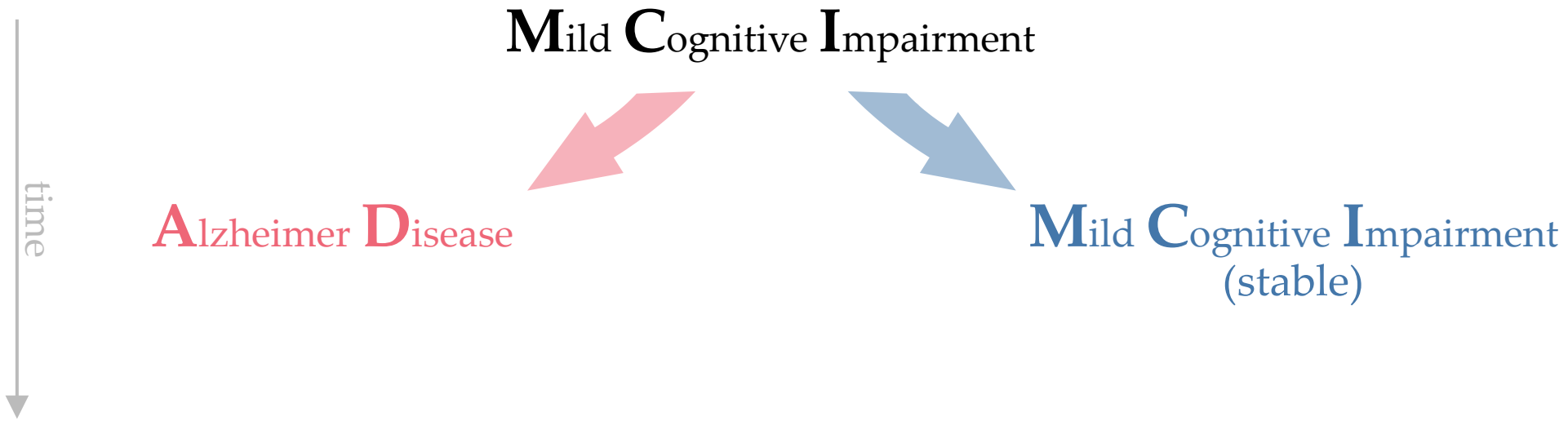
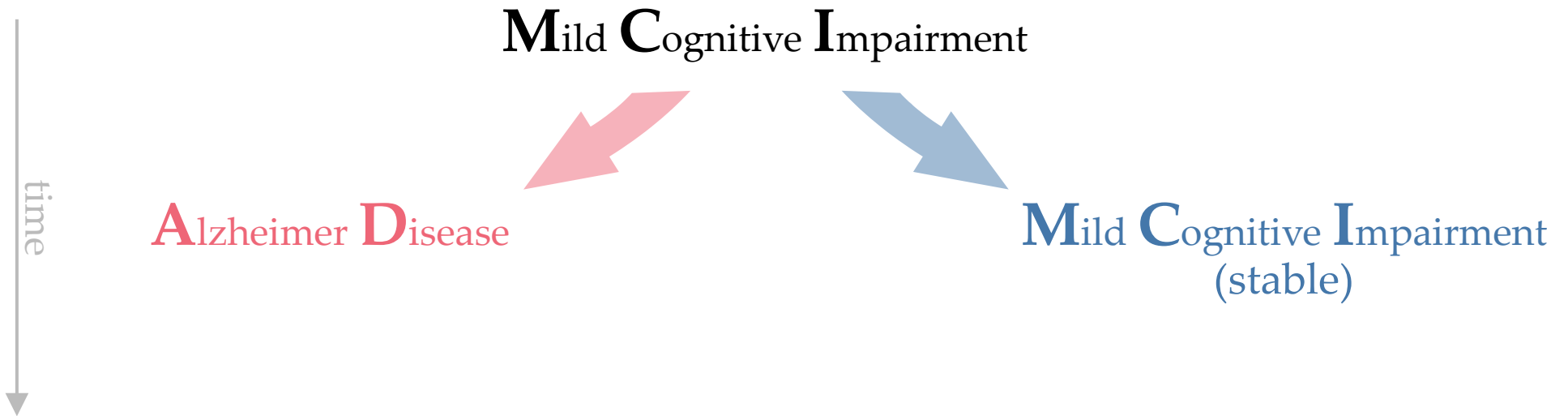




Mild Cognitive Impairment





♂ Gender

♂ AGE

🕒 ANARTERR

🕒 GDTOTAL

🕒 RAVLT

🕒 CATANIMSC

🕒 TRABSCOR

🕒 AVDELTOT

🕒 TRAASCOR

🕒 AVDEL30MIN

🧠 LRHHC

🧠 LRLV

🧬 Apoe4

👤 FAQ

♂ Gender

♂ AGE

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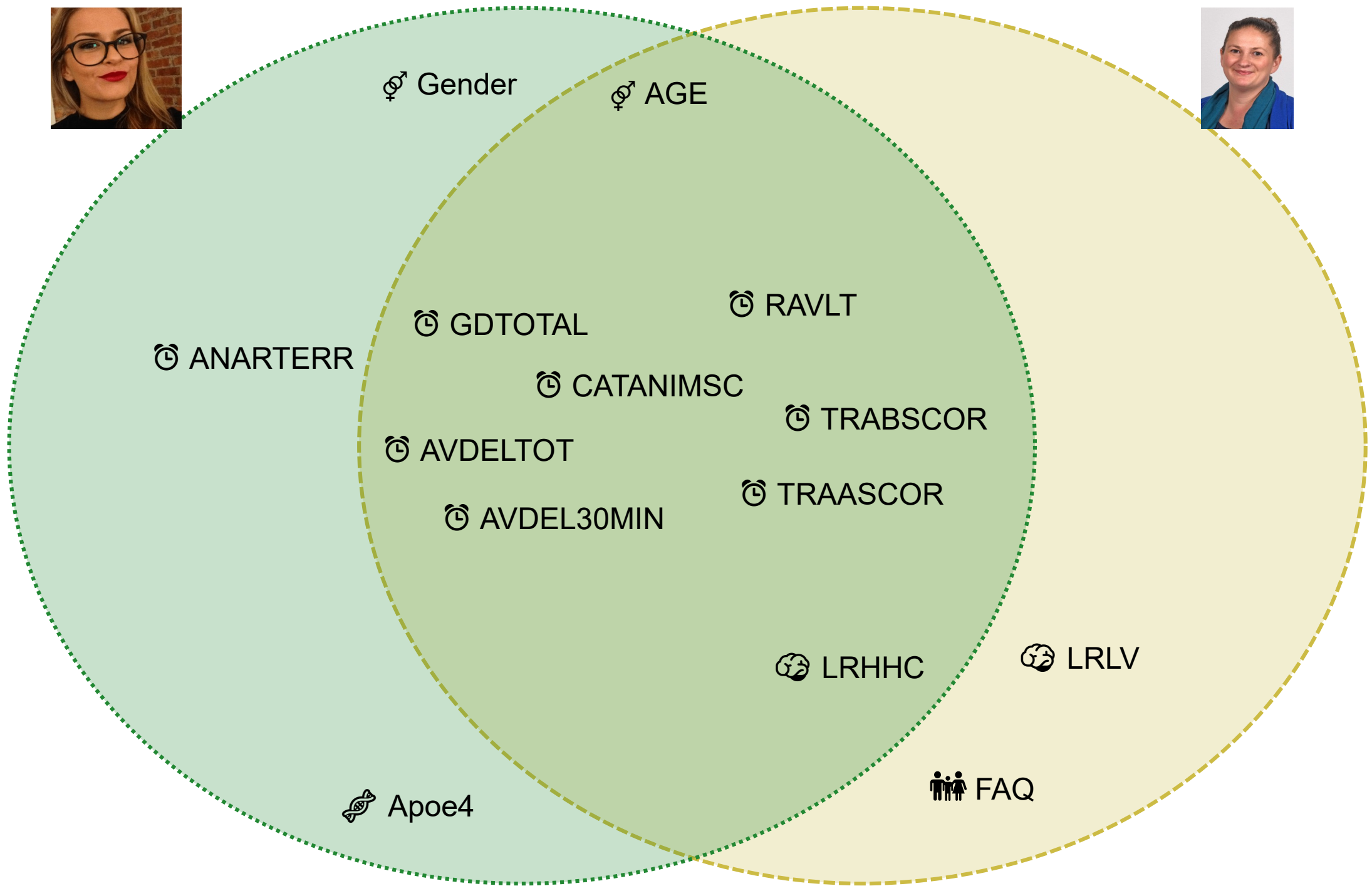
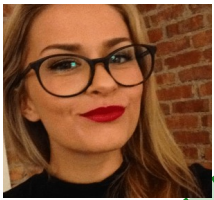
🧠 LRHHC

🧠 LRLV

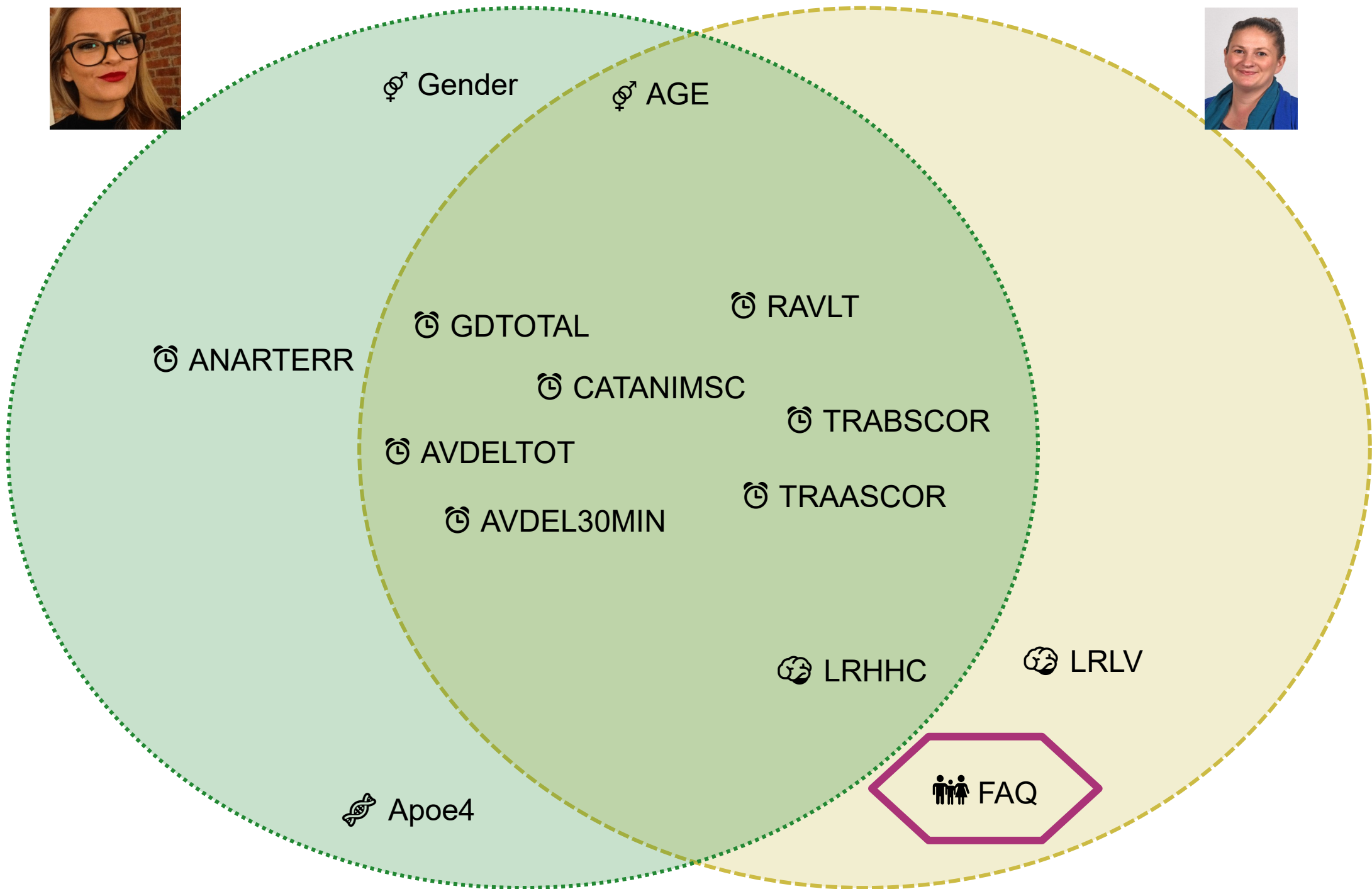
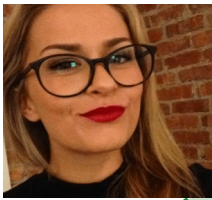
🧬 Apoe4

