```
In[1]:= << "christoffelsymbols.m"
(* First index is upper index Table[FS[cc[[ii,;;,;;]]==T[cc[[ii,;;,;;]]]],{ii,1,4}] *)
```

```
In[29]:= showf[assumptions_, simp_ : FullSimplify] := ((Assuming[assumptions, Expand //@ simp@PowerExpand[#]] // MF) &);
```

```
In[•]:= (* Show matrix expressions and power expansions*)
show[assumptions_, power_, simp_ : FullSimplify] := ({Assuming[assumptions, simp@PowerExpand[#]] // MF, "\n",
      Assuming[assumptions, simp@PowerExpand[Series[#, {c, Infinity, power}]]] // MF} &);
shows[assumptions_, power_, simp_ : FullSimplify] := ((
      Assuming[assumptions, simp@PowerExpand[Series[#, {c, Infinity, power}]]] // MF) &);
showf[assumptions_, simp_ : FullSimplify] := ((Assuming[assumptions, Expand //@ simp@PowerExpand[#]] // MF) &);
show1[assumptions_, simp_ : Identity] := ((Assuming[assumptions, simp[#]] // MF) &);
show2[assumptions_, power_, simp_ : Identity] := ((
      Assuming[assumptions, simp[Series[#, {c, Infinity, power}]]] // MF) &);
```

```
In[2]:= coords = {t, x, y, z}
```

Out[2]= {t, x, y, z}

```
In[3]:= (* Flat metric *)
(gg0 = DiagonalMatrix[{-c^2, 1, 1, 1}]) // MF
```

Out[3]//MatrixForm=

$$\begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[•]:= (*D[G*M/Sqrt[x^2+y^2+z^2],{{x,y,z}}] *)
```

```
In[•]:= (* -W is the potential gravitational energy: W=GM/r
   that is, F_g(downwards)=grad W
*)
```

```
In[4]:= (* Rotating metric from poissonetal *)
(gg = DiagonalMatrix@Diagonal@{{-c^2*(1-2*W/c^2 +O[c, +Infinity]^4(*-2*((*Ψ[t,x,y,z]*)-W[t,x,y,z]^2)/c^4*)),
         -4*Wx/c^2+O[c, +Infinity]^4, -4*Wy/c^2+O[c, +Infinity]^4, -4*Wz/c^2+O[c, +Infinity]^4},
       {-4*Wx/c^2+O[c, +Infinity]^4,
        1+2*W/c^2+O[c, +Infinity]^4, 0, 0},
       {-4*Wy/c^2+O[c, +Infinity]^4, 0, 1+2*W/c^2+O[c, +Infinity]^4, 0},
       {-4*Wz/c^2+O[c, +Infinity]^4, 0, 0, 1+2*W/c^2+O[c, +Infinity]^4}}) // MF
(*(gg=DiagonalMatrix[{-c^2*(1+2*Φ[t,r]/c^2),1+2*Λ[t,r]/c^2,r^2,r^2*Sin[θ]^2}])//MF*)
```

Out[4]//MatrixForm=

$$\begin{pmatrix} -c^2+2W+O[\frac{1}{c}]^2 & 0 & 0 & 0 \\ 0 & 1+\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 & 0 \\ 0 & 0 & 1+\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 \\ 0 & 0 & 0 & 1+\frac{2W}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[•]:= Inverse[gg] // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -(\frac{1}{c})^2-\frac{2W}{c^4}+O[\frac{1}{c}]^6 & 0 & 0 & 0 \\ 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 & 0 \\ 0 & 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 \\ 0 & 0 & 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[•]:= (*(gg=DiagonalMatrix@Diagonal[gg])//MF*)
```

```
In[8]:= (* functions to temporarily remove coord-dep *)
tW[xx_] := (xx /. {W → W[t, x, y, z], Wx → Wx[t, x, y, z], Wy → Wy[t, x, y, z], Wz → Wz[t, x, y, z]});
itW[xx_] := (xx /. {W[t, x, y, z] → W, Wx[t, x, y, z] → Wx, Wy[t, x, y, z] → Wy, Wz[t, x, y, z] → Wz});
```

```
In[10]:= assut = {c > 0, Element[a, Reals], Element[v, Reals], Element[t, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals],
    Element[vx, Reals], Element[vy, Reals], Element[vz, Reals], Element[n, Reals], Element[r, Reals], Element[θ, Reals], Element[ϕ, Reals], Abs[v] < c, -c < vx < c, -c < ux < c, r > 0, 0 < θ < Pi,
    Normal@gg[[1, 1]] / c^2 < 0, Normal@gg[[2, 2]] > 0, Normal@gg[[3, 3]] > 0, Normal@gg[[4, 4]] > 0, n > 0, Element[jx, Reals], Element[jy, Reals], Element[jz, Reals], Element[sxx, Reals], Element[sxy, Reals], Element[sxz, Reals], Element[syy, Reals], Element[syz, Reals], Element[szz, Reals],
    -Normal@Det[gg] > 0, β > 0};

assutt = {c > 0, Element[a, Reals], Element[v, Reals], Element[t, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals],
    Element[v_x, Reals], Element[v_y, Reals], Element[v_z, Reals], Element[n, Reals], Element[r, Reals], Element[θ, Reals], Element[ϕ, Reals], Abs[v] < c, -c < v_x < c, -c < u_x < c, r > 0, 0 < θ < Pi,
    β > 0, Normal@ggt[[1, 1]] / c^2 < 0, Normal@ggt[[2, 2]] > 0, Normal@ggt[[3, 3]] > 0, Normal@ggt[[4, 4]] > 0,
    -Normal@Det[ggt] > 0};
```

```
In[15]:= (igg = Assuming[assut, FullSimplify@PowerExpand[Inverse[gg]]]) // MF
```

Out[15]//MatrixForm=

$$\begin{pmatrix} -(\frac{1}{c})^2-\frac{2W}{c^4}+O[\frac{1}{c}]^6 & 0 & 0 & 0 \\ 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 & 0 \\ 0 & 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 \\ 0 & 0 & 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[•]:= (*show[assut,2]@ChristoffelSymbol[gg,coords][[2]]*)
```

```
In[16]:= (* volume element *)
(dg = Assuming[assut, FullSimplify@PowerExpand[Sqrt[-Det[gg]] / c]]) // MF
```

Out[16]//MatrixForm=

$$1+\frac{2W}{c^2}+O[\frac{1}{c}]^4$$

```
In[17]:= (* Christoffel symbols  *)
cc = Assuming[assut, FullSimplify@PowerExpand[itW[ChristoffelSymbol[ggt, coords]]]];
```

```
In[•]:=
```

(* Matter current *)

```
In[18]:= (* matter-current 3-covector *)
NJ = {n, jx, jy, jz};
```

```
In[19]:= (* norm of matter 3-covector *)
Assuming[assut, Expand //@ FS@PowerExpand[Sqrt[-NJ.gg.NJ] / c]]
```

Out[19]= $n+\frac{-\frac{jx^2}{2n}-\frac{jy^2}{2n}-\frac{jz^2}{2n}-nW}{c^2}+O[\frac{1}{c}]^4$

```
In[20]:= (* matter associated 1-vector *)
(NJvec = Assuming[assut, Expand //@ FS@PowerExpand[NJ / dg]]) // MF
```

Out[20]//MatrixForm=

$$\begin{pmatrix} n-\frac{2(nW)}{c^2}+O[\frac{1}{c}]^4 \\ jx-\frac{2(jx\,W)}{c^2}+O[\frac{1}{c}]^4 \\ jy-\frac{2(jy\,W)}{c^2}+O[\frac{1}{c}]^4 \\ jz-\frac{2(jz\,W)}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[21]:= (* matter associated 4-vel vector *)
(uu = Assuming[assut, Expand //@ FS@PowerExpand[c*NJvec / Sqrt[-NJvec.gg.NJvec]]]) // MF
```

Out[21]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{jx^2}{2n^2}+\frac{jy^2}{2n^2}+\frac{jz^2}{2n^2}+W}{c^2}+O[\frac{1}{c}]^4 \\ \frac{jx}{n}+\frac{\frac{jx^3}{2n^3}+\frac{jx\,jy^2}{2n^3}+\frac{jx\,jz^2}{2n^3}+\frac{jx\,W}{n}}{c^2}+O[\frac{1}{c}]^4 \\ \frac{jy}{n}+\frac{\frac{jx^2\,jy}{2n^3}+\frac{jy^3}{2n^3}+\frac{jy\,jz^2}{2n^3}+\frac{jy\,W}{n}}{c^2}+O[\frac{1}{c}]^4 \\ \frac{jz}{n}+\frac{\frac{jx^2\,jz}{2n^3}+\frac{jy^2\,jz}{2n^3}+\frac{jz^3}{2n^3}+\frac{jz\,W}{n}}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[22]:= (* it is normalized *)
uu.gg.uu
```

Out[22]= $-c^2+O[\frac{1}{c}]^2$

```
In[23]:= (* matter associated 1-covector *)
(NJcov = Assuming[assut, Expand //@ FS@PowerExpand[gg.(NJ / dg)]]) // MF
```

Out[23]//MatrixForm=

$$\begin{pmatrix} -n\,c^2+4\,n\,W+O[\frac{1}{c}]^2 \\ jx+O[\frac{1}{c}]^4 \\ jy+O[\frac{1}{c}]^4 \\ jz+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[25]:= (* scalar product with de/de_x (for momentum x-component) *)
uu.gg.{0, 1, 0, 0}
```

Out[25]= $\frac{jx}{n}+\frac{\frac{jx^3}{2n^3}+\frac{jx\,jy^2}{2n^3}+\frac{jx\,jz^2}{2n^3}+\frac{3\,jx\,W}{n}}{c^2}+O[\frac{1}{c}]^4$

In[26]:=
```
(* scalar product with de/de_i (for momentum i-component) *)
uu.gg // MF
```
Out[26]//MatrixForm=

$$\begin{pmatrix} -c^2+\left(-\frac{jx^2}{2\,n^2}-\frac{jy^2}{2\,n^2}-\frac{jz^2}{2\,n^2}+W\right)+O\left[\frac{1}{c}\right]^2 \\ \frac{jx}{n}+\frac{\frac{jx^3}{2\,n^3}+\frac{jx\,jy^2}{2\,n^3}+\frac{jx\,jz^2}{2\,n^3}+\frac{3\,jx\,W}{n}}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n}+\frac{\frac{jx^2\,jy}{2\,n^3}+\frac{jy^3}{2\,n^3}+\frac{jy\,jz^2}{2\,n^3}+\frac{3\,jy\,W}{n}}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n}+\frac{\frac{jx^2\,jz}{2\,n^3}+\frac{jy^2\,jz}{2\,n^3}+\frac{jz^3}{2\,n^3}+\frac{3\,jz\,W}{n}}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[30]:=
```
(* retransform matter 4-vel to matter 3-covector *)
showf[assut][uu * dg / c * Sqrt[-NJvec.gg.NJvec]]
```
Out[30]//MatrixForm=

$$\begin{pmatrix} n+O\left[\frac{1}{c}\right]^5 \\ jx+O\left[\frac{1}{c}\right]^5 \\ jy+O\left[\frac{1}{c}\right]^5 \\ jz+O\left[\frac{1}{c}\right]^5 \end{pmatrix}$$

In[31]:=
```
(* simplification to x-directed matter flux and velocity *)
assutjx = Join[assut, {jy == 0, jz == 0, uy == 0, uz == 0}];
```

```
(* *** This is a list of vector fields V used in defining balance laws from the energy-momentum tensor, from the equation d(T.V)=-T ∧. ∇V *** *)
```

In[32]:=
```
(* 3-vector of surface parallel to yz moving with velocity V    surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
yzVxsurface = {-(Vx * Ax + Vy * Ay + Vz * Az), Ax, Ay, Az} * Δt;
```

In[36]:=
```
(* flux of matter across surface *)
FS[yzVxsurface.NJ / (Δt)]
```
Out[36]= $Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz)$

In[37]:=
```
(* replace matter flux in terms of velocity*)
replaceJu = {jx → ux * n, jy → uy * n, jz → uz * n}
```
Out[37]= $\{jx\to n\,ux,\ jy\to n\,uy,\ jz\to n\,uz\}$

In[38]:=
```
(* collect velocity magnitude*)
replaceuUnorm = {ux ^ 2 → U ^ 2 - uy ^ 2 - uz ^ 2, ux ^ 3 → ux * (U ^ 2 - uy ^ 2 - uz ^ 2), jx ^ 2 → J ^ 2 - jy ^ 2 - jz ^ 2, jx ^ 3 → jx * (J ^ 2 - jy ^ 2 - jz ^ 2)}
```
Out[38]= $\{ux^2\to U^2-uy^2-uz^2,\ ux^3\to ux\,(U^2-uy^2-uz^2),\ jx^2\to J^2-jy^2-jz^2,\ jx^3\to jx\,(J^2-jy^2-jz^2)\}$

In[41]:=
```
FS[uu /. replaceuUnorm] // MF
```
Out[41]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{J^2}{2\,n^2}+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jx}{n}+\frac{jx\,(J^2+2\,n^2\,W)}{2\,n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n}+\frac{jy\,(J^2+2\,n^2\,W)}{2\,n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n}+\frac{jz\,(J^2+2\,n^2\,W)}{2\,n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[42]:=
```
FS[uu /. replaceJu] // MF
```
Out[42]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{1}{2}\,(ux^2+uy^2+uz^2)+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ ux+\frac{ux\,(ux^2+uy^2+uz^2+2\,W)}{2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ uy+\frac{uy\,(ux^2+uy^2+uz^2+2\,W)}{2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ uz+\frac{uz\,(ux^2+uy^2+uz^2+2\,W)}{2\,c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[43]:=
```
FS[(uu /. replaceJu) /. replaceuUnorm] // MF
```
Out[43]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{U^2}{2}+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ ux+\frac{\frac{U^2\,ux}{2}+ux\,W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ uy+\frac{\frac{U^2\,uy}{2}+uy\,W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ uz+\frac{\frac{U^2\,uz}{2}+uz\,W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[52]:=
```
(* Project along u velocity *)
proju = Assuming[assut, Expand //@ FS@PowerExpand[-Outer[Times, uu, gg.uu] / c ^ 2]];
projperpu = Assuming[assut, Expand //@ FS@PowerExpand[IdentityMatrix[4] - proju]];
testproj[ass_, x_] := showf[ass] /@ {Assuming[ass, Expand //@ FS@PowerExpand[proju.x.proju == x]], projperpu.x.proju == x, proju.x.projperpu == x, projperpu.x.projperpu == x}

(* Project along u velocity *)
proju = Assuming[assut, FS[-Outer[Times, uu, gg.uu] / c ^ 2]];
projperpu = Assuming[assut, FS[IdentityMatrix[4] - proju]];
testproj[ass_, x_] := showf[ass] /@ {Assuming[ass, Expand //@ FS@PowerExpand[proju.x.proju == x]], projperpu.x.proju == x, proju.x.projperpu == x, projperpu.x.projperpu == x}
```

In[44]:=
```
(* aux 4-velocity *)
auu = {temp, aux, auy, auz};
solu = temp /. Solve[Normal[auu.gg.auu] == -c ^ 2, temp][[2]]
```
Out[45]= $\frac{\sqrt{aux^2+auy^2+auz^2+c^2}}{\sqrt{c^2-2\,W}}$

In[48]:=
```
(auu = Assuming[assut, FS[auu /. {temp → solu}]]) // MF
```
Out[48]//MatrixForm=

$$\begin{pmatrix} \sqrt{\frac{aux^2+auy^2+auz^2+c^2}{c^2-2\,W}} \\ aux \\ auy \\ auz \end{pmatrix}$$

In[49]:=
```
auu.gg.auu
```
Out[49]= $-c^2+O\left[\frac{1}{c}\right]^2$

(* Construction of energy–momentum tensor *)

In[50]:=
```
(* definition of heat-flux, orthogonal to matter-current *)
Qtemp = {qt, qx, qy, qz};
```

In[55]:=
```
(proju.Qtemp) // MF
```
Out[55]//MatrixForm=

$$\begin{pmatrix} qt+\frac{\left(\frac{jx^2}{n^2}+\frac{jy^2}{n^2}+\frac{jz^2}{n^2}\right)qt-\frac{jx\,qx}{n}-\frac{jy\,qy}{n}-\frac{jz\,qz}{n}}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jx\,qt}{n}+\frac{\left(\frac{jx^3}{n^3}+\frac{jx\,jy^2}{n^3}+\frac{jx\,jz^2}{n^3}\right)qt-\frac{jx^2\,qx}{n^2}-\frac{jx\,jy\,qy}{n^2}-\frac{jx\,jz\,qz}{n^2}}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy\,qt}{n}+\frac{\left(\frac{jx^2\,jy}{n^3}+\frac{jy^3}{n^3}+\frac{jy\,jz^2}{n^3}\right)qt-\frac{jx\,jy\,qx}{n^2}-\frac{jy^2\,qy}{n^2}-\frac{jy\,jz\,qz}{n^2}}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz\,qt}{n}+\frac{\left(\frac{jx^2\,jz}{n^3}+\frac{jy^2\,jz}{n^3}+\frac{jz^3}{n^3}\right)qt-\frac{jx\,jz\,qx}{n^2}-\frac{jy\,jz\,qy}{n^2}-\frac{jz^2\,qz}{n^2}}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[56]:=
```
qsol = Solve[Normal[proju.Qtemp] == 0, qt][[1]]
```
Out[56]= $\left\{qt\to\frac{n\,(jx\,qx+jy\,qy+jz\,qz)}{jx^2+jy^2+jz^2+c^2\,n^2}\right\}$

In[57]:=
```
(Q = Assuming[assut, FS[Qtemp /. qsol]]) // MF
```
Out[57]//MatrixForm=

$$\begin{pmatrix} \frac{n\,(jx\,qx+jy\,qy+jz\,qz)}{jx^2+jy^2+jz^2+c^2\,n^2} \\ qx \\ qy \\ qz \end{pmatrix}$$

In[58]:=
```
{Normal@Series[Q.NJ, {c, Infinity, 1}] == 0}
```
Out[58]= $\{jx\,qx+jy\,qy+jz\,qz==0\}$

In[59]:=
```
assutQ = Join[assut, {Normal@Series[Q.NJ, {c, Infinity, 1}] == 0}];
```

In[62]:=
```
Assuming[assutQ, FS@{proju.Q, projperpu.Q == Q}]
```
Out[62]= $\left\{\left\{O\left[\frac{1}{c}\right]^6,\ O\left[\frac{1}{c}\right]^6,\ O\left[\frac{1}{c}\right]^6,\ O\left[\frac{1}{c}\right]^6\right\},\ True\right\}$

In[65]:=
```
(* non-symmetric heat-tensor *)
Assuming[assutQ, FS[Qtens = Assuming[assut, Expand //@ FS@PowerExpand[Outer[Times, Q, gg.uu / c ^ 2]]]]] // MF
```

Out[65]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \\ -qx+\frac{-\frac{(jx^2+jy^2+jz^2)qx}{2n^2}+qx\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{jx\,qx}{n\,c^2}+\frac{jx\,qx(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jy\,qx}{n\,c^2}+\frac{jy\,qx(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jz\,qx}{n\,c^2}+\frac{jz\,qx(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 \\ -qy+\frac{-\frac{(jx^2+jy^2+jz^2)qy}{2n^2}+qy\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{jx\,qy}{n\,c^2}+\frac{jx\,qy(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jy\,qy}{n\,c^2}+\frac{jy\,qy(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jz\,qy}{n\,c^2}+\frac{jz\,qy(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 \\ -qz+\frac{-\frac{(jx^2+jy^2+jz^2)qz}{2n^2}+qz\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{jx\,qz}{n\,c^2}+\frac{jx\,qz(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jy\,qz}{n\,c^2}+\frac{jy\,qz(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jz\,qz}{n\,c^2}+\frac{jz\,qz(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 \end{pmatrix}$$

In[66]:=
```
Assuming[assutQ, FS[T[Qtens.Inverse[gg]].gg - Qtens]] // MF
```

Out[66]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^6 & \frac{qx}{c^2}+\frac{\frac{(jx^2+jy^2+jz^2)qx}{2n^2}+3\,qx\,W}{c^4}+O[\frac{1}{c}]^6 & \frac{qy}{c^2}+\frac{\frac{(jx^2+jy^2+jz^2)qy}{2n^2}+3\,qy\,W}{c^4}+O[\frac{1}{c}]^6 & \frac{qz}{c^2}+\frac{\frac{(jx^2+jy^2+jz^2)qz}{2n^2}+3\,qz\,W}{c^4}+O[\frac{1}{c}]^6 \\ qx+\frac{\frac{(jx^2+jy^2+jz^2)qx}{2n^2}-qx\,W}{c^2}+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^6 & \frac{-jy\,qx+jx\,qy}{n\,c^2}+\frac{(-jy\,qx+jx\,qy)(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{-jz\,qx+jx\,qz}{n\,c^2}+\frac{(-jz\,qx+jx\,qz)(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 \\ qy+\frac{\frac{(jx^2+jy^2+jz^2)qy}{2n^2}-qy\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{jy\,qx-jx\,qy}{n\,c^2}+\frac{(jy\,qx-jx\,qy)(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & O[\frac{1}{c}]^6 & \frac{-jz\,qy+jy\,qz}{n\,c^2}+\frac{(-jz\,qy+jy\,qz)(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 \\ qz+\frac{\frac{(jx^2+jy^2+jz^2)qz}{2n^2}-qz\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{jz\,qx-jx\,qz}{n\,c^2}+\frac{(jz\,qx-jx\,qz)(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jz\,qy-jy\,qz}{n\,c^2}+\frac{(jz\,qy-jy\,qz)(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & O[\frac{1}{c}]^6 \end{pmatrix}$$

In[69]:=
```
(* definition of momentum-flux, orthogonal to matter-current *)
Ptemp = {pt, px, py, pz};
```

In[70]:=
```
Assuming[assut, FS[Ptemp.proju]] // MF
```

Out[70]//MatrixForm=

$$\begin{pmatrix} \frac{n\,pt+jx\,px+jy\,py+jz\,pz}{n}+\frac{(jx^2+jy^2+jz^2)(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^3c^2}+O[\frac{1}{c}]^4 \\ -\frac{jx(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^2c^2}-\frac{jx(n\,pt+jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4}+O[\frac{1}{c}]^6 \\ -\frac{jy(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^2c^2}-\frac{jy(n\,pt+jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4}+O[\frac{1}{c}]^6 \\ -\frac{jz(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^2c^2}-\frac{jz(n\,pt+jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4}+O[\frac{1}{c}]^6 \end{pmatrix}$$

