In[1]:=
```
<< "christoffelsymbols.m"
(* First index is upper index Table[FS[cc[[ii,;;,;;]]==T[cc[[ii,;;,;;]]]],{ii,1,4}] *)
```

In[2]:=
```
(*showf[assumptions_,simp_:FullSimplify]:=((Assuming[assumptions,Expand//@simp@PowerExpand[#]]//MF)&);*)
```

In[3]:=
```
(* Show matrix expressions and power expansions*)
(*
show[assumptions_,power_,simp_:FullSimplify]:=({Assuming[assumptions,simp@PowerExpand[#]]//MF,"\n",
    Assuming[assumptions,simp@PowerExpand[Series[#,{c,Infinity,power}]]]//MF}&);
shows[assumptions_,power_,simp_:FullSimplify]:=((
    Assuming[assumptions,simp@PowerExpand[Series[#,{c,Infinity,power}]]]//MF)&);
showf[assumptions_,simp_:FullSimplify]:=((Assuming[assumptions,Expand//@simp@PowerExpand[#]]//MF)&);
show1[assumptions_,simp_:Identity]:=((Assuming[assumptions,simp[#]]//MF)&);
show2[assumptions_,power_,simp_:Identity]:=((
    Assuming[assumptions,simp[Series[#,{c,Infinity,power}]]]//MF)&);
*)
```

In[4]:=
```
coords = {t, x, y, z}
```

Out[4]= $\{t, x, y, z\}$

In[5]:=
```
(* Flat metric *)
(gg0 = DiagonalMatrix[{-c^2, 1, 1, 1}]) // MF
```

Out[5]//MatrixForm=

$$\begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[6]:=
```
(*D[G*M/Sqrt[x^2+y^2+z^2],{{x,y,z}}] *)
```

In[7]:=
```
(* -W is the potential gravitational energy: W=GM/r
  that is, F_g(downwards)=grad W
*)
```

In[8]:=
```
(* Rotating metric from poissonetal *)
(gg = {{-c^2*(1-2*W/c^2 +2*W^2/c^4)+O[c, +Infinity]^3,
     -4*Wx/c^2+O[c, +Infinity]^4, -4*Wy/c^2+O[c, +Infinity]^4, -4*Wz/c^2+O[c, +Infinity]^4},
    {-4*Wx/c^2+O[c, +Infinity]^4,
     1+2*W/c^2+O[c, +Infinity]^4, 0, 0},
    {-4*Wy/c^2+O[c, +Infinity]^4, 0, 1+2*W/c^2+O[c, +Infinity]^4, 0},
    {-4*Wz/c^2+O[c, +Infinity]^4, 0, 0, 1+2*W/c^2+O[c, +Infinity]^4}}) // MF
(*(gg=DiagonalMatrix[{-c^2*(1+2*Φ[t,r]/c^2),1+2*Λ[t,r]/c^2,r^2,r^2*Sin[θ]^2}])//MF*)
```

Out[8]//MatrixForm=

$$\begin{pmatrix} -c^2+2W-\frac{2W^2}{c^2}+O\left[\frac{1}{c}\right]^3 & -\frac{4Wx}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{4Wy}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{4Wz}{c^2}+O\left[\frac{1}{c}\right]^4 \\ -\frac{4Wx}{c^2}+O\left[\frac{1}{c}\right]^4 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 0 \\ -\frac{4Wy}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 \\ -\frac{4Wz}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 0 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[9]:=
```
(gg = (gg /. {Wx → O[c, +Infinity]*c, Wy → O[c, +Infinity]*c, Wz → O[c, +Infinity]*c})) // MF
```

Out[9]//MatrixForm=

$$\begin{pmatrix} -c^2+2W-\frac{2W^2}{c^2}+O\left[\frac{1}{c}\right]^3 & O\left[\frac{1}{c}\right]^2 & O\left[\frac{1}{c}\right]^2 & O\left[\frac{1}{c}\right]^2 \\ O\left[\frac{1}{c}\right]^2 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 0 \\ O\left[\frac{1}{c}\right]^2 & 0 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 \\ O\left[\frac{1}{c}\right]^2 & 0 & 0 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[10]:=
```
Inverse[gg] // MF
```

Out[10]//MatrixForm=

$$\begin{pmatrix} -\left(\frac{1}{c}\right)^2-\frac{2W}{c^4}+O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 \\ O\left[\frac{1}{c}\right]^4 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 \\ O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^6 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^6 \\ O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[11]:=
```
(*(gg=DiagonalMatrix@Diagonal[gg])//MF*)
```

In[12]:=
```
(* functions to temporarily remove coord-dep *)
tW[xx_] := (xx /. {W → W[t, x, y, z], Wx → Wx[t, x, y, z], Wy → Wy[t, x, y, z], Wz → Wz[t, x, y, z]});
itW[xx_] := (xx /. {W[t, x, y, z] → W, Wx[t, x, y, z] → Wx, Wy[t, x, y, z] → Wy, Wz[t, x, y, z] → Wz});
(ggt = tW[gg]) // MF;
```

In[15]:=
```
assut = {c > 0, Element[a, Reals], Element[v, Reals], Element[t, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals],
   Element[vx, Reals], Element[vy, Reals], Element[vz, Reals], Element[n, Reals], Element[r, Reals], Element[θ, Reals], Element[ϕ, Reals], Abs[v] < c, -c < vx < c, -c < ux < c, r > 0, 0 < θ < Pi,
   Normal@gg[[1, 1]] / c^2 < 0, Normal@gg[[2, 2]] > 0, Normal@gg[[3, 3]] > 0, Normal@gg[[4, 4]] > 0, n > 0, Element[jx, Reals], Element[jy, Reals], Element[jz, Reals], Element[sxx, Reals], Element[sxy, Reals], Element[sxz, Reals], Element[syy, Reals], Element[syz, Reals], Element[szz, Reals],
   -Normal@Det[gg] > 0, β > 0};

assutt = {c > 0, Element[a, Reals], Element[v, Reals], Element[t, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals],
   Element[vx, Reals], Element[vy, Reals], Element[vz, Reals], Element[n, Reals], Element[r, Reals], Element[θ, Reals], Element[ϕ, Reals], Abs[v] < c, -c < vx < c, -c < ux < c, r > 0, 0 < θ < Pi,
   β > 0, Normal@ggt[[1, 1]] / c^2 < 0, Normal@ggt[[2, 2]] > 0, Normal@ggt[[3, 3]] > 0, Normal@ggt[[4, 4]] > 0,
   -Normal@Det[ggt] > 0};
```

In[17]:=
```
(igg = Assuming[assut, FullSimplify@PowerExpand[Inverse[gg]]]) // MF
```

Out[17]//MatrixForm=

$$\begin{pmatrix} -\left(\frac{1}{c}\right)^2-\frac{2W}{c^4}+O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 \\ O\left[\frac{1}{c}\right]^4 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 \\ O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^6 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^6 \\ O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[18]:=
```
(*show[assut,2]@ChristoffelSymbol[gg,coords][[2]]*)
```

In[19]:=
```
(* volume element *)
(dg = Assuming[assut, FullSimplify@PowerExpand[Sqrt[-Det[gg]] / c]]) // MF
```

Out[19]//MatrixForm=

$$1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4$$

In[20]:=
```
(* Christoffel symbols  *)
cc = Assuming[assut, FullSimplify@PowerExpand[itW[ChristoffelSymbol[ggt, coords]]]];
```

In[21]:=
```
(* 3-vector of moving surface parallel to yz moving with velocity V   *)
surface = {-(Vx*Ax+Vy*Ay+Vz*Az), Ax, Ay, Az}*Δt;
```

In[22]:=

(* Matter current *)

In[23]:=
```
(* matter-current 3-covector *)
NJ = {n, jx, jy, jz};
```

In[24]:=
```
(* norm of matter 3-covector *)
Assuming[assut, FS[Sqrt[-NJ.gg.NJ] / c]]
```

Out[24]= $n-\frac{jx^2+jy^2+jz^2+2n^2W}{2nc^2}+O\left[\frac{1}{c}\right]^4$

In[25]:=
```
(* matter associated 1-vector *)
(NJvec = Assuming[assut, FS[NJ / dg]]) // MF
```

Out[25]//MatrixForm=

$$\begin{pmatrix} n-\frac{2(nW)}{c^2}+O\left[\frac{1}{c}\right]^4 \\ jx-\frac{2(jxW)}{c^2}+O\left[\frac{1}{c}\right]^4 \\ jy-\frac{2(jyW)}{c^2}+O\left[\frac{1}{c}\right]^4 \\ jz-\frac{2(jzW)}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[26]:=
```
(* matter associated 4-vel vector *)
(uu = Assuming[assut, FS[c*NJvec / Sqrt[-NJvec.gg.NJvec]]]) // MF
```

Out[26]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{jx^2+jy^2+jz^2}{2n^2}+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jx}{n}+\frac{jx\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n}+\frac{jy\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n}+\frac{jz\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[27]:=
```
(* replace matter flux in terms of velocity*)
replaceJu = {jx → ux*n, jy → uy*n, jz → uz*n}
```

Out[27]= $\{jx \to n\,ux,\ jy \to n\,uy,\ jz \to n\,uz\}$

In[28]:=
```
(* collect velocity magnitude*)
replaceuUnorm = {ux^2 → U^2-uy^2-uz^2, ux^3 → ux*(U^2-uy^2-uz^2), jx^2 → J^2-jy^2-jz^2, jx^3 → jx*(J^2-jy^2-jz^2)}
```

Out[28]= $\{ux^2 \to U^2-uy^2-uz^2,\ ux^3 \to ux\left(U^2-uy^2-uz^2\right),\ jx^2 \to J^2-jy^2-jz^2,\ jx^3 \to jx\left(J^2-jy^2-jz^2\right)\}$

In[29]:=
```
Assuming[assut, FS[uu /. replaceJu]] // MF
```

Out[29]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{1}{2}(ux^2+uy^2+uz^2)+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ ux+\frac{ux\left(ux^2+uy^2+uz^2+2W\right)}{2c^2}+O\left[\frac{1}{c}\right]^4 \\ uy+\frac{uy\left(ux^2+uy^2+uz^2+2W\right)}{2c^2}+O\left[\frac{1}{c}\right]^4 \\ uz+\frac{uz\left(ux^2+uy^2+uz^2+2W\right)}{2c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[30]:=
```
(* it is normalized *)
FS[uu.gg.uu]
```

Out[30]= $-c^2+O\left[\frac{1}{c}\right]^2$

In[31]:=
```
(* matter associated 1-covector *)
(NJcov = Assuming[assut, FS[gg.(NJ / dg)]]) // MF
```

Out[31]//MatrixForm=

$$\begin{pmatrix} -n\,c^2+4\,n\,W+O\left[\frac{1}{c}\right]^2 \\ jx+O\left[\frac{1}{c}\right]^2 \\ jy+O\left[\frac{1}{c}\right]^2 \\ jz+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[32]:=
```
(* scalar product with de/de_x (for momentum x-component) *)
Simplify[uu.gg.{0, 1, 0, 0}]
```

Out[32]= $\frac{jx}{n}+O\left[\frac{1}{c}\right]^2$

In[33]:=
```
(* scalar product with de/de_i (for momentum i-component) *)
Simplify[uu.gg] // MF
```

Out[33]//MatrixForm=

$$\begin{pmatrix} -c^2+\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)+O\left[\frac{1}{c}\right]^2 \\ \frac{jx}{n}+O\left[\frac{1}{c}\right]^2 \\ \frac{jy}{n}+O\left[\frac{1}{c}\right]^2 \\ \frac{jz}{n}+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[34]:=
```
(* retransform matter 4-vel to matter 3-covector *)
Assuming[assut, FS[uu * dg / c * Sqrt[-NJvec.gg.NJvec]]] // MF
```

Out[34]//MatrixForm=

$$\begin{pmatrix} n+O\left[\frac{1}{c}\right]^4 \\ jx+O\left[\frac{1}{c}\right]^4 \\ jy+O\left[\frac{1}{c}\right]^4 \\ jz+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[35]:=
```
(* simplification to x-directed matter flux and velocity *)
assutjx = Join[assut, {jy == 0, jz == 0, uy == 0, uz == 0}];
```

In[36]:=
```
(* flux of matter across surface *)
Simplify[surface.NJ / (Δt)]
```

Out[36]= $Ax\,(jx-n\,Vx)+Ay\,(jy-n\,Vy)+Az\,(jz-n\,Vz)$

In[37]:=
```
(* normalized zero-flux velocity is same as U *)
vnoflux = {1, jx / n, jy / n, jz / n};
FS[c * vnoflux / Sqrt[-vnoflux.gg.vnoflux] == uu]
```

Out[38]= True

In[39]:=
```
FS[uu /. replaceuUnorm] // MF
```

Out[39]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{J^2}{2n^2}+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jx}{n}+\frac{jx\left(J^2+2n^2W\right)}{2n^3c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n}+\frac{jy\left(J^2+2n^2W\right)}{2n^3c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n}+\frac{jz\left(J^2+2n^2W\right)}{2n^3c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[40]:=
```
FS[uu /. replaceJu] // MF
```

Out[40]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{1}{2}(ux^2+uy^2+uz^2)+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ ux+\frac{ux\left(ux^2+uy^2+uz^2+2W\right)}{2c^2}+O\left[\frac{1}{c}\right]^4 \\ uy+\frac{uy\left(ux^2+uy^2+uz^2+2W\right)}{2c^2}+O\left[\frac{1}{c}\right]^4 \\ uz+\frac{uz\left(ux^2+uy^2+uz^2+2W\right)}{2c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[41]:=

In[42]:=
```
(* Project along u velocity *)
proju = Assuming[assut, Expand //@ FS@PowerExpand[-Outer[Times, uu, gg.uu] / c ^ 2]];
projperpu = Assuming[assut, Expand //@ FS@PowerExpand[IdentityMatrix[4] - proju]];
testproj[ass_, x_] := showf[ass] /@ {Assuming[ass, Expand //@ FS@PowerExpand[proju.x.proju == x]], projperpu.x.proju == x, proju.x.projperpu == x, projperpu.x.projperpu == x}
```

In[45]:=
```
(* Project along u velocity *)
proju = Assuming[assut, FS[-Outer[Times, uu, gg.uu] / c ^ 2]];
projperpu = Assuming[assut, FS[IdentityMatrix[4] - proju]];
testproj[ass_, x_] := showf[ass] /@ {Assuming[ass, Expand //@ FS@PowerExpand[proju.x.proju == x]], projperpu.x.proju == x, proju.x.projperpu == x, projperpu.x.projperpu == x}
```

In[48]:=
```
(* aux 4-velocity *)
auu = {temp, aux, auy, auz};
solu = FS[temp /. Solve[Normal[auu.gg.auu] == -c ^ 2, temp][[2]]]
```

Out[49]= $\frac{\sqrt{aux^2+auy^2+auz^2+c^2}}{\sqrt{c^2-2W}}$

In[50]:=
```
(auu = Assuming[assut, FS[auu /. {temp → solu}]]) // MF
```

Out[50]//MatrixForm=

$$\begin{pmatrix} \frac{\sqrt{aux^2+auy^2+auz^2+c^2}}{\sqrt{c^2-2W}} \\ aux \\ auy \\ auz \end{pmatrix}$$

In[51]:=
```
FS[auu.gg.auu]
```

Out[51]= $-c^2+O\left[\frac{1}{c}\right]^2$

In[52]:=
```
(* 4-velocity and matter current with explicit coordinate dependence *)
tjv[xx_] := (xx /. {n → n[t, x, y, z], jx → ux[t, x, y, z] * n[t, x, y, z], jy → uy[t, x, y, z] * n[t, x, y, z], jz → uz[t, x, y, z] * n[t, x, y, z]});
tjn[xx_] := (xx /. {n → n[t, x, y, z], jx → jx[t, x, y, z], jy → jy[t, x, y, z], jz → jz[t, x, y, z]});
itjn[xx_] := (xx /. {n[t, x, y, z] → n, jx[t, x, y, z] → jx, jy[t, x, y, z] → jy, jz[t, x, y, z] → jz});
itjv[xx_] := (xx /. {n[t, x, y, z] → n, ux[t, x, y, z] → jx / n, uy[t, x, y, z] → jy / n, uz[t, x, y, z] → jz / n});
repjn = {D[n[t, x, y, z], t] → D[jx[t, x, y, z], x] + D[jy[t, x, y, z], y] + D[jz[t, x, y, z], z]};
MF /@ {uut = Assuming[assut, FS[tjn[tW[uu]]]], uuv = Assuming[assut, FS[tjv[tW[uu]]]]}
```

Out[57]=

$$\left\{\begin{pmatrix} 1+\frac{\frac{jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2}{2\,n[t,x,y,z]^2}+W[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jx[t,x,y,z]}{n[t,x,y,z]}+\frac{jx[t,x,y,z]\left(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z]\right)}{2\,n[t,x,y,z]^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy[t,x,y,z]}{n[t,x,y,z]}+\frac{jy[t,x,y,z]\left(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z]\right)}{2\,n[t,x,y,z]^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz[t,x,y,z]}{n[t,x,y,z]}+\frac{jz[t,x,y,z]\left(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z]\right)}{2\,n[t,x,y,z]^3\,c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} 1+\frac{\frac{1}{2}\left(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2\right)+W[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \\ ux[t,x,y,z]+\frac{ux[t,x,y,z]\left(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z]\right)}{2c^2}+O\left[\frac{1}{c}\right]^4 \\ uy[t,x,y,z]+\frac{uy[t,x,y,z]\left(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z]\right)}{2c^2}+O\left[\frac{1}{c}\right]^4 \\ uz[t,x,y,z]+\frac{uz[t,x,y,z]\left(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z]\right)}{2c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[58]:=

In[59]:=

(* Construction of energy–momentum tensor *)

In[60]:=
```
(* definition of heat-flux, orthogonal to matter-current *)
Qtemp = {qt, qx, qy, qz};
```

In[61]:=
```
(proju.Qtemp) // MF
```

Out[61]//MatrixForm=

$$\begin{pmatrix} qt+\frac{\frac{(jx^2+jy^2+jz^2)\,qt}{n^2}-\frac{jx\,qx}{n}-\frac{jy\,qy}{n}-\frac{jz\,qz}{n}}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jx\,qt}{n}+\frac{\frac{jx\,(jx^2+jy^2+jz^2)\,qt}{n^3}-\frac{jx^2\,qx}{n^2}-\frac{jx\,jy\,qy}{n^2}-\frac{jx\,jz\,qz}{n^2}}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy\,qt}{n}+\frac{\frac{jy\,(jx^2+jy^2+jz^2)\,qt}{n^3}-\frac{jx\,jy\,qx}{n^2}-\frac{jy^2\,qy}{n^2}-\frac{jy\,jz\,qz}{n^2}}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz\,qt}{n}+\frac{\frac{jz\,(jx^2+jy^2+jz^2)\,qt}{n^3}-\frac{jx\,jz\,qx}{n^2}-\frac{jy\,jz\,qy}{n^2}-\frac{jz^2\,qz}{n^2}}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[62]:=
```
qsol = Solve[Normal[proju.Qtemp] == 0, qt][[1]]
```

Out[62]= $\left\{qt \to \frac{n\,(jx\,qx+jy\,qy+jz\,qz)}{jx^2+jy^2+jz^2+c^2\,n^2}\right\}$

In[63]:=
```
(Q = Assuming[assut, FS[Qtemp /. qsol]]) // MF
```
Out[63]//MatrixForm=

$$\begin{pmatrix} \frac{n\left(jx\,qx+jy\,qy+jz\,qz\right)}{jx^2+jy^2+jz^2+c^2\,n^2} \\ qx \\ qy \\ qz \end{pmatrix}$$

In[64]:=
```
{Normal@Series[Q.NJ, {c, Infinity, 1}] == 0}
```
Out[64]= $\left\{jx\,qx+jy\,qy+jz\,qz == 0\right\}$

In[65]:=
```
assutQ = Join[assut, {Normal@Series[Q.NJ, {c, Infinity, 1}] == 0}];
```
In[66]:=
```
Assuming[assutQ, FS@{proju.Q, projperpu.Q == Q}]
```
Out[66]= $\left\{\left\{O\left[\frac{1}{c}\right]^4, O\left[\frac{1}{c}\right]^4, O\left[\frac{1}{c}\right]^4, O\left[\frac{1}{c}\right]^4\right\}, \text{True}\right\}$

In[67]:=
```
(* non-symmetric heat-tensor *)
Assuming[assutQ, FS[Qtens = Assuming[assut, Expand //@ FS@PowerExpand[Outer[Times, Q, gg.uu / c^2]]]]] // MF
```
Out[67]//MatrixForm=

$$\begin{pmatrix} O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 \\ -qx+\frac{-\frac{\left(jx^2+jy^2+jz^2\right)qx}{2n^2}+qx\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jx\,qx}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jy\,qx}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jz\,qx}{n\,c^2}+O\left[\frac{1}{c}\right]^4 \\ -qy+\frac{-\frac{\left(jx^2+jy^2+jz^2\right)qy}{2n^2}+qy\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jx\,qy}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jy\,qy}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jz\,qy}{n\,c^2}+O\left[\frac{1}{c}\right]^4 \\ -qz+\frac{-\frac{\left(jx^2+jy^2+jz^2\right)qz}{2n^2}+qz\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jx\,qz}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jy\,qz}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jz\,qz}{n\,c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[68]:=
```
Assuming[assutQ, FS[T[Qtens.Inverse[gg]].gg - Qtens]] // MF
```
Out[68]//MatrixForm=

$$\begin{pmatrix} O\left[\frac{1}{c}\right]^4 & \frac{qx}{c^2}+\frac{\frac{\left(jx^2+jy^2+jz^2\right)qx}{2n^2}+3\,qx\,W}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{qy}{c^2}+\frac{\frac{\left(jx^2+jy^2+jz^2\right)qy}{2n^2}+3\,qy\,W}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{qz}{c^2}+\frac{\frac{\left(jx^2+jy^2+jz^2\right)qz}{2n^2}+3\,qz\,W}{c^4}+O\left[\frac{1}{c}\right]^6 \\ qx+\frac{\frac{\left(jx^2+jy^2+jz^2\right)qx}{2n^2}-qx\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & \frac{-jy\,qx+jx\,qy}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & \frac{-jz\,qx+jx\,qz}{n\,c^2}+O\left[\frac{1}{c}\right]^4 \\ qy+\frac{\frac{\left(jx^2+jy^2+jz^2\right)qy}{2n^2}-qy\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jy\,qx-jx\,qy}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & \frac{-jz\,qy+jy\,qz}{n\,c^2}+O\left[\frac{1}{c}\right]^4 \\ qz+\frac{\frac{\left(jx^2+jy^2+jz^2\right)qz}{2n^2}-qz\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jz\,qx-jx\,qz}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jz\,qy-jy\,qz}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[69]:=
```
(* definition of momentum-flux, orthogonal to matter-current *)
Ptemp = {pt, px, py, pz};
```
In[70]:=
```
Assuming[assut, FS[Ptemp.proju]] // MF
```
Out[70]//MatrixForm=

$$\begin{pmatrix} \frac{n\,pt+jx\,px+jy\,py+jz\,pz}{n}+\frac{\left(jx^2+jy^2+jz^2\right)\left(n\,pt+jx\,px+jy\,py+jz\,pz\right)}{n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\ -\frac{jx\left(n\,pt+jx\,px+jy\,py+jz\,pz\right)}{n^2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ -\frac{jy\left(n\,pt+jx\,px+jy\,py+jz\,pz\right)}{n^2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ -\frac{jz\left(n\,pt+jx\,px+jy\,py+jz\,pz\right)}{n^2\,c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[71]:=
```
psol = Solve[Normal[Ptemp.proju] == 0, pt][[1]]
```
Out[71]= $\left\{pt \to -\frac{jx\,px+jy\,py+jz\,pz}{n}\right\}$

In[72]:=
```
(P = Assuming[assut, FS[Ptemp /. psol]]) // MF
```
Out[72]//MatrixForm=

$$\begin{pmatrix} -\frac{jx\,px+jy\,py+jz\,pz}{n} \\ px \\ py \\ pz \end{pmatrix}$$

In[73]:=
```
{FS[Normal@Series[P.uu, {c, Infinity, 1}]] == 0}
```
Out[73]= {True}

In[74]:=
```
Assuming[assutQ, FS@{P.proju, P.projperpu == P}]
```
Out[74]= $\left\{\left\{O\left[\frac{1}{c}\right]^4, O\left[\frac{1}{c}\right]^4, O\left[\frac{1}{c}\right]^4, O\left[\frac{1}{c}\right]^4\right\}, \text{True}\right\}$

