C**: ???**/**//

increasing importance

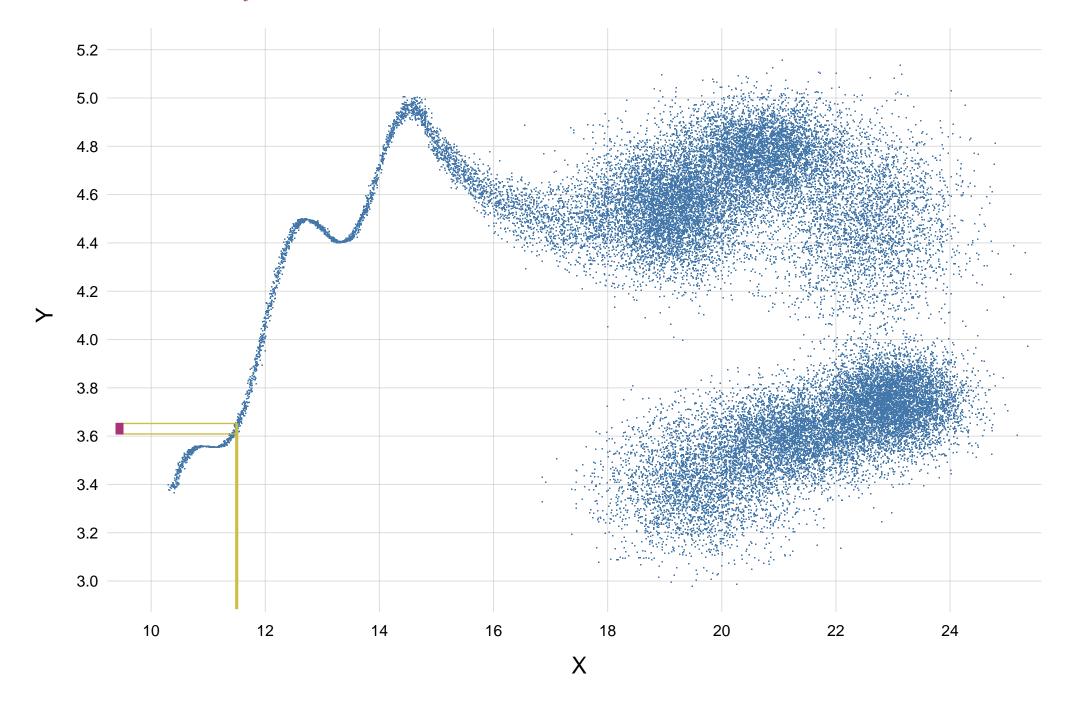
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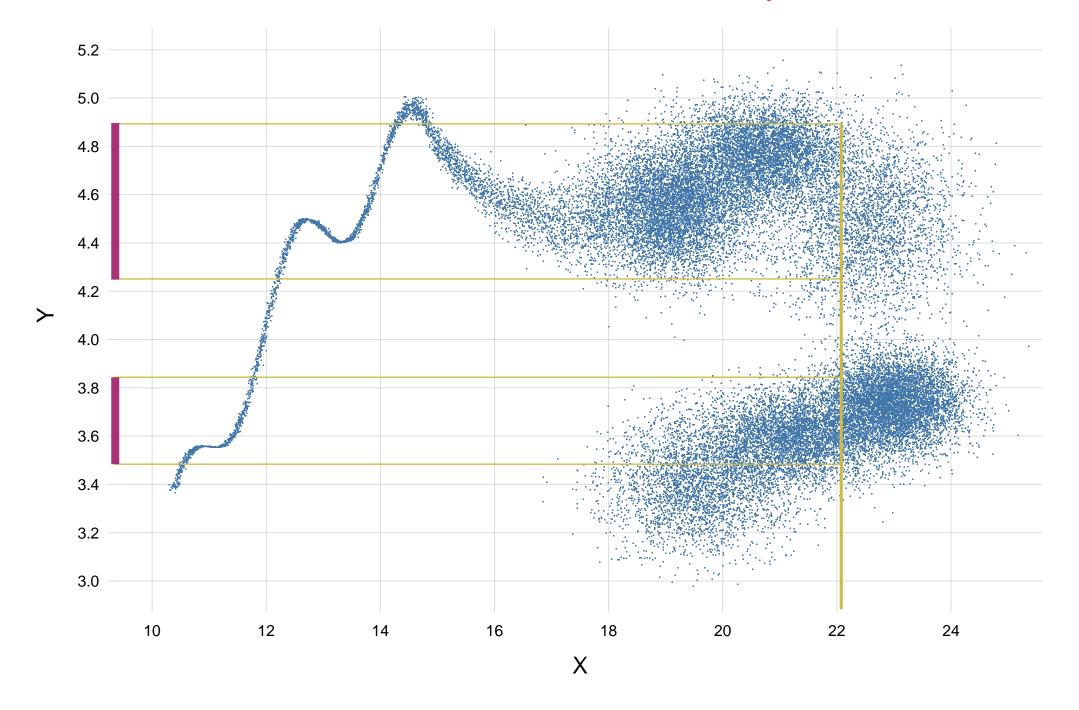
 \rightarrow If we have to discard one clue, it's most important that we keep \mathbf{C}



