Open Science Framework DOI:10.31219/osf.io/***

Introduction and teaching of relativity theory

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25 March 2023; updated 25 March 2023 [draft]

Notes on a possible way to introduce and teach general, Lorentzian, and Newtonian relativity.

1 Motivation

The way Lorentzian and general relativity are introduced in many books leaves some uneasiness as regards some logical steps in the underlying reasoning – although I feel no doubt about the internal logical consistency of the results. For example, if we come to the conclusion that metre sticks shortens or are seen shortened by some observer, then is our initial reasoning about some experiments, such as the Michelson-Morley one, still logically valid? How were those meters initially defined? If general relativity makes the notion of rigid body very tricky, then where does our initial reasoning involving rigid rods logically stand?

The present note tries to develop the theory from some postulates and notions that can be induced from experiments – at least from thought experiments – while avoiding logical loop-holes. It is only tentative, a step, probably still affected by inconsistencies.

The presentation is, for the moment, mostly in form of main points and self-reminders.

2 Initial notions and postulates

We start with the notion of "event", something that has very small extension in space and time. Intuitive, qualitative notions of "extension" should suffice for such a definition. Spacetime is introduced as the four-dimensional topological space made of such events. For the moment the only thing we can say about events is whether they are coincident (or very close, in a qualitative sense) or not.

Next we introduce the notion of a persistent clock. "Persistent" in the sense that it is not just an event, but rather a continuous sequence of events, which has a very small extension in space. In spacetime it can be represented by a one-dimensional curve, which is called a "worldline". "Clock" in the sense that there is some kind of periodicity in this sequence of events. Note how tricky the notion of "periodicity" is, though: how do we establish that some phenomenon is periodic, without having a notion of clock? For the moment we must thus take "periodic" as a primitive notion. It's the refinement of our own biological feeling of "equal time intervals". We call such a persistent clock an "observer".

Next we make an experimentally verifiable assumption. Whenever two observers coincide – they have coincident worldlines – and their clocks are of the same making, then they notice and exact equality of the periods of the two clocks. The absolute times shown by the clocks – the number of periods counted from some initial moment – may be different, but the period intervals are the same.

Let us now imagine that two such observers are coincident in some initial and final parts of their worldlines, but not in all their internal parts. Let us also assume that in their initial, coincident part, their clocks are synchronized, in the sense that they give the same absolute time, not just same time intervals. Experimentally it is observed that if the worldlines diverge and then come to coincide again, at the point of reunion the two clocks will have generally different absolute times (though both future times with respect to the initial point of worldline separation), but they have again same time intervals in this new coincident part of the curve. Generalizing to three or more observers, in general we would find three or more different absolute times at the point of reunion.

This experimental fact is at variance with Newtonian relativity, which instead assumes that in such a situation the two clocks would still have identical absolute times upon reuniting.

We also take as primitive the possibility of sending light signals from a given observer, using some kind of device.

Another experimentally verifiable assumption is that if two coincident observers send a light signal in the same direction, using light-sending devices of the same making, then the two signals also have coincident worldlines.

This experimental assumption is extended, however. Suppose we have two observers with non-coincident worldlines, which however intersect in one event. At that event, both observers send light signals in the same direction. Then also in this case the two light signals have coincident worldlines. This is an experimentally verifiable assumption.

This last assumption is extremely important because it selects, in spacetime, a special set of worldlines and of “lightcones”. A lightcone is the set of worldlines of light signals stemming from the same event but having different directions.

The last assumption could have been restated as saying that the velocity of a light signal is independent of the velocity of its source. But we have thus far happily avoided the tricky notions of “velocity” and “distance”.

✎ Next: introduce radar coordinates. Emphasis on the fact that they can only be local. Notion of “constant distance or position” with respect to an observer. *Definition* of local velocity.