In[75]:=
```
(* non-symmetric momentum-tensor *)
Assuming[assut, FS[Ptens = Assuming[assut, FS[Outer[Times, uu, P / c^2]]]]] // MF
```
Out[75]//MatrixForm=

$$\begin{pmatrix} -\frac{jx\,px+jy\,py+jz\,pz}{n\,c^2}-\frac{\left(jx\,px+jy\,py+jz\,pz\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2\,n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{px}{c^2}+\frac{px\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{py}{c^2}+\frac{py\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{pz}{c^2}+\frac{pz\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O\left[\frac{1}{c}\right]^6 \\ -\frac{jx\left(jx\,px+jy\,py+jz\,pz\right)}{n^2\,c^2}-\frac{jx\left(jx\,px+jy\,py+jz\,pz\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2\,n^4\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jx\,px}{n\,c^2}+\frac{jx\,px\left(jx^2+jy^2+jz^2+2n^2W\right)}{2\,n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jx\,py}{n\,c^2}+\frac{jx\,py\left(jx^2+jy^2+jz^2+2n^2W\right)}{2\,n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jx\,pz}{n\,c^2}+\frac{jx\,pz\left(jx^2+jy^2+jz^2+2n^2W\right)}{2\,n^3\,c^4}+O\left[\frac{1}{c}\right]^6 \\ -\frac{jy\left(jx\,px+jy\,py+jz\,pz\right)}{n^2\,c^2}-\frac{jy\left(jx\,px+jy\,py+jz\,pz\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2\,n^4\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jy\,px}{n\,c^2}+\frac{jy\,px\left(jx^2+jy^2+jz^2+2n^2W\right)}{2\,n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jy\,py}{n\,c^2}+\frac{jy\,py\left(jx^2+jy^2+jz^2+2n^2W\right)}{2\,n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jy\,pz}{n\,c^2}+\frac{jy\,pz\left(jx^2+jy^2+jz^2+2n^2W\right)}{2\,n^3\,c^4}+O\left[\frac{1}{c}\right]^6 \\ -\frac{jz\left(jx\,px+jy\,py+jz\,pz\right)}{n^2\,c^2}-\frac{jz\left(jx\,px+jy\,py+jz\,pz\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2\,n^4\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jz\,px}{n\,c^2}+\frac{jz\,px\left(jx^2+jy^2+jz^2+2n^2W\right)}{2\,n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jz\,py}{n\,c^2}+\frac{jz\,py\left(jx^2+jy^2+jz^2+2n^2W\right)}{2\,n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jz\,pz}{n\,c^2}+\frac{jz\,pz\left(jx^2+jy^2+jz^2+2n^2W\right)}{2\,n^3\,c^4}+O\left[\frac{1}{c}\right]^6 \end{pmatrix}$$

In[76]:=
```
FS[T[Ptens.Inverse[gg]].gg - Ptens] // MF
```
Out[76]//MatrixForm=

$$\begin{pmatrix} O\left[\frac{1}{c}\right]^4 & -\frac{px}{c^2}+\frac{\frac{jx\left(jx\,px+jy\,py+jz\,pz\right)}{n^2}-px\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{py}{c^2}+\frac{\frac{jy\left(jx\,px+jy\,py+jz\,pz\right)}{n^2}-py\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{pz}{c^2}+\frac{\frac{jz\left(jx\,px+jy\,py+jz\,pz\right)}{n^2}-pz\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O\left[\frac{1}{c}\right]^6 \\ -px+\frac{jx^2\,px+2\,jx\left(jy\,py+jz\,pz\right)-px\left(jy^2+jz^2-6\,n^2\,W\right)}{2\,n^2\,c^2}+O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & \frac{jy\,px-jx\,py}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jz\,px-jx\,pz}{n\,c^2}+O\left[\frac{1}{c}\right]^4 \\ -py+\frac{-\frac{\left(jx^2+jy^2+jz^2\right)py}{2n^2}+\frac{jy\left(jx\,px+jy\,py+jz\,pz\right)}{n^2}+3\,py\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{-jy\,px+jx\,py}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & \frac{jz\,py-jy\,pz}{n\,c^2}+O\left[\frac{1}{c}\right]^4 \\ -pz+\frac{-\frac{\left(jx^2+jy^2+jz^2\right)pz}{2n^2}+\frac{jz\left(jx\,px+jy\,py+jz\,pz\right)}{n^2}+3\,pz\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{-jz\,px+jx\,pz}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & \frac{-jz\,py+jy\,pz}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[77]:=
```
(* definition of stress, orthogonal to matter-current *)
(Stemp = {{stt, stx, sty, stz}, {sxt, sxx, sxy, sxz}, {syt, syx, syy, syz}, {szt, szx, szy, szz}}) // MF
```
Out[77]//MatrixForm=

$$\begin{pmatrix} stt & stx & sty & stz \\ sxt & sxx & sxy & sxz \\ syt & syx & syy & syz \\ szt & szx & szy & szz \end{pmatrix}$$

In[78]:=
```
(Stempsym = Assuming[assut, FS[(T[Stemp.Inverse[gg]].gg + Stemp) / 2]]) // MF
```
Out[78]//MatrixForm=

$$\begin{pmatrix} stt+O\left[\frac{1}{c}\right]^2 & \frac{stx}{2}-\frac{sxt}{2c^2}+O\left[\frac{1}{c}\right]^4 & \frac{sty}{2}-\frac{syt}{2c^2}+O\left[\frac{1}{c}\right]^4 & \frac{stz}{2}-\frac{szt}{2c^2}+O\left[\frac{1}{c}\right]^4 \\ -\frac{stx\,c^2}{2}+\frac{1}{2}\left(sxt+4\,stx\,W\right)+O\left[\frac{1}{c}\right]^2 & sxx+O\left[\frac{1}{c}\right]^2 & \frac{sxy+syx}{2}+O\left[\frac{1}{c}\right]^2 & \frac{sxz+szx}{2}+O\left[\frac{1}{c}\right]^2 \\ -\frac{sty\,c^2}{2}+\frac{1}{2}\left(syt+4\,sty\,W\right)+O\left[\frac{1}{c}\right]^2 & \frac{sxy+syx}{2}+O\left[\frac{1}{c}\right]^2 & syy+O\left[\frac{1}{c}\right]^2 & \frac{syz+szy}{2}+O\left[\frac{1}{c}\right]^2 \\ -\frac{stz\,c^2}{2}+\frac{1}{2}\left(szt+4\,stz\,W\right)+O\left[\frac{1}{c}\right]^2 & \frac{sxz+szx}{2}+O\left[\frac{1}{c}\right]^2 & \frac{syz+szy}{2}+O\left[\frac{1}{c}\right]^2 & szz+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[79]:=
```
FS[proju.Stemp.proju] // MF
```
Out[79]//MatrixForm=

$$\begin{pmatrix} \frac{n\,stt+jx\,stx+jy\,sty+jz\,stz}{n}+\frac{2\,jx^3\,stx+2\,jy^3\,sty+jx^2\left(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx\right)-jy\,n^2\,syt+jy^2\left(2\,n\,stt+2\,jz\,stz-n\,syy\right)+jx\left(2\,jy^2\,stx-n^2\,sxt-jy\,n\left(sxy+syx\right)+jz\left(2\,jz\,stx-n\left(sxz+szx\right)\right)\right)+jy\,jz\left(2\,jz\,sty-n\left(syz+szy\right)\right)+jz\left(2\,jz^2\,stz-n^2\,szt+jz\,n\left(2\,stt-szz\right)\right)}{n^3\,c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{jx\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^2\,c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{jy\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^2\,c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{jz\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jx\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^2}+\frac{jx\left(2\,jx^3\,stx+2\,jy^3\,sty+jx^2\left(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx\right)-jy\,n^2\,syt+jy^2\left(2\,n\,stt+2\,jz\,stz-n\,syy\right)+jx\left(2\,jy^2\,stx-n^2\,sxt-jy\,n\left(sxy+syx\right)+jz\left(2\,jz\,stx-n\left(sxz+szx\right)\right)\right)+jy\,jz\left(2\,jz\,sty-n\left(syz+szy\right)\right)+jz\left(2\,jz^2\,stz-n^2\,szt+jz\,n\left(2\,stt-szz\right)\right)\right)}{n^4\,c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{jx^2\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^3\,c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{jx\,jy\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^3\,c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{jx\,jz\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^2}+\frac{jy\left(2\,jx^3\,stx+2\,jy^3\,sty+jx^2\left(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx\right)-jy\,n^2\,syt+jy^2\left(2\,n\,stt+2\,jz\,stz-n\,syy\right)+jx\left(2\,jy^2\,stx-n^2\,sxt-jy\,n\left(sxy+syx\right)+jz\left(2\,jz\,stx-n\left(sxz+szx\right)\right)\right)+jy\,jz\left(2\,jz\,sty-n\left(syz+szy\right)\right)+jz\left(2\,jz^2\,stz-n^2\,szt+jz\,n\left(2\,stt-szz\right)\right)\right)}{n^4\,c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{jx\,jy\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^3\,c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{jy^2\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^3\,c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{jy\,jz\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^2}+\frac{jz\left(2\,jx^3\,stx+2\,jy^3\,sty+jx^2\left(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx\right)-jy\,n^2\,syt+jy^2\left(2\,n\,stt+2\,jz\,stz-n\,syy\right)+jx\left(2\,jy^2\,stx-n^2\,sxt-jy\,n\left(sxy+syx\right)+jz\left(2\,jz\,stx-n\left(sxz+szx\right)\right)\right)+jy\,jz\left(2\,jz\,sty-n\left(syz+szy\right)\right)+jz\left(2\,jz^2\,stz-n^2\,szt+jz\,n\left(2\,stt-szz\right)\right)\right)}{n^4\,c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{jx\,jz\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^3\,c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{jy\,jz\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^3\,c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{jz^2\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[80]:=
```
ssol = Solve[{Normal[proju.Stemp.proju] == 0, Normal[projperpu.Stemp.projperpu] == Stemp}, {stt, stx, sty, stz, sxt, syt, szt}][[1]]
```
Out[80]= $\left\{stt \to -\frac{jx^2\,sxx+jx\,jy\,sxy+jx\,jz\,sxz+jx\,jy\,syx+jy^2\,syy+jy\,jz\,syz+jx\,jz\,szx+jy\,jz\,szy+jz^2\,szz}{jx^2+jy^2+jz^2+c^2\,n^2},\ stx \to \frac{n\left(jx\,sxx+jy\,syx+jz\,szx\right)}{jx^2+jy^2+jz^2+c^2\,n^2},\ sty \to \frac{n\left(jx\,sxy+jy\,syy+jz\,szy\right)}{jx^2+jy^2+jz^2+c^2\,n^2},\ stz \to \frac{n\left(jx\,sxz+jy\,syz+jz\,szz\right)}{jx^2+jy^2+jz^2+c^2\,n^2},\ sxt \to -\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n},\ syt \to -\frac{jx\,syx+jy\,syy+jz\,syz}{n},\ szt \to -\frac{jx\,szx+jy\,szy+jz\,szz}{n}\right\}$

In[81]:=
```
ssol = Solve[Normal[proju.Stemp] == 0 && Normal[Stemp.proju] == 0, {stt, stx, sty, stz, sxt, syt, szt}][[1]]
```
Out[81]= $\left\{stt \to -\frac{jx^2\,sxx+jx\,jy\,sxy+jx\,jz\,sxz+jx\,jy\,syx+jy^2\,syy+jy\,jz\,syz+jx\,jz\,szx+jy\,jz\,szy+jz^2\,szz}{jx^2+jy^2+jz^2+c^2\,n^2},\ stx \to \frac{n\left(jx\,sxx+jy\,syx+jz\,szx\right)}{jx^2+jy^2+jz^2+c^2\,n^2},\ sty \to \frac{n\left(jx\,sxy+jy\,syy+jz\,szy\right)}{jx^2+jy^2+jz^2+c^2\,n^2},\ stz \to \frac{n\left(jx\,sxz+jy\,syz+jz\,szz\right)}{jx^2+jy^2+jz^2+c^2\,n^2},\ sxt \to -\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n},\ syt \to -\frac{jx\,syx+jy\,syy+jz\,syz}{n},\ szt \to -\frac{jx\,szx+jy\,szy+jz\,szz}{n}\right\}$

In[82]:=
```
(S = Assuming[assut, FS[(Stemp /. ssol)]]) // MF
```
Out[82]//MatrixForm=

$$\begin{pmatrix} -\frac{jx^2\,sxx+jx\,jy\left(sxy+syx\right)+jy^2\,syy+jx\,jz\left(sxz+szx\right)+jy\,jz\left(syz+szy\right)+jz^2\,szz}{jx^2+jy^2+jz^2+c^2\,n^2} & \frac{n\left(jx\,sxx+jy\,syx+jz\,szx\right)}{jx^2+jy^2+jz^2+c^2\,n^2} & \frac{n\left(jx\,sxy+jy\,syy+jz\,szy\right)}{jx^2+jy^2+jz^2+c^2\,n^2} & \frac{n\left(jx\,sxz+jy\,syz+jz\,szz\right)}{jx^2+jy^2+jz^2+c^2\,n^2} \\ -\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n} & sxx & sxy & sxz \\ -\frac{jx\,syx+jy\,syy+jz\,syz}{n} & syx & syy & syz \\ -\frac{jx\,szx+jy\,szy+jz\,szz}{n} & szx & szy & szz \end{pmatrix}$$

In[83]:=
```
MF /@ FS@{proju.S.proju, Assuming[assut, FS[projperpu.S.projperpu - S]]}
```
Out[83]=

$$\left\{\begin{pmatrix} O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 \\ O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 \\ O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 \\ O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^6 \end{pmatrix}, \begin{pmatrix} O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 \\ O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 \\ O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 \\ O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[84]:=
```
FS[T[S.Inverse[gg]].gg - S] // MF
```
Out[84]//MatrixForm=

$$\begin{pmatrix} O\left[\frac{1}{c}\right]^4 & \frac{jy\,sxy+jz\,sxz-jy\,syx-jz\,szx}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jx\left(-sxy+syx\right)+jz\left(syz-szy\right)}{n\,c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jx\left(-sxz+szx\right)+jy\left(-syz+szy\right)}{n\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy\,sxy+jz\,sxz-jy\,syx-jz\,szx}{n}+O\left[\frac{1}{c}\right]^2 & O\left[\frac{1}{c}\right]^4 & \left(-sxy+syx\right)+O\left[\frac{1}{c}\right]^4 & \left(-sxz+szx\right)+O\left[\frac{1}{c}\right]^4 \\ \frac{jx\left(-sxy+syx\right)+jz\left(syz-szy\right)}{n}+O\left[\frac{1}{c}\right]^2 & \left(sxy-syx\right)+O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & \left(-syz+szy\right)+O\left[\frac{1}{c}\right]^4 \\ \frac{jx\left(-sxz+szx\right)+jy\left(-syz+szy\right)}{n}+O\left[\frac{1}{c}\right]^2 & \left(sxz-szx\right)+O\left[\frac{1}{c}\right]^4 & \left(syz-szy\right)+O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[85]:=
```
(* define "dust" 4-stress *)
(dust = Assuming[assut, FS[(ρ * c ^ 2 + ϵ) * Outer[Times, NJ, gg.uu / c ^ 2]]]) // MF
```

Out[85]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-n\epsilon+n\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac1c]^2 & jx\,\rho+O[\tfrac1c]^2 & jy\,\rho+O[\tfrac1c]^2 & jz\,\rho+O[\tfrac1c]^2 \\ -jx\,\rho c^2+\left(-jx\,\epsilon+jx\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac1c]^2 & \frac{jx^2\rho}{n}+O[\tfrac1c]^2 & \frac{jx\,jy\,\rho}{n}+O[\tfrac1c]^2 & \frac{jx\,jz\,\rho}{n}+O[\tfrac1c]^2 \\ -jy\,\rho c^2+\left(-jy\,\epsilon+jy\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac1c]^2 & \frac{jx\,jy\,\rho}{n}+O[\tfrac1c]^2 & \frac{jy^2\rho}{n}+O[\tfrac1c]^2 & \frac{jy\,jz\,\rho}{n}+O[\tfrac1c]^2 \\ -jz\,\rho c^2+\left(-jz\,\epsilon+jz\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac1c]^2 & \frac{jx\,jz\,\rho}{n}+O[\tfrac1c]^2 & \frac{jy\,jz\,\rho}{n}+O[\tfrac1c]^2 & \frac{jz^2\rho}{n}+O[\tfrac1c]^2 \end{pmatrix}$$

In[86]:=
```
FS[T[dust.Inverse[gg]] == dust.Inverse[gg]]
```

Out[86]= $\{\{O[\tfrac1c]^4, O[\tfrac1c]^2, O[\tfrac1c]^2, O[\tfrac1c]^2\}, \{O[\tfrac1c]^2, O[\tfrac1c]^2, O[\tfrac1c]^2, O[\tfrac1c]^2\}, \{O[\tfrac1c]^2, O[\tfrac1c]^2, O[\tfrac1c]^2, O[\tfrac1c]^2\}, \{O[\tfrac1c]^2, O[\tfrac1c]^2, O[\tfrac1c]^2, O[\tfrac1c]^2\}\} == \{\{O[\tfrac1c]^4, O[\tfrac1c]^2, O[\tfrac1c]^2, O[\tfrac1c]^2\}, \{O[\tfrac1c]^4, O[\tfrac1c]^2, O[\tfrac1c]^2, O[\tfrac1c]^2\}, \{O[\tfrac1c]^4, O[\tfrac1c]^2, O[\tfrac1c]^2, O[\tfrac1c]^2\}, \{O[\tfrac1c]^4, O[\tfrac1c]^2, O[\tfrac1c]^2, O[\tfrac1c]^2\}\}$

In[87]:=
```
FS[T[dust].gg == gg.dust]
```

Out[87]= True

In[88]:=
```
MF /@ FS@{proju.dust.proju == dust, projperpu.dust.projperpu}
```

Out[88]=

$$\left\{\text{True}, \begin{pmatrix} O[\tfrac1c]^4 & O[\tfrac1c]^4 & O[\tfrac1c]^4 & O[\tfrac1c]^4 \\ O[\tfrac1c]^2 & O[\tfrac1c]^2 & O[\tfrac1c]^2 & O[\tfrac1c]^2 \\ O[\tfrac1c]^2 & O[\tfrac1c]^2 & O[\tfrac1c]^2 & O[\tfrac1c]^2 \\ O[\tfrac1c]^2 & O[\tfrac1c]^2 & O[\tfrac1c]^2 & O[\tfrac1c]^2 \end{pmatrix}\right\}$$

In[89]:=
```
showf[assut][dust2 = Assuming[assut, Expand //@ FS@PowerExpand[(ρ * c ^ 2 + ϵ) * Outer[Times, NJ, gg.auu / c ^ 2]]]];
```

In[90]:=
```
(* define generic 4-stress *)
(EPS = Assuming[assut, FS[dust + Qtens + Ptens + S]]) // MF
```

Out[90]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-n\epsilon+n\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac1c]^2 & jx\,\rho+O[\tfrac1c]^2 & jy\,\rho+O[\tfrac1c]^2 & jz\,\rho+O[\tfrac1c]^2 \\ -jx\,\rho c^2+\left(-qx-\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n}-jx\,\epsilon+jx\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac1c]^2 & \left(sxx+\frac{jx^2\rho}{n}\right)+O[\tfrac1c]^2 & \left(sxy+\frac{jx\,jy\,\rho}{n}\right)+O[\tfrac1c]^2 & \left(sxz+\frac{jx\,jz\,\rho}{n}\right)+O[\tfrac1c]^2 \\ -jy\,\rho c^2+\left(-qy-\frac{jx\,syx+jy\,syy+jz\,syz}{n}-jy\,\epsilon+jy\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac1c]^2 & \left(syx+\frac{jx\,jy\,\rho}{n}\right)+O[\tfrac1c]^2 & \left(syy+\frac{jy^2\rho}{n}\right)+O[\tfrac1c]^2 & \left(syz+\frac{jy\,jz\,\rho}{n}\right)+O[\tfrac1c]^2 \\ -jz\,\rho c^2+\left(-qz-\frac{jx\,szx+jy\,szy+jz\,szz}{n}-jz\,\epsilon+jz\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac1c]^2 & \left(szx+\frac{jx\,jz\,\rho}{n}\right)+O[\tfrac1c]^2 & \left(szy+\frac{jy\,jz\,\rho}{n}\right)+O[\tfrac1c]^2 & \left(szz+\frac{jz^2\rho}{n}\right)+O[\tfrac1c]^2 \end{pmatrix}$$

In[91]:=
```
Assuming[assut, FS[(EPS /. replaceJu)]] // MF
```

Out[91]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-\frac12 n\left(2\epsilon+\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O[\tfrac1c]^2 & n\,ux\,\rho+O[\tfrac1c]^2 & n\,uy\,\rho+O[\tfrac1c]^2 & n\,uz\,\rho+O[\tfrac1c]^2 \\ -n\,ux\,\rho c^2+\left(-qx-sxy\,uy-sxz\,uz-ux\left(sxx+n\epsilon\right)-\frac12 n\,ux\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O[\tfrac1c]^2 & \left(sxx+n\,ux^2\rho\right)+O[\tfrac1c]^2 & \left(sxy+n\,ux\,uy\,\rho\right)+O[\tfrac1c]^2 & \left(sxz+n\,ux\,uz\,\rho\right)+O[\tfrac1c]^2 \\ -n\,uy\,\rho c^2+\left(-qy-syx\,ux-syz\,uz-uy\left(syy+n\epsilon\right)-\frac12 n\,uy\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O[\tfrac1c]^2 & \left(syx+n\,ux\,uy\,\rho\right)+O[\tfrac1c]^2 & \left(syy+n\,uy^2\rho\right)+O[\tfrac1c]^2 & \left(syz+n\,uy\,uz\,\rho\right)+O[\tfrac1c]^2 \\ -n\,uz\,\rho c^2+\left(-qz-szx\,ux-szy\,uy-uz\left(szz+n\epsilon\right)-\frac12 n\,uz\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O[\tfrac1c]^2 & \left(szx+n\,ux\,uz\,\rho\right)+O[\tfrac1c]^2 & \left(szy+n\,uy\,uz\,\rho\right)+O[\tfrac1c]^2 & \left(szz+n\,uz^2\rho\right)+O[\tfrac1c]^2 \end{pmatrix}$$

In[92]:=
```
Assuming[assut, FS[(EPS /. replaceJu) /. replaceuUnorm]] // MF
```

Out[92]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-n\epsilon-\frac12 n\,U^2\rho+n\,W\rho\right)+O[\tfrac1c]^2 & n\,ux\,\rho+O[\tfrac1c]^2 & n\,uy\,\rho+O[\tfrac1c]^2 & n\,uz\,\rho+O[\tfrac1c]^2 \\ -n\,ux\,\rho c^2+\left(-qx-sxy\,uy-sxz\,uz-\frac12 ux\left(2\left(sxx+n\epsilon\right)+n\left(U^2-2W\right)\rho\right)\right)+O[\tfrac1c]^2 & \left(sxx+n\left(U^2-uy^2-uz^2\right)\rho\right)+O[\tfrac1c]^2 & \left(sxy+n\,ux\,uy\,\rho\right)+O[\tfrac1c]^2 & \left(sxz+n\,ux\,uz\,\rho\right)+O[\tfrac1c]^2 \\ -n\,uy\,\rho c^2+\left(-qy-syx\,ux-syz\,uz-\frac12 uy\left(2\left(syy+n\epsilon\right)+n\left(U^2-2W\right)\rho\right)\right)+O[\tfrac1c]^2 & \left(syx+n\,ux\,uy\,\rho\right)+O[\tfrac1c]^2 & \left(syy+n\,uy^2\rho\right)+O[\tfrac1c]^2 & \left(syz+n\,uy\,uz\,\rho\right)+O[\tfrac1c]^2 \\ -n\,uz\,\rho c^2+\left(-qz-szx\,ux-szy\,uy-\frac12 uz\left(2\left(szz+n\epsilon\right)+n\left(U^2-2W\right)\rho\right)\right)+O[\tfrac1c]^2 & \left(szx+n\,ux\,uz\,\rho\right)+O[\tfrac1c]^2 & \left(szy+n\,uy\,uz\,\rho\right)+O[\tfrac1c]^2 & \left(szz+n\,uz^2\rho\right)+O[\tfrac1c]^2 \end{pmatrix}$$

In[93]:=
```
FS[T[EPS.Inverse[gg]].gg - EPS] // MF
```

Out[93]//MatrixForm=

$$\begin{pmatrix} O[\tfrac1c]^2 & O[\tfrac1c]^2 & O[\tfrac1c]^2 & O[\tfrac1c]^2 \\ O[\tfrac1c]^0 & O[\tfrac1c]^2 & (-sxy+syx)+O[\tfrac1c]^2 & (-sxz+szx)+O[\tfrac1c]^2 \\ O[\tfrac1c]^0 & (sxy-syx)+O[\tfrac1c]^2 & O[\tfrac1c]^2 & (-syz+szy)+O[\tfrac1c]^2 \\ O[\tfrac1c]^0 & (sxz-szx)+O[\tfrac1c]^2 & (syz-szy)+O[\tfrac1c]^2 & O[\tfrac1c]^2 \end{pmatrix}$$