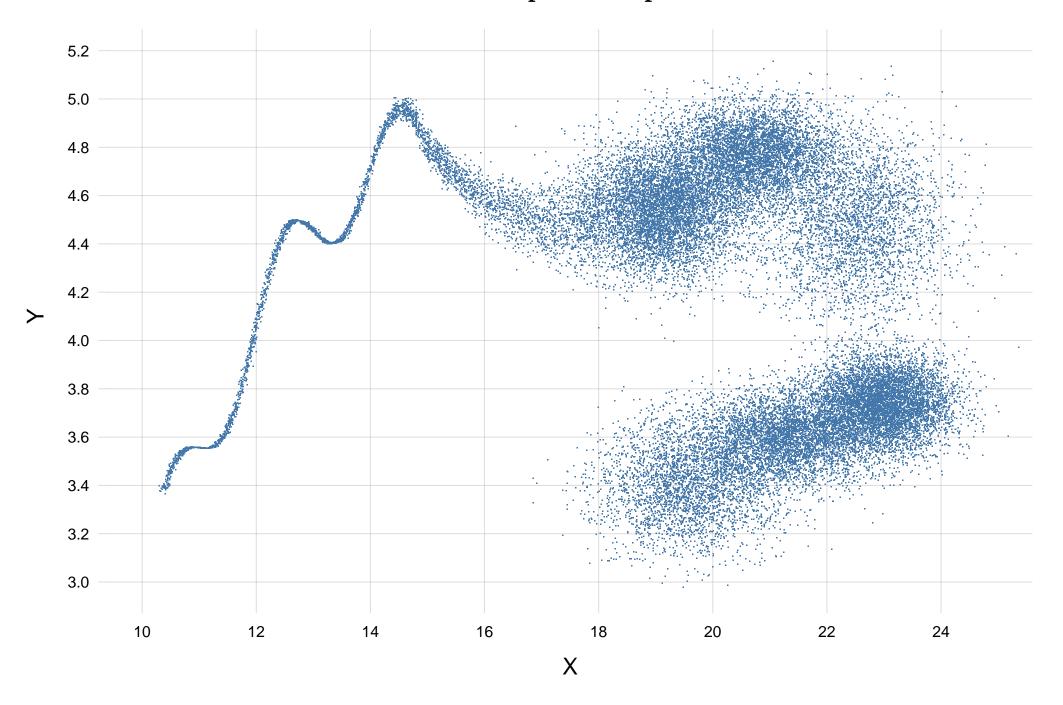


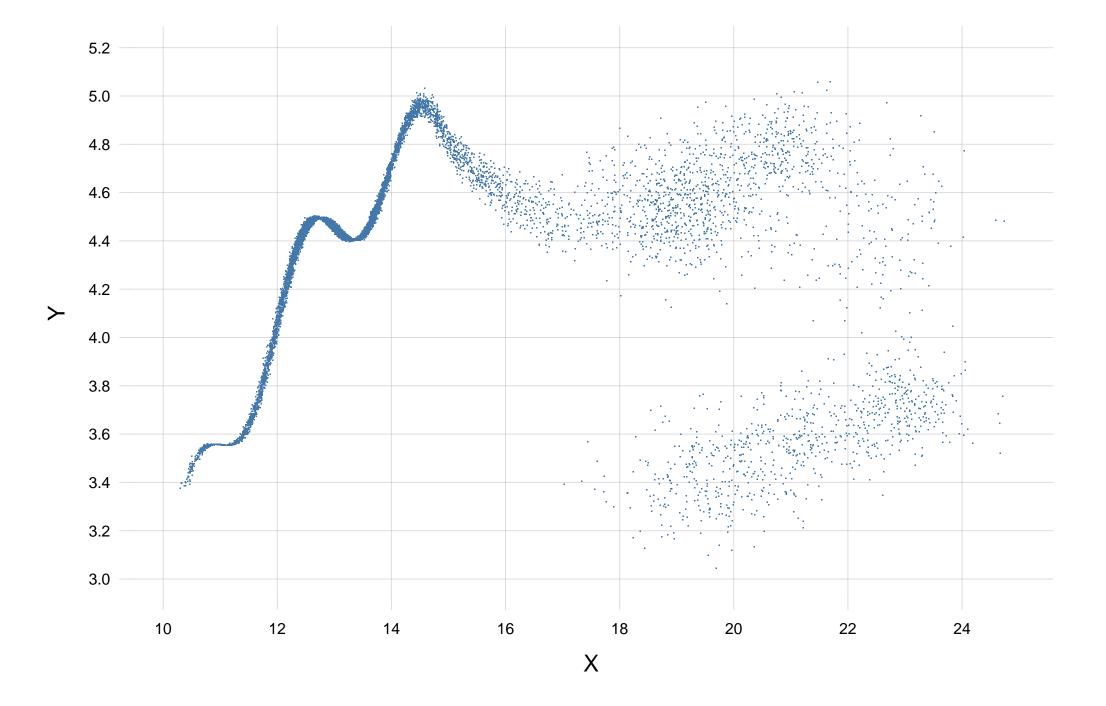
 $x = 11.5 \implies y \approx 3.60 - 3.65$

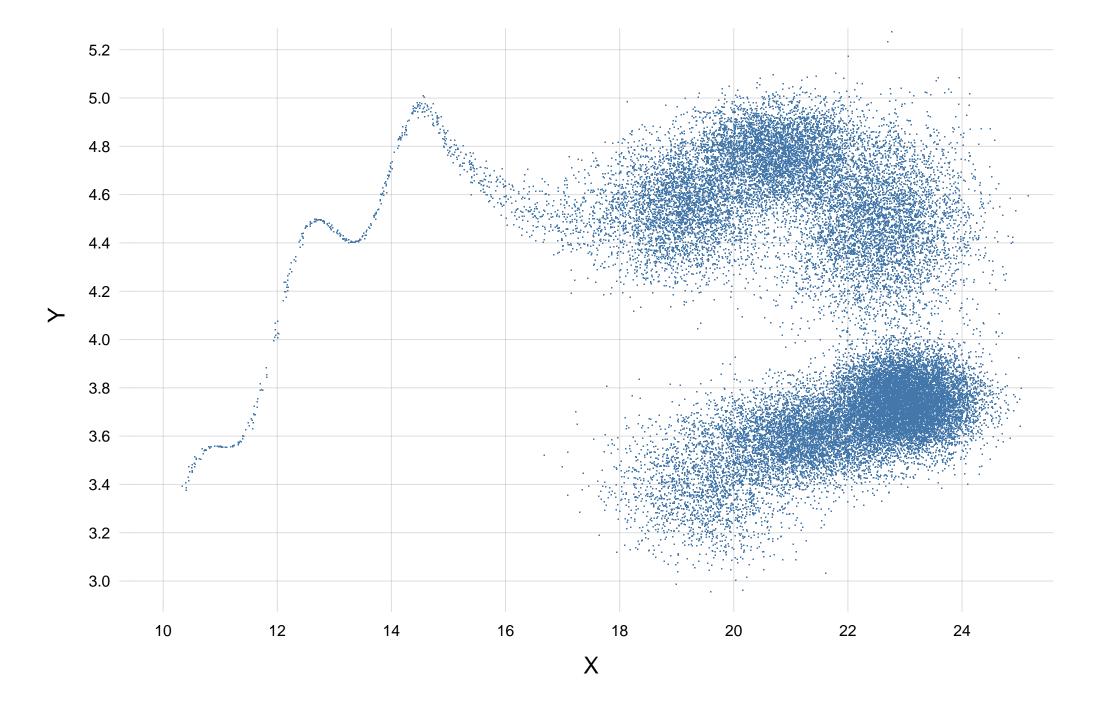




What is the 'overall predictive power' of X?







