In[1]:=
```
<< "christoffelsymbols.m"
(* First index is upper index Table[FS[cc[[ii,;;,;;]]==T[cc[[ii,;;,;;]]]],{ii,1,4}] *)
```

In[2]:=
```
showf[assumptions_, simp_ : FullSimplify] := ((Assuming[assumptions, Expand //@ simp@PowerExpand[#]] // MF) &);
```

In[3]:=
```
(* Show matrix expressions and power expansions*)
show[assumptions_, power_, simp_ : FullSimplify] := ({Assuming[assumptions, simp@PowerExpand[#]] // MF, "\n",
      Assuming[assumptions, simp@PowerExpand[Series[#, {c, Infinity, power}]]] // MF} &);
shows[assumptions_, power_, simp_ : FullSimplify] := ((
      Assuming[assumptions, simp@PowerExpand[Series[#, {c, Infinity, power}]]] // MF) &);
showf[assumptions_, simp_ : FullSimplify] := ((Assuming[assumptions, Expand //@ simp@PowerExpand[#]] // MF) &);
show1[assumptions_, simp_ : Identity] := ((Assuming[assumptions, simp[#]] // MF) &);
show2[assumptions_, power_, simp_ : Identity] := ((
      Assuming[assumptions, simp[Series[#, {c, Infinity, power}]]] // MF) &);
```

In[8]:=
```
coords = {t, x, y, z}
```
Out[8]=
```
{t, x, y, z}
```

In[9]:=
```
(* Flat metric *)
(gg0 = DiagonalMatrix[{-c^2, 1, 1, 1}]) // MF
```
Out[9]//MatrixForm=
$$\begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[10]:=
```
(*D[G*M/Sqrt[x^2+y^2+z^2],{{x,y,z}}] *)
```

In[11]:=
```
(* -W is the potential gravitational energy: W=GM/r
   that is, F_g(downwards)=grad W
*)
```

In[12]:=
```
(* Rotating metric from poissonetal *)
(gg = DiagonalMatrix@Diagonal@{{-c^2*(1-2*W/c^2 +O[c, +Infinity]^4(*-2*((*Ψ[t,x,y,z]*)-W[t,x,y,z]^2)/c^4*))},
        -4*Wx/c^2+O[c, +Infinity]^4, -4*Wy/c^2+O[c, +Infinity]^4, -4*Wz/c^2+O[c, +Infinity]^4},
       {-4*Wx/c^2+O[c, +Infinity]^4,
        1+2*W/c^2+O[c, +Infinity]^4, 0, 0},
       {-4*Wy/c^2+O[c, +Infinity]^4, 0, 1+2*W/c^2+O[c, +Infinity]^4, 0},
       {-4*Wz/c^2+O[c, +Infinity]^4, 0, 0, 1+2*W/c^2+O[c, +Infinity]^4}}) // MF
(*(gg=DiagonalMatrix[{-c^2*(1+2*Φ[t,r]/c^2),1+2*Λ[t,r]/c^2,r^2,r^2*Sin[θ]^2}])//MF*)
```
Out[12]//MatrixForm=
$$\begin{pmatrix} -c^2+2\,W+O\left[\frac{1}{c}\right]^2 & 0 & 0 & 0 \\ 0 & 1+\frac{2\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 0 \\ 0 & 0 & 1+\frac{2\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 \\ 0 & 0 & 0 & 1+\frac{2\,W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[13]:=
```
Inverse[gg] // MF
```
Out[13]//MatrixForm=
$$\begin{pmatrix} -\left(\frac{1}{c}\right)^2-\frac{2\,W}{c^4}+O\left[\frac{1}{c}\right]^6 & 0 & 0 & 0 \\ 0 & 1-\frac{2\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 0 \\ 0 & 0 & 1-\frac{2\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 \\ 0 & 0 & 0 & 1-\frac{2\,W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[14]:=
```
(*(gg=DiagonalMatrix@Diagonal[gg])//MF*)
```

In[130]:=
```
(* functions to temporarily remove coord-dep *)
tW[xx_] := (xx /. {W → W[t, x, y, z], Wx → Wx[t, x, y, z], Wy → Wy[t, x, y, z], Wz → Wz[t, x, y, z]});
itW[xx_] := (xx /. {W[t, x, y, z] → W, Wx[t, x, y, z] → Wx, Wy[t, x, y, z] → Wy, Wz[t, x, y, z] → Wz});
(ggt = tW[gg]) // MF;
```

In[133]:=
```
assut = {c > 0, Element[a, Reals], Element[v, Reals], Element[t, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals],
    Element[vx, Reals], Element[vy, Reals], Element[vz, Reals], Element[n, Reals], Element[r, Reals], Element[θ, Reals], Element[ϕ, Reals], Abs[v] < c, -c < vx < c, -c < ux < c, r > 0, 0 < θ < Pi,
    Normal@gg[[1, 1]] / c^2 < 0, Normal@gg[[2, 2]] > 0, Normal@gg[[3, 3]] > 0, Normal@gg[[4, 4]] > 0, n > 0, Element[jx, Reals], Element[jy, Reals], Element[jz, Reals], Element[sxx, Reals], Element[sxy, Reals], Element[sxz, Reals], Element[syy, Reals], Element[syz, Reals], Element[szz, Reals],
    -Normal@Det[gg] > 0, β > 0};

assutt = {c > 0, Element[a, Reals], Element[v, Reals], Element[t, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals],
    Element[vx, Reals], Element[vy, Reals], Element[vz, Reals], Element[n, Reals], Element[r, Reals], Element[θ, Reals], Element[ϕ, Reals], Abs[v] < c, -c < vx < c, -c < ux < c, r > 0, 0 < θ < Pi,
    β > 0, Normal@ggt[[1, 1]] / c^2 < 0, Normal@ggt[[2, 2]] > 0, Normal@ggt[[3, 3]] > 0, Normal@ggt[[4, 4]] > 0,
    -Normal@Det[ggt] > 0};
```

In[135]:=
```
(igg = Assuming[assut, FullSimplify@PowerExpand[Inverse[gg]]]) // MF
```
Out[135]//MatrixForm=
$$\begin{pmatrix} -\left(\frac{1}{c}\right)^2-\frac{2\,W}{c^4}+O\left[\frac{1}{c}\right]^6 & 0 & 0 & 0 \\ 0 & 1-\frac{2\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 0 \\ 0 & 0 & 1-\frac{2\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 \\ 0 & 0 & 0 & 1-\frac{2\,W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[136]:=
```
(*show[assut,2]@ChristoffelSymbol[gg,coords][[2]]*)
```

In[137]:=
```
(* volume element *)
(dg = Assuming[assut, FullSimplify@PowerExpand[Sqrt[-Det[gg]] / c]]) // MF
```
Out[137]//MatrixForm=
$$1+\frac{2\,W}{c^2}+O\left[\frac{1}{c}\right]^4$$

In[138]:=
```
(* Christoffel symbols  *)
cc = Assuming[assut, FullSimplify@PowerExpand[itW[ChristoffelSymbol[ggt, coords]]]];
```

In[139]:=

In[140]:=
```
(* 3-vector of moving surface parallel to yz moving with velocity V   *)
surface = {-(Vx * Ax + Vy * Ay + Vz * Az), Ax, Ay, Az} * Δt;
```

In[141]:=

(* Matter current *)

In[142]:=
```
(* matter-current 3-covector *)
NJ = {n, jx, jy, jz};
```

In[331]:=
```
(* norm of matter 3-covector *)
Assuming[assut, FS[Sqrt[-NJ.gg.NJ] / c]]
```
Out[331]=
$$n-\frac{jx^2+jy^2+jz^2+2\,n^2\,W}{2\,n\,c^2}+O\left[\frac{1}{c}\right]^4$$

In[332]:=
```
(* matter associated 1-vector *)
(NJvec = Assuming[assut, FS[NJ / dg]]) // MF
```
Out[332]//MatrixForm=
$$\begin{pmatrix} n-\frac{2\,(n\,W)}{c^2}+O\left[\frac{1}{c}\right]^4 \\ jx-\frac{2\,(jx\,W)}{c^2}+O\left[\frac{1}{c}\right]^4 \\ jy-\frac{2\,(jy\,W)}{c^2}+O\left[\frac{1}{c}\right]^4 \\ jz-\frac{2\,(jz\,W)}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[333]:=
```
(* matter associated 4-vel vector *)
(uu = Assuming[assut, FS[c * NJvec / Sqrt[-NJvec.gg.NJvec]]]) // MF
```
Out[333]//MatrixForm=
$$\begin{pmatrix} 1+\frac{\frac{jx^2+jy^2+jz^2}{2\,n^2}+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jx}{n}+\frac{jx\left(jx^2+jy^2+jz^2+2\,n^2\,W\right)}{2\,n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n}+\frac{jy\left(jx^2+jy^2+jz^2+2\,n^2\,W\right)}{2\,n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n}+\frac{jz\left(jx^2+jy^2+jz^2+2\,n^2\,W\right)}{2\,n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[338]:=
```
Assuming[assut, FS[uu /. replaceJu]] // MF
```
Out[338]//MatrixForm=
$$\begin{pmatrix} 1+\frac{\frac{1}{2}\,(ux^2+uy^2+uz^2)+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ ux+\frac{ux\left(ux^2+uy^2+uz^2+2\,W\right)}{2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ uy+\frac{uy\left(ux^2+uy^2+uz^2+2\,W\right)}{2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ uz+\frac{uz\left(ux^2+uy^2+uz^2+2\,W\right)}{2\,c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[146]:=
```
(* it is normalized *)
uu.gg.uu
```
Out[146]=
$$-c^2+O\left[\frac{1}{c}\right]^2$$

In[334]:=
```
(* matter associated 1-covector *)
(NJcov = Assuming[assut, FS[gg.(NJ / dg)]]) // MF
```
Out[334]//MatrixForm=
$$\begin{pmatrix} -n\,c^2+4\,n\,W+O\left[\frac{1}{c}\right]^2 \\ jx+O\left[\frac{1}{c}\right]^4 \\ jy+O\left[\frac{1}{c}\right]^4 \\ jz+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[148]:=
```
(* scalar product with de/de_x (for momentum x-component) *)
uu.gg.{0, 1, 0, 0}
```

Out[148]=

$$\frac{jx}{n}+\frac{\frac{jx^3}{2\,n^3}+\frac{jx\,jy^2}{2\,n^3}+\frac{jx\,jz^2}{2\,n^3}+\frac{3\,jx\,W}{n}}{c^2}+O\left[\frac{1}{c}\right]^4$$

In[149]:=
```
(* scalar product with de/de_i (for momentum i-component) *)
uu.gg // MF
```

Out[149]//MatrixForm=

$$\begin{pmatrix}-c^2+\left(-\frac{jx^2}{2\,n^2}-\frac{jy^2}{2\,n^2}-\frac{jz^2}{2\,n^2}+W\right)+O\left[\frac{1}{c}\right]^2\\ \frac{jx}{n}+\frac{\frac{jx^3}{2\,n^3}+\frac{jx\,jy^2}{2\,n^3}+\frac{jx\,jz^2}{2\,n^3}+\frac{3\,jx\,W}{n}}{c^2}+O\left[\frac{1}{c}\right]^4\\ \frac{jy}{n}+\frac{\frac{jx^2\,jy}{2\,n^3}+\frac{jy^3}{2\,n^3}+\frac{jy\,jz^2}{2\,n^3}+\frac{3\,jy\,W}{n}}{c^2}+O\left[\frac{1}{c}\right]^4\\ \frac{jz}{n}+\frac{\frac{jx^2\,jz}{2\,n^3}+\frac{jy^2\,jz}{2\,n^3}+\frac{jz^3}{2\,n^3}+\frac{3\,jz\,W}{n}}{c^2}+O\left[\frac{1}{c}\right]^4\end{pmatrix}$$

In[337]:=
```
(* retransform matter 4-vel to matter 3-covector *)
Assuming[assut, FS[uu * dg / c * Sqrt[-NJvec.gg.NJvec]]] // MF
```

Out[337]//MatrixForm=

$$\begin{pmatrix}n+O\left[\frac{1}{c}\right]^4\\ jx+O\left[\frac{1}{c}\right]^4\\ jy+O\left[\frac{1}{c}\right]^4\\ jz+O\left[\frac{1}{c}\right]^4\end{pmatrix}$$

In[151]:=
```
(* simplification to x-directed matter flux and velocity *)
assutjx = Join[assut, {jy == 0, jz == 0, uy == 0, uz == 0}];
```

In[152]:=
```
(* flux of matter across surface *)
Simplify[surface.NJ / (Δt)]
```

Out[152]=

$$Ax\left(jx-n\,Vx\right)+Ay\left(jy-n\,Vy\right)+Az\left(jz-n\,Vz\right)$$

In[153]:=
```
(* normalized zero-flux velocity is same as U *)
vnoflux = {1, jx / n, jy / n, jz / n};
FS[c * vnoflux / Sqrt[-vnoflux.gg.vnoflux] == uu]
```

Out[154]=

True

In[155]:=
```
(* replace matter flux in terms of velocity*)
replaceJu = {jx → ux * n, jy → uy * n, jz → uz * n}
```

Out[155]=

$$\{jx\to n\,ux,\ jy\to n\,uy,\ jz\to n\,uz\}$$

In[156]:=
```
(* collect velocity magnitude*)
replaceuUnorm = {ux^2 → U^2 - uy^2 - uz^2, ux^3 → ux * (U^2 - uy^2 - uz^2), jx^2 → J^2 - jy^2 - jz^2, jx^3 → jx * (J^2 - jy^2 - jz^2)}
```

Out[156]=

$$\left\{ux^2\to U^2-uy^2-uz^2,\ ux^3\to ux\left(U^2-uy^2-uz^2\right),\ jx^2\to J^2-jy^2-jz^2,\ jx^3\to jx\left(J^2-jy^2-jz^2\right)\right\}$$

In[157]:=
```
FS[uu /. replaceuUnorm] // MF
```

Out[157]//MatrixForm=

$$\begin{pmatrix}1+\frac{\frac{J^2}{2\,n^2}+W}{c^2}+O\left[\frac{1}{c}\right]^4\\ \frac{jx}{n}+\frac{jx\left(3^2+2\,n^2\,W\right)}{2\,n^3\,c^2}+O\left[\frac{1}{c}\right]^4\\ \frac{jy}{n}+\frac{jy\left(3^2+2\,n^2\,W\right)}{2\,n^3\,c^2}+O\left[\frac{1}{c}\right]^4\\ \frac{jz}{n}+\frac{jz\left(3^2+2\,n^2\,W\right)}{2\,n^3\,c^2}+O\left[\frac{1}{c}\right]^4\end{pmatrix}$$