In[71]:=
```
psol = Solve[Normal[Ptemp.proju] == 0, pt][[1]]
```

Out[71]= $\left\{pt\to-\frac{jx\,px+jy\,py+jz\,pz}{n}\right\}$

In[72]:=
```
(P = Assuming[assut, FS[Ptemp /. psol]]) // MF
```

Out[72]//MatrixForm=

$$\begin{pmatrix} -\frac{jx\,px+jy\,py+jz\,pz}{n} \\ px \\ py \\ pz \end{pmatrix}$$

In[73]:=
```
{FS[Normal@Series[P.uu, {c, Infinity, 1}]] == 0}
```

Out[73]= {True}

In[74]:=
```
Assuming[assutQ, FS@{P.proju, P.projperpu == P}]
```

Out[74]= $\left\{\left\{O[\frac{1}{c}]^4, O[\frac{1}{c}]^6, O[\frac{1}{c}]^6, O[\frac{1}{c}]^6\right\}, True\right\}$

In[78]:=
```
(* non-symmetric momentum-tensor *)
Assuming[assut, FS[Ptens = Assuming[assut, FS[Outer[Times, uu, P / c ^ 2]]]]] // MF
```

Out[78]//MatrixForm=

$$\begin{pmatrix} -\frac{jx\,px+jy\,py+jz\,pz}{n\,c^2}-\frac{(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{px}{c^2}+\frac{\frac{(jx^2+jy^2+jz^2)px}{2n^2}+px\,W}{c^4}+O[\frac{1}{c}]^6 & \frac{py}{c^2}+\frac{\frac{(jx^2+jy^2+jz^2)py}{2n^2}+py\,W}{c^4}+O[\frac{1}{c}]^6 & \frac{pz}{c^2}+\frac{\frac{(jx^2+jy^2+jz^2)pz}{2n^2}+pz\,W}{c^4}+O[\frac{1}{c}]^6 \\ -\frac{jx(jx\,px+jy\,py+jz\,pz)}{n^2c^2}-\frac{jx(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^4c^4}+O[\frac{1}{c}]^6 & \frac{jx\,px}{n\,c^2}+\frac{jx\,px(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jx\,py}{n\,c^2}+\frac{jx\,py(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jx\,pz}{n\,c^2}+\frac{jx\,pz(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 \\ -\frac{jy(jx\,px+jy\,py+jz\,pz)}{n^2c^2}-\frac{jy(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^4c^4}+O[\frac{1}{c}]^6 & \frac{jy\,px}{n\,c^2}+\frac{jy\,px(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jy\,py}{n\,c^2}+\frac{jy\,py(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jy\,pz}{n\,c^2}+\frac{jy\,pz(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 \\ -\frac{jz(jx\,px+jy\,py+jz\,pz)}{n^2c^2}-\frac{jz(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^4c^4}+O[\frac{1}{c}]^6 & \frac{jz\,px}{n\,c^2}+\frac{jz\,px(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jz\,py}{n\,c^2}+\frac{jz\,py(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jz\,pz}{n\,c^2}+\frac{jz\,pz(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 \end{pmatrix}$$

In[82]:=
```
FS[T[Ptens.Inverse[gg]].gg - Ptens] // MF
```

Out[82]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^6 & -\frac{px}{c^2}+\frac{jx^2\,px+2\,jx(jy\,py+jz\,pz)-px(jy^2+jz^2+2n^2W)}{2n^2c^4}+O[\frac{1}{c}]^6 & -\frac{py}{c^2}+\frac{-\frac{(jx^2+jy^2+jz^2)py}{2n^2}+\frac{jy(jx\,px+jy\,py+jz\,pz)}{n^2}-py\,W}{c^4}+O[\frac{1}{c}]^6 & -\frac{pz}{c^2}+\frac{-\frac{(jx^2+jy^2+jz^2)pz}{2n^2}+\frac{jz(jx\,px+jy\,py+jz\,pz)}{n^2}-pz\,W}{c^4}+O[\frac{1}{c}]^6 \\ -px+\frac{-\frac{(jx^2+jy^2+jz^2)px}{2n^2}+\frac{jx(jx\,px+jy\,py+jz\,pz)}{n^2}+3\,px\,W}{c^2}+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^6 & \frac{jy\,px-jx\,py}{n\,c^2}+\frac{(jy\,px-jx\,py)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jz\,px-jx\,pz}{n\,c^2}+\frac{(jz\,px-jx\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 \\ -py+\frac{-\frac{(jx^2+jy^2+jz^2)py}{2n^2}+\frac{jy(jx\,px+jy\,py+jz\,pz)}{n^2}+3\,py\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{-jy\,px+jx\,py}{n\,c^2}+\frac{(-jy\,px+jx\,py)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & O[\frac{1}{c}]^6 & \frac{jz\,py-jy\,pz}{n\,c^2}+\frac{(jz\,py-jy\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 \\ -pz+\frac{-\frac{(jx^2+jy^2+jz^2)pz}{2n^2}+\frac{jz(jx\,px+jy\,py+jz\,pz)}{n^2}+3\,pz\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{-jz\,px+jx\,pz}{n\,c^2}+\frac{(-jz\,px+jx\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{-jz\,py+jy\,pz}{n\,c^2}+\frac{(-jz\,py+jy\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & O[\frac{1}{c}]^6 \end{pmatrix}$$

In[83]:=
```
(* definition of stress, orthogonal to matter-current *)
(Stemp = {{stt, stx, sty, stz}, {sxt, sxx, sxy, sxz}, {syt, syx, syy, syz}, {szt, szx, szy, szz}}) // MF
```

Out[83]//MatrixForm=

$$\begin{pmatrix} stt & stx & sty & stz \\ sxt & sxx & sxy & sxz \\ syt & syx & syy & syz \\ szt & szx & szy & szz \end{pmatrix}$$

In[88]:=
```
(Stempsym = Assuming[assut, FS[(T[Stemp.Inverse[gg]].gg + Stemp) / 2]]) // MF
```

Out[88]//MatrixForm=

$$\begin{pmatrix} stt+O[\frac{1}{c}]^4 & \frac{stx}{2}-\frac{sxt}{2c^2}-\frac{2(sxt\,W)}{c^4}+O[\frac{1}{c}]^6 & \frac{sty}{2}-\frac{syt}{2c^2}-\frac{2(syt\,W)}{c^4}+O[\frac{1}{c}]^6 & \frac{stz}{2}-\frac{szt}{2c^2}-\frac{2(szt\,W)}{c^4}+O[\frac{1}{c}]^6 \\ -\frac{stx\,c^2}{2}+\frac{1}{2}(sxt+4\,stx\,W)+O[\frac{1}{c}]^2 & sxx+O[\frac{1}{c}]^4 & \frac{sxy+syx}{2}+O[\frac{1}{c}]^4 & \frac{sxz+szx}{2}+O[\frac{1}{c}]^4 \\ -\frac{sty\,c^2}{2}+\frac{1}{2}(syt+4\,sty\,W)+O[\frac{1}{c}]^2 & \frac{sxy+syx}{2}+O[\frac{1}{c}]^4 & syy+O[\frac{1}{c}]^4 & \frac{syz+szy}{2}+O[\frac{1}{c}]^4 \\ -\frac{stz\,c^2}{2}+\frac{1}{2}(szt+4\,stz\,W)+O[\frac{1}{c}]^2 & \frac{sxz+szx}{2}+O[\frac{1}{c}]^4 & \frac{syz+szy}{2}+O[\frac{1}{c}]^4 & szz+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[90]:=
```
FS[proju.Stemp.proju] // MF
```

Out[90]//MatrixForm=

$$\begin{pmatrix} \frac{n\,stt+jx\,stx+jy\,sty+jz\,stz}{n}+\frac{2\,jx^3\,stx+2\,jy^3\,sty+jx^2(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx)-jy\,n^2\,syt+jy^2(2\,n\,stt+2\,jz\,stz-n\,syy)+jx(2\,jy^2\,stx-n^2\,sxt-jy\,n(sxy+syx)+jz(2\,jz\,stx-n(sxz+szx)))+jy\,jz(2\,jz\,sty-n(syz+szy))+jz(2\,jz^2\,stz-n^2\,szt+jz\,n(2\,stt-szz))}{n^3c^2}+O[\frac{1}{c}]^4 & -\frac{jx(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2c^2}+\frac{jx(-2\,jx^3\,stx-2\,jy^3\,sty+jx^2(-2(n\,stt+jy\,sty+jz\,stz)+n\,sxx)+jy^2(-2\,jz\,stz+n(-2\,stt+syy))+jz(-2\,jz^2\,stz+n^2\,szt+jz\,n(-2\,stt+szz))-4\,n^2(n\,stt+jz\,stz)W+jx(-2 \ldots}{n^4c^4} \ldots \\ \frac{jx(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2}+\frac{jx(2\,jx^3\,stx+2\,jy^3\,sty+jx^2(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx)-jy\,n^2\,syt+jy^2(2\,n\,stt+2\,jz\,stz-n\,syy)+jx(2\,jy^2\,stx-n^2\,sxt-jy\,n(sxy+syx)+jz(2\,jz\,stx-n(sxz+szx)))+jy\,jz(2\,jz\,sty-n(syz+szy))+jz(2\,jz^2\,stz-n^2\,szt+jz\,n(2\,stt-szz)))}{n^4c^2}+O[\frac{1}{c}]^4 & -\frac{jx^2(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^3c^2}+\frac{jx^2(-2\,jx^3\,stx-2\,jy^3\,sty+jx^2(-2(n\,stt+jy\,sty+jz\,stz)+n\,sxx)+jy^2(-2\,jz\,stz+n(-2\,stt+syy))+jz(-2\,jz^2\,stz+n^2\,szt+jz\,n(-2\,stt+szz))-4\,n^2(n\,stt+jz\,stz)W+jx(\ldots}{n^5c^4} \ldots \\ \frac{jy(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2}+\frac{jy(2\,jx^3\,stx+2\,jy^3\,sty+jx^2(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx)-jy\,n^2\,syt+jy^2(2\,n\,stt+2\,jz\,stz-n\,syy)+jx(2\,jy^2\,stx-n^2\,sxt-jy\,n(sxy+syx)+jz(2\,jz\,stx-n(sxz+szx)))+jy\,jz(2\,jz\,sty-n(syz+szy))+jz(2\,jz^2\,stz-n^2\,szt+jz\,n(2\,stt-szz)))}{n^4c^2}+O[\frac{1}{c}]^4 & -\frac{jx\,jy(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^3c^2}+\frac{jx\,jy(-2\,jx^3\,stx-2\,jy^3\,sty+jx^2(-2(n\,stt+jy\,sty+jz\,stz)+n\,sxx)+jy^2(-2\,jz\,stz+n(-2\,stt+syy))+jz(-2\,jz^2\,stz+n^2\,szt+jz\,n(-2\,stt+szz))-4\,n^2(n\,stt+jz\,stz)W+ \ldots}{n^5c^4} \ldots \\ \frac{jz(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2}+\frac{jz(2\,jx^3\,stx+2\,jy^3\,sty+jx^2(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx)-jy\,n^2\,syt+jy^2(2\,n\,stt+2\,jz\,stz-n\,syy)+jx(2\,jy^2\,stx-n^2\,sxt-jy\,n(sxy+syx)+jz(2\,jz\,stx-n(sxz+szx)))+jy\,jz(2\,jz\,sty-n(syz+szy))+jz(2\,jz^2\,stz-n^2\,szt+jz\,n(2\,stt-szz)))}{n^4c^2}+O[\frac{1}{c}]^4 & -\frac{jx\,jz(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^3c^2}+\frac{jx\,jz(-2\,jx^3\,stx-2\,jy^3\,sty+jx^2(-2(n\,stt+jy\,sty+jz\,stz)+n\,sxx)+jy^2(-2\,jz\,stz+n(-2\,stt+syy))+jz(-2\,jz^2\,stz+n^2\,szt+jz\,n(-2\,stt+szz))-4\,n^2(n\,stt+jz\,stz)W+ \ldots}{n^5c^4} \ldots \end{pmatrix}$$

In[91]:=
```
ssol = Solve[{Normal[proju.Stemp.proju] == 0, Normal[projperpu.Stemp.projperpu] == Stemp}, {stt, stx, sty, stz, sxt, syt, szt}][[1]]
```

Out[91]= $\left\{stt\to-\frac{jx^2\,sxx+jx\,jy\,sxy+jx\,jz\,sxz+jx\,jy\,syx+jy^2\,syy+jy\,jz\,syz+jx\,jz\,szx+jy\,jz\,szy+jz^2\,szz}{jx^2+jy^2+jz^2+c^2n^2},\ stx\to\frac{n(jx\,sxx+jy\,syx+jz\,szx)}{jx^2+jy^2+jz^2+c^2n^2},\ sty\to\frac{n(jx\,sxy+jy\,syy+jz\,szy)}{jx^2+jy^2+jz^2+c^2n^2},\ stz\to\frac{n(jx\,sxz+jy\,syz+jz\,szz)}{jx^2+jy^2+jz^2+c^2n^2},\ sxt\to-\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n},\ syt\to-\frac{jx\,syx+jy\,syy+jz\,syz}{n},\ szt\to-\frac{jx\,szx+jy\,szy+jz\,szz}{n}\right\}$

In[92]:=
```
ssol = Solve[Normal[proju.Stemp] == 0 && Normal[Stemp.proju] == 0, {stt, stx, sty, stz, sxt, syt, szt}][[1]]
```

Out[92]= $\left\{stt\to-\frac{jx^2\,sxx+jx\,jy\,sxy+jx\,jz\,sxz+jx\,jy\,syx+jy^2\,syy+jy\,jz\,syz+jx\,jz\,szx+jy\,jz\,szy+jz^2\,szz}{jx^2+jy^2+jz^2+c^2n^2},\ stx\to\frac{n(jx\,sxx+jy\,syx+jz\,szx)}{jx^2+jy^2+jz^2+c^2n^2},\ sty\to\frac{n(jx\,sxy+jy\,syy+jz\,szy)}{jx^2+jy^2+jz^2+c^2n^2},\ stz\to\frac{n(jx\,sxz+jy\,syz+jz\,szz)}{jx^2+jy^2+jz^2+c^2n^2},\ sxt\to-\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n},\ syt\to-\frac{jx\,syx+jy\,syy+jz\,syz}{n},\ szt\to-\frac{jx\,szx+jy\,szy+jz\,szz}{n}\right\}$

In[94]:=
```
(S = Assuming[assut, FS[(Stemp /. ssol)]]) // MF
```

Out[94]//MatrixForm=

$$\begin{pmatrix} -\frac{jx^2\,sxx+jx\,jy(sxy+syx)+jy^2\,syy+jx\,jz(sxz+szx)+jy\,jz(syz+szy)+jz^2\,szz}{jx^2+jy^2+jz^2+c^2n^2} & \frac{n(jx\,sxx+jy\,syx+jz\,szx)}{jx^2+jy^2+jz^2+c^2n^2} & \frac{n(jx\,sxy+jy\,syy+jz\,szy)}{jx^2+jy^2+jz^2+c^2n^2} & \frac{n(jx\,sxz+jy\,syz+jz\,szz)}{jx^2+jy^2+jz^2+c^2n^2} \\ -\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n} & sxx & sxy & sxz \\ -\frac{jx\,syx+jy\,syy+jz\,syz}{n} & syx & syy & syz \\ -\frac{jx\,szx+jy\,szy+jz\,szz}{n} & szx & szy & szz \end{pmatrix}$$

In[95]:=
```
MF /@ FS@{proju.S.proju, Assuming[assut, Expand //@ FS@PowerExpand[projperpu.S.projperpu - S]]}
```

Out[95]=

$$\left\{\begin{pmatrix} O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \\ O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \\ O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \\ O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \end{pmatrix},\begin{pmatrix} -\frac{(jx^2\,sxx+jx\,jy(sxy+syx)+jy^2\,syy+jx\,jz(sxz+szx)+jy\,jz(syz+szy)+jz^2\,szz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4}+O[\frac{1}{c}]^6 & \frac{(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4n^2W)}{n^3c^4}+O[\frac{1}{c}]^6 & \frac{(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4n^2W)}{n^3c^4}+O[\frac{1}{c}]^6 & \frac{(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4n^2W)}{n^3c^4}+O[\frac{1}{c}]^6 \\ O[\frac{1}{c}]^4 & \frac{jx(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4}+O[\frac{1}{c}]^6 & \frac{jx(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4}+O[\frac{1}{c}]^6 & \frac{jx(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4}+O[\frac{1}{c}]^6 \\ O[\frac{1}{c}]^4 & \frac{jy(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4}+O[\frac{1}{c}]^6 & \frac{jy(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4}+O[\frac{1}{c}]^6 & \frac{jy(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4}+O[\frac{1}{c}]^6 \\ O[\frac{1}{c}]^4 & \frac{jz(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4}+O[\frac{1}{c}]^6 & \frac{jz(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4}+O[\frac{1}{c}]^6 & \frac{jz(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4}+O[\frac{1}{c}]^6 \end{pmatrix}\right\}$$

In[98]:=
```
FS[T[S.Inverse[gg]].gg - S] // MF
```

Out[98]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^6 & \frac{jy\,sxy+jz\,sxz-jy\,syx-jz\,szx}{n\,c^2}+\frac{\frac{(jx^2+jy^2+jz^2)(jx\,sxx+jy\,syx+jz\,szx)}{n^3}+\frac{4(jx\,sxx+jy\,sxy+jz\,sxz)W}{n}}{c^4}+O[\frac{1}{c}]^6 & \frac{jx(-sxy+syx)+jz(syz-szy)}{n\,c^2}+\frac{\frac{(jx^2+jy^2+jz^2)(jx\,sxy+jy\,syy+jz\,szy)}{n^3}+\frac{4(jx\,syx+jy\,syy+jz\,syz)W}{n}}{c^4}+O[\frac{1}{c}]^6 & \frac{jx(-sxz+szx)+jy(-syz+szy)}{n\,c^2}+\frac{\frac{(jx^2+jy^2+jz^2)(jx\,sxz+jy\,syz+jz\,szz)}{n^3}+\frac{4(jx\,szx+jy\,szy+jz\,szz)W}{n}}{c^4}+O[\frac{1}{c}]^6 \\ \frac{jy\,sxy+jz\,sxz-jy\,syx-jz\,szx}{n}+\frac{(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4n^2W)}{n^3c^2}+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & (-sxy+syx)+O[\frac{1}{c}]^4 & (-sxz+szx)+O[\frac{1}{c}]^4 \\ \frac{jx(-sxy+syx)+jz(syz-szy)}{n}+\frac{(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4n^2W)}{n^3c^2}+O[\frac{1}{c}]^4 & (sxy-syx)+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & (-syz+szy)+O[\frac{1}{c}]^4 \\ \frac{jx(-sxz+szx)+jy(-syz+szy)}{n}+\frac{(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4n^2W)}{n^3c^2}+O[\frac{1}{c}]^4 & (sxz-szx)+O[\frac{1}{c}]^4 & (syz-szy)+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}$$

In[101]:=
```
(* define "dust" 4-stress *)
(dust = Assuming[assut, FS[(ρ * c ^ 2 + ϵ) * Outer[Times, NJ, gg.uu / c ^ 2]]]) // MF
```

Out[101]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & jx\,\rho+\frac{\frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2}+jx(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jy\,\rho+\frac{\frac{jy(jx^2+jy^2+jz^2)\rho}{2n^2}+jy(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jz\,\rho+\frac{\frac{jz(jx^2+jy^2+jz^2)\rho}{2n^2}+jz(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -jx\,\rho\,c^2+\left(-\frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2}+jx(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & \frac{jx^2\rho}{n}+\frac{jx^2((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jx\,jy\,\rho}{n}+\frac{jx\,jy((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jx\,jz\,\rho}{n}+\frac{jx\,jz((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 \\ -jy\,\rho\,c^2+\left(-\frac{jy(jx^2+jy^2+jz^2)\rho}{2n^2}+jy(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & \frac{jx\,jy\,\rho}{n}+\frac{jx\,jy((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jy^2\rho}{n}+\frac{jy^2((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jy\,jz\,\rho}{n}+\frac{jy\,jz((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 \\ -jz\,\rho\,c^2+\left(-\frac{jz(jx^2+jy^2+jz^2)\rho}{2n^2}+jz(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & \frac{jx\,jz\,\rho}{n}+\frac{jx\,jz((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jy\,jz\,\rho}{n}+\frac{jy\,jz((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jz^2\rho}{n}+\frac{jz^2((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[102]:=
```
FS[T[dust.Inverse[gg]] == dust.Inverse[gg]]
```
Out[102]=
```
True
```

In[103]:=
```
FS[T[dust].gg == gg.dust]
```
Out[103]=
```
True
```

In[104]:=
```
MF /@ FS@{proju.dust.proju == dust, projperpu.dust.projperpu}
```
Out[104]=
```
{True, ( O[1/c]^4  O[1/c]^4  O[1/c]^4  O[1/c]^4
         O[1/c]^4  O[1/c]^4  O[1/c]^4  O[1/c]^4
         O[1/c]^4  O[1/c]^4  O[1/c]^4  O[1/c]^4
         O[1/c]^4  O[1/c]^4  O[1/c]^4  O[1/c]^4 )}
```

```
showf[assut][dust2 = Assuming[assut, Expand //@ FS@PowerExpand[(ρ * c ^ 2 + ϵ) * Outer[Times, NJ, gg.auu / c ^ 2]]]];
```