The 'predictive power' of X depends on P(X)

△ Careful with 'balancing'! △

Information Theory

The Bell System Technical Journal

Vol. XXVII July, 1948 No. 3

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By C. E. SHANNON

Introduction

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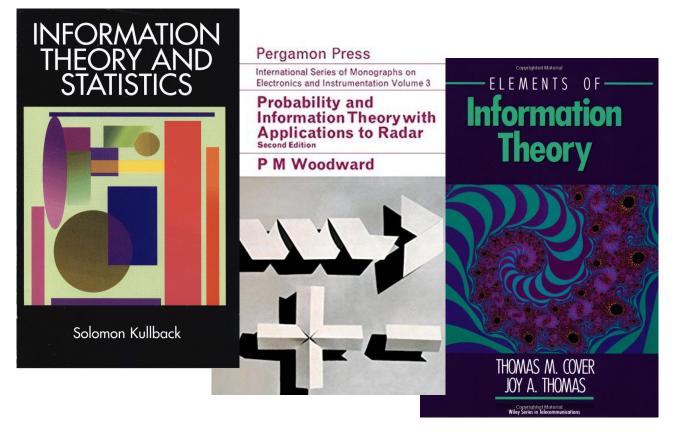
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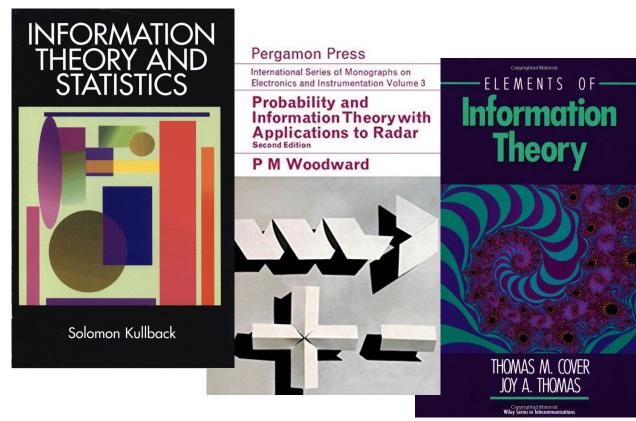
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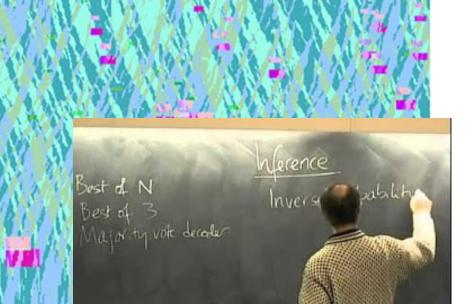
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David J.C. MacKay

Information Theory, Inference, and Learning Algorithms https://www.inference.org.uk/itila/book.html https://youtube.com/playlist?list=PLruBu5BI5n4aFpG32iMbdWoRVAA-Vcso6



Cambridge University Press, 2003

'predictive power' of X for Y := Mutual information between <math>Y and X (mean transinformation content)

$$I(X;Y) := \int p(y|x) p(x) \log \left[\frac{p(y|x)}{p(y)} \right] dy dx$$

$$I(Y; X_1, X_2) \geq I(Y; X_1)$$

$$I(Y; X_1, X_2) \geq I(Y; X_2)$$

but
$$I(Y; X_1, X_2) \neq I(Y; X_1) + I(Y; X_2)$$



Edition 1.0 2008-03

INTERNATIONAL STANDARD

NORME INTERNATIONALE

Quantities and units -

Part 13: Information science and technology

Grandeurs et unités -

Partie 13: Science et technologies de l'information





Edition 1.0 2008-03

INTERNATIONAL STANDARD

INFORMATION SCIENCE AND TECHNOLOGY				QUANTITIES	
Item No.	Name	Symbol	Definition	Remarks	
13-24 (<i>902</i>)	information content fr quantité (f) d'information	I(x)	$I(x) = \operatorname{lb} \frac{1}{p(x)} \operatorname{Sh} = \operatorname{lg} \frac{1}{p(x)} \operatorname{Hart} =$ $\operatorname{ln} \frac{1}{p(x)} \operatorname{nat}$ where $p(x)$ is the probability of event x	See ISO/IEC 2382-16, item 16.03.02. See also IEC 60027-3.	
13-25 (<i>903</i>)	entropy fr entropie (f)	H	$H(X) = \sum_{i=1}^{n} p(x_i)I(x_i)$ for the set $X = \{x_1,, x_n\}$ where $p(x_i)$ is the probability and $I(x_i)$ is the information content of event x_i	See ISO/IEC 2382-16, item 16.03.03.	
13-30 (<i>908</i>)	joint information content fr quantité (f) d'information conjointe	I(x, y)	$I(x, y) = \operatorname{lb} \frac{1}{p(x, y)} \operatorname{Sh} = \operatorname{lg} \frac{1}{p(x, y)} \operatorname{Hart} =$ $\operatorname{ln} \frac{1}{p(x, y)} \operatorname{nat}$ where $p(x, y)$ is the joint probability of events x and y		
13-35 (<i>912</i>)	transinformation content fr transinformation (f)	T(x,y)	T(x,y) = I(x) + I(y) - I(x,y) where $I(x)$ and $I(y)$ are the information contents (13-24) of events x and y , respectively, and $I(x,y)$ is their joint information content (13-30)	See ISO/IEC 2382-16, item 16.04.07.	
13-36 (<i>913</i>)	mean transinformation content fr transinformation (f) moyenne	T	$T(X,Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} p(x_i, y_j) T(x_i, y_j)$ for the sets $X = \{x_1,, x_n\}, Y = \{y_1,, y_m\},$ where $p(x_i, y_j)$ is the joint probability of events x_i and y_j , and $T(x_i, y_j)$ is their transinformation content (item 13-35)	See ISO/IEC 2382-16, item 16.04.08.	