👤 FAQ

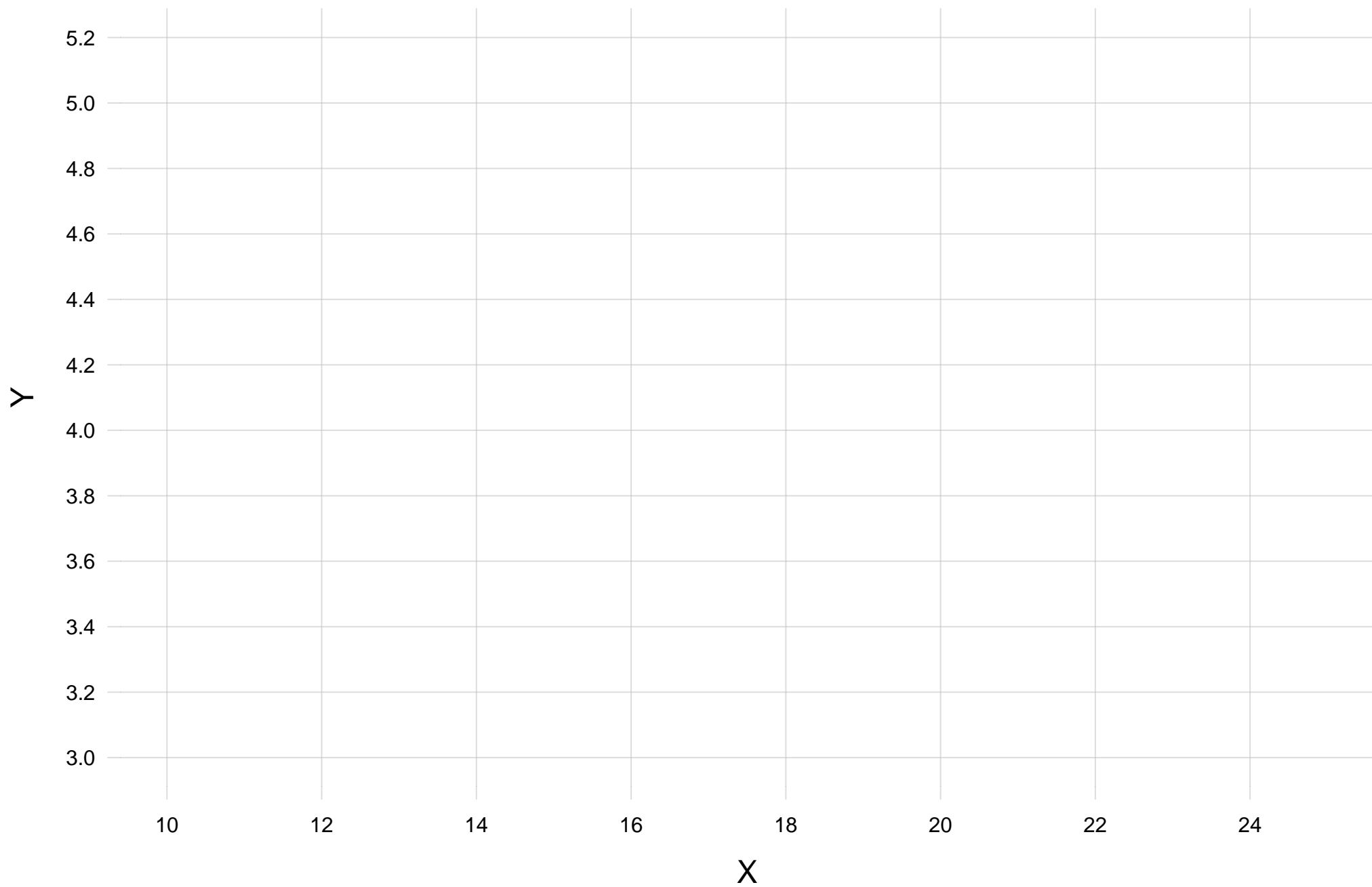
How 'good' are these features at prognosing the later onset of Alzheimer?

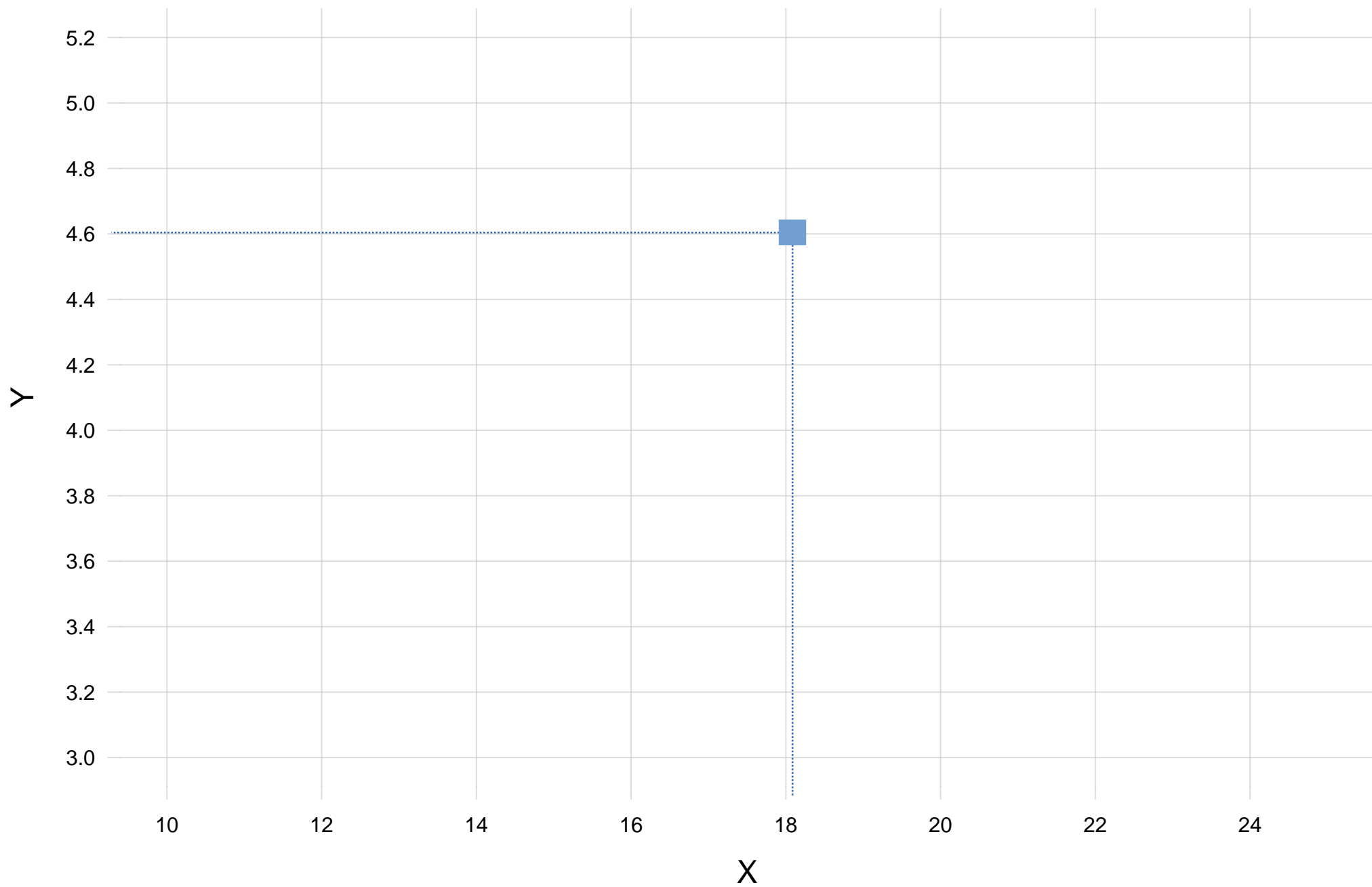


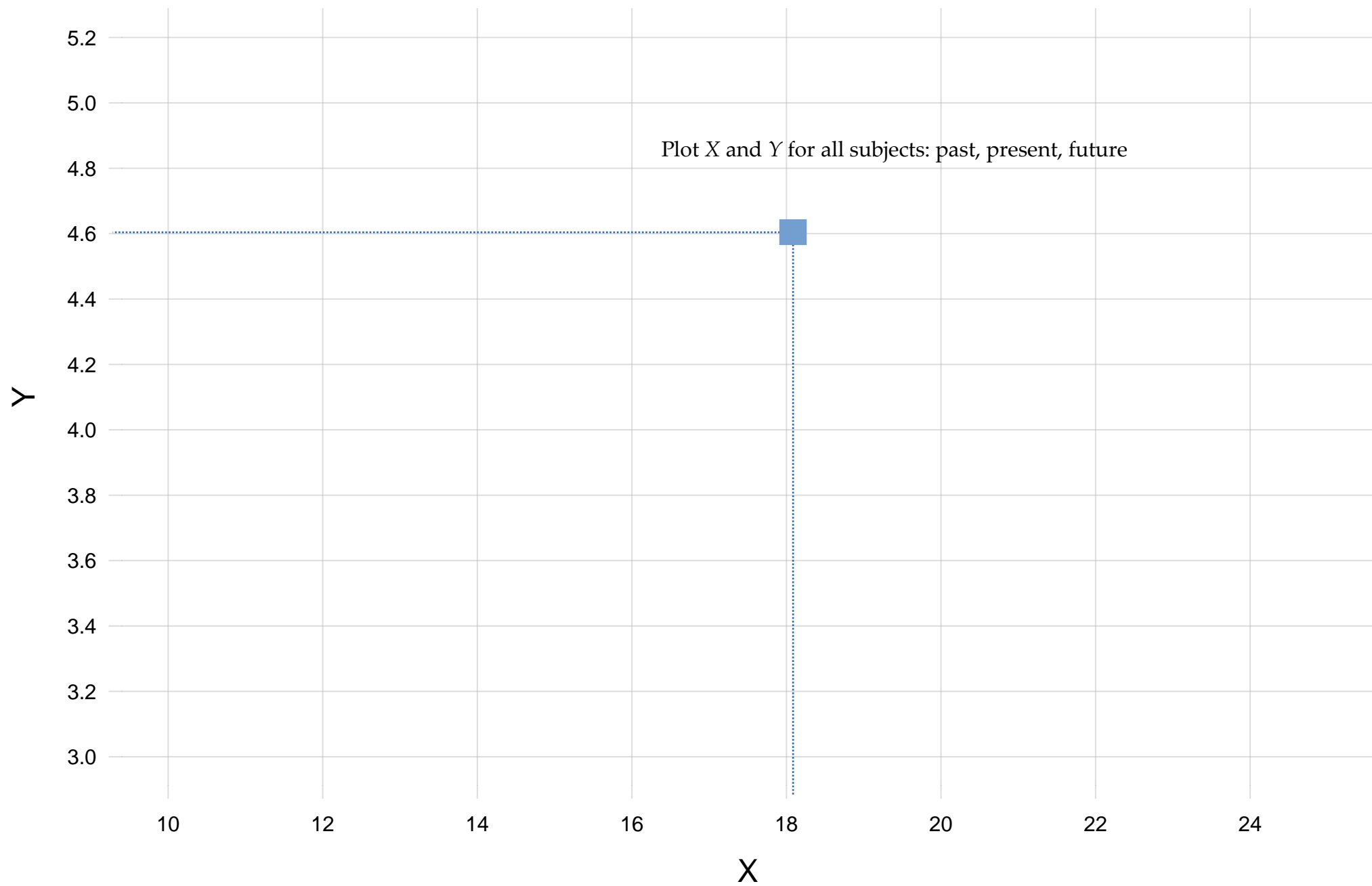
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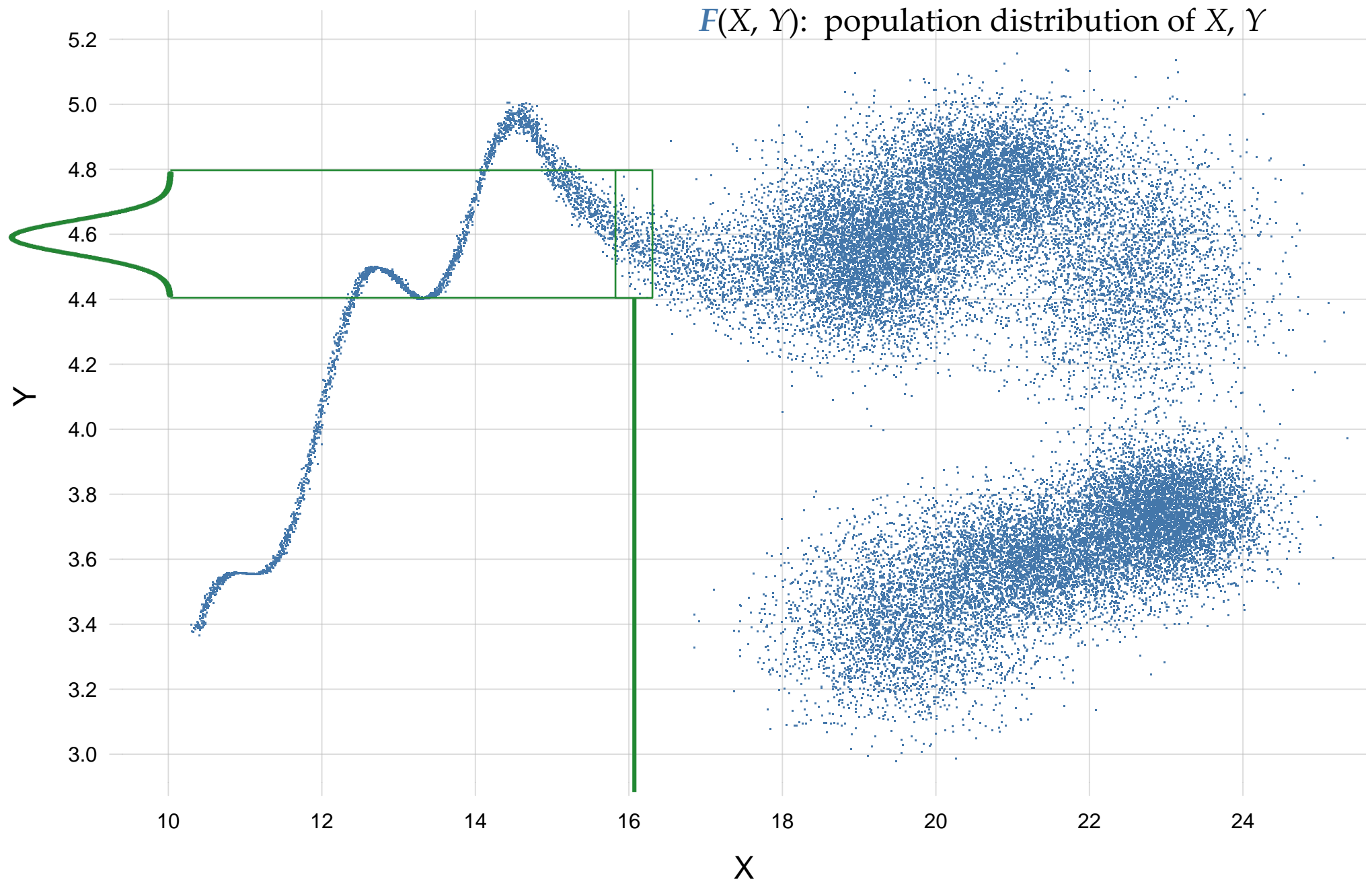