In[105]:=
```
(* define generic 4-stress *)
(EPS = Assuming[assut, FS[dust + Qtens + Ptens + S]]) // MF
```
Out[105]//MatrixForm=
```
( -n ρ c^2 + (-(jx^2+jy^2+jz^2) ρ/(2 n) + n (-ϵ + W ρ)) + O[1/c]^2
      jx ρ + (px + (jx sxx+jy syx+jz szx)/n + jx (jx^2+jy^2+jz^2) ρ/(2 n^2) + jx (ϵ+3 W ρ))/c^2 + O[1/c]^4
      jy ρ + (py + (jx sxy+jy syy+jz szy)/n + jy (jx^2+jy^2+jz^2) ρ/(2 n^2) + jy (ϵ+3 W ρ))/c^2 + O[1/c]^4
      jz ρ + (pz + (jx sxz+jy syz+jz szz)/n + jz (jx^2+jy^2+jz^2) ρ/(2 n^2) + jz (ϵ+3 W ρ))/c^2 + O[1/c]^4
  -jx ρ c^2 - (2 n (jx sxx+jy sxy+jz sxz+n (qx+jx ϵ))+jx (jx^2+jy^2+jz^2-2 n^2 W) ρ)/(2 n^2) + O[1/c]^2
      (sxx + jx^2 ρ/n) + jx (jx (jx^2+jy^2+jz^2) ρ+2 n^2 (px+qx+jx ϵ+3 jx W ρ))/(2 n^3 c^2) + O[1/c]^4
      (sxy + jx jy ρ/n) + ((jx py+jy qx+jx jy ϵ)/n + jx jy (jx^2+jy^2+jz^2+6 n^2 W) ρ/(2 n^3))/c^2 + O[1/c]^4
      (sxz + jx jz ρ/n) + ((jx pz+jz qx+jx jz ϵ)/n + jx jz (jx^2+jy^2+jz^2+6 n^2 W) ρ/(2 n^3))/c^2 + O[1/c]^4
  -jy ρ c^2 - (2 n (jx syx+jy syy+jz syz+n (qy+jy ϵ))+jy (jx^2+jy^2+jz^2-2 n^2 W) ρ)/(2 n^2) + O[1/c]^2
      (syx + jx jy ρ/n) + ((jy px+jx qy+jx jy ϵ)/n + jx jy (jx^2+jy^2+jz^2+6 n^2 W) ρ/(2 n^3))/c^2 + O[1/c]^4
      (syy + jy^2 ρ/n) + jy (jy (jx^2+jy^2+jz^2) ρ+2 n^2 (py+qy+jy ϵ+3 jy W ρ))/(2 n^3 c^2) + O[1/c]^4
      (syz + jy jz ρ/n) + ((jy pz+jz qy+jy jz ϵ)/n + jy jz (jx^2+jy^2+jz^2+6 n^2 W) ρ/(2 n^3))/c^2 + O[1/c]^4
  -jz ρ c^2 - (2 n (jx szx+jy szy+jz szz+n (qz+jz ϵ))+jz (jx^2+jy^2+jz^2-2 n^2 W) ρ)/(2 n^2) + O[1/c]^2
      (szx + jx jz ρ/n) + ((jz px+jx qz+jx jz ϵ)/n + jx jz (jx^2+jy^2+jz^2+6 n^2 W) ρ/(2 n^3))/c^2 + O[1/c]^4
      (szy + jy jz ρ/n) + ((jz py+jy qz+jy jz ϵ)/n + jy jz (jx^2+jy^2+jz^2+6 n^2 W) ρ/(2 n^3))/c^2 + O[1/c]^4
      (szz + jz^2 ρ/n) + jz (jz (jx^2+jy^2+jz^2) ρ+2 n^2 (pz+qz+jz ϵ+3 jz W ρ))/(2 n^3 c^2) + O[1/c]^4 )
```

In[106]:=
```
FS[T[EPS.Inverse[gg]].gg - EPS] // MF
```
Out[106]//MatrixForm=
```
( O[1/c]^2
      (n (-px+qx)+jy (sxy-syx)+jz (sxz-szx))/(n c^2) + O[1/c]^4
      (n (-py+qy)+jx (-sxy+syx)+jz (syz-szy))/(n c^2) + O[1/c]^4
      (n (-pz+qz)+jx (-sxz+szx)+jy (-syz+szy))/(n c^2) + O[1/c]^4
  (n (-px+qx)+jy (sxy-syx)+jz (sxz-szx))/n + O[1/c]^2
      O[1/c]^4
      (-sxy + syx) + (jy px-jx py-jy qx+jx qy)/(n c^2) + O[1/c]^4
      (-sxz + szx) + (jz px-jx pz-jz qx+jx qz)/(n c^2) + O[1/c]^4
  (n (-py+qy)+jx (-sxy+syx)+jz (syz-szy))/n + O[1/c]^2
      (sxy - syx) + (jy (-px+qx)+jx (py-qy))/(n c^2) + O[1/c]^4
      O[1/c]^4
      (-syz + szy) + (jz py-jy pz-jz qy+jy qz)/(n c^2) + O[1/c]^4
  (n (-pz+qz)+jx (-sxz+szx)+jy (-syz+szy))/n + O[1/c]^2
      (sxz - szx) + (jz (-px+qx)+jx (pz-qz))/(n c^2) + O[1/c]^4
      (syz - szy) + (jz (-py+qy)+jy (pz-qz))/(n c^2) + O[1/c]^4
      O[1/c]^4 )
```

In[108]:=
```
(* Conditions for symmetry of total 4-stress tensor *)
subsym = {px → qx, py → qy, pz → qz, syx → sxy, szx → sxz, szy → syz};
```

In[109]:=
```
(EPSsym = Assuming[assut, FS[EPS /. subsym]]) // MF
```
Out[109]//MatrixForm=
```
( -n ρ c^2 + (-(jx^2+jy^2+jz^2) ρ/(2 n) + n (-ϵ + W ρ)) + O[1/c]^2
      jx ρ + (qx + (jx sxx+jy sxy+jz sxz)/n + jx (jx^2+jy^2+jz^2) ρ/(2 n^2) + jx (ϵ+3 W ρ))/c^2 + O[1/c]^4
      jy ρ + (qy + (jx sxy+jy syy+jz syz)/n + jy (jx^2+jy^2+jz^2) ρ/(2 n^2) + jy (ϵ+3 W ρ))/c^2 + O[1/c]^4
      jz ρ + (qz + (jx sxz+jy syz+jz szz)/n + jz (jx^2+jy^2+jz^2) ρ/(2 n^2) + jz (ϵ+3 W ρ))/c^2 + O[1/c]^4
  -jx ρ c^2 - (2 n (jx sxx+jy sxy+jz sxz+n (qx+jx ϵ))+jx (jx^2+jy^2+jz^2-2 n^2 W) ρ)/(2 n^2) + O[1/c]^2
      (sxx + jx^2 ρ/n) + jx (jx (jx^2+jy^2+jz^2) ρ+2 n^2 (2 qx+jx (ϵ+3 W ρ)))/(2 n^3 c^2) + O[1/c]^4
      (sxy + jx jy ρ/n) + ((jy qx+jx qy+jx jy ϵ)/n + jx jy (jx^2+jy^2+jz^2+6 n^2 W) ρ/(2 n^3))/c^2 + O[1/c]^4
      (sxz + jx jz ρ/n) + ((jz qx+jx qz+jx jz ϵ)/n + jx jz (jx^2+jy^2+jz^2+6 n^2 W) ρ/(2 n^3))/c^2 + O[1/c]^4
  -jy ρ c^2 - (2 n (jx sxy+jy syy+jz syz+n (qy+jy ϵ))+jy (jx^2+jy^2+jz^2-2 n^2 W) ρ)/(2 n^2) + O[1/c]^2
      (sxy + jx jy ρ/n) + ((jy qx+jx qy+jx jy ϵ)/n + jx jy (jx^2+jy^2+jz^2+6 n^2 W) ρ/(2 n^3))/c^2 + O[1/c]^4
      (syy + jy^2 ρ/n) + jy (jy (jx^2+jy^2+jz^2) ρ+2 n^2 (2 qy+jy (ϵ+3 W ρ)))/(2 n^3 c^2) + O[1/c]^4
      (syz + jy jz ρ/n) + ((jz qy+jy qz+jy jz ϵ)/n + jy jz (jx^2+jy^2+jz^2+6 n^2 W) ρ/(2 n^3))/c^2 + O[1/c]^4
  -jz ρ c^2 - (2 n (jx sxz+jy syz+jz szz+n (qz+jz ϵ))+jz (jx^2+jy^2+jz^2-2 n^2 W) ρ)/(2 n^2) + O[1/c]^2
      (sxz + jx jz ρ/n) + ((jz qx+jx qz+jx jz ϵ)/n + jx jz (jx^2+jy^2+jz^2+6 n^2 W) ρ/(2 n^3))/c^2 + O[1/c]^4
      (syz + jy jz ρ/n) + ((jz qy+jy qz+jy jz ϵ)/n + jy jz (jx^2+jy^2+jz^2+6 n^2 W) ρ/(2 n^3))/c^2 + O[1/c]^4
      (szz + jz^2 ρ/n) + jz (jz (jx^2+jy^2+jz^2) ρ+2 n^2 (2 qz+jz (ϵ+3 W ρ)))/(2 n^3 c^2) + O[1/c]^4 )
```

In[111]:=
```
FS[T[EPSsym.Inverse[gg]].gg - EPSsym] // MF
```
Out[111]//MatrixForm=
```
( O[1/c]^2  O[1/c]^4  O[1/c]^4  O[1/c]^4
  O[1/c]^2  O[1/c]^4  O[1/c]^4  O[1/c]^4
  O[1/c]^2  O[1/c]^4  O[1/c]^4  O[1/c]^4
  O[1/c]^2  O[1/c]^4  O[1/c]^4  O[1/c]^4 )
```

In[113]:=
```
(* in terms of matter velocity *)
Assuming[assut, FS[EPS /. replaceJu]] // MF
```
Out[113]//MatrixForm=
```
( -n ρ c^2 - 1/2 n (2 ϵ + (ux^2 + uy^2 + uz^2 - 2 W) ρ) + O[1/c]^2
      n ux ρ + (px+sxx ux+syx uy+szx uz+n ux ϵ+1/2 n ux (ux^2+uy^2+uz^2+6 W) ρ)/c^2 + O[1/c]^4
      n uy ρ + (py+sxy ux+syy uy+szy uz+n uy ϵ+1/2 n uy (ux^2+uy^2+uz^2+6 W) ρ)/c^2 + O[1/c]^4
      n uz ρ + (pz+sxz ux+syz uy+szz uz+n uz ϵ+1/2 n uz (ux^2+uy^2+uz^2+6 W) ρ)/c^2 + O[1/c]^4
  -n ux ρ c^2 + (-qx - sxy uy - sxz uz - ux (sxx + n ϵ) - 1/2 n ux (ux^2 + uy^2 + uz^2 - 2 W) ρ) + O[1/c]^2
      (sxx + n ux^2 ρ) + ux (2 (px+qx+n ux ϵ)+n ux (ux^2+uy^2+uz^2+6 W) ρ)/(2 c^2) + O[1/c]^4
      (sxy + n ux uy ρ) + (py ux+qx uy+1/2 n ux uy (2 ϵ+(ux^2+uy^2+uz^2+6 W) ρ))/c^2 + O[1/c]^4
      (sxz + n ux uz ρ) + (pz ux+qx uz+1/2 n ux uz (2 ϵ+(ux^2+uy^2+uz^2+6 W) ρ))/c^2 + O[1/c]^4
  -n uy ρ c^2 + (-qy - syx ux - syz uz - uy (syy + n ϵ) - 1/2 n uy (ux^2 + uy^2 + uz^2 - 2 W) ρ) + O[1/c]^2
      (syx + n ux uy ρ) + (qy ux+px uy+1/2 n ux uy (2 ϵ+(ux^2+uy^2+uz^2+6 W) ρ))/c^2 + O[1/c]^4
      (syy + n uy^2 ρ) + uy (2 (py+qy+n uy ϵ)+n uy (ux^2+uy^2+uz^2+6 W) ρ)/(2 c^2) + O[1/c]^4
      (syz + n uy uz ρ) + (pz uy+qy uz+1/2 n uy uz (2 ϵ+(ux^2+uy^2+uz^2+6 W) ρ))/c^2 + O[1/c]^4
  -n uz ρ c^2 + (-qz - szx ux - szy uy - uz (szz + n ϵ) - 1/2 n uz (ux^2 + uy^2 + uz^2 - 2 W) ρ) + O[1/c]^2
      (szx + n ux uz ρ) + (qz ux+px uz+1/2 n ux uz (2 ϵ+(ux^2+uy^2+uz^2+6 W) ρ))/c^2 + O[1/c]^4
      (szy + n uy uz ρ) + (qz uy+py uz+1/2 n uy uz (2 ϵ+(ux^2+uy^2+uz^2+6 W) ρ))/c^2 + O[1/c]^4
      (szz + n uz^2 ρ) + uz (2 (pz+qz+n uz ϵ)+n uz (ux^2+uy^2+uz^2+6 W) ρ)/(2 c^2) + O[1/c]^4 )
```

(* Balanced quantities constructed from energy–momentum tensor, and their supplies *)

In[165]:=
```
(* Symmetrized energy-stress tensor, with explicit dep. on coords *)
(TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]]) // MF; (TTxsym = tW[tjv[(EPSsym + T[EPSsym.Inverse[gg]].gg) / 2]]) // MF
```
Out[165]//MatrixForm=
```
( -ρ n[t, x, y, z] c^2 + 1/2 (-ρ (n[t,x,y,z]^2 ux[t,x,y,z]^2+n[t,x,y,z]^2 uy[t,x,y,z]^2+n[t,x,y,z]^2 uz[t,x,y,z]^2)/n[t,x,y,z] + 2 n[t, x, y, z] (-ϵ + ρ W[t, x, y, z])) + O[1/c]^2
  -ρ n[t, x, y, z] × ux[t, x, y, z] c^2 + 1/2 (-qx - (sxx n[t,x,y,z]×ux[t,x,y,z]+sxy n[t,x,y,z]×uy[t,x,y,z]+sxz n[t,x,y,z]×uz[t,x,y,z])/n[t,x,y,z] - ρ ux[t,x,y,z] (n[t,x,y,z]^2 ux[t,x,y,z]^2+n[t,x,y,z]^2 uy[t,x,y,z]^2+n[t,x,y,z]^2 uz[t,x,y,z]^2)/(2 n[t,x,y,z]) + 4 ρ n[t, x, y, z] × ux[t, x, y, z] × W[t, x, y, z] - n[t, x, y, z] × ux[t, x, y, z] (ϵ + 3 ρ W[t, x, y, z]) - 2 n[t,x,y,z] (sxx n[t,x,y,z]·ux[t,x,y,z]+n[t,x,y,z] (qx+ϵ n[t,x,y,z]·ux[t,x,y,z] …
  -ρ n[t, x, y, z] × uy[t, x, y, z] c^2 + 1/2 (-qy - (sxy n[t,x,y,z]×ux[t,x,y,z]+syy n[t,x,y,z]×uy[t,x,y,z]+syz n[t,x,y,z]×uz[t,x,y,z])/n[t,x,y,z] - ρ uy[t,x,y,z] (n[t,x,y,z]^2 ux[t,x,y,z]^2+n[t,x,y,z]^2 uy[t,x,y,z]^2+n[t,x,y,z]^2 uz[t,x,y,z]^2)/(2 n[t,x,y,z]) + 4 ρ n[t, x, y, z] × uy[t, x, y, z] × W[t, x, y, z] - n[t, x, y, z] × uy[t, x, y, z] (ϵ + 3 ρ W[t, x, y, z]) - 2 n[t,x,y,z] (sxy n[t,x,y,z]·ux[t,x,y,z]+syy n[t,x,y,z]·uy[t,x,y,z]+n[t,x,y,z] (q …
  -ρ n[t, x, y, z] × uz[t, x, y, z] c^2 + 1/2 (-qz - (sxz n[t,x,y,z]×ux[t,x,y,z]+syz n[t,x,y,z]×uy[t,x,y,z]+szz n[t,x,y,z]×uz[t,x,y,z])/n[t,x,y,z] - ρ uz[t,x,y,z] (n[t,x,y,z]^2 ux[t,x,y,z]^2+n[t,x,y,z]^2 uy[t,x,y,z]^2+n[t,x,y,z]^2 uz[t,x,y,z]^2)/(2 n[t,x,y,z]) + 4 ρ n[t, x, y, z] × uz[t, x, y, z] × W[t, x, y, z] - n[t, x, y, z] × uz[t, x, y, z] (ϵ + 3 ρ W[t, x, y, z]) - 2 n[t,x,y,z] (sxz n[t,x,y,z]·ux[t,x,y,z]+syz n[t,x,y,z]·uy[t,x,y,z]+szz n[t,x …
```

In[135]:=
```
(* covariant derivatives of 4-velocity, for later use *)
tjv[xx_] := (xx /. {n → n[t, x, y, z], jx → ux[t, x, y, z] * n[t, x, y, z], jy → uy[t, x, y, z] * n[t, x, y, z], jz → uz[t, x, y, z] * n[t, x, y, z]});
tjn[xx_] := (xx /. {n → n[t, x, y, z], jx → jx[t, x, y, z], jy → jy[t, x, y, z], jz → jz[t, x, y, z]});
itjn[xx_] := (xx /. {n[t, x, y, z] → n, jx[t, x, y, z] → jx, jy[t, x, y, z] → jy, jz[t, x, y, z] → jz});
itjv[xx_] := (xx /. {n[t, x, y, z] → n, ux[t, x, y, z] → jx / n, uy[t, x, y, z] → jy / n, uz[t, x, y, z] → jz / n});
repjn = {D[n[t, x, y, z], t] → D[jx[t, x, y, z], x] + D[jy[t, x, y, z], y] + D[jz[t, x, y, z], z]};
MF /@ {uut = Assuming[assut, FS[tjn[tW[uu]]]], uuv = Assuming[assut, FS[tjv[tW[uu]]]]}
```
Out[140]=
```
{( 1 + ((jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2)/(2 n[t,x,y,z]^2) + W[t,x,y,z])/c^2 + O[1/c]^4
   jx[t,x,y,z]/n[t,x,y,z] + jx[t,x,y,z] (jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2 n[t,x,y,z]^2 W[t,x,y,z])/(2 n[t,x,y,z]^3 c^2) + O[1/c]^4
   jy[t,x,y,z]/n[t,x,y,z] + jy[t,x,y,z] (jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2 n[t,x,y,z]^2 W[t,x,y,z])/(2 n[t,x,y,z]^3 c^2) + O[1/c]^4
   jz[t,x,y,z]/n[t,x,y,z] + jz[t,x,y,z] (jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2 n[t,x,y,z]^2 W[t,x,y,z])/(2 n[t,x,y,z]^3 c^2) + O[1/c]^4 ),
 ( 1 + (1/2 (ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2)+W[t,x,y,z])/c^2 + O[1/c]^4
   ux[t, x, y, z] + ux[t,x,y,z] (ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2 W[t,x,y,z])/(2 c^2) + O[1/c]^4
   uy[t, x, y, z] + uy[t,x,y,z] (ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2 W[t,x,y,z])/(2 c^2) + O[1/c]^4
   uz[t, x, y, z] + uz[t,x,y,z] (ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2 W[t,x,y,z])/(2 c^2) + O[1/c]^4 )}
```

In[145]:=
```
(Duu = Assuming[assut, FS[(D[Normal@uut, {coords}] + Sum[uut[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]) // MF
```
Out[145]//MatrixForm=
```
( (n[t,x,y,z] (jx[t,x,y,z] (-n[t,x,y,z] W^(0,1,0,0)[t,x,y,z]+jx^(1,0,0,0)[t,x,y,z])+jy[t,x,y,z] (-n[t,x,y,z] W^(0,0,1,0)[t,x,y,z]+jy^(1,0,0,0)[t,x,y,z])+jz[t,x,y,z] (-n[t,x,y,z] W^(0,0,0,1)[t,x,y,z]+jz^(1,0,0,0)[t,x,y,z]))-(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2) n^(1,0,0,0)[t,x,y,z])/(n[t,x,y,z]^3 c^2) + O[1/c]^4
      (n[t,x,y,z] (jx[t,x,y,z] jx^(0,1,0,0)[t,x,y,z]+jy[t,x,y,z] jy^(0,1,0,0)[t,x,y,z]+jz[t,x,y,z] jz^(0,1,0,0)[t,x,y,z])-(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2) n^(0,1,0,0)[t,x,y,z])/(n[t,x,y,z]^3 c^2) + O[1/c]^4 …
  (-W^(0,1,0,0)[t, x, y, z] + (n[t,x,y,z] jx^(1,0,0,0)[t,x,y,z]-jx[t,x,y,z] n^(1,0,0,0)[t,x,y,z])/n[t,x,y,z]^2) + O[1/c]^2
      (n[t,x,y,z] jx^(0,1,0,0)[t,x,y,z]-jx[t,x,y,z] n^(0,1,0,0)[t,x,y,z])/n[t,x,y,z]^2 + …
  (-W^(0,0,1,0)[t, x, y, z] + (n[t,x,y,z] jy^(1,0,0,0)[t,x,y,z]-jy[t,x,y,z] n^(1,0,0,0)[t,x,y,z])/n[t,x,y,z]^2) + O[1/c]^2
      (n[t,x,y,z] jy^(0,1,0,0)[t,x,y,z]-jy[t,x,y,z] n^(0,1,0,0)[t,x,y,z])/n[t,x,y,z]^2 + (-2 jx[t,x,y,z] n[t,x,y,z]^2 W^(0,0,1,0)[t,x,y,z]+n[t,x,y,z] (jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2 n[t,x,y,z]^2 W[t,x,y,z]) jy^(0,1,0,0) …
  (-W^(0,0,0,1)[t, x, y, z] + (n[t,x,y,z] jz^(1,0,0,0)[t,x,y,z]-jz[t,x,y,z] n^(1,0,0,0)[t,x,y,z])/n[t,x,y,z]^2) + O[1/c]^2
      (n[t,x,y,z] jz^(0,1,0,0)[t,x,y,z]-jz[t,x,y,z] n^(0,1,0,0)[t,x,y,z])/n[t,x,y,z]^2 + (jx[t,x,y,z] (-2 n[t,x,y,z]^3 W^(0,0,0,1)[t,x,y,z]+2 jz[t,x,y,z] n[t,x,y,z] jx^(0,1,0,0)[t,x,y,z]…
```

In[146]:=
```
(Duv = Assuming[assut, FS[(D[Normal@uuv, {coords}] + Sum[uuv[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]) // MF
```
Out[146]//MatrixForm=
```
( (ux[t,x,y,z] (-W^(0,1,0,0)[t,x,y,z]+ux^(1,0,0,0)[t,x,y,z])+uy[t,x,y,z] (-W^(0,0,1,0)[t,x,y,z]+uy^(1,0,0,0)[t,x,y,z])+uz[t,x,y,z] (-W^(0,0,0,1)[t,x,y,z]+uz^(1,0,0,0)[t,x,y,z]))/c^2 + O[1/c]^4
      (ux[t,x,y,z] ux^(0,1,0,0)[t,x,y,z]+uy[t,x,y,z] uy^(0,1,0,0)[t,x,y,z]+uz[t,x,y,z] uz^(0,1,0,0)[t,x,y,z])/c^2 + O[1/c]^4
  (-W^(0,1,0,0)[t, x, y, z] + ux^(1,0,0,0)[t, x, y, z]) + O[1/c]^2
      ux^(0,1,0,0)[t, x, y, z] + (uy[t,x,y,z]^2 ux^(0,1,0,0)[t,x,y,z]+uz[t,x,y,z]^2 ux^(0,1,0,0)[t,x,y,z]+(3 ux[t, …
  (-W^(0,0,1,0)[t, x, y, z] + uy^(1,0,0,0)[t, x, y, z]) + O[1/c]^2
      uy^(0,1,0,0)[t, x, y, z] + (-2 ux[t,x,y,z] (W^(0,0,1,0)[t,x,y,z]-uy[t,x,y,z] ux^(0,1,0,0)[t,x,y,z])+ux[t,x …
  (-W^(0,0,0,1)[t, x, y, z] + uz^(1,0,0,0)[t, x, y, z]) + O[1/c]^2
      uz^(0,1,0,0)[t, x, y, z] + (-ux[t,x,y,z] W^(0,0,0,1)[t,x,y,z]+1/2 (ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2 … + O[ …
```