In[94]:=
```
(* Conditions for symmetry of total 4-stress tensor *)
subsym = {px → qx, py → qy, pz → qz, syx → sxy, szx → sxz, szy → syz};
```

In[95]:=
```
(EPSsym = Assuming[assut, FS[EPS /. subsym]]) // MF
```

Out[95]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-n\epsilon+n\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac1c]^2 & jx\,\rho+O[\tfrac1c]^2 & jy\,\rho+O[\tfrac1c]^2 & jz\,\rho+O[\tfrac1c]^2 \\ -jx\,\rho c^2+\left(-qx-\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n}-jx\,\epsilon+jx\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac1c]^2 & \left(sxx+\frac{jx^2\rho}{n}\right)+O[\tfrac1c]^2 & \left(sxy+\frac{jx\,jy\,\rho}{n}\right)+O[\tfrac1c]^2 & \left(sxz+\frac{jx\,jz\,\rho}{n}\right)+O[\tfrac1c]^2 \\ -jy\,\rho c^2+\left(-qy-\frac{jx\,sxy+jy\,syy+jz\,syz}{n}-jy\,\epsilon+jy\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac1c]^2 & \left(sxy+\frac{jx\,jy\,\rho}{n}\right)+O[\tfrac1c]^2 & \left(syy+\frac{jy^2\rho}{n}\right)+O[\tfrac1c]^2 & \left(syz+\frac{jy\,jz\,\rho}{n}\right)+O[\tfrac1c]^2 \\ -jz\,\rho c^2+\left(-qz-\frac{jx\,sxz+jy\,syz+jz\,szz}{n}-jz\,\epsilon+jz\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac1c]^2 & \left(sxz+\frac{jx\,jz\,\rho}{n}\right)+O[\tfrac1c]^2 & \left(syz+\frac{jy\,jz\,\rho}{n}\right)+O[\tfrac1c]^2 & \left(szz+\frac{jz^2\rho}{n}\right)+O[\tfrac1c]^2 \end{pmatrix}$$

In[96]:=
```
FS[T[EPSsym.Inverse[gg]].gg - EPSsym] // MF
```

Out[96]//MatrixForm=

$$\begin{pmatrix} O[\tfrac1c]^2 & O[\tfrac1c]^2 & O[\tfrac1c]^2 & O[\tfrac1c]^2 \\ O[\tfrac1c]^0 & O[\tfrac1c]^2 & O[\tfrac1c]^2 & O[\tfrac1c]^2 \\ O[\tfrac1c]^0 & O[\tfrac1c]^2 & O[\tfrac1c]^2 & O[\tfrac1c]^2 \\ O[\tfrac1c]^0 & O[\tfrac1c]^2 & O[\tfrac1c]^2 & O[\tfrac1c]^2 \end{pmatrix}$$

In[97]:=
```
Assuming[assut, FS[(EPSsym /. replaceJu) /. replaceuUnorm]] // MF
```

Out[97]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-n\epsilon-\frac12 n\,U^2\rho+n\,W\rho\right)+O[\tfrac1c]^2 & n\,ux\,\rho+O[\tfrac1c]^2 & n\,uy\,\rho+O[\tfrac1c]^2 & n\,uz\,\rho+O[\tfrac1c]^2 \\ -n\,ux\,\rho c^2+\left(-qx-sxy\,uy-sxz\,uz-\frac12 ux\left(2\left(sxx+n\epsilon\right)+n\left(U^2-2W\right)\rho\right)\right)+O[\tfrac1c]^2 & \left(sxx+n\left(U^2-uy^2-uz^2\right)\rho\right)+O[\tfrac1c]^2 & \left(sxy+n\,ux\,uy\,\rho\right)+O[\tfrac1c]^2 & \left(sxz+n\,ux\,uz\,\rho\right)+O[\tfrac1c]^2 \\ -n\,uy\,\rho c^2+\left(-qy-sxy\,ux-syz\,uz-\frac12 uy\left(2\left(syy+n\epsilon\right)+n\left(U^2-2W\right)\rho\right)\right)+O[\tfrac1c]^2 & \left(sxy+n\,ux\,uy\,\rho\right)+O[\tfrac1c]^2 & \left(syy+n\,uy^2\rho\right)+O[\tfrac1c]^2 & \left(syz+n\,uy\,uz\,\rho\right)+O[\tfrac1c]^2 \\ -n\,uz\,\rho c^2+\left(-qz-sxz\,ux-syz\,uy-\frac12 uz\left(2\left(szz+n\epsilon\right)+n\left(U^2-2W\right)\rho\right)\right)+O[\tfrac1c]^2 & \left(sxz+n\,ux\,uz\,\rho\right)+O[\tfrac1c]^2 & \left(syz+n\,uy\,uz\,\rho\right)+O[\tfrac1c]^2 & \left(szz+n\,uz^2\rho\right)+O[\tfrac1c]^2 \end{pmatrix}$$

In[98]:=

(* Balanced quantities constructed from energy–momentum tensor, and their supplies *)

In[99]:=
```
(* Symmetrized energy-stress tensor, with explicit dep. on coords *)
(TTx = Assuming[assut, FS[tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]]]]) // MF
```

Out[99]//MatrixForm=

$$\begin{pmatrix} -\rho\,n[t,x,y,z]\,c^2-\frac12 n[t,x,y,z]\left(2\epsilon+\rho\left(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2-2W[t,x,y,z]\right)\right)+O[\tfrac1c]^2 & \rho\,n[t,x,y,z]\,ux[t,x,y,z]+O[\tfrac1c]^2 & \rho\,n[t,x,y,z]\,uy[t,x,y,z]+O[\tfrac1c]^2 & \rho\,n[t,x,y,z]\,uz[t,x,y,z]+O[\tfrac1c]^2 \\ -\rho\,n[t,x,y,z]\,ux[t,x,y,z]\,c^2+O[\tfrac1c]^0 & \left(sxx+\rho\,n[t,x,y,z]\,ux[t,x,y,z]^2\right)+O[\tfrac1c]^2 & \frac12\left(sxy+syx+2\rho\,n[t,x,y,z]\,ux[t,x,y,z]\,uy[t,x,y,z]\right)+O[\tfrac1c]^2 & \frac12\left(sxz+szx+2\rho\,n[t,x,y,z]\,ux[t,x,y,z]\,uz[t,x,y,z]\right)+O[\tfrac1c]^2 \\ -\rho\,n[t,x,y,z]\,uy[t,x,y,z]\,c^2+O[\tfrac1c]^0 & \frac12\left(sxy+syx+2\rho\,n[t,x,y,z]\,ux[t,x,y,z]\,uy[t,x,y,z]\right)+O[\tfrac1c]^2 & \left(syy+\rho\,n[t,x,y,z]\,uy[t,x,y,z]^2\right)+O[\tfrac1c]^2 & \frac12\left(syz+szy+2\rho\,n[t,x,y,z]\,uy[t,x,y,z]\,uz[t,x,y,z]\right)+O[\tfrac1c]^2 \\ -\rho\,n[t,x,y,z]\,uz[t,x,y,z]\,c^2+O[\tfrac1c]^0 & \frac12\left(sxz+szx+2\rho\,n[t,x,y,z]\,ux[t,x,y,z]\,uz[t,x,y,z]\right)+O[\tfrac1c]^2 & \frac12\left(syz+szy+2\rho\,n[t,x,y,z]\,uy[t,x,y,z]\,uz[t,x,y,z]\right)+O[\tfrac1c]^2 & \left(szz+\rho\,n[t,x,y,z]\,uz[t,x,y,z]^2\right)+O[\tfrac1c]^2 \end{pmatrix}$$

In[100]:=
```
(Duu = Assuming[assut, FS[(D[Normal@uut, {coords}] + Sum[uut[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]) // MF
```

Out[100]//MatrixForm=

$$\frac{n[t,x,y,z]\left(jx[t,x,y,z]\left(-n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+jx^{(1,0,0,0)}[t,x,y,z]\right)+jy[t,x,y,z]\left(-n[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]+jy^{(1,0,0,0)}[t,x,y,z]\right)+jz[t,x,y,z]\left(-n[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]+jz^{(1,0,0,0)}[t,x,y,z]\right)\right)-\left(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2\right)n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^3\,c^2}+O[\tfrac1c]^4$$

$$\left(-W^{(0,1,0,0)}[t,x,y,z]+\frac{n[t,x,y,z]\,jx^{(1,0,0,0)}[t,x,y,z]-jx[t,x,y,z]\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}\right)+\ldots$$

$$\left(-W^{(0,0,1,0)}[t,x,y,z]+\frac{n[t,x,y,z]\,jy^{(1,0,0,0)}[t,x,y,z]-jy[t,x,y,z]\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}\right)+\ldots$$

$$\left(-W^{(0,0,0,1)}[t,x,y,z]+\frac{n[t,x,y,z]\,jz^{(1,0,0,0)}[t,x,y,z]-jz[t,x,y,z]\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}\right)+\ldots$$

In[101]:=
```
(Duv = Assuming[assut, FS[(D[Normal@uuv, {coords}] + Sum[uuv[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]) // MF
```

Out[101]//MatrixForm=

$$\frac{ux[t,x,y,z]\left(-W^{(0,1,0,0)}[t,x,y,z]+ux^{(1,0,0,0)}[t,x,y,z]\right)+uy[t,x,y,z]\left(-W^{(0,0,1,0)}[t,x,y,z]+uy^{(1,0,0,0)}[t,x,y,z]\right)+uz[t,x,y,z]\left(-W^{(0,0,0,1)}[t,x,y,z]+uz^{(1,0,0,0)}[t,x,y,z]\right)}{c^2}+O[\tfrac1c]^4$$

$$\left(-W^{(0,1,0,0)}[t,x,y,z]+ux^{(1,0,0,0)}[t,x,y,z]\right)+\frac{\left(8W-ux[t,x,y,z]^2-uy[t,x,y,z]^2-uz[t,x,y,z]^2-2W[t,x,y,z]\right)W^{(0,1,0,0)}[t,x,y,z]+3\,ux[t,x,y,z]^2\,ux^{(1,0,0,0)}[t,x,y,z]+\left(uy[t,x,y,z]^2+uz[t,x,y,z]^2+2W[t,x,y,z]\right)ux^{(1,0,0,0)}[t,x,y,z]+2\,ux[t,x,y,z]\left(uy[t,x,y,z]\,uy^{(1,0,0,0)}[t,x,y,z]+uz[t,x,y,z]\,uz^{(1,0,0,0)}[t,x,y,z]+2W^{(1,0,0,0)}[t,x,y,z]\right)}{2c^2}+O[\tfrac1c]^3 \quad \ldots$$

$$\left(-W^{(0,0,1,0)}[t,x,y,z]+uy^{(1,0,0,0)}[t,x,y,z]\right)+\ldots+O[\tfrac1c]^3 \quad \ldots$$

$$\left(-W^{(0,0,0,1)}[t,x,y,z]+uz^{(1,0,0,0)}[t,x,y,z]\right)+\ldots+O[\tfrac1c]^3 \quad \ldots$$

(* Energy *)

In[102]:=
```
(* Energy current and supply according to 4-velocity *)
pvec = uu; Dpvec = Duv;
MF@(MF /@ {Efluxuu = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Esupplyuu = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[103]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} -n\rho c^2-n\epsilon+O[\tfrac1c]^2 \\ \left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\rho c^2+\left(-Ax\left(qx+\left(jx-n\,Vx\right)\epsilon\right)-Ay\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)-Az\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)+O[\tfrac1c]^2 \end{pmatrix} \\ \frac12\left(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z]+(syz+szy)\left(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\right)+2\left(syy\,uy^{(0,0,1,0)}[t,x,y,z]+sxx\,ux^{(0,1,0,0)}[t,x,y,z]\right)+(sxy+syx)\left(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z]\right)+(sxz+szx)\left(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\right)\right)+O[\tfrac1c]^2 \end{pmatrix}$$

In[104]:=
```
MF@(MF /@ (FS[({Efluxuu, Esupplyuu} /. replaceJu) /. replaceuUnorm]))
```

Out[104]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} -n\rho c^2-n\epsilon+O[\tfrac1c]^2 \\ n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\epsilon\right)+O[\tfrac1c]^2 \end{pmatrix} \\ \frac12\left(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z]+(syz+szy)\left(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\right)+2\left(syy\,uy^{(0,0,1,0)}[t,x,y,z]+sxx\,ux^{(0,1,0,0)}[t,x,y,z]\right)+(sxy+syx)\left(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z]\right)+(sxz+szx)\left(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\right)\right)+O[\tfrac1c]^2 \end{pmatrix}$$

In[105]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[105]=

$$\left\{\begin{pmatrix} \frac{jx}{n}+\frac{qx}{n\rho c^2}+O[\tfrac1c]^4 \\ \frac{jy}{n}+\frac{qy}{n\rho c^2}+O[\tfrac1c]^4 \\ \frac{jz}{n}+\frac{qz}{n\rho c^2}+O[\tfrac1c]^4 \end{pmatrix}, \begin{pmatrix} ux+\frac{qx}{n\rho c^2}+O[\tfrac1c]^4 \\ uy+\frac{qy}{n\rho c^2}+O[\tfrac1c]^4 \\ uz+\frac{qz}{n\rho c^2}+O[\tfrac1c]^4 \end{pmatrix}\right\}$$

In[106]:=
```
(* Energy current and supply according to t-vector *)
pvec = - {1, 0, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ ({Efluxt = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Esupplyt = Assuming[assut, FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]]}))
```

Out[107]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} n\,\rho\,c^2 + n\left(\epsilon + \frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n^2} - W\,\rho\right) + O[\frac{1}{c}]^2 \\ \left(Ay\,jy + Az\,jz + Ax\,(jx - n\,Vx) - n\,(Ay\,Vy + Az\,Vz)\right)\rho\,c^2 + \frac{2\,n\left(Ax\left(jx\,sxx+jy\,sxy+jz\,sxz+n\left(qx+(jx-n\,Vx)\,\epsilon\right)\right)+Ay\left(jx\,syx+jy\,syy+jz\,syz+n\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)\right)+Az\left(jx\,szx+jy\,szy+jz\,szz+n\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)\right)+\left(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz)\right)\left(jx^2+jy^2+jz^2-2\,n^2\,W\right)\rho}{2\,n^2} + O[\frac{1}{c}]^2 \end{pmatrix} \\ -n\,\rho\,W^{(1,0,0,0)}[t,x,y,z] + O[\frac{1}{c}]^2\end{pmatrix}$$

In[108]:=
```
MF@(MF /@ (FS[({Efluxt, Esupplyt} /. replaceJu) /. replaceuUnorm]))
```

Out[108]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} n\,\rho\,c^2 + n\left(\epsilon + \frac{1}{2}\left(U^2 - 2\,W\right)\rho\right) + O[\frac{1}{c}]^2 \\ n\left(Ax\,(ux - Vx) + Ay\,(uy - Vy) + Az\,(uz - Vz)\right)\rho\,c^2 + \left(Ax\left(qx + sxx\,ux + sxy\,uy + sxz\,uz + n\,(ux - Vx)\,\epsilon\right) + Ay\left(qy + syx\,ux + syy\,uy + syz\,uz + n\,(uy - Vy)\,\epsilon\right) + Az\left(qz + szx\,ux + szy\,uy + szz\,uz + n\,(uz - Vz)\,\epsilon\right) + \frac{1}{2}\,n\left(Ax\,(ux - Vx) + Ay\,(uy - Vy) + Az\,(uz - Vz)\right)\left(U^2 - 2\,W\right)\rho\right) + O[\frac{1}{c}]^2 \end{pmatrix} \\ -n\,\rho\,W^{(1,0,0,0)}[t,x,y,z] + O[\frac{1}{c}]^2\end{pmatrix}$$

In[109]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[109]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\,\rho\,c^2} + O[\frac{1}{c}]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\,\rho\,c^2} + O[\frac{1}{c}]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\,\rho\,c^2} + O[\frac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\,\rho\,c^2} + O[\frac{1}{c}]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\,\rho\,c^2} + O[\frac{1}{c}]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\,\rho\,c^2} + O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

In[110]:=
```
(* Energy current and supply according to norm. t-vector *)
pvec = - c * {1, 0, 0, 0} / Sqrt[-gg[[1, 1]]]; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Efluxnt = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Esupplynt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[111]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} n\,\rho\,c^2 + \left(n\,\epsilon + \frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n}\right) + O[\frac{1}{c}]^2 \\ \left(Ay\,jy + Az\,jz + Ax\,(jx - n\,Vx) - n\,(Ay\,Vy + Az\,Vz)\right)\rho\,c^2 + \frac{2\,n\left(Ax\left(jx\,sxx+jy\,sxy+jz\,sxz+n\left(qx+(jx-n\,Vx)\,\epsilon\right)\right)+Ay\left(jx\,syx+jy\,syy+jz\,syz+n\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)\right)+Az\left(jx\,szx+jy\,szy+jz\,szz+n\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)\right)+(jx^2+jy^2+jz^2)\left(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz)\right)\rho}{2\,n^2} + O[\frac{1}{c}]^2 \end{pmatrix} \\ \rho\left(jz\,W^{(0,0,0,1)}[t,x,y,z] + jy\,W^{(0,0,1,0)}[t,x,y,z] + jx\,W^{(0,1,0,0)}[t,x,y,z]\right) + O[\frac{1}{c}]^2\end{pmatrix}$$

In[112]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[112]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\,\rho\,c^2} + O[\frac{1}{c}]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\,\rho\,c^2} + O[\frac{1}{c}]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\,\rho\,c^2} + O[\frac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\,\rho\,c^2} + O[\frac{1}{c}]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\,\rho\,c^2} + O[\frac{1}{c}]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\,\rho\,c^2} + O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

In[113]:=
```
(* Energy current and supply according to cov. t-vector *)
pvec = c^2 * igg.{1, 0, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Efluxcovt = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Esupplycovt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[114]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} n\,\rho\,c^2 + \left(\frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n} + n\,(\epsilon + W\,\rho)\right) + O[\frac{1}{c}]^2 \\ \left(Ay\,jy + Az\,jz + Ax\,(jx - n\,Vx) - n\,(Ay\,Vy + Az\,Vz)\right)\rho\,c^2 + \frac{2\,n\left(Ax\left(jx\,sxx+jy\,sxy+jz\,sxz+n\left(qx+(jx-n\,Vx)\,\epsilon\right)\right)+Ay\left(jx\,syx+jy\,syy+jz\,syz+n\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)\right)+Az\left(jx\,szx+jy\,szy+jz\,szz+n\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)\right)+\left(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz)\right)\left(jx^2+jy^2+jz^2+2\,n^2\,W\right)\rho}{2\,n^2} + O[\frac{1}{c}]^2 \end{pmatrix} \\ \rho\left(2\,jz\,W^{(0,0,0,1)}[t,x,y,z] + 2\,jy\,W^{(0,0,1,0)}[t,x,y,z] + 2\,jx\,W^{(0,1,0,0)}[t,x,y,z] + n\,W^{(1,0,0,0)}[t,x,y,z]\right) + O[\frac{1}{c}]^2\end{pmatrix}$$

In[115]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[115]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\,\rho\,c^2} + O[\frac{1}{c}]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\,\rho\,c^2} + O[\frac{1}{c}]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\,\rho\,c^2} + O[\frac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\,\rho\,c^2} + O[\frac{1}{c}]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\,\rho\,c^2} + O[\frac{1}{c}]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\,\rho\,c^2} + O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

(* Momentum *)

In[116]:=
```
(* Momentum current and supply according to x-vector *)
pvec = {0, 1, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Pfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Psupplyx = Assuming[assut, FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]]})
```

Out[117]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} jx\,\rho + O[\frac{1}{c}]^2 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + \frac{jx\left(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz)\right)\rho}{n}\right) + O[\frac{1}{c}]^2 \end{pmatrix} \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + \frac{\left(3\,(jx^2+jy^2+jz^2)\,\rho+2\,n\left(sxx+syy+szz+n\,\epsilon-n\,W\,\rho\right)\right)W^{(0,1,0,0)}[t,x,y,z]}{2\,n\,c^2} + O[\frac{1}{c}]^3\end{pmatrix}$$

In[118]:=
```
MF@(MF /@ (FS[({Pfluxx, Psupplyx} /. replaceJu) /. replaceuUnorm]))
```

Out[118]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} n\,ux\,\rho + O[\frac{1}{c}]^2 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + n\,ux\left(Ax\,(ux - Vx) + Ay\,(uy - Vy) + Az\,(uz - Vz)\right)\rho\right) + O[\frac{1}{c}]^2 \end{pmatrix} \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + \frac{\left(2\left(sxx+syy+szz+n\,\epsilon\right)+n\left(3\,U^2-2\,W\right)\rho\right)W^{(0,1,0,0)}[t,x,y,z]}{2\,c^2} + O[\frac{1}{c}]^3\end{pmatrix}$$

In[119]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[119]=

$$\left\{\begin{pmatrix} \left(\frac{jx}{n} + \frac{sxx}{jx\,\rho}\right) + O[\frac{1}{c}]^2 \\ \left(\frac{jy}{n} + \frac{syx}{jx\,\rho}\right) + O[\frac{1}{c}]^2 \\ \left(\frac{jz}{n} + \frac{szx}{jx\,\rho}\right) + O[\frac{1}{c}]^2 \end{pmatrix}, \begin{pmatrix} \left(ux + \frac{sxx}{n\,ux\,\rho}\right) + O[\frac{1}{c}]^2 \\ \left(uy + \frac{syx}{n\,ux\,\rho}\right) + O[\frac{1}{c}]^2 \\ \left(uz + \frac{szx}{n\,ux\,\rho}\right) + O[\frac{1}{c}]^2 \end{pmatrix}\right\}$$

In[120]:=
```
(* Momentum current and supply according to cov. x-vector *)
pvec = igg.{0, 1, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Pfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Psupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[121]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} jx\,\rho + O[\frac{1}{c}]^2 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + \frac{jx\left(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz)\right)\rho}{n}\right) + O[\frac{1}{c}]^2 \end{pmatrix} \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + \frac{-2\left(n\,(sxz+szx)+2\,jx\,jz\,\rho\right)W^{(0,0,0,1)}[t,x,y,z]-2\left(n\,(sxy+syx)+2\,jx\,jy\,\rho\right)W^{(0,0,1,0)}[t,x,y,z]+\left(2\,n\left(-sxx+syy+szz+n\,\epsilon\right)-jx^2\,\rho+3\left(jy^2+jz^2-2\,n^2\,W\right)\rho\right)W^{(0,1,0,0)}[t,x,y,z]-4\,jx\,n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]}{2\,n\,c^2} + O[\frac{1}{c}]^3\end{pmatrix}$$

In[122]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[122]=

$$\left\{\begin{pmatrix} \left(\frac{jx}{n} + \frac{sxx}{jx\,\rho}\right) + O[\frac{1}{c}]^2 \\ \left(\frac{jy}{n} + \frac{syx}{jx\,\rho}\right) + O[\frac{1}{c}]^2 \\ \left(\frac{jz}{n} + \frac{szx}{jx\,\rho}\right) + O[\frac{1}{c}]^2 \end{pmatrix}, \begin{pmatrix} \left(ux + \frac{sxx}{n\,ux\,\rho}\right) + O[\frac{1}{c}]^2 \\ \left(uy + \frac{syx}{n\,ux\,\rho}\right) + O[\frac{1}{c}]^2 \\ \left(uz + \frac{szx}{n\,ux\,\rho}\right) + O[\frac{1}{c}]^2 \end{pmatrix}\right\}$$

(* Angular momentum *)