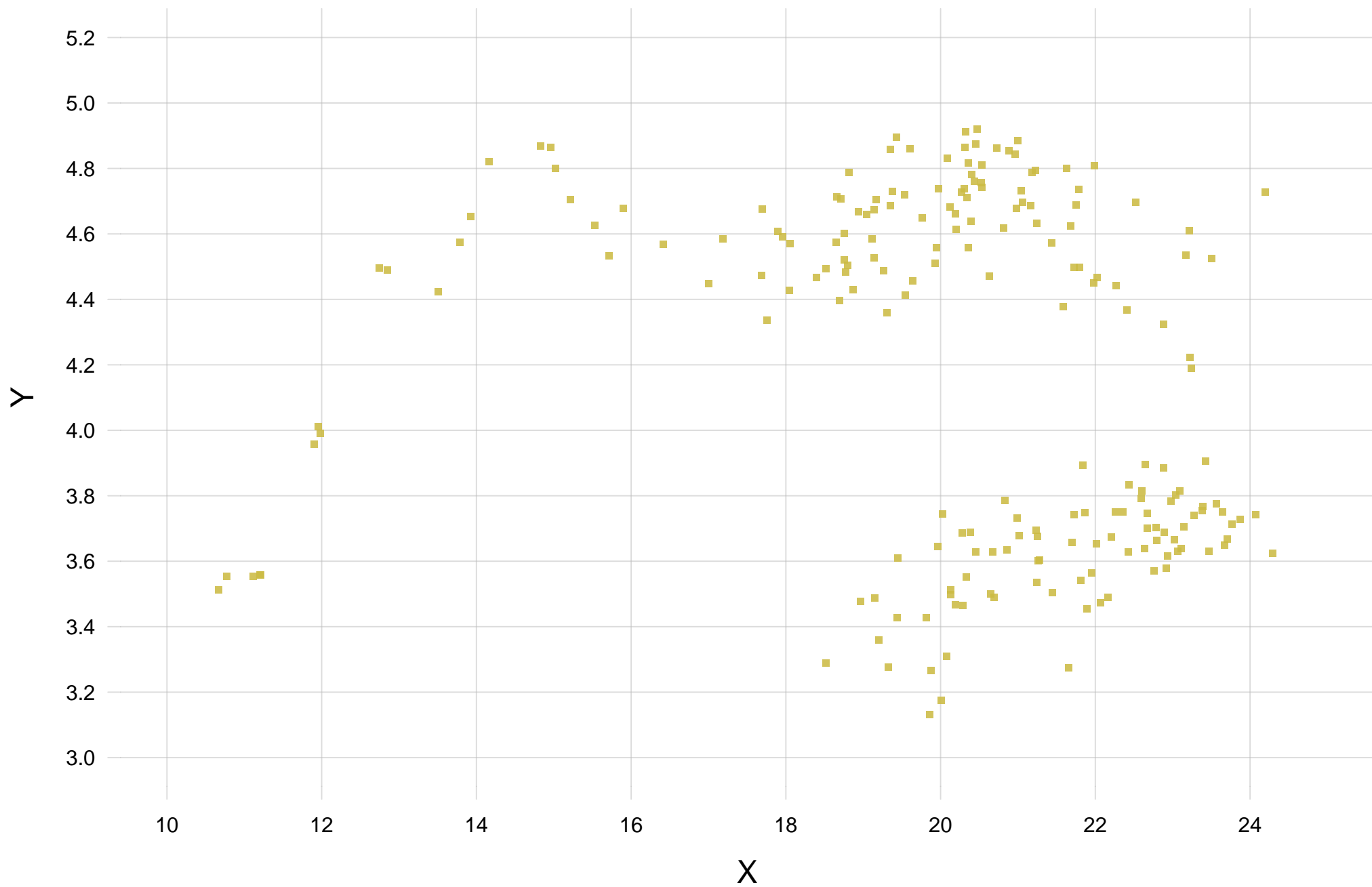




New patient: $X = 16$

$\Rightarrow Y \approx 4.5-4.7$





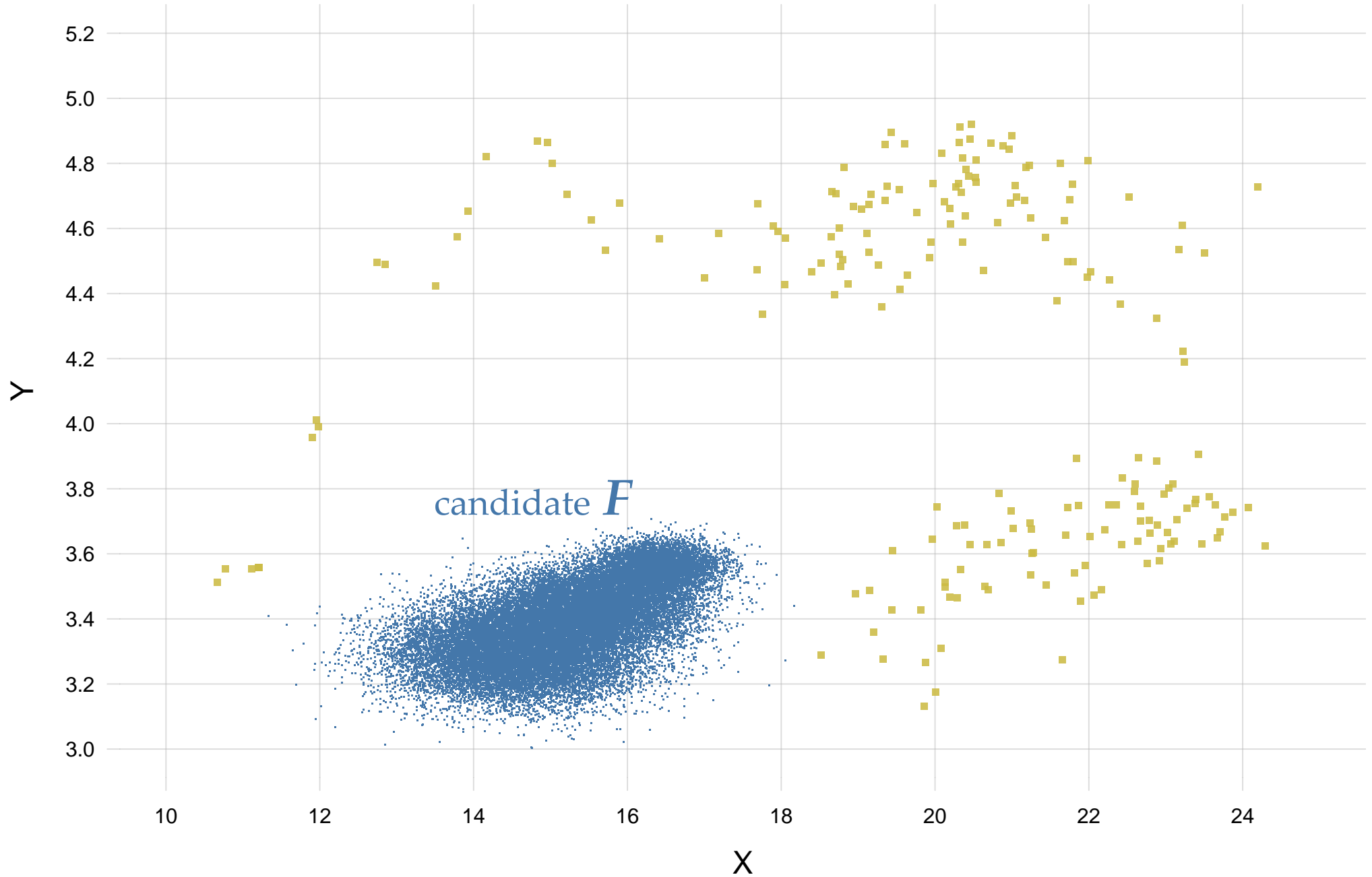
$$P(y \mid x) = \int F(y \mid x) \, p(F \mid \text{data}) \, dF$$

probability = average over all possible population distributions

$$p(F \mid \text{data}) \propto \underbrace{F(y_1, x_1) \times F(y_2, x_2) \times F(y_3, x_3) \times \dots}_{\text{how well the candidate distribution fits the data}} \times \underbrace{p(F \mid \text{prior info})}_{\text{extra-data knowledge}}$$

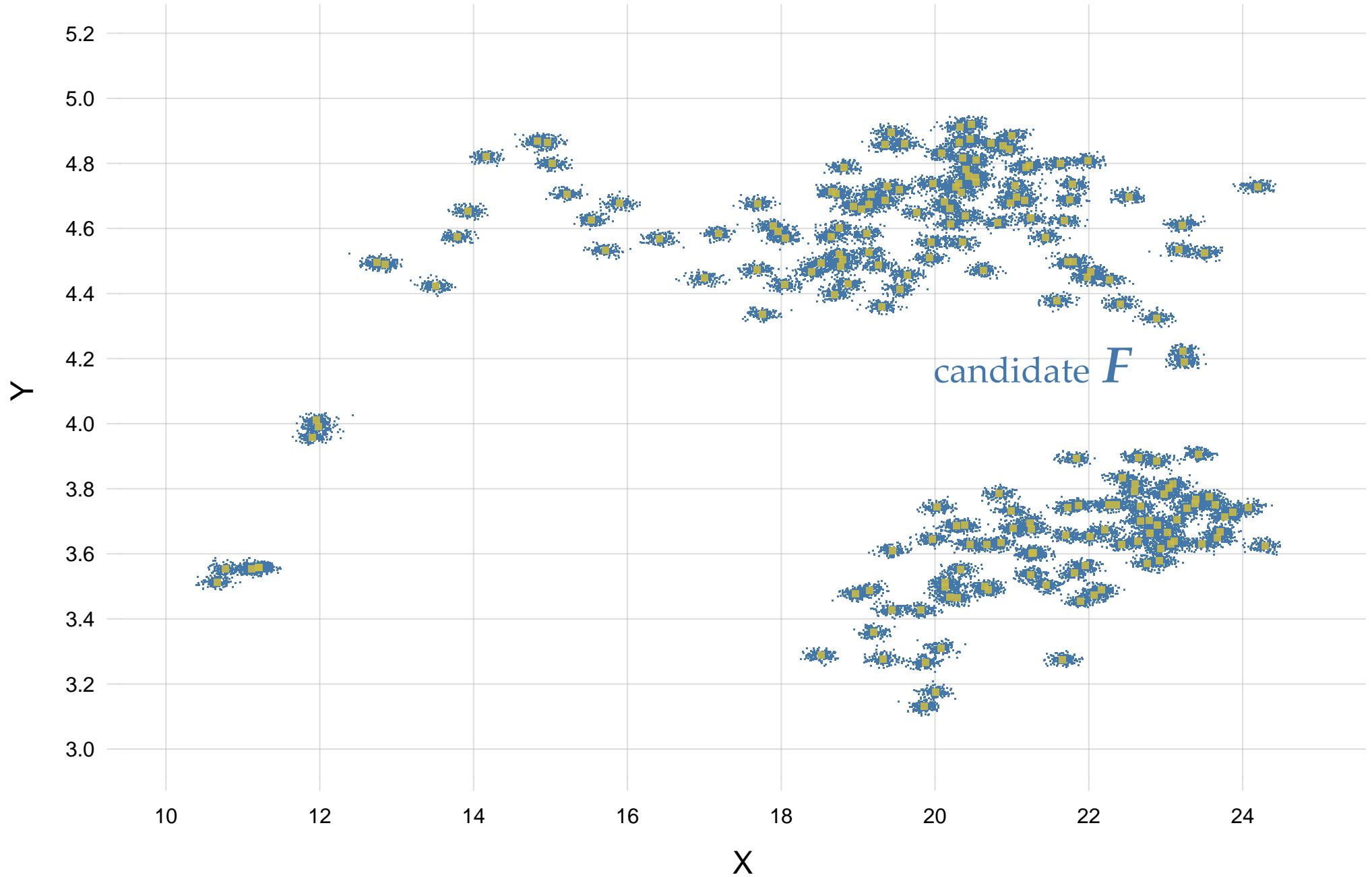
poor candidate: doesn't fit the data

$$\underbrace{F(y_1, x_1) \times F(y_2, x_2) \times F(y_3, x_3) \times \dots}_{\text{low}} \times \underbrace{p(F \mid \text{prior info})}_{\text{high}}$$



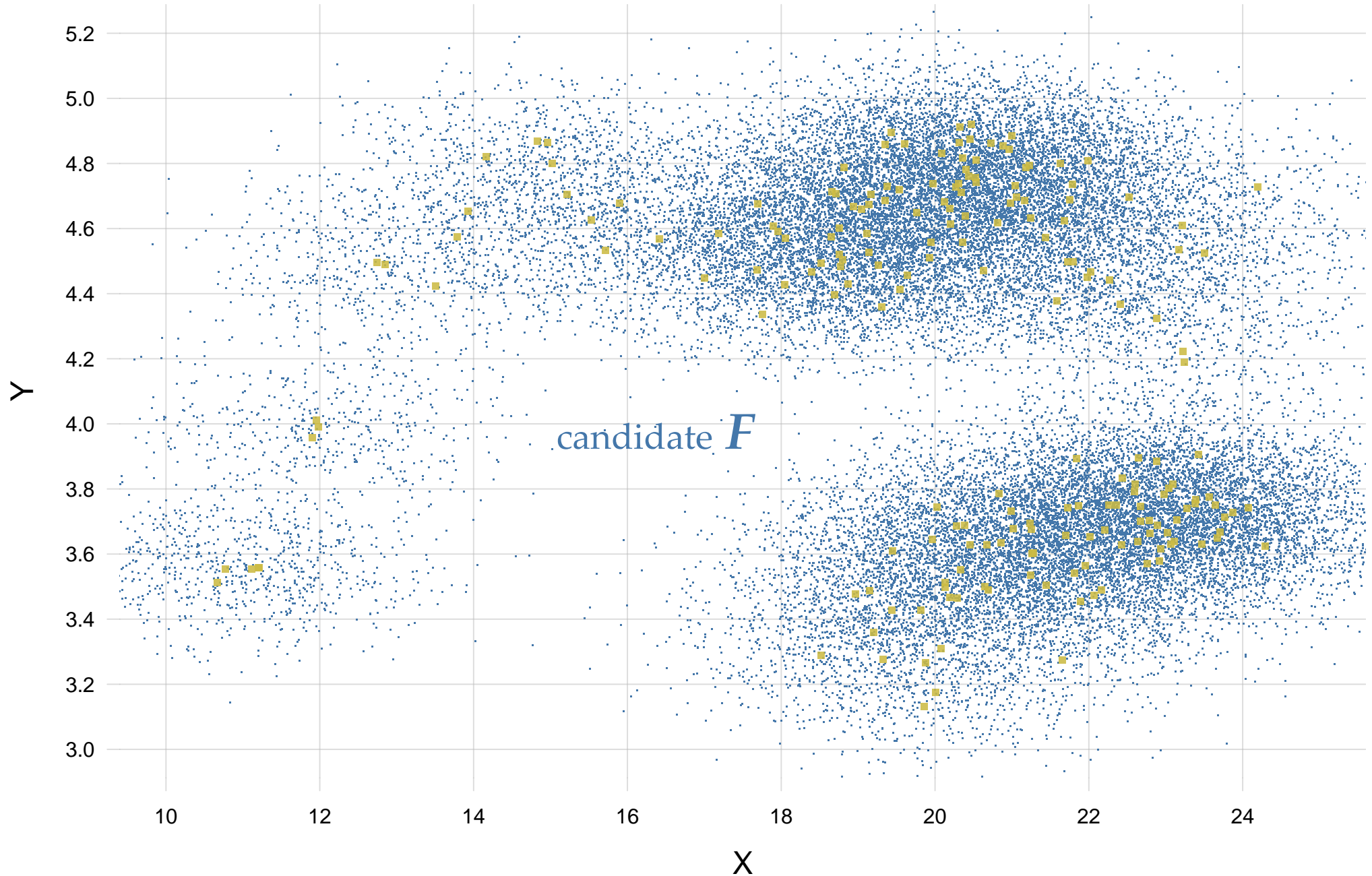
poor candidate: biologically implausible