In[278]:=

```
(* Energy current and supply according to 4-velocity *)
pvec = uu; Dpvec = Duv;
MF@(MF /@ {Efluxuu = FS[({{1, 0, 0, 0}, yzVxsurface / (Δt)}.EPS.pvec)], , Esupplysymuu = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Esupplyuu = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```
Out[279]//MatrixForm=
```
( ( -n ρ c^2 - n ϵ + O[1/c]^2
    (-Ax jx - Ay jy - Az jz + Ax n Vx + Ay n Vy + Az n Vz) ρ c^2 + (-Ax (qx + (jx - n Vx) ϵ) - Ay (qy + jy ϵ - n Vy ϵ) - Az (qz + jz ϵ - n Vz ϵ)) + O[1/c]^2 )
  Null
  (szz uz^(0,0,0,1)[t, x, y, z] + syy uy^(0,0,1,0)[t, x, y, z] + syz (uy^(0,0,0,1)[t, x, y, z] + uz^(0,0,1,0)[t, x, y, z]) + sxx ux^(0,1,0,0)[t, x, y, z] + sxy (ux^(0,0,1,0)[t, x, y, z] + uy^(0,1,0,0)[t, x, y, z]) + sxz (ux^(0,0,0,1)[t, x, y, z] + uz^(0,1,0,0)[t, x, y, z])) + O[1/c]^2
  Null
  1/2 (2 szz uz^(0,0,0,1)[t, x, y, z] + (syz + szy) (uy^(0,0,0,1)[t, x, y, z] + uz^(0,0,1,0)[t, x, y, z]) + 2 (syy uy^(0,0,1,0)[t, x, y, z] + sxx ux^(0,1,0,0)[t, x, y, z]) + (sxy + syx) (ux^(0,0,1,0)[t, x, y, z] + uy^(0,1,0,0)[t, x, y, z]) + (sxz + szx) (ux^(0,0,0,1)[t, x, y, z] + uz^(0,1,0,0)[t, x, y, z])) + O[1/c]^2 )
```

In[280]:=
```
MF@(MF /@ (FS[({Efluxuu, , Esupplysymuu, , Esupplyuu} /. replaceJu) /. replaceuUnorm]))
```
Out[280]//MatrixForm=
```
( ( -n ρ c^2 - n ϵ + O[1/c]^2
    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c^2 + (-Ax qx - Ay qy - Az qz + n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ϵ) + O[1/c]^2 )
  Null
  (szz uz^(0,0,0,1)[t, x, y, z] + syy uy^(0,0,1,0)[t, x, y, z] + syz (uy^(0,0,0,1)[t, x, y, z] + uz^(0,0,1,0)[t, x, y, z]) + sxx ux^(0,1,0,0)[t, x, y, z] + sxy (ux^(0,0,1,0)[t, x, y, z] + uy^(0,1,0,0)[t, x, y, z]) + sxz (ux^(0,0,0,1)[t, x, y, z] + uz^(0,1,0,0)[t, x, y, z])) + O[1/c]^2
  Null
  1/2 (2 szz uz^(0,0,0,1)[t, x, y, z] + (syz + szy) (uy^(0,0,0,1)[t, x, y, z] + uz^(0,0,1,0)[t, x, y, z]) + 2 (syy uy^(0,0,1,0)[t, x, y, z] + sxx ux^(0,1,0,0)[t, x, y, z]) + (sxy + syx) (ux^(0,0,1,0)[t, x, y, z] + uy^(0,1,0,0)[t, x, y, z]) + (sxz + szx) (ux^(0,0,0,1)[t, x, y, z] + uz^(0,1,0,0)[t, x, y, z])) + O[1/c]^2 )
```

In[281]:=
```
(* Energy current and supply according to t-vector *)
pvec = -{1, 0, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ ({Efluxt = FS[(({1, 0, 0, 0}, yzVxsurface / (Δt)).EPS.pvec)], , Esupplysymt = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Esupplyt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]}))
```

Out[282]//MatrixForm=
```
n ρ c^2 + ((jx^2+jy^2+jz^2) ρ/(2 n) + n (ϵ - W ρ)) + O[1/c]^2
(Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) ρ c^2 + (2 n (Ax (jx sxx+jy sxy+jz sxz+n (qx+(jx-n Vx) ϵ)) + Ay (jx syx+jy syy+jz syz+n (qy+jy ϵ-n Vy ϵ)) + Az (jx szx+jy szy+jz szz+n (qz+jz ϵ-n Vz ϵ))) + (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jx^2+jy^2+jz^2-2 n^2 W) ρ)/(2 n^2) + O[1/c]^2
Null
-n ρ W^(1,0,0,0)[t, x, y, z] + O[1/c]^2
Null
-n ρ W^(1,0,0,0)[t, x, y, z] + O[1/c]^2
```

In[283]:=
```
MF@(MF /@ (FS[({Efluxt, , Esupplysymt, , Esupplyt} /. replaceJu) /. replaceuUnorm]))
```

Out[283]//MatrixForm=
```
n ρ c^2 + n (ϵ + 1/2 (U^2 - 2 W) ρ) + O[1/c]^2
n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) ρ c^2 + (Ax (qx + sxx ux + sxy uy + sxz uz + n (ux - Vx) ϵ) + Ay (qy + syx ux + syy uy + syz uz + n (uy - Vy) ϵ) + Az (qz + szx ux + szy uy + szz uz + n (uz - Vz) ϵ) + 1/2 n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (U^2 - 2 W) ρ) + O[1/c]^2
Null
-n ρ W^(1,0,0,0)[t, x, y, z] + O[1/c]^2
Null
-n ρ W^(1,0,0,0)[t, x, y, z] + O[1/c]^2
```

In[284]:=
```
(* Energy current and supply according to norm. t-vector *)
pvec = -c * {1, 0, 0, 0} / Sqrt[-gg[[1, 1]]]; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Efluxnt = FS[(({1, 0, 0, 0}, yzVxsurface / (Δt)).EPS.pvec)], , Esupplysymnt = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Esupplynt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[285]//MatrixForm=
```
n ρ c^2 + (n ϵ + (jx^2+jy^2+jz^2) ρ/(2 n)) + O[1/c]^2
(Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) ρ c^2 + (2 n (Ax (jx sxx+jy sxy+jz sxz+n (qx+(jx-n Vx) ϵ)) + Ay (jx syx+jy syy+jz syz+n (qy+jy ϵ-n Vy ϵ)) + Az (jx szx+jy szy+jz szz+n (qz+jz ϵ-n Vz ϵ))) + (jx^2+jy^2+jz^2) (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) ρ)/(2 n^2) + O[1/c]^2
Null
ρ (jz W^(0,0,0,1)[t, x, y, z] + jy W^(0,0,1,0)[t, x, y, z] + jx W^(0,1,0,0)[t, x, y, z]) + O[1/c]^2
Null
ρ (jz W^(0,0,0,1)[t, x, y, z] + jy W^(0,0,1,0)[t, x, y, z] + jx W^(0,1,0,0)[t, x, y, z]) + O[1/c]^2
```

In[286]:=
```
(* Energy current and supply according to cov. t-vector *)
pvec = c^2 * igg.{1, 0, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Efluxcovt = FS[(({1, 0, 0, 0}, yzVxsurface / (Δt)).EPS.pvec)], , Esupplysymcovt = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Esupplycovt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[287]//MatrixForm=
```
n ρ c^2 + ((jx^2+jy^2+jz^2) ρ/(2 n) + n (ϵ + W ρ)) + O[1/c]^2
(Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) ρ c^2 + (2 n (Ax (jx sxx+jy sxy+jz sxz+n (qx+(jx-n Vx) ϵ)) + Ay (jx syx+jy syy+jz syz+n (qy+jy ϵ-n Vy ϵ)) + Az (jx szx+jy szy+jz szz+n (qz+jz ϵ-n Vz ϵ))) + (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jx^2+jy^2+jz^2+2 n^2 W) ρ)/(2 n^2) + O[1/c]^2
Null
ρ (2 jz W^(0,0,0,1)[t, x, y, z] + 2 jy W^(0,0,1,0)[t, x, y, z] + 2 jx W^(0,1,0,0)[t, x, y, z] + n W^(1,0,0,0)[t, x, y, z]) + O[1/c]^2
Null
ρ (2 jz W^(0,0,0,1)[t, x, y, z] + 2 jy W^(0,0,1,0)[t, x, y, z] + 2 jx W^(0,1,0,0)[t, x, y, z] + n W^(1,0,0,0)[t, x, y, z]) + O[1/c]^2
```

In[288]:=
```
(* Momentum current and supply according to x-vector *)
pvec = {0, 1, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Pfluxx = FS[(({1, 0, 0, 0}, yzVxsurface / (Δt)).EPS.pvec)], , Psupplysymx = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Psupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[289]//MatrixForm=
```
jx ρ + (px + (jx sxx+jy sxy+jz sxz)/n + jx (jx^2+jy^2+jz^2) ρ/(2 n^2) + jx (ϵ+3 W ρ))/c^2 + O[1/c]^4
(Ax sxx + Ay syx + Az szx + jx (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) ρ/n) + (-n (Ax Vx+Ay Vy+Az Vz) (2 n (jx sxx+jy syx+jz szx+n (px+jx ϵ)) + jx (jx^2+jy^2+jz^2+6 n^2 W) ρ) + Ay (2 n^2 (jx qy+jy (px+jx ϵ)) + jx jy (jx^2+jy^2+jz^2+6 n^2 W) ρ) + Az (2 n^2 (jx qz+jz (px+jx ϵ)) + jx jz (jx^2+jy^2+jz^2+6 n^2 W) ρ) + Ax jx (jx (jx^2+jy^2+jz^2) ρ + 2 n^2 (px+qx+jx ϵ+3 jx W ρ)))/(2 n^3 c^2) + O[1/c]^4
Null
n ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
Null
n ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
```

In[290]:=
```
MF@(MF /@ (FS[({Pfluxx, , Psupplysymx, , Psupplyx} /. replaceJu) /. replaceuUnorm]))
```

Out[290]//MatrixForm=
```
n ux ρ + (px + sxx ux + sxy uy + sxz uz + n ux ϵ + 1/2 n ux (U^2+6 W) ρ)/c^2 + O[1/c]^4
(Ax sxx + Ay syx + Az szx + n ux (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) ρ) + (Ax (qx ux + px (ux-Vx) - (sxx ux+syx uy+szx uz) Vx + n ux (ux-Vx) ϵ) + Ay (qy ux + px (uy-Vy) - (sxx ux+sxy uy+sxz uz) Vy + n ux (uy-Vy) ϵ) + Az (qz ux + px (uz-Vz) - (sxx ux+sxy uy+sxz uz) Vz + n ux (uz-Vz) ϵ) + 1/2 Ax n ux (ux-Vx) (U^2+6 W) ρ + 1/2 n ux (Ay (uy-Vy)+Az (uz-Vz)) (U^2+6 W) ρ)/c^2 + O[1/c]^4
Null
n ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
Null
n ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
```

In[291]:=
```
(* Momentum current and supply according to cov. x-vector *)
pvec = igg.{0, 1, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Pfluxx = FS[(({1, 0, 0, 0}, yzVxsurface / (Δt)).EPS.pvec)], , Psupplysymx = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Psupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[292]//MatrixForm=
```
jx ρ + (px + (jx sxx+jy sxy+jz sxz)/n + jx (jx^2+jy^2+jz^2) ρ/(2 n^2) + jx (ϵ+W ρ))/c^2 + O[1/c]^4
(Ax sxx + Ay syx + Az szx + jx (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) ρ/n) + ((Ax (jx px+jy sxy+jz sxz) ... )/n + jx (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jx^2+jy^2+jz^2-2 n^2 W) ρ/(2 n^2))/c^2 + O[1/c]^4
Null
n ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
Null
n ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
```

In[293]:=
```
(* Ang.momentum current and supply according to yz-vector *)
pvec = y * {0, 0, 0, 1} - z * {0, 0, 1, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Lfluxx = FS[(({1, 0, 0, 0}, yzVxsurface / (Δt)).EPS.pvec)], , Lsupplysymx = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Lsupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[294]//MatrixForm=
```
(jz y - jy z) ρ + (2 n ((jx sxz+jy syz+jz szz) y - (jx sxy+jy syy+jz szy) z + n (pz y + jz y ϵ - z (py+jy ϵ))) + (jx^2+jy^2+jz^2+6 n^2 W) (jz y - jy z) ρ)/(2 n^2 c^2) + O[1/c]^4
((Ax sxz + Ay syz + Az szz) y - (Ax sxy + Ay syy + Az szy) z + (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jz y - jy z) ρ/n) + (-z (-n (Ax Vx+Ay Vy+Az Vz) (2 n (jx sxy+jy syy+jz szy+n (py+jy ϵ)) + jy (jx^2+jy^2+jz^2+6 n^2 W) ρ) + Ax (2 n^2 (jy qx+jx (py+jy ϵ)) + jx jy (jx^2+jy^2+jz^2+6 n^2 W) ρ) + Az (2 n^2 (jy qz+jz (py+jy ϵ)) + jy jz (jx^2+jy^2+jz^2+6 n^2 W) ρ) + Ay jy (jy (jx^2+jy^2+jz^2) ρ + 2 n^2 (py+qy+jy ϵ+3 jy W ρ))) + y (-n (Ax Vx+Ay Vy+Az Vz) (2 n (jx sxz+jy syz+jz szz+n (pz+jz ϵ)) + jz (jx^2+jy^2 ...)/(2 n^3 c^2)
Null
n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]^2
Null
n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]^2
```

In[295]:=
```
(* Ang.momentum current and supply according to cov. yz-vector *)
pvec = igg.(y * {0, 0, 0, 1} - z * {0, 0, 1, 0}); Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Lfluxcx = FS[(({1, 0, 0, 0}, yzVxsurface / (Δt)).EPS.pvec)], , Lsupplysymcx = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Lsupplycx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[296]//MatrixForm=
```
(jz y - jy z) ρ + (2 n ((jx sxz+jy syz+jz szz) y - (jx sxy+jy syy+jz szy) z + n (pz y + jz y ϵ - z (py+jy ϵ))) + (jx^2+jy^2+jz^2+2 n^2 W) (jz y - jy z) ρ)/(2 n^2 c^2) + O[1/c]^4
((Ax sxz + Ay syz + Az szz) y - (Ax sxy + Ay syy + Az szy) z + (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jz y - jy z) ρ/n) + (4 n^2 W y (-n (Ax sxz+Ay syz+Az szz) + jz (-Ax jx - Ay jy - Az jz + Ax n Vx + Ay n Vy + Az n Vz) ρ) + 4 n^2 W z (n (Ax sxy+Ay syy+Az szy) + jy (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) ρ) - z (-n (Ax Vx+Ay Vy+Az Vz) (2 n (jx sxy+jy syy+jz szy+n (py+jy ϵ)) + jy (jx^2+jy^2+jz^2+6 n^2 W) ρ) + Ax (2 n^2 (jy qx+jx (py+jy ϵ)) + jx jy (jx^2+jy^2+jz^2+6 n^2 W) ρ) + Az (2 n^2 (jy qz+jz (...
Null
n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]^2
Null
n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]^2
```

In[297]:=
```
(* Ang.boost-momentum current and supply according to tx-vector *)
pvec = t * {0, 1, 0, 0} + x * {1, 0, 0, 0} / c^2; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Bfluxx = FS[(({1, 0, 0, 0}, yzVxsurface / (Δt)).EPS.pvec)], , Bsupplysymx = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Bsupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[298]//MatrixForm=
```
(jx t - n x) ρ + (x (-(jx^2+jy^2+jz^2) ρ/(2 n) + n (-ϵ + W ρ)) + t (px + (jx sxx+jy syx+jz szx)/n + jx (jx^2+jy^2+jz^2) ρ/(2 n^2) + jx (ϵ+3 W ρ)))/c^2 + O[1/c]^4
((Ax sxx + Ay syx + Az szx) t + (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jx t - n x) ρ/n) + (-n x (2 n (Ax (jx sxx+jy sxy+jz sxz+n (qx+(jx-n Vx) ϵ)) + Ay (jx syx+jy syy+jz syz+n (qy+jy ϵ-n Vy ϵ)) + Az (jx szx+jy szy+jz szz+n (qz+jz ϵ-n Vz ϵ))) + (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jx^2+jy^2+jz^2-2 n^2 W) ρ) + t (-n (Ax Vx+Ay Vy+Az Vz) (2 n (jx sxx+jy syx+jz szx+n (px+jx ϵ)) + jx (jx^2+jy^2+jz^2+6 n^2 W) ρ) + Ay (2 n^2 (jx qy+jy (px+jx ϵ)) + jx jy (jx^2+jy^2+jz^2+6 n^2 W) ρ) + Az (2 n^2 (jx qz+jz (...)/(2 n^3 c^2)
Null
n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
Null
n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
```

In[299]:=
```
(* Ang.boost-momentum current and supply according to cov. tx-vector *)
pvec = igg.(t * {0, 1, 0, 0} - x * {1, 0, 0, 0}); Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Bfluxcx = FS[(({1, 0, 0, 0}, yzVxsurface / (Δt)).EPS.pvec)], , Bsupplysymcx = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Bsupplycx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[300]//MatrixForm=
```
(jx t - n x) ρ + (2 n ((n px + jx sxx + jy syx + jz szx) t + n (jx t - n x) ϵ) + (jx^2+jy^2+jz^2+2 n^2 W) (jx t - n x) ρ)/(2 n^2 c^2) + O[1/c]^4
((Ax sxx + Ay syx + Az szx) t + (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jx t - n x) ρ/n) + (-2 Ay n^2 (t (-jx qy + n px Vy + jx sxx Vy + jz szx Vy + 2 n syx W) + (n qy + jx syx + jz syz) x + jy (-px t + syx t Vy + syy x)) - 2 Az n^2 (t (-jx qz + n px Vz + jx sxx Vz + jy syx Vz + 2 n szx W) + (n qz + jx szx + jy szy) x + jz (-px t + szx t Vz + szz x)) + 2 Ay n^2 (jy - n Vy) (jx t - n x) ϵ + 2 Az n^2 (jz - n Vz) (jx t - n x) ϵ + 2 Ax n^2 (jx t (px + qx - sxx Vx) - t (n px Vx + jy syx Vx + jz szx Vx + 2 n sxx W) - jx sxx x - (n qx + jy sxy + jz sxz) x + (j ...)/(2 n^3 c^2)
Null
n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
Null
n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
```

```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[supply.TTx]], {supply, {Dtxyzvec[[1]], Dtxyzvec[[2]], {0, 0, 0, 0}, Dgtxyzvec[[1]] * c^2, Dgtxyzvec[[2]], {0, 0, 0, 0}, DLxvec, Dxboost / c, {0, 0, 0, 0}, DgLxvec, Dgxboost, {0, 0, 0, 0}, Duu, Dtvecnorm}}]]
```

In[∘]:=
```
(* coordinate/internal/coordinate-proper energy and x-momentum, content and fluxes (TRANSPOSED) *)
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, yzVxsurface/(Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, igg[[1]]*c^2, igg[[2]], {0, 0, 0, 0}, Lxvec, xboost/c, {0, 0, 0, 0}, gLxvec, gxboost, {0, 0, 0, 0}, uu, tvecnorm}])]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(-\epsilon+W\rho)\right)+O[\tfrac{1}{c}]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2+\left(-\frac{Ax(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz)+Ay(n\,qy+jx\,syx+jy\,syy+jz\,syz)+Az(n\,qz+jx\,szx+jy\,szy+jz\,szz)}{n}+(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\epsilon-\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}\right)+O[\tfrac{1}{c}]^2 \\
jx\,\rho+O[\tfrac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O[\tfrac{1}{c}]^2 \\
0 & 0 \\
n\rho c^2+\left(\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(\epsilon+W\rho)\right)+O[\tfrac{1}{c}]^2 & (Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho c^2+\frac{2n\left(Ax(jx\,sxx+jy\,sxy+jz\,sxz+n(qx+(jx-n\,Vx)\epsilon))+Ay(jx\,syx+jy\,syy+jz\,syz+n(qy+jy\,\epsilon-n\,Vy\,\epsilon))+Az(jx\,szx+jy\,szy+jz\,szz+n(qz+jz\,\epsilon-n\,Vz\,\epsilon))\right)+(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jx^2+jy^2+jz^2+2n^2W)\rho}{2n^2}+O[\tfrac{1}{c}]^2 \\
jx\,\rho+O[\tfrac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O[\tfrac{1}{c}]^2 \\
0 & 0 \\
(jz\,y-jy\,z)\rho+O[\tfrac{1}{c}]^2 & \left((Ax\,sxz+Ay\,syz+Az\,szz)y-(Ax\,sxy+Ay\,syy+Az\,szy)z+\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jz\,y-jy\,z)\rho}{n}\right)+O[\tfrac{1}{c}]^2 \\
(jx\,t-n\,x)\rho+O[\tfrac{1}{c}]^2 & \left((Ax\,sxx+Ay\,syx+Az\,szx)t+\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jx\,t-n\,x)\rho}{n}\right)+O[\tfrac{1}{c}]^2 \\
0 & 0 \\
(jz\,y-jy\,z)\rho+O[\tfrac{1}{c}]^2 & \left((Ax\,sxz+Ay\,syz+Az\,szz)y-(Ax\,sxy+Ay\,syy+Az\,szy)z+\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jz\,y-jy\,z)\rho}{n}\right)+O[\tfrac{1}{c}]^2 \\
(-jx\,t\,\rho+n\,x\,\rho)+O[\tfrac{1}{c}]^2 & \left(-((Ax\,sxx+Ay\,syx+Az\,szx)t)-\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jx\,t-n\,x)\rho}{n}\right)+O[\tfrac{1}{c}]^2 \\
0 & 0 \\
-n\rho c^2-n\epsilon+O[\tfrac{1}{c}]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2+\left(-Ax(qx+(jx-n\,Vx)\epsilon)-Ay(qy+jy\,\epsilon-n\,Vy\,\epsilon)-Az(qz+jz\,\epsilon-n\,Vz\,\epsilon)\right)+O[\tfrac{1}{c}]^2 \\
-n\rho c^2+\left(-n\epsilon-\frac{(jx^2+jy^2+jz^2)\rho}{2n}\right)+O[\tfrac{1}{c}]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2+\left(-\frac{Ax(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz)+Ay(n\,qy+jx\,syx+jy\,syy+jz\,syz)+Az(n\,qz+jx\,szx+jy\,szy+jz\,szz)}{n}+(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\epsilon-\frac{(jx^2+jy^2+jz^2)(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{2n^2}\right)+O[\tfrac{1}{c}]^2
\end{pmatrix}$$