In[158]:=
```
FS[uu /. replaceJu] // MF
```

Out[158]//MatrixForm=

$$\begin{pmatrix}1+\frac{\frac{1}{2}\left(ux^2+uy^2+uz^2\right)+W}{c^2}+O\left[\frac{1}{c}\right]^4\\ ux+\frac{ux\left(ux^2+uy^2+uz^2+2\,W\right)}{2\,c^2}+O\left[\frac{1}{c}\right]^4\\ uy+\frac{uy\left(ux^2+uy^2+uz^2+2\,W\right)}{2\,c^2}+O\left[\frac{1}{c}\right]^4\\ uz+\frac{uz\left(ux^2+uy^2+uz^2+2\,W\right)}{2\,c^2}+O\left[\frac{1}{c}\right]^4\end{pmatrix}$$

In[160]:=

In[161]:=
```
(* Project along u velocity *)
proju = Assuming[assut, Expand //@ FS@PowerExpand[-Outer[Times, uu, gg.uu] / c^2]];
projperpu = Assuming[assut, Expand //@ FS@PowerExpand[IdentityMatrix[4] - proju]];
testproj[ass_, x_] := showf[ass] /@ {Assuming[ass, Expand //@ FS@PowerExpand[proju.x.proju == x]], projperpu.x.proju == x, proju.x.projperpu == x, projperpu.x.projperpu == x}
```

In[164]:=
```
(* Project along u velocity *)
proju = Assuming[assut, FS[-Outer[Times, uu, gg.uu] / c^2]];
projperpu = Assuming[assut, FS[IdentityMatrix[4] - proju]];
testproj[ass_, x_] := showf[ass] /@ {Assuming[ass, Expand //@ FS@PowerExpand[proju.x.proju == x]], projperpu.x.proju == x, proju.x.projperpu == x, projperpu.x.projperpu == x}
```

In[167]:=
```
(* aux 4-velocity *)
auu = {temp, aux, auy, auz};
solu = temp /. Solve[Normal[auu.gg.auu] == -c^2, temp][[2]]
```

Out[168]=

$$\frac{\sqrt{aux^2+auy^2+auz^2+c^2}}{\sqrt{c^2-2\,W}}$$

In[169]:=
```
(auu = Assuming[assut, FS[auu /. {temp → solu}]]) // MF
```

Out[169]//MatrixForm=

$$\begin{pmatrix}\sqrt{\frac{aux^2+auy^2+auz^2+c^2}{c^2-2\,W}}\\ aux\\ auy\\ auz\end{pmatrix}$$

In[170]:=
```
auu.gg.auu
```

Out[170]=

$$-c^2+O\left[\frac{1}{c}\right]^2$$

In[211]:=
```
(* 4-velocity and matter current with explicit coordinate dependence *)
tjv[xx_] := (xx /. {n → n[t, x, y, z], jx → ux[t, x, y, z] * n[t, x, y, z], jy → uy[t, x, y, z] * n[t, x, y, z], jz → uz[t, x, y, z] * n[t, x, y, z]});
tjn[xx_] := (xx /. {n → n[t, x, y, z], jx → jx[t, x, y, z], jy → jy[t, x, y, z], jz → jz[t, x, y, z]});
itjn[xx_] := (xx /. {n[t, x, y, z] → n, jx[t, x, y, z] → jx, jy[t, x, y, z] → jy, jz[t, x, y, z] → jz});
itjv[xx_] := (xx /. {n[t, x, y, z] → n, ux[t, x, y, z] → jx / n, uy[t, x, y, z] → jy / n, uz[t, x, y, z] → jz / n});
repjn = {D[n[t, x, y, z], t] → D[jx[t, x, y, z], x] + D[jy[t, x, y, z], y] + D[jz[t, x, y, z], z]};
MF /@ {uut = Assuming[assut, FS[tjn[tW[uu]]]], uuv = Assuming[assut, FS[tjv[tW[uu]]]]}
```

Out[216]=

$$\left\{\begin{pmatrix}1+\frac{jx[t,x,y,z]^2-jy[t,x,y,z]^2-jz[t,x,y,z]^2}{2\,n[t,x,y,z]^2}+W[t,x,y,z]\\ \frac{jx[t,x,y,z]}{n[t,x,y,z]}+\frac{jx[t,x,y,z]\left(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z]\right)}{2\,n[t,x,y,z]^3\,c^2}+O\left[\frac{1}{c}\right]^4\\ \frac{jy[t,x,y,z]}{n[t,x,y,z]}+\frac{jy[t,x,y,z]\left(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z]\right)}{2\,n[t,x,y,z]^3\,c^2}+O\left[\frac{1}{c}\right]^4\\ \frac{jz[t,x,y,z]}{n[t,x,y,z]}+\frac{jz[t,x,y,z]\left(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z]\right)}{2\,n[t,x,y,z]^3\,c^2}+O\left[\frac{1}{c}\right]^4\end{pmatrix},\begin{pmatrix}1+\frac{\frac{1}{2}\left(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2\right)+W[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4\\ ux[t,x,y,z]+\frac{ux[t,x,y,z]\left(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z]\right)}{2\,c^2}+O\left[\frac{1}{c}\right]^4\\ uy[t,x,y,z]+\frac{uy[t,x,y,z]\left(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z]\right)}{2\,c^2}+O\left[\frac{1}{c}\right]^4\\ uz[t,x,y,z]+\frac{uz[t,x,y,z]\left(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z]\right)}{2\,c^2}+O\left[\frac{1}{c}\right]^4\end{pmatrix}\right\}$$

In[171]:=

In[217]:=

(* Construction of energy–momentum tensor *)

In[218]:=
```
(* definition of heat-flux, orthogonal to matter-current *)
Qtemp = {qt, qx, qy, qz};
```

In[219]:=
```
(proju.Qtemp) // MF
```

Out[219]//MatrixForm=

$$\begin{pmatrix}qt+\frac{\frac{\left(jx^2-jy^2-jz^2\right)qt}{n^2}-\frac{jx\,qx}{n}-\frac{jy\,qy}{n}-\frac{jz\,qz}{n}}{c^2}+O\left[\frac{1}{c}\right]^4\\ \frac{jx\,qt}{n}+\frac{\frac{jx\left(jx^2-jy^2-jz^2\right)qt}{n^3}-\frac{jx^2\,qx}{n^2}-\frac{jx\,jy\,qy}{n^2}-\frac{jx\,jz\,qz}{n^2}}{c^2}+O\left[\frac{1}{c}\right]^4\\ \frac{jy\,qt}{n}+\frac{\frac{jy\left(jx^2-jy^2-jz^2\right)qt}{n^3}-\frac{jx\,jy\,qx}{n^2}-\frac{jy^2\,qy}{n^2}-\frac{jy\,jz\,qz}{n^2}}{c^2}+O\left[\frac{1}{c}\right]^4\\ \frac{jz\,qt}{n}+\frac{\frac{jz\left(jx^2-jy^2-jz^2\right)qt}{n^3}-\frac{jx\,jz\,qx}{n^2}-\frac{jy\,jz\,qy}{n^2}-\frac{jz^2\,qz}{n^2}}{c^2}+O\left[\frac{1}{c}\right]^4\end{pmatrix}$$

In[220]:=
```
qsol = Solve[Normal[proju.Qtemp] == 0, qt][[1]]
```

Out[220]=

$$\left\{qt\to\frac{n\left(jx\,qx+jy\,qy+jz\,qz\right)}{jx^2+jy^2+jz^2+c^2\,n^2}\right\}$$

In[221]:=
```
(Q = Assuming[assut, FS[Qtemp /. qsol]]) // MF
```

Out[221]//MatrixForm=

$$\begin{pmatrix}\frac{n\left(jx\,qx+jy\,qy+jz\,qz\right)}{jx^2+jy^2+jz^2+c^2\,n^2}\\ qx\\ qy\\ qz\end{pmatrix}$$

In[222]:=
```
{Normal@Series[Q.NJ, {c, Infinity, 1}] == 0}
```
Out[222]=
$$\{jx\,qx + jy\,qy + jz\,qz == 0\}$$

In[223]:=
```
assutQ = Join[assut, {Normal@Series[Q.NJ, {c, Infinity, 1}] == 0}];
```

In[224]:=
```
Assuming[assutQ, FS@{proju.Q, projperpu.Q == Q}]
```
Out[224]=
$$\left\{\left\{O\left[\frac{1}{c}\right]^6, O\left[\frac{1}{c}\right]^6, O\left[\frac{1}{c}\right]^6, O\left[\frac{1}{c}\right]^6\right\}, \text{True}\right\}$$

In[225]:=
```
(* non-symmetric heat-tensor *)
Assuming[assutQ, FS[Qtens = Assuming[assut, Expand //@ FS@PowerExpand[Outer[Times, Q, gg.uu / c ^ 2]]]]] // MF
```
Out[225]//MatrixForm=
$$\begin{pmatrix}
O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \\
-qx + \frac{-\frac{(jx^2+jy^2+jz^2)qx}{2n^2}+qx\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{jx\,qx}{n\,c^2} + \frac{jx\,qx(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jy\,qx}{n\,c^2} + \frac{jy\,qx(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jz\,qx}{n\,c^2} + \frac{jz\,qx(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 \\
-qy + \frac{-\frac{(jx^2+jy^2+jz^2)qy}{2n^2}+qy\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{jx\,qy}{n\,c^2} + \frac{jx\,qy(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jy\,qy}{n\,c^2} + \frac{jy\,qy(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jz\,qy}{n\,c^2} + \frac{jz\,qy(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 \\
-qz + \frac{-\frac{(jx^2+jy^2+jz^2)qz}{2n^2}+qz\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{jx\,qz}{n\,c^2} + \frac{jx\,qz(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jy\,qz}{n\,c^2} + \frac{jy\,qz(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jz\,qz}{n\,c^2} + \frac{jz\,qz(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6
\end{pmatrix}$$

In[226]:=
```
Assuming[assutQ, FS[T[Qtens.Inverse[gg]].gg - Qtens]] // MF
```
Out[226]//MatrixForm=
$$\begin{pmatrix}
O[\frac{1}{c}]^6 & \frac{qx}{c^2} + \frac{\frac{(jx^2+jy^2+jz^2)qx}{2n^2}+3\,qx\,W}{c^4} + O[\frac{1}{c}]^6 & \frac{qy}{c^2} + \frac{\frac{(jx^2+jy^2+jz^2)qy}{2n^2}+3\,qy\,W}{c^4} + O[\frac{1}{c}]^6 & \frac{qz}{c^2} + \frac{\frac{(jx^2+jy^2+jz^2)qz}{2n^2}+3\,qz\,W}{c^4} + O[\frac{1}{c}]^6 \\
qx + \frac{\frac{(jx^2+jy^2+jz^2)qx}{2n^2}-qx\,W}{c^2} + O[\frac{1}{c}]^4 & O[\frac{1}{c}]^6 & \frac{-jy\,qx+jx\,qy}{n\,c^2} + \frac{(-jy\,qx+jx\,qy)(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{-jz\,qx+jx\,qz}{n\,c^2} + \frac{(-jz\,qx+jx\,qz)(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 \\
qy + \frac{\frac{(jx^2+jy^2+jz^2)qy}{2n^2}-qy\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{jy\,qx-jx\,qy}{n\,c^2} + \frac{(jy\,qx-jx\,qy)(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & O[\frac{1}{c}]^6 & \frac{-jz\,qy+jy\,qz}{n\,c^2} + \frac{(-jz\,qy+jy\,qz)(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 \\
qz + \frac{\frac{(jx^2+jy^2+jz^2)qz}{2n^2}-qz\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{jz\,qx-jx\,qz}{n\,c^2} + \frac{(jz\,qx-jx\,qz)(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jz\,qy-jy\,qz}{n\,c^2} + \frac{(jz\,qy-jy\,qz)(jx^2+jy^2+jz^2+6n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & O[\frac{1}{c}]^6
\end{pmatrix}$$

In[227]:=
```
(* definition of momentum-flux, orthogonal to matter-current *)
Ptemp = {pt, px, py, pz};
```

In[228]:=
```
Assuming[assut, FS[Ptemp.proju]] // MF
```
Out[228]//MatrixForm=
$$\begin{pmatrix}
\frac{n\,pt+jx\,px+jy\,py+jz\,pz}{n} + \frac{(jx^2+jy^2+jz^2)(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^3c^2} + O[\frac{1}{c}]^4 \\
-\frac{jx(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^2c^2} - \frac{jx(n\,pt+jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4} + O[\frac{1}{c}]^6 \\
-\frac{jy(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^2c^2} - \frac{jy(n\,pt+jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4} + O[\frac{1}{c}]^6 \\
-\frac{jz(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^2c^2} - \frac{jz(n\,pt+jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4} + O[\frac{1}{c}]^6
\end{pmatrix}$$

In[229]:=
```
psol = Solve[Normal[Ptemp.proju] == 0, pt][[1]]
```
Out[229]=
$$\left\{pt \to -\frac{jx\,px + jy\,py + jz\,pz}{n}\right\}$$

In[230]:=
```
(P = Assuming[assut, FS[Ptemp /. psol]]) // MF
```
Out[230]//MatrixForm=
$$\begin{pmatrix} -\frac{jx\,px+jy\,py+jz\,pz}{n} \\ px \\ py \\ pz \end{pmatrix}$$

In[231]:=
```
{FS[Normal@Series[P.uu, {c, Infinity, 1}]] == 0}
```
Out[231]=
{True}

In[232]:=
```
Assuming[assutQ, FS@{P.proju, P.projperpu == P}]
```
Out[232]=
$$\left\{\left\{O\left[\frac{1}{c}\right]^4, O\left[\frac{1}{c}\right]^6, O\left[\frac{1}{c}\right]^6, O\left[\frac{1}{c}\right]^6\right\}, \text{True}\right\}$$