In[123]:=
```
(* Ang.momentum current and supply according to yz-vector *)
pvec = y * {0, 0, 0, 1} - z * {0, 0, 1, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Lfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Lsupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[124]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} \left(jz\,y\,\rho - jy\,z\,\rho\right) + O[\frac{1}{c}]^2 \\ \left(\left(Ax\,sxz + Ay\,syz + Az\,szz\right)y - \left(Ax\,sxy + Ay\,syy + Az\,szy\right)z + \frac{\left(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz)\right)(jz\,y-jy\,z)\,\rho}{n}\right) + O[\frac{1}{c}]^2 \end{pmatrix} \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z] - z\,W^{(0,0,1,0)}[t,x,y,z]\right) + O[\frac{1}{c}]^2\end{pmatrix}$$

In[125]:=
```
MF@(MF /@ (FS[({Lfluxx, Lsupplyx} /. replaceJu) /. replaceuUnorm]))
```

Out[125]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} n\left(uz\,y - uy\,z\right)\rho + O[\frac{1}{c}]^2 \\ \left(\left(Ax\,sxz + Ay\,syz + Az\,szz\right)y - \left(Ax\,sxy + Ay\,syy + Az\,szy\right)z + n\left(Ax\,(ux - Vx) + Ay\,(uy - Vy) + Az\,(uz - Vz)\right)\left(uz\,y - uy\,z\right)\rho\right) + O[\frac{1}{c}]^2 \end{pmatrix} \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z] - z\,W^{(0,0,1,0)}[t,x,y,z]\right) + O[\frac{1}{c}]^2\end{pmatrix}$$

In[126]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[126]=

$$\left\{\begin{pmatrix} \left(\frac{jx}{n} + \frac{sxz\,y-sxy\,z}{jz\,y\,\rho-jy\,z\,\rho}\right) + O[\frac{1}{c}]^2 \\ \left(\frac{jy}{n} + \frac{syz\,y-syy\,z}{jz\,y\,\rho-jy\,z\,\rho}\right) + O[\frac{1}{c}]^2 \\ \left(\frac{jz}{n} + \frac{szz\,y-szy\,z}{jz\,y\,\rho-jy\,z\,\rho}\right) + O[\frac{1}{c}]^2 \end{pmatrix}, \begin{pmatrix} \left(ux + \frac{sxz\,y-sxy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right) + O[\frac{1}{c}]^2 \\ \left(uy + \frac{syz\,y-syy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right) + O[\frac{1}{c}]^2 \\ \left(uz + \frac{szz\,y-szy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right) + O[\frac{1}{c}]^2 \end{pmatrix}\right\}$$

In[127]:=
```
(* Ang.momentum current and supply according to yz-vector as g_{ab} x^{a} (Kopeinik & al.) *)
pvec = (gg.coords)[[3]] * {0, 0, 0, 1} - (gg.coords)[[4]] * {0, 0, 1, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Lfluxkx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Lsupplykx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[128]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} \left(jz\,y\,\rho - jy\,z\,\rho\right) + O[\frac{1}{c}]^2 \\ \left(\left(Ax\,sxz + Ay\,syz + Az\,szz\right)y - \left(Ax\,sxy + Ay\,syy + Az\,szy\right)z + \frac{\left(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz)\right)(jz\,y-jy\,z)\,\rho}{n}\right) + O[\frac{1}{c}]^2 \end{pmatrix} \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z] - z\,W^{(0,0,1,0)}[t,x,y,z]\right) + O[\frac{1}{c}]^2\end{pmatrix}$$

In[129]:=
```
MF@(MF /@ (FS[({Lfluxkx, Lsupplykx} /. replaceJu) /. replaceuUnorm]))
```

Out[129]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} n\left(uz\,y - uy\,z\right)\rho + O[\frac{1}{c}]^2 \\ \left(\left(Ax\,sxz + Ay\,syz + Az\,szz\right)y - \left(Ax\,sxy + Ay\,syy + Az\,szy\right)z + n\left(Ax\,(ux - Vx) + Ay\,(uy - Vy) + Az\,(uz - Vz)\right)\left(uz\,y - uy\,z\right)\rho\right) + O[\frac{1}{c}]^2 \end{pmatrix} \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z] - z\,W^{(0,0,1,0)}[t,x,y,z]\right) + O[\frac{1}{c}]^2\end{pmatrix}$$

In[130]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[130]=

$$\left\{\begin{pmatrix}\left(\frac{jx}{n}+\frac{sxz\,y-sxy\,z}{jz\,y\,\rho-jy\,z\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(\frac{jy}{n}+\frac{syz\,y-syy\,z}{jz\,y\,\rho-jy\,z\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(\frac{jz}{n}+\frac{szz\,y-szy\,z}{jz\,y\,\rho-jy\,z\,\rho}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix},\begin{pmatrix}\left(ux+\frac{sxz\,y-sxy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(uy+\frac{syz\,y-syy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(uz+\frac{szz\,y-szy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}\right\}$$

In[131]:=
```
(* Ang.momentum current and supply according to cov. yz-vector *)
pvec = igg.(y * {0, 0, 0, 1} - z * {0, 0, 1, 0}); Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Lfluxcx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Lsupplycx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[132]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix}\left(jz\,y\,\rho-jy\,z\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ \left(\left(Ax\,sxz+Ay\,syz+Az\,szz\right)y-\left(Ax\,sxy+Ay\,syy+Az\,szy\right)z+\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jz\,y-jy\,z\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}\\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[133]:=
```
MF@(MF /@ (FS[({Lfluxcx, Lsupplycx} /. replaceJu) /. replaceuUnorm]))
```

Out[133]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix}n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2\\ \left(\left(Ax\,sxz+Ay\,syz+Az\,szz\right)y-\left(Ax\,sxy+Ay\,syy+Az\,szy\right)z+n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}\\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[134]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[134]=

$$\left\{\begin{pmatrix}\left(\frac{jx}{n}+\frac{sxz\,y-sxy\,z}{jz\,y\,\rho-jy\,z\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(\frac{jy}{n}+\frac{syz\,y-syy\,z}{jz\,y\,\rho-jy\,z\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(\frac{jz}{n}+\frac{szz\,y-szy\,z}{jz\,y\,\rho-jy\,z\,\rho}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix},\begin{pmatrix}\left(ux+\frac{sxz\,y-sxy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(uy+\frac{syz\,y-syy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(uz+\frac{szz\,y-szy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}\right\}$$

(* Boost momentum *)

In[135]:=
```
(* Ang.boost-momentum current and supply according to tx-vector *)
pvec = t * {0, 1, 0, 0} + x * {1, 0, 0, 0} / c^2; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Bfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Bsupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[136]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix}\left(jx\,t\,\rho-n\,x\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ \left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t+\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx\,t-n\,x\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}\\ n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[137]:=
```
MF@(MF /@ (FS[({Bfluxx, Bsupplyx} /. replaceJu) /. replaceuUnorm]))
```

Out[137]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix}n\left(t\,ux-x\right)\rho+O\left[\frac{1}{c}\right]^2\\ \left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t+n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}\\ n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[138]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[138]=

$$\left\{\begin{pmatrix}\left(\frac{jx}{n}+\frac{sxx\,t}{jx\,t\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(\frac{jy}{n}+\frac{syx\,t}{jx\,t\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(\frac{jz}{n}+\frac{szx\,t}{jx\,t\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix},\begin{pmatrix}\left(ux+\frac{sxx\,t}{n\,t\,ux\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(uy+\frac{syx\,t}{n\,t\,ux\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(uz+\frac{szx\,t}{n\,t\,ux\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}\right\}$$

In[139]:=
```
(* Ang.boost-momentum current and supply according to cov. tx-vector as g_{ab} x^{a} (Kopeinik & al.) *)
pvec = -((gg.coords)[[1]] * {0, 1, 0, 0} - (gg.coords)[[2]] * {1, 0, 0, 0}) / c^2; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Bfluxkx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Bsupplykx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[140]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix}\left(jx\,t\,\rho-n\,x\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ \left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t+\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx\,t-n\,x\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}\\ n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[141]:=
```
MF@(MF /@ (FS[({Bfluxkx, Bsupplykx} /. replaceJu) /. replaceuUnorm]))
```

Out[141]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix}n\left(t\,ux-x\right)\rho+O\left[\frac{1}{c}\right]^2\\ \left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t+n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}\\ n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[142]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[142]=

$$\left\{\begin{pmatrix}\left(\frac{jx}{n}+\frac{sxx\,t}{jx\,t\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(\frac{jy}{n}+\frac{syx\,t}{jx\,t\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(\frac{jz}{n}+\frac{szx\,t}{jx\,t\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix},\begin{pmatrix}\left(ux+\frac{sxx\,t}{n\,t\,ux\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(uy+\frac{syx\,t}{n\,t\,ux\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(uz+\frac{szx\,t}{n\,t\,ux\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}\right\}$$

In[143]:=
```
(* Ang.boost-momentum current and supply according to cov. tx-vector *)
pvec = igg.(t * {0, 1, 0, 0} - x * {1, 0, 0, 0}); Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Bfluxcx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Bsupplycx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[144]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix}\left(jx\,t\,\rho-n\,x\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ \left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t+\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx\,t-n\,x\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}\\ n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[145]:=
```
MF@(MF /@ (FS[({Bfluxcx, Bsupplycx} /. replaceJu) /. replaceuUnorm]))
```

Out[145]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix}n\left(t\,ux-x\right)\rho+O\left[\frac{1}{c}\right]^2\\ \left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t+n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}\\ n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[146]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[146]=

$$\left\{\begin{pmatrix}\left(\frac{jx}{n}+\frac{sxx\,t}{jx\,t\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(\frac{jy}{n}+\frac{syx\,t}{jx\,t\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(\frac{jz}{n}+\frac{szx\,t}{jx\,t\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix},\begin{pmatrix}\left(ux+\frac{sxx\,t}{n\,t\,ux\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(uy+\frac{syx\,t}{n\,t\,ux\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\\ \left(uz+\frac{szx\,t}{n\,t\,ux\,\rho-n\,x\,\rho}\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}\right\}$$

```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[supply.TTx]], {supply, {Dtxyzvec[[1]], Dtxyzvec[[2]], {0, 0, 0, 0}, Dgtxyzvec[[1]] * c^2, Dgtxyzvec[[2]], {0, 0, 0, 0}, DLxvec, Dxboost / c, {0, 0, 0, 0}, DgLxvec, Dgxboost, {0, 0, 0, 0}, Duu, Dtvecnorm}}]]
```

```
(* coordinate/internal/coordinate-proper energy and x-momentum, content and fluxes (TRANSPOSED) *)
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, igg[[1]] * c^2, igg[[2]], {0, 0, 0, 0}, Lxvec, xboost / c, {0, 0, 0, 0}, gLxvec, gxboost, {0, 0, 0, 0}, uu, tvecnorm}])]]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-n\,\rho\,c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n\left(-\epsilon+W\rho\right)\right)+O\left[\frac{1}{c}\right]^2 & \left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\rho\,c^2+\left(-\frac{Ax\left(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz\right)+Ay\left(n\,qy+jx\,syx+jy\,syy+jz\,syz\right)+Az\left(n\,qz+jx\,szx+jy\,szy+jz\,szz\right)}{n}+\left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\epsilon-\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx^2+jy^2+jz^2-2n^2W\right)\rho}{2n^2}\right)+O\left[\frac{1}{c}\right]^2\\
jx\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2\\
0 & 0\\
n\,\rho\,c^2+\left(\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n\left(\epsilon+W\rho\right)\right)+O\left[\frac{1}{c}\right]^2 & \left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho\,c^2+\frac{2n\left(Ax\left(jx\,sxx+jy\,sxy+jz\,sxz+n\left(qx+\left(jx-n\,Vx\right)\epsilon\right)\right)+Ay\left(jx\,syx+jy\,syy+jz\,syz+n\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)\right)+Az\left(jx\,szx+jy\,szy+jz\,szz+n\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)\right)+\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx^2+jy^2+jz^2+2n^2W\right)\rho}{2n^2}+O\left[\frac{1}{c}\right]^2\\
jx\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2\\
0 & 0\\
\left(jz\,y-jy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(\left(Ax\,sxz+Ay\,syz+Az\,szz\right)y-\left(Ax\,sxy+Ay\,syy+Az\,szy\right)z+\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jz\,y-jy\,z\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2\\
\left(jx\,t-n\,x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t+\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx\,t-n\,x\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2\\
0 & 0\\
\left(jz\,y-jy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(\left(Ax\,sxz+Ay\,syz+Az\,szz\right)y-\left(Ax\,sxy+Ay\,syy+Az\,szy\right)z+\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jz\,y-jy\,z\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2\\
\left(-jx\,t\,\rho+n\,x\,\rho\right)+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx\,t-n\,x\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2\\
0 & 0\\
-n\,\rho\,c^2-n\,\epsilon+O\left[\frac{1}{c}\right]^2 & \left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\rho\,c^2+\left(-Ax\left(qx+\left(jx-n\,Vx\right)\epsilon\right)-Ay\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)-Az\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)+O\left[\frac{1}{c}\right]^2\\
-n\,\rho\,c^2+\left(-n\,\epsilon-\frac{(jx^2+jy^2+jz^2)\rho}{2n}\right)+O\left[\frac{1}{c}\right]^2 & \left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\rho\,c^2+\left(-\frac{Ax\left(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz\right)+Ay\left(n\,qy+jx\,syx+jy\,syy+jz\,syz\right)+Az\left(n\,qz+jx\,szx+jy\,szy+jz\,szz\right)}{n}+\left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\epsilon-\frac{(jx^2+jy^2+jz^2)\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{2n^2}\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

```
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, igg[[1]] * c ^ 2, igg[[2]], {0, 0, 0, 0}, Lxvec, xboost / c, {0, 0, 0, 0}, gLxvec, gxboost, {0, 0, 0, 0}, uu, tvecnorm}]) /. replaceJu]]]
```

Out[•]//MatrixForm=

$$\left(\begin{array}{ll}
-n\rho c^2-\frac12 n\left(2\epsilon+\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O[\tfrac1c]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac12 n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O[\tfrac1c]^2 \\
n\,ux\,\rho+O[\tfrac1c]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O[\tfrac1c]^2 \\
0 & 0 \\
n\rho c^2+\left(n\epsilon+\frac12 n\left(ux^2+uy^2+uz^2+2W\right)\rho\right)+O[\tfrac1c]^2 & n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho c^2+\left(Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)+Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)+Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon\right)+\frac12 n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(ux^2+uy^2+uz^2+2W\right)\rho\right)+O[\tfrac1c]^2 \\
n\,ux\,\rho+O[\tfrac1c]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O[\tfrac1c]^2 \\
0 & 0 \\
n\left(uz\,y-uy\,z\right)\rho+O[\tfrac1c]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O[\tfrac1c]^2 \\
n\left(t\,ux-x\right)\rho+O[\tfrac1c]^2 & \left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O[\tfrac1c]^2 \\
0 & 0 \\
n\left(uz\,y-uy\,z\right)\rho+O[\tfrac1c]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O[\tfrac1c]^2 \\
n\left(-t\,ux+x\right)\rho+O[\tfrac1c]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O[\tfrac1c]^2 \\
0 & 0 \\
-n\rho c^2-n\epsilon+O[\tfrac1c]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\epsilon\right)+O[\tfrac1c]^2 \\
-n\rho c^2-\frac12 n\left(2\epsilon+\left(ux^2+uy^2+uz^2\right)\rho\right)+O[\tfrac1c]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac12 n\left(ux^2+uy^2+uz^2\right)\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O[\tfrac1c]^2
\end{array}\right)$$

```
In[•]:= (* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[supply.TTx]], {supply, {Dtxyzvec[[1]], Dtxyzvec[[2]], {0, 0, 0, 0}, Dgtxyzvec[[1]] * c ^ 2, Dgtxyzvec[[2]], {0, 0, 0, 0}, DLxvec, Dxboost / c, {0, 0, 0, 0}, DgLxvec, Dgxboost, {0, 0, 0, 0}, Duu, Dtvecnorm}}]]
```

Out[•]//MatrixForm=

$$\left(\begin{array}{l}
\rho\, n[t,x,y,z]\, W^{(1,0,0,0)}[t,x,y,z]+O[\tfrac1c]^2 \\
\rho\, n[t,x,y,z]\, W^{(0,1,0,0)}[t,x,y,z]+O[\tfrac1c]^2 \\
0 \\
\left(2\rho\, n[t,x,y,z]\times uz[t,x,y,z]\, W^{(0,0,0,1)}[t,x,y,z]+2\rho\, n[t,x,y,z]\times uy[t,x,y,z]\, W^{(0,0,1,0)}[t,x,y,z]+2\rho\, n[t,x,y,z]\times ux[t,x,y,z]\, W^{(0,1,0,0)}[t,x,y,z]+\rho\, n[t,x,y,z]\, W^{(1,0,0,0)}[t,x,y,z]\right)+O[\tfrac1c]^2 \\
\rho\, n[t,x,y,z]\, W^{(0,1,0,0)}[t,x,y,z]+O[\tfrac1c]^2 \\
0 \\
\left(y\rho\, n[t,x,y,z]\, W^{(0,0,0,1)}[t,x,y,z]-z\rho\, n[t,x,y,z]\, W^{(0,0,1,0)}[t,x,y,z]\right)+O[\tfrac1c]^2 \\
t\rho\, n[t,x,y,z]\, W^{(0,1,0,0)}[t,x,y,z]+O[\tfrac1c]^2 \\
0 \\
\left(y\rho\, n[t,x,y,z]\, W^{(0,0,0,1)}[t,x,y,z]-z\rho\, n[t,x,y,z]\, W^{(0,0,1,0)}[t,x,y,z]\right)+O[\tfrac1c]^2 \\
-t\rho\, n[t,x,y,z]\, W^{(0,1,0,0)}[t,x,y,z]+O[\tfrac1c]^2 \\
0 \\
\left(\frac{sxz\, jx^{(0,0,0,1)}[t,x,y,z]}{2n[t,x,y,z]}+\frac{szx\, jx^{(0,0,0,1)}[t,x,y,z]}{2n[t,x,y,z]}-\frac{\rho\, jx[t,x,y,z]\times uz[t,x,y,z]\, jx^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}+\rho\, ux[t,x,y,z]\times uz[t,x,y,z]\, jx^{(0,0,0,1)}[t,x,y,z]+\frac{syz\, jy^{(0,0,0,1)}[t,x,y,z]}{2n[t,x,y,z]}+\frac{szy\, jy^{(0,0,0,1)}[t,x,y,z]}{2n[t,x,y,z]}-\frac{\rho\, jy[t,x,y,z]\times uz[t,x,y,z]\, jy^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}+\rho\, uy[t,x,y,z]\times uz[t,x,y,z]\, jy^{(0,0,0,1)}[t,x,y,z]+\frac{szz\, jz^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}-\frac{\rho\, jz[t,x,y,z]\times uz[t,x,y,z]\, jz^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}+\rho\, uz[t,x,y,z]^2\, jz^{(0,0,0,1)}\right. \\
\left(-\rho\, n[t,x,y,z]\times uz[t,x,y,z]\, W^{(0,0,0,1)}[t,x,y,z]-\rho\, n[t,x,y,z]\times uy[t,x,y,z]\, W^{(0,0,1,0)}[t,x,y,z]-\rho\, n[t,x,y,z]\times ux[t,x,y,z]\, W^{(0,1,0,0)}[t,x,y,z]\right)+O[\tfrac1c]^2
\end{array}\right)$$

```
In[•]:= (* covariant derivatives of coordinate 4-vectors (equivalent to Christoffel symbols), for later use *)
(Dtxyzvec = Table[Assuming[assut, Expand //@ FS@PowerExpand[(D[IdentityMatrix[4][[aa]], {coords}] + Sum[IdentityMatrix[4][[aa]][[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]], {aa, 1, 4}]);
```

```
In[•]:= (* normalized coordinate-t 4-vector*)
tvecnorm = Assuming[assut, Expand //@ FS@PowerExpand[c * {1, 0, 0, 0} / Sqrt[-gg[[1, 1]]]]]
```

Out[•]= $\left\{1+\frac{W}{c^2}+O[\tfrac1c]^4,\ 0,\ 0,\ 0\right\}$

```
In[•]:= (* and its covariant derivative *)
(Dtvecnorm = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[tvecnorm], {coords}] + Sum[tW[tvecnorm][[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]) // MF
```

Out[•]//MatrixForm=

$$\left(\begin{array}{llll}
O[\tfrac1c]^4 & O[\tfrac1c]^4 & O[\tfrac1c]^4 & O[\tfrac1c]^4 \\
-W^{(0,1,0,0)}[t,x,y,z]+O[\tfrac1c]^2 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & 0 & 0 \\
-W^{(0,0,1,0)}[t,x,y,z]+O[\tfrac1c]^2 & 0 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & 0 \\
-W^{(0,0,0,1)}[t,x,y,z]+O[\tfrac1c]^2 & 0 & 0 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4
\end{array}\right)$$

```
In[•]:= (* "raised" coordinate 4-covectors *)
(gtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[igg.IdentityMatrix[4]]]) // MF
```

Out[•]//MatrixForm=

$$\left(\begin{array}{llll}
-\left(\frac1c\right)^2-\frac{2W}{c^4}+O[\tfrac1c]^6 & 0 & 0 & 0 \\
0 & 1-\frac{2W}{c^2}+O[\tfrac1c]^4 & 0 & 0 \\
0 & 0 & 1-\frac{2W}{c^2}+O[\tfrac1c]^4 & 0 \\
0 & 0 & 0 & 1-\frac{2W}{c^2}+O[\tfrac1c]^4
\end{array}\right)$$

```
In[•]:= (* and their covariant derivatives *)
(Dgtxyzvec = Table[Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[igg[[aa]]], {coords}] + Sum[tW[igg[[aa]]][[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]], {aa, 1, 4}]);
```

```
In[•]:= (* x-component of rot vector *)
Lxvec = Assuming[assut, Expand //@ FS@PowerExpand[{0, 0, -z, y}]]
```

Out[•]= {0, 0, -z, y}

```
In[•]:= (* and its covariant derivative *)
showf[assut][DLxvec = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[Lxvec], {coords}] + Sum[tW[Lxvec][[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\left(\begin{array}{llll}
\frac{-y W^{(0,0,0,1)}[t,x,y,z]+z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & 0 & -\frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\tfrac1c]^6 & \frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\tfrac1c]^6 \\
0 & \frac{y W^{(0,0,0,1)}[t,x,y,z]-z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & \frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & -\frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 \\
-\frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & -\frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & \frac{y W^{(0,0,0,1)}[t,x,y,z]-z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & -1+\frac{-z W^{(0,0,0,1)}[t,x,y,z]-y W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 \\
\frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & \frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & 1+\frac{z W^{(0,0,0,1)}[t,x,y,z]+y W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & \frac{y W^{(0,0,0,1)}[t,x,y,z]-z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4
\end{array}\right)$$

```
In[•]:= (* "raised" x-component of rot co-vector *)
gLxvec = Assuming[assut, Expand //@ FS@PowerExpand[igg.{0, 0, -z, y}]]
```

Out[•]= $\left\{0,\ 0,\ -z+\frac{2Wz}{c^2}+O[\tfrac1c]^4,\ y-\frac{2(Wy)}{c^2}+O[\tfrac1c]^4\right\}$

```
In[•]:= (* and its covariant derivative *)
showf[assut][DgLxvec = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[gLxvec], {coords}] + Sum[tW[gLxvec][[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\left(\begin{array}{llll}
\frac{-y W^{(0,0,0,1)}[t,x,y,z]+z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & 0 & -\frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\tfrac1c]^6 & \frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\tfrac1c]^6 \\
0 & \frac{y W^{(0,0,0,1)}[t,x,y,z]-z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & \frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & -\frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 \\
\frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & \frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & \frac{y W^{(0,0,0,1)}[t,x,y,z]-z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & -1+\frac{2W[t,x,y,z]+z W^{(0,0,0,1)}[t,x,y,z]-y W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 \\
-\frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & -\frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & 1+\frac{-2W[t,x,y,z]+z W^{(0,0,0,1)}[t,x,y,z]-y W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4 & \frac{-y W^{(0,0,0,1)}[t,x,y,z]-z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac1c]^4
\end{array}\right)$$

```
In[•]:= (* x-component of boost vector *)
xboost = Assuming[assut, Expand //@ FS@PowerExpand[{x / c, t * c, 0, 0}]]
```

Out[•]= $\left\{\frac{x}{c},\ c\,t,\ 0,\ 0\right\}$

```
In[•]:= (* and its covariant derivative *)
showf[assut][Dxboost = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[xboost], {coords}] + Sum[tW[xboost][[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\left(\begin{array}{llll}
-\frac{t W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\tfrac1c]^3 & \frac1c+\frac{-x W^{(0,1,0,0)}[t,x,y,z]+t W^{(1,0,0,0)}[t,x,y,z]}{c^3}+O[\tfrac1c]^5 & -\frac{x W^{(0,0,1,0)}[t,x,y,z]}{c^3}+O[\tfrac1c]^5 & -\frac{x W^{(0,0,0,1)}[t,x,y,z]}{c^3}+O[\tfrac1c]^5 \\
c+\frac{-x W^{(0,1,0,0)}[t,x,y,z]+t W^{(1,0,0,0)}[t,x,y,z]}{c}+O[\tfrac1c]^3 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\tfrac1c]^3 & \frac{t W^{(0,0,1,0)}[t,x,y,z]}{c}+O[\tfrac1c]^3 & \frac{t W^{(0,0,0,1)}[t,x,y,z]}{c}+O[\tfrac1c]^3 \\
-\frac{x W^{(0,0,1,0)}[t,x,y,z]}{c}+O[\tfrac1c]^3 & -\frac{t W^{(0,0,1,0)}[t,x,y,z]}{c}+O[\tfrac1c]^3 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\tfrac1c]^3 & 0 \\
-\frac{x W^{(0,0,0,1)}[t,x,y,z]}{c}+O[\tfrac1c]^3 & -\frac{t W^{(0,0,0,1)}[t,x,y,z]}{c}+O[\tfrac1c]^3 & 0 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\tfrac1c]^3
\end{array}\right)$$