$$\underbrace{F(y_1, x_1) \times F(y_2, x_2) \times F(y_3, x_3) \times \dots}_{\text{high}} \times \underbrace{p(F \mid \text{prior info})}_{\text{low}}$$



reasonable candidate

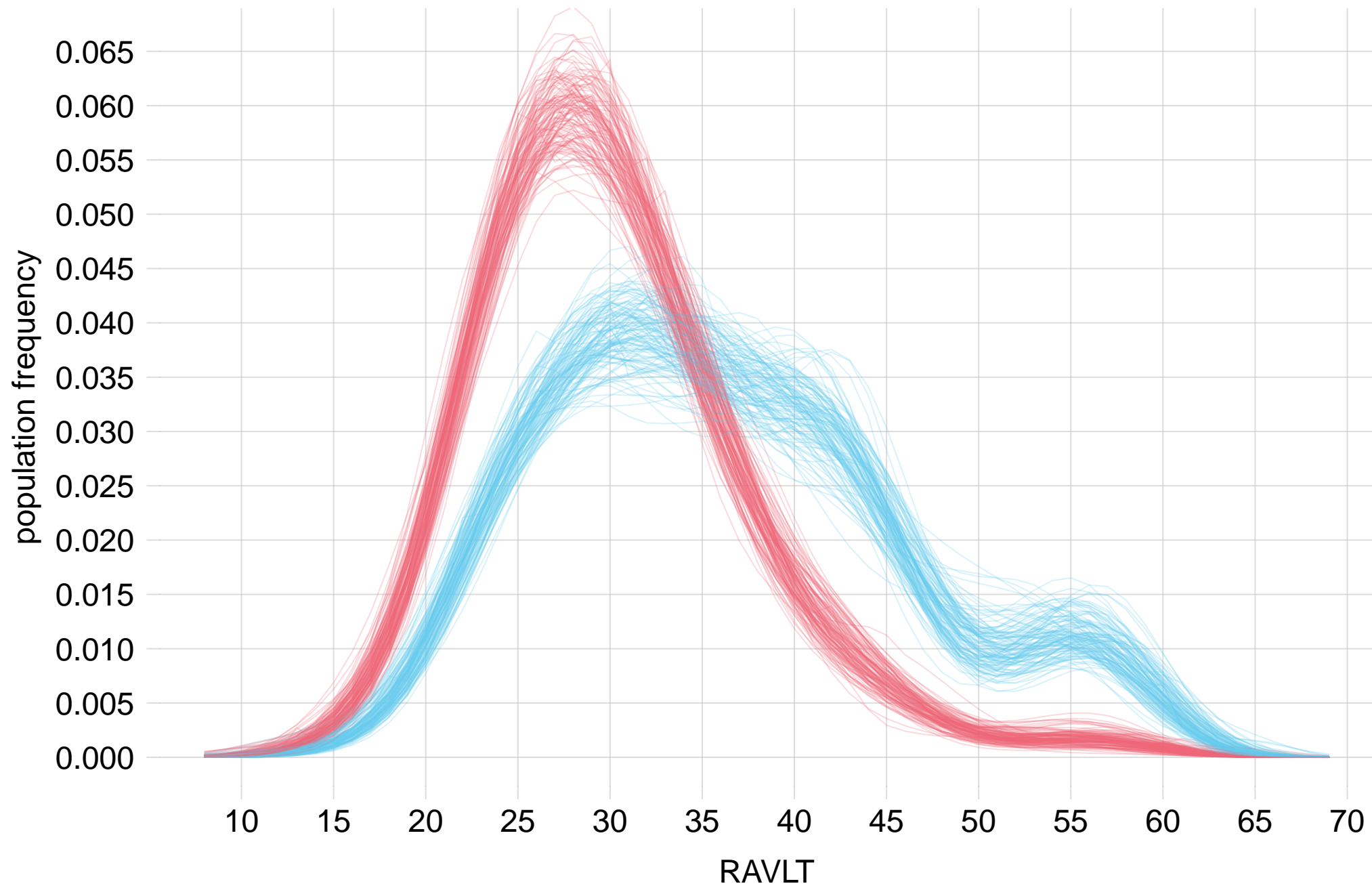
$$\underbrace{F(y_1, x_1) \times F(y_2, x_2) \times F(y_3, x_3) \times \dots}_{\text{high}} \times \underbrace{p(F \mid \text{prior info})}_{\text{high}}$$