In[233]:=
```
(* non-symmetric momentum-tensor *)
Assuming[assut, FS[Ptens = Assuming[assut, FS[Outer[Times, uu, P / c ^ 2]]]]] // MF
```
Out[233]//MatrixForm=
$$\begin{pmatrix}
-\frac{jx\,px+jy\,py+jz\,pz}{n\,c^2} - \frac{(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{px}{c^2} + \frac{\frac{(jx^2+jy^2+jz^2)px}{2n^2}+px\,W}{c^4} + O[\frac{1}{c}]^6 & \frac{py}{c^2} + \frac{\frac{(jx^2+jy^2+jz^2)py}{2n^2}+py\,W}{c^4} + O[\frac{1}{c}]^6 & \frac{pz}{c^2} + \frac{\frac{(jx^2+jy^2+jz^2)pz}{2n^2}+pz\,W}{c^4} + O[\frac{1}{c}]^6 \\
-\frac{jx(jx\,px+jy\,py+jz\,pz)}{n^2c^2} - \frac{jx(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^4c^4} + O[\frac{1}{c}]^6 & \frac{jx\,px}{n\,c^2} + \frac{jx\,px(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jx\,py}{n\,c^2} + \frac{jx\,py(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jx\,pz}{n\,c^2} + \frac{jx\,pz(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 \\
-\frac{jy(jx\,px+jy\,py+jz\,pz)}{n^2c^2} - \frac{jy(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^4c^4} + O[\frac{1}{c}]^6 & \frac{jy\,px}{n\,c^2} + \frac{jy\,px(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jy\,py}{n\,c^2} + \frac{jy\,py(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jy\,pz}{n\,c^2} + \frac{jy\,pz(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 \\
-\frac{jz(jx\,px+jy\,py+jz\,pz)}{n^2c^2} - \frac{jz(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^4c^4} + O[\frac{1}{c}]^6 & \frac{jz\,px}{n\,c^2} + \frac{jz\,px(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jz\,py}{n\,c^2} + \frac{jz\,py(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jz\,pz}{n\,c^2} + \frac{jz\,pz(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6
\end{pmatrix}$$

In[234]:=
```
FS[T[Ptens.Inverse[gg]].gg - Ptens] // MF
```
Out[234]//MatrixForm=
$$\begin{pmatrix}
O[\frac{1}{c}]^6 & -\frac{px}{c^2} + \frac{jx^2\,px+2\,jx(jy\,py+jz\,pz)-px(jy^2+jz^2+2n^2W)}{2n^2c^4} + O[\frac{1}{c}]^6 & -\frac{py}{c^2} + \frac{-\frac{(jx^2+jy^2+jz^2)py}{2n^2}+\frac{jy(jx\,px+jy\,py+jz\,pz)}{n^2}-py\,W}{c^4} + O[\frac{1}{c}]^6 & -\frac{pz}{c^2} + \frac{-\frac{(jx^2+jy^2+jz^2)pz}{2n^2}+\frac{jz(jx\,px+jy\,py+jz\,pz)}{n^2}-pz\,W}{c^4} + O[\frac{1}{c}]^6 \\
-px + \frac{-\frac{(jx^2+jy^2+jz^2)px}{2n^2}+\frac{jx(jx\,px+jy\,py+jz\,pz)}{n^2}+3\,px\,W}{c^2} + O[\frac{1}{c}]^4 & O[\frac{1}{c}]^6 & \frac{jy\,px-jx\,py}{n\,c^2} + \frac{(jy\,px-jx\,py)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{jz\,px-jx\,pz}{n\,c^2} + \frac{(jz\,px-jx\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 \\
-py + \frac{-\frac{(jx^2+jy^2+jz^2)py}{2n^2}+\frac{jy(jx\,px+jy\,py+jz\,pz)}{n^2}+3\,py\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{-jy\,px+jx\,py}{n\,c^2} + \frac{(-jy\,px+jx\,py)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & O[\frac{1}{c}]^6 & \frac{jz\,py-jy\,pz}{n\,c^2} + \frac{(jz\,py-jy\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 \\
-pz + \frac{-\frac{(jx^2+jy^2+jz^2)pz}{2n^2}+\frac{jz(jx\,px+jy\,py+jz\,pz)}{n^2}+3\,pz\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{-jz\,px+jx\,pz}{n\,c^2} + \frac{(-jz\,px+jx\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & \frac{-jz\,py+jy\,pz}{n\,c^2} + \frac{(-jz\,py+jy\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4} + O[\frac{1}{c}]^6 & O[\frac{1}{c}]^6
\end{pmatrix}$$

In[235]:=
```
(* definition of stress, orthogonal to matter-current *)
(Stemp = {{stt, stx, sty, stz}, {sxt, sxx, sxy, sxz}, {syt, syx, syy, syz}, {szt, szx, szy, szz}}) // MF
```
Out[235]//MatrixForm=
$$\begin{pmatrix} stt & stx & sty & stz \\ sxt & sxx & sxy & sxz \\ syt & syx & syy & syz \\ szt & szx & szy & szz \end{pmatrix}$$

In[236]:=
```
(Stempsym = Assuming[assut, FS[(T[Stemp.Inverse[gg]].gg + Stemp) / 2]]) // MF
```
Out[236]//MatrixForm=
$$\begin{pmatrix}
stt + O[\frac{1}{c}]^4 & \frac{stx}{2} - \frac{sxt}{2c^2} - \frac{2(sxt\,W)}{c^4} + O[\frac{1}{c}]^6 & \frac{sty}{2} - \frac{syt}{2c^2} - \frac{2(syt\,W)}{c^4} + O[\frac{1}{c}]^6 & \frac{stz}{2} - \frac{szt}{2c^2} - \frac{2(szt\,W)}{c^4} + O[\frac{1}{c}]^6 \\
-\frac{stx\,c^2}{2} + \frac{1}{2}(sxt + 4\,stx\,W) + O[\frac{1}{c}]^2 & sxx + O[\frac{1}{c}]^4 & \frac{sxy+syx}{2} + O[\frac{1}{c}]^4 & \frac{sxz+szx}{2} + O[\frac{1}{c}]^4 \\
-\frac{sty\,c^2}{2} + \frac{1}{2}(syt + 4\,sty\,W) + O[\frac{1}{c}]^2 & \frac{sxy+syx}{2} + O[\frac{1}{c}]^4 & syy + O[\frac{1}{c}]^4 & \frac{syz+szy}{2} + O[\frac{1}{c}]^4 \\
-\frac{stz\,c^2}{2} + \frac{1}{2}(szt + 4\,stz\,W) + O[\frac{1}{c}]^2 & \frac{sxz+szx}{2} + O[\frac{1}{c}]^4 & \frac{syz+szy}{2} + O[\frac{1}{c}]^4 & szz + O[\frac{1}{c}]^4
\end{pmatrix}$$

In[237]:=
```
FS[proju.Stemp.proju] // MF
```
Out[237]//MatrixForm=
$$\begin{pmatrix}
\frac{n\,stt+jx\,stx+jy\,sty+jz\,stz}{n} + \frac{2\,jx^3\,stx+2\,jy^3\,sty+jx^2(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx)-jy\,n^2\,syt+jy^2(2\,n\,stt+2\,jz\,stz-n\,syy)+jx(2\,jy^2\,stx-n^2\,sxt-jy\,n(sxy+syx)+jz(2\,jz\,stx-n(sxz+szx)))+jy\,jz(2\,jz\,sty-n(syz+szy))+jz(2\,jz^2\,stz-n^2\,szt+jz\,n(2\,stt-szz))}{n^3c^2} + O[\frac{1}{c}]^4 & -\frac{jx(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2c^2} + \frac{jx(-2\,jx^3\,stx-2\,jy^3\,sty+jx^2(-2(n\,stt+jy\,sty+jz\,stz)+n\,sxx)+jy^2(-2\,jz\,stz+n(-2\,stt+syy))+jz(-2\,jz^2\,stz+n^2\,szt+jz\,n(-2\,stt+szz))-4\,n^2(n\,stt+jz\,stz)W+jx(-2 \cdots}{n^4c^4} & \cdots \\
\frac{jx(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2} + \frac{jx(2\,jx^3\,stx+2\,jy^3\,sty+jx^2(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx)-jy\,n^2\,syt+jy^2(2\,n\,stt+2\,jz\,stz-n\,syy)+jx(2\,jy^2\,stx-n^2\,sxt-jy\,n(sxy+syx)+jz(2\,jz\,stx-n(sxz+szx)))+jy\,jz(2\,jz\,sty-n(syz+szy))+jz(2\,jz^2\,stz-n^2\,szt+jz\,n(2\,stt-szz)))}{n^4c^2} + O[\frac{1}{c}]^4 & -\frac{jx^2(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^3c^2} + \frac{jx^2(-2\,jx^3\,stx-2\,jy^3\,sty+jx^2(-2(n\,stt+jy\,sty+jz\,stz)+n\,sxx)+jy^2(-2\,jz\,stz+n(-2\,stt+syy))+jz(-2\,jz^2\,stz+n^2\,szt+jz\,n(-2\,stt+szz))-4\,n^2(n\,stt+jz\,stz)W+jx(\cdots}{n^5c^4} & \cdots \\
\frac{jy(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2} + \frac{jy(2\,jx^3\,stx+2\,jy^3\,sty+jx^2(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx)-jy\,n^2\,syt+jy^2(2\,n\,stt+2\,jz\,stz-n\,syy)+jx(2\,jy^2\,stx-n^2\,sxt-jy\,n(sxy+syx)+jz(2\,jz\,stx-n(sxz+szx)))+jy\,jz(2\,jz\,sty-n(syz+szy))+jz(2\,jz^2\,stz-n^2\,szt+jz\,n(2\,stt-szz)))}{n^4c^2} + O[\frac{1}{c}]^4 & -\frac{jx\,jy(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^3c^2} + \frac{jx\,jy(-2\,jx^3\,stx-2\,jy^3\,sty+jx^2(-2(n\,stt+jy\,sty+jz\,stz)+n\,sxx)+jy^2(-2\,jz\,stz+n(-2\,stt+syy))+jz(-2\,jz^2\,stz+n^2\,szt+jz\,n(-2\,stt+szz))-4\,n^2(n\,stt+jz\,stz)W+ \cdots}{n^5c^4} & \cdots \\
\frac{jz(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2} + \frac{jz(2\,jx^3\,stx+2\,jy^3\,sty+jx^2(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx)-jy\,n^2\,syt+jy^2(2\,n\,stt+2\,jz\,stz-n\,syy)+jx(2\,jy^2\,stx-n^2\,sxt-jy\,n(sxy+syx)+jz(2\,jz\,stx-n(sxz+szx)))+jy\,jz(2\,jz\,sty-n(syz+szy))+jz(2\,jz^2\,stz-n^2\,szt+jz\,n(2\,stt-szz)))}{n^4c^2} + O[\frac{1}{c}]^4 & -\frac{jx\,jz(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^3c^2} + \frac{jx\,jz(-2\,jx^3\,stx-2\,jy^3\,sty+jx^2(-2(n\,stt+jy\,sty+jz\,stz)+n\,sxx)+jy^2(-2\,jz\,stz+n(-2\,stt+syy))+jz(-2\,jz^2\,stz+n^2\,szt+jz\,n(-2\,stt+szz))-4\,n^2(n\,stt+jz\,stz)W+ \cdots}{n^5c^4} & \cdots
\end{pmatrix}$$

In[238]:=
```
ssol = Solve[{Normal[proju.Stemp.proju] == 0, Normal[projperpu.Stemp.projperpu] == Stemp}, {stt, stx, sty, stz, sxt, syt, szt}][[1]]
```
Out[238]=
$$\left\{stt \to -\left(\left(jx^2\,sxx + jx\,jy\,sxy + jx\,jz\,sxz + jx\,jy\,syx + jy^2\,syy + jy\,jz\,syz + jx\,jz\,szx + jy\,jz\,szy + jz^2\,szz\right) / \left(jx^2 + jy^2 + jz^2 + c^2\,n^2\right)\right),\right.$$
$$stx \to \frac{n(jx\,sxx + jy\,syx + jz\,szx)}{jx^2 + jy^2 + jz^2 + c^2\,n^2},\ sty \to \frac{n(jx\,sxy + jy\,syy + jz\,szy)}{jx^2 + jy^2 + jz^2 + c^2\,n^2},\ stz \to \frac{n(jx\,sxz + jy\,syz + jz\,szz)}{jx^2 + jy^2 + jz^2 + c^2\,n^2},\ sxt \to -\frac{jx\,sxx + jy\,sxy + jz\,sxz}{n},\ syt \to -\frac{jx\,syx + jy\,syy + jz\,syz}{n},\ \left.szt \to -\frac{jx\,szx + jy\,szy + jz\,szz}{n}\right\}$$

In[239]:=
```
ssol = Solve[Normal[proju.Stemp] == 0 && Normal[Stemp.proju] == 0, {stt, stx, sty, stz, sxt, syt, szt}][[1]]
```
Out[239]=
$$\left\{stt \to -\left(\left(jx^2\,sxx + jx\,jy\,sxy + jx\,jz\,sxz + jx\,jy\,syx + jy^2\,syy + jy\,jz\,syz + jx\,jz\,szx + jy\,jz\,szy + jz^2\,szz\right) / \left(jx^2 + jy^2 + jz^2 + c^2\,n^2\right)\right),\right.$$
$$stx \to \frac{n(jx\,sxx + jy\,syx + jz\,szx)}{jx^2 + jy^2 + jz^2 + c^2\,n^2},\ sty \to \frac{n(jx\,sxy + jy\,syy + jz\,szy)}{jx^2 + jy^2 + jz^2 + c^2\,n^2},\ stz \to \frac{n(jx\,sxz + jy\,syz + jz\,szz)}{jx^2 + jy^2 + jz^2 + c^2\,n^2},\ sxt \to -\frac{jx\,sxx + jy\,sxy + jz\,sxz}{n},\ syt \to -\frac{jx\,syx + jy\,syy + jz\,syz}{n},\ \left.szt \to -\frac{jx\,szx + jy\,szy + jz\,szz}{n}\right\}$$

In[240]:=
```
(S = Assuming[assut, FS[(Stemp /. ssol)]]) // MF
```
Out[240]//MatrixForm=
$$\begin{pmatrix}
-\frac{jx^2\,sxx+jx\,jy(sxy+syx)+jy^2\,syy+jx\,jz(sxz+szx)+jy\,jz(syz+szy)+jz^2\,szz}{jx^2+jy^2+jz^2+c^2n^2} & \frac{n(jx\,sxx+jy\,syx+jz\,szx)}{jx^2+jy^2+jz^2+c^2n^2} & \frac{n(jx\,sxy+jy\,syy+jz\,szy)}{jx^2+jy^2+jz^2+c^2n^2} & \frac{n(jx\,sxz+jy\,syz+jz\,szz)}{jx^2+jy^2+jz^2+c^2n^2} \\
-\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n} & sxx & sxy & sxz \\
-\frac{jx\,syx+jy\,syy+jz\,syz}{n} & syx & syy & syz \\
-\frac{jx\,szx+jy\,szy+jz\,szz}{n} & szx & szy & szz
\end{pmatrix}$$