In[•]:=
```
(* "raised" x-component of boost co-vector *)
gxboost = Assuming[assut, Expand //@ FS@PowerExpand[igg.{x, -t, 0, 0}]]
```

Out[•]=
$$\left\{-\frac{x}{c^2}-\frac{2\left(W\,x\right)}{c^4}+O\left[\frac{1}{c}\right]^6,\ -t+\frac{2\,t\,W}{c^2}+O\left[\frac{1}{c}\right]^4,\ 0,\ 0\right\}$$

In[•]:=
```
(* and its covariant derivative *)
showf[assut][Dgxboost = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[gxboost], {coords}] + Sum[tW[gxboost][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} \frac{t\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -(\frac{1}{c})^2+\frac{-2\,W[t,x,y,z]-x\,W^{(0,1,0,0)}[t,x,y,z]-t\,W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 & -\frac{x\,W^{(0,0,1,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 & -\frac{x\,W^{(0,0,0,1)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 \\ -1+\frac{2\,W[t,x,y,z]+x\,W^{(0,1,0,0)}[t,x,y,z]+t\,W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t\,W^{(0,0,0,1)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ \frac{x\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{t\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 \\ \frac{x\,W^{(0,0,0,1)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t\,W^{(0,0,0,1)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 & -\frac{t\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
(* content and flux of coordinatevector-energy and coordinatevector-momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n}+n\left(-\epsilon+W\,\rho\right)\right)+O[\frac{1}{c}]^2 & \left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\rho\,c^2+\left(-\frac{Ax\left(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz\right)+Ay\left(n\,qy+jx\,syx+jy\,syy+jz\,syz\right)+Az\left(n\,qz+jx\,szx+jy\,szy+jz\,szz\right)}{n}+\left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\epsilon-\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx^2+jy^2+jz^2-2\,n^2\,W\right)\rho}{2\,n^2}\right)+O[\frac{1}{c}]^2 \\ jx\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jy\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+\frac{jy\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jz\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+\frac{jz\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[Dtxyzvec[[aa,  ;; ,  ;;]].TTx]], {aa, 1, 4}]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} \rho\,n[t,x,y,z]\,W^{(1,0,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ \rho\,n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ \rho\,n[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ \rho\,n[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]+O[\frac{1}{c}]^2 \end{pmatrix}$$

```
(* content and flux of raised coordinatecovector-energy and coordinatecovector-momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

```
(* content and flux of coord-energy and momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxEPS = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n}+n\left(-\epsilon+W\,\rho\right)\right)+O[\frac{1}{c}]^2 & \left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\rho\,c^2+\left(-\frac{Ax\left(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz\right)+Ay\left(n\,qy+jx\,syx+jy\,syy+jz\,syz\right)+Az\left(n\,qz+jx\,szx+jy\,szy+jz\,szz\right)}{n}+\left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\epsilon-\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx^2+jy^2+jz^2-2\,n^2\,W\right)\rho}{2\,n^2}\right)+O[\frac{1}{c}]^2 \\ jx\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jy\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+\frac{jy\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jz\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+\frac{jz\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O[\frac{1}{c}]^2 \end{pmatrix}$$

```
(* content and flux of coord-energy and momentum for dust (TRANSPOSED) *)
shows[assut, 1][T[fluxdust = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.dust2]]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(aux^2+auy^2+auz^2-2\,W\right)\rho\right)+O[\frac{1}{c}]^2 & \left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\rho\,c^2-\frac{1}{2}\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(2\,\epsilon+\left(aux^2+auy^2+auz^2-2\,W\right)\rho\right)+O[\frac{1}{c}]^2 \\ aux\,n\,\rho+O[\frac{1}{c}]^2 & aux\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho+O[\frac{1}{c}]^2 \\ auy\,n\,\rho+O[\frac{1}{c}]^2 & auy\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho+O[\frac{1}{c}]^2 \\ auz\,n\,\rho+O[\frac{1}{c}]^2 & auz\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
shows[assut, 1][T[fluxEPS /. replaceJu]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O[\frac{1}{c}]^2 & n\left(Ax\left(-ux+Vx\right)+Ay\left(-uy+Vy\right)+Az\left(-uz+Vz\right)\right)\rho\,c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\left(ux-Vx\right)\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n\left(uy-Vy\right)\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n\left(uz-Vz\right)\epsilon\right)-\frac{1}{2}\,n\left(Ax\left(ux-Vx\right)+Ay\left(uy-Vy\right)+Az\left(uz-Vz\right)\right)\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O[\frac{1}{c}]^2 \\ n\,ux\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax\left(ux-Vx\right)+Ay\left(uy-Vy\right)+Az\left(uz-Vz\right)\right)\rho\right)+O[\frac{1}{c}]^2 \\ n\,uy\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+n\,uy\left(Ax\left(ux-Vx\right)+Ay\left(uy-Vy\right)+Az\left(uz-Vz\right)\right)\rho\right)+O[\frac{1}{c}]^2 \\ n\,uz\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+n\,uz\left(Ax\left(ux-Vx\right)+Ay\left(uy-Vy\right)+Az\left(uz-Vz\right)\right)\rho\right)+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* momentum flux = A.σ + P A.(u-V)*)
fluxPS = ({Ax, Ay, Az}.(S[[2 ;; 4, 2 ;; 4]]).{1, 0, 0} + EPS[[1, 2]] * ({Ax, Ay, Az}.({jx, jy, jz} / n - {Vx, Vy, Vz})))
```

Out[•]=
$$\left(Ax\,sxx+Ay\,syx+Az\,szx+jx\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\rho\right)+\frac{1}{c^2}\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\left(px+\frac{jx\,sxx}{n}+\frac{jy\,syx}{n}+\frac{jz\,szx}{n}+jx\,\epsilon+\frac{jx^3\,\rho}{2\,n^2}+\frac{jx\,jy^2\,\rho}{2\,n^2}+\frac{jx\,jz^2\,\rho}{2\,n^2}+3\,jx\,W\,\rho\right)+O\left[\frac{1}{c}\right]^4$$

In[•]:=
```
shows[assut, 1][Expand //@ FS@PowerExpand[fluxEPS[[2, 2]] - fluxPS]]
```

Out[•]//MatrixForm=
$$O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* energy flux = A.q + A.σ.u + E A.(u-V)*)
fluxE = -({Ax, Ay, Az}.{qx, qy, qz} + {Ax, Ay, Az}.(S[[2 ;; 4, 2 ;; 4]] + Qtens[[2 ;; 4, 2 ;; 4]]).{jx, jy, jz} / n + (-EPS[[1, 1]]) * {Ax, Ay, Az}.({jx, jy, jz} / n - {Vx, Vy, Vz}))
```

Out[•]=
$$-n\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\rho\,c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz-\frac{1}{n}\left(jx\left(Ax\,sxx+Ay\,syx+Az\,szx\right)+jy\left(Ax\,sxy+Ay\,syy+Az\,szy\right)+jz\left(Ax\,sxz+Ay\,syz+Az\,szz\right)\right)-\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\left(n\,\epsilon+\frac{jx^2\,\rho}{2\,n}+\frac{jy^2\,\rho}{2\,n}+\frac{jz^2\,\rho}{2\,n}-n\,W\,\rho\right)\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[fluxEPS[[2, 1]] - fluxE]]
```

Out[•]//MatrixForm=
$$O\left[\frac{1}{c}\right]^2$$

```
(* matter flux n A.(u-V) *)
shows[assut, 1][fluxNJ = Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.NJ /. replaceJu]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} n \\ n\left(Ax\left(ux-Vx\right)+Ay\left(uy-Vy\right)+Az\left(uz-Vz\right)\right) \end{pmatrix}$$

```
(* content and flux of coord-energy and momentum assuming no matter flux (transposed) *)
shows[Join[assut, {((surface / Δt).NJ) == 0} /. replaceJu], 1][T@fluxEPS /. replaceJu]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O[\frac{1}{c}]^2 & \left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz\right)\right)+O[\frac{1}{c}]^2 \\ n\,ux\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx\right)+O[\frac{1}{c}]^2 \\ n\,uy\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy\right)+O[\frac{1}{c}]^2 \\ n\,uz\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz\right)+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=

```
(* coordinate/internal/coordinate-proper energy and x-momentum, content and fluxes (TRANSPOSED) *)
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, Lxvec, Lxvec2, xboost / c, xboost2, uu, ntvec}]) /. replaceJu]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O[\frac{1}{c}]^2 & n\left(Ax\left(-ux+Vx\right)+Ay\left(-uy+Vy\right)+Az\left(-uz+Vz\right)\right)\rho\,c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\left(ux-Vx\right)\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n\left(uy-Vy\right)\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n\left(uz-Vz\right)\epsilon\right)-\frac{1}{2}\,n\left(Ax\left(ux-Vx\right)+Ay\left(uy-Vy\right)+Az\left(uz-Vz\right)\right)\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O[\frac{1}{c}]^2 \\ n\,ux\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax\left(ux-Vx\right)+Ay\left(uy-Vy\right)+Az\left(uz-Vz\right)\right)\rho\right)+O[\frac{1}{c}]^2 \\ n\left(uz\,y-uy\,z\right)\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax\left(ux-Vx\right)+Ay\left(uy-Vy\right)+Az\left(uz-Vz\right)\right)\left(uz\,y-uy\,z\right)\rho\right)+O[\frac{1}{c}]^2 \\ n\left(uz\,y-uy\,z\right)\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax\left(ux-Vx\right)+Ay\left(uy-Vy\right)+Az\left(uz-Vz\right)\right)\left(uz\,y-uy\,z\right)\rho\right)+O[\frac{1}{c}]^2 \\ -n\left(t\,ux+x\right)\rho+O[\frac{1}{c}]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-n\left(Ax\left(ux-Vx\right)+Ay\left(uy-Vy\right)+Az\left(uz-Vz\right)\right)\left(t\,ux+x\right)\rho\right)+O[\frac{1}{c}]^2 \\ n\left(-t\,ux+x\right)\rho+O[\frac{1}{c}]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-n\left(Ax\left(ux-Vx\right)+Ay\left(uy-Vy\right)+Az\left(uz-Vz\right)\right)\left(t\,ux-x\right)\rho\right)+O[\frac{1}{c}]^2 \\ -n\,\rho\,c^2-n\,\epsilon+O[\frac{1}{c}]^2 & n\left(Ax\left(-ux+Vx\right)+Ay\left(-uy+Vy\right)+Az\left(-uz+Vz\right)\right)\rho\,c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+n\left(Ax\left(-ux+Vx\right)+Ay\left(-uy+Vy\right)+Az\left(-uz+Vz\right)\right)\epsilon\right)+O[\frac{1}{c}]^2 \\ -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(ux^2+uy^2+uz^2\right)\rho\right)+O[\frac{1}{c}]^2 & n\left(Ax\left(-ux+Vx\right)+Ay\left(-uy+Vy\right)+Az\left(-uz+Vz\right)\right)\rho\,c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\left(ux-Vx\right)\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n\left(uy-Vy\right)\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n\left(uz-Vz\right)\epsilon\right)-\frac{1}{2}\,n\left(ux^2+uy^2+uz^2\right)\left(Ax\left(ux-Vx\right)+Ay\left(uy-Vy\right)+Az\left(uz-Vz\right)\right)\rho\right)+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
show2[assut, 1][T[variousfluxes /. {Vy → 0, Vz → 0, uy → 0, uz → 0}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & Ax\,n(-ux+Vx)\rho c^2+\left(-Ay\left(qy+syx\,ux\right)-Az\left(qz+szx\,ux\right)-Ax\left(qx+sxx\,ux+n(ux-Vx)\epsilon\right)-\frac{1}{2}Ax\,n(ux-Vx)\left(ux^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+Ax\,n\,ux(ux-Vx)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z\right)+O\left[\frac{1}{c}\right]^2 \\
O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z\right)+O\left[\frac{1}{c}\right]^2 \\
-n\left(t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-Ax\,n(ux-Vx)\left(t\,ux+x\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
n\left(-t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-Ax\,n(ux-Vx)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
-n\rho c^2-n\epsilon+O\left[\frac{1}{c}\right]^2 & Ax\,n(-ux+Vx)\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+Ax\,n(-ux+Vx)\epsilon\right)+O\left[\frac{1}{c}\right]^2 \\
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+ux^2\rho\right)+O\left[\frac{1}{c}\right]^2 & Ax\,n(-ux+Vx)\rho c^2+\left(-Ay\left(qy+syx\,ux\right)-Az\left(qz+szx\,ux\right)-Ax\left(qx+sxx\,ux+n(ux-Vx)\epsilon\right)-\frac{1}{2}Ax\,n\,ux^2(ux-Vx)\rho\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[•]:=
```
(* velocity of energy *)
shows[assut, 5][(EPS.{1, 0, 0, 0})[[2 ;; 4]] / (EPS.{1, 0, 0, 0})[[1]] /. replaceJu]
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
ux+\frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\rho c^2}+O\left[\frac{1}{c}\right]^4 \\
uy+\frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\rho c^2}+O\left[\frac{1}{c}\right]^4 \\
uz+\frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\rho c^2}+O\left[\frac{1}{c}\right]^4
\end{pmatrix}$$

In[•]:=
```
temp = SeriesCoefficient[tt.{1, 0, 0, 0}, {c, Infinity, -2}];
shows[assut, 5][(tt.{1, 0, 0, 0} - temp * c^2)[[2 ;; 4]] / (tt.{1, 0, 0, 0} - temp * c^2)[[1]] /. j2v]
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
\left(ux+\frac{2\left(qx+sxx\,ux+sxy\,uy+sxz\,uz\right)}{2n\epsilon+n\left(ux^2+uy^2+uz^2-2W\right)\rho}\right)+O\left[\frac{1}{c}\right]^2 \\
\left(uy+\frac{2\left(qy+syx\,ux+syy\,uy+syz\,uz\right)}{2n\epsilon+n\left(ux^2+uy^2+uz^2-2W\right)\rho}\right)+O\left[\frac{1}{c}\right]^2 \\
\left(uz+\frac{2\left(qz+szx\,ux+szy\,uy+szz\,uz\right)}{2n\epsilon+n\left(ux^2+uy^2+uz^2-2W\right)\rho}\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[•]:=
```
showf[assut][{variousfluxes[[ ;; , 1]] - variousfluxes[[ ;; , 7]], variousfluxes[[ ;; , 1]] - variousfluxes[[ ;; , 8]], variousfluxes[[ ;; , 7]] - variousfluxes[[ ;; , 8]]}]
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
\left(-\frac{1}{2}n\,ux^2\rho-\frac{1}{2}n\,uy^2\rho-\frac{1}{2}n\,uz^2\rho+nW\rho\right)+O\left[\frac{1}{c}\right]^2 & \left(-Ax\,sxx\,ux-Ay\,syx\,ux-Az\,szx\,ux-Ax\,sxy\,uy-Ay\,syy\,uy-Az\,szy\,uy-Ax\,sxz\,uz-Ay\,syz\,uz-Az\,szz\,uz-\frac{1}{2}Ax\,n\,ux^3\rho-\frac{1}{2}Ay\,n\,ux^2\,uy\,\rho-\frac{1}{2}Ax\,n\,ux\,uy^2\rho-\frac{1}{2}Ay\,n\,uy^3\rho-\frac{1}{2}Az\,n\,ux^2\,uz\,\rho-\frac{1}{2}Az\,n\,uy^2\,uz\,\rho-\frac{1}{2}Ax\,n\,ux\,uz^2\rho-\frac{1}{2}Ay\,n\,uy\,uz^2\rho-\frac{1}{2}Az\,n\,uz^3\rho+\frac{1}{2}Ax\,n\,ux^2\,Vx\,\rho+\frac{1}{2}Ax\,n\,uy^2\,Vx\,\rho+\frac{1}{2}Ax\,n\ldots \right. \\
nW\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,n\,ux\,W\rho+Ay\,n\,uy\,W\rho+Az\,n\,uz\,W\rho-Ax\,n\,Vx\,W\rho-Ay\,n\,Vy\,W\rho-Az\,n\,Vz\,W\rho\right)+O\left[\frac{1}{c}\right]^2 \\
\left(\frac{1}{2}n\,ux^2\rho+\frac{1}{2}n\,uy^2\rho+\frac{1}{2}n\,uz^2\rho\right)+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx\,ux+Ay\,syx\,ux+Az\,szx\,ux+Ax\,sxy\,uy+Ay\,syy\,uy+Az\,szy\,uy+Ax\,sxz\,uz+Ay\,syz\,uz+Az\,szz\,uz+\frac{1}{2}Ax\,n\,ux^3\rho+\frac{1}{2}Ay\,n\,ux^2\,uy\,\rho+\frac{1}{2}Ax\,n\,ux\,uy^2\rho+\frac{1}{2}Ay\,n\,uy^3\rho+\frac{1}{2}Az\,n\,ux^2\,uz\,\rho+\frac{1}{2}Az\,n\,uy^2\,uz\,\rho+\frac{1}{2}Ax\,n\,ux\,uz^2\rho+\frac{1}{2}Ay\,n\,uy\,uz^2\rho+\frac{1}{2}Az\,n\,uz^3\rho-\frac{1}{2}Ax\,n\,ux^2\,Vx\,\rho-\frac{1}{2}Ax\,n\,uy^2\,Vx\,\rho-\frac{1}{2}Ax\,n\ldots \right.
\end{pmatrix}$$

In[•]:=
```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
show2[assut, 2][FS[(itjv[Tr[#.TTx]]) /. replaceJu] & /@ {Dxyzvec[[1, ;; , ;;]], Dxyzvec[[2, ;; , ;;]], DLxvec, DLxvec2, Dxboost / c, Dxboost2, Duv, Dntvec} // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
n\rho W^{(1,0,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\
n\rho W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\
n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\
n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\
-n\rho\left(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\
-n\,t\,\rho W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\
\frac{1}{2}\left(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z]+(syz+szy)\left(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\right)+2\left(syy\,uy^{(0,0,1,0)}[t,x,y,z]+sxx\,ux^{(0,1,0,0)}[t,x,y,z]\right)+(sxy+syx)\left(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z]\right)+(sxz+szx)\left(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\right)\right)+O\left[\frac{1}{c}\right]^2 \\
-n\rho\left(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.EPSsym.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, Lxvec, Lxvec2, xboost / c, xboost2, uu, ntvec}]) /. replaceJu]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+sxy\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+sxz\,ux+syz\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac{1}{2}n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,sxy+Az\,sxz+n\,ux\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,syz\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,syz\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
-n\left(t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,sxy+Az\,sxz\right)t\right)-n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(t\,ux+x\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
n\left(-t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,sxy+Az\,sxz\right)t\right)-n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
-n\rho c^2-n\epsilon+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\epsilon\right)+O\left[\frac{1}{c}\right]^2 \\
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2+uy^2+uz^2\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+sxy\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+sxz\,ux+syz\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac{1}{2}n\left(ux^2+uy^2+uz^2\right)\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[•]:=
```
TTx = tW[tjv[(EPSsym + T[EPSsym.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. replaceJu] & /@ {Dxyzvec[[1, ;; , ;;]], Dxyzvec[[2, ;; , ;;]], DLxvec, DLxvec2, Dxboost / c, Dxboost2, Duv, Dntvec} // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
n\rho W^{(1,0,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\
n\rho W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\
n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\
n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\
-n\rho\left(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\
-n\,t\,\rho W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\
\left(szz\,uz^{(0,0,0,1)}[t,x,y,z]+syy\,uy^{(0,0,1,0)}[t,x,y,z]+syz\left(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\right)+sxx\,ux^{(0,1,0,0)}[t,x,y,z]+sxy\left(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z]\right)+sxz\left(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\right)\right)+O\left[\frac{1}{c}\right]^2 \\
-n\rho\left(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[•]:=
```
(* 2-vector of surface parallel to yz  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(yzsurface = ((T[{{0, 0, Ly, 0}}].{{0, 0, 0, Lz}} - T[{{0, 0, 0, Lz}}].{{0, 0, Ly, 0}}) /. {Ly → Ayz / Lz})) // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Ayz \\ 0 & 0 & -Ayz & 0 \end{pmatrix}$$

In[•]:=
```
(* 2-vector of surface parallel to tx  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(txsurface = (-(T[{{Δt, 0, 0, 0}}].{{0, Lx, 0, 0}} - T[{{0, Lx, 0, 0}}].{{Δt, 0, 0, 0}}))) // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -Lx\,\Delta t & 0 & 0 \\ Lx\,\Delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[•]:=
```
(* 2-vector of surface parallel to ty  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(tysurface = (-(T[{{Δt, 0, 0, 0}}].{{0, 0, Ly, 0}} - T[{{0, 0, Ly, 0}}].{{Δt, 0, 0, 0}}))) // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -Ly\,\Delta t & 0 \\ 0 & 0 & 0 & 0 \\ Ly\,\Delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[•]:=
```
(* 2-vector of surface parallel to y moving to x  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(yVxsurface = (-(T[{{1, Vx, 0, 0}} * Δt].{{0, 0, Ly, 0}} - T[{{0, 0, Ly, 0}}].{{1, Vx, 0, 0}} * Δt))) // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -Ly\,\Delta t & 0 \\ 0 & 0 & -Ly\,Vx\,\Delta t & 0 \\ Ly\,\Delta t & Ly\,Vx\,\Delta t & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[•]:=
```
(Tr[T[txsurface].txsurface]) // MF
```

Out[•]//MatrixForm=

$$2\,Lx^2\,\Delta t^2$$

In[•]:=
```
(* Faraday tensor *)
repE = {Ex → Ex * c * Sqrt[μ * ϵ], Ey → Ey * c * Sqrt[μ * ϵ], Ez → Ez * c * Sqrt[μ * ϵ]};
fftemp = {{0, -Ex, -Ey, -Ez}, {0, 0, Bz, -By}, {0, 0, 0, Bx}, {0, 0, 0, 0}} /. repE;
showf[assut][F = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -c\,Ex\sqrt{\epsilon}\sqrt{\mu} & -c\,Ey\sqrt{\epsilon}\sqrt{\mu} & -c\,Ez\sqrt{\epsilon}\sqrt{\mu} \\ c\,Ex\sqrt{\epsilon}\sqrt{\mu} & 0 & Bz & -By \\ c\,Ey\sqrt{\epsilon}\sqrt{\mu} & -Bz & 0 & Bx \\ c\,Ez\sqrt{\epsilon}\sqrt{\mu} & By & -Bx & 0 \end{pmatrix}$$

In[•]:=
```
(FS[{Tr[yzsurface.T[F]], Tr[T[txsurface].F], Tr[T[tysurface].F], Tr[T[yVxsurface].F]} / 2]) // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix} Ayz\,Bx \\ c\,Ex\,Lx\,\Delta t\sqrt{\epsilon}\sqrt{\mu} \\ c\,Ey\,Ly\,\Delta t\sqrt{\epsilon}\sqrt{\mu} \\ Ly\,\Delta t\left(-Bz\,Vx+c\,Ey\sqrt{\epsilon}\sqrt{\mu}\right) \end{pmatrix}$$

In[•]:=
```
(* charge-current-potential tensor *)
fftemp = {{0, -Hx, -Hy, -Hz}, {0, 0, Dz, -Dy}, {0, 0, 0, Dx}, {0, 0, 0, 0}};
showf[assut][H = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -Hx & -Hy & -Hz \\ Hx & 0 & Dz & -Dy \\ Hy & -Dz & 0 & Dx \\ Hz & Dy & -Dx & 0 \end{pmatrix}$$

In[∘]:=
```
showf[assut][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
  ]]]
```
Out[∘]//MatrixForm=

(⋯ 1 ⋯)

Full expression not available (original memory size: 0.7 MB)

In[∘]:=
```
shows[assut, 1][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
  ]]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix}
-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & \frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{(-Bz\,Ex+Bx\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{(By\,Ex-Bx\,Ey)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
\frac{(-Bz\,Ey+By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(Bz\,Ey-By\,Ez)W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{-Bx^2+By^2+Bz^2+(-Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & -\frac{Bx\,By+Ex\,Ey\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & -\frac{Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 \\
\frac{(Bz\,Ex-Bx\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(-Bz\,Ex+Bx\,Ez)W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & -\frac{Bx\,By+Ex\,Ey\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & \frac{Bx^2-By^2+Bz^2+(Ex^2-Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & -\frac{By\,Bz+Ey\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 \\
\frac{(-By\,Ex+Bx\,Ey)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(By\,Ex-Bx\,Ey)W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & -\frac{Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & -\frac{By\,Bz+Ey\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & \frac{Bx^2+By^2-Bz^2+(Ex^2+Ey^2-Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2
\end{pmatrix}$$

In[∘]:=
```
showf[assut][T[tte.Inverse[gg]].gg - tte]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix}
O[\frac{1}{c}]^4 & O[\frac{1}{c}]^5 & O[\frac{1}{c}]^5 & O[\frac{1}{c}]^5 \\
O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\
O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\
O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4
\end{pmatrix}$$