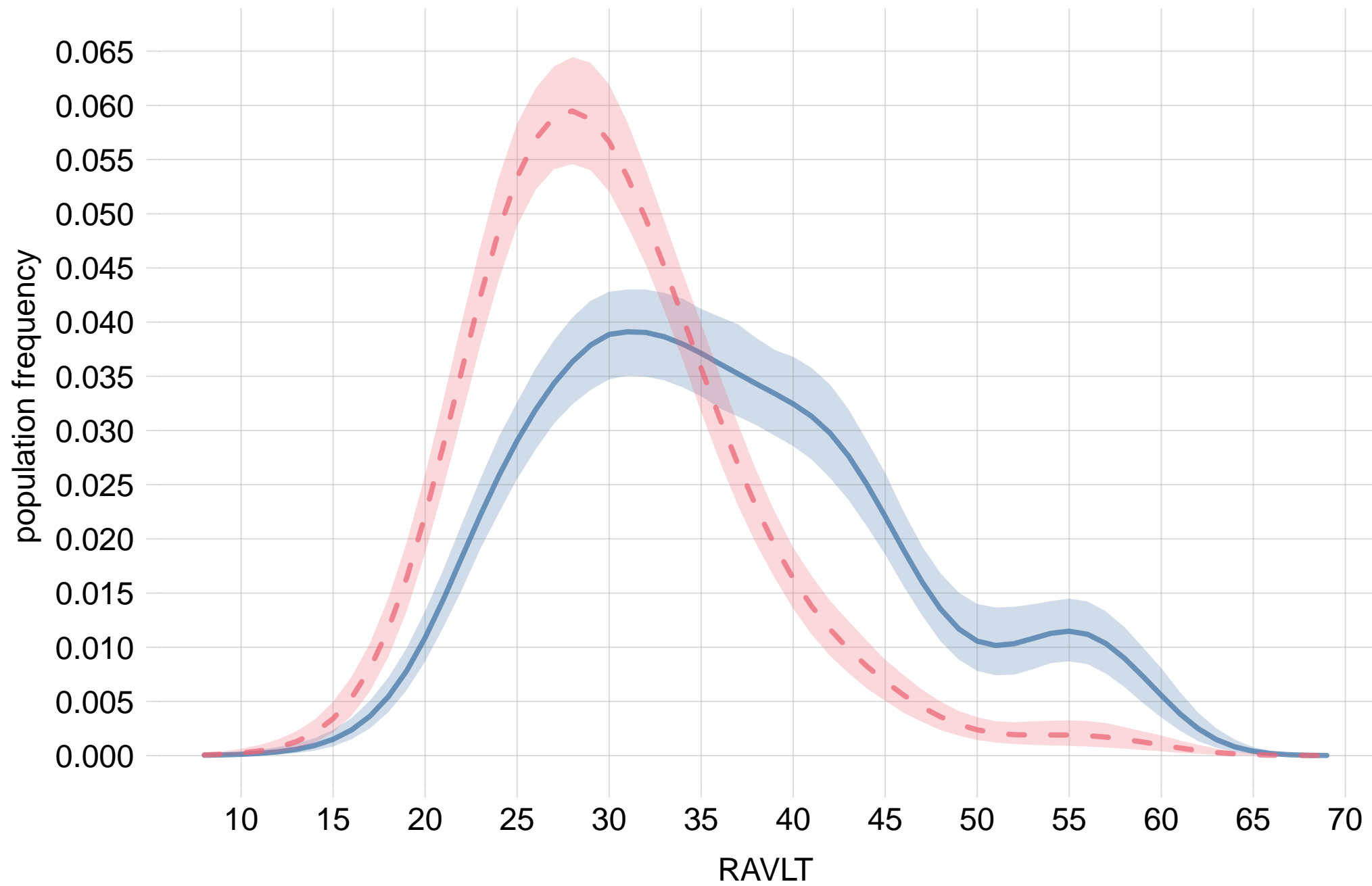


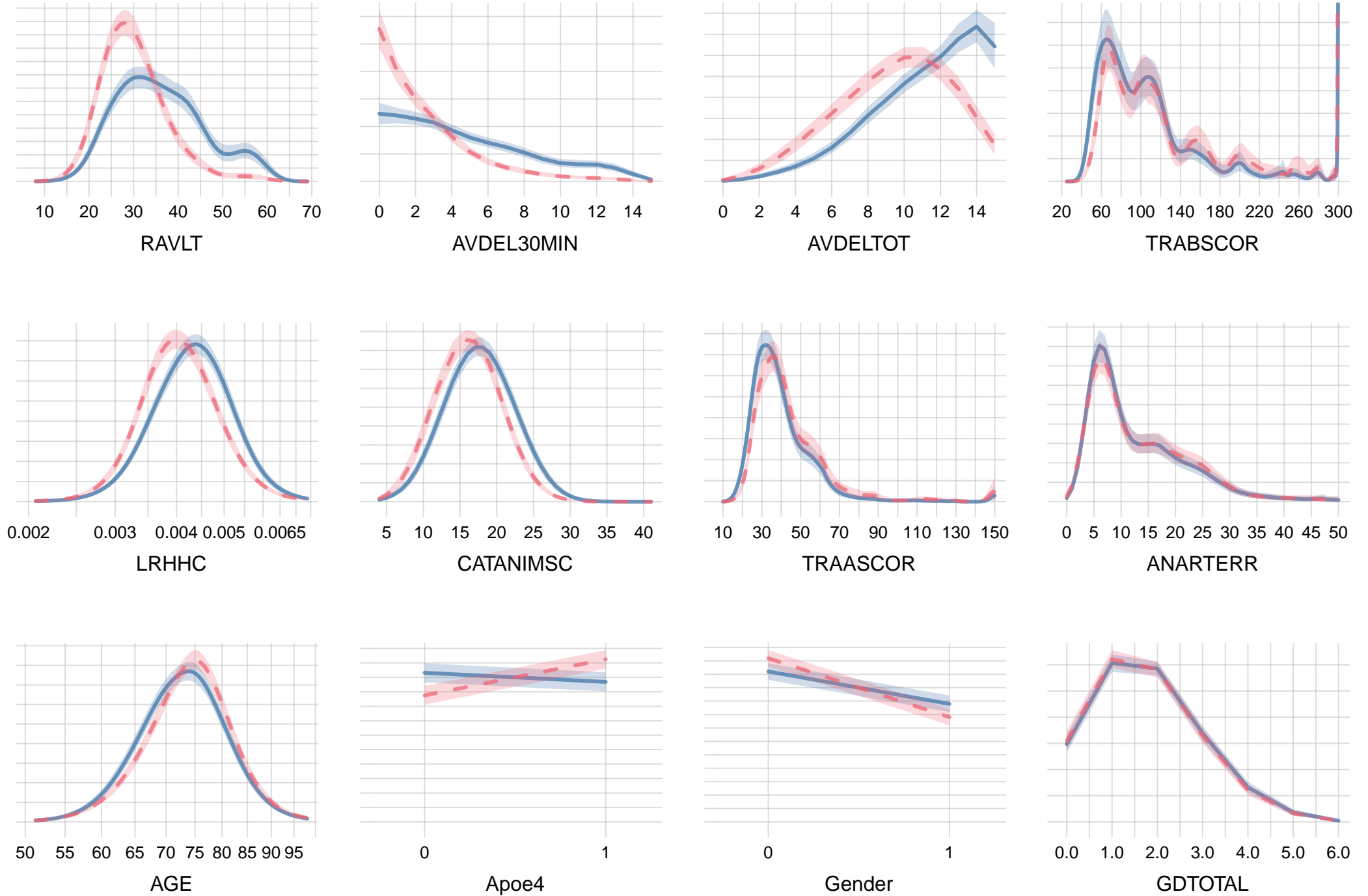
intuition → *mathematics*

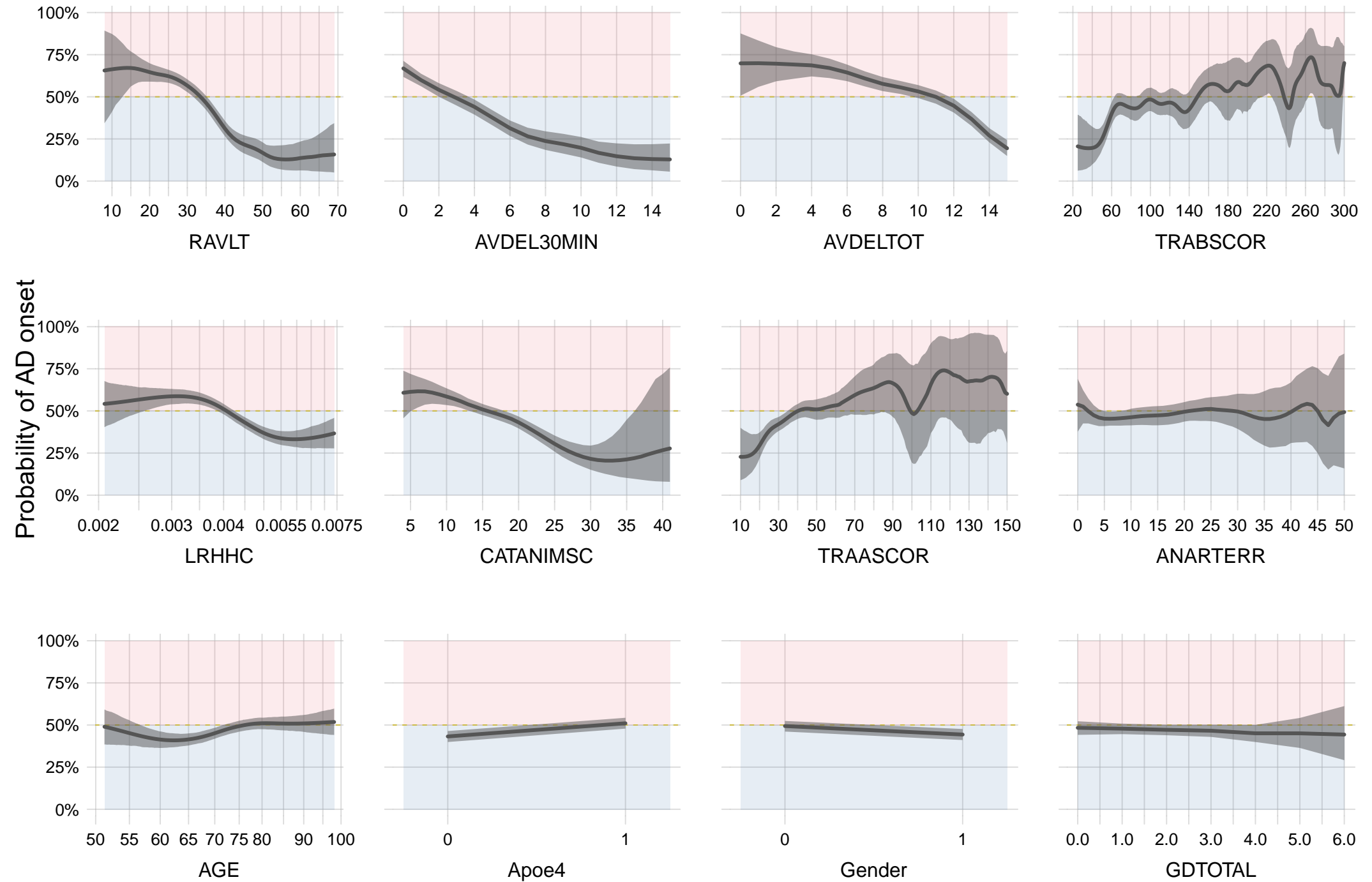
first principles \rightarrow *mathematics* \rightarrow *intuition*

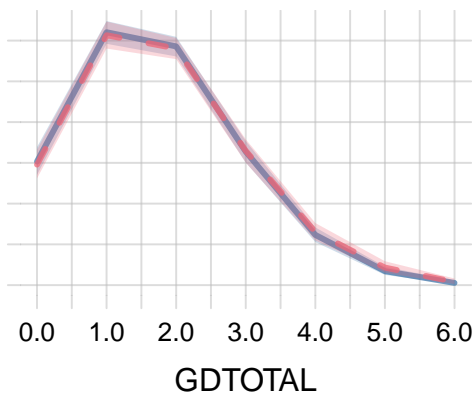
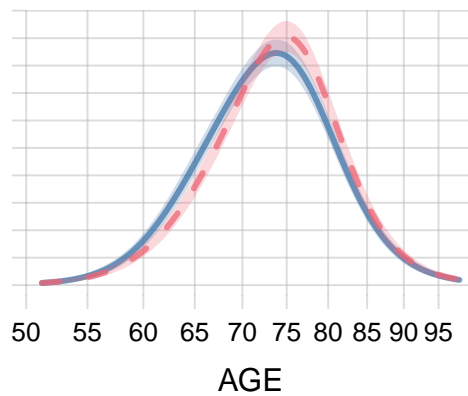
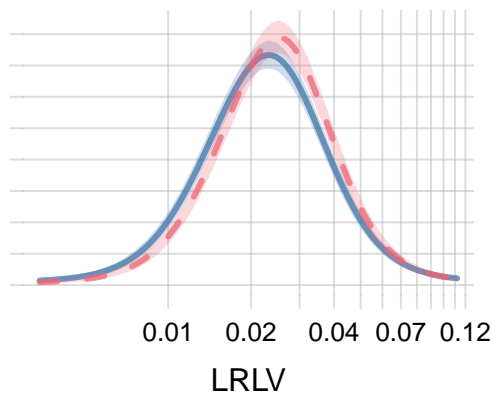
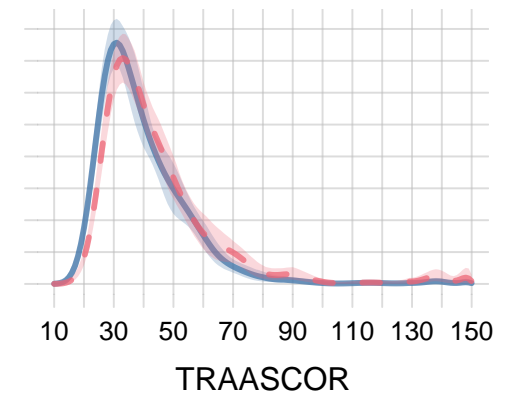
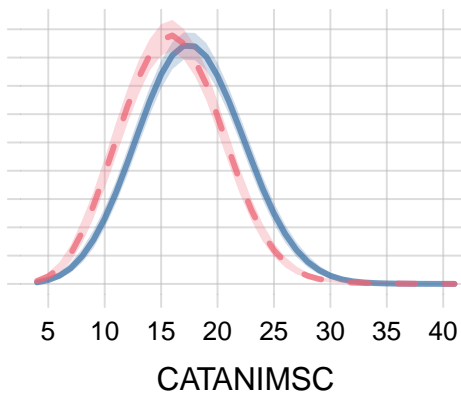
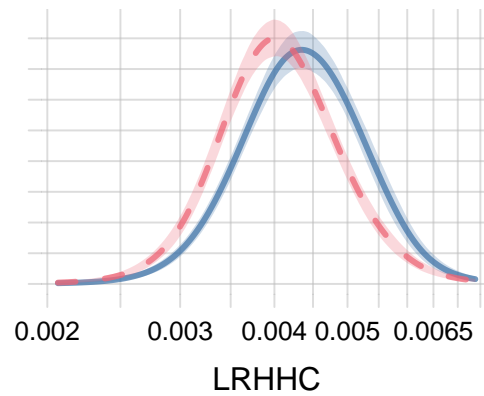
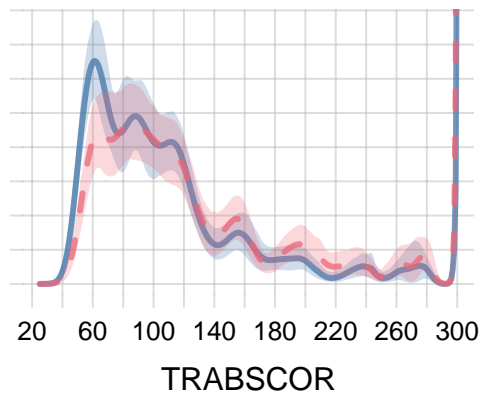
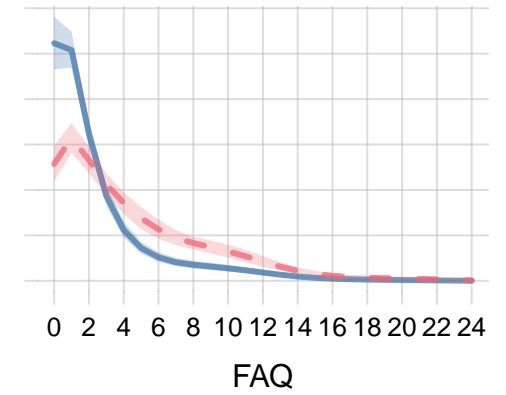
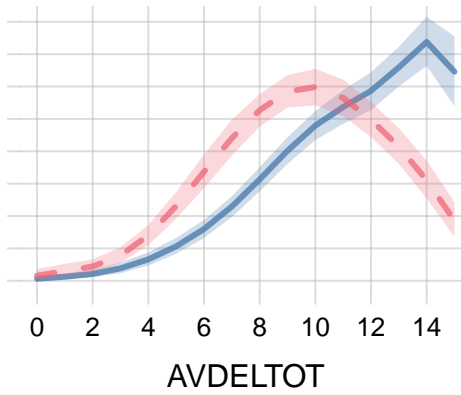
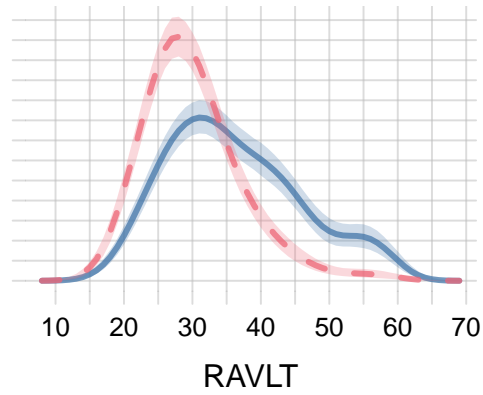
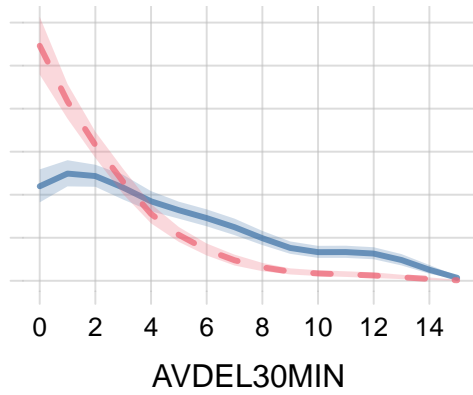
(‘Bayesian’)

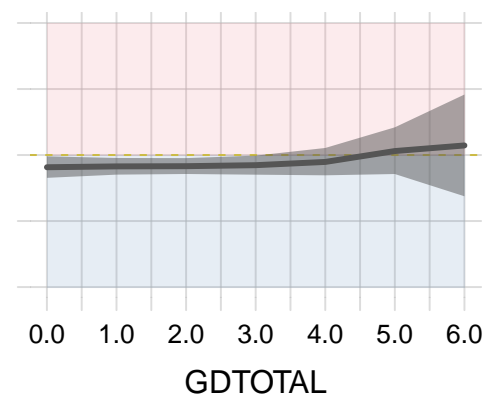
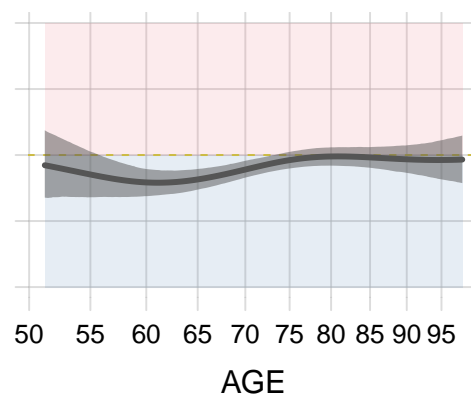
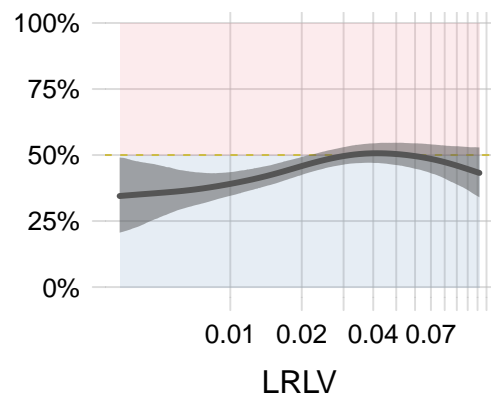
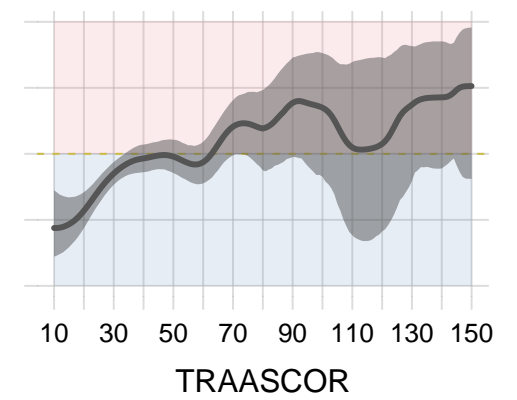
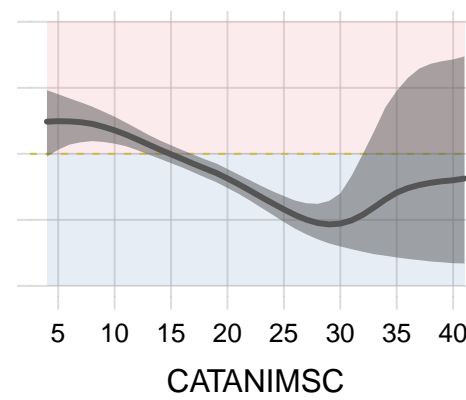
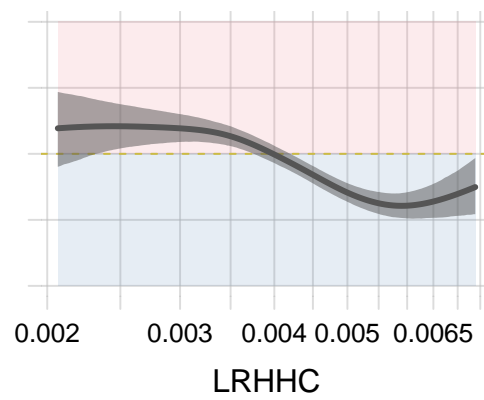
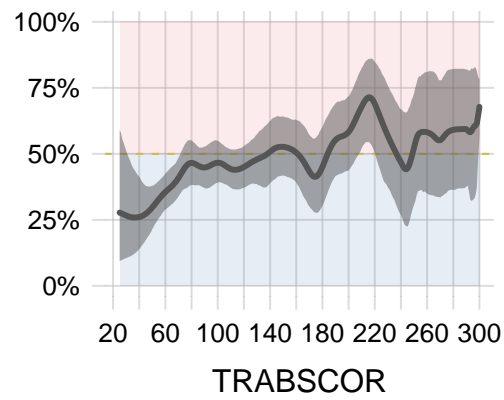
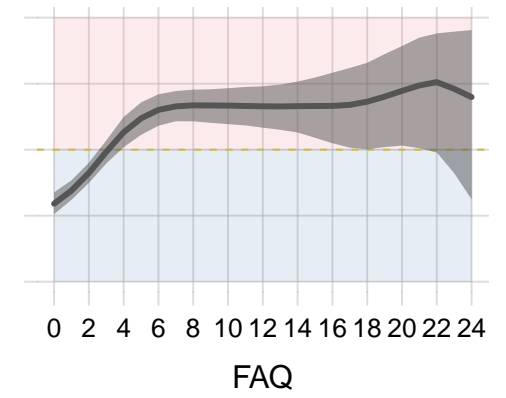
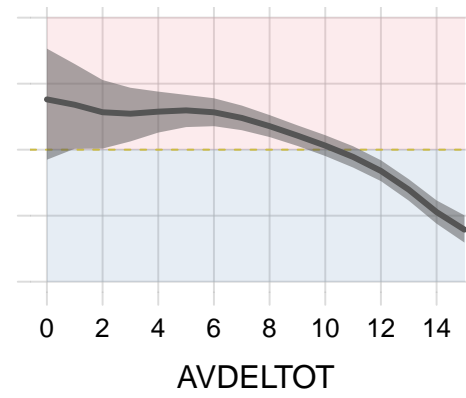
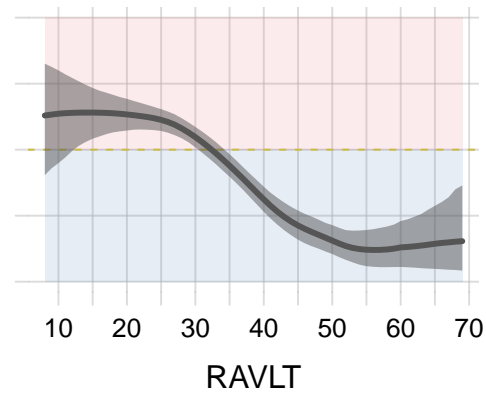
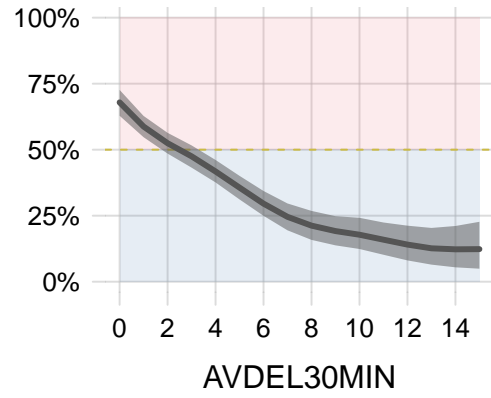