In[241]:=
```
MF /@ FS@{proju.S.proju, Assuming[assut, Expand //@ FS@PowerExpand[projperpu.S.projperpu - S]]}
```
Out[241]=
$$\left\{\begin{pmatrix}
O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \\
O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \\
O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \\
O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8
\end{pmatrix},
\begin{pmatrix}
-\frac{(jx^2\,sxx+jx\,jy(sxy+syx)+jy^2\,syy+jx\,jz(sxz+szx)+jy\,jz(syz+szy)+jz^2\,szz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4} + O[\frac{1}{c}]^6 & \frac{(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4n^2W)}{n^3c^4} + O[\frac{1}{c}]^6 & \frac{(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4n^2W)}{n^3c^4} + O[\frac{1}{c}]^6 & \frac{(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4n^2W)}{n^3c^4} + O[\frac{1}{c}]^6 \\
O[\frac{1}{c}]^4 & \frac{jx(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4} + O[\frac{1}{c}]^6 & \frac{jx(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4} + O[\frac{1}{c}]^6 & \frac{jx(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4} + O[\frac{1}{c}]^6 \\
O[\frac{1}{c}]^4 & \frac{jy(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4} + O[\frac{1}{c}]^6 & \frac{jy(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4} + O[\frac{1}{c}]^6 & \frac{jy(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4} + O[\frac{1}{c}]^6 \\
O[\frac{1}{c}]^4 & \frac{jz(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4} + O[\frac{1}{c}]^6 & \frac{jz(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4} + O[\frac{1}{c}]^6 & \frac{jz(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4n^2W)}{n^4c^4} + O[\frac{1}{c}]^6
\end{pmatrix}\right\}$$

In[242]:=
```
FS[T[S.Inverse[gg]].gg - S] // MF
```
Out[242]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^6 & \frac{jy\,sxy+jz\,sxz-jy\,syx-jz\,szx}{n\,c^2}+\frac{\frac{(jx^2+jy^2+jz^2)(jx\,sxx+jy\,syx+jz\,szx)}{n^3}+\frac{4(jx\,sxx+jy\,syx+jz\,szx)W}{n}}{c^4}+O[\frac{1}{c}]^6 & \frac{jx(-sxy+syx)+jz(syz-szy)}{n\,c^2}+\frac{\frac{(jx^2+jy^2+jz^2)(jx\,sxy+jy\,syy+jz\,szy)}{n^3}+\frac{4(jx\,sxy+jy\,syy+jz\,syz)W}{n}}{c^4}+O[\frac{1}{c}]^6 & \frac{jx(-sxz+szx)+jy(-syz+szy)}{n\,c^2}+\frac{\frac{(jx^2+jy^2+jz^2)(jx\,sxz+jy\,syz+jz\,szz)}{n^3}+\frac{4(jx\,szx+jy\,szy+jz\,szz)W}{n}}{c^4}+O[\frac{1}{c}]^6 \\ \frac{jy\,sxy+jz\,sxz-jy\,syx-jz\,szx}{n}+\frac{(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4n^2W)}{n^3c^2}+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & (-sxy+syx)+O[\frac{1}{c}]^4 & (-sxz+szx)+O[\frac{1}{c}]^4 \\ \frac{jx(-sxy+syx)+jz(syz-szy)}{n}+\frac{(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4n^2W)}{n^3c^2}+O[\frac{1}{c}]^4 & (sxy-syx)+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & (-syz+szy)+O[\frac{1}{c}]^4 \\ \frac{jx(-sxz+szx)+jy(-syz+szy)}{n}+\frac{(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4n^2W)}{n^3c^2}+O[\frac{1}{c}]^4 & (sxz-szx)+O[\frac{1}{c}]^4 & (syz-szy)+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}$$

In[243]:=
```
(* define "dust" 4-stress *)
(dust = Assuming[assut, FS[(ρ * c^2 + ϵ) * Outer[Times, NJ, gg.uu / c^2]]]) // MF
```
Out[243]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & jx\,\rho+\frac{\frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2}+jx(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jy\,\rho+\frac{\frac{jy(jx^2+jy^2+jz^2)\rho}{2n^2}+jy(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jz\,\rho+\frac{\frac{jz(jx^2+jy^2+jz^2)\rho}{2n^2}+jz(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -jx\,\rho c^2+\left(-\frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2}+jx(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & \frac{jx^2\rho}{n}+\frac{jx^2((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jx\,jy\,\rho}{n}+\frac{jx\,jy((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jx\,jz\,\rho}{n}+\frac{jx\,jz((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 \\ -jy\,\rho c^2+\left(-\frac{jy(jx^2+jy^2+jz^2)\rho}{2n^2}+jy(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & \frac{jx\,jy\,\rho}{n}+\frac{jx\,jy((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jy^2\rho}{n}+\frac{jy^2((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jy\,jz\,\rho}{n}+\frac{jy\,jz((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 \\ -jz\,\rho c^2+\left(-\frac{jz(jx^2+jy^2+jz^2)\rho}{2n^2}+jz(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & \frac{jx\,jz\,\rho}{n}+\frac{jx\,jz((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jy\,jz\,\rho}{n}+\frac{jy\,jz((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jz^2\rho}{n}+\frac{jz^2((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[244]:=
```
FS[T[dust.Inverse[gg]] == dust.Inverse[gg]]
```
Out[244]=

True

In[245]:=
```
FS[T[dust].gg == gg.dust]
```
Out[245]=

True

In[246]:=
```
MF /@ FS@{proju.dust.proju == dust, projperpu.dust.projperpu}
```
Out[246]=

$$\left\{\text{True}, \begin{pmatrix} O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

In[247]:=
```
showf[assut][dust2 = Assuming[assut, Expand //@ FS@PowerExpand[(ρ * c^2 + ϵ) * Outer[Times, NJ, gg.auu / c^2]]]];
```

In[339]:=
```
(* define generic 4-stress *)
(EPS = Assuming[assut, FS[dust + Qtens + Ptens + S]]) // MF
```
Out[339]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & jx\,\rho+\frac{px+\frac{jx\,sxx+jy\,syx+jz\,szx}{n}+\frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2}+jx(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jy\,\rho+\frac{py+\frac{jx\,sxy+jy\,syy+jz\,szy}{n}+\frac{jy(jx^2+jy^2+jz^2)\rho}{2n^2}+jy(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jz\,\rho+\frac{pz+\frac{jx\,sxz+jy\,syz+jz\,szz}{n}+\frac{jz(jx^2+jy^2+jz^2)\rho}{2n^2}+jz(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -jx\,\rho c^2-\frac{2n(jx\,sxx+jy\,sxy+jz\,sxz+n(qx+jx\,\epsilon))+jx(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}+O[\frac{1}{c}]^2 & \left(sxx+\frac{jx^2\rho}{n}\right)+\frac{jx(jx(jx^2+jy^2+jz^2)\rho+2n^2(px+qx+jx\,\epsilon+3\,jx\,W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \left(sxy+\frac{jx\,jy\,\rho}{n}\right)+\frac{\frac{jx\,py+jy\,qx+jx\,jy\,\epsilon}{n}+\frac{jx\,jy(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(sxz+\frac{jx\,jz\,\rho}{n}\right)+\frac{\frac{jx\,pz+jz\,qx+jx\,jz\,\epsilon}{n}+\frac{jx\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 \\ -jy\,\rho c^2-\frac{2n(jx\,syx+jy\,syy+jz\,syz+n(qy+jy\,\epsilon))+jy(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}+O[\frac{1}{c}]^2 & \left(syx+\frac{jx\,jy\,\rho}{n}\right)+\frac{\frac{jy\,px+jx\,qy+jx\,jy\,\epsilon}{n}+\frac{jx\,jy(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(syy+\frac{jy^2\rho}{n}\right)+\frac{jy(jy(jx^2+jy^2+jz^2)\rho+2n^2(py+qy+jy\,\epsilon+3\,jy\,W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \left(syz+\frac{jy\,jz\,\rho}{n}\right)+\frac{\frac{jy\,pz+jz\,qy+jy\,jz\,\epsilon}{n}+\frac{jy\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 \\ -jz\,\rho c^2-\frac{2n(jx\,szx+jy\,szy+jz\,szz+n(qz+jz\,\epsilon))+jz(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}+O[\frac{1}{c}]^2 & \left(szx+\frac{jx\,jz\,\rho}{n}\right)+\frac{\frac{jz\,px+jx\,qz+jx\,jz\,\epsilon}{n}+\frac{jx\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(szy+\frac{jy\,jz\,\rho}{n}\right)+\frac{\frac{jz\,py+jy\,qz+jy\,jz\,\epsilon}{n}+\frac{jy\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(szz+\frac{jz^2\rho}{n}\right)+\frac{jz(jz(jx^2+jy^2+jz^2)\rho+2n^2(pz+qz+jz\,\epsilon+3\,jz\,W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[344]:=
```
Assuming[assut, FS[(EPS /. replaceJu)]] // MF
```
Out[344]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-\frac{1}{2}n(2\epsilon+(ux^2+uy^2+uz^2-2W)\rho)+O[\frac{1}{c}]^2 & n\,ux\,\rho+\frac{px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon+\frac{1}{2}n\,ux(ux^2+uy^2+uz^2+6W)\rho}{c^2}+O[\frac{1}{c}]^4 & n\,uy\,\rho+\frac{py+sxy\,ux+syy\,uy+szy\,uz+n\,uy\,\epsilon+\frac{1}{2}n\,uy(ux^2+uy^2+uz^2+6W)\rho}{c^2}+O[\frac{1}{c}]^4 & n\,uz\,\rho+\frac{pz+sxz\,ux+syz\,uy+szz\,uz+n\,uz\,\epsilon+\frac{1}{2}n\,uz(ux^2+uy^2+uz^2+6W)\rho}{c^2}+O[\frac{1}{c}]^4 \\ -n\,ux\,\rho c^2+\left(-qx-sxy\,uy-sxz\,uz-ux(sxx+n\epsilon)-\frac{1}{2}n\,ux(ux^2+uy^2+uz^2-2W)\rho\right)+O[\frac{1}{c}]^2 & (sxx+n\,ux^2\rho)+\frac{ux(2(px+qx+n\,ux\,\epsilon)+n\,ux(ux^2+uy^2+uz^2+6W)\rho)}{2c^2}+O[\frac{1}{c}]^4 & (sxy+n\,ux\,uy\,\rho)+\frac{py\,ux+qx\,uy+\frac{1}{2}n\,ux\,uy(2\epsilon+(ux^2+uy^2+uz^2+6W)\rho)}{c^2}+O[\frac{1}{c}]^4 & (sxz+n\,ux\,uz\,\rho)+\frac{pz\,ux+qx\,uz+\frac{1}{2}n\,ux\,uz(2\epsilon+(ux^2+uy^2+uz^2+6W)\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -n\,uy\,\rho c^2+\left(-qy-syx\,ux-syz\,uz-uy(syy+n\epsilon)-\frac{1}{2}n\,uy(ux^2+uy^2+uz^2-2W)\rho\right)+O[\frac{1}{c}]^2 & (syx+n\,ux\,uy\,\rho)+\frac{qy\,ux+px\,uy+\frac{1}{2}n\,ux\,uy(2\epsilon+(ux^2+uy^2+uz^2+6W)\rho)}{c^2}+O[\frac{1}{c}]^4 & (syy+n\,uy^2\rho)+\frac{uy(2(py+qy+n\,uy\,\epsilon)+n\,uy(ux^2+uy^2+uz^2+6W)\rho)}{2c^2}+O[\frac{1}{c}]^4 & (syz+n\,uy\,uz\,\rho)+\frac{pz\,uy+qy\,uz+\frac{1}{2}n\,uy\,uz(2\epsilon+(ux^2+uy^2+uz^2+6W)\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -n\,uz\,\rho c^2+\left(-qz-szx\,ux-szy\,uy-uz(szz+n\epsilon)-\frac{1}{2}n\,uz(ux^2+uy^2+uz^2-2W)\rho\right)+O[\frac{1}{c}]^2 & (szx+n\,ux\,uz\,\rho)+\frac{qz\,ux+px\,uz+\frac{1}{2}n\,ux\,uz(2\epsilon+(ux^2+uy^2+uz^2+6W)\rho)}{c^2}+O[\frac{1}{c}]^4 & (szy+n\,uy\,uz\,\rho)+\frac{qz\,uy+py\,uz+\frac{1}{2}n\,uy\,uz(2\epsilon+(ux^2+uy^2+uz^2+6W)\rho)}{c^2}+O[\frac{1}{c}]^4 & (szz+n\,uz^2\rho)+\frac{uz(2(pz+qz+n\,uz\,\epsilon)+n\,uz(ux^2+uy^2+uz^2+6W)\rho)}{2c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[343]:=
```
Assuming[assut, FS[(EPS /. replaceJu) /. replaceuUnorm]] // MF
```
Out[343]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-n\epsilon-\frac{1}{2}nU^2\rho+nW\rho\right)+O[\frac{1}{c}]^2 & n\,ux\,\rho+\frac{px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon+\frac{1}{2}n\,ux(U^2+6W)\rho}{c^2}+O[\frac{1}{c}]^4 & n\,uy\,\rho+\frac{py+sxy\,ux+syy\,uy+szy\,uz+n\,uy\,\epsilon+\frac{1}{2}n\,uy(U^2+6W)\rho}{c^2}+O[\frac{1}{c}]^4 & n\,uz\,\rho+\frac{pz+sxz\,ux+syz\,uy+szz\,uz+n\,uz\,\epsilon+\frac{1}{2}n\,uz(U^2+6W)\rho}{c^2}+O[\frac{1}{c}]^4 \\ -n\,ux\,\rho c^2+\left(-qx-sxy\,uy-sxz\,uz-\frac{1}{2}ux(2(sxx+n\epsilon)+n(U^2-2W)\rho)\right)+O[\frac{1}{c}]^2 & (sxx+n(U^2-uy^2-uz^2)\rho)+\frac{ux(2(px+qx+n\,ux\,\epsilon)+n\,ux(U^2+6W)\rho)}{2c^2}+O[\frac{1}{c}]^4 & (sxy+n\,ux\,uy\,\rho)+\frac{py\,ux+qx\,uy+\frac{1}{2}n\,ux\,uy(2\epsilon+U^2\rho+6W\rho)}{c^2}+O[\frac{1}{c}]^4 & (sxz+n\,ux\,uz\,\rho)+\frac{pz\,ux+qx\,uz+\frac{1}{2}n\,ux\,uz(2\epsilon+U^2\rho+6W\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -n\,uy\,\rho c^2+\left(-qy-syx\,ux-syz\,uz-\frac{1}{2}uy(2(syy+n\epsilon)+n(U^2-2W)\rho)\right)+O[\frac{1}{c}]^2 & (syx+n\,ux\,uy\,\rho)+\frac{qy\,ux+px\,uy+\frac{1}{2}n\,ux\,uy(2\epsilon+U^2\rho+6W\rho)}{c^2}+O[\frac{1}{c}]^4 & (syy+n\,uy^2\rho)+\frac{uy(2(py+qy+n\,uy\,\epsilon)+n\,uy(U^2+6W)\rho)}{2c^2}+O[\frac{1}{c}]^4 & (syz+n\,uy\,uz\,\rho)+\frac{pz\,uy+qy\,uz+\frac{1}{2}n\,uy\,uz(2\epsilon+U^2\rho+6W\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -n\,uz\,\rho c^2+\left(-qz-szx\,ux-szy\,uy-\frac{1}{2}uz(2(szz+n\epsilon)+n(U^2-2W)\rho)\right)+O[\frac{1}{c}]^2 & (szx+n\,ux\,uz\,\rho)+\frac{qz\,ux+px\,uz+\frac{1}{2}n\,ux\,uz(2\epsilon+U^2\rho+6W\rho)}{c^2}+O[\frac{1}{c}]^4 & (szy+n\,uy\,uz\,\rho)+\frac{qz\,uy+py\,uz+\frac{1}{2}n\,uy\,uz(2\epsilon+U^2\rho+6W\rho)}{c^2}+O[\frac{1}{c}]^4 & (szz+n\,uz^2\rho)+\frac{uz(2(pz+qz+n\,uz\,\epsilon)+n\,uz(U^2+6W)\rho)}{2c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[249]:=
```
FS[T[EPS.Inverse[gg]].gg - EPS] // MF
```
Out[249]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^2 & \frac{n(-px+qx)+jy(sxy-syx)+jz(sxz-szx)}{n\,c^2}+O[\frac{1}{c}]^4 & \frac{n(-py+qy)+jx(-sxy+syx)+jz(syz-szy)}{n\,c^2}+O[\frac{1}{c}]^4 & \frac{n(-pz+qz)+jx(-sxz+szx)+jy(-syz+szy)}{n\,c^2}+O[\frac{1}{c}]^4 \\ \frac{n(-px+qx)+jy(sxy-syx)+jz(sxz-szx)}{n}+O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & (-sxy+syx)+\frac{jy\,px-jx\,py-jy\,qx+jx\,qy}{n\,c^2}+O[\frac{1}{c}]^4 & (-sxz+szx)+\frac{jz\,px-jx\,pz-jz\,qx+jx\,qz}{n\,c^2}+O[\frac{1}{c}]^4 \\ \frac{n(-py+qy)+jx(-sxy+syx)+jz(syz-szy)}{n}+O[\frac{1}{c}]^2 & (sxy-syx)+\frac{jy(-px+qx)+jx(py-qy)}{n\,c^2}+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & (-syz+szy)+\frac{jz\,py-jy\,pz-jz\,qy+jy\,qz}{n\,c^2}+O[\frac{1}{c}]^4 \\ \frac{n(-pz+qz)+jx(-sxz+szx)+jy(-syz+szy)}{n}+O[\frac{1}{c}]^2 & (sxz-szx)+\frac{jz(-px+qx)+jx(pz-qz)}{n\,c^2}+O[\frac{1}{c}]^4 & (syz-szy)+\frac{jz(-py+qy)+jy(pz-qz)}{n\,c^2}+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}$$