In[∘]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.tte.T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix}
-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & \frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{(Ax\,Vx+Ay\,Vy+Az\,Vz)(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)}{2\,\mu0}+\frac{2(Az\,By\,Ex-Ay\,Bz\,Ex-Az\,Bx\,Ey+Ax\,Bz\,Ey+Ay\,Bx\,Ez-Ax\,By\,Ez)W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+\frac{(Bz\,Ey\,ux-By\,Ez\,ux-Bz\,Ex\,uy+Bx\,Ez\,uy+By\,Ex\,uz-Bx\,Ey\,uz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{Ax(-2\,Bx(By\,uy+Bz\,uz)+Bx^2(-ux+Vx)+(By^2+Bz^2)(ux+Vx)+(-2\,Ex(Ey\,uy+Ez\,uz)+Ex^2(-ux+Vx)+(Ey^2+Ez^2)(ux+Vx))\,\epsilon0\,\mu0)+Ay(-2\,Bx\,By\,ux-2\,By\,Bz\,uz+By^2(-uy+Vy)+Bx^2(uy+Vy)+Bz^2(uy+Vy)+(-2\,Ex\,Ey\,ux-2\,Ey\,Ez\,uz+Ey^2(-uy+Vy)+Ex^2(uy+Vy)+Ez^2(uy+Vy))\,\epsilon0\,\mu0)+Az(-2\,Bx\,Bz\,ux-2\,By\,Bz\,uy+Bz^2(-uz+Vz)+Bx^2 \cdots}{2\,\mu0} \cdots \\
-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & \frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{(Ax\,Vx+Ay\,Vy+Az\,Vz)(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)}{2\,\mu0}+\frac{(Az\,By\,Ex-Ay\,Bz\,Ex-Az\,Bx\,Ey+Ax\,Bz\,Ey+Ay\,Bx\,Ez-Ax\,By\,Ez)W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
\frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{-2(Ay\,Bx\,By+Az\,Bx\,Bz+Ay\,Ex\,Ey\,\epsilon0\,\mu0+Az\,Ex\,Ez\,\epsilon0\,\mu0)+Ax(-Bx^2+By^2+Bz^2+(-Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)}{2\,\mu0}-\frac{(Bz\,Ey-By\,Ez)(Ax\,Vx+Ay\,Vy+Az\,Vz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+(-2(Ax\,Ex+Ay\,Ey)Ez\,y+Az(Ex^2+Ey^2-Ez^2)y+2\,Ey(Ax\,Ex+Az\,Ez)z-Ay(Ex^2-Ey^2+Ez^2)z)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{(Ax\,Vx+Ay\,Vy+Az\,Vz)(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+(-2(Ax\,Ex+Ay\,Ey)Ez\,y+Az(Ex^2+Ey^2-Ez^2)y+2\,Ey(Ax\,Ex+Az\,Ez)z-Ay(Ex^2-Ey^2+Ez^2)z)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{(Ax\,Vx+Ay\,Vy+Az\,Vz)(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{t(2(Ay\,Bx\,By+Az\,Bx\,Bz+Ay\,Ex\,Ey\,\epsilon0\,\mu0+Az\,Ex\,Ez\,\epsilon0\,\mu0)+Ax(Bx^2-By^2-Bz^2+(Ex^2-Ey^2-Ez^2)\,\epsilon0\,\mu0))}{2\,\mu0}+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{t(2(Ay\,Bx\,By+Az\,Bx\,Bz+Ay\,Ex\,Ey\,\epsilon0\,\mu0+Az\,Ex\,Ez\,\epsilon0\,\mu0)+Ax(Bx^2-By^2-Bz^2+(Ex^2-Ey^2-Ez^2)\,\epsilon0\,\mu0))}{2\,\mu0}+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(Az\,By\,Ex-Ay\,Bz\,Ex-Az\,Bx\,Ey+Ax\,Bz\,Ey+Ay\,Bx\,Ez-Ax\,By\,Ez)x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2
\end{pmatrix}$$

In[∘]:=
```
TTx = tW[tjv[(tte + T[tte.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```
Out[∘]//MatrixForm=

$$\begin{pmatrix}
\frac{(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)W^{(1,0,0,0)}[t,x,y,z]}{\mu0\,c^2}+O[\frac{1}{c}]^3 \\
\frac{-2(Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0)ux^{(0,0,0,1)}[t,x,y,z]-2(By\,Bz+Ey\,Ez\,\epsilon0\,\mu0)uy^{(0,0,0,1)}[t,x,y,z]+Bx^2\,uz^{(0,0,0,1)}[t,x,y,z]+By^2\,uz^{(0,0,0,1)}[t,x,y,z]-Bz^2\,uz^{(0,0,0,1)}[t,x,y,z]+Ex^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ey^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-Ez^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-2\,Bx\,By\,ux^{(0,0,1,0)}[t,x,y,z]-2\,Ex\,Ey\,\epsilon0\,\mu0\,ux^{(0,0,1,0)} \cdots}{2\,\mu0} \\
\frac{\sqrt{\epsilon0}((-By\,Ex+Bx\,Ey)W^{(0,0,0,1)}[t,x,y,z]+(Bz\,Ex-Bx\,Ez)W^{(0,0,1,0)}[t,x,y,z]+(-Bz\,Ey+By\,Ez)W^{(0,1,0,0)}[t,x,y,z])}{\sqrt{\mu0}\,c}+\frac{(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)W^{(1,0,0,0)}[t,x,y,z]}{2\,\mu0\,c^2}+O[\frac{1}{c}]^3 \\
\frac{(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)W^{(0,1,0,0)}[t,x,y,z]}{\mu0\,c^2}+O[\frac{1}{c}]^3 \\
\frac{(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z])}{\mu0\,c^2}+O[\frac{1}{c}]^3 \\
\frac{2((Bz^2\,y-By\,Bz\,z+Ez(Ez\,y-Ey\,z)\,\epsilon0\,\mu0)W^{(0,0,0,1)}[t,x,y,z]+(By\,Bz\,y-By^2\,z+Ey(Ez\,y-Ey\,z)\,\epsilon0\,\mu0)W^{(0,0,1,0)}[t,x,y,z]+(Bx(Bz\,y-By\,z)+Ex(Ez\,y-Ey\,z)\,\epsilon0\,\mu0)W^{(0,1,0,0)}[t,x,y,z])}{\mu0\,c^2}+O[\frac{1}{c}]^3 \\
\frac{2(-Bz\,Ey+By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}-\frac{t(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)W^{(0,1,0,0)}[t,x,y,z]}{\mu0\,c^2}+O[\frac{1}{c}]^3 \\
-\frac{2(t((Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0)W^{(0,0,0,1)}[t,x,y,z]+(Bx\,By+Ex\,Ey\,\epsilon0\,\mu0)W^{(0,0,1,0)}[t,x,y,z]+(Bx^2+Ex^2\,\epsilon0\,\mu0)W^{(0,1,0,0)}[t,x,y,z]))}{\mu0\,c^2}+O[\frac{1}{c}]^3
\end{pmatrix}$$

In[∘]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.(tte + ttsym).T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix}
-n\,\rho\,c^2+\left(-n\,\epsilon-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{1}{2}n(ux^2+uy^2+uz^2-2\,W)\rho\right)+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\rho\,c^2+\frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{-2\,Ax(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,ux\,\epsilon)\mu0-2\,Ay(qy+sxy\,ux+syy\,uy+syz\,uz+n\,uy\,\epsilon)\mu0-2\,Az(qz+sxz\,ux+syz\,uy+szz\,uz+n\,uz\,\epsilon)\mu0+Ax\,Vx(Bx^2+By^2+Bz^2+2\,n\,\epsilon\,\mu0+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)+Ay\,Vy(Bx^2+By^2+Bz^2+2\,n\,\epsilon\,\mu0+(Ex^2 \cdots}{2\,\mu0} \cdots \\
-n\,\rho\,c^2-\frac{Bx^2+By^2+Bz^2+2\,n\,\epsilon\,\mu0+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+\frac{(Bz\,Ey\,ux-By\,Ez\,ux-Bz\,Ex\,uy+Bx\,Ez\,uy+By\,Ex\,uz-Bx\,Ey\,uz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\rho\,c^2+\frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{Ay(-2\,Bx\,By\,ux-2\,By\,Bz\,uz+By^2(-uy+Vy)+Bx^2(uy+Vy)+Bz^2(uy+Vy)-2(qy+n(uy-Vy)\epsilon)\mu0+(-2\,Ex\,Ey\,ux-2\,Ey\,Ez\,uz+Ey^2(-uy+Vy)+Ex^2(uy+Vy)+Ez^2(uy+Vy))\,\epsilon0\,\mu0)+Ax(-2\,Bx(By\,uy+Bz\,uz)+Bx^2(-ux+Vx)+(By^2+Bz^2)(ux+Vx) \cdots}{2\,\mu0} \cdots \\
-n\,\rho\,c^2+\left(-n\,\epsilon-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{1}{2}n(ux^2+uy^2+uz^2)\rho\right)+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\rho\,c^2+\frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{-2\,Ax(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,ux\,\epsilon)\mu0-2\,Ay(qy+sxy\,ux+syy\,uy+syz\,uz+n\,uy\,\epsilon)\mu0-2\,Az(qz+sxz\,ux+syz\,uy+szz\,uz+n\,uz\,\epsilon)\mu0+Ax\,Vx(Bx^2+By^2+Bz^2+2\,n\,\epsilon\,\mu0+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)+Ay\,Vy(Bx^2+By^2+Bz^2+2\,n\,\epsilon\,\mu0+(Ex^2 \cdots}{2\,\mu0} \cdots \\
n\,ux\,\rho+\frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{-2\,Ay(Bx\,By-sxy\,\mu0+Ex\,Ey\,\epsilon0\,\mu0+n\,ux(-uy+Vy)\mu0\,\rho)+Ax(-Bx^2+By^2+Bz^2+\mu0(2\,sxx+(-Ex^2+Ey^2+Ez^2)\,\epsilon0+2\,n\,ux(ux-Vx)\rho))+2\,Az(-Bx\,Bz+\mu0(sxz-Ex\,Ez\,\epsilon0+n\,ux(uz-Vz)\rho))}{2\,\mu0}-\frac{(Bz\,Ey-By\,Ez)(Ax\,Vx+Ay\,Vy+Az\,Vz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
n(uz\,y-uy\,z)\rho+\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+2\,Ax\,sxz\,y\,\mu0+2\,Ay\,syz\,y\,\mu0+2\,Az\,szz\,y\,\mu0-2\,Ax\,sxy\,z\,\mu0-2\,Ay\,syy\,z\,\mu0-2\,Az\,syz\,z\,\mu0+Az\,Ex^2\,y\,\epsilon0\,\mu0+Az\,Ey^2\,y\,\epsilon0\,\mu0-2\,Ax\,Ex\,Ez\,y\,\epsilon0\,\mu0-2\,Ay\,Ey\,Ez\,y\,\epsilon0\,\mu0-Az\,Ez^2\,y\,\epsilon0\,\mu0-Ay\,Ex^2\,z\,\epsilon0\,\mu0+2\,Ax\,Ex\,Ey\,z\,\epsilon0\,\mu0+Ay\,Ey^2\,z\,\epsilon0\,\mu0+2\,Az\,Ey\,Ez\,z\,\epsilon0\,\mu0-Ay\,Ez^2\,z\,\epsilon0 \cdots}{2\,\mu0} \cdots \\
n(uz\,y-uy\,z)\rho+\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+2\,Ax\,sxz\,y\,\mu0+2\,Ay\,syz\,y\,\mu0+2\,Az\,szz\,y\,\mu0-2\,Ax\,sxy\,z\,\mu0-2\,Ay\,syy\,z\,\mu0-2\,Az\,syz\,z\,\mu0+Az\,Ex^2\,y\,\epsilon0\,\mu0+Az\,Ey^2\,y\,\epsilon0\,\mu0-2\,Ax\,Ex\,Ez\,y\,\epsilon0\,\mu0-2\,Ay\,Ey\,Ez\,y\,\epsilon0\,\mu0-Az\,Ez^2\,y\,\epsilon0\,\mu0-Ay\,Ex^2\,z\,\epsilon0\,\mu0+2\,Ax\,Ex\,Ey\,z\,\epsilon0\,\mu0+Ay\,Ey^2\,z\,\epsilon0\,\mu0+2\,Az\,Ey\,Ez\,z\,\epsilon0\,\mu0-Ay\,Ez^2\,z\,\epsilon0 \cdots}{2\,\mu0} \cdots \\
-n(t\,ux+x)\rho+\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \left(\frac{t(Ax\,Bx^2+2\,Ay\,Bx\,By-Ax\,By^2+2\,Az\,Bx\,Bz-Ax\,Bz^2-(2\,Ax\,sxx+2\,Ay\,sxy+2\,Az\,sxz-2\,Ex(Ay\,Ey+Az\,Ez)\,\epsilon0+Ax(-Ex^2+Ey^2+Ez^2)\,\epsilon0)\mu0)}{2\,\mu0}-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux+x)\rho\right)+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
n(-t\,ux+x)\rho+\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \left(\frac{t(Ax\,Bx^2+2\,Ay\,Bx\,By-Ax\,By^2+2\,Az\,Bx\,Bz-Ax\,Bz^2-(2\,Ax\,sxx+2\,Ay\,sxy+2\,Az\,sxz-2\,Ex(Ay\,Ey+Az\,Ez)\,\epsilon0+Ax(-Ex^2+Ey^2+Ez^2)\,\epsilon0)\mu0)}{2\,\mu0}-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux-x)\rho\right)+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(Az\,By\,Ex-Ay\,Bz\,Ex-Az\,Bx\,Ey+Ax\,Bz\,Ey+Ay\,Bx\,Ez-Ax\,By\,Ez)x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2
\end{pmatrix}$$

In[∘]:=
```
TTx = tW[tjv[((tte + ttsym) + T[(tte + ttsym).Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```
Out[∘]//MatrixForm=

$$\begin{pmatrix}
n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
\frac{-2(Bx\,Bz-sxz\,\mu0+Ex\,Ez\,\epsilon0\,\mu0)ux^{(0,0,0,1)}[t,x,y,z]-2(By\,Bz-syz\,\mu0+Ey\,Ez\,\epsilon0\,\mu0)uy^{(0,0,0,1)}[t,x,y,z]+Bx^2\,uz^{(0,0,0,1)}[t,x,y,z]+By^2\,uz^{(0,0,0,1)}[t,x,y,z]-Bz^2\,uz^{(0,0,0,1)}[t,x,y,z]+2\,szz\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ex^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ey^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-Ez^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)} \cdots}{\cdots} \\
-n\,\rho(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z])+\frac{\sqrt{\epsilon0}((-By\,Ex+Bx\,Ey)W^{(0,0,0,1)}[t,x,y,z]+(Bz\,Ex-Bx\,Ez)W^{(0,0,1,0)}[t,x,y,z]+(-Bz\,Ey+By\,Ez)W^{(0,1,0,0)}[t,x,y,z])}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
n\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
n\,\rho(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z])+O[\frac{1}{c}]^2 \\
n\,\rho(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z])+O[\frac{1}{c}]^2 \\
-n\,\rho(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z])+\frac{2(-Bz\,Ey+By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
-n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2
\end{pmatrix}$$

In[∘]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.(tt + tte).T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix}
-n\,\rho\,c^2+\left(-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{1}{2}n(ux^2+uy^2+uz^2-2\,W+2\,\epsilon)\rho\right)+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\rho\,c^2+\frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{Ax(-2(qx+sxx\,ux+sxy\,uy+sxz\,uz)\mu0+Vx(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)+n(-ux+Vx)(ux^2+uy^2+uz^2-2\,W+2\,\epsilon)\mu0\,\rho)+Ay(-2(qy+sxy\,ux+syy\,uy+syz\,uz)\mu0+Vy(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)+n(-uy+Vy)(ux^2+uy^2+ \cdots}{2\,\mu0} \cdots \\
-n\,\rho\,c^2-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0+2\,n\,\epsilon\,\mu0\,\rho}{2\,\mu0}+\frac{(Bz\,Ey\,ux-By\,Ez\,ux-Bz\,Ex\,uy+Bx\,Ez\,uy+By\,Ex\,uz-Bx\,Ey\,uz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\rho\,c^2+\frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{Ax(Bz^2\,ux-2\,Bx(By\,uy+Bz\,uz)+Bx^2\,Vx+Bx^2(-ux+Vx)+By^2(ux+Vx)-2\,qx\,\mu0-Ex^2\,ux\,\epsilon0\,\mu0+Ey^2\,ux\,\epsilon0\,\mu0+Ez^2\,ux\,\epsilon0\,\mu0-2\,Ex\,Ey\,uy\,\epsilon0\,\mu0-2\,Ex\,Ez\,uz\,\epsilon0\,\mu0+Ex^2\,Vx\,\epsilon0\,\mu0+Ey^2\,Vx\,\epsilon0\,\mu0+Ez^2\,Vx\,\epsilon0\,\mu0+2\,n(-ux+Vx)\epsilon\,\mu0\,\rho)+Ay(-2\,Bx \cdots}{2\,\mu0} \cdots \\
-n\,\rho\,c^2+\left(-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{1}{2}n(ux^2+uy^2+uz^2+2\,\epsilon)\rho\right)+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\rho\,c^2+\frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{Ax(-2(qx+sxx\,ux+sxy\,uy+sxz\,uz)\mu0+Vx(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)+n(-ux+Vx)(ux^2+uy^2+uz^2+2\,\epsilon)\mu0\,\rho)+Ay(-2(qy+sxy\,ux+syy\,uy+syz\,uz)\mu0+Vy(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)+n(-uy+Vy)(ux^2+uy^2+ \cdots}{2\,\mu0} \cdots \\
n\,ux\,\rho+\frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{-2\,Ay(Bx\,By-syx\,\mu0+Ex\,Ey\,\epsilon0\,\mu0+n\,ux(-uy+Vy)\mu0\,\rho)+Ax(-Bx^2+By^2+Bz^2+\mu0(2\,sxx+(-Ex^2+Ey^2+Ez^2)\,\epsilon0+2\,n\,ux(ux-Vx)\rho))+2\,Az(-Bx\,Bz+\mu0(szx-Ex\,Ez\,\epsilon0+n\,ux(uz-Vz)\rho))}{2\,\mu0}-\frac{(Bz\,Ey-By\,Ez)(Ax\,Vx+Ay\,Vy+Az\,Vz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
n(uz\,y-uy\,z)\rho+\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+2\,Ax\,sxz\,y\,\mu0+2\,Ay\,syz\,y\,\mu0+2\,Az\,szz\,y\,\mu0-2\,Ax\,sxy\,z\,\mu0-2\,Ay\,syy\,z\,\mu0-2\,Az\,syz\,z\,\mu0+Az\,Ex^2\,y\,\epsilon0\,\mu0+Az\,Ey^2\,y\,\epsilon0\,\mu0-2\,Ax\,Ex\,Ez\,y\,\epsilon0\,\mu0-2\,Ay\,Ey\,Ez\,y\,\epsilon0\,\mu0-Az\,Ez^2\,y\,\epsilon0\,\mu0-Ay\,Ex^2\,z\,\epsilon0\,\mu0+2\,Ax\,Ex\,Ey\,z\,\epsilon0\,\mu0+Ay\,Ey^2\,z\,\epsilon0\,\mu0+2\,Az\,Ey\,Ez\,z\,\epsilon0\,\mu0-Ay\,Ez^2\,z\,\epsilon0 \cdots}{2\,\mu0} \cdots \\
n(uz\,y-uy\,z)\rho+\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+2\,Ax\,sxz\,y\,\mu0+2\,Ay\,syz\,y\,\mu0+2\,Az\,szz\,y\,\mu0-2\,Ax\,sxy\,z\,\mu0-2\,Ay\,syy\,z\,\mu0-2\,Az\,syz\,z\,\mu0+Az\,Ex^2\,y\,\epsilon0\,\mu0+Az\,Ey^2\,y\,\epsilon0\,\mu0-2\,Ax\,Ex\,Ez\,y\,\epsilon0\,\mu0-2\,Ay\,Ey\,Ez\,y\,\epsilon0\,\mu0-Az\,Ez^2\,y\,\epsilon0\,\mu0-Ay\,Ex^2\,z\,\epsilon0\,\mu0+2\,Ax\,Ex\,Ey\,z\,\epsilon0\,\mu0+Ay\,Ey^2\,z\,\epsilon0\,\mu0+2\,Az\,Ey\,Ez\,z\,\epsilon0\,\mu0-Ay\,Ez^2\,z\,\epsilon0 \cdots}{2\,\mu0} \cdots \\
-n(t\,ux+x)\rho+\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \left(\frac{t(Ax\,Bx^2+2\,Ay\,Bx\,By-Ax\,By^2+2\,Az\,Bx\,Bz-Ax\,Bz^2-(2\,Ax\,sxx+2\,Ay\,syx+2\,Az\,szx-2\,Ex(Ay\,Ey+Az\,Ez)\,\epsilon0+Ax(-Ex^2+Ey^2+Ez^2)\,\epsilon0)\mu0)}{2\,\mu0}-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux+x)\rho\right)+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
n(-t\,ux+x)\rho+\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \left(\frac{t(Ax\,Bx^2+2\,Ay\,Bx\,By-Ax\,By^2+2\,Az\,Bx\,Bz-Ax\,Bz^2-(2\,Ax\,sxx+2\,Ay\,syx+2\,Az\,szx-2\,Ex(Ay\,Ey+Az\,Ez)\,\epsilon0+Ax(-Ex^2+Ey^2+Ez^2)\,\epsilon0)\mu0)}{2\,\mu0}-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux-x)\rho\right)+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(Az\,By\,Ex-Ay\,Bz\,Ex-Az\,Bx\,Ey+Ax\,Bz\,Ey+Ay\,Bx\,Ez-Ax\,By\,Ez)x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2
\end{pmatrix}$$

In[∘]:=
```
TTx = tW[tjv[((tt + tte) + T[(tt + tte).Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 4][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```
Out[∘]//MatrixForm=

$$\begin{pmatrix}
n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
\frac{(-2\,Bx\,Bz+(sxz+szx-2\,Ex\,Ez\,\epsilon0)\mu0)ux^{(0,0,0,1)}[t,x,y,z]+(-2\,By\,Bz+(syz+szy-2\,Ey\,Ez\,\epsilon0)\mu0)uy^{(0,0,0,1)}[t,x,y,z]+Bx^2\,uz^{(0,0,0,1)}[t,x,y,z]+By^2\,uz^{(0,0,0,1)}[t,x,y,z]-Bz^2\,uz^{(0,0,0,1)}[t,x,y,z]+2\,szz\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ex^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ey^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-Ez^2\,\epsilon0 \cdots}{\cdots} \\
-n\,\rho(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z])+\frac{\sqrt{\epsilon0}((-By\,Ex+Bx\,Ey)W^{(0,0,0,1)}[t,x,y,z]+(Bz\,Ex-Bx\,Ez)W^{(0,0,1,0)}[t,x,y,z]+(-Bz\,Ey+By\,Ez)W^{(0,1,0,0)}[t,x,y,z])}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
n\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
n\,\rho(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z])+O[\frac{1}{c}]^2 \\
n\,\rho(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z])+O[\frac{1}{c}]^2 \\
-n\,\rho(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z])+\frac{2(-Bz\,Ey+By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\
-n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2
\end{pmatrix}$$

```
(* Flux of energy is given by: *)
```

```
In[]:= shows[assut, 1][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.tt /. {Ax → 0, Ay → 0, Vz → 0, jz → 0} /. j2v]]
```
Out[]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-\frac{1}{2}n\left(ux^2+uy^2-2W+2\epsilon\right)\rho+O[\tfrac{1}{c}]^2 & n\,ux\,\rho+O[\tfrac{1}{c}]^2 & n\,uy\,\rho+O[\tfrac{1}{c}]^2 & O[\tfrac{1}{c}]^2 \\ -Az\left(qz+sxz\,ux+syz\,uy\right)+O[\tfrac{1}{c}]^2 & Az\,sxz+O[\tfrac{1}{c}]^2 & Az\,syz+O[\tfrac{1}{c}]^2 & Az\,szz+O[\tfrac{1}{c}]^2 \end{pmatrix}$$