UNITS INFORMATION SCIENCE AND TECHNOLOGY				
Item No.	Name	Symbol	Definition	Conversion factors and remarks
13-24.a	shannon	Sh	value of the quantity when the argument is equal to 2	1 Sh ≈ 0,693 nat ≈ 0,301 Hart
13-24.b	hartley	Hart	value of the quantity when the argument is equal to 10	1 Hart ≈ 3,322 Sh ≈ 2,303 nat
13-24.c	natural unit of information	nat	value of the quantity when the argument is equal to e	1 nat ≈ 1,433 Sh ≈ 0,434 Hart
13-25.a	shannon	Sh		
13-25.b	hartley	Hart		
13-25.c	natural unit of information	nat		
13-30.a	shannon	Sh		
13-30.b	hartley	Hart		
13-30.c	natural unit of information	nat		
13-35.a	shannon	Sh		
13-35.b	hartley	Hart		
13-35.c	natural unit of information	nat		
13-36.a	shannon	Sh		In practice, the unit "shannon per character" is generally used, and
13-36.b	hartley	Hart		sometimes the units "hartley per character" and "natural unit per character".
13-36.c	natural unit of information	nat		

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In N=100 new prognoses:

- we are **completely certain** about 22
- we are **completely uncertain** about 100-22 = 78
- \rightarrow approx 22+78/2 = 61 correct prognoses (TP+TN)

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 ± 0.8 $\sqrt{N}\%$ correct prognoses

