


Kinematics and dynamics from a modern perspective

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The kinematics of mechanical, thermodynamic, and electromagnetic phenomena is developed in such a way as to be used for Newtonian, Lorentzian, and general relativity. The same is done, as much as possible, for their dynamics as well

1 Scribbles and memos

1.1 Twisted objects

Denote the wedge product by juxtaposition: $dt \wedge dx =: dt \, dx$ and so on; abbreviate $dt \, dx =: dtx$ and so on; and denote twisted differential forms by \tilde{d} .

Where an ordered set of coordinate functions (t, x, y, z) is chosen, the twisted unit $\tilde{1}$ is defined. It has unit magnitude and outer-orientation $txyz$, and the property $\tilde{1} \cdot \tilde{1} = 1$. In general it is only defined in chart domain, where coordinate functions can be defined, but not globally. We obtain twisted vectors and covectors by multiplying their non-twisted counterparts by the twisted unit. For a vector or covector ω we have that the orientation of its twisted counterpart is such that (see Burke 1983)

$$\{\{\omega\}, \{\omega\}\} = \{\tilde{1}\} . \quad (1)$$

Said otherwise, in the product $\tilde{1} \cdot \omega$ the *right* side of the orientation of $\tilde{1}$ cancels out with the orientation of ω . This rule must be respected even if we invert the product order, so $\tilde{1} \cdot \omega \equiv \omega \cdot \tilde{1}$.

Multiplying a twisted vector or covector by the twisted unit, we obtain its non-twisted counterpart. The resulting orientation is obtained by cancelling out the orientation of the twisted object and the *left* side of the orientation of $\tilde{1}$.

We have:

$$\{\underline{1}\} = txyz ; \quad (2)$$

$$\{\underline{dt}\} = -xyz , \quad \{\underline{dx}\} = tyz , \quad \{\underline{dy}\} = tzx , \quad \{\underline{dz}\} = txy ; \quad (3)$$

$$\{\underline{dtx}\} = yz , \quad \{\underline{dty}\} = zx , \quad \{\underline{dtz}\} = xy , \quad (4)$$

$$\{\underline{dxy}\} = tz , \quad \{\underline{dyz}\} = tx , \quad \{\underline{dzx}\} = ty ; \quad (5)$$

$$\{\underline{dtyz}\} = -x , \quad \{\underline{dtzx}\} = -y , \quad \{\underline{dtxy}\} = -z , \quad \{\underline{dxyz}\} = t ; \quad (6)$$

$$\{\underline{dtxyz}\} = +1 . \quad (7)$$

The minus signs appear in the odd ranks when we have t and an even number of other coordinates after the “ \underline{d} ”. These minus signs flip if we keep t always to the right, with orientation $xyzt$.

Note that considering, say, the *function* x , we have

$$\{\underline{x}\} = \begin{cases} \{txyz\} & \text{if } x > 0 , \\ -\{txyz\} & \text{if } x < 0 . \end{cases} \quad (8)$$

1.2 Charge and current densities

We can represent charge density and current density in one geometrical entity. Consider an ordered coordinate system (t, x, y, z) . The charge-current density is

$$\rho \underline{dxyz} - j_x \underline{dtyz} - j_y \underline{dtzx} - j_z \underline{dtxy} \equiv \rho \underline{dxyz} - dt (j_x \underline{dyz} + j_y \underline{dzx} + j_z \underline{dxy}) . \quad (9)$$

It has the dimensions of charge (current · time). The minus signs appear so that the x -component of the current, for example, is positive when $j_x > 0$ and so on.

This object automatically give us net volume charge when integrated over a three-dimensional region at constant time, or the net flux of charge when integrated over a two-dimensional surface – possibly even moving – over a lapse of time. Consider for example a 3-volume V at constant time, having *outer* orientation in the positive t direction and parameterized by

$$(u, v, w) \mapsto (t_0, u, v, w) . \quad (10)$$

On it, the basis twisted 1-forms map to

$$\begin{aligned} \underset{-xyz}{dt}|_V = 0, \quad \underset{tyz}{dx}|_V = \underset{vw}{du}, \quad \underset{tzy}{dy}|_V = \underset{wu}{dv}, \quad \underset{txy}{dz}|_V = \underset{uv}{dw}, \\ \underset{t}{dxyz}|_V = \underset{+}{duvw}. \end{aligned} \quad (11)$$

Then we have

$$(\rho \underset{\sim}{d}xyz - j_x \underset{\sim}{d}tyz - j_y \underset{\sim}{d}tzy - j_z \underset{\sim}{d}txy)|_V = \rho \underset{+}{duvw} \quad (12)$$

and the current density gives no contribution.

Bibliography

(“de X” is listed under D, “van X” under V, and so on, regardless of national conventions.)

Burke, W. L. (1983): *Manifestly parity invariant electromagnetic theory and twisted tensors*. J. Math. Phys. **24**¹, 65–69. [DOI:10.1063/1.525603](https://doi.org/10.1063/1.525603).