In[250]:=
```
(* Conditions for symmetry of total 4-stress tensor *)
subsym = {px → qx, py → qy, pz → qz, syx → sxy, szx → sxz, szy → syz};
```

In[251]:=
```
(EPSsym = Assuming[assut, FS[EPS /. subsym]]) // MF
```
Out[251]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & jx\,\rho+\frac{qx+\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n}+\frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2}+jx(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jy\,\rho+\frac{qy+\frac{jx\,sxy+jy\,syy+jz\,syz}{n}+\frac{jy(jx^2+jy^2+jz^2)\rho}{2n^2}+jy(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jz\,\rho+\frac{qz+\frac{jx\,sxz+jy\,syz+jz\,szz}{n}+\frac{jz(jx^2+jy^2+jz^2)\rho}{2n^2}+jz(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -jx\,\rho c^2-\frac{2n(jx\,sxx+jy\,sxy+jz\,sxz+n(qx+jx\,\epsilon))+jx(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}+O[\frac{1}{c}]^2 & \left(sxx+\frac{jx^2\rho}{n}\right)+\frac{jx(jx(jx^2+jy^2+jz^2)\rho+2n^2(2\,qx+jx(\epsilon+3W\rho)))}{2n^3c^2}+O[\frac{1}{c}]^4 & \left(sxy+\frac{jx\,jy\,\rho}{n}\right)+\frac{\frac{jy\,qx+jx\,qy+jx\,jy\,\epsilon}{n}+\frac{jx\,jy(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(sxz+\frac{jx\,jz\,\rho}{n}\right)+\frac{\frac{jz\,qx+jx\,qz+jx\,jz\,\epsilon}{n}+\frac{jx\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 \\ -jy\,\rho c^2-\frac{2n(jx\,sxy+jy\,syy+jz\,syz+n(qy+jy\,\epsilon))+jy(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}+O[\frac{1}{c}]^2 & \left(sxy+\frac{jx\,jy\,\rho}{n}\right)+\frac{\frac{jy\,qx+jx\,qy+jx\,jy\,\epsilon}{n}+\frac{jx\,jy(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(syy+\frac{jy^2\rho}{n}\right)+\frac{jy(jy(jx^2+jy^2+jz^2)\rho+2n^2(2\,qy+jy(\epsilon+3W\rho)))}{2n^3c^2}+O[\frac{1}{c}]^4 & \left(syz+\frac{jy\,jz\,\rho}{n}\right)+\frac{\frac{jz\,qy+jy\,qz+jy\,jz\,\epsilon}{n}+\frac{jy\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 \\ -jz\,\rho c^2-\frac{2n(jx\,sxz+jy\,syz+jz\,szz+n(qz+jz\,\epsilon))+jz(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}+O[\frac{1}{c}]^2 & \left(sxz+\frac{jx\,jz\,\rho}{n}\right)+\frac{\frac{jz\,qx+jx\,qz+jx\,jz\,\epsilon}{n}+\frac{jx\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(syz+\frac{jy\,jz\,\rho}{n}\right)+\frac{\frac{jz\,qy+jy\,qz+jy\,jz\,\epsilon}{n}+\frac{jy\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(szz+\frac{jz^2\rho}{n}\right)+\frac{jz(jz(jx^2+jy^2+jz^2)\rho+2n^2(2\,qz+jz(\epsilon+3W\rho)))}{2n^3c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[252]:=
```
FS[T[EPSsym.Inverse[gg]].gg - EPSsym] // MF
```
Out[252]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}$$

In[345]:=
```
Assuming[assut, FS[(EPSsym /. replaceJu) /. replaceuUnorm]] // MF
```
Out[345]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-n\epsilon-\frac{1}{2}nU^2\rho+nW\rho\right)+O[\frac{1}{c}]^2 & n\,ux\,\rho+\frac{qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,ux\,\epsilon+\frac{1}{2}n\,ux(U^2+6W)\rho}{c^2}+O[\frac{1}{c}]^4 & n\,uy\,\rho+\frac{qy+sxy\,ux+syy\,uy+syz\,uz+n\,uy\,\epsilon+\frac{1}{2}n\,uy(U^2+6W)\rho}{c^2}+O[\frac{1}{c}]^4 & n\,uz\,\rho+\frac{qz+sxz\,ux+syz\,uy+szz\,uz+n\,uz\,\epsilon+\frac{1}{2}n\,uz(U^2+6W)\rho}{c^2}+O[\frac{1}{c}]^4 \\ -n\,ux\,\rho c^2+\left(-qx-sxy\,uy-sxz\,uz-\frac{1}{2}ux(2(sxx+n\epsilon)+n(U^2-2W)\rho)\right)+O[\frac{1}{c}]^2 & (sxx+n(U^2-uy^2-uz^2)\rho)+\frac{ux(4\,qx+n\,ux(2\epsilon+(U^2+6W)\rho))}{2c^2}+O[\frac{1}{c}]^4 & (sxy+n\,ux\,uy\,\rho)+\frac{qy\,ux+qx\,uy+\frac{1}{2}n\,ux\,uy(2\epsilon+U^2\rho+6W\rho)}{c^2}+O[\frac{1}{c}]^4 & (sxz+n\,ux\,uz\,\rho)+\frac{qz\,ux+qx\,uz+\frac{1}{2}n\,ux\,uz(2\epsilon+U^2\rho+6W\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -n\,uy\,\rho c^2+\left(-qy-sxy\,ux-syz\,uz-\frac{1}{2}uy(2(syy+n\epsilon)+n(U^2-2W)\rho)\right)+O[\frac{1}{c}]^2 & (sxy+n\,ux\,uy\,\rho)+\frac{qy\,ux+qx\,uy+\frac{1}{2}n\,ux\,uy(2\epsilon+U^2\rho+6W\rho)}{c^2}+O[\frac{1}{c}]^4 & (syy+n\,uy^2\rho)+\frac{uy(4\,qy+n\,uy(2\epsilon+(U^2+6W)\rho))}{2c^2}+O[\frac{1}{c}]^4 & (syz+n\,uy\,uz\,\rho)+\frac{qz\,uy+qy\,uz+\frac{1}{2}n\,uy\,uz(2\epsilon+U^2\rho+6W\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -n\,uz\,\rho c^2+\left(-qz-sxz\,ux-syz\,uy-\frac{1}{2}uz(2(szz+n\epsilon)+n(U^2-2W)\rho)\right)+O[\frac{1}{c}]^2 & (sxz+n\,ux\,uz\,\rho)+\frac{qz\,ux+qx\,uz+\frac{1}{2}n\,ux\,uz(2\epsilon+U^2\rho+6W\rho)}{c^2}+O[\frac{1}{c}]^4 & (syz+n\,uy\,uz\,\rho)+\frac{qz\,uy+qy\,uz+\frac{1}{2}n\,uy\,uz(2\epsilon+U^2\rho+6W\rho)}{c^2}+O[\frac{1}{c}]^4 & (szz+n\,uz^2\rho)+\frac{uz(4\,qz+n\,uz(2\epsilon+(U^2+6W)\rho))}{2c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[254]:=

(* Balanced quantities constructed from energy–momentum tensor, and their supplies *)

In[347]:=
```
(* Symmetrized energy-stress tensor, with explicit dep. on coords *)
(TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]]) // MF
```
Out[347]//MatrixForm=

$$-\rho\,n[t,x,y,z]\,c^2+\frac{1}{2}\left(-\frac{\rho\,(n[t,x,y,z]^2\,ux[t,x,y,z]^2+n[t,x,y,z]^2\,uy[t,x,y,z]^2+n[t,x,y,z]^2\,uz[t,x,y,z]^2)}{n[t,x,y,z]}+2\,n[t,x,y,z]\,(-\epsilon+\rho\,W[t,x,y,z])\right)+O[\tfrac{1}{c}]^2$$

$$-\rho\,n[t,x,y,z]\times ux[t,x,y,z]\,c^2+\frac{1}{2}\Big(-px-\frac{sxx\,n[t,x,y,z]\times ux[t,x,y,z]+syx\,n[t,x,y,z]\times uy[t,x,y,z]+szx\,n[t,x,y,z]\times uz[t,x,y,z]}{n[t,x,y,z]}-\frac{\rho\,ux[t,x,y,z]\,(n[t,x,y,z]^2\,ux[t,x,y,z]^2+n[t,x,y,z]^2\,uy[t,x,y,z]^2+n[t,x,y,z]^2\,uz[t,x,y,z]^2)}{2\,n[t,x,y,z]}+4\,\rho\,n[t,x,y,z]\times ux[t,x,y,z]\times W[t,x,y,z]-n[t,x,y,z]\times ux[t,x,y,z]\,(\epsilon+3\,\rho\,W[t,x,y,z])-\frac{2\,n[t,x,y,z]\,(sxx\,n[t,x,y,z]\times ux[t,x,y,z]+n[t,x,y,z]\,(qx+\epsilon\,n[t,x,y,z]\times ux[t,x,y,z])+\ldots}{\ldots}$$

$$-\rho\,n[t,x,y,z]\times uy[t,x,y,z]\,c^2+\frac{1}{2}\Big(-py-\frac{sxy\,n[t,x,y,z]\times ux[t,x,y,z]+syy\,n[t,x,y,z]\times uy[t,x,y,z]+szy\,n[t,x,y,z]\times uz[t,x,y,z]}{n[t,x,y,z]}-\frac{\rho\,uy[t,x,y,z]\,(n[t,x,y,z]^2\,ux[t,x,y,z]^2+n[t,x,y,z]^2\,uy[t,x,y,z]^2+n[t,x,y,z]^2\,uz[t,x,y,z]^2)}{2\,n[t,x,y,z]}+4\,\rho\,n[t,x,y,z]\times uy[t,x,y,z]\times W[t,x,y,z]-n[t,x,y,z]\times uy[t,x,y,z]\,(\epsilon+3\,\rho\,W[t,x,y,z])-\frac{2\,n[t,x,y,z]\,(syx\,n[t,x,y,z]\times ux[t,x,y,z]+syy\,n[t,x,y,z]\times uy[t,x,y,z]+n[t,x,y,z]\,(q\ldots}{\ldots}$$

$$-\rho\,n[t,x,y,z]\times uz[t,x,y,z]\,c^2+\frac{1}{2}\Big(-pz-\frac{sxz\,n[t,x,y,z]\times ux[t,x,y,z]+syz\,n[t,x,y,z]\times uy[t,x,y,z]+szz\,n[t,x,y,z]\times uz[t,x,y,z]}{n[t,x,y,z]}-\frac{\rho\,uz[t,x,y,z]\,(n[t,x,y,z]^2\,ux[t,x,y,z]^2+n[t,x,y,z]^2\,uy[t,x,y,z]^2+n[t,x,y,z]^2\,uz[t,x,y,z]^2)}{2\,n[t,x,y,z]}+4\,\rho\,n[t,x,y,z]\times uz[t,x,y,z]\times W[t,x,y,z]-n[t,x,y,z]\times uz[t,x,y,z]\,(\epsilon+3\,\rho\,W[t,x,y,z])-\frac{2\,n[t,x,y,z]\,(szx\,n[t,x,y,z]\times ux[t,x,y,z]+szy\,n[t,x,y,z]\times uy[t,x,y,z]+szz\,n[t,x,y,\ldots}{\ldots}$$

