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Kinematics and dynamics from a modern perspective

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The kinematics of mechanical, thermodynamic, and electromagnetic phenomena is developed in such a way as to be used for Newtonian, Lorentzian, and general relativity. The same is done, as much as possible, for their dynamics as well

1 Scribbles and memos

1.1 Twisted objects

Denote the wedge product by juxtaposition: $\mathrm{d}t \wedge \mathrm{d}x =: \mathrm{d}t\, \mathrm{d}x$ and so on; abbreviate $\mathrm{d}t\, \mathrm{d}x =: \mathrm{d}tx$ and so on; and denote twisted differential forms by $\tilde{\mathrm{d}}$.

Where an ordered set of coordinate functions (t, x, y, z) is chosen, the twisted unit $\tilde{1}$ is defined. It has unit magnitude and outer-orientation $txyz$, and the property $\tilde{1} \cdot \tilde{1} = 1$. In general it is only defined in chart domain, where coordinate functions can be defined, but not globally. We obtain twisted vectors and covectors by multiplying their non-twisted counterparts by the twisted unit. For a vector or covector ω we have that the orientation of its twisted counterpart is such that (see Burke 1983)

$$\big\{\{\tilde{\omega}\}, \{\omega\}\big\} = \big\{\tilde{1}\big\} \,. \tag{1}$$

Said otherwise, in the product $\tilde{1} \cdot \omega$ the *right* side of the orientation of $\tilde{1}$ cancels out with the orientation of ω. This rule must be respected even if we invert the product order, so $\tilde{1} \cdot \omega \equiv \omega \cdot \tilde{1}$.

Multiplying a twisted vector or covector by the twisted unit, we obtain its non-twisted counterpart. The resulting orientation is obtained by cancelling out the orientation of the twisted object and the *left* side of the orientation of $\tilde{1}$.

We have:

$$\{\tilde{1}\} = txyz \; ; \tag{2}$$

$$\{\tilde{\mathrm{d}}t\} = -xyz \,, \quad \{\tilde{\mathrm{d}}x\} = tyz \,, \quad \{\tilde{\mathrm{d}}y\} = tzx \,, \quad \{\tilde{\mathrm{d}}z\} = txy \; ; \tag{3}$$

$$\{\tilde{\mathrm{d}}tx\} = yz \,, \quad \{\tilde{\mathrm{d}}ty\} = zx \,, \quad \{\tilde{\mathrm{d}}tz\} = xy \,, \tag{4}$$

$$\{\tilde{\mathrm{d}}xy\} = tz \,, \quad \{\tilde{\mathrm{d}}yz\} = tx \,, \quad \{\tilde{\mathrm{d}}zx\} = ty \; ; \tag{5}$$

$$\{\tilde{\mathrm{d}}tyz\} = -x \,, \quad \{\tilde{\mathrm{d}}tzx\} = -y \,, \quad \{\tilde{\mathrm{d}}txy\} = -z \,, \quad \{\tilde{\mathrm{d}}xyz\} = t \; ; \tag{6}$$

$$\{\tilde{\mathrm{d}}txyz\} = +1 \,. \tag{7}$$

The minus signs appear when we have t and an even number of other coordinates after the "$\tilde{\mathrm{d}}$".

Note that considering, say, the *function* t, we have

$$\{\tilde{t}\} = \begin{cases} \{txyz\} & \text{if } t > 0 \,, \\ -\{txyz\} & \text{if } t < 0 \,. \end{cases} \tag{8}$$

1.2 Charge and current densities

We can represent charge density and current density in one geometrical entity. Consider an ordered coordinate system (t, x, y, z). The charge-current density is

$$\rho\, \tilde{\mathrm{d}}xyz - j_x\, \tilde{\mathrm{d}}tyz - j_y\, \tilde{\mathrm{d}}tzx - j_z\, \tilde{\mathrm{d}}txy \equiv \\ \rho\, \tilde{\mathrm{d}}xyz - \mathrm{d}t\, \left(j_x\, \tilde{\mathrm{d}}yz + j_y\, \tilde{\mathrm{d}}zx + j_z\, \tilde{\mathrm{d}}xy\right) \,. \tag{9}$$

It has the dimensions of charge (current · time). The minus signs appear so that the x-component of the current, for example, is positive when $j_x > 0$, and so on.

This object automatically give us net volume charge when integrated over a three-dimensional region at constant time, or the net flux of charge when integrated over a two-dimensional surface – possibly even moving – over a lapse of time.

Bibliography

("de X" is listed under D, "van X" under V, and so on, regardless of national conventions.)

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