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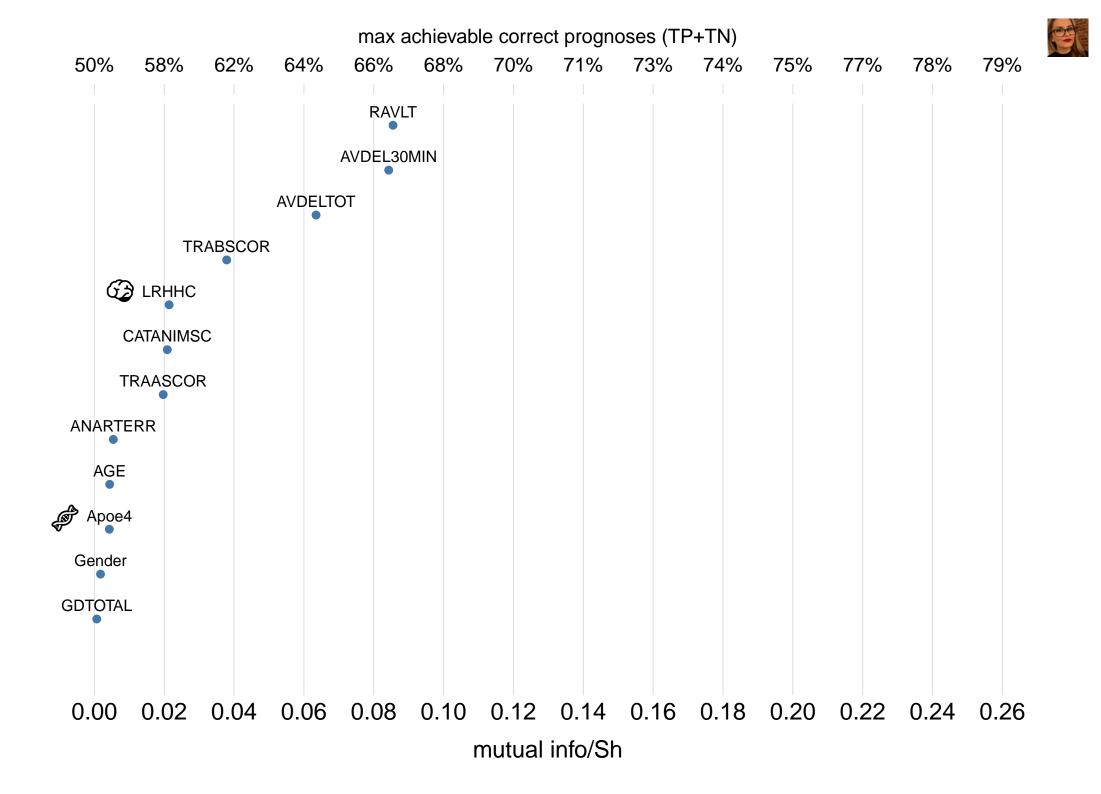
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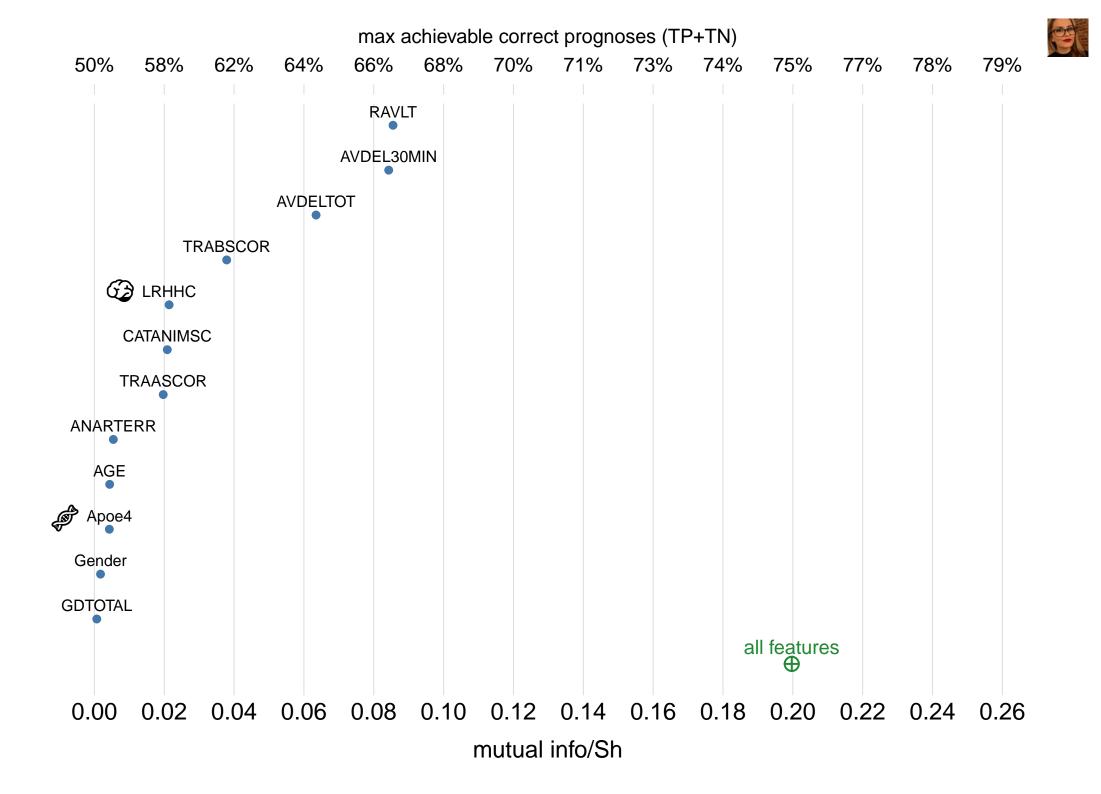
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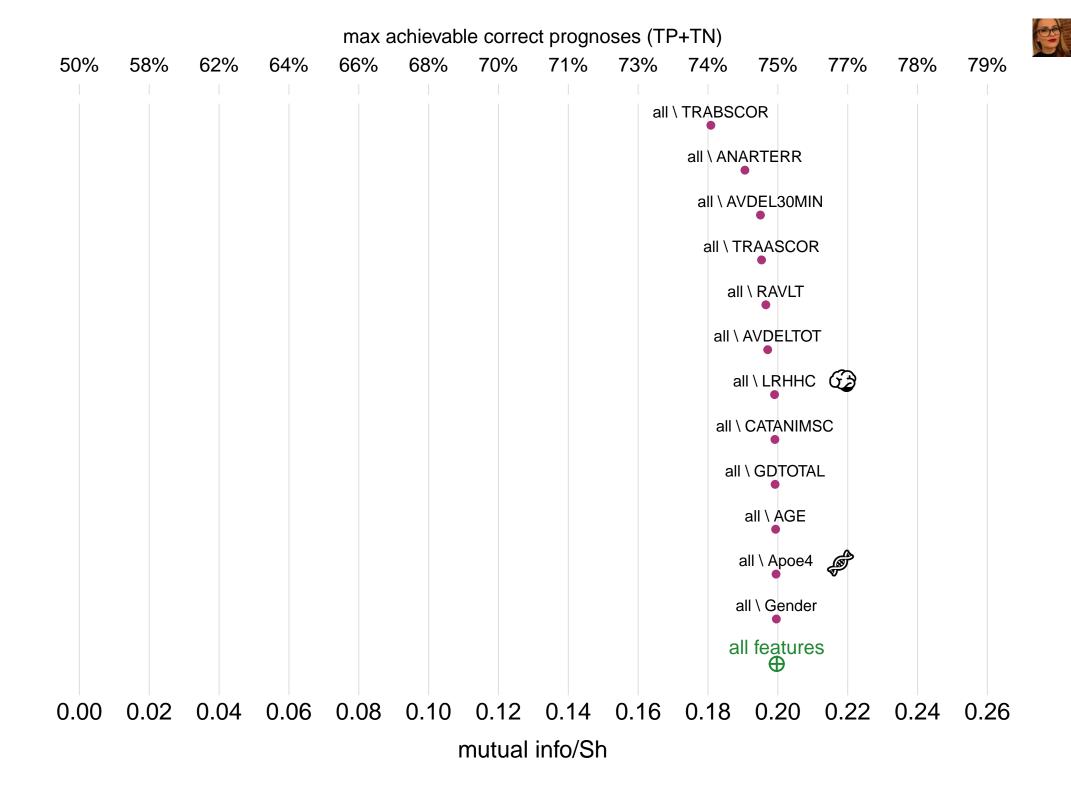