How to quantify the ‘prognostic power’ of a set of features?

Prediction problem:

guess the six digits of the winning lottery ticket ???????

Clue A: ✓✓✓✓??

Clue B: ✓✓✓?✓?

Clue C: ???✓✓✓

What is the ‘importance’ or ‘predictive power’ of each clue?

Scenario 1: we can use **only one** clue

Clue A: ✓✓✓✓??

Clue B: ✓✓✓?✓?

Clue C: ???✓✓✓



Best: **A** or **B** (each gives $1/81$ winning chance)

Worst: **C** (gives $1/729$ winning chance)

Scenario 2: we can use **all** clues

Clue A: ✓✓✓✓??

Clue B: ✓✓✓?✓?

Clue C: ???✓✓✓

→ We fully know the winning number! 💰

Scenario 2: what happens if we **discard** clues?

Clue A: ✓✓✓✓??

Clue B: ✓✓✓?✓?

Clue C: ???✓✓✓

Scenario 2: what happens if we **discard** clues?

Clue A: ✓✓✓✓??

Clue B: ✓✓✓?✓?

Clue C: ???✓✓✓

- Discard A: still 100% win \Rightarrow A has 'importance=0'

Scenario 2: what happens if we **discard** clues?

Clue A: ✓✓✓✓??

Clue B: ✓✓✓?✓?

Clue C: ???✓✓✓

- Discard **A**: still 100% win \Rightarrow **A** has 'importance=0'
- Discard **B**: still 100% win \Rightarrow **B** has 'importance= 0'

Scenario 2: what happens if we **discard** clues?

Clue A: ✓✓✓✓??

Clue B: ✓✓✓?✓?

Clue C: ???✓✓✓

- Discard **A**: still 100% win \Rightarrow **A** has 'importance=0'
- Discard **B**: still 100% win \Rightarrow **B** has 'importance= 0'
- Discard **A and B**: 1/9 winning chance

Scenario 2: what happens if we **discard** clues?

Clue A: ✓✓✓✓??

Clue B: ✓✓✓?✓?

Clue C: ???✓✓✓

- Discard **A**: still 100% win \Rightarrow **A** has 'importance=0'
- Discard **B**: still 100% win \Rightarrow **B** has 'importance=0'
- Discard **A and B**: 1/9 winning chance
 \Rightarrow **A and B** together have 'importance>0'

Scenario 2: what happens if we **discard** clues?

Clue A: ✓✓✓✓??

Clue B: ✓✓✓?✓?

Clue C: ???✓✓✓

- Discard **A**: still 100% win \Rightarrow **A** has 'importance=0'
- Discard **B**: still 100% win \Rightarrow **B** has 'importance=0'
- Discard **A and B**: 1/9 winning chance
 \Rightarrow **A and B** together have 'importance>0'

$$'0 + 0 \neq 0'$$

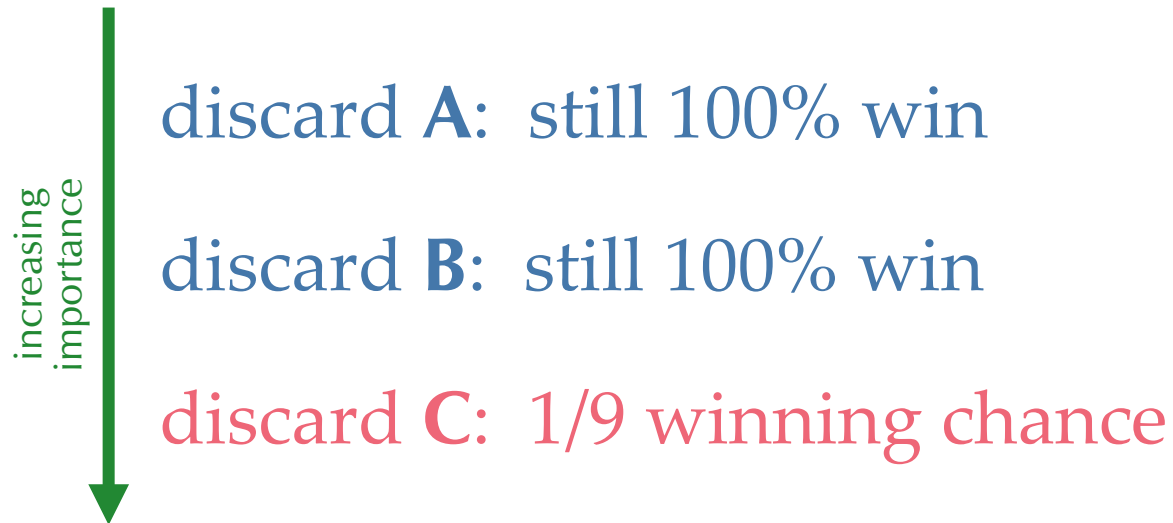
‘Importance’ or ‘predictive power’ is *not* an *additive* property

Scenario 3: we have to **discard one** clue. Which?

Clue A: ✓✓✓✓??

Clue B: ✓✓✓?✓?