In[∘]:=
```
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, yzVxsurface/(Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, igg[[1]]*c^2, igg[[2]], {0, 0, 0, 0}, Lxvec, xboost/c, {0, 0, 0, 0}, gLxvec, gxboost, {0, 0, 0, 0}, uu, tvecnorm}]) /. replaceJu]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+(ux^2+uy^2+uz^2-2W)\rho\right)+O[\tfrac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\rho c^2+\left(-Ax(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon)-Ay(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon)-Az(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon)-\frac{1}{2}n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(ux^2+uy^2+uz^2-2W)\rho\right)+O[\tfrac{1}{c}]^2 \\
n\,ux\,\rho+O[\tfrac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))\rho\right)+O[\tfrac{1}{c}]^2 \\
0 & 0 \\
n\rho c^2+\left(n\epsilon+\frac{1}{2}n(ux^2+uy^2+uz^2+2W)\rho\right)+O[\tfrac{1}{c}]^2 & n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))\rho c^2+\left(Ax(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon)+Ay(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon)+Az(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon)+\frac{1}{2}n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(ux^2+uy^2+uz^2+2W)\rho\right)+O[\tfrac{1}{c}]^2 \\
n\,ux\,\rho+O[\tfrac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))\rho\right)+O[\tfrac{1}{c}]^2 \\
0 & 0 \\
n(uz\,y-uy\,z)\rho+O[\tfrac{1}{c}]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(uz\,y-uy\,z)\rho\right)+O[\tfrac{1}{c}]^2 \\
n(t\,ux-x)\rho+O[\tfrac{1}{c}]^2 & \left((Ax\,sxx+Ay\,syx+Az\,szx)t+n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux-x)\rho\right)+O[\tfrac{1}{c}]^2 \\
0 & 0 \\
n(uz\,y-uy\,z)\rho+O[\tfrac{1}{c}]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(uz\,y-uy\,z)\rho\right)+O[\tfrac{1}{c}]^2 \\
n(-t\,ux+x)\rho+O[\tfrac{1}{c}]^2 & \left(-((Ax\,sxx+Ay\,syx+Az\,szx)t)-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux-x)\rho\right)+O[\tfrac{1}{c}]^2 \\
0 & 0 \\
-n\rho c^2-n\epsilon+O[\tfrac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\epsilon\right)+O[\tfrac{1}{c}]^2 \\
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+(ux^2+uy^2+uz^2)\rho\right)+O[\tfrac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\rho c^2+\left(-Ax(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon)-Ay(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon)-Az(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon)-\frac{1}{2}n(ux^2+uy^2+uz^2)(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))\rho\right)+O[\tfrac{1}{c}]^2
\end{pmatrix}$$

In[∘]:=
```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg)/2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand//@FS@PowerExpand[Tr[supply.TTx]], {supply, {Dtxyzvec[[1]], Dtxyzvec[[2]], {0, 0, 0, 0}, Dgtxyzvec[[1]]*c^2, Dgtxyzvec[[2]], {0, 0, 0, 0}, DLxvec, Dxboost/c, {0, 0, 0, 0}, DgLxvec, Dgxboost, {0, 0, 0, 0}, Duu, Dtvecnorm}}]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
\rho\,n[t,x,y,z]\,W^{(1,0,0,0)}[t,x,y,z]+O[\tfrac{1}{c}]^2 \\
\rho\,n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+O[\tfrac{1}{c}]^2 \\
0 \\
\left(2\rho\,n[t,x,y,z]\,uz[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]+2\rho\,n[t,x,y,z]\,uy[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]+2\rho\,n[t,x,y,z]\,ux[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+\rho\,n[t,x,y,z]\,W^{(1,0,0,0)}[t,x,y,z]\right)+O[\tfrac{1}{c}]^2 \\
\rho\,n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+O[\tfrac{1}{c}]^2 \\
0 \\
\left(y\,\rho\,n[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]-z\,\rho\,n[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]\right)+O[\tfrac{1}{c}]^2 \\
t\,\rho\,n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+O[\tfrac{1}{c}]^2 \\
0 \\
\left(y\,\rho\,n[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]-z\,\rho\,n[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]\right)+O[\tfrac{1}{c}]^2 \\
-t\,\rho\,n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+O[\tfrac{1}{c}]^2 \\
0 \\
\left(\frac{sxz\,jx^{(0,0,0,1)}[t,x,y,z]}{2\,n[t,x,y,z]}+\frac{szx\,jx^{(0,0,0,1)}[t,x,y,z]}{2\,n[t,x,y,z]}-\frac{\rho\,jx[t,x,y,z]\,uz[t,x,y,z]\,jx^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}+\rho\,ux[t,x,y,z]\,uz[t,x,y,z]\,jx^{(0,0,0,1)}[t,x,y,z]+\frac{syz\,jy^{(0,0,0,1)}[t,x,y,z]}{2\,n[t,x,y,z]}+\frac{szy\,jy^{(0,0,0,1)}[t,x,y,z]}{2\,n[t,x,y,z]}-\frac{\rho\,jy[t,x,y,z]\,uz[t,x,y,z]\,jy^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}+\rho\,uy[t,x,y,z]\,uz[t,x,y,z]\,jy^{(0,0,0,1)}[t,x,y,z]+\frac{szz\,jz^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}-\frac{\rho\,jz[t,x,y,z]\,uz[t,x,y,z]\,jz^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}+\rho\,uz[t,x,y,z]^2\,jz^{(0,0,0,1)}\right. \\
\left(-\rho\,n[t,x,y,z]\,uz[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]-\rho\,n[t,x,y,z]\,uy[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]-\rho\,n[t,x,y,z]\,ux[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]\right)+O[\tfrac{1}{c}]^2
\end{pmatrix}$$

In[∘]:=
```
(* covariant derivatives of coordinate 4-vectors (equivalent to Christoffel symbols), for later use *)
(Dtxyzvec = Table[Assuming[assut, Expand//@FS@PowerExpand[(D[IdentityMatrix[4][[aa]], {coords}] + Sum[IdentityMatrix[4][[aa]][[ii]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]], {aa, 1, 4}]);
```

In[∘]:=
```
(* normalized coordinate-t 4-vector*)
tvecnorm = Assuming[assut, Expand//@FS@PowerExpand[c*{1, 0, 0, 0}/Sqrt[-gg[[1, 1]]]]]
```

Out[∘]= $\left\{1+\frac{W}{c^2}+O\left[\frac{1}{c}\right]^4, 0, 0, 0\right\}$

In[∘]:=
```
(* and its covariant derivative *)
(Dtvecnorm = Assuming[assut, Expand//@FS@PowerExpand[(D[Normal@tW[tvecnorm], {coords}] + Sum[tW[tvecnorm][[ii]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]) // MF
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 \\
-W^{(0,1,0,0)}[t,x,y,z]+O[\tfrac{1}{c}]^2 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4 & 0 & 0 \\
-W^{(0,0,1,0)}[t,x,y,z]+O[\tfrac{1}{c}]^2 & 0 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4 & 0 \\
-W^{(0,0,0,1)}[t,x,y,z]+O[\tfrac{1}{c}]^2 & 0 & 0 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4
\end{pmatrix}$$

In[∘]:=
```
(* "raised" coordinate 4-covectors *)
(gtxyzvec = Assuming[assut, Expand//@FS@PowerExpand[igg.IdentityMatrix[4]]]) // MF
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
-(\tfrac{1}{c})^2-\frac{2W}{c^4}+O[\tfrac{1}{c}]^6 & 0 & 0 & 0 \\
0 & 1-\frac{2W}{c^2}+O[\tfrac{1}{c}]^4 & 0 & 0 \\
0 & 0 & 1-\frac{2W}{c^2}+O[\tfrac{1}{c}]^4 & 0 \\
0 & 0 & 0 & 1-\frac{2W}{c^2}+O[\tfrac{1}{c}]^4
\end{pmatrix}$$

In[∘]:=
```
(* and their covariant derivatives *)
(Dgtxyzvec = Table[Assuming[assut, Expand//@FS@PowerExpand[(D[Normal@tW[igg[[aa]]], {coords}] + Sum[tW[igg[[aa]][[ii]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]], {aa, 1, 4}]);
```

In[∘]:=
```
(* x-component of rot vector *)
Lxvec = Assuming[assut, Expand//@FS@PowerExpand[{0, 0, -z, y}]]
```

Out[∘]= {0, 0, -z, y}

In[∘]:=
```
(* and its covariant derivative *)
showf[assut][DLxvec = Assuming[assut, Expand//@FS@PowerExpand[(D[Normal@tW[Lxvec], {coords}] + Sum[tW[Lxvec][[ii]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix}
\frac{-y\,W^{(0,0,0,1)}[t,x,y,z]+z\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4 & 0 & -\frac{z\,W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\tfrac{1}{c}]^6 & \frac{y\,W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\tfrac{1}{c}]^6 \\
0 & \frac{y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4 & \frac{z\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4 & -\frac{y\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4 \\
-\frac{z\,W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4 & -\frac{z\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4 & \frac{y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4 & -1+\frac{-z\,W^{(0,0,0,1)}[t,x,y,z]-y\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4 \\
\frac{y\,W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4 & \frac{y\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4 & 1+\frac{z\,W^{(0,0,0,1)}[t,x,y,z]+y\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4 & \frac{y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4
\end{pmatrix}$$

In[∘]:=
```
(* "raised" x-component of rot co-vector *)
gLxvec = Assuming[assut, Expand//@FS@PowerExpand[igg.{0, 0, -z, y}]]
```

Out[∘]= $\left\{0, 0, -z+\frac{2Wz}{c^2}+O\left[\frac{1}{c}\right]^4, y-\frac{2(Wy)}{c^2}+O\left[\frac{1}{c}\right]^4\right\}$

In[•]:=
```
(* and its covariant derivative *)
showf[assut][DgLxvec = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[gLxvec], {coords}] + Sum[tW[gLxvec]⟦ii⟧ * cc⟦ ;; , ;; , ii⟧, {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{-y W^{(0,0,0,1)}[t,x,y,z]+z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 & -\frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 & \frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 \\ 0 & \frac{y W^{(0,0,0,1)}[t,x,y,z]-z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ \frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{y W^{(0,0,0,1)}[t,x,y,z]+z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -1+\frac{2 W[t,x,y,z]+z W^{(0,0,0,1)}[t,x,y,z]-y W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ -\frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 1+\frac{-2 W[t,x,y,z]+z W^{(0,0,0,1)}[t,x,y,z]-y W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{-y W^{(0,0,0,1)}[t,x,y,z]-z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[•]:=
```
(* x-component of boost vector *)
xboost = Assuming[assut, Expand //@ FS@PowerExpand[{x / c, t * c, 0, 0}]]
```

Out[•]= $\left\{\frac{x}{c}, c\,t, 0, 0\right\}$

In[•]:=
```
(* and its covariant derivative *)
showf[assut][Dxboost = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[xboost], {coords}] + Sum[tW[xboost]⟦ii⟧ * cc⟦ ;; , ;; , ii⟧, {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -\frac{t W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{1}{c}+\frac{-x W^{(0,1,0,0)}[t,x,y,z]+t W^{(1,0,0,0)}[t,x,y,z]}{c^3}+O[\frac{1}{c}]^5 & -\frac{x W^{(0,0,1,0)}[t,x,y,z]}{c^3}+O[\frac{1}{c}]^5 & -\frac{x W^{(0,0,0,1)}[t,x,y,z]}{c^3}+O[\frac{1}{c}]^5 \\ c+\frac{-x W^{(0,1,0,0)}[t,x,y,z]+t W^{(1,0,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{t W^{(0,0,1,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{t W^{(0,0,0,1)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 \\ -\frac{x W^{(0,0,1,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & -\frac{t W^{(0,0,1,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & 0 \\ -\frac{x W^{(0,0,0,1)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & -\frac{t W^{(0,0,0,1)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & 0 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 \end{pmatrix}$$

In[•]:=
```
(* "raised" x-component of boost co-vector *)
gxboost = Assuming[assut, Expand //@ FS@PowerExpand[igg.{x, -t, 0, 0}]]
```

Out[•]= $\left\{-\frac{x}{c^2}-\frac{2 (W x)}{c^4}+O[\frac{1}{c}]^6, -t+\frac{2 t W}{c^2}+O[\frac{1}{c}]^4, 0, 0\right\}$

In[•]:=
```
(* and its covariant derivative *)
showf[assut][Dgxboost = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[gxboost], {coords}] + Sum[tW[gxboost]⟦ii⟧ * cc⟦ ;; , ;; , ii⟧, {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -(\frac{1}{c})^2+\frac{-2 W[t,x,y,z]-x W^{(0,1,0,0)}[t,x,y,z]-t W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 & -\frac{x W^{(0,0,1,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 & -\frac{x W^{(0,0,0,1)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 \\ -1+\frac{2 W[t,x,y,z]+x W^{(0,1,0,0)}[t,x,y,z]+t W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t W^{(0,0,0,1)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ \frac{x W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{t W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 \\ \frac{x W^{(0,0,0,1)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t W^{(0,0,0,1)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 & -\frac{t W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[•]:=
```
(* content and flux of coordinatevector-energy and coordinatevector-momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, yzVxsurface / (Δt)}.EPS]]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n \rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2 n}+n(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2+\left(-\frac{Ax(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz)+Ay(n\,qy+jx\,syx+jy\,syy+jz\,syz)+Az(n\,qz+jx\,szx+jy\,szy+jz\,szz)}{n}+(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\epsilon-\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}\right)+O[\frac{1}{c}]^2 \\ jx\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jy\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+\frac{jy(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jz\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+\frac{jz(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords⟦aa,;;,;;⟧].gg+Dcoords⟦aa,;;,;;⟧).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[Dtxyzvec⟦aa, ;; , ;;⟧.TTx]], {aa, 1, 4}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \rho\, n[t, x, y, z]\, W^{(1,0,0,0)}[t, x, y, z]+O[\frac{1}{c}]^2 \\ \rho\, n[t, x, y, z]\, W^{(0,1,0,0)}[t, x, y, z]+O[\frac{1}{c}]^2 \\ \rho\, n[t, x, y, z]\, W^{(0,0,1,0)}[t, x, y, z]+O[\frac{1}{c}]^2 \\ \rho\, n[t, x, y, z]\, W^{(0,0,0,1)}[t, x, y, z]+O[\frac{1}{c}]^2 \end{pmatrix}$$

```
(* content and flux of raised coordinatecovector-energy and coordinatecovector-momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, yzVxsurface / (Δt)}.EPS]]]]
```

In[•]:=
```
(* content and flux of coord-energy and momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxEPS = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, yzVxsurface / (Δt)}.EPS]]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n \rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2 n}+n(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2+\left(-\frac{Ax(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz)+Ay(n\,qy+jx\,syx+jy\,syy+jz\,syz)+Az(n\,qz+jx\,szx+jy\,szy+jz\,szz)}{n}+(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\epsilon-\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}\right)+O[\frac{1}{c}]^2 \\ jx\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jy\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+\frac{jy(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jz\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+\frac{jz(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* content and flux of coord-energy and momentum for dust (TRANSPOSED) *)
shows[assut, 1][T[fluxdust = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, yzVxsurface / (Δt)}.dust2]]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-\frac{1}{2}n\left(2\epsilon+(aux^2+auy^2+auz^2-2W)\rho\right)+O[\frac{1}{c}]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2-\frac{1}{2}\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)\left(2\epsilon+(aux^2+auy^2+auz^2-2W)\rho\right)+O[\frac{1}{c}]^2 \\ aux\,n\,\rho+O[\frac{1}{c}]^2 & aux\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)\rho+O[\frac{1}{c}]^2 \\ auy\,n\,\rho+O[\frac{1}{c}]^2 & auy\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)\rho+O[\frac{1}{c}]^2 \\ auz\,n\,\rho+O[\frac{1}{c}]^2 & auz\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)\rho+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
shows[assut, 1][T[fluxEPS /. replaceJu]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-\frac{1}{2}n\left(2\epsilon+(ux^2+uy^2+uz^2-2W)\rho\right)+O[\frac{1}{c}]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac{1}{2}n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)(ux^2+uy^2+uz^2-2W)\rho\right)+O[\frac{1}{c}]^2 \\ n\,ux\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O[\frac{1}{c}]^2 \\ n\,uy\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+n\,uy\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O[\frac{1}{c}]^2 \\ n\,uz\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+n\,uz\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* momentum flux = A.σ + P A.(u-V)*)
fluxPS = ({Ax, Ay, Az}.(S⟦2 ;; 4, 2 ;; 4⟧).{1, 0, 0} + EPS⟦1, 2⟧ * ({Ax, Ay, Az}.({jx, jy, jz} / n - {Vx, Vy, Vz})))
```

Out[•]= $\left(Ax\,sxx+Ay\,syx+Az\,szx+jx\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\rho\right)+\frac{1}{c^2}\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\left(px+\frac{jx\,sxx}{n}+\frac{jy\,syx}{n}+\frac{jz\,szx}{n}+jx\,\epsilon+\frac{jx^3\rho}{2n^2}+\frac{jx\,jy^2\rho}{2n^2}+\frac{jx\,jz^2\rho}{2n^2}+3\,jx\,W\rho\right)+O[\frac{1}{c}]^4$

In[•]:=
```
shows[assut, 1][Expand //@ FS@PowerExpand[fluxEPS⟦2, 2⟧ - fluxPS]]
```

Out[•]//MatrixForm=

$O[\frac{1}{c}]^2$

In[•]:=
```
(* energy flux = A.q + A.σ.u + E A.(u-V)*)
fluxE = -({Ax, Ay, Az}.{qx, qy, qz} + {Ax, Ay, Az}.(S⟦2 ;; 4, 2 ;; 4⟧ + Qtens⟦2 ;; 4, 2 ;; 4⟧).{jx, jy, jz} / n + (-EPS⟦1, 1⟧) * {Ax, Ay, Az}.({jx, jy, jz} / n - {Vx, Vy, Vz}))
```

Out[•]= $-n\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz-\frac{1}{n}\left(jx(Ax\,sxx+Ay\,syx+Az\,szx)+jy(Ax\,sxy+Ay\,syy+Az\,szy)+jz(Ax\,sxz+Ay\,syz+Az\,szz)\right)-\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\left(n\,\epsilon+\frac{jx^2\rho}{2n}+\frac{jy^2\rho}{2n}+\frac{jz^2\rho}{2n}-n\,W\rho\right)\right)+O[\frac{1}{c}]^2$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[fluxEPS⟦2, 1⟧ - fluxE]]
```

Out[•]//MatrixForm=

$O[\frac{1}{c}]^2$

In[•]:=
```
(* matter flux n A.(u-V) *)
shows[assut, 1][fluxNJ = Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, yzVxsurface / (Δt)}.NJ /. replaceJu]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} n \\ n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right) \end{pmatrix}$$

```
In[•]:= (* content and flux of coord-energy and momentum assuming no matter flux (transposed) *)
shows[Join[assut, {((yzVxsurface/Δt).NJ) == 0} /. replaceJu], 1][T@fluxEPS /. replaceJu]
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & \left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz\right)\right)+O\left[\frac{1}{c}\right]^2\\
n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx\right)+O\left[\frac{1}{c}\right]^2\\
n\,uy\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy\right)+O\left[\frac{1}{c}\right]^2\\
n\,uz\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}
$$

```
In[•]:=
```

```
In[•]:= (* coordinate/internal/coordinate-proper energy and x-momentum, content and fluxes (TRANSPOSED) *)
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, yzVxsurface/(Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, Lxvec, Lxvec2, xboost/c, xboost2, uu, ntvec}]) /. replaceJu]]]
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac{1}{2}n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
-n\left(t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(t\,ux+x\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\left(-t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
-n\rho c^2-n\epsilon+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\epsilon\right)+O\left[\frac{1}{c}\right]^2\\
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2+uy^2+uz^2\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac{1}{2}n\left(ux^2+uy^2+uz^2\right)\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}
$$

```
In[•]:= show2[assut, 1][T[variousfluxes /. {Vy → 0, Vz → 0, uy → 0, uz → 0}]]
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & Ax\,n(-ux+Vx)\rho c^2+\left(-Ay\left(qy+syx\,ux\right)-Az\left(qz+szx\,ux\right)-Ax\left(qx+sxx\,ux+n(ux-Vx)\epsilon\right)-\frac{1}{2}Ax\,n(ux-Vx)\left(ux^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+Ax\,n\,ux(ux-Vx)\rho\right)+O\left[\frac{1}{c}\right]^2\\
O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z\right)+O\left[\frac{1}{c}\right]^2\\
O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z\right)+O\left[\frac{1}{c}\right]^2\\
-n\left(t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-Ax\,n(ux-Vx)\left(t\,ux+x\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\left(-t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-Ax\,n(ux-Vx)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
-n\rho c^2-n\epsilon+O\left[\frac{1}{c}\right]^2 & Ax\,n(-ux+Vx)\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+Ax\,n(-ux+Vx)\epsilon\right)+O\left[\frac{1}{c}\right]^2\\
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+ux^2\rho\right)+O\left[\frac{1}{c}\right]^2 & Ax\,n(-ux+Vx)\rho c^2+\left(-Ay\left(qy+syx\,ux\right)-Az\left(qz+szx\,ux\right)-Ax\left(qx+sxx\,ux+n(ux-Vx)\epsilon\right)-\frac{1}{2}Ax\,n\,ux^2(ux-Vx)\rho\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}
$$

```
In[•]:= (* velocity of energy *)
shows[assut, 5][(EPS.{1, 0, 0, 0})[[2 ;; 4]]/(EPS.{1, 0, 0, 0})[[1]] /. replaceJu]
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
ux+\frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\rho c^2}+O\left[\frac{1}{c}\right]^4\\
uy+\frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\rho c^2}+O\left[\frac{1}{c}\right]^4\\
uz+\frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\rho c^2}+O\left[\frac{1}{c}\right]^4
\end{pmatrix}
$$

