

Report for “Dimensional analysis in general relativity and differential geometry”, by P.G.L. Porta Mana.

In the submitted manuscript the author has illustrated how to perform dimensional analysis in applications of differential geometry. Although most of the parts in this manuscript the aim has been achieved, there are some misleading parts. I recommend that the submitted manuscript is suitable for publication ONLY IF the author revises the manuscript in respect to ALL of Main Issues raised by me.

Main Issues

1. This manuscript focuses on a so-called dimensional analysis in physical subjects that can be written in terms of differential geometry. In particular such analysis for tensors, without integration, was carried out, except for Eq. (20).

In physics, as emphasized in this manuscript, forms, vectors, and tensors in general have (physical) dimensions. Then integrals of forms with appropriate chains give real numbers with dimensions. In addition, curves, areas, and volumes have dimensions ($\mathbb{L}, \mathbb{L}^2, \mathbb{L}^3$, respectively). The dimension of a curve C is denoted by S as in Section 7 of the submitted manuscript. Thus please clarify a reason why the dimension of a chain does not affect that of an integral as in Eq.(20). Anyway c in Eq.(20) has not been defined.

2. There are some similar discussions in the literature. I found the following:

... *In this paper, we have only dealt with scalar quantities. It is straightforward to extend to multi-component entities, such as vectors, tensors, differential forms and so on. There, we should not forget to assign dimensions to basis vectors and basis covectors appropriately, not only to their components. For example, in the polar coordinate, using the natural cobasis $\mathbf{n}_r = \nabla r, \mathbf{n}_\theta = \nabla \theta, \mathbf{n}_\phi = \nabla \phi$, an electric vector field can be represented as $\mathbf{E} = E_r \mathbf{n}_r + E_\theta \mathbf{n}_\theta + E_\phi \mathbf{n}_\phi$. The dimension of each element is as follows: $[\mathbf{E}] = [\mathbf{E}_r] = VL^{-1}$, $[\mathbf{E}_\theta] = [\mathbf{E}_\phi] = V$, $[\mathbf{n}_r] = 1$, $[\mathbf{n}_\theta] = [\mathbf{n}_\phi] = [\nabla] = L^{-1}$. where V represents the dimension of voltage. (We assume the EUS containing the MKSA.) With the cobasis, the metric tensor is represented as $g = \sum_{ij} g_{ij} \mathbf{n}_i \otimes \mathbf{n}_j$, where the dimensions are $[g] = [g_{rr}] = 1$, $[g_{\theta\theta}] = [g_{\phi\phi}] = L^2$. As shown in these examples, the coefficients could have different dimensions. But dimensions of (co)vectors rectify them and yield the proper dimension for the vectorial or tensorial quantities, which is independent of coordinate system. These are also good examples of the Buckingham Pi theorem.* ...

Section X, Conclusion of [M. Kitano, “Mathematical structure of unit systems”, J. Math. Phys., **54**, 052901 (2013).]

- (a) Please mention this (and/or similar) paper(s) so that the reader can learn more on this topic. If the author finds more appropriate paper(s), then it is not necessary to cite the paper above.
- (b) Please mention explicitly the similarity and differences between this manuscript and the literature like above.

General Points

The following are suggestions so that the reader avoids misunderstandings.

- The word “Riemann tensor” in this manuscript is ambiguous for some audience, so if the author does not mind, please change it to “Riemann curvature tensor”.
- What is $\mathbf{A}^{\text{T}\alpha\beta}$ in Eq. (12)?
Many readers can see the transpose of components of tensors. But at least in mathematics this notion is not common. Although this is briefly explained in the last part of Section2, please add a few words more on this. I guess that the bases and co-bases are also affected by this operation $\mathbf{A} \mapsto \mathbf{A}^{\text{T}\alpha\beta}$.

I, as a referee, believe that the revised manuscript, if the author intends to revise this manuscript, is more suitable for publication. I hope that the reader can easily understand the paper as a result of this revision.