```
In[]:= shows[assut, 1][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A ∗ Δt)}.tt /. {jx → 0, Vx → 0} /. j2v]]
```
Out[]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-\frac{1}{2}n\left(uy^2+uz^2-2W+2\epsilon\right)\rho+O[\tfrac{1}{c}]^2 & O[\tfrac{1}{c}]^2 & n\,uy\,\rho+O[\tfrac{1}{c}]^2 & n\,uz\,\rho+O[\tfrac{1}{c}]^2 \\ \left(-qx-sxy\,uy-sxz\,uz\right)+O[\tfrac{1}{c}]^2 & sxx+O[\tfrac{1}{c}]^2 & sxy+O[\tfrac{1}{c}]^2 & sxz+O[\tfrac{1}{c}]^2 \end{pmatrix}$$

```
In[]:= shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A ∗ Δt)}.tt /. j2v]]
```
Out[]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-\frac{1}{2}n\left(ux^2+uy^2+uz^2-2W+2\epsilon\right)\rho+O[\tfrac{1}{c}]^2 & n\,ux\,\rho+\frac{qx+sxx\,ux+sxy\,uy+sxz\,uz+\frac{1}{2}n\left(-8Wx+ux\left(ux^2+uy^2+uz^2+6W+2\epsilon\right)\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 & n\,uy\,\rho+\frac{qy+sxy\,ux+syy\,uy+syz\,uz+\frac{1}{2}n\left(-8Wy+uy\left(ux^2+uy^2+uz^2+6W+2\epsilon\right)\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 & n\,uz\,\rho+\frac{qz+sxz\,ux+syz\,uy+szz\,uz+\frac{1}{2}n\left(-8Wz+uz\left(ux^2+uy^2+uz^2+6W+2\epsilon\right)\right)\rho}{} \\ n\left(-ux+Vx\right)\rho c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}n\left(ux-Vx\right)\left(ux^2+uy^2+uz^2-2W+2\epsilon\right)\rho\right)+O[\tfrac{1}{c}]^2 & \left(sxx+n\,ux\left(ux-Vx\right)\rho\right)+\frac{qx\left(2ux-Vx\right)-\left(sxx\,ux+sxy\,uy+sxz\,uz\right)Vx+\frac{1}{2}n\left(ux-Vx\right)\left(ux^3-8Wx+ux\left(uy^2+uz^2+6W+2\epsilon\right)\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 & \left(sxy+n\,uy\left(ux-Vx\right)\rho\right)+\frac{qx\,uy+qy\left(ux-Vx\right)-\left(sxy\,ux+syy\,uy+syz\,uz\right)Vx+\frac{1}{2}n\left(ux-Vx\right)\left(-8Wy+uy\left(ux^2+uy^2+uz^2+6W+2\epsilon\right)\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 & \left(sxz+n\,uz\left(ux-Vx\right)\rho\right)+\frac{qx\,uz+qz\left(ux-Vx\right)-\left(sxz\,ux+syz\,uy+szz\,uz\right)}{} \end{pmatrix}$$

```
In[]:= shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A ∗ Δt)}.tt /. j2v /. {sxx → SXX − n ∗ ux ∗ (ux − Vx) ∗ ρ, sxy → SXY − n ∗ uy ∗ (ux − Vx) ∗ ρ, sxz → SXZ − n ∗ uz ∗ (ux − Vx) ∗ ρ}]]
```
Out[]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-\frac{1}{2}n\left(ux^2+uy^2+uz^2-2W+2\epsilon\right)\rho+O[\tfrac{1}{c}]^2 & n\,ux\,\rho+\frac{qx+SXX\,ux+SXY\,uy+SXZ\,uz-\frac{1}{2}n\left(ux^3-2ux^2Vx-2\left(uy^2+uz^2\right)Vx+8Wx+ux\left(uy^2+uz^2-6W-2\epsilon\right)\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 & n\,uy\,\rho+\frac{qy+SXY\,ux+syy\,uy+syz\,uz+\frac{1}{2}n\left(-8Wy+uy\left(-ux^2+uy^2+uz^2+2\,ux\,Vx+6W+2\epsilon\right)\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 & n\,uz\,\rho+\frac{qz+SXZ\,ux+syz\,uy+szz\,uz+\frac{1}{2}n\left(-8Wz+uz\left(-ux^2+uy^2+uz^2+2\,ux\,Vx+6W+2\epsilon\right)\right)\rho}{c^2}+O[\tfrac{1}{c}] \\ n\left(-ux+Vx\right)\rho c^2+\left(-qx-SXX\,ux-SXY\,uy-SXZ\,uz+\frac{1}{2}n\left(ux-Vx\right)\left(ux^2+uy^2+uz^2+2W-2\epsilon\right)\rho\right)+O[\tfrac{1}{c}]^2 & SXX+\frac{qx\left(2ux-Vx\right)-\left(SXX\,ux+SXY\,uy+SXZ\,uz\right)Vx+\frac{1}{2}n\left(ux-Vx\right)\left(ux^3+2ux^2Vx+2\left(uy^2+uz^2\right)Vx-8Wx+ux\left(uy^2+uz^2+6W+2\epsilon\right)\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 & SXY+\frac{qx\,uy+qy\left(ux-Vx\right)-\left(SXY\,ux+syy\,uy+syz\,uz\right)Vx+\frac{1}{2}n\left(ux-Vx\right)\left(-8Wy+uy\left(ux^2+uy^2+uz^2+2\,ux\,Vx+6W+2\epsilon\right)\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 & SXZ+\frac{qx\,uz+qz\left(ux-Vx\right)-\left(SXZ\,ux+syz\,uy+szz\,uz\right)Vx+\frac{1}{2}n\left(ux-Vx\right)\left(-8Wz+uz\left(ux^2+uy^2+uz^2+2\,ux\,V\right.\right.}{c^2} \end{pmatrix}$$

```
In[]:= (* matter flux in same direction as imaginary moving surface, different velocity *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A ∗ Δt)}.tt /. {jy → 0, jz → 0} /. j2v]]
```
Out[]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-\frac{1}{2}n\left(ux^2-2W+2\epsilon\right)\rho+O[\tfrac{1}{c}]^2 & n\,ux\,\rho+\frac{qx+sxx\,ux+\frac{1}{2}n\left(ux^3+6\,ux\,W-8Wx+2\,ux\,\epsilon\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 & \frac{qy+sxy\,ux-4\,n\,Wy\,\rho}{c^2}+O[\tfrac{1}{c}]^3 & \frac{qz+sxz\,ux-4\,n\,Wz\,\rho}{c^2}+O[\tfrac{1}{c}]^3 \\ n\left(-ux+Vx\right)\rho c^2+\left(-qx-sxx\,ux-\frac{1}{2}n\left(ux-Vx\right)\left(ux^2-2W+2\epsilon\right)\rho\right)+O[\tfrac{1}{c}]^2 & \left(sxx+n\,ux\left(ux-Vx\right)\rho\right)+\frac{2\,qx\,ux-\left(qx+sxx\,ux\right)Vx+\frac{1}{2}n\left(ux-Vx\right)\left(ux^3-8Wx+2\,ux\left(3W+\epsilon\right)\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 & sxy+\frac{qy\left(ux-Vx\right)-sxy\,ux\,Vx+4\,n\left(-ux+Vx\right)Wy\,\rho}{c^2}+O[\tfrac{1}{c}]^3 & sxz+\frac{qz\left(ux-Vx\right)-sxz\,ux\,Vx+4\,n\left(-ux+Vx\right)Wz\,\rho}{c^2}+O[\tfrac{1}{c}]^3 \end{pmatrix}$$

```
In[]:= (* imaginary moving surface, no matter flux through it *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A ∗ Δt)}.tt /. j2v /. {ux → Vx}]]
```
Out[]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-\frac{1}{2}n\left(uy^2+uz^2+Vx^2-2W+2\epsilon\right)\rho+O[\tfrac{1}{c}]^2 & n\,Vx\,\rho+\frac{qx+sxy\,uy+sxz\,uz+sxx\,Vx+\frac{1}{2}n\left(-8Wx+Vx\left(uy^2+uz^2+Vx^2+6W+2\epsilon\right)\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 & n\,uy\,\rho+\frac{qy+syy\,uy+syz\,uz+sxy\,Vx+\frac{1}{2}n\left(-8Wy+uy\left(uy^2+uz^2+Vx^2+6W+2\epsilon\right)\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 & n\,uz\,\rho+\frac{qz+syz\,uy+szz\,uz+sxz\,Vx+\frac{1}{2}n\left(-8Wz+uz\left(uy^2+uz^2+Vx^2+6W+2\epsilon\right)\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 \\ \left(-qx-sxy\,uy-sxz\,uz-sxx\,Vx\right)+O[\tfrac{1}{c}]^2 & sxx-\frac{Vx\left(-qx+sxy\,uy+sxz\,uz+sxx\,Vx\right)}{c^2}+O[\tfrac{1}{c}]^3 & sxy+\frac{qx\,uy-Vx\left(syy\,uy+syz\,uz+sxy\,Vx\right)}{c^2}+O[\tfrac{1}{c}]^3 & sxz+\frac{qx\,uz-Vx\left(syz\,uy+szz\,uz+sxz\,Vx\right)}{c^2}+O[\tfrac{1}{c}]^3 \end{pmatrix}$$

```
(* imaginary moving surface, no matter flux through it and no transversal matter motion *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A ∗ Δt)}.tt /. {jy → 0, jz → 0} /. j2v /. {ux → Vx}]]
```
Out[]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-\frac{1}{2}n\left(Vx^2-2W+2\epsilon\right)\rho+O[\tfrac{1}{c}]^2 & n\,Vx\,\rho+\frac{qx+sxx\,Vx+\frac{1}{2}n\left(Vx^3+6\,Vx\,W-8Wx+2\,Vx\,\epsilon\right)\rho}{c^2}+O[\tfrac{1}{c}]^3 & \frac{qy+sxy\,Vx-4\,n\,Wy\,\rho}{c^2}+O[\tfrac{1}{c}]^3 & \frac{qz+sxz\,Vx-4\,n\,Wz\,\rho}{c^2}+O[\tfrac{1}{c}]^3 \\ \left(-qx-sxx\,Vx\right)+O[\tfrac{1}{c}]^2 & sxx+\frac{Vx\left(qx-sxx\,Vx\right)}{c^2}+O[\tfrac{1}{c}]^3 & sxy-\frac{sxy\,Vx^2}{c^2}+O[\tfrac{1}{c}]^3 & sxz-\frac{sxz\,Vx^2}{c^2}+O[\tfrac{1}{c}]^3 \end{pmatrix}$$

```
In[]:= (* imaginary moving surface, matter at rest in coordinates *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A ∗ Δt)}.tt /. {jx → 0, jy → 0, jz → 0}]]
```
Out[]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+n\left(W-\epsilon\right)\rho+O[\tfrac{1}{c}]^2 & \frac{qx-4\,n\,Wx\,\rho}{c^2}+O[\tfrac{1}{c}]^3 & \frac{qy-4\,n\,Wy\,\rho}{c^2}+O[\tfrac{1}{c}]^3 & \frac{qz-4\,n\,Wz\,\rho}{c^2}+O[\tfrac{1}{c}]^3 \\ n\,Vx\,\rho c^2+\left(-qx+n\,Vx\left(-W+\epsilon\right)\rho\right)+O[\tfrac{1}{c}]^2 & sxx+\frac{-qx\,Vx+4\,n\,Vx\,Wx\,\rho}{c^2}+O[\tfrac{1}{c}]^3 & sxy+\frac{-qy\,Vx+4\,n\,Vx\,Wy\,\rho}{c^2}+O[\tfrac{1}{c}]^3 & sxz+\frac{-qz\,Vx+4\,n\,Vx\,Wz\,\rho}{c^2}+O[\tfrac{1}{c}]^3 \end{pmatrix}$$

```
In[]:= showf[assut][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A ∗ Δt)}.tt /. {jx → n ∗ Vx, jy → 0, jz → 0}]]
```
Out[]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-\frac{1}{2}n\,Vx^2\rho+n\,W\rho-n\,\epsilon\,\rho\right)+O[\tfrac{1}{c}]^2 & n\,Vx\,\rho+\frac{qx+sxx\,Vx+\frac{1}{2}n\,Vx^3\rho+3\,n\,Vx\,W\rho-4\,n\,Wx\,\rho+n\,Vx\,\epsilon\,\rho}{c^2}+O[\tfrac{1}{c}]^4 & \frac{qy+sxy\,Vx-4\,n\,Wy\,\rho}{c^2}+O[\tfrac{1}{c}]^4 & \frac{qz+sxz\,Vx-4\,n\,Wz\,\rho}{c^2}+O[\tfrac{1}{c}]^4 \\ \left(-qx-sxx\,Vx\right)+O[\tfrac{1}{c}]^2 & sxx+\frac{qx\,Vx-sxx\,Vx^2}{c^2}+O[\tfrac{1}{c}]^4 & sxy-\frac{sxy\,Vx^2}{c^2}+O[\tfrac{1}{c}]^4 & sxz-\frac{sxz\,Vx^2}{c^2}+O[\tfrac{1}{c}]^4 \end{pmatrix}$$

```
In[]:= (* COORDINATE ENERGY *)
```

```
In[]:= (* energy 3-form when projected along coord. axes *)
showf[assut][Expand //@ FS@PowerExpand[tt.{1, 0, 0, 0}]]
```
Out[]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-\frac{jx^2\rho}{2n}-\frac{jy^2\rho}{2n}-\frac{jz^2\rho}{2n}+n\,W\rho-n\,\epsilon\,\rho\right)+O[\tfrac{1}{c}]^2 \\ -jx\,\rho c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\rho}{2n^2}-\frac{jx\,jy^2\rho}{2n^2}-\frac{jx\,jz^2\rho}{2n^2}+jx\,W\rho-jx\,\epsilon\,\rho\right)+O[\tfrac{1}{c}]^2 \\ -jy\,\rho c^2+\left(-qy-\frac{jx\,sxy}{n}-\frac{jy\,syy}{n}-\frac{jz\,syz}{n}-\frac{jx^2\,jy\,\rho}{2n^2}-\frac{jy^3\rho}{2n^2}-\frac{jy\,jz^2\rho}{2n^2}+jy\,W\rho-jy\,\epsilon\,\rho\right)+O[\tfrac{1}{c}]^2 \\ -jz\,\rho c^2+\left(-qz-\frac{jx\,sxz}{n}-\frac{jy\,syz}{n}-\frac{jz\,szz}{n}-\frac{jx^2\,jz\,\rho}{2n^2}-\frac{jy^2\,jz\,\rho}{2n^2}-\frac{jz^3\rho}{2n^2}+jz\,W\rho-jz\,\epsilon\,\rho\right)+O[\tfrac{1}{c}]^2 \end{pmatrix}$$

```
In[]:= (* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.{1, 0, 0, 0} /. j2v]]
```
Out[]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-\frac{1}{2}n\,ux^2\rho-\frac{1}{2}n\,uy^2\rho-\frac{1}{2}n\,uz^2\rho+n\,W\rho-n\,\epsilon\,\rho\right)+O[\tfrac{1}{c}]^2 \\ -n\,ux\,\rho c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}n\,ux^3\rho-\frac{1}{2}n\,ux\,uy^2\rho-\frac{1}{2}n\,ux\,uz^2\rho+n\,ux\,W\rho-n\,ux\,\epsilon\,\rho\right)+O[\tfrac{1}{c}]^2 \\ -n\,uy\,\rho c^2+\left(-qy-sxy\,ux-syy\,uy-syz\,uz-\frac{1}{2}n\,ux^2\,uy\,\rho-\frac{1}{2}n\,uy^3\rho-\frac{1}{2}n\,uy\,uz^2\rho+n\,uy\,W\rho-n\,uy\,\epsilon\,\rho\right)+O[\tfrac{1}{c}]^2 \\ -n\,uz\,\rho c^2+\left(-qz-sxz\,ux-syz\,uy-szz\,uz-\frac{1}{2}n\,ux^2\,uz\,\rho-\frac{1}{2}n\,uy^2\,uz\,\rho-\frac{1}{2}n\,uz^3\rho+n\,uz\,W\rho-n\,uz\,\epsilon\,\rho\right)+O[\tfrac{1}{c}]^2 \end{pmatrix}$$

```
In[]:= (* flux of coord. energy across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A ∗ Δt)]]
```
Out[]//MatrixForm=

$$\left(-jx\,\rho+n\,Vx\,\rho\right)c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\rho}{2n^2}-\frac{jx\,jy^2\rho}{2n^2}-\frac{jx\,jz^2\rho}{2n^2}+\frac{jx^2\,Vx\,\rho}{2n}+\frac{jy^2\,Vx\,\rho}{2n}+\frac{jz^2\,Vx\,\rho}{2n}+jx\,W\rho-n\,Vx\,W\rho-jx\,\epsilon\,\rho+n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[]:= showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A ∗ Δt) /. j2vr]]
```
Out[]//MatrixForm=

$$-Lx\,n\,\rho c^2+\left(-qx-sxx\,ux-\frac{1}{2}Lx\,n\,ux^2\rho+Lx\,n\,W\rho-Lx\,n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[]:= showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A ∗ Δt) /. {jx → 0, jy → 0, jz → 0}]]
```
Out[]//MatrixForm=

$$n\,Vx\,\rho c^2+\left(-qx-n\,Vx\,W\rho+n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[]:= (* in terms of matter flux*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A ∗ Δt) /. repjf]]
```
Out[]//MatrixForm=

$$-JX\,\rho c^2+\left(-qx-\frac{JX\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-sxx\,Vx-\frac{JX^3\rho}{2n^2}-\frac{JX\,jy^2\rho}{2n^2}-\frac{JX\,jz^2\rho}{2n^2}-\frac{JX^2\,Vx\,\rho}{n}-\frac{1}{2}JX\,Vx^2\rho+JX\,W\rho-JX\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[]:= showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A ∗ Δt) /. repjf]]
```
Out[]//MatrixForm=

$$-JX\,\rho c^2+\left(-qx-\frac{JX\,sxx}{n}-sxx\,Vx-\frac{JX^3\rho}{2n^2}-\frac{JX^2\,Vx\,\rho}{n}-\frac{1}{2}JX\,Vx^2\rho+JX\,W\rho-JX\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[]:= (* in terms of matter flux & matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A ∗ Δt) /. repjf /. j2v]]
```
Out[]//MatrixForm=

$$-JX\,\rho c^2+\left(-qx-\frac{JX\,sxx}{n}-sxy\,uy-sxz\,uz-sxx\,Vx-\frac{JX^3\rho}{2n^2}-\frac{1}{2}JX\,uy^2\rho-\frac{1}{2}JX\,uz^2\rho-\frac{JX^2\,Vx\,\rho}{n}-\frac{1}{2}JX\,Vx^2\rho+JX\,W\rho-JX\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[]:= showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A ∗ Δt) /. repjf /. {JX → 0, jy → 0, jz → 0}]]
```
Out[]//MatrixForm=

$$\left(-qx-sxx\,Vx\right)+O\left[\frac{1}{c}\right]^2$$

```
In[]:= (* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A ∗ Δt) /. relv]]
```
Out[]//MatrixForm=

$$-n\,Vx\,\rho c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^2\,Vx\,\rho}{2n}-\frac{jy^2\,Vx\,\rho}{2n}-\frac{jz^2\,Vx\,\rho}{2n}+n\,Vx\,W\rho-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[]:= (* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A ∗ Δt) /. j2vr]]
```
Out[]//MatrixForm=

$$-n\,Vx\,\rho c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}n\,ux^2\,Vx\,\rho-\frac{1}{2}n\,uy^2\,Vx\,\rho-\frac{1}{2}n\,uz^2\,Vx\,\rho+n\,Vx\,W\rho-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[]:= (* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A ∗ Δt) /. j2vr /. {Vx → 0}]]
```
Out[]//MatrixForm=

$$\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz\right)+O\left[\frac{1}{c}\right]^2$$

```
In[]:= (* supply term for coord. energy *)
TTx = tW[Normal[tt]]; shows[assut, 2][Expand //@ FS@PowerExpand[Tr[1 / 2 ∗ Normal@(Inverse[gg].T[Dcoords〚1, ;; , ;;〛].gg + Dcoords〚1, ;; , ;;〛).TTx]]]
```
Out[]//MatrixForm=

$$n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]+\frac{\left(3\left(jx^2+jy^2+jz^2\right)\rho+2\,n\left(sxx+syy+szz+n\,\epsilon\,\rho\right)-2\,n^2\rho\,W[t,x,y,z]\right)W^{(1,0,0,0)}[t,x,y,z]}{2\,n\,c^2}+O\left[\frac{1}{c}\right]^3$$

```
In[]:= (* INTERNAL ENERGY *)
```

```
In[]:= (* energy 3-form when projected along matter 4-velocity, "internal energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.uu]]
```
Out[]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-n\,\epsilon\,\rho+O[\tfrac{1}{c}]^2 \\ -jx\,\rho c^2+\left(-qx-jx\,\epsilon\,\rho\right)+O[\tfrac{1}{c}]^2 \\ -jy\,\rho c^2+\left(-qy-jy\,\epsilon\,\rho\right)+O[\tfrac{1}{c}]^2 \\ -jz\,\rho c^2+\left(-qz-jz\,\epsilon\,\rho\right)+O[\tfrac{1}{c}]^2 \end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.uu /. j2vr]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2 - n\,\epsilon\,\rho + O\left[\frac{1}{c}\right]^2 \\ -n\,ux\,\rho\,c^2 + \left(-qx - n\,ux\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uy\,\rho\,c^2 + \left(-qy - n\,uy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uz\,\rho\,c^2 + \left(-qz - n\,uz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* flux of internal energy across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt)]]
```
Out[∘]//MatrixForm=

$$\left(-jx\,\rho + n\,vx\,\rho\right)c^2 + \left(-qx - jx\,\epsilon\,\rho + n\,vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. relv]]
```
Out[∘]//MatrixForm=

$$-n\,Vx\,\rho\,c^2 + \left(-qx - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. j2vr]]
```
Out[∘]//MatrixForm=

$$-n\,Vx\,\rho\,c^2 + \left(-qx - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[∘]//MatrixForm=

$$-qx + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* supply term for internal energy (should be reversed in sign; remember that stress is compressive, not tensile) *)
TTx = tW[tjv@tt]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Duv].gg + Duv).TTx]]]
```
Out[∘]//MatrixForm=

$$\left(sxz\,vx^{(0,0,0,1)}[t,x,y,z] + syz\,vy^{(0,0,0,1)}[t,x,y,z] + szz\,vz^{(0,0,0,1)}[t,x,y,z] + sxy\,vx^{(0,0,1,0)}[t,x,y,z] + syy\,vy^{(0,0,1,0)}[t,x,y,z] + syz\,vz^{(0,0,1,0)}[t,x,y,z] + sxx\,vx^{(0,1,0,0)}[t,x,y,z] + sxy\,vy^{(0,1,0,0)}[t,x,y,z] + sxz\,vz^{(0,1,0,0)}[t,x,y,z]\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* difference between "coord. energy" and "internal energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - uu)]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{jx^2\rho}{2n} - \frac{jy^2\rho}{2n} - \frac{jz^2\rho}{2n} + n\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^3\rho}{2n^2} - \frac{jx\,jy^2\rho}{2n^2} - \frac{jx\,jz^2\rho}{2n^2} + jx\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\,sxy}{n} - \frac{jy\,syy}{n} - \frac{jz\,syz}{n} - \frac{jx^2\,jy\,\rho}{2n^2} - \frac{jy^3\rho}{2n^2} - \frac{jy\,jz^2\rho}{2n^2} + jy\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\,sxz}{n} - \frac{jy\,syz}{n} - \frac{jz\,szz}{n} - \frac{jx^2\,jz\,\rho}{2n^2} - \frac{jy^2\,jz\,\rho}{2n^2} - \frac{jz^3\rho}{2n^2} + jz\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - uu) /. j2vr]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{1}{2}n\,ux^2\rho - \frac{1}{2}n\,uy^2\rho - \frac{1}{2}n\,uz^2\rho + n\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2}n\,ux^3\rho - \frac{1}{2}n\,ux\,uy^2\rho - \frac{1}{2}n\,ux\,uz^2\rho + n\,ux\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-sxy\,ux - syy\,uy - syz\,uz - \frac{1}{2}n\,ux^2\,uy\,\rho - \frac{1}{2}n\,uy^3\rho - \frac{1}{2}n\,uy\,uz^2\rho + n\,uy\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-sxz\,ux - syz\,uy - szz\,uz - \frac{1}{2}n\,ux^2\,uz\,\rho - \frac{1}{2}n\,uy^2\,uz\,\rho - \frac{1}{2}n\,uz^3\rho + n\,uz\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt)]]
```
Out[∘]//MatrixForm=