```
In[•]:= temp = SeriesCoefficient[tt.{1, 0, 0, 0}, {c, Infinity, -2}];
shows[assut, 5][(tt.{1, 0, 0, 0} - temp*c^2)[[2 ;; 4]]/(tt.{1, 0, 0, 0} - temp*c^2)[[1]] /. j2v]
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
\left(ux+\frac{2\left(qx+sxx\,ux+sxy\,uy+sxz\,uz\right)}{2n\epsilon+n\left(ux^2+uy^2+uz^2-2W\right)\rho}\right)+O\left[\frac{1}{c}\right]^2\\
\left(uy+\frac{2\left(qy+syx\,ux+syy\,uy+syz\,uz\right)}{2n\epsilon+n\left(ux^2+uy^2+uz^2-2W\right)\rho}\right)+O\left[\frac{1}{c}\right]^2\\
\left(uz+\frac{2\left(qz+szx\,ux+szy\,uy+szz\,uz\right)}{2n\epsilon+n\left(ux^2+uy^2+uz^2-2W\right)\rho}\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}
$$

```
In[•]:= showf[assut][{variousfluxes[[ ;; , 1]] - variousfluxes[[ ;; , 7]], variousfluxes[[ ;; , 1]] - variousfluxes[[ ;; , 8]], variousfluxes[[ ;; , 7]] - variousfluxes[[ ;; , 8]]}]
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
\left(-\frac{3}{2}n\,ux^2\rho-\frac{1}{2}n\,uy^2\rho-\frac{1}{2}n\,uz^2\rho+n W\rho\right)+O\left[\frac{1}{c}\right]^2 & (-Ax\,sxx\,ux-Ay\,syx\,ux-Az\,szx\,ux-Ax\,sxy\,uy-Ay\,syy\,uy-Az\,szy\,uy-Ax\,sxz\,uz-Ay\,syz\,uz-Az\,szz\,uz-\frac{1}{2}Ax\,n\,ux^3\rho-\frac{1}{2}Ay\,n\,ux^2\,uy\,\rho-\frac{1}{2}Ax\,n\,ux\,uy^2\rho-\frac{1}{2}Ay\,n\,uy^3\rho-\frac{1}{2}Az\,n\,ux^2\,uz\,\rho-\frac{1}{2}Az\,n\,uy^2\,uz\,\rho-\frac{1}{2}Ax\,n\,ux\,uz^2\rho-\frac{1}{2}Ay\,n\,uy\,uz^2\rho-\frac{1}{2}Az\,n\,uz^3\rho+\frac{1}{2}Ax\,n\,ux^2\,Vx\,\rho+\frac{1}{2}Ax\,n\,uy^2\,Vx\,\rho+\frac{1}{2}Ax\,n\ldots\\
n W\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,n\,ux\,W\rho+Ay\,n\,uy\,W\rho+Az\,n\,uz\,W\rho-Ax\,n\,Vx\,W\rho-Ay\,n\,Vy\,W\rho-Az\,n\,Vz\,W\rho\right)+O\left[\frac{1}{c}\right]^2\\
\left(\frac{3}{2}n\,ux^2\rho+\frac{1}{2}n\,uy^2\rho+\frac{1}{2}n\,uz^2\rho\right)+O\left[\frac{1}{c}\right]^2 & (Ax\,sxx\,ux+Ay\,syx\,ux+Az\,szx\,ux+Ax\,sxy\,uy+Ay\,syy\,uy+Az\,szy\,uy+Ax\,sxz\,uz+Ay\,syz\,uz+Az\,szz\,uz+\frac{1}{2}Ax\,n\,ux^3\rho+\frac{1}{2}Ay\,n\,ux^2\,uy\,\rho+\frac{1}{2}Ax\,n\,ux\,uy^2\rho+\frac{1}{2}Ay\,n\,uy^3\rho+\frac{1}{2}Az\,n\,ux^2\,uz\,\rho+\frac{1}{2}Az\,n\,uy^2\,uz\,\rho+\frac{1}{2}Ax\,n\,ux\,uz^2\rho+\frac{1}{2}Ay\,n\,uy\,uz^2\rho+\frac{1}{2}Az\,n\,uz^3\rho-\frac{1}{2}Ax\,n\,ux^2\,Vx\,\rho-\frac{1}{2}Ax\,n\,uy^2\,Vx\,\rho-\frac{1}{2}Ax\,n\,u\ldots
\end{pmatrix}
$$

```
In[•]:= (* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg)/2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
show2[assut, 2][FS[(itjv[Tr[#.TTx]]) /. replaceJu]] & /@ {Dxyzvec[[1, ;; , ;;]], Dxyzvec[[2, ;; , ;;]], DLxvec, DLxvec2, Dxboost/c, Dxboost2, Duv, Dntvec} // MF
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
n\rho W^{(1,0,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\\
n\rho W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\\
n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\\
n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\\
-n\rho\left(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\\
-n\,t\,\rho W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\\
\frac{1}{2}\left(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z]+(syz+szy)\left(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\right)+2\left(syy\,uy^{(0,0,1,0)}[t,x,y,z]+sxx\,ux^{(0,1,0,0)}[t,x,y,z]\right)+(sxy+syx)\left(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z]\right)+(sxz+szx)\left(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\right)\right)+O\left[\frac{1}{c}\right]^2\\
-n\rho\left(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}
$$

```
In[•]:= shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, yzVxsurface/(Δt)}.EPSsym.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, Lxvec, Lxvec2, xboost/c, xboost2, uu, ntvec}]) /. replaceJu]]]
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+sxy\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+sxz\,ux+syz\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac{1}{2}n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,sxy+Az\,sxz+n\,ux\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,syz\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,syz\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
-n\left(t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,sxy+Az\,sxz\right)t\right)-n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(t\,ux+x\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\left(-t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,sxy+Az\,sxz\right)t\right)-n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
-n\rho c^2-n\epsilon+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\epsilon\right)+O\left[\frac{1}{c}\right]^2\\
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2+uy^2+uz^2\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+sxy\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+sxz\,ux+syz\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac{1}{2}n\left(ux^2+uy^2+uz^2\right)\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}
$$

```
In[•]:= TTx = tW[tjv[(EPSsym + T[EPSsym.Inverse[gg]].gg)/2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. replaceJu] & /@ {Dxyzvec[[1, ;; , ;;]], Dxyzvec[[2, ;; , ;;]], DLxvec, DLxvec2, Dxboost/c, Dxboost2, Duv, Dntvec} // MF
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
n\rho W^{(1,0,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\\
n\rho W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\\
n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\\
n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\\
-n\rho\left(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\\
-n\,t\,\rho W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\\
\left(szz\,uz^{(0,0,0,1)}[t,x,y,z]+syy\,uy^{(0,0,1,0)}[t,x,y,z]+syz\left(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\right)+sxx\,ux^{(0,1,0,0)}[t,x,y,z]+sxy\left(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z]\right)+sxz\left(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\right)\right)+O\left[\frac{1}{c}\right]^2\\
-n\rho\left(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}
$$

```
In[•]:= (* 2-vector of surface parallel to yz  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(yzsurface = ((T[{{0, 0, Ly, 0}}].{{0, 0, 0, Lz}} - T[{{0, 0, 0, Lz}}].{{0, 0, Ly, 0}}) /. {Ly → Ayz/Lz})) // MF
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
0 & 0 & 0 & 0\\
0 & 0 & 0 & 0\\
0 & 0 & 0 & Ayz\\
0 & 0 & -Ayz & 0
\end{pmatrix}
$$

```
In[•]:= (* 2-vector of surface parallel to tx  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(txsurface = (-(T[{{Δt, 0, 0, 0}}].{{0, Lx, 0, 0}} - T[{{0, Lx, 0, 0}}].{{Δt, 0, 0, 0}}))) // MF
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
0 & -Lx\,\Delta t & 0 & 0\\
Lx\,\Delta t & 0 & 0 & 0\\
0 & 0 & 0 & 0\\
0 & 0 & 0 & 0
\end{pmatrix}
$$

```
In[•]:= (* 2-vector of surface parallel to ty  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(tysurface = (-(T[{{Δt, 0, 0, 0}}].{{0, 0, Ly, 0}} - T[{{0, 0, Ly, 0}}].{{Δt, 0, 0, 0}}))) // MF
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
0 & 0 & -Ly\,\Delta t & 0\\
0 & 0 & 0 & 0\\
Ly\,\Delta t & 0 & 0 & 0\\
0 & 0 & 0 & 0
\end{pmatrix}
$$

```
In[•]:= (* 2-vector of surface parallel to y moving to x  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(yVxsurface = (-(T[{{1, Vx, 0, 0}}*Δt].{{0, 0, Ly, 0}} - T[{{0, 0, Ly, 0}}].{{1, Vx, 0, 0}}*Δt))) // MF
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
0 & 0 & -Ly\,\Delta t & 0\\
0 & 0 & -Ly\,Vx\,\Delta t & 0\\
Ly\,\Delta t & Ly\,Vx\,\Delta t & 0 & 0\\
0 & 0 & 0 & 0
\end{pmatrix}
$$

```
In[•]:= (Tr[T[txsurface].txsurface]) // MF
```

Out[•]//MatrixForm=

$$2\,Lx^2\,\Delta t^2$$

In[ ]:=
```
(* Faraday tensor *)
repE = {Ex → Ex*c*Sqrt[μ*ϵ], Ey → Ey*c*Sqrt[μ*ϵ], Ez → Ez*c*Sqrt[μ*ϵ]};
fftemp = {{0, -Ex, -Ey, -Ez}, {0, 0, Bz, -By}, {0, 0, 0, Bx}, {0, 0, 0, 0}} /. repE;
showf[assut][F = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & -c\,Ex\sqrt{\epsilon}\sqrt{\mu} & -c\,Ey\sqrt{\epsilon}\sqrt{\mu} & -c\,Ez\sqrt{\epsilon}\sqrt{\mu} \\ c\,Ex\sqrt{\epsilon}\sqrt{\mu} & 0 & Bz & -By \\ c\,Ey\sqrt{\epsilon}\sqrt{\mu} & -Bz & 0 & Bx \\ c\,Ez\sqrt{\epsilon}\sqrt{\mu} & By & -Bx & 0 \end{pmatrix}$$

In[ ]:=
```
(FS[{Tr[yzsurface.T[F]], Tr[T[txsurface].F], Tr[T[tysurface].F], Tr[T[yVxsurface].F]} / 2]) // MF
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} Ayz\,Bx \\ c\,Ex\,Lx\,\Delta t\sqrt{\epsilon}\sqrt{\mu} \\ c\,Ey\,Ly\,\Delta t\sqrt{\epsilon}\sqrt{\mu} \\ Ly\,\Delta t\left(-Bz\,Vx + c\,Ey\sqrt{\epsilon}\sqrt{\mu}\right) \end{pmatrix}$$

In[ ]:=
```
(* charge-current-potential tensor *)
fftemp = {{0, -Hx, -Hy, -Hz}, {0, 0, Dz, -Dy}, {0, 0, 0, Dx}, {0, 0, 0, 0}};
showf[assut][H = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & -Hx & -Hy & -Hz \\ Hx & 0 & Dz & -Dy \\ Hy & -Dz & 0 & Dx \\ Hz & Dy & -Dx & 0 \end{pmatrix}$$

In[ ]:=
```
showf[assut][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
  ]]]
```

Out[ ]//MatrixForm=

(··· 1 ···)

Full expression not available (original memory size: 0.7 MB)

In[ ]:=
```
shows[assut, 1][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
  ]]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & \frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{(-Bz\,Ex+Bx\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{(By\,Ex-Bx\,Ey)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ \frac{(-Bz\,Ey+By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(Bz\,Ey-By\,Ez)\,W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{-Bx^2+By^2+Bz^2+(-Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & -\frac{Bx\,By+Ex\,Ey\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & -\frac{Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 \\ \frac{(Bz\,Ex-Bx\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(-Bz\,Ex+Bx\,Ez)\,W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & -\frac{Bx\,By+Ex\,Ey\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & \frac{Bx^2-By^2+Bz^2+(Ex^2-Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & -\frac{By\,Bz+Ey\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 \\ \frac{(-By\,Ex+Bx\,Ey)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(By\,Ex-Bx\,Ey)\,W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & -\frac{Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & -\frac{By\,Bz+Ey\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & \frac{Bx^2+By^2-Bz^2+(Ex^2+Ey^2-Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[ ]:=
```
showf[assut][T[tte.Inverse[gg]].gg - tte]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^4 & O[\frac{1}{c}]^5 & O[\frac{1}{c}]^5 & O[\frac{1}{c}]^5 \\ O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}$$

In[ ]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.tte.T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```

Out[ ]//MatrixForm=

$$-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 \qquad \frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{(Ax\,Vx+Ay\,Vy+Az\,Vz)(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)}{2\,\mu0}+\frac{2(Az\,By\,Ex-Ay\,Bz\,Ex-Az\,Bx\,Ey+Ax\,Bz\,Ey+Ay\,Bx\,Ez-Ax\,By\,Ez)\,W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2$$

$$-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+\frac{(Bz\,Ey\,ux-By\,Ez\,ux-Bz\,Ex\,uy+Bx\,Ez\,uy+By\,Ex\,uz-Bx\,Ey\,uz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{Ax(-2\,Bx(By\,uy+Bz\,uz)+Bx^2(-ux+Vx)+\ldots}{2\,\mu0}\ldots$$

$$-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 \qquad \frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{(Ax\,Vx+Ay\,Vy+Az\,Vz)(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)}{2\,\mu0}+\frac{(Az\,By\,Ex-Az\,Bz\,Ex-Az\,Bx\,Ey+Ay\,Bz\,Ey+Ay\,Bx\,Ez-Ax\,By\,Ez)\,W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2$$

$$\frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \frac{-2(Ay\,Bx\,By+Az\,Bx\,Bz+Ay\,Ex\,Ey\,\epsilon0\,\mu0+\ldots)+Ax(-Bx^2+By^2+Bz^2+(-Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)}{2\,\mu0}-\frac{(Bz\,Ey-By\,Ez)(Ax\,Vx+Ay\,Vy+Az\,Vz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2$$

$$\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+(-2(Ax\,Ex+Ay\,Ey)\,Ez\,y+Az(Ex^2+Ey^2-Ez^2)\,y+2\,Ey(Ax\,Ex+Az\,Ez)\,z-Ay(Ex^2-Ey^2+Ez^2)\,z)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{(Ax\,Vx+Ay\,Vy+Az\,Vz)(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2$$

$$\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+(-2(Ax\,Ex+Ay\,Ey)\,Ez\,y+Az(Ex^2+Ey^2-Ez^2)\,y+2\,Ey(Ax\,Ex+Az\,Ez)\,z-Ay(Ex^2-Ey^2+Ez^2)\,z)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{(Ax\,Vx+Ay\,Vy+Az\,Vz)(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2$$

$$\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \frac{t(2(Ay\,Bx\,By+Az\,Bx\,Bz+Ay\,Ex\,Ey\,\epsilon0\,\mu0+Az\,Ex\,Ez\,\epsilon0\,\mu0)+Ax(Bx^2-By^2-Bz^2+(Ex^2-Ey^2-Ez^2)\,\epsilon0\,\mu0))}{2\,\mu0}+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\,x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2$$

$$\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \frac{t(2(Ay\,Bx\,By+Az\,Bx\,Bz+Ay\,Ex\,Ey\,\epsilon0\,\mu0+Az\,Ex\,Ez\,\epsilon0\,\mu0)+Ax(Bx^2-By^2-Bz^2+(Ex^2-Ey^2-Ez^2)\,\epsilon0\,\mu0))}{2\,\mu0}+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(Az\,By\,Ex-Ay\,Bz\,Ex-Az\,Bx\,Ey+Ax\,Bz\,Ey+Ay\,Bx\,Ez-Ax\,By\,Ez)\,x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2$$

In[ ]:=
```
TTx = tW[tjv[(tte + T[tte.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```

Out[ ]//MatrixForm=

$$\frac{(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)\,W^{(1,0,0,0)}[t,x,y,z]}{\mu0\,c^2}+O[\frac{1}{c}]^3$$

$$\frac{-2(Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0)\,ux^{(0,0,0,1)}[t,x,y,z]-2(By\,Bz+Ey\,Ez\,\epsilon0\,\mu0)\,uy^{(0,0,0,1)}[t,x,y,z]+Bx^2\,uz^{(0,0,0,1)}[t,x,y,z]+By^2\,uz^{(0,0,0,1)}[t,x,y,z]-Bz^2\,uz^{(0,0,0,1)}[t,x,y,z]+Ex^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ey^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-Ez^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-2\,Bx\,By\,ux^{(0,0,1,0)}[t,x,y,z]-2\,Ex\,Ey\,\epsilon0\,\mu0\,ux^{(0,0,1,0)}[t,\ldots}{2\,\mu0}$$

$$\frac{\sqrt{\epsilon0}\,((-By\,Ex+Bx\,Ey)\,W^{(0,0,0,1)}[t,x,y,z]+(Bz\,Ex-Bx\,Ez)\,W^{(0,0,1,0)}[t,x,y,z]+(-Bz\,Ey+By\,Ez)\,W^{(0,1,0,0)}[t,x,y,z])}{\sqrt{\mu0}\,c}+\frac{(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)\,W^{(1,0,0,0)}[t,x,y,z]}{2\,\mu0\,c^2}+O[\frac{1}{c}]^3$$

$$\frac{(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)\,W^{(0,1,0,0)}[t,x,y,z]}{\mu0\,c^2}+O[\frac{1}{c}]^3$$

$$\frac{(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z])}{\mu0\,c^2}+O[\frac{1}{c}]^3$$

$$\frac{2((Bz^2\,y-By\,Bz\,z+Ez(Ez\,y-Ey\,z)\,\epsilon0\,\mu0)\,W^{(0,0,0,1)}[t,x,y,z]+(By\,Bz\,y-By^2\,z+Ey(Ez\,y-Ey\,z)\,\epsilon0\,\mu0)\,W^{(0,0,1,0)}[t,x,y,z]+(Bx(Bz\,y-By\,z)+Ex(Ez\,y-Ey\,z)\,\epsilon0\,\mu0)\,W^{(0,1,0,0)}[t,x,y,z])}{\mu0\,c^2}+O[\frac{1}{c}]^3$$

$$\frac{2(-Bz\,Ey+By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}-\frac{t(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)\,W^{(0,1,0,0)}[t,x,y,z]}{\mu0\,c^2}+O[\frac{1}{c}]^3$$

$$-\frac{2(t((Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0)\,W^{(0,0,0,1)}[t,x,y,z]+(Bx\,By+Ex\,Ey\,\epsilon0\,\mu0)\,W^{(0,0,1,0)}[t,x,y,z])+(Bx^2+Ex^2\,\epsilon0\,\mu0)\,W^{(0,1,0,0)}[t,x,y,z])}{\mu0\,c^2}+O[\frac{1}{c}]^3$$

In[ ]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.(tte + ttsym).T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```

Out[ ]//MatrixForm=

$$-n\,\rho\,c^2+\left(-n\,\epsilon-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{1}{2}\,n(ux^2+uy^2+uz^2-2\,W)\,\rho\right)+O[\frac{1}{c}]^2 \qquad n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\,\rho\,c^2+\frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\ldots$$

$$-n\,\rho\,c^2-\frac{Bx^2+By^2+Bz^2+2\,n\,\epsilon\,\mu0\,\rho+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+\frac{(Bz\,Ey\,ux-By\,Ez\,ux-Bz\,Ex\,uy+Bx\,Ez\,uy+By\,Ex\,uz-Bx\,Ey\,uz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\,\rho\,c^2+\ldots$$

$$-n\,\rho\,c^2+\left(-n\,\epsilon-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{1}{2}\,n(ux^2+uy^2+uz^2)\,\rho\right)+O[\frac{1}{c}]^2 \qquad n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\,\rho\,c^2+\ldots$$

$$n\,ux\,\rho+\frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \ldots-\frac{(Bz\,Ey-By\,Ez)(Ax\,Vx+Ay\,Vy+Az\,Vz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2$$

$$n(uz\,y-uy\,z)\,\rho+\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+2\,Ax\,sxz\,y\,\mu0+2\,Ay\,syz\,y\,\mu0+2\,Az\,szz\,y\,\mu0-2\,Ax\,sxy\,z\,\mu0\ldots}{2\,\mu0}$$

$$n(uz\,y-uy\,z)\,\rho+\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+2\,Ax\,sxz\,y\,\mu0+2\,Ay\,syz\,y\,\mu0+2\,Az\,szz\,y\,\mu0-2\,Ax\,sxy\,z\,\mu0\ldots}{2\,\mu0}$$

$$-n(t\,ux+x)\,\rho+\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \left(\frac{t(Ax\,Bx^2+2\,Ay\,Bx\,By-Ax\,By^2+2\,Az\,Bx\,Bz-Ax\,Bz^2-(2\,Ax\,sxx+2\,Ay\,sxy+2\,Az\,sxz-2\,Ex(Ay\,Ey+Az\,Ez)\,\epsilon0+Ax(-Ex^2+Ey^2+Ez^2)\,\epsilon0)\,\mu0)}{2\,\mu0}-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux+x)\,\rho\right)+\ldots+O[\frac{1}{c}]^2$$

$$n(-t\,ux+x)\,\rho+\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \left(\frac{t(Ax\,Bx^2+2\,Ay\,Bx\,By-Ax\,By^2+2\,Az\,Bx\,Bz-Ax\,Bz^2-(2\,Ax\,sxx+2\,Ay\,sxy+2\,Az\,sxz-2\,Ex(Ay\,Ey+Az\,Ez)\,\epsilon0+Ax(-Ex^2+Ey^2+Ez^2)\,\epsilon0)\,\mu0)}{2\,\mu0}-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux-x)\,\rho\right)+\ldots+O[\frac{1}{c}]^2$$