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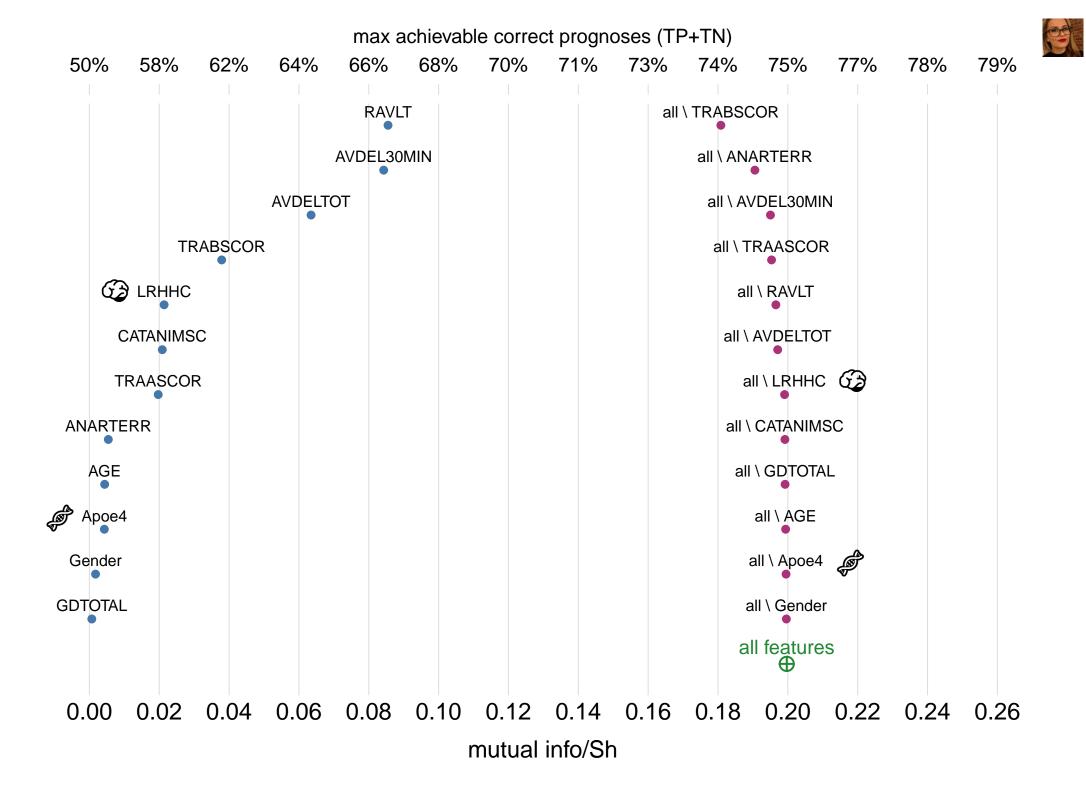
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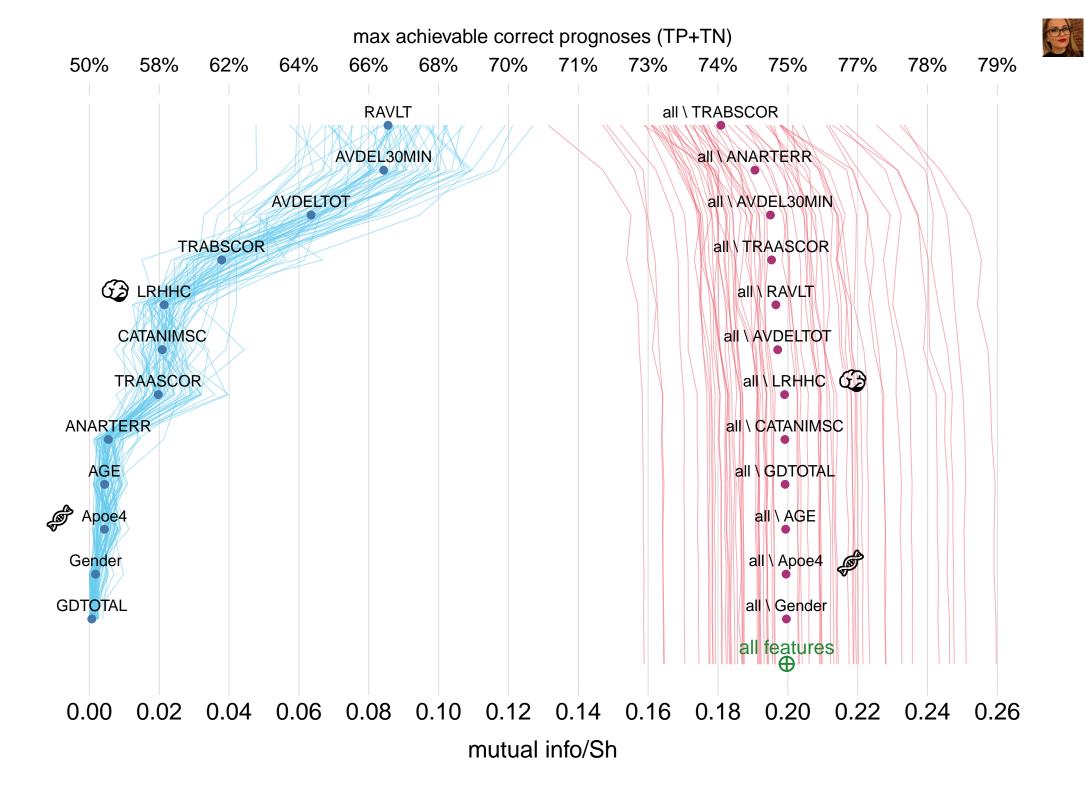
Maximum accuracy attainable by *any* algorithm which uses only feature set *X*