$$\left(-\frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^3\rho}{2n^2} - \frac{jx\,jy^2\rho}{2n^2} - \frac{jx\,jz^2\rho}{2n^2} + \frac{jx^2\,vx\,\rho}{2n} + \frac{jy^2\,vx\,\rho}{2n} + \frac{jz^2\,vx\,\rho}{2n} + jx\,W\,\rho - n\,vx\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. relv]]
```
Out[∘]//MatrixForm=

$$\left(-\frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^2\,Vx\,\rho}{2n} - \frac{jy^2\,Vx\,\rho}{2n} - \frac{jz^2\,Vx\,\rho}{2n} + n\,Vx\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. j2vr]]
```
Out[∘]//MatrixForm=

$$\left(-sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2}n\,ux^2\,Vx\,\rho - \frac{1}{2}n\,uy^2\,Vx\,\rho - \frac{1}{2}n\,uz^2\,Vx\,\rho + n\,Vx\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[∘]//MatrixForm=

$$\left(-sxx\,ux - sxy\,uy - sxz\,uz\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* PROPER-TIME COORD ENERGY*)
```

In[∘]:=
```
(* energy 3-form when projected along normalized coord-t
  note how the gravitational term is missing *)
showf[assut][Expand //@ FS@PowerExpand[tt.vtn]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2 + \left(-\frac{jx^2\rho}{2n} - \frac{jy^2\rho}{2n} - \frac{jz^2\rho}{2n} - n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jx\,\rho\,c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^3\rho}{2n^2} - \frac{jx\,jy^2\rho}{2n^2} - \frac{jx\,jz^2\rho}{2n^2} - jx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jy\,\rho\,c^2 + \left(-qy - \frac{jx\,sxy}{n} - \frac{jy\,syy}{n} - \frac{jz\,syz}{n} - \frac{jx^2\,jy\,\rho}{2n^2} - \frac{jy^3\rho}{2n^2} - \frac{jy\,jz^2\rho}{2n^2} - jy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jz\,\rho\,c^2 + \left(-qz - \frac{jx\,sxz}{n} - \frac{jy\,syz}{n} - \frac{jz\,szz}{n} - \frac{jx^2\,jz\,\rho}{2n^2} - \frac{jy^2\,jz\,\rho}{2n^2} - \frac{jz^3\rho}{2n^2} - jz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.vtn /. j2vr]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2 + \left(-\frac{1}{2}n\,ux^2\rho - \frac{1}{2}n\,uy^2\rho - \frac{1}{2}n\,uz^2\rho - n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,ux\,\rho\,c^2 + \left(-qx - sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2}n\,ux^3\rho - \frac{1}{2}n\,ux\,uy^2\rho - \frac{1}{2}n\,ux\,uz^2\rho - n\,ux\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uy\,\rho\,c^2 + \left(-qy - sxy\,ux - syy\,uy - syz\,uz - \frac{1}{2}n\,ux^2\,uy\,\rho - \frac{1}{2}n\,uy^3\rho - \frac{1}{2}n\,uy\,uz^2\rho - n\,uy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uz\,\rho\,c^2 + \left(-qz - sxz\,ux - syz\,uy - szz\,uz - \frac{1}{2}n\,ux^2\,uz\,\rho - \frac{1}{2}n\,uy^2\,uz\,\rho - \frac{1}{2}n\,uz^3\rho - n\,uz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* flux of normalized-coord-t energy across surface *)
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt)]]
```
Out[∘]//MatrixForm=

$$\left(-jx\,\rho + n\,vx\,\rho\right)c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jx^3\rho}{2n^2} + \frac{jx^2\,vx\,\rho}{2n} - jx\,\epsilon\,\rho + n\,vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(*in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. relv]]
```
Out[∘]//MatrixForm=

$$-n\,Vx\,\rho\,c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^2\,Vx\,\rho}{2n} - \frac{jy^2\,Vx\,\rho}{2n} - \frac{jz^2\,Vx\,\rho}{2n} - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. j2vr]]
```
Out[∘]//MatrixForm=

$$-n\,Vx\,\rho\,c^2 + \left(-qx - sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2}n\,ux^2\,Vx\,\rho - \frac{1}{2}n\,uy^2\,Vx\,\rho - \frac{1}{2}n\,uz^2\,Vx\,\rho - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[∘]//MatrixForm=

$$\left(-qx - sxx\,ux - sxy\,uy - sxz\,uz\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* supply term for normalized-coord-t energy
  we obtain the "power generated by the gravity field" *)
TTx = tW[tjv@tt]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Dvtn].gg + Dvtn).TTx]]]
```
Out[∘]//MatrixForm=

$$\left(-\rho\,n[t,x,y,z] \times vz[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z] - \rho\,n[t,x,y,z] \times vy[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z] - \rho\,n[t,x,y,z] \times vx[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* difference between "coord. energy" and "proper-time coord. energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - vtn)]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} n\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ jx\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ jy\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ jz\,W\,\rho + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - vtn) /. j2vr]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} n\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ n\,ux\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ n\,uy\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ n\,uz\,W\,\rho + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt)]]
```
Out[•]//MatrixForm=

$$(\text{jx}\, W\, \rho - n\, \text{vx}\, W\, \rho) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. relv]]
```
Out[•]//MatrixForm=

$$n\, \text{Vx}\, W\, \rho + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=

$$n\, \text{Vx}\, W\, \rho + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=

$$O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* difference between "internal energy" and "proper-time coord. energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.(uu - vtn)]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix}
\left(\frac{\text{jx}^2 \rho}{2 n} + \frac{\text{jy}^2 \rho}{2 n} + \frac{\text{jz}^2 \rho}{2 n}\right) + O\left[\frac{1}{c}\right]^2 \\
\left(\frac{\text{jx}\,\text{sxx}}{n} + \frac{\text{jy}\,\text{sxy}}{n} + \frac{\text{jz}\,\text{sxz}}{n} + \frac{\text{jx}^3 \rho}{2 n^2} + \frac{\text{jx}\,\text{jy}^2 \rho}{2 n^2} + \frac{\text{jx}\,\text{jz}^2 \rho}{2 n^2}\right) + O\left[\frac{1}{c}\right]^2 \\
\left(\frac{\text{jx}\,\text{sxy}}{n} + \frac{\text{jy}\,\text{syy}}{n} + \frac{\text{jz}\,\text{syz}}{n} + \frac{\text{jx}^2 \text{jy}\, \rho}{2 n^2} + \frac{\text{jy}^3 \rho}{2 n^2} + \frac{\text{jy}\,\text{jz}^2 \rho}{2 n^2}\right) + O\left[\frac{1}{c}\right]^2 \\
\left(\frac{\text{jx}\,\text{sxz}}{n} + \frac{\text{jy}\,\text{syz}}{n} + \frac{\text{jz}\,\text{szz}}{n} + \frac{\text{jx}^2 \text{jz}\, \rho}{2 n^2} + \frac{\text{jy}^2 \text{jz}\, \rho}{2 n^2} + \frac{\text{jz}^3 \rho}{2 n^2}\right) + O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.(uu - vtn) /. j2vr]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix}
\left(\frac{1}{2} n\, \text{ux}^2 \rho + \frac{1}{2} n\, \text{uy}^2 \rho + \frac{1}{2} n\, \text{uz}^2 \rho\right) + O\left[\frac{1}{c}\right]^2 \\
\left(\text{sxx}\,\text{ux} + \text{sxy}\,\text{uy} + \text{sxz}\,\text{uz} + \frac{1}{2} n\, \text{ux}^3 \rho + \frac{1}{2} n\, \text{ux}\,\text{uy}^2 \rho + \frac{1}{2} n\, \text{ux}\,\text{uz}^2 \rho\right) + O\left[\frac{1}{c}\right]^2 \\
\left(\text{sxy}\,\text{ux} + \text{syy}\,\text{uy} + \text{syz}\,\text{uz} + \frac{1}{2} n\, \text{ux}^2 \text{uy}\, \rho + \frac{1}{2} n\, \text{uy}^3 \rho + \frac{1}{2} n\, \text{uy}\,\text{uz}^2 \rho\right) + O\left[\frac{1}{c}\right]^2 \\
\left(\text{sxz}\,\text{ux} + \text{syz}\,\text{uy} + \text{szz}\,\text{uz} + \frac{1}{2} n\, \text{ux}^2 \text{uz}\, \rho + \frac{1}{2} n\, \text{uy}^2 \text{uz}\, \rho + \frac{1}{2} n\, \text{uz}^3 \rho\right) + O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[•]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - vtn) / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(\frac{\text{jx}\,\text{sxx}}{n} + \frac{\text{jy}\,\text{sxy}}{n} + \frac{\text{jz}\,\text{sxz}}{n} + \frac{\text{jx}^3 \rho}{2 n^2} + \frac{\text{jx}\,\text{jy}^2 \rho}{2 n^2} + \frac{\text{jx}\,\text{jz}^2 \rho}{2 n^2} - \frac{\text{jx}^2 \text{vx}\, \rho}{2 n} - \frac{\text{jy}^2 \text{vx}\, \rho}{2 n} - \frac{\text{jz}^2 \text{vx}\, \rho}{2 n}\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - vtn) / (A * Δt) /. relv]]
```
Out[•]//MatrixForm=

$$\left(\frac{\text{jx}\,\text{sxx}}{n} + \frac{\text{jy}\,\text{sxy}}{n} + \frac{\text{jz}\,\text{sxz}}{n} + \frac{\text{jx}^2 \text{Vx}\, \rho}{2 n} + \frac{\text{jy}^2 \text{Vx}\, \rho}{2 n} + \frac{\text{jz}^2 \text{Vx}\, \rho}{2 n}\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - uu) / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=

$$O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - uu) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=

$$O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* Faraday tensor *)
repE = {Ex → Ex * c * Sqrt[μ0 * ϵ0], Ey → Ey * c * Sqrt[μ0 * ϵ0], Ez → Ez * c * Sqrt[μ0 * ϵ0]};
fftemp = {{0, -Ex, -Ey, -Ez}, {0, 0, Bz, -By}, {0, 0, 0, Bx}, {0, 0, 0, 0}} /. repE;
showf[assut][ffdd = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix}
0 & -c\, \text{Ex} \sqrt{\epsilon 0} \sqrt{\mu 0} & -c\, \text{Ey} \sqrt{\epsilon 0} \sqrt{\mu 0} & -c\, \text{Ez} \sqrt{\epsilon 0} \sqrt{\mu 0} \\
c\, \text{Ex} \sqrt{\epsilon 0} \sqrt{\mu 0} & 0 & \text{Bz} & -\text{By} \\
c\, \text{Ey} \sqrt{\epsilon 0} \sqrt{\mu 0} & -\text{Bz} & 0 & \text{Bx} \\
c\, \text{Ez} \sqrt{\epsilon 0} \sqrt{\mu 0} & \text{By} & -\text{Bx} & 0
\end{pmatrix}$$

In[•]:=
```
showf[assut][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]])).gg * dg)
  ]]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix}
\left(-\frac{\text{Ex}^2 \epsilon 0}{2} - \frac{\text{Ey}^2 \epsilon 0}{2} - \frac{\text{Ez}^2 \epsilon 0}{2} - \frac{\text{Bx}^2}{2 \mu 0} - \frac{\text{By}^2}{2 \mu 0} - \frac{\text{Bz}^2}{2 \mu 0}\right) + \frac{-\text{Ex}^2 W \epsilon 0 - \text{Ey}^2 W \epsilon 0 - \text{Ez}^2 W \epsilon 0 + \frac{\text{Bx}^2 W}{\mu 0} + \frac{\text{By}^2 W}{\mu 0} + \frac{\text{Bz}^2 W}{\mu 0}}{c^2} + O\left[\frac{1}{c}\right]^4 \\
\left(-\frac{\text{Bz}\,\text{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\text{By}\,\text{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{\frac{2\, \text{Bz}\,\text{Ey}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2\, \text{By}\,\text{Ez}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{-4\, \text{Ey}^2 \text{Wx}\, \epsilon 0 - 4\, \text{Ez}^2 \text{Wx}\, \epsilon 0 + 4\, \text{Ex}\,\text{Ey}\,\text{Wy}\, \epsilon 0 + 4\, \text{Ex}\,\text{Ez}\,\text{Wz}\, \epsilon 0}{c^2} + O\left[\frac{1}{c}\right]^3 \\
\left(\frac{\text{Bz}\,\text{Ex} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{\text{Bx}\,\text{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{-\frac{2\, \text{Bz}\,\text{Ex}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{2\, \text{Bx}\,\text{Ez}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{4\, \text{Ex}\,\text{Ey}\,\text{Wx}\, \epsilon 0 - 4\, \text{Ex}^2 \text{Wy}\, \epsilon 0 - 4\, \text{Ez}^2 \text{Wy}\, \epsilon 0 + 4\, \text{Ey}\,\text{Ez}\,\text{Wz}\, \epsilon 0}{c^2} + O\left[\frac{1}{c}\right]^3 \\
\left(-\frac{\text{By}\,\text{Ex} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\text{Bx}\,\text{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{\frac{2\, \text{By}\,\text{Ex}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2\, \text{Bx}\,\text{Ey}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{4\, \text{Ex}\,\text{Ez}\,\text{Wx}\, \epsilon 0 + 4\, \text{Ey}\,\text{Ez}\,\text{Wy}\, \epsilon 0 - 4\, \text{Ex}^2 \text{Wz}\, \epsilon 0 - 4\, \text{Ey}^2 \text{Wz}\, \epsilon 0}{c^2} + O\left[\frac{1}{c}\right]^3
\end{pmatrix}$$

$$\begin{pmatrix}
\frac{\frac{\text{Bz}\,\text{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{\text{By}\,\text{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{\frac{2\, \text{Bz}\,\text{Ey}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2\, \text{By}\,\text{Ez}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + \frac{-\frac{4\, \text{By}^2 \text{Wx}}{\mu 0} - \frac{4\, \text{Bz}^2 \text{Wx}}{\mu 0} + \frac{4\, \text{Bx}\,\text{By}\,\text{Wy}}{\mu 0} + \frac{4\, \text{Bx}\,\text{Bz}\,\text{Wz}}{\mu 0}}{c^4} + O\left[\frac{1}{c}\right]^5 \\
\left(-\frac{\text{Ex}^2 \epsilon 0}{2} + \frac{\text{Ey}^2 \epsilon 0}{2} + \frac{\text{Ez}^2 \epsilon 0}{2} - \frac{\text{Bx}^2}{2 \mu 0} + \frac{\text{By}^2}{2 \mu 0} + \frac{\text{Bz}^2}{2 \mu 0}\right) + \frac{-\text{Ex}^2 W \epsilon 0 + \text{Ey}^2 W \epsilon 0 + \text{Ez}^2 W \epsilon 0 + \frac{\text{Bx}^2 W}{\mu 0} - \frac{\text{By}^2 W}{\mu 0} - \frac{\text{Bz}^2 W}{\mu 0}}{c^2} + \frac{-\frac{4\, \text{Bz}\,\text{Ex}\,\text{Wy} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\, \text{Bx}\,\text{Ez}\,\text{Wy} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\, \text{By}\,\text{Ex}\,\text{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\, \text{Bx}\,\text{Ey}\,\text{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O\left[\frac{1}{c}\right]^4 \\
\left(-\text{Ex}\,\text{Ey}\, \epsilon 0 - \frac{\text{Bx}\,\text{By}}{\mu 0}\right) + \frac{-2\, \text{Ex}\,\text{Ey}\, W \epsilon 0 + \frac{2\, \text{Bx}\,\text{By}\, W}{\mu 0}}{c^2} + \frac{\frac{4\, \text{Bz}\,\text{Ex}\,\text{Wx} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\, \text{By}\,\text{Ez}\,\text{Wy} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\, \text{Bx}\,\text{Ex}\,\text{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\, \text{By}\,\text{Ey}\,\text{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O\left[\frac{1}{c}\right]^4 \\
\left(-\text{Ex}\,\text{Ez}\, \epsilon 0 - \frac{\text{Bx}\,\text{Bz}}{\mu 0}\right) + \frac{-2\, \text{Ex}\,\text{Ez}\, W \epsilon 0 + \frac{2\, \text{Bx}\,\text{Bz}\, W}{\mu 0}}{c^2} + \frac{-\frac{4\, \text{By}\,\text{Ex}\,\text{Wx} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\, \text{Bx}\,\text{Ex}\,\text{Wy} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\, \text{Bz}\,\text{Ez}\,\text{Wy} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\, \text{Bz}\,\text{Ey}\,\text{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O\left[\frac{1}{c}\right]^4
\end{pmatrix}$$

$$\begin{pmatrix}
\frac{-\frac{\text{Bz}\,\text{Ex} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\text{Bx}\,\text{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{-\frac{2\, \text{Bz}\,\text{Ex}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{2\, \text{Bx}\,\text{Ez}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + \frac{\frac{4\, \text{Bx}\,\text{By}\,\text{Wx}}{\mu 0} - \frac{4\, \text{Bx}^2 \text{Wy}}{\mu 0} - \frac{4\, \text{Bz}^2 \text{Wy}}{\mu 0} + \frac{4\, \text{By}\,\text{Bz}\,\text{Wz}}{\mu 0}}{c^4} + O\left[\frac{1}{c}\right]^5 \\
\left(-\text{Ex}\,\text{Ey}\, \epsilon 0 - \frac{\text{Bx}\,\text{By}}{\mu 0}\right) + \frac{-2\, \text{Ex}\,\text{Ey}\, W \epsilon 0 + \frac{2\, \text{Bx}\,\text{By}\, W}{\mu 0}}{c^2} + \frac{[illegible]}{c^3} + O\left[\frac{1}{c}\right]^4 \\
\left(\frac{\text{Ex}^2 \epsilon 0}{2} - \frac{\text{Ey}^2 \epsilon 0}{2} + \frac{\text{Ez}^2 \epsilon 0}{2} + \frac{\text{Bx}^2}{2 \mu 0} - \frac{\text{By}^2}{2 \mu 0} + \frac{\text{Bz}^2}{2 \mu 0}\right) + \frac{\text{Ex}^2 W \epsilon 0 - \text{Ey}^2 W \epsilon 0 + \text{Ez}^2 W \epsilon 0 - \frac{\text{Bx}^2 W}{\mu 0} + \frac{\text{By}^2 W}{\mu 0} - \frac{\text{Bz}^2 W}{\mu 0}}{c^2} + \frac{[illegible]}{c^3} + O\left[\frac{1}{c}\right]^4 \\
\left(-\text{Ey}\,\text{Ez}\, \epsilon 0 - \frac{\text{By}\,\text{Bz}}{\mu 0}\right) + \frac{-2\, \text{Ey}\,\text{Ez}\, W \epsilon 0 + \frac{2\, \text{By}\,\text{Bz}\, W}{\mu 0}}{c^2} + \frac{[illegible]}{c^3} + O\left[\frac{1}{c}\right]^4
\end{pmatrix}$$

$$\begin{pmatrix}
\frac{\text{By}\,\text{Ex} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{\text{Bx}\,\text{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \ldots \\
\left(-\text{Ex}\,\text{Ez}\, \epsilon 0 - \frac{\text{Bx}\,[illegible]}{} \ldots \right. \\
\left(-\text{Ey}\,\text{Ez}\, \epsilon 0 - \frac{\text{By}\,[illegible]}{\mu 0} \ldots \right. \\
\left(\frac{\text{Ex}^2 \epsilon 0}{2} + \frac{\text{Ey}^2 \epsilon 0}{2} - \ldots \right.
\end{pmatrix}$$

In[•]:=
```
(* COORDINATE EM ENERGY *)
```

In[•]:=
```
(* energy 3-form when projected along coord. axes *)
showf[assut][Expand //@ FS@PowerExpand[tte.{1, 0, 0, 0}]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix}
\left(-\frac{\text{Ex}^2 \epsilon 0}{2} - \frac{\text{Ey}^2 \epsilon 0}{2} - \frac{\text{Ez}^2 \epsilon 0}{2} - \frac{\text{Bx}^2}{2 \mu 0} - \frac{\text{By}^2}{2 \mu 0} - \frac{\text{Bz}^2}{2 \mu 0}\right) + \frac{-\text{Ex}^2 W \epsilon 0 - \text{Ey}^2 W \epsilon 0 - \text{Ez}^2 W \epsilon 0 + \frac{\text{Bx}^2 W}{\mu 0} + \frac{\text{By}^2 W}{\mu 0} + \frac{\text{Bz}^2 W}{\mu 0}}{c^2} + O\left[\frac{1}{c}\right]^4 \\
\left(-\frac{\text{Bz}\,\text{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\text{By}\,\text{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{\frac{2\, \text{Bz}\,\text{Ey}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2\, \text{By}\,\text{Ez}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{-4\, \text{Ey}^2 \text{Wx}\, \epsilon 0 - 4\, \text{Ez}^2 \text{Wx}\, \epsilon 0 + 4\, \text{Ex}\,\text{Ey}\,\text{Wy}\, \epsilon 0 + 4\, \text{Ex}\,\text{Ez}\,\text{Wz}\, \epsilon 0}{c^2} + O\left[\frac{1}{c}\right]^3 \\
\left(\frac{\text{Bz}\,\text{Ex} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{\text{Bx}\,\text{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{-\frac{2\, \text{Bz}\,\text{Ex}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{2\, \text{Bx}\,\text{Ez}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{4\, \text{Ex}\,\text{Ey}\,\text{Wx}\, \epsilon 0 - 4\, \text{Ex}^2 \text{Wy}\, \epsilon 0 - 4\, \text{Ez}^2 \text{Wy}\, \epsilon 0 + 4\, \text{Ey}\,\text{Ez}\,\text{Wz}\, \epsilon 0}{c^2} + O\left[\frac{1}{c}\right]^3 \\
\left(-\frac{\text{By}\,\text{Ex} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\text{Bx}\,\text{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{\frac{2\, \text{By}\,\text{Ex}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2\, \text{Bx}\,\text{Ey}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{4\, \text{Ex}\,\text{Ez}\,\text{Wx}\, \epsilon 0 + 4\, \text{Ey}\,\text{Ez}\,\text{Wy}\, \epsilon 0 - 4\, \text{Ex}^2 \text{Wz}\, \epsilon 0 - 4\, \text{Ey}^2 \text{Wz}\, \epsilon 0}{c^2} + O\left[\frac{1}{c}\right]^3
\end{pmatrix}$$

In[•]:=
```
(* flux of coord. energy across surface *)
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tte.{1, 0, 0, 0} / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(-\frac{\text{Bz}\,\text{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\text{By}\,\text{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \left(\frac{1}{2} \text{Ex}^2 \text{vx}\, \epsilon 0 + \frac{1}{2} \text{Ey}^2 \text{vx}\, \epsilon 0 + \frac{1}{2} \text{Ez}^2 \text{vx}\, \epsilon 0 + \frac{\text{Bx}^2 \text{vx}}{2 \mu 0} + \frac{\text{By}^2 \text{vx}}{2 \mu 0} + \frac{\text{Bz}^2 \text{vx}}{2 \mu 0}\right) + \frac{\frac{2\, \text{Bz}\,\text{Ey}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2\, \text{By}\,\text{Ez}\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{\text{Ex}^2 \text{vx}\, W \epsilon 0 + \text{Ey}^2 \text{vx}\, W \epsilon 0 + \text{Ez}^2 \text{vx}\, W \epsilon 0 - 4\, \text{Ey}^2 \text{Wx}\, \epsilon 0 - 4\, \text{Ez}^2 \text{Wx}\, \epsilon 0 + 4\, \text{Ex}\,\text{Ey}\,\text{Wy}\, \epsilon 0 + 4\, \text{Ex}\,\text{Ez}\,\text{Wz}\, \epsilon 0 - \frac{\text{Bx}^2 \text{vx}\, W}{\mu 0} - \frac{\text{By}^2 \text{vx}\, W}{\mu 0} - \frac{\text{Bz}^2 \text{vx}\, W}{\mu 0}}{c^2} + O\left[\frac{1}{c}\right]^3$$

In[•]:=
```
(* supply term for coord. energy *)
TTx = tW[tjv@tte]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Dcoords[[1, ;; , ;;]]].gg + Dcoords[[1, ;; , ;;]]).TTx]]]
```
Out[•]//MatrixForm=

$$\frac{\text{Ex}^2 \epsilon 0\, W^{(1,0,0,0)}[t, x, y, z] + \text{Ey}^2 \epsilon 0\, W^{(1,0,0,0)}[t, x, y, z] + \text{Ez}^2 \epsilon 0\, W^{(1,0,0,0)}[t, x, y, z] + \frac{\text{Bx}^2 W^{(1,0,0,0)}[t,x,y,z]}{\mu 0} + \frac{\text{By}^2 W^{(1,0,0,0)}[t,x,y,z]}{\mu 0} + \frac{\text{Bz}^2 W^{(1,0,0,0)}[t,x,y,z]}{\mu 0}}{c^2} + O\left[\frac{1}{c}\right]^3$$