In[348]:=
```
(Duu = Assuming[assut, FS[(D[Normal@uut, {coords}] + Sum[uut[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]) // MF
```
Out[348]//MatrixForm=

$$\frac{n[t,x,y,z]\,(jx[t,x,y,z]\,(-n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+jx^{(1,0,0,0)}[t,x,y,z])+jy[t,x,y,z]\,(-n[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]+jy^{(1,0,0,0)}[t,x,y,z])+jz[t,x,y,z]\,(-n[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]+jz^{(1,0,0,0)}[t,x,y,z]))-(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2)\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^3\,c^2}+O[\tfrac{1}{c}]^4$$

$$\frac{n[t,x,y,z]\,(jx[t,x,y,z]\,jx^{(0,1,0,0)}[t,x,y,z]+jy[t,x,y,z]\,jy^{(0,1,0,0)}[t,x,y,z]+jz[t,x,y,z]\,jz^{(0,1,0,0)}[t,x,y,z])-(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2)\,n^{(0,1,0,0)}[t,x,y,z]}{n[t,x,y,z]^3\,c^2}+O[\tfrac{1}{c}]^4$$

$$\left(-W^{(0,1,0,0)}[t,x,y,z]+\frac{n[t,x,y,z]\,jx^{(1,0,0,0)}[t,x,y,z]-jx[t,x,y,z]\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}\right)+O[\tfrac{1}{c}]^2 \qquad \frac{n[t,x,y,z]\,jx^{(0,1,0,0)}[t,x,y,z]-jx[t,x,y,z]\,n^{(0,1,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}+\frac{2\,jy[t,x,y,z]\,(n[t,x,y,z]^3\,W^{(0,0,1,0)}[t,x,y,z]+jx[t,x,y,z]\times n[t,x,y,z]\,jy^{(0,1,0,0)}[t,x,y,z])+2\,jz[t,x,y,z]\,(n[t,x,y,z]^3\,W^{(0,0,0,1)}[t,x,y,z]+jx[t,x,y,z]\times n[t,x,y,z]\,jz^{(0,1,0,0)}[t,x,y,z])+jy[t,x,y,\ldots}{\ldots}$$

$$\left(-W^{(0,0,1,0)}[t,x,y,z]+\frac{n[t,x,y,z]\,jy^{(1,0,0,0)}[t,x,y,z]-jy[t,x,y,z]\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}\right)+O[\tfrac{1}{c}]^2 \qquad \frac{n[t,x,y,z]\,jy^{(0,1,0,0)}[t,x,y,z]-jy[t,x,y,z]\,n^{(0,1,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}+\frac{-2\,jx[t,x,y,z]\,n[t,x,y,z]^3\,W^{(0,0,1,0)}[t,x,y,z]+n[t,x,y,z]\,(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z])\,jy^{(0,1,0,0)}[\ldots}{\ldots}$$

$$\left(-W^{(0,0,0,1)}[t,x,y,z]+\frac{n[t,x,y,z]\,jz^{(1,0,0,0)}[t,x,y,z]-jz[t,x,y,z]\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}\right)+O[\tfrac{1}{c}]^2 \qquad \frac{n[t,x,y,z]\,jz^{(0,1,0,0)}[t,x,y,z]-jz[t,x,y,z]\,n^{(0,1,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}+\frac{jx[t,x,y,z]\,(-2\,n[t,x,y,z]^3\,W^{(0,0,0,1)}[t,x,y,z]+2\,jz[t,x,y,z]\times n[t,x,y,z]\,jx^{(0,1,0,0)}[t,x,y,z])+2\,jy[t,x,y,z]\times jz[t,x,y,z]\times n[t,x,y,z]\,jy^{(0\ldots}{\ldots}$$

In[349]:=
```
(Duv = Assuming[assut, FS[(D[Normal@uuv, {coords}] + Sum[uuv[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]) // MF
```
Out[349]//MatrixForm=

$$\frac{ux[t,x,y,z]\,(-W^{(0,1,0,0)}[t,x,y,z]+ux^{(1,0,0,0)}[t,x,y,z])+uy[t,x,y,z]\,(-W^{(0,0,1,0)}[t,x,y,z]+uy^{(1,0,0,0)}[t,x,y,z])+uz[t,x,y,z]\,(-W^{(0,0,0,1)}[t,x,y,z]+uz^{(1,0,0,0)}[t,x,y,z])}{c^2}+O[\tfrac{1}{c}]^4 \qquad \frac{ux[t,x,y,z]\,ux^{(0,1,0,0)}[t,x,y,z]+uy[t,x,y,z]\,uy^{(0,1,0,0)}[t,x,y,z]+uz[t,x,y,z]\,uz^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4$$

$$(-W^{(0,1,0,0)}[t,x,y,z]+ux^{(1,0,0,0)}[t,x,y,z])+O[\tfrac{1}{c}]^2 \qquad ux^{(0,1,0,0)}[t,x,y,z]+\frac{uy[t,x,y,z]^2\,ux^{(0,1,0,0)}[t,x,y,z]+uz[t,x,y,z]^2\,ux^{(0,1,0,0)}[t,x,y,z]+(3\,ux[t,x,y,z]^2+2\,W[t,x,y,z])\,ux^{(0,1,0,0)}[t,x,y,z]+2\,uy[t,x,y,z]\,(W^{(0,0,1,0)}[t,x,y,z]+ux[t,x,y,z]\,uy^{(0,1,0,0)}[t,x,y,z])+2\,uz[t,x,y,z]\,(W^{(0,0,0,1)}[t,x,y,z]+ux[t,x,y,z]\,uz^{(0,1,0,0)}[t,x,y,z])+4\,ux[t,x,y,z]\,W^{(0,1,0,0)}[t,x,\ldots}{2\,c^2}$$

$$(-W^{(0,0,1,0)}[t,x,y,z]+uy^{(1,0,0,0)}[t,x,y,z])+O[\tfrac{1}{c}]^2 \qquad uy^{(0,1,0,0)}[t,x,y,z]+\frac{-2\,ux[t,x,y,z]\,(W^{(0,0,1,0)}[t,x,y,z]-uy[t,x,y,z]\,ux^{(0,1,0,0)}[t,x,y,z])+ux[t,x,y,z]^2\,uy^{(0,1,0,0)}[t,x,y,z]+3\,uy[t,x,y,z]^2\,uy^{(0,1,0,0)}[t,x,y,z]+(uz[t,x,y,z]^2+2\,W[t,x,y,z])\,uy^{(0,1,0,0)}[t,x,y,z]+2\,uy[t,x,y,z]\,(uz[t,x,y,z]\,uz^{(0,1,0,0)}[t,x,y,z]+2\,W^{(0,1,0,0)}[t,x,y,z])}{2\,c^2}+O[\tfrac{1}{c}]^4$$

$$(-W^{(0,0,0,1)}[t,x,y,z]+uz^{(1,0,0,0)}[t,x,y,z])+O[\tfrac{1}{c}]^2 \qquad uz^{(0,1,0,0)}[t,x,y,z]+\frac{-ux[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]+\frac{1}{2}(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z])\,uz^{(0,1,0,0)}[t,x,y,z]+uz[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+uz[t,x,y,z]\,(ux[t,x,y,z]\,ux^{(0,1,0,0)}[t,x,y,z]+uy[t,x,y,z]\,uy^{(0,1,0,0)}[t,x,y,z]+uz[t,x,y,z]\,uz^{(0,1,0,0)}[t,x,y,z]+W^{(0,1,0,0)}[t,x,y,z])}{c^2}+O[\ldots$$

In[359]:=

```
(* Energy current and supply according to 4-velocity *)
pvec = uu; Dpvec = Duv;
MF@(MF/@{Efluxuu = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Esupplyuu = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[360]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} -n\,\rho\,c^2 - n\,\epsilon + O\left[\frac{1}{c}\right]^2 \\ \left(-Ax\,jx - Ay\,jy - Az\,jz + Ax\,n\,Vx + Ay\,n\,Vy + Az\,n\,Vz\right)\rho\,c^2 + \left(-Ax\left(qx + \left(jx - n\,Vx\right)\epsilon\right) - Ay\left(qy + jy\,\epsilon - n\,Vy\,\epsilon\right) - Az\left(qz + jz\,\epsilon - n\,Vz\,\epsilon\right)\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix} \\ \frac{1}{2}\left(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z] + (syz+szy)\left(uy^{(0,0,0,1)}[t,x,y,z] + uz^{(0,0,1,0)}[t,x,y,z]\right) + 2\left(syy\,uy^{(0,0,1,0)}[t,x,y,z] + sxx\,ux^{(0,1,0,0)}[t,x,y,z]\right) + (sxy+syx)\left(ux^{(0,0,1,0)}[t,x,y,z] + uy^{(0,1,0,0)}[t,x,y,z]\right) + (sxz+szx)\left(ux^{(0,0,0,1)}[t,x,y,z] + uz^{(0,1,0,0)}[t,x,y,z]\right)\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[361]:=

```
MF@(MF/@(FS[({Efluxuu, Esupplyuu}/. replaceJu)/. replaceuUnorm]))
```

Out[361]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} -n\,\rho\,c^2 - n\,\epsilon + O\left[\frac{1}{c}\right]^2 \\ n\left(Ax\left(-ux+Vx\right) + Ay\left(-uy+Vy\right) + Az\left(-uz+Vz\right)\right)\rho\,c^2 + \left(-Ax\,qx - Ay\,qy - Az\,qz + n\left(Ax\left(-ux+Vx\right) + Ay\left(-uy+Vy\right) + Az\left(-uz+Vz\right)\right)\epsilon\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix} \\ \frac{1}{2}\left(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z] + (syz+szy)\left(uy^{(0,0,0,1)}[t,x,y,z] + uz^{(0,0,1,0)}[t,x,y,z]\right) + 2\left(syy\,uy^{(0,0,1,0)}[t,x,y,z] + sxx\,ux^{(0,1,0,0)}[t,x,y,z]\right) + (sxy+syx)\left(ux^{(0,0,1,0)}[t,x,y,z] + uy^{(0,1,0,0)}[t,x,y,z]\right) + (sxz+szx)\left(ux^{(0,0,0,1)}[t,x,y,z] + uz^{(0,1,0,0)}[t,x,y,z]\right)\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[362]:=

```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]]/. replaceJu] // MF}
```

Out[362]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{qx}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n} + \frac{qy}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n} + \frac{qz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uy + \frac{qy}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uz + \frac{qz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[363]:=

```
(* Energy current and supply according to t-vector *)
pvec = -{1, 0, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF/@{Efluxt = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Esupplyt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[364]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\,\rho\,c^2 + \left(\frac{\left(jx^2+jy^2+jz^2\right)\rho}{2\,n} + n\left(\epsilon - W\,\rho\right)\right) + O\left[\frac{1}{c}\right]^2 \\ \left(Ay\,jy + Az\,jz + Ax\left(jx - n\,Vx\right) - n\left(Ay\,Vy + Az\,Vz\right)\right)\rho\,c^2 + \frac{2\,n\left(Ax\left(jx\,sxx+jy\,sxy+jz\,sxz+n\left(qx+\left(jx-n\,Vx\right)\epsilon\right)\right)+Ay\left(jx\,syx+jy\,syy+jz\,syz+n\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)\right)+Az\left(jx\,szx+jy\,szy+jz\,szz+n\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)\right)+\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx^2+jy^2+jz^2-2\,n^2\,W\right)\rho}{2\,n^2} + O\left[\frac{1}{c}\right]^2 \end{pmatrix} \\ -n\,\rho\,W^{(1,0,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[365]:=

```
MF@(MF/@(FS[({Efluxt, Esupplyt}/. replaceJu)/. replaceuUnorm]))
```

Out[365]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\,\rho\,c^2 + n\left(\epsilon + \frac{1}{2}\left(U^2 - 2\,W\right)\rho\right) + O\left[\frac{1}{c}\right]^2 \\ n\left(Ax\left(ux - Vx\right) + Ay\left(uy - Vy\right) + Az\left(uz - Vz\right)\right)\rho\,c^2 + \left(Ax\left(qx + sxx\,ux + sxy\,uy + sxz\,uz + n\left(ux - Vx\right)\epsilon\right) + Ay\left(qy + syx\,ux + syy\,uy + syz\,uz + n\left(uy - Vy\right)\epsilon\right) + Az\left(qz + szx\,ux + szy\,uy + szz\,uz + n\left(uz - Vz\right)\epsilon\right) + \frac{1}{2}\,n\left(Ax\left(ux - Vx\right) + Ay\left(uy - Vy\right) + Az\left(uz - Vz\right)\right)\left(U^2 - 2\,W\right)\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix} \\ -n\,\rho\,W^{(1,0,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[366]:=

```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]]/. replaceJu] // MF}
```

