

Differential-geometric thermodynamics [draft]

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1 Background

Background knowledge and point of view is from Astarita¹ and Samohýl & Pekař².

A system is defined as a set of observables about a body.

We introduce a manifold of states of the system. A state represents the information that, given at time t together with some boundary conditions, allows us to determine the evolution of all observables of interest at later times. It thus also allows us to determine the state at later times.

The manifold of states is usually coordinatized by a subset of the system's observable, for example temperature T and volume V . Here we avoid as much as possible the use of specific coordinates. This allows for a presentation that doesn't rely on partial derivatives.

A process is time-dependent sequence of states, represented by a function from a real interval to the state manifold.

Some process are admissible, others inadmissible. Each inadmissible process is physically impossible not for dynamical reasons, but for kinematic ones. For example, it may contradict the very definition of state.

An important example is a process in a phase space, which we can coordinatize with position and momentum, (x, mv) . A process $t \mapsto (x(t), mv(t))$ for which $dx(t)/dt \neq v(t)$ is obviously inadmissible, because the rate of change of position isn't equal to velocity – which by definition is the rate of change of position. Admissible processes are therefore only those for which $dx(t)/dt = v(t)$.

¹ Astarita 1990. ² Samohýl & Pekař 2014.

Bibliography

(“de X ” is listed under D , “van X ” under V , and so on, regardless of national conventions.)

Astarita, G. (1990): *Thermodynamics: An Advanced Textbook for Chemical Engineers*, 2nd pr. (Springer, New York). First publ. 1989.

Samohýl, I., Pekař, M. (2014): *The Thermodynamics of Linear Fluids and Fluid Mixtures*. (Springer, Cham). First published as **samohyl1987**.