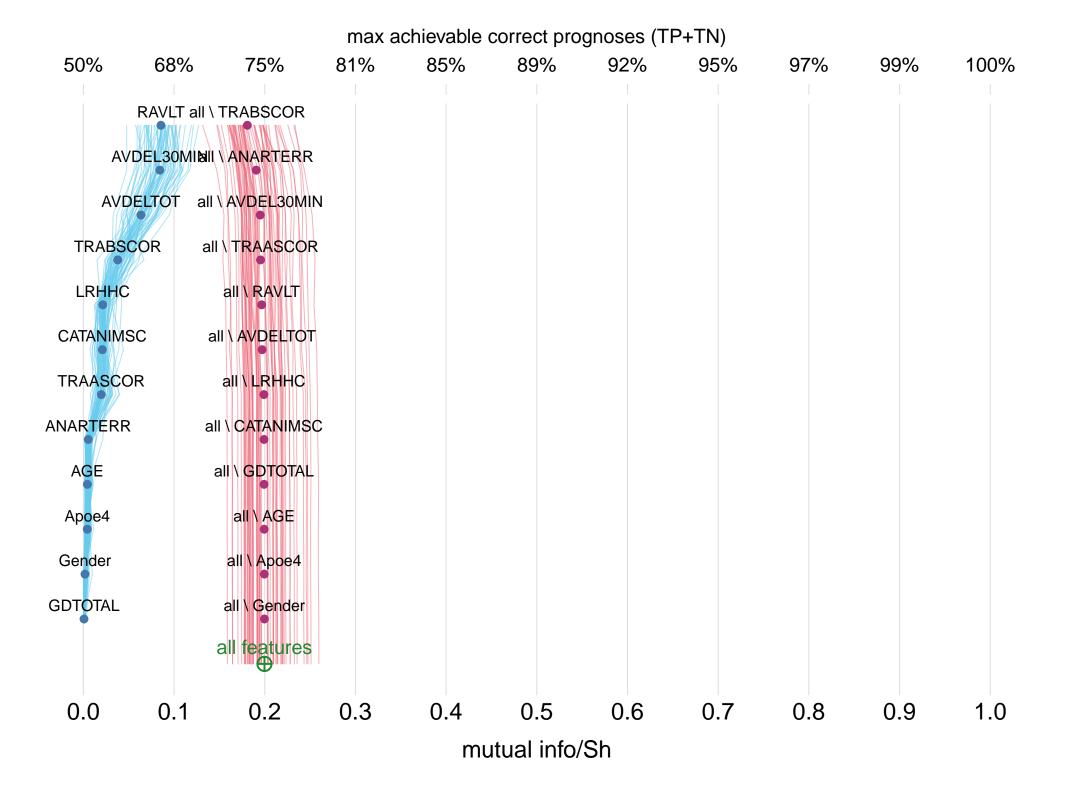


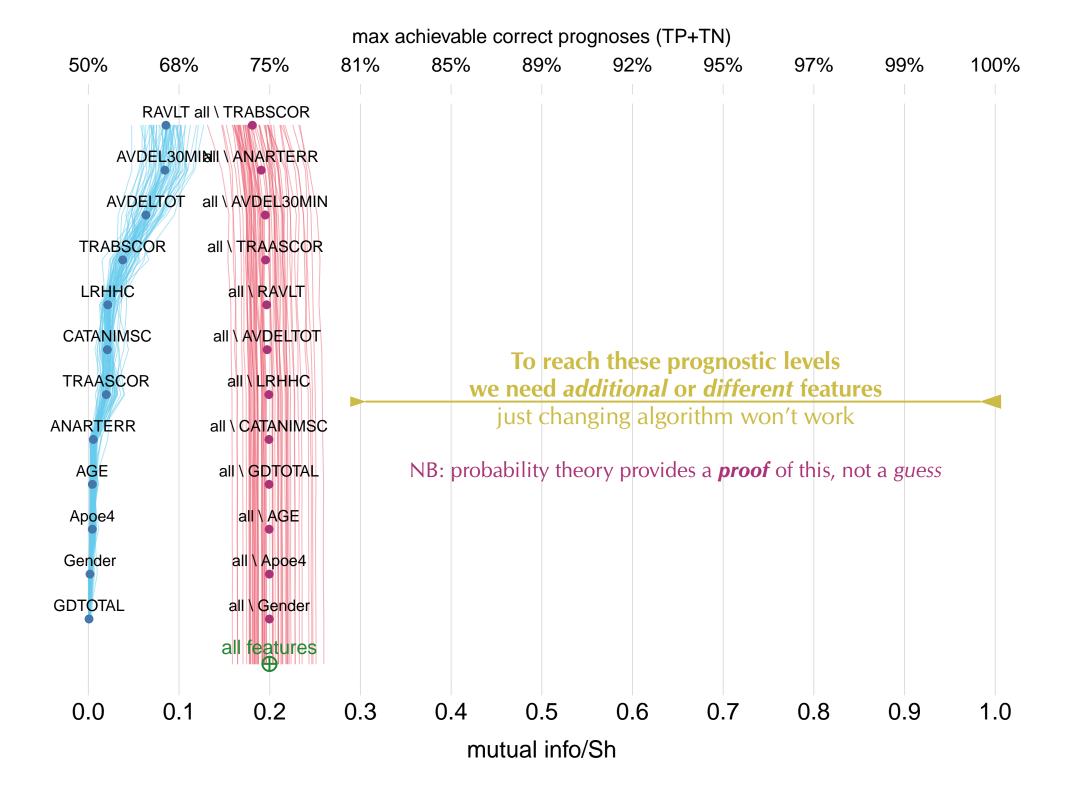


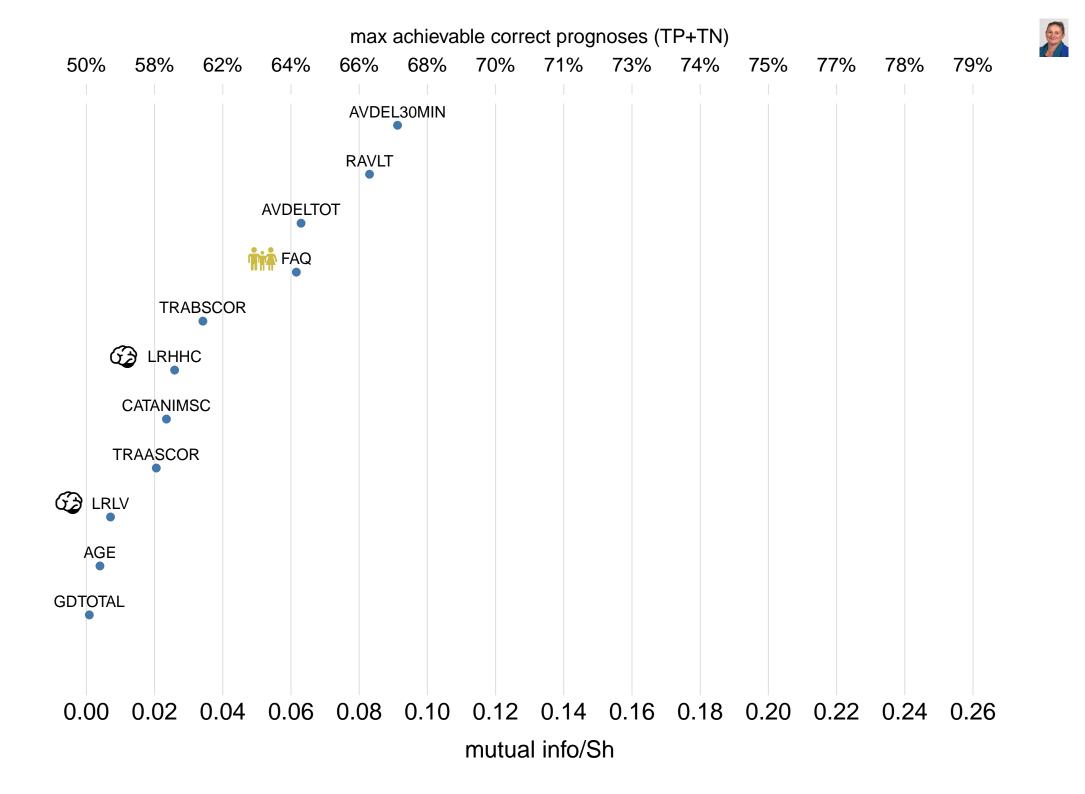