Out[366]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[367]:=

```
(* Energy current and supply according to norm. t-vector *)
pvec = -c*{1, 0, 0, 0}/Sqrt[-gg[[1, 1]]]; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF/@{Efluxnt = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Esupplynt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[368]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\,\rho\,c^2 + \left(n\,\epsilon + \frac{\left(jx^2+jy^2+jz^2\right)\rho}{2\,n}\right) + O\left[\frac{1}{c}\right]^2 \\ \left(Ay\,jy + Az\,jz + Ax\left(jx - n\,Vx\right) - n\left(Ay\,Vy + Az\,Vz\right)\right)\rho\,c^2 + \frac{2\,n\left(Ax\left(jx\,sxx+jy\,sxy+jz\,sxz+n\left(qx+\left(jx-n\,Vx\right)\epsilon\right)\right)+Ay\left(jx\,syx+jy\,syy+jz\,syz+n\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)\right)+Az\left(jx\,szx+jy\,szy+jz\,szz+n\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)\right)+\left(jx^2+jy^2+jz^2\right)\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{2\,n^2} + O\left[\frac{1}{c}\right]^2 \\ \rho\left(jz\,W^{(0,0,0,1)}[t,x,y,z] + jy\,W^{(0,0,1,0)}[t,x,y,z] + jx\,W^{(0,1,0,0)}[t,x,y,z]\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[369]:=

```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]]/. replaceJu] // MF}
```

Out[369]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[370]:=

```
(* Energy current and supply according to cov. t-vector *)
pvec = c^2*igg.{1, 0, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF/@{Efluxcovt = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Esupplycovt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[371]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\,\rho\,c^2 + \left(\frac{\left(jx^2+jy^2+jz^2\right)\rho}{2\,n} + n\left(\epsilon + W\,\rho\right)\right) + O\left[\frac{1}{c}\right]^2 \\ \left(Ay\,jy + Az\,jz + Ax\left(jx - n\,Vx\right) - n\left(Ay\,Vy + Az\,Vz\right)\right)\rho\,c^2 + \frac{2\,n\left(Ax\left(jx\,sxx+jy\,sxy+jz\,sxz+n\left(qx+\left(jx-n\,Vx\right)\epsilon\right)\right)+Ay\left(jx\,syx+jy\,syy+jz\,syz+n\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)\right)+Az\left(jx\,szx+jy\,szy+jz\,szz+n\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)\right)+\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx^2+jy^2+jz^2-2\,n^2\,W\right)\rho}{2\,n^2} + O\left[\frac{1}{c}\right]^2 \\ \rho\left(2\,jz\,W^{(0,0,0,1)}[t,x,y,z] + 2\,jy\,W^{(0,0,1,0)}[t,x,y,z] + 2\,jx\,W^{(0,1,0,0)}[t,x,y,z] + n\,W^{(1,0,0,0)}[t,x,y,z]\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[372]:=

```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]]/. replaceJu] // MF}
```

Out[372]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[373]:=

```
(* Momentum current and supply according to x-vector *)
pvec = {0, 1, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF/@{Pfluxx = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Psupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[374]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} jx\,\rho + \frac{px + \frac{jx\,sxx+jy\,syx+jz\,szx}{n} + \frac{jx\left(jx^2+jy^2+jz^2\right)\rho}{2\,n^2} + jx\left(\epsilon + 3\,W\,\rho\right)}{c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + \frac{jx\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right) + \frac{-n\left(Ax\,Vx+Ay\,Vy+Az\,Vz\right)\left(2\,n\left(jx\,sxx+jy\,syx+jz\,szx+n\left(px+jx\,\epsilon\right)\right)+jx\left(jx^2+jy^2+jz^2+6\,n^2\,W\right)\rho\right)+Ay\left(2\,n^2\left(jx\,qy+jy\left(px+jx\,\epsilon\right)\right)+jx\,jy\left(jx^2+jy^2+jz^2+6\,n^2\,W\right)\rho\right)+Az\left(2\,n^2\left(jx\,qz+jz\left(px+jx\,\epsilon\right)\right)+jx\,jz\left(jx^2+jy^2+jz^2+6\,n^2\,W\right)\rho\right)+Ax\,jx\left(jx\left(jx^2+jy^2+jz^2\right)\rho+2\,n^2\left(px+qx+jx\,\epsilon+3\,jx\,W\,\rho\right)\right)}{2\,n^3\,c^2} + O\left[\frac{1}{c}\right]^4 \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[375]:=

```
MF@(MF/@(FS[({Pfluxx, Psupplyx}/. replaceJu)/. replaceuUnorm]))
```

Out[375]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\,ux\,\rho + \frac{px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon+\frac{1}{2}\,n\,ux\left(U^2+6\,W\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + n\,ux\left(Ax\left(ux - Vx\right) + Ay\left(uy - Vy\right) + Az\left(uz - Vz\right)\right)\rho\right) + \frac{Ax\left(qx\,ux+px\left(ux-Vx\right)-\left(sxx\,ux+syx\,uy+szx\,uz\right)Vx+n\,ux\left(ux-Vx\right)\epsilon\right)+Ay\left(qy\,ux+px\left(uy-Vy\right)-\left(sxx\,ux+syx\,uy+szx\,uz\right)Vy+n\,ux\left(uy-Vy\right)\epsilon\right)+Az\left(qz\,ux+px\left(uz-Vz\right)-\left(sxx\,ux+syx\,uy+szx\,uz\right)Vz+n\,ux\left(uz-Vz\right)\epsilon\right)+\frac{1}{2}\,Ax\,n\,ux\left(ux-Vx\right)\left(U^2+6\,W\right)\rho+\frac{1}{2}\,n\,ux\left(Ay\left(uy-Vy\right)+Az\left(uz-Vz\right)\right)\left(U^2+6\,W\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^4 \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[376]:=

```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]]/. replaceJu] // MF}
```

Out[376]=

$$\left\{\begin{pmatrix} \left(\frac{jx}{n} + \frac{sxx}{jx\,\rho}\right) - \frac{2\,n\,sxx\left(jx\,sxx+jy\,syx+jz\,szx+n\left(px+jx\,\epsilon\right)\right)+jx\left(3\,jx^2\,sxx+2\,jx\left(-n\,qx+jy\,syx+jz\,szx\right)+sxx\left(jy^2+jz^2+6\,n^2\,W\right)\right)\rho}{2\left(jx^2\,n^2\,\rho^2\right)c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(\frac{jy}{n} + \frac{syx}{jx\,\rho}\right) - \frac{2\,n\,syx\left(jx\,sxx+jy\,syx+jz\,szx+n\left(px+jx\,\epsilon\right)\right)+jx\left(-2\,jx\,n\,qy+2\,jx\,jy\,sxx+jx^2\,syx+3\,jy^2\,syx+jz^2\,syx+2\,jy\,jz\,sxz+6\,n^2\,syx\,W\right)\rho}{2\left(jx^2\,n^2\,\rho^2\right)c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(\frac{jz}{n} + \frac{szx}{jx\,\rho}\right) - \frac{2\,n\,szx\left(jx\,sxx+jy\,syx+jz\,szx+n\left(px+jx\,\epsilon\right)\right)+jx\left(-2\,jx\,n\,qz+2\,jx\,jz\,sxx+2\,jy\,jz\,syx+jx^2\,szx+jy^2\,szx+3\,jz^2\,szx+6\,n^2\,szx\,W\right)\rho}{2\left(jx^2\,n^2\,\rho^2\right)c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} \left(ux + \frac{sxx}{n\,ux\,\rho}\right) - \frac{2\,sxx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\,ux\left(-2\,qx\,ux+2\,ux\left(syx\,uy+szx\,uz\right)+sxx\left(3\,ux^2+uy^2+uz^2+6\,W\right)\right)\rho}{2\left(n^2\,ux^2\,\rho^2\right)c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(uy + \frac{syx}{n\,ux\,\rho}\right) - \frac{2\,syx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\,ux\left(-2\,qy\,ux+2\,uy\left(sxx\,ux+szx\,uz\right)+syx\left(ux^2+3\,uy^2+uz^2+6\,W\right)\right)\rho}{2\left(n^2\,ux^2\,\rho^2\right)c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(uz + \frac{szx}{n\,ux\,\rho}\right) - \frac{2\,szx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\,ux\left(-2\,qz\,ux+2\left(sxx\,ux+syx\,uy\right)uz+szx\left(ux^2+uy^2+3\,uz^2+6\,W\right)\right)\rho}{2\left(n^2\,ux^2\,\rho^2\right)c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[377]:=

```
(* Momentum current and supply according to cov. x-vector *)
pvec = igg.{0, 1, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF/@{Pfluxx = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Psupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[378]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} jx\,\rho + \frac{px + \frac{jx\,sxx+jy\,syx+jz\,szx}{n} + \frac{jx\left(jx^2+jy^2+jz^2\right)\rho}{2\,n^2} + jx\left(\epsilon + W\,\rho\right)}{c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + \frac{jx\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right) + \frac{Ax\left(-\left(\left(n\,px+jy\,syx+jz\,szx\right)Vx\right)-2\,n\,sxx\,W+jx\left(px+qx-Vx\left(sxx+n\,\epsilon\right)\right)+Ay\left(-\left(\left(n\,px+jz\,szx\right)Vy\right)-2\,n\,syx\,W+jy\left(px-syx\,Vy+jx\,\epsilon\right)+jx\left(qy-Vy\left(sxx+n\,\epsilon\right)\right)\right)+Az\left(-\left(\left(n\,px+jy\,syx\right)Vz\right)-2\,n\,szx\,W+jz\left(px-szx\,Vz+jx\,\epsilon\right)+jx\left(qz-Vz\left(sxx+n\,\epsilon\right)\right)\right)+\frac{jx\left(Ay\,jy+Az\,jz-Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx^2+jy^2+jz^2-2\,n^2\,W\right)\rho}{2\,n^3}}{c^2} + O\left[\frac{1}{c}\right]^4 \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[379]:=

```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]]/. replaceJu] // MF}
```

Out[379]=

$$\left\{\begin{pmatrix} \left(\frac{jx}{n} + \frac{sxx}{jx\,\rho}\right) - \frac{2\,n\,sxx\left(jx\,sxx+jy\,syx+jz\,szx+n\left(px+jx\,\epsilon\right)\right)+jx\left(3\,jx^2\,sxx+2\,jx\left(-n\,qx+jy\,syx+jz\,szx\right)+sxx\left(jy^2+jz^2+6\,n^2\,W\right)\right)\rho}{2\left(jx^2\,n^2\,\rho^2\right)c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(\frac{jy}{n} + \frac{syx}{jx\,\rho}\right) - \frac{2\,n\,syx\left(jx\,sxx+jy\,syx+jz\,szx+n\left(px+jx\,\epsilon\right)\right)+jx\left(-2\,jx\,n\,qy+2\,jx\,jy\,sxx+jx^2\,syx+3\,jy^2\,syx+jz^2\,syx+2\,jy\,jz\,szx+6\,n^2\,syx\,W\right)\rho}{2\left(jx^2\,n^2\,\rho^2\right)c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(\frac{jz}{n} + \frac{szx}{jx\,\rho}\right) - \frac{2\,n\,szx\left(jx\,sxx+jy\,syx+jz\,szx+n\left(px+jx\,\epsilon\right)\right)+jx\left(-2\,jx\,n\,qz+2\,jx\,jz\,sxx+2\,jy\,jz\,syx+jx^2\,szx+jy^2\,szx+3\,jz^2\,szx+6\,n^2\,szx\,W\right)\rho}{2\left(jx^2\,n^2\,\rho^2\right)c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} \left(ux + \frac{sxx}{n\,ux\,\rho}\right) - \frac{2\,sxx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\,ux\left(-2\,qx\,ux+2\,ux\left(syx\,uy+szx\,uz\right)+sxx\left(3\,ux^2+uy^2+uz^2+6\,W\right)\right)\rho}{2\left(n^2\,ux^2\,\rho^2\right)c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(uy + \frac{syx}{n\,ux\,\rho}\right) - \frac{2\,syx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\,ux\left(-2\,qy\,ux+2\,uy\left(sxx\,ux+szx\,uz\right)+syx\left(ux^2+3\,uy^2+uz^2+6\,W\right)\right)\rho}{2\left(n^2\,ux^2\,\rho^2\right)c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(uz + \frac{szx}{n\,ux\,\rho}\right) - \frac{2\,szx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\,ux\left(-2\,qz\,ux+2\left(sxx\,ux+syx\,uy\right)uz+szx\left(ux^2+uy^2+3\,uz^2+6\,W\right)\right)\rho}{2\left(n^2\,ux^2\,\rho^2\right)c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[380]:=

```
(* Ang.momentum current and supply according to yz-vector *)
pvec = y*{0, 0, 0, 1} - z*{0, 0, 1, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF/@{Lfluxx = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Lsupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[381]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} \left(jz\,y - jy\,z\right)\rho + \frac{2\,n\left(\left(jx\,sxz+jy\,syz+jz\,szz\right)y-\left(jx\,sxy+jy\,syy+jz\,szy\right)z+n\left(pz\,y+jz\,y\,\epsilon-z\left(py+jy\,\epsilon\right)\right)\right)+\left(jx^2+jy^2+jz^2+6\,n^2\,W\right)\left(jz\,y-jy\,z\right)\rho}{2\,n^2\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(\left(Ax\,sxz + Ay\,syz + Az\,szz\right)y - \left(Ax\,sxy + Ay\,syy + Az\,szy\right)z + \frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jz\,y-jy\,z\right)\rho}{n}\right) + \frac{-z\left(-n\left(Ax\,Vx+Ay\,Vy+Az\,Vz\right)\left(2\,n\left(jx\,sxy+jy\,syy+jz\,szy+n\left(py+jy\,\epsilon\right)\right)+jy\left(jx^2+jy^2+jz^2+6\,n^2\,W\right)\rho\right)+Ax\left(2\,n^2\left(jy\,qx+jx\left(py+jy\,\epsilon\right)\right)+jx\,jy\left(jx^2+jy^2+jz^2+6\,n^2\,W\right)\rho\right)+Az\left(2\,n^2\left(jy\,qz+jz\left(py+jy\,\epsilon\right)\right)+jy\,jz\left(jx^2+jy^2+jz^2+6\,n^2\,W\right)\rho\right)+Ay\,jy\left(jy\left(jx^2+jy^2+jz^2\right)\rho+2\,n^2\left(py+qy+jy\,\epsilon+3\,jy\,W\,\rho\right)\right)+y\left(-n\left(Ax\,Vx+Ay\,Vy+Az\,Vz\right)\left(2\,n\left(jx\,sxz+jy\,syz+jz\,szz+n\left(pz+jz\,\epsilon\right)\right)+jz\left(jx^2+jy^2\right.\right.\right.}{2\,n^3\,c^2} \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z] - z\,W^{(0,0,1,0)}[t,x,y,z]\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[382]:=