In[ ]:=
```
TTx = tW[tjv[((tte + ttsym) + T[(tte + ttsym).Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```

Out[ ]//MatrixForm=

$$n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2$$

$$\frac{-2(Bx\,Bz-sxz\,\mu0+Ex\,Ez\,\epsilon0\,\mu0)\,ux^{(0,0,0,1)}[t,x,y,z]-2(By\,Bz-syz\,\mu0+Ey\,Ez\,\epsilon0\,\mu0)\,uy^{(0,0,0,1)}[t,x,y,z]+Bx^2\,uz^{(0,0,0,1)}[t,x,y,z]+By^2\,uz^{(0,0,0,1)}[t,x,y,z]-Bz^2\,uz^{(0,0,0,1)}[t,x,y,z]+2\,szz\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ex^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ey^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-Ez^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-2\,Bx\,By\ldots}{2\,\mu0}$$

$$-n\,\rho\left(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z]\right)+\frac{\sqrt{\epsilon0}\,((-By\,Ex+Bx\,Ey)\,W^{(0,0,0,1)}[t,x,y,z]+(Bz\,Ex-Bx\,Ez)\,W^{(0,0,1,0)}[t,x,y,z]+(-Bz\,Ey+By\,Ez)\,W^{(0,1,0,0)}[t,x,y,z])}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2$$

$$n\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2$$

$$n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O[\frac{1}{c}]^2$$

$$n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O[\frac{1}{c}]^2$$

$$-n\,\rho\left(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z]\right)+\frac{2(-Bz\,Ey+By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2$$

$$-n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2$$

In[ ]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.(tt + tte).T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```

Out[ ]//MatrixForm=

$$-n\,\rho\,c^2+\left(-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{1}{2}\,n(ux^2+uy^2+uz^2-2\,W+2\,\epsilon)\,\rho\right)+O[\frac{1}{c}]^2 \qquad n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\,\rho\,c^2+\ldots$$

$$-n\,\rho\,c^2-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0+2\,n\,\epsilon\,\mu0\,\rho}{2\,\mu0}+\frac{(Bz\,Ey\,ux-By\,Ez\,ux-Bz\,Ex\,uy+Bx\,Ez\,uy+By\,Ex\,uz-Bx\,Ey\,uz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\,\rho\,c^2+\ldots$$

$$-n\,\rho\,c^2+\left(-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{1}{2}\,n(ux^2+uy^2+uz^2+2\,\epsilon)\,\rho\right)+O[\frac{1}{c}]^2 \qquad n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\,\rho\,c^2+\ldots$$

$$n\,ux\,\rho+\frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \ldots-\frac{(Bz\,Ey-By\,Ez)(Ax\,Vx+Ay\,Vy+Az\,Vz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2$$

$$n(uz\,y-uy\,z)\,\rho+\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \ldots$$

$$n(uz\,y-uy\,z)\,\rho+\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \ldots$$

$$-n(t\,ux+x)\,\rho+\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \left(\frac{t(Ax\,Bx^2+2\,Ay\,Bx\,By-Ax\,By^2+2\,Az\,Bx\,Bz-Ax\,Bz^2-(2\,Ax\,sxx+2\,Ay\,sxy+2\,Az\,sxz-2\,Ex(Ay\,Ey+Az\,Ez)\,\epsilon0+Ax(-Ex^2+Ey^2+Ez^2)\,\epsilon0)\,\mu0)}{2\,\mu0}-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux+x)\,\rho\right)+\ldots+O[\frac{1}{c}]^2$$

$$n(-t\,ux+x)\,\rho+\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \qquad \left(\frac{t(Ax\,Bx^2+2\,Ay\,Bx\,By-Ax\,By^2+2\,Az\,Bx\,Bz-Ax\,Bz^2-(2\,Ax\,sxx+2\,Ay\,sxy+2\,Az\,sxz-2\,Ex(Ay\,Ey+Az\,Ez)\,\epsilon0+Ax(-Ex^2+Ey^2+Ez^2)\,\epsilon0)\,\mu0)}{2\,\mu0}-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux-x)\,\rho\right)+\ldots+O[\frac{1}{c}]^2$$

In[•]:=
```
TTx = tW[tjv[((tt + tte) + T[(tt + tte).Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 4][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```
Out[•]//MatrixForm=

$$\begin{pmatrix} n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\ \left(-2\,Bx\,Bz+\left(sxz+szx-2\,Ex\,Ez\,\epsilon0\right)\mu0\right)ux^{(0,0,0,1)}[t,x,y,z]+\left(-2\,By\,Bz+\left(syz+szy-2\,Ey\,Ez\,\epsilon0\right)\mu0\right)uy^{(0,0,0,1)}[t,x,y,z]+Bx^2\,uz^{(0,0,0,1)}[t,x,y,z]+By^2\,uz^{(0,0,0,1)}[t,x,y,z]-Bz^2\,uz^{(0,0,0,1)}[t,x,y,z]+2\,szz\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ex^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ey^2\,\epsilon0\,\mu0\,u\cdots \\ -n\,\rho\left(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z]\right)+\frac{\sqrt{\epsilon0}\left(\left(-By\,Ex+Bx\,Ey\right)W^{(0,0,0,1)}[t,x,y,z]+\left(Bz\,Ex-Bx\,Ez\right)W^{(0,0,1,0)}[t,x,y,z]+\left(-Bz\,Ey+By\,Ez\right)W^{(0,1,0,0)}[t,x,y,z]\right)}{\sqrt{\mu0}\,c}+O\left[\frac{1}{c}\right]^2 \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,\rho\left(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z]\right)+\frac{2\left(-Bz\,Ey+By\,Ez\right)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O\left[\frac{1}{c}\right]^2 \\ -n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

```
(* Flux of energy is given by: *)
```

In[•]:=
```
shows[assut, 1][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.tt /. {Ax → 0, Ay → 0, Vz → 0, jz → 0} /. j2v]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(ux^2+uy^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & n\,uy\,\rho+O\left[\frac{1}{c}\right]^2 & O\left[\frac{1}{c}\right]^2 \\ -Az\left(qz+sxz\,ux+syz\,uy\right)+O\left[\frac{1}{c}\right]^2 & Az\,sxz+O\left[\frac{1}{c}\right]^2 & Az\,syz+O\left[\frac{1}{c}\right]^2 & Az\,szz+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
shows[assut, 1][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → 0, Vx → 0} /. j2v]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(uy^2+uz^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & O\left[\frac{1}{c}\right]^2 & n\,uy\,\rho+O\left[\frac{1}{c}\right]^2 & n\,uz\,\rho+O\left[\frac{1}{c}\right]^2 \\ \left(-qx-sxy\,uy-sxz\,uz\right)+O\left[\frac{1}{c}\right]^2 & sxx+O\left[\frac{1}{c}\right]^2 & sxy+O\left[\frac{1}{c}\right]^2 & sxz+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(ux^2+uy^2+uz^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & n\,ux\,\rho+\frac{qx+sxx\,ux+sxy\,uy+sxz\,uz+\frac{1}{2}\,n\left(-8\,Wx+ux\left(ux^2+uy^2+uz^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & n\,uy\,\rho+\frac{qy+sxy\,ux+syy\,uy+syz\,uz+\frac{1}{2}\,n\left(-8\,Wy+uy\left(ux^2+uy^2+uz^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & n\,uz\,\rho+\frac{qz+sxz\,ux+syz\,uy+szz\,uz+\frac{1}{2}\,n\left(-8\,Wz+uz\left(ux^2+uy^2+uz^2+6\,W+2\,\epsilon\right)\right)\rho\cdots}{c^2}\cdots \\ n\left(-ux+Vx\right)\rho\,c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\left(ux-Vx\right)\left(ux^2+uy^2+uz^2-2\,W+2\,\epsilon\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & \left(sxx+n\,ux\left(ux-Vx\right)\rho\right)+\frac{qx\left(2\,ux-Vx\right)-\left(sxx\,ux+sxy\,uy+sxz\,uz\right)Vx+\frac{1}{2}\,n\left(ux-Vx\right)\left(ux^3-8\,Wx+ux\left(uy^2+uz^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \left(sxy+n\,uy\left(ux-Vx\right)\rho\right)+\frac{qx\,uy+qy\left(ux-Vx\right)-\left(sxy\,ux+syy\,uy+syz\,uz\right)Vx+\frac{1}{2}\,n\left(ux-Vx\right)\left(-8\,Wy+uy\left(ux^2+uy^2+uz^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \left(sxz+n\,uz\left(ux-Vx\right)\rho\right)+\frac{qx\,uz+qz\left(ux-Vx\right)-\left(sxz\,ux+syz\,uy+szz\,uz\right)\cdots}{c^2}\cdots \end{pmatrix}$$

In[•]:=
```
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v /. {sxx → SXX - n * ux * (ux - Vx) * ρ, sxy → SXY - n * uy * (ux - Vx) * ρ, sxz → SXZ - n * uz * (ux - Vx) * ρ}]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(ux^2+uy^2+uz^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & n\,ux\,\rho+\frac{qx+SXX\,ux+SXY\,uy+SXZ\,uz-\frac{1}{2}\,n\left(ux^3-2\,ux^2\,Vx-2\left(uy^2+uz^2\right)Vx+8\,Wx+ux\left(uy^2+uz^2-6\,W-2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & n\,uy\,\rho+\frac{qy+SXY\,ux+syy\,uy+syz\,uz+\frac{1}{2}\,n\left(-8\,Wy+uy\left(-ux^2+uy^2+uz^2+2\,ux\,Vx+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & n\,uz\,\rho+\frac{qz+SXZ\,ux+syz\,uy+szz\,uz+\frac{1}{2}\,n\left(-8\,Wz+uz\left(-ux^2+uy^2+uz^2+2\,ux\,Vx+6\,W+2\,\epsilon\right)\right)\rho\cdots}{c^2}\cdots \\ n\left(-ux+Vx\right)\rho\,c^2+\left(-qx-SXX\,ux-SXY\,uy-SXZ\,uz+\frac{1}{2}\,n\left(ux-Vx\right)\left(ux^2+uy^2+uz^2+2\,W-2\,\epsilon\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & SXX+\frac{qx\left(2\,ux-Vx\right)-\left(SXX\,ux+SXY\,uy+SXZ\,uz\right)Vx+\frac{1}{2}\,n\left(ux-Vx\right)\left(ux^3+2\,ux^2\,Vx+2\left(uy^2+uz^2\right)Vx-8\,Wx+ux\left(uy^2+uz^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & SXY+\frac{qx\,uy+qy\left(ux-Vx\right)-\left(SXY\,ux+syy\,uy+syz\,uz\right)Vx+\frac{1}{2}\,n\left(ux-Vx\right)\left(-8\,Wy+uy\left(-ux^2+uy^2+uz^2+2\,ux\,Vx+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & SXZ+\frac{qx\,uz+qz\left(ux-Vx\right)-\left(SXZ\,ux+syz\,uy+szz\,uz\right)Vx+\frac{1}{2}\,n\left(ux-Vx\right)\left(-8\,Wz+uz\left(-ux^2+uy^2+uz^2+2\,ux\cdots\right.\right.}{c^2}\cdots \end{pmatrix}$$

In[•]:=
```
(* matter flux in same direction as imaginary moving surface, different velocity *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jy → 0, jz → 0} /. j2v]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(ux^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & n\,ux\,\rho+\frac{qx+sxx\,ux+\frac{1}{2}\,n\left(ux^3+6\,ux\,W-8\,Wx+2\,ux\,\epsilon\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \frac{qy+sxy\,ux-4\,n\,Wy\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \frac{qz+sxz\,ux-4\,n\,Wz\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 \\ n\left(-ux+Vx\right)\rho\,c^2+\left(-qx-sxx\,ux-\frac{1}{2}\,n\left(ux-Vx\right)\left(ux^2-2\,W+2\,\epsilon\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & \left(sxx+n\,ux\left(ux-Vx\right)\rho\right)+\frac{2\,qx\,ux-\left(qx+sxx\,ux\right)Vx+\frac{1}{2}\,n\left(ux-Vx\right)\left(ux^3-8\,Wx+2\,ux\left(3\,W+\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & sxy+\frac{qy\left(ux-Vx\right)-sxy\,ux\,Vx+4\,n\left(-ux+Vx\right)Wy\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & sxz+\frac{qz\left(ux-Vx\right)-sxz\,ux\,Vx+4\,n\left(-ux+Vx\right)Wz\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:=
```
(* imaginary moving surface, no matter flux through it *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v /. {ux → Vx}]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(uy^2+uz^2+Vx^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & n\,Vx\,\rho+\frac{qx+sxy\,uy+sxz\,uz+sxx\,Vx+\frac{1}{2}\,n\left(-8\,Wx+Vx\left(uy^2+uz^2+Vx^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & n\,uy\,\rho+\frac{qy+syy\,uy+syz\,uz+sxy\,Vx+\frac{1}{2}\,n\left(-8\,Wy+uy\left(uy^2+uz^2+Vx^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & n\,uz\,\rho+\frac{qz+syz\,uy+szz\,uz+sxz\,Vx+\frac{1}{2}\,n\left(-8\,Wz+uz\left(uy^2+uz^2+Vx^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 \\ \left(-qx-sxy\,uy-sxz\,uz-sxx\,Vx\right)+O\left[\frac{1}{c}\right]^2 & sxx-\frac{Vx\left(-qx+sxy\,uy+sxz\,uz+sxx\,Vx\right)}{c^2}+O\left[\frac{1}{c}\right]^3 & sxy+\frac{qx\,uy-Vx\left(syy\,uy+syz\,uz+sxy\,Vx\right)}{c^2}+O\left[\frac{1}{c}\right]^3 & sxz+\frac{qx\,uz-Vx\left(syz\,uy+szz\,uz+sxz\,Vx\right)}{c^2}+O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

```
(* imaginary moving surface, no matter flux through it and no transversal matter motion *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jy → 0, jz → 0} /. j2v /. {ux → Vx}]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(Vx^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & n\,Vx\,\rho+\frac{qx+sxx\,Vx+\frac{1}{2}\,n\left(Vx^3+6\,Vx\,W-8\,Wx+2\,Vx\,\epsilon\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \frac{qy+sxy\,Vx-4\,n\,Wy\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \frac{qz+sxz\,Vx-4\,n\,Wz\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 \\ \left(-qx-sxx\,Vx\right)+O\left[\frac{1}{c}\right]^2 & sxx+\frac{Vx\left(qx-sxx\,Vx\right)}{c^2}+O\left[\frac{1}{c}\right]^3 & sxy-\frac{sxy\,Vx^2}{c^2}+O\left[\frac{1}{c}\right]^3 & sxz-\frac{sxz\,Vx^2}{c^2}+O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:=
```
(* imaginary moving surface, matter at rest in coordinates *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → 0, jy → 0, jz → 0}]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+n\left(W-\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & \frac{qx-4\,n\,Wx\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \frac{qy-4\,n\,Wy\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \frac{qz-4\,n\,Wz\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 \\ n\,Vx\,\rho\,c^2+\left(-qx+n\,Vx\left(-W+\epsilon\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & sxx+\frac{-qx\,Vx+4\,n\,Vx\,Wx\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & sxy+\frac{-qy\,Vx+4\,n\,Vx\,Wy\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & sxz+\frac{-qz\,Vx+4\,n\,Vx\,Wz\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → n * Vx, jy → 0, jz → 0}]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{1}{2}\,n\,Vx^2\,\rho+n\,W\,\rho-n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 & n\,Vx\,\rho+\frac{qx+sxx\,Vx+\frac{1}{2}\,n\,Vx^3\,\rho+3\,n\,Vx\,W\,\rho-4\,n\,Wx\,\rho+n\,Vx\,\epsilon\,\rho}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{qy+sxy\,Vx-4\,n\,Wy\,\rho}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{qz+sxz\,Vx-4\,n\,Wz\,\rho}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \left(-qx-sxx\,Vx\right)+O\left[\frac{1}{c}\right]^2 & sxx+\frac{qx\,Vx-sxx\,Vx^2}{c^2}+O\left[\frac{1}{c}\right]^4 & sxy-\frac{sxy\,Vx^2}{c^2}+O\left[\frac{1}{c}\right]^4 & sxz-\frac{sxz\,Vx^2}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[•]:=
```
(* COORDINATE ENERGY *)
```

In[•]:=
```
(* energy 3-form when projected along coord. axes *)
showf[assut][Expand //@ FS@PowerExpand[tt.{1, 0, 0, 0}]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{jx^2\,\rho}{2\,n}-\frac{jy^2\,\rho}{2\,n}-\frac{jz^2\,\rho}{2\,n}+n\,W\,\rho-n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -jx\,\rho\,c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\,\rho}{2\,n^2}-\frac{jx\,jy^2\,\rho}{2\,n^2}-\frac{jx\,jz^2\,\rho}{2\,n^2}+jx\,W\,\rho-jx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -jy\,\rho\,c^2+\left(-qy-\frac{jx\,sxy}{n}-\frac{jy\,syy}{n}-\frac{jz\,syz}{n}-\frac{jx^2\,jy\,\rho}{2\,n^2}-\frac{jy^3\,\rho}{2\,n^2}-\frac{jy\,jz^2\,\rho}{2\,n^2}+jy\,W\,\rho-jy\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -jz\,\rho\,c^2+\left(-qz-\frac{jx\,sxz}{n}-\frac{jy\,syz}{n}-\frac{jz\,szz}{n}-\frac{jx^2\,jz\,\rho}{2\,n^2}-\frac{jy^2\,jz\,\rho}{2\,n^2}-\frac{jz^3\,\rho}{2\,n^2}+jz\,W\,\rho-jz\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.{1, 0, 0, 0} /. j2v]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{1}{2}\,n\,ux^2\,\rho-\frac{1}{2}\,n\,uy^2\,\rho-\frac{1}{2}\,n\,uz^2\,\rho+n\,W\,\rho-n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,ux\,\rho\,c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^3\,\rho-\frac{1}{2}\,n\,ux\,uy^2\,\rho-\frac{1}{2}\,n\,ux\,uz^2\,\rho+n\,ux\,W\,\rho-n\,ux\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,uy\,\rho\,c^2+\left(-qy-sxy\,ux-syy\,uy-syz\,uz-\frac{1}{2}\,n\,ux^2\,uy\,\rho-\frac{1}{2}\,n\,uy^3\,\rho-\frac{1}{2}\,n\,uy\,uz^2\,\rho+n\,uy\,W\,\rho-n\,uy\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,uz\,\rho\,c^2+\left(-qz-sxz\,ux-syz\,uy-szz\,uz-\frac{1}{2}\,n\,ux^2\,uz\,\rho-\frac{1}{2}\,n\,uy^2\,uz\,\rho-\frac{1}{2}\,n\,uz^3\,\rho+n\,uz\,W\,\rho-n\,uz\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* flux of coord. energy across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(-jx\,\rho+n\,Vx\,\rho\right)c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\,\rho}{2\,n^2}-\frac{jx\,jy^2\,\rho}{2\,n^2}-\frac{jx\,jz^2\,\rho}{2\,n^2}+\frac{jx^2\,Vx\,\rho}{2\,n}+\frac{jy^2\,Vx\,\rho}{2\,n}+\frac{jz^2\,Vx\,\rho}{2\,n}+jx\,W\,\rho-n\,Vx\,W\,\rho-jx\,\epsilon\,\rho+n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt)] /. j2vr]
```
Out[•]//MatrixForm=

$$-Lx\,n\,\rho\,c^2+\left(-qx-sxx\,ux-\frac{1}{2}\,Lx\,n\,ux^2\,\rho+Lx\,n\,W\,\rho-Lx\,n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. {jx → 0, jy → 0, jz → 0}]]
```
Out[•]//MatrixForm=

$$n\,Vx\,\rho\,c^2+\left(-qx-n\,Vx\,W\,\rho+n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of matter flux*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf]]
```
Out[•]//MatrixForm=

$$-JX\,\rho\,c^2+\left(-qx-\frac{JX\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-sxx\,Vx-\frac{JX^3\,\rho}{2\,n^2}-\frac{JX\,jy^2\,\rho}{2\,n^2}-\frac{JX\,jz^2\,\rho}{2\,n^2}-\frac{JX^2\,Vx\,\rho}{n}-\frac{1}{2}\,JX\,Vx^2\,\rho+JX\,W\,\rho-JX\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf]]
```
Out[•]//MatrixForm=

$$-JX\,\rho\,c^2+\left(-qx-\frac{JX\,sxx}{n}-sxx\,Vx-\frac{JX^3\,\rho}{2\,n^2}-\frac{JX^2\,Vx\,\rho}{n}-\frac{1}{2}\,JX\,Vx^2\,\rho+JX\,W\,\rho-JX\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of matter flux & matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf /. j2v]]
```
Out[•]//MatrixForm=

$$-JX\,\rho\,c^2+\left(-qx-\frac{JX\,sxx}{n}-sxy\,uy-sxz\,uz-sxx\,Vx-\frac{JX^3\,\rho}{2\,n^2}-\frac{1}{2}\,JX\,uy^2\,\rho-\frac{1}{2}\,JX\,uz^2\,\rho-\frac{JX^2\,Vx\,\rho}{n}-\frac{1}{2}\,JX\,Vx^2\,\rho+JX\,W\,\rho-JX\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf /. {JX → 0, jy → 0, jz → 0}]]
```
Out[•]//MatrixForm=

$$\left(-qx-sxx\,Vx\right)+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. relv]]
```

Out[•]//MatrixForm=

$$-n\,Vx\,\rho\,c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^2\,Vx\,\rho}{2\,n}-\frac{jy^2\,Vx\,\rho}{2\,n}-\frac{jz^2\,Vx\,\rho}{2\,n}+n\,Vx\,W\,\rho-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr]]
```

Out[•]//MatrixForm=

$$-n\,Vx\,\rho\,c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^2\,Vx\,\rho-\frac{1}{2}\,n\,uy^2\,Vx\,\rho-\frac{1}{2}\,n\,uz^2\,Vx\,\rho+n\,Vx\,W\,\rho-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr /. {Vx → 0}]]
```

Out[•]//MatrixForm=

$$\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz\right)+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (* supply term for coord. energy *)
TTx = tW[Normal[tt]]; shows[assut, 2][Expand //@ FS@PowerExpand[Tr[1 / 2 * Normal@(Inverse[gg].T[Dcoords[[1, ;; , ;;]]].gg + Dcoords[[1, ;; , ;;]]).TTx]]]
```

Out[•]//MatrixForm=

$$n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]+\frac{\left(3\left(jx^2+jy^2+jz^2\right)\rho+2\,n\left(sxx+syy+szz+n\,\epsilon\,\rho\right)-2\,n^2\,\rho\,W[t,x,y,z]\right)W^{(1,0,0,0)}[t,x,y,z]}{2\,n\,c^2}+O\left[\frac{1}{c}\right]^3$$