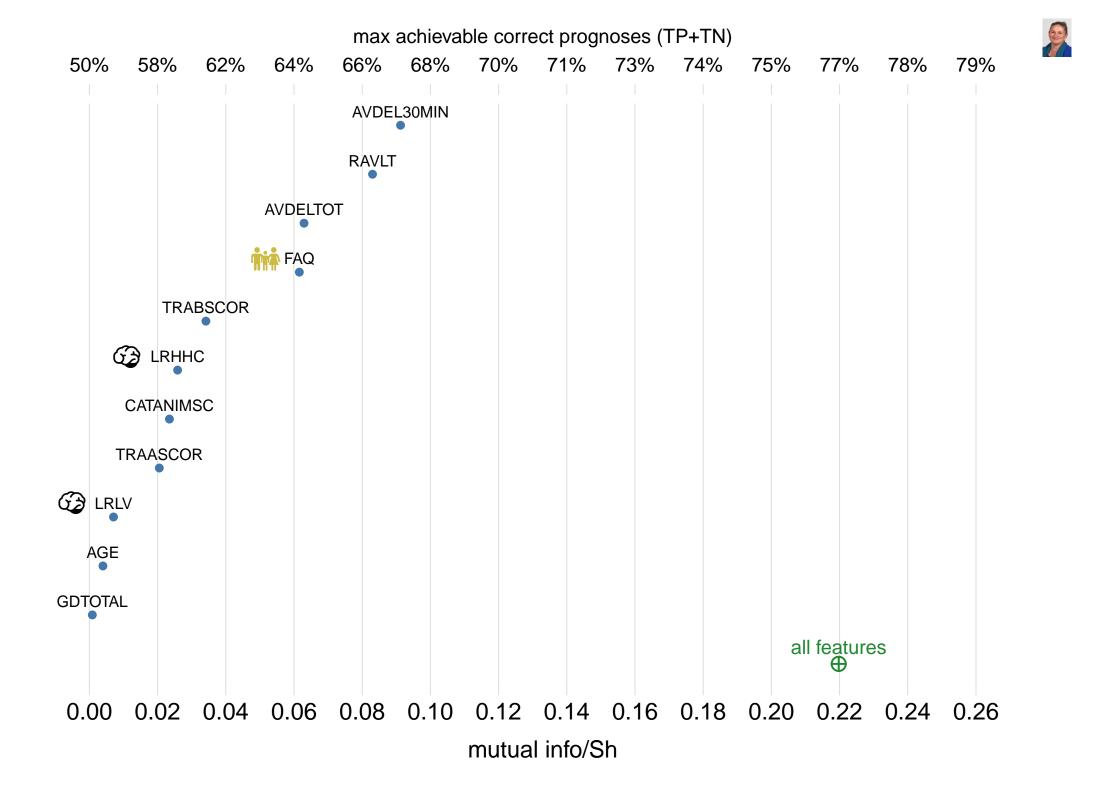


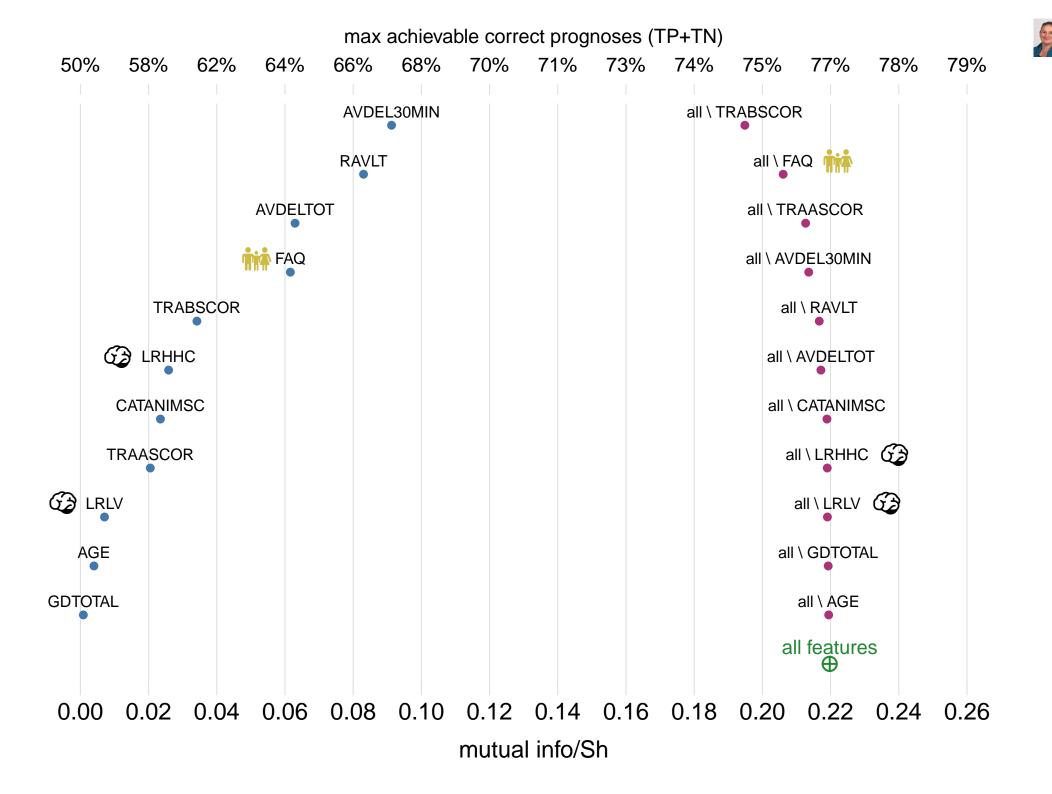


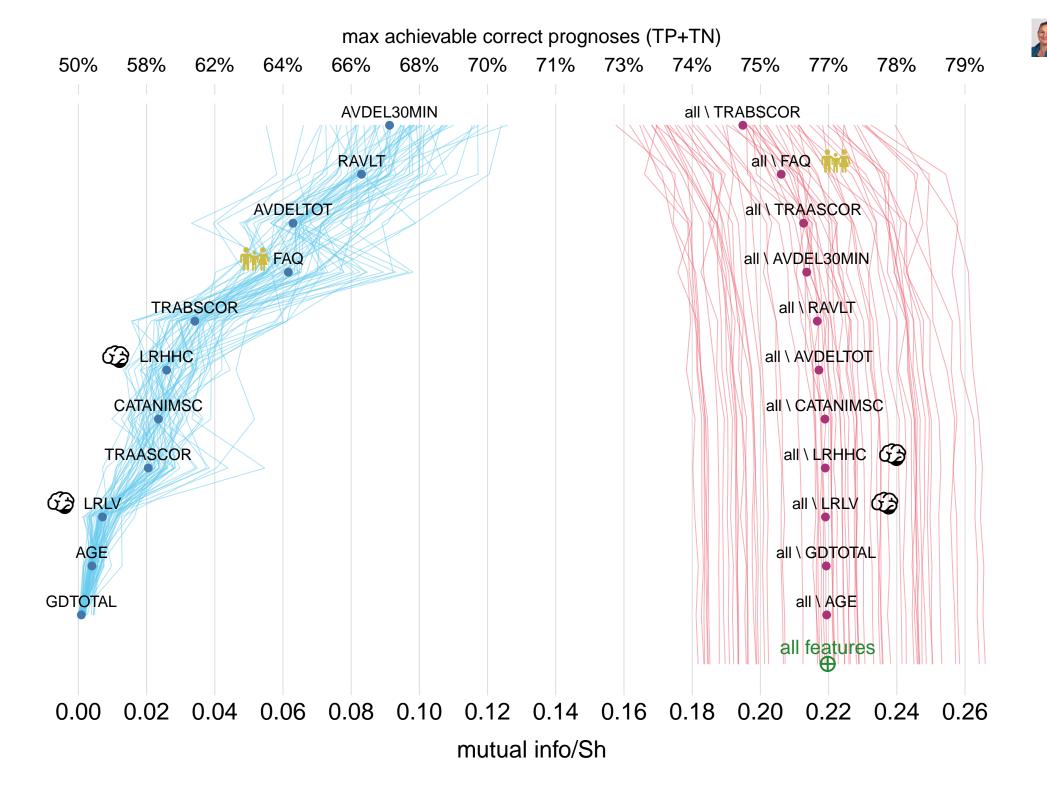














Alexandra's results