Clue C: ???✓✓✓



→ If we have to discard one clue, it's most important that we keep **C**

Scenario 1:
choose one clue



A B
C

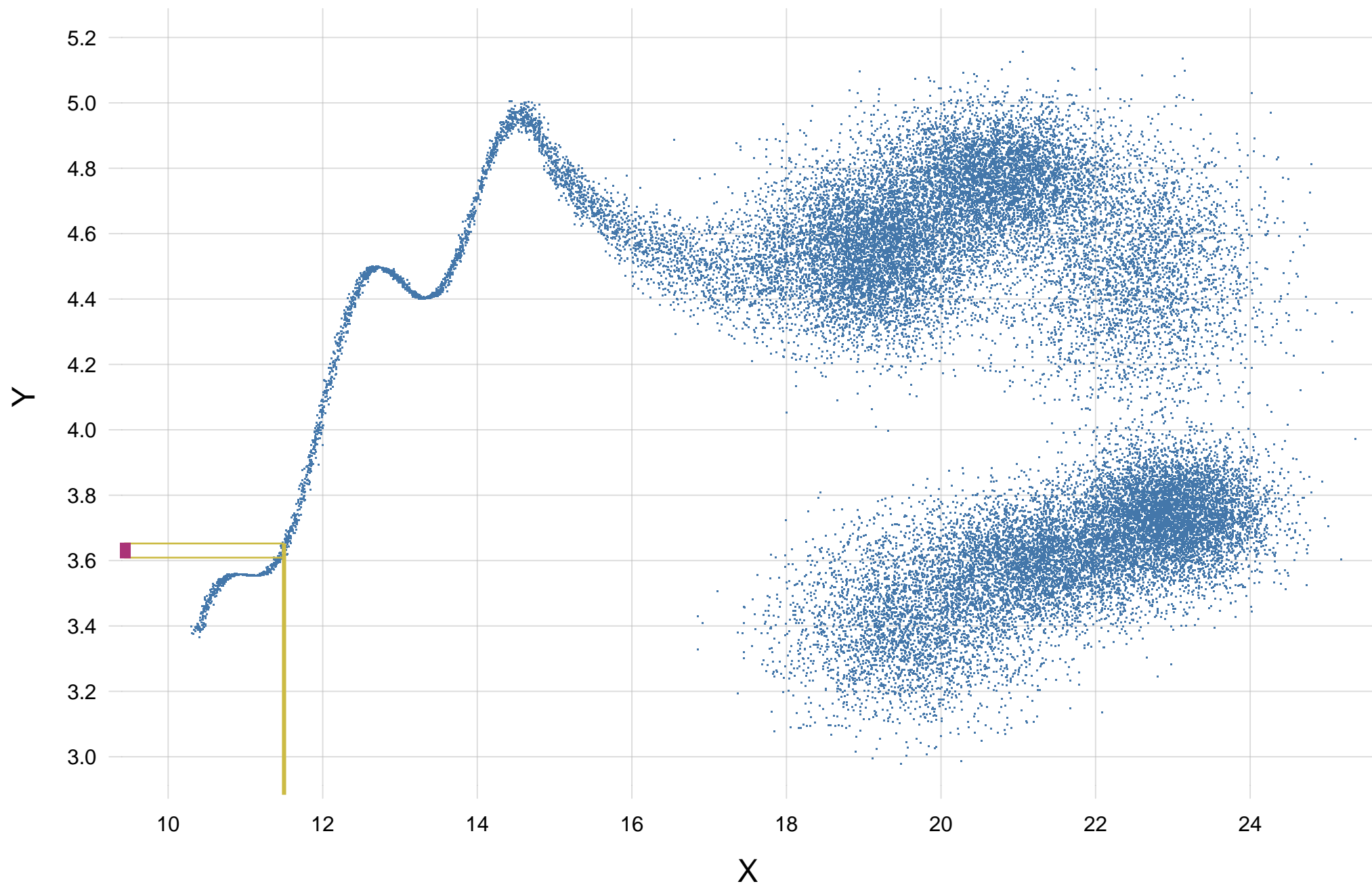
Scenario 3:
discard one clue



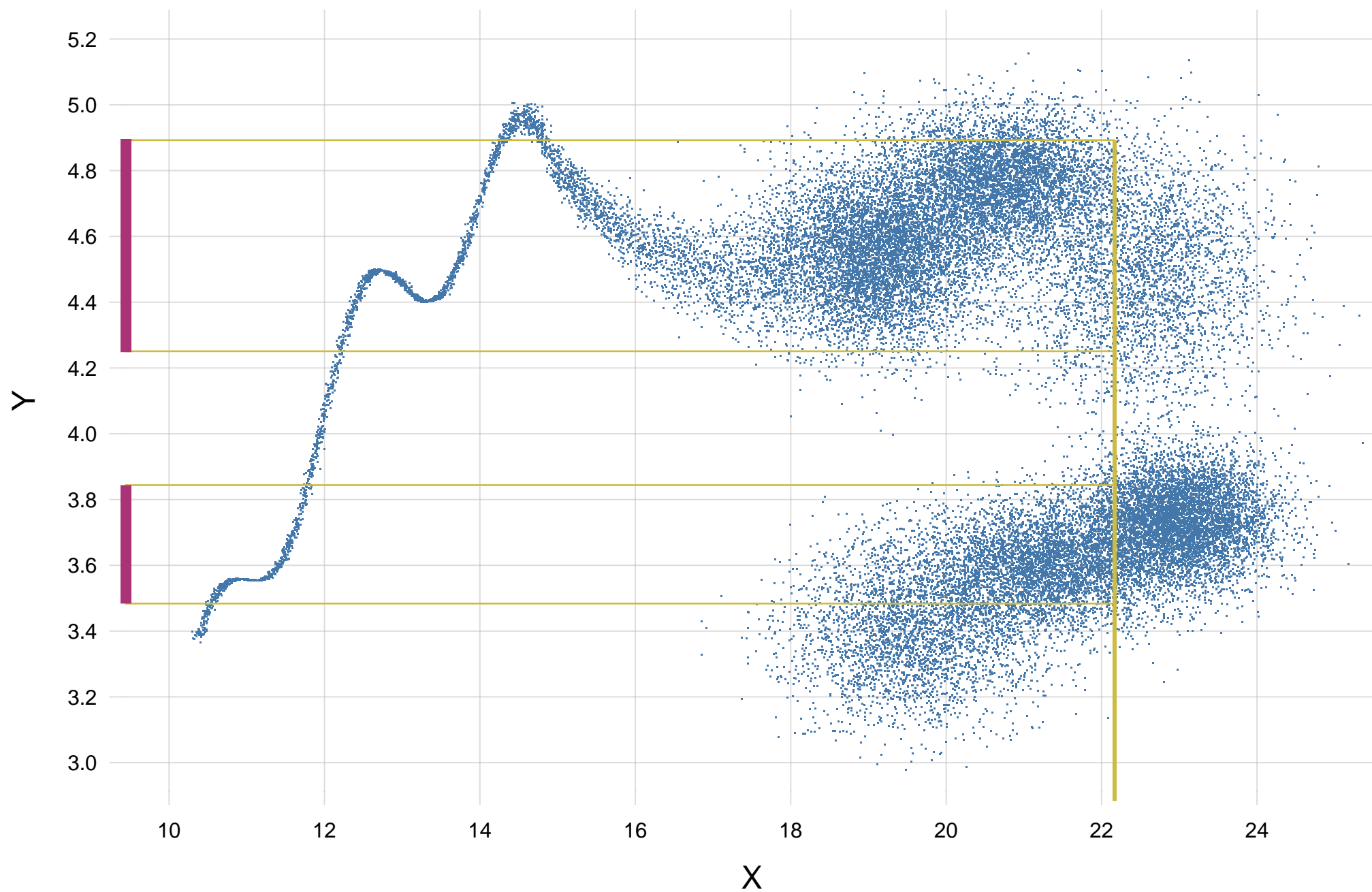
C
A B

‘Importance’ or ‘predictive power’ is *context-dependent*

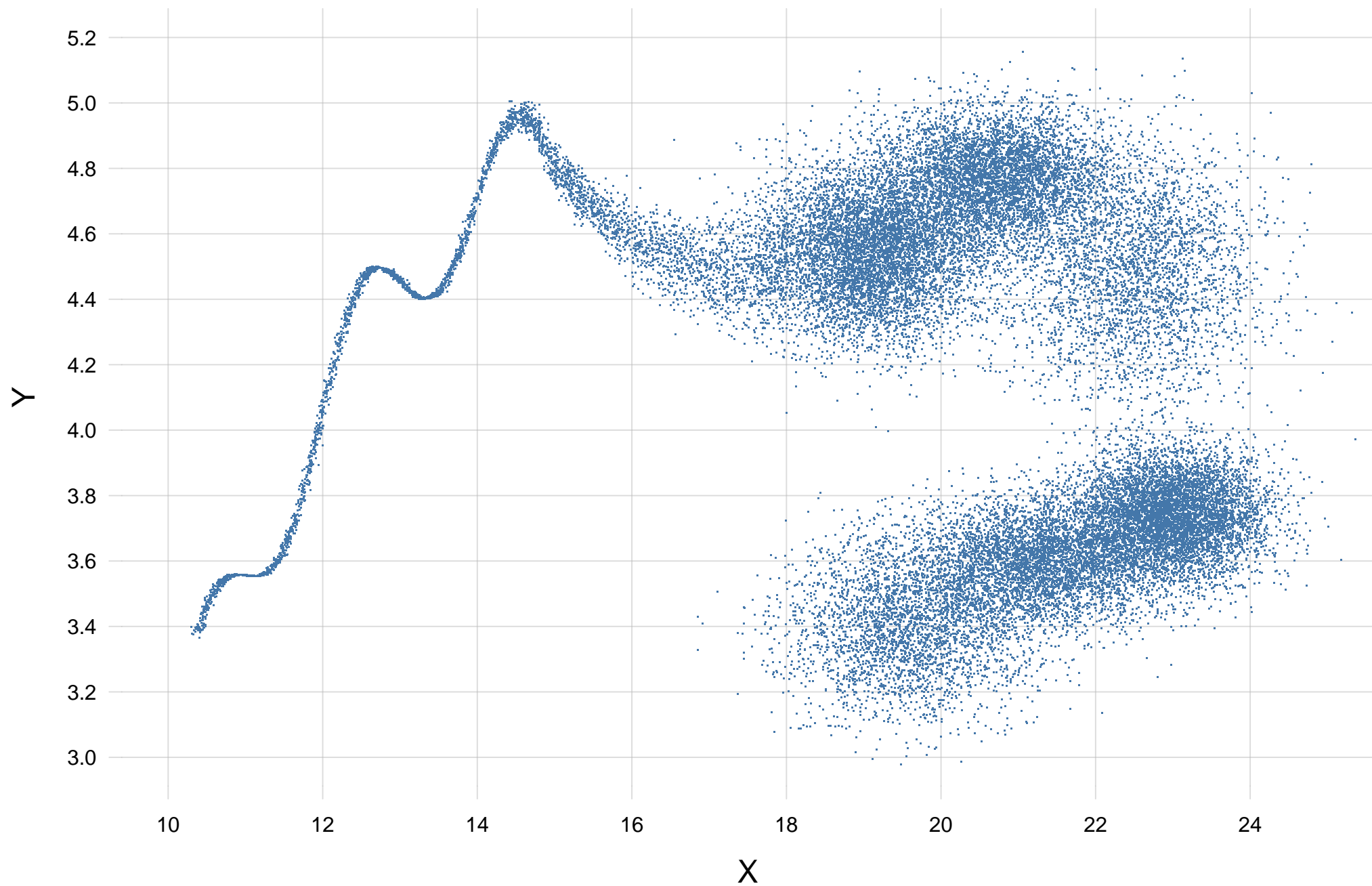
$$x = 11.5 \Rightarrow y \approx 3.60-3.65$$

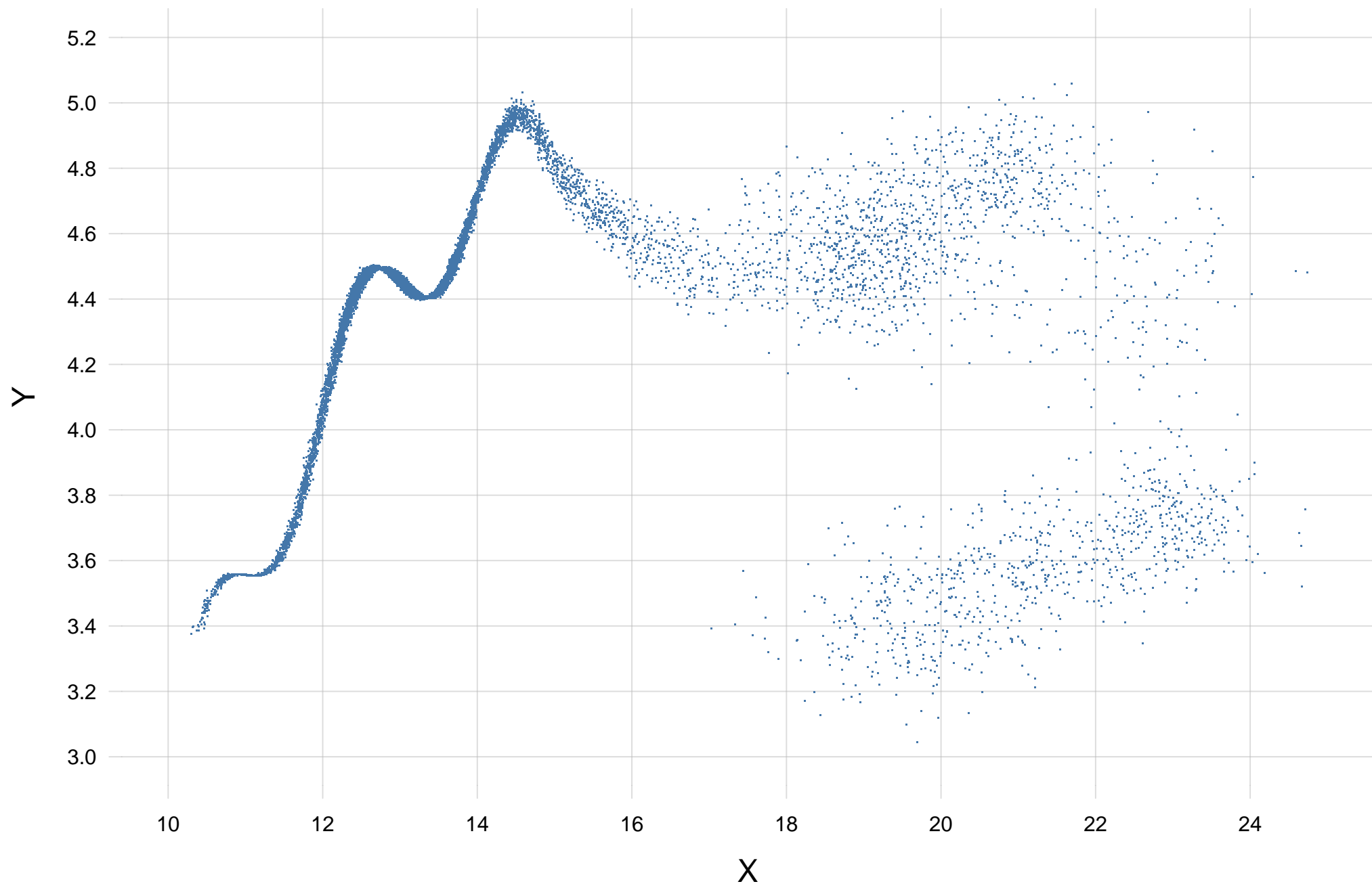


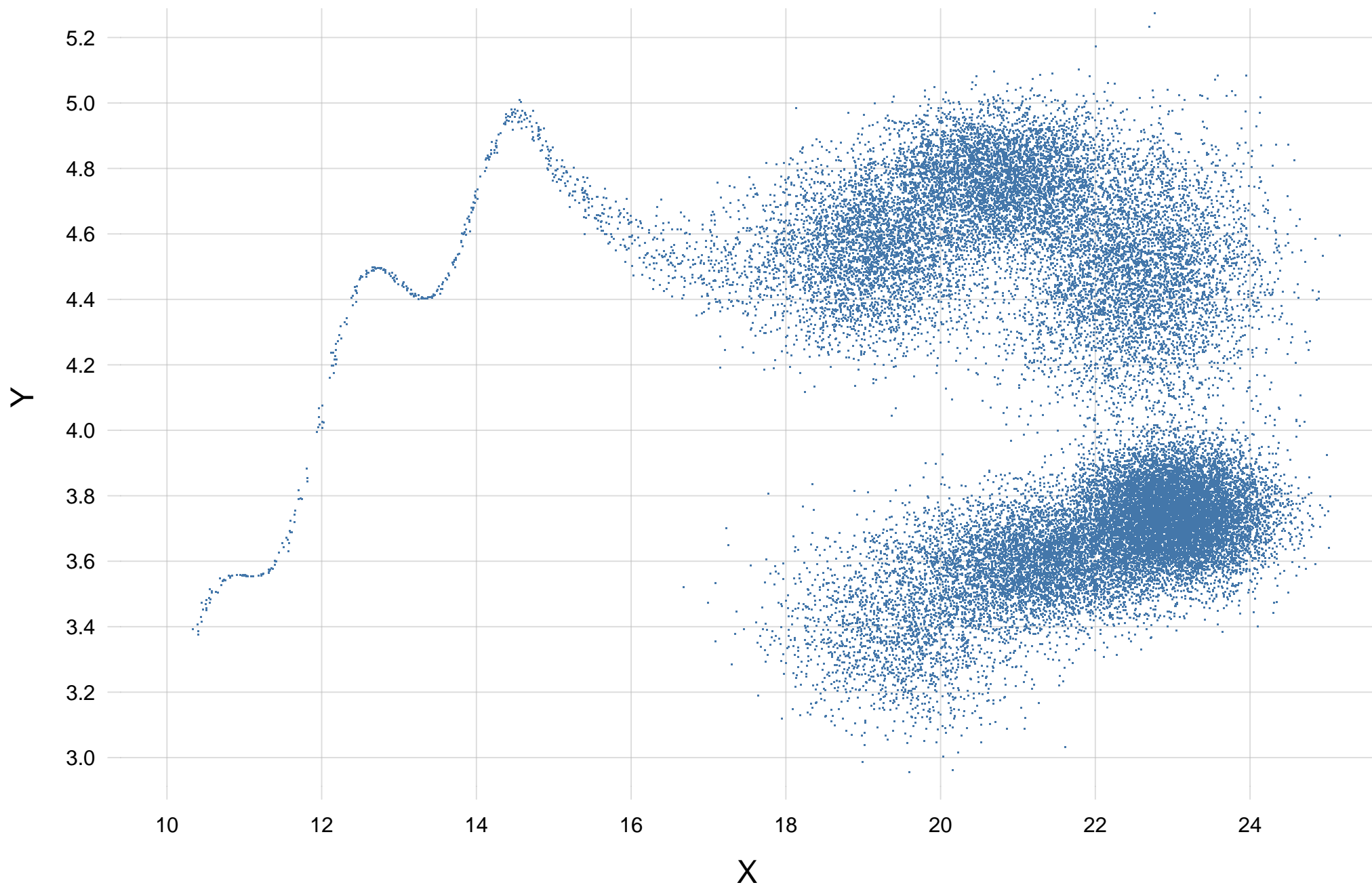
$x = 22 \Rightarrow y \approx 3.50\text{--}3.85 \text{ or } 4.25\text{--}4.90$



What is the 'overall predictive power' of X ?