```
In[•]:= (* INTERNAL ENERGY *)
```

```
In[•]:= (* energy 3-form when projected along matter 4-velocity, "internal energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.uu]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-n\,\epsilon\,\rho+O\left[\frac{1}{c}\right]^2 \\ -jx\,\rho\,c^2+\left(-qx-jx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -jy\,\rho\,c^2+\left(-qy-jy\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -jz\,\rho\,c^2+\left(-qz-jz\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

```
In[•]:= (* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.uu /. j2vr]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-n\,\epsilon\,\rho+O\left[\frac{1}{c}\right]^2 \\ -n\,ux\,\rho\,c^2+\left(-qx-n\,ux\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,uy\,\rho\,c^2+\left(-qy-n\,uy\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,uz\,\rho\,c^2+\left(-qz-n\,uz\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

```
In[•]:= (* flux of internal energy across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt)]]
```

Out[•]//MatrixForm=

$$\left(-jx\,\rho+n\,vx\,\rho\right)c^2+\left(-qx-jx\,\epsilon\,\rho+n\,vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. relv]]
```

Out[•]//MatrixForm=

$$-n\,Vx\,\rho\,c^2+\left(-qx-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. j2vr]]
```

Out[•]//MatrixForm=

$$-n\,Vx\,\rho\,c^2+\left(-qx-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. j2vr /. {Vx → 0}]]
```

Out[•]//MatrixForm=

$$-qx+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (* supply term for internal energy (should be reversed in sign; remember that stress is compressive, not tensile) *)
TTx = tW[tjv@tt]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Duv].gg + Duv).TTx]]]
```

Out[•]//MatrixForm=

$$\left(sxz\,vx^{(0,0,0,1)}[t,x,y,z]+syz\,vy^{(0,0,0,1)}[t,x,y,z]+szz\,vz^{(0,0,0,1)}[t,x,y,z]+sxy\,vx^{(0,0,1,0)}[t,x,y,z]+syy\,vy^{(0,0,1,0)}[t,x,y,z]+syz\,vz^{(0,0,1,0)}[t,x,y,z]+sxx\,vx^{(0,1,0,0)}[t,x,y,z]+sxy\,vy^{(0,1,0,0)}[t,x,y,z]+sxz\,vz^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (* difference between "coord. energy" and "internal energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - uu)]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{jx^2\,\rho}{2\,n}-\frac{jy^2\,\rho}{2\,n}-\frac{jz^2\,\rho}{2\,n}+n\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\,\rho}{2\,n^2}-\frac{jx\,jy^2\,\rho}{2\,n^2}-\frac{jx\,jz^2\,\rho}{2\,n^2}+jx\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\,sxy}{n}-\frac{jy\,syy}{n}-\frac{jz\,syz}{n}-\frac{jx^2\,jy\,\rho}{2\,n^2}-\frac{jy^3\,\rho}{2\,n^2}-\frac{jy\,jz^2\,\rho}{2\,n^2}+jy\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\,sxz}{n}-\frac{jy\,syz}{n}-\frac{jz\,szz}{n}-\frac{jx^2\,jz\,\rho}{2\,n^2}-\frac{jy^2\,jz\,\rho}{2\,n^2}-\frac{jz^3\,\rho}{2\,n^2}+jz\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

```
In[•]:= (* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - uu) /. j2vr]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{1}{2}\,n\,ux^2\,\rho-\frac{1}{2}\,n\,uy^2\,\rho-\frac{1}{2}\,n\,uz^2\,\rho+n\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ \left(-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^3\,\rho-\frac{1}{2}\,n\,ux\,uy^2\,\rho-\frac{1}{2}\,n\,ux\,uz^2\,\rho+n\,ux\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ \left(-sxy\,ux-syy\,uy-syz\,uz-\frac{1}{2}\,n\,ux^2\,uy\,\rho-\frac{1}{2}\,n\,uy^3\,\rho-\frac{1}{2}\,n\,uy\,uz^2\,\rho+n\,uy\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ \left(-sxz\,ux-syz\,uy-szz\,uz-\frac{1}{2}\,n\,ux^2\,uz\,\rho-\frac{1}{2}\,n\,uy^2\,uz\,\rho-\frac{1}{2}\,n\,uz^3\,\rho+n\,uz\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

```
In[•]:= (* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt)]]
```

Out[•]//MatrixForm=

$$\left(-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\,\rho}{2\,n^2}-\frac{jx\,jy^2\,\rho}{2\,n^2}-\frac{jx\,jz^2\,\rho}{2\,n^2}+\frac{jx^2\,vx\,\rho}{2\,n}+\frac{jy^2\,vx\,\rho}{2\,n}+\frac{jz^2\,vx\,\rho}{2\,n}+jx\,W\,\rho-n\,vx\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. relv]]
```

Out[•]//MatrixForm=

$$\left(-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^2\,Vx\,\rho}{2\,n}-\frac{jy^2\,Vx\,\rho}{2\,n}-\frac{jz^2\,Vx\,\rho}{2\,n}+n\,Vx\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. j2vr]]
```

Out[•]//MatrixForm=

$$\left(-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^2\,Vx\,\rho-\frac{1}{2}\,n\,uy^2\,Vx\,\rho-\frac{1}{2}\,n\,uz^2\,Vx\,\rho+n\,Vx\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. j2vr /. {Vx → 0}]]
```

Out[•]//MatrixForm=

$$\left(-sxx\,ux-sxy\,uy-sxz\,uz\right)+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (* PROPER-TIME COORD ENERGY*)
```

```
In[•]:= (* energy 3-form when projected along normalized coord-t
          note how the gravitational term is missing *)
showf[assut][Expand //@ FS@PowerExpand[tt.vtn]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{jx^2\,\rho}{2\,n}-\frac{jy^2\,\rho}{2\,n}-\frac{jz^2\,\rho}{2\,n}-n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -jx\,\rho\,c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\,\rho}{2\,n^2}-\frac{jx\,jy^2\,\rho}{2\,n^2}-\frac{jx\,jz^2\,\rho}{2\,n^2}-jx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -jy\,\rho\,c^2+\left(-qy-\frac{jx\,sxy}{n}-\frac{jy\,syy}{n}-\frac{jz\,syz}{n}-\frac{jx^2\,jy\,\rho}{2\,n^2}-\frac{jy^3\,\rho}{2\,n^2}-\frac{jy\,jz^2\,\rho}{2\,n^2}-jy\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -jz\,\rho\,c^2+\left(-qz-\frac{jx\,sxz}{n}-\frac{jy\,syz}{n}-\frac{jz\,szz}{n}-\frac{jx^2\,jz\,\rho}{2\,n^2}-\frac{jy^2\,jz\,\rho}{2\,n^2}-\frac{jz^3\,\rho}{2\,n^2}-jz\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

```
In[•]:= (* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.vtn /. j2vr]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{1}{2}\,n\,ux^2\,\rho-\frac{1}{2}\,n\,uy^2\,\rho-\frac{1}{2}\,n\,uz^2\,\rho-n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,ux\,\rho\,c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^3\,\rho-\frac{1}{2}\,n\,ux\,uy^2\,\rho-\frac{1}{2}\,n\,ux\,uz^2\,\rho-n\,ux\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,uy\,\rho\,c^2+\left(-qy-sxy\,ux-syy\,uy-syz\,uz-\frac{1}{2}\,n\,ux^2\,uy\,\rho-\frac{1}{2}\,n\,uy^3\,\rho-\frac{1}{2}\,n\,uy\,uz^2\,\rho-n\,uy\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,uz\,\rho\,c^2+\left(-qz-sxz\,ux-syz\,uy-szz\,uz-\frac{1}{2}\,n\,ux^2\,uz\,\rho-\frac{1}{2}\,n\,uy^2\,uz\,\rho-\frac{1}{2}\,n\,uz^3\,\rho-n\,uz\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

```
In[•]:= (* flux of normalized-coord-t energy across surface *)
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt)]]
```

Out[•]//MatrixForm=

$$\left(-jx\,\rho+n\,vx\,\rho\right)c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jx^3\,\rho}{2\,n^2}+\frac{jx^2\,vx\,\rho}{2\,n}-jx\,\epsilon\,\rho+n\,vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

```
In[•]:= (*in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. relv]]
```

Out[•]//MatrixForm=

$$-n\,Vx\,\rho\,c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^2\,Vx\,\rho}{2\,n}-\frac{jy^2\,Vx\,\rho}{2\,n}-\frac{jz^2\,Vx\,\rho}{2\,n}-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. j2vr]]
```
Out[∘]//MatrixForm=

$$-n\,Vx\,\rho\,c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^2\,Vx\,\rho-\frac{1}{2}\,n\,uy^2\,Vx\,\rho-\frac{1}{2}\,n\,uz^2\,Vx\,\rho-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[∘]//MatrixForm=

$$\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* supply term for normalized-coord-t energy
   we obtain the "power generated by the gravity field" *)
TTx = tW[tjv@tt]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Dvtn].gg + Dvtn).TTx]]]
```
Out[∘]//MatrixForm=

$$\left(-\rho\,n[t,x,y,z]\times vz[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]-\rho\,n[t,x,y,z]\times vy[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]-\rho\,n[t,x,y,z]\times vx[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* difference between "coord. energy" and "proper-time coord. energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - vtn)]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} n\,W\,\rho+O[\frac{1}{c}]^2 \\ jx\,W\,\rho+O[\frac{1}{c}]^2 \\ jy\,W\,\rho+O[\frac{1}{c}]^2 \\ jz\,W\,\rho+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - vtn) /. j2vr]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} n\,W\,\rho+O[\frac{1}{c}]^2 \\ n\,ux\,W\,\rho+O[\frac{1}{c}]^2 \\ n\,uy\,W\,\rho+O[\frac{1}{c}]^2 \\ n\,uz\,W\,\rho+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[∘]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt)]]
```
Out[∘]//MatrixForm=

$$\left(jx\,W\,\rho-n\,vx\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. relv]]
```
Out[∘]//MatrixForm=

$$n\,Vx\,W\,\rho+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. j2vr]]
```
Out[∘]//MatrixForm=

$$n\,Vx\,W\,\rho+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[∘]//MatrixForm=

$$O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* difference between "internal energy" and "proper-time coord. energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.(uu - vtn)]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} \left(\frac{jx^2\rho}{2n}+\frac{jy^2\rho}{2n}+\frac{jz^2\rho}{2n}\right)+O[\frac{1}{c}]^2 \\ \left(\frac{jx\,sxx}{n}+\frac{jy\,sxy}{n}+\frac{jz\,sxz}{n}+\frac{jx^3\rho}{2n^2}+\frac{jx\,jy^2\rho}{2n^2}+\frac{jx\,jz^2\rho}{2n^2}\right)+O[\frac{1}{c}]^2 \\ \left(\frac{jx\,sxy}{n}+\frac{jy\,syy}{n}+\frac{jz\,syz}{n}+\frac{jx^2\,jy\,\rho}{2n^2}+\frac{jy^3\rho}{2n^2}+\frac{jy\,jz^2\rho}{2n^2}\right)+O[\frac{1}{c}]^2 \\ \left(\frac{jx\,sxz}{n}+\frac{jy\,syz}{n}+\frac{jz\,szz}{n}+\frac{jx^2\,jz\,\rho}{2n^2}+\frac{jy^2\,jz\,\rho}{2n^2}+\frac{jz^3\rho}{2n^2}\right)+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.(uu - vtn) /. j2vr]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} \left(\frac{1}{2}n\,ux^2\rho+\frac{1}{2}n\,uy^2\rho+\frac{1}{2}n\,uz^2\rho\right)+O[\frac{1}{c}]^2 \\ \left(sxx\,ux+sxy\,uy+sxz\,uz+\frac{1}{2}n\,ux^3\rho+\frac{1}{2}n\,ux\,uy^2\rho+\frac{1}{2}n\,ux\,uz^2\rho\right)+O[\frac{1}{c}]^2 \\ \left(sxy\,ux+syy\,uy+syz\,uz+\frac{1}{2}n\,ux^2\,uy\,\rho+\frac{1}{2}n\,uy^3\rho+\frac{1}{2}n\,uy\,uz^2\rho\right)+O[\frac{1}{c}]^2 \\ \left(sxz\,ux+syz\,uy+szz\,uz+\frac{1}{2}n\,ux^2\,uz\,\rho+\frac{1}{2}n\,uy^2\,uz\,\rho+\frac{1}{2}n\,uz^3\rho\right)+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[∘]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - vtn) / (A * Δt)]]
```
Out[∘]//MatrixForm=

$$\left(\frac{jx\,sxx}{n}+\frac{jy\,sxy}{n}+\frac{jz\,sxz}{n}+\frac{jx^3\rho}{2n^2}+\frac{jx\,jy^2\rho}{2n^2}+\frac{jx\,jz^2\rho}{2n^2}-\frac{jx^2\,vx\,\rho}{2n}-\frac{jy^2\,vx\,\rho}{2n}-\frac{jz^2\,vx\,\rho}{2n}\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - vtn) / (A * Δt) /. relv]]
```
Out[∘]//MatrixForm=

$$\left(\frac{jx\,sxx}{n}+\frac{jy\,sxy}{n}+\frac{jz\,sxz}{n}+\frac{jx^2\,Vx\,\rho}{2n}+\frac{jy^2\,Vx\,\rho}{2n}+\frac{jz^2\,Vx\,\rho}{2n}\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - uu) / (A * Δt) /. j2vr]]
```
Out[∘]//MatrixForm=

$$O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - uu) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[∘]//MatrixForm=

$$O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* Faraday tensor *)
repE = {Ex → Ex * c * Sqrt[μ0 * ϵ0], Ey → Ey * c * Sqrt[μ0 * ϵ0], Ez → Ez * c * Sqrt[μ0 * ϵ0]};
fftemp = {{0, -Ex, -Ey, -Ez}, {0, 0, Bz, -By}, {0, 0, 0, Bx}, {0, 0, 0, 0}} /. repE;
showf[assut][ffdd = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & -c\,Ex\sqrt{\epsilon0}\sqrt{\mu0} & -c\,Ey\sqrt{\epsilon0}\sqrt{\mu0} & -c\,Ez\sqrt{\epsilon0}\sqrt{\mu0} \\ c\,Ex\sqrt{\epsilon0}\sqrt{\mu0} & 0 & Bz & -By \\ c\,Ey\sqrt{\epsilon0}\sqrt{\mu0} & -Bz & 0 & Bx \\ c\,Ez\sqrt{\epsilon0}\sqrt{\mu0} & By & -Bx & 0 \end{pmatrix}$$

In[∘]:=
```
showf[assut][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
  ]]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{Ex^2\epsilon0}{2}-\frac{Ey^2\epsilon0}{2}-\frac{Ez^2\epsilon0}{2}-\frac{Bx^2}{2\mu0}-\frac{By^2}{2\mu0}-\frac{Bz^2}{2\mu0}\right)+\frac{-Ex^2W\epsilon0-Ey^2W\epsilon0-Ez^2W\epsilon0+\frac{Bx^2W}{\mu0}+\frac{By^2W}{\mu0}+\frac{Bz^2W}{\mu0}}{c^2}+O[\frac{1}{c}]^4 \\ \left(-\frac{Bz\,Ey\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{By\,Ez\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\frac{\frac{2\,Bz\,Ey\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{2\,By\,Ez\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{-4\,Ey^2\,Wx\,\epsilon0-4\,Ez^2\,Wx\,\epsilon0+4\,Ex\,Ey\,Wy\,\epsilon0+4\,Ex\,Ez\,Wz\,\epsilon0}{c^2}+O[\frac{1}{c}]^3 \\ \left(\frac{Bz\,Ex\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{Bx\,Ez\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\frac{-\frac{2\,Bz\,Ex\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{2\,Bx\,Ez\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{4\,Ex\,Ey\,Wx\,\epsilon0-4\,Ex^2\,Wy\,\epsilon0-4\,Ez^2\,Wy\,\epsilon0+4\,Ey\,Ez\,Wz\,\epsilon0}{c^2}+O[\frac{1}{c}]^3 \\ \left(-\frac{By\,Ex\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{Bx\,Ey\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\frac{\frac{2\,By\,Ex\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{2\,Bx\,Ey\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{4\,Ex\,Ez\,Wx\,\epsilon0+4\,Ey\,Ez\,Wy\,\epsilon0-4\,Ex^2\,Wz\,\epsilon0-4\,Ey^2\,Wz\,\epsilon0}{c^2}+O[\frac{1}{c}]^3 \end{pmatrix}$$

$$\begin{pmatrix} \frac{\frac{Bz\,Ey\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{By\,Ez\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{\frac{2\,Bz\,Ey\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{2\,By\,Ez\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c^3}+\frac{-\frac{4\,By^2\,Wx}{\mu0}-\frac{4\,Bz^2\,Wx}{\mu0}+\frac{4\,Bx\,By\,Wy}{\mu0}+\frac{4\,Bx\,Bz\,Wz}{\mu0}}{c^4}+O[\frac{1}{c}]^5 \\ \left(-\frac{Ex^2\epsilon0}{2}+\frac{Ey^2\epsilon0}{2}+\frac{Ez^2\epsilon0}{2}-\frac{Bx^2}{2\mu0}+\frac{By^2}{2\mu0}+\frac{Bz^2}{2\mu0}\right)+\frac{-Ex^2W\epsilon0+Ey^2W\epsilon0+Ez^2W\epsilon0+\frac{Bx^2W}{\mu0}-\frac{By^2W}{\mu0}-\frac{Bz^2W}{\mu0}}{c^2}+\ldots \\ \left(-Ex\,Ey\,\epsilon0-\frac{Bx\,By}{\mu0}\right)+\frac{-2\,Ex\,Ey\,W\,\epsilon0+\frac{2\,Bx\,By\,W}{\mu0}}{c^2}+\frac{\frac{4\,Bz\,Ex\,Wx\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{4\,By\,Ez\,Wy\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{4\,Bx\,Ex\,Wz\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{4\,By\,Ey\,Wz\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c^3}+O[\frac{1}{c}]^4 \\ \left(-Ex\,Ez\,\epsilon0-\frac{Bx\,Bz}{\mu0}\right)+\frac{-2\,Ex\,Ez\,W\,\epsilon0+\frac{2\,Bx\,Bz\,W}{\mu0}}{c^2}+\frac{-\frac{4\,By\,Ex\,Wx\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{4\,Bx\,Ex\,Wy\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{4\,Bz\,Ez\,Wy\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{4\,Bz\,Ey\,Wz\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c^3}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[∘]:=
```
(* COORDINATE EM ENERGY *)
```

In[∘]:=
```
(* energy 3-form when projected along coord. axes *)
showf[assut][Expand //@ FS@PowerExpand[tte.{1, 0, 0, 0}]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{Ex^2\epsilon0}{2}-\frac{Ey^2\epsilon0}{2}-\frac{Ez^2\epsilon0}{2}-\frac{Bx^2}{2\mu0}-\frac{By^2}{2\mu0}-\frac{Bz^2}{2\mu0}\right)+\frac{-Ex^2W\epsilon0-Ey^2W\epsilon0-Ez^2W\epsilon0+\frac{Bx^2W}{\mu0}+\frac{By^2W}{\mu0}+\frac{Bz^2W}{\mu0}}{c^2}+O[\frac{1}{c}]^4 \\ \left(-\frac{Bz\,Ey\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{By\,Ez\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\frac{\frac{2\,Bz\,Ey\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{2\,By\,Ez\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{-4\,Ey^2\,Wx\,\epsilon0-4\,Ez^2\,Wx\,\epsilon0+4\,Ex\,Ey\,Wy\,\epsilon0+4\,Ex\,Ez\,Wz\,\epsilon0}{c^2}+O[\frac{1}{c}]^3 \\ \left(\frac{Bz\,Ex\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{Bx\,Ez\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\frac{-\frac{2\,Bz\,Ex\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{2\,Bx\,Ez\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{4\,Ex\,Ey\,Wx\,\epsilon0-4\,Ex^2\,Wy\,\epsilon0-4\,Ez^2\,Wy\,\epsilon0+4\,Ey\,Ez\,Wz\,\epsilon0}{c^2}+O[\frac{1}{c}]^3 \\ \left(-\frac{By\,Ex\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{Bx\,Ey\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\frac{\frac{2\,By\,Ex\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{2\,Bx\,Ey\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{4\,Ex\,Ez\,Wx\,\epsilon0+4\,Ey\,Ez\,Wy\,\epsilon0-4\,Ex^2\,Wz\,\epsilon0-4\,Ey^2\,Wz\,\epsilon0}{c^2}+O[\frac{1}{c}]^3 \end{pmatrix}$$

In[∘]:=
```
(* flux of coord. energy across surface *)
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tte.{1, 0, 0, 0} / (A * Δt)]]
```
Out[∘]//MatrixForm=

$$\left(-\frac{Bz\,Ey\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{By\,Ez\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\left(\frac{1}{2}Ex^2\,vx\,\epsilon0+\frac{1}{2}Ey^2\,vx\,\epsilon0+\frac{1}{2}Ez^2\,vx\,\epsilon0+\frac{Bx^2\,vx}{2\mu0}+\frac{By^2\,vx}{2\mu0}+\frac{Bz^2\,vx}{2\mu0}\right)+\frac{\frac{2\,Bz\,Ey\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{2\,By\,Ez\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{Ex^2\,vx\,W\,\epsilon0+Ey^2\,vx\,W\,\epsilon0+Ez^2\,vx\,W\,\epsilon0-4\,Ey^2\,Wx\,\epsilon0-4\,Ez^2\,Wx\,\epsilon0+4\,Ex\,Ey\,Wy\,\epsilon0+4\,Ex\,Ez\,Wz\,\epsilon0-\frac{Bx^2\,vx\,W}{\mu0}-\frac{By^2\,vx\,W}{\mu0}-\frac{Bz^2\,vx\,W}{\mu0}}{c^2}+O\left[\frac{1}{c}\right]^3$$

In[•]:=
```
(* supply term for coord. energy *)
TTx = tW[tjv@tte]; showf[assut][Expand //@ FS@PowerExpand[Tr[1/2 * (Inverse[gg].T[Dcoords[[1, ;; , ;;]]].gg + Dcoords[[1, ;; , ;;]]).TTx]]]
```

Out[•]//MatrixForm=

$$\frac{Ex^2\,\epsilon 0\,W^{(1,0,0,0)}[t,x,y,z] + Ey^2\,\epsilon 0\,W^{(1,0,0,0)}[t,x,y,z] + Ez^2\,\epsilon 0\,W^{(1,0,0,0)}[t,x,y,z] + \frac{Bx^2\,W^{(1,0,0,0)}[t,x,y,z]}{\mu 0} + \frac{By^2\,W^{(1,0,0,0)}[t,x,y,z]}{\mu 0} + \frac{Bz^2\,W^{(1,0,0,0)}[t,x,y,z]}{\mu 0}}{c^2} + O\left[\frac{1}{c}\right]^3$$