The ‘predictive power’ of X depends on $P(X)$

⚠ Careful with ‘balancing’! ⚠

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By C. E. SHANNON

INTRODUCTION

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The fundamental problem of communication is that of reproducing at

Information Theory

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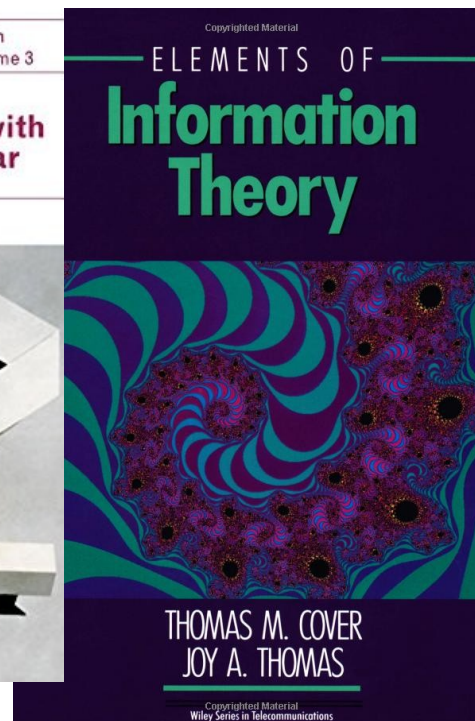
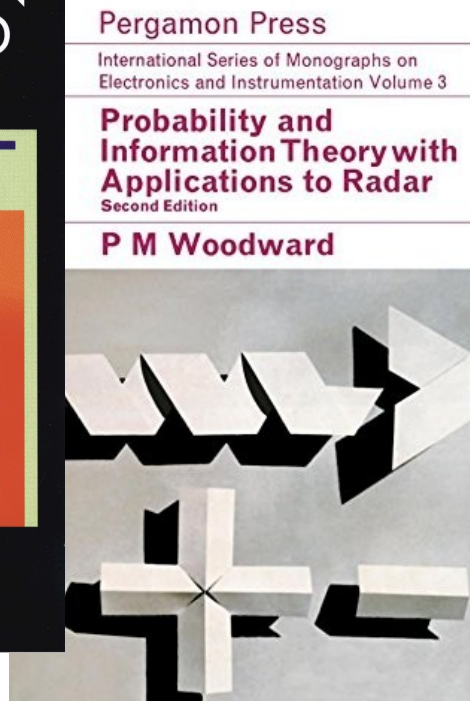
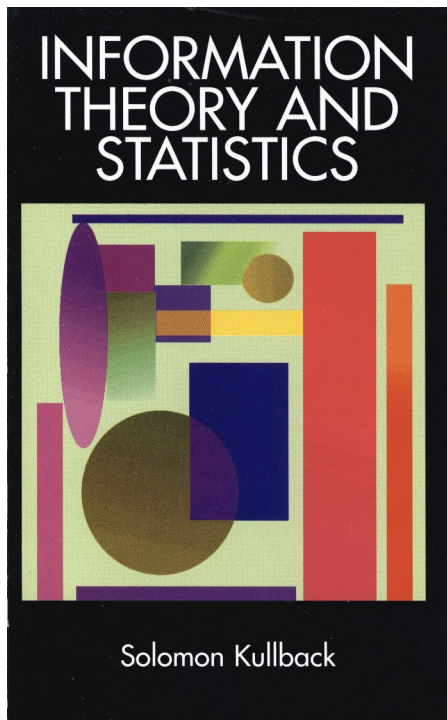
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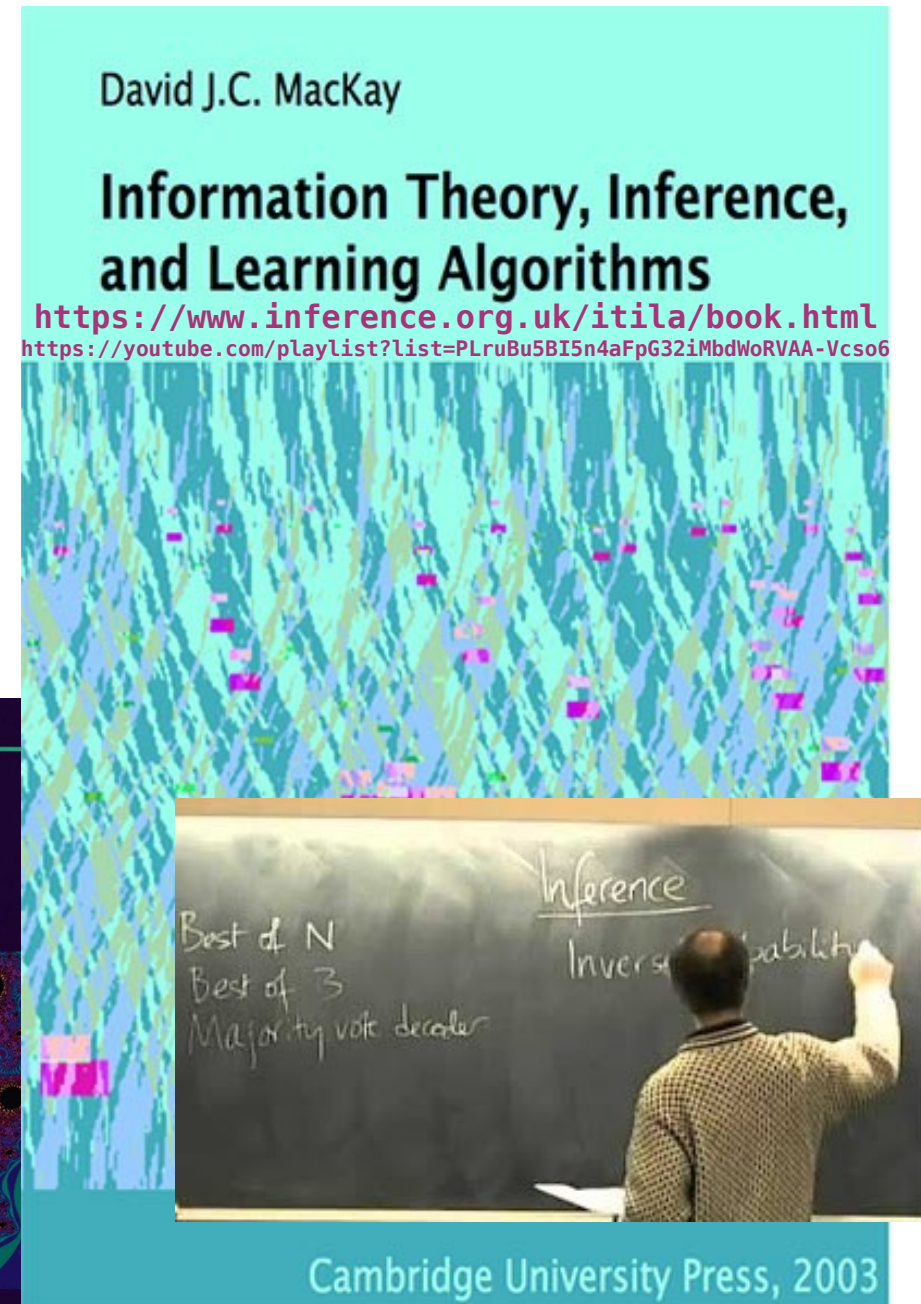
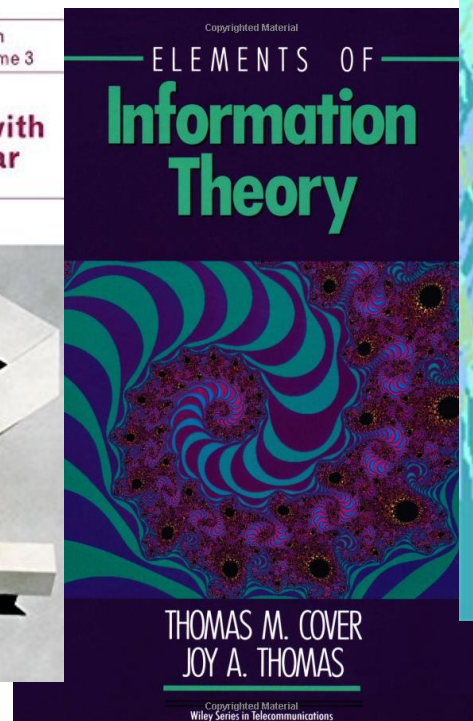
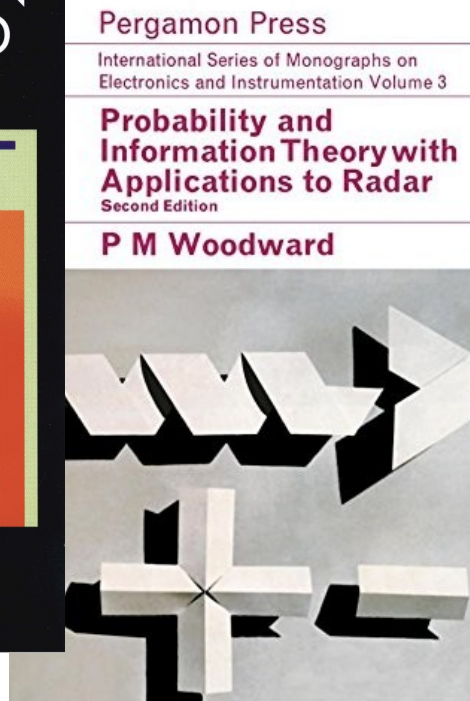
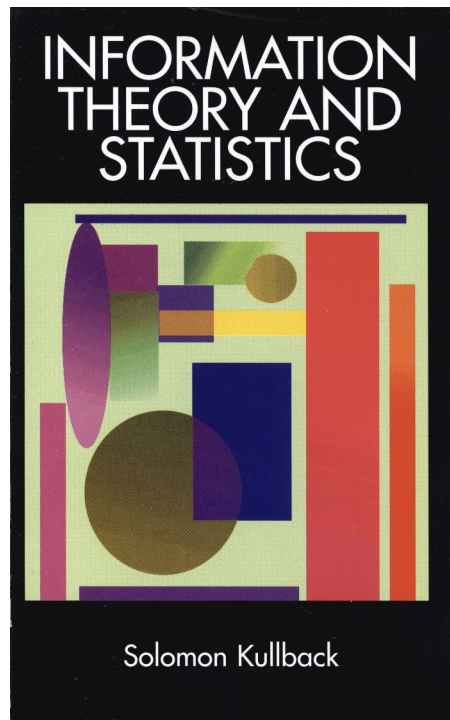
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The fundamental problem of communication is that of reproducing at



‘predictive power’ of X for Y \coloneqq **Mutual information** between Y and X
(mean transinformation content)

$$I(X; Y) := \int p(y|x) p(x) \log \left[\frac{p(y|x)}{p(y)} \right] dy dx$$

$$I(Y; X_1, X_2) \geq I(Y; X_1)$$

$$I(Y; X_1, X_2) \geq I(Y; X_2)$$

$$\text{but } I(Y; X_1, X_2) \neq I(Y; X_1) + I(Y; X_2)$$

INTERNATIONAL STANDARD

NORME INTERNATIONALE

**Quantities and units –
Part 13: Information science and technology**

**Grandeurs et unités –
Partie 13: Science et technologies de l'information**

INTERNATIONAL STANDARD

INFORMATION SCIENCE AND TECHNOLOGY			QUANTITIES	
Item No.	Name	Symbol	Definition	Remarks
13-24 (902)	information content <i>fr</i> quantité (f) d'information	$I(x)$	$I(x) = \lg \frac{1}{p(x)} \text{ Sh} = \lg \frac{1}{p(x)} \text{ Hart} =$ $\ln \frac{1}{p(x)} \text{ nat}$ <p>where $p(x)$ is the probability of event x</p>	See ISO/IEC 2382-16, item 16.03.02. See also IEC 60027-3.
13-25 (903)	entropy <i>fr</i> entropie (f)	H	$H(X) = -\sum_{i=1}^n p(x_i) \lg p(x_i)$ <p>for the set $X = \{x_1, \dots, x_n\}$ where $p(x_i)$ is the probability and $I(x_i)$ is the information content of event x_i</p>	See ISO/IEC 2382-16, item 16.03.03.
13-30 (908)	joint information content <i>fr</i> quantité (f) d'information conjointe	$I(x, y)$	$I(x, y) = \lg \frac{1}{p(x, y)} \text{ Sh} = \lg \frac{1}{p(x, y)} \text{ Hart} =$ $\ln \frac{1}{p(x, y)} \text{ nat}$ <p>where $p(x, y)$ is the joint probability of events x and y</p>	
13-35 (912)	transinformation content <i>fr</i> transinformation (f)	$T(x, y)$	$T(x, y) = I(x) + I(y) - I(x, y)$ <p>where $I(x)$ and $I(y)$ are the information contents (13-24) of events x and y, respectively, and $I(x, y)$ is their joint information content (13-30)</p>	See ISO/IEC 2382-16, item 16.04.07.
13-36 (913)	mean transinformation content <i>fr</i> transinformation (f) moyenne	T	$T(X, Y) = \sum_{i=1}^n \sum_{j=1}^m p(x_i, y_j) T(x_i, y_j)$ <p>for the sets $X = \{x_1, \dots, x_n\}$, $Y = \{y_1, \dots, y_m\}$, where $p(x_i, y_j)$ is the joint probability of events x_i and y_j, and $T(x_i, y_j)$ is their transinformation content (item 13-35)</p>	See ISO/IEC 2382-16, item 16.04.08.

UNITS INFORMATION SCIENCE AND TECHNOLOGY				
Item No.	Name	Symbol	Definition	Conversion factors and remarks
13-24.a	shannon	Sh	value of the quantity when the argument is equal to 2	1 Sh \approx 0,693 nat \approx 0,301 Hart
13-24.b	hartley	Hart	value of the quantity when the argument is equal to 10	1 Hart \approx 3,322 Sh \approx 2,303 nat
13-24.c	natural unit of information	nat	value of the quantity when the argument is equal to e	1 nat \approx 1,433 Sh \approx 0,434 Hart
13-25.a	shannon	Sh		
13-25.b	hartley	Hart		
13-25.c	natural unit of information	nat		
13-30.a	shannon	Sh		
13-30.b	hartley	Hart		
13-30.c	natural unit of information	nat		
13-35.a	shannon	Sh		
13-35.b	hartley	Hart		
13-35.c	natural unit of information	nat		
13-36.a	shannon	Sh		In practice, the unit "shannon per character" is generally used, and sometimes the units "hartley per character" and "natural unit per character".
13-36.b	hartley	Hart		
13-36.c	natural unit of information	nat		

If Y is binary:

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Maximum accuracy attainable
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