```
MF@(MF/@(FS[({Lfluxx, Lsupplyx}/. replaceJu)/. replaceuUnorm]))
```

Out[382]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\left(uz\,y - uy\,z\right)\rho + \frac{pz\,y+sxz\,ux\,y+syz\,uy\,y+szz\,uz\,y-\left(py+sxy\,ux\right)z-syy\,uy\,z-szy\,uz\,z+n\,uz\,y\,\epsilon-n\,uy\,z\,\epsilon+\frac{1}{2}\,n\left(U^2+6\,W\right)\left(uz\,y-uy\,z\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(\left(Ax\,sxz + Ay\,syz + Az\,szz\right)y - \left(Ax\,sxy + Ay\,syy + Az\,szy\right)z + n\left(Ax\left(ux - Vx\right) + Ay\left(uy - Vy\right) + Az\left(uz - Vz\right)\right)\left(uz\,y - uy\,z\right)\rho\right) + \frac{Ax\left(qx\,uz\,y+pz\left(ux-Vx\right)y-sxz\,ux\,Vx\,y-syz\,uy\,Vx\,y-szz\,uz\,Vx\,y-py\,ux\,z-qx\,uy\,z+py\,Vx\,z+sxy\,ux\,Vx\,z+syy\,uy\,Vx\,z+szy\,uz\,Vx\,z+n\left(ux-Vx\right)\left(uz\,y-uy\,z\right)\epsilon\right)+Az\left(qz\,uz\,y+pz\left(uz-Vz\right)y-sxz\,ux\,Vz\,y-syz\,uy\,Vz\,y-szz\,uz\,Vz\,y-qz\,uy\,z-py\,uz\,z+py\,Vz\,z+sxy\,ux\,Vz\,z+syy\,uy\,Vz\,z+szy\,uz\,Vz\,z+n\left(uz-Vz\right)\left(uz\,y-uy\,z\right)\epsilon\right)+Ay\left(qy\,uz\,y+pz\left(uy-Vy\right)y\right.}{c^2} \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z] - z\,W^{(0,0,1,0)}[t,x,y,z]\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[383]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]] /. replaceJu] // MF}
```

Out[383]=

[illegible]

$$\left(\begin{array}{l} \left(ux+\frac{sxz\,y-sxy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right)+\frac{-\left(\left(2\left(pz\,y+(sxz\,ux+syz\,uy+szz\,uz)\,y-(sxy\,ux+syy\,uy+szy\,uz)\,z+n\,uz\,y\,\epsilon-z\,(py+n\,uy\,\epsilon)\right)+n\left(ux^2+uy^2+uz^2+6W\right)(uz\,y-uy\,z)\,\rho\right)(sxz\,y+n\,ux\,uz\,y\,\rho-z\,(sxy+n\,ux\,uy\,\rho))\right)+n\,(uz\,y-uy\,z)\,\rho\,(2\,pz\,ux\,y-2\,py\,ux\,z+(uz\,y-uy\,z)\,(2\,qx+n\,ux\,(2\,\epsilon+(ux^2+\ldots}{2\,n^2\,(uz\,y-uy\,z)^2\,\rho^2\,c^2}\ldots \\ \left(uy+\frac{syz\,y-syy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right)-\frac{2\,pz\,y\,(syz\,y-syy\,z)+2\,(syz\,y-syy\,z)\,(sxz\,ux\,y+syz\,uy\,y+szz\,uz\,y-(py+sxy\,ux+syy\,uy)\,z-szy\,uz\,z+n\,uz\,y\,\epsilon-n\,uy\,z\,\epsilon)+n\,(uz\,y-uy\,z)\,(2\,(sxz\,ux\,uy-qy\,uz+szz\,uy\,uz)\,y+syz\,(ux^2+3\,uy^2+uz^2+6W)\,y-(2\,uy\,(-qy+sxy\,ux+szy\,uz)+syy\,(ux^2+\ldots}{2\,(n^2\,(uz\,y-uy\,z)^2\,\rho^2)\,c^2}\ldots \\ \left(uz+\frac{szz\,y-szy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right)-\frac{2\,pz\,y\,(szz\,y-szy\,z)+2\,(szz\,y-szy\,z)\,(sxz\,ux\,y+syz\,uy\,y+szz\,uz\,y-(py+sxy\,ux+syy\,uy)\,z-szy\,uz\,z+n\,uz\,y\,\epsilon-n\,uy\,z\,\epsilon)+n\,(uz\,y-uy\,z)\,(2\,(-qz+sxz\,ux+syz\,uy)\,uz\,y+szz\,(ux^2+uy^2+3\,uz^2+6W)\,y-(-2\,qz\,uy+2\,sxy\,ux\,uz+2\,syy\,uy\,uz+szy\,(u\ldots}{2\,(n^2\,(uz\,y-uy\,z)^2\,\rho^2)\,c^2}\ldots \end{array}\right)$$

In[384]:=
```
(* Ang.momentum current and supply according to cov. yz-vector *)
pvec = igg.(y*{0, 0, 0, 1} - z*{0, 0, 1, 0}); Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF/@{Lfluxcx = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Lsupplycx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[385]//MatrixForm=

$$\left(\begin{array}{l} (jz\,y-jy\,z)\,\rho+\frac{2\,n\left((jx\,sxz+jy\,syz+jz\,szz)\,y-(jx\,sxy+jy\,syy+jz\,szy)\,z+n\,(pz\,y+jz\,y\,\epsilon-z\,(py+jy\,\epsilon))\right)+(jx^2+jy^2+jz^2+2\,n^2\,W)\,(jz\,y-jy\,z)\,\rho}{2\,n^2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \left((Ax\,sxz+Ay\,syz+Az\,szz)\,y-(Ax\,sxy+Ay\,syy+Az\,szy)\,z+\frac{(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jz\,y-jy\,z)\,\rho}{n}\right)+\frac{4\,n^2\,W\,y\,(-n\,(Ax\,sxz+Ay\,syz+Az\,szz)+jz\,(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\,\rho)+4\,n^2\,W\,z\,(n\,(Ax\,sxy+\ldots}{\ldots} \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \end{array}\right)$$

In[386]:=
```
MF@(MF/@(FS[({Lfluxcx, Lsupplycx} /. replaceJu) /. replaceuUnorm]))
```

Out[386]//MatrixForm=

$$\left(\begin{array}{l} n\,(uz\,y-uy\,z)\,\rho+\frac{pz\,y+sxz\,ux\,y+syz\,uy\,y+szz\,uz\,y-(py+sxy\,ux+syy\,uy)\,z-szy\,uz\,z+n\,uz\,y\,\epsilon-n\,uy\,z\,\epsilon+\frac{1}{2}\,n\,(U^2+2W)\,(uz\,y-uy\,z)\,\rho}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \left((Ax\,sxz+Ay\,syz+Az\,szz)\,y-(Ax\,sxy+Ay\,syy+Az\,szy)\,z+n\,(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz))\,(uz\,y-uy\,z)\,\rho\right)+\frac{Ax\,(qx\,uz\,y+pz\,(ux-Vx)\,y-sxz\,ux\,Vx\,y-syz\,uy\,Vx\,y-szz\,uz\,Vx\,y-2\,sxz\,W\,y-py\,ux\,z-qx\,uy\,z+py\,Vx\,z+sxy\,ux\,Vx\,z+syy\,uy\,Vx\,z+szy\,uz\,Vx\,z+2\,sxy\,W\,z+n\,(ux-Vx)\,(uz\,y-uy\,z)\,\epsilon)+Az\,(qz\,uz\,y+\ldots}{c^2} \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \end{array}\right)$$

In[387]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]] /. replaceJu] // MF}
```

Out[387]=

[illegible]

$$\left(\begin{array}{l} \left(ux+\frac{sxz\,y-sxy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right)-\frac{2\,pz\,y\,(sxz\,y-sxy\,z)+2\,(sxz\,y-sxy\,z)\,(sxz\,ux\,y+syz\,uy\,y+szz\,uz\,y-(py+sxy\,ux+syy\,uy)\,z-szy\,uz\,z+n\,uz\,y\,\epsilon-n\,uy\,z\,\epsilon)+n\,(uz\,y-uy\,z)\,(2\,(syz\,ux\,uy-qx\,uz+szz\,ux\,uz)\,y+sxz\,(3\,ux^2+uy^2+uz^2+6W)\,y-(-2\,qx\,uy+2\,syy\,ux\,uy+2\,szy\,ux\,uz+\ldots}{2\,(n^2\,(uz\,y-uy\,z)^2\,\rho^2)\,c^2}\ldots \\ \left(uy+\frac{syz\,y-syy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right)-\frac{2\,pz\,y\,(syz\,y-syy\,z)+2\,(syz\,y-syy\,z)\,(sxz\,ux\,y+syz\,uy\,y+szz\,uz\,y-(py+sxy\,ux+syy\,uy)\,z-szy\,uz\,z+n\,uz\,y\,\epsilon-n\,uy\,z\,\epsilon)+n\,(uz\,y-uy\,z)\,(2\,(sxz\,ux\,uy-qy\,uz+szz\,uy\,uz)\,y+syz\,(ux^2+3\,uy^2+uz^2+6W)\,y-(2\,uy\,(-qy+sxy\,ux+szy\,uz)+syy\,(ux^2+\ldots}{2\,(n^2\,(uz\,y-uy\,z)^2\,\rho^2)\,c^2}\ldots \\ \left(uz+\frac{szz\,y-szy\,z}{n\,uz\,y\,\rho-n\,uy\,z\,\rho}\right)-\frac{2\,pz\,y\,(szz\,y-szy\,z)+2\,(szz\,y-szy\,z)\,(sxz\,ux\,y+syz\,uy\,y+szz\,uz\,y-(py+sxy\,ux+syy\,uy)\,z-szy\,uz\,z+n\,uz\,y\,\epsilon-n\,uy\,z\,\epsilon)+n\,(uz\,y-uy\,z)\,(2\,(-qz+sxz\,ux+syz\,uy)\,uz\,y+szz\,(ux^2+uy^2+3\,uz^2+6W)\,y-(-2\,qz\,uy+2\,sxy\,ux\,uz+2\,syy\,uy\,uz+szy\,(u\ldots}{2\,(n^2\,(uz\,y-uy\,z)^2\,\rho^2)\,c^2}\ldots \end{array}\right)$$

In[388]:=
```
(* Ang.boost-momentum current and supply according to tx-vector *)
pvec = t*{0, 1, 0, 0} + x*{1, 0, 0, 0}/c^2; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF/@{Bfluxx = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Bsupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[389]//MatrixForm=

$$\left(\begin{array}{l} (jx\,t-n\,x)\,\rho+\frac{x\left(-\frac{(jx^2+jy^2+jz^2)\,\rho}{2n}+n\,(-\epsilon+W\rho)\right)+t\left(px+\frac{jx\,sxx+jy\,syx+jz\,szx}{n}+\frac{jx\,(jx^2+jy^2+jz^2)\,\rho}{2\,n^2}+jx\,(\epsilon+3\,W\rho)\right)}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \left((Ax\,sxx+Ay\,syx+Az\,szx)\,t+\frac{(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jx\,t-n\,x)\,\rho}{n}\right)+\frac{-n\,x\,(2\,n\,(Ax\,(jx\,sxx+jy\,sxy+jz\,sxz+n\,(qx+(jx-n\,Vx)\,\epsilon))+Ay\,(jx\,syx+jy\,syy+jz\,syz+n\,(qy+jy\,\epsilon-n\,Vy\,\epsilon))+Az\,(jx\,szx+jy\,szy+jz\,szz+n\,(qz+jz\,\epsilon-n\,Vz\,\epsilon)))+(Ay\ldots}{2\,n^3\,c^2} \\ n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \end{array}\right)$$

In[390]:=
```
MF@(MF/@(FS[({Bfluxx, Bsupplyx} /. replaceJu) /. replaceuUnorm]))
```

Out[390]//MatrixForm=

$$\left(\begin{array}{l} n\,(t\,ux-x)\,\rho+\frac{px\,t+sxx\,t\,ux+syx\,t\,uy+szx\,t\,uz+n\,t\,ux\,\epsilon-n\,x\,\epsilon+\frac{1}{2}\,n\,(t\,U^2\,ux+6\,t\,ux\,W-U^2\,x+2\,W\,x)\,\rho}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \left((Ax\,sxx+Ay\,syx+Az\,szx)\,t+n\,(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz))\,(t\,ux-x)\,\rho\right)+\frac{\ldots}{c^2} \\ n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \end{array}\right)$$

In[391]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]] /. replaceJu] // MF}
```

Out[391]=

[illegible]

$$\left(\begin{array}{l} \left(ux+\frac{sxx\,t}{n\,t\,ux\,\rho-n\,x\,\rho}\right)+\frac{-2\,sxx\,t\,(t\,(px+sxx\,ux+syx\,uy+szx\,uz)+n\,(t\,ux-x)\,\epsilon)+n\,(2\,qx\,(-t\,ux+x)^2-2\,(t\,ux-x)\,(syx\,t\,ux\,uy+szx\,t\,ux\,uz+sxy\,uy\,x+sxz\,uz\,x)+sxx\,(-t^2\,ux\,(3\,ux^2+uy^2+uz^2+6W)+t\,(ux^2+uy^2+uz^2-2W)\,x+2\,ux\,x^2))\,\rho}{2\,n^2\,(-t\,ux+x)^2\,\rho^2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \left(uy+\frac{syx\,t}{n\,t\,ux\,\rho-n\,x\,\rho}\right)-\frac{2\,syx\,t\,(t\,(px+sxx\,ux+syx\,uy+szx\,uz)+n\,(t\,ux-x)\,\epsilon)+n\,(-2\,qy\,(-t\,ux+x)^2+2\,(t\,ux-x)\,(sxx\,t\,ux\,uy+szx\,t\,uy\,uz+syy\,uy\,x+syz\,uz\,x)+syx\,(t^2\,ux\,(ux^2+3\,uy^2+uz^2+6W)+t\,(ux^2-3\,uy^2-uz^2+2W)\,x-2\,ux\,x^2))\,\rho}{2\,(n^2\,(-t\,ux+x)^2\,\rho^2)\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \left(uz+\frac{szx\,t}{n\,t\,ux\,\rho-n\,x\,\rho}\right)-\frac{2\,szx\,t\,(t\,(px+sxx\,ux+syx\,uy+szx\,uz)+n\,(t\,ux-x)\,\epsilon)+n\,(-2\,qz\,(-t\,ux+x)^2+2\,(t\,ux-x)\,(sxx\,t\,ux\,uz+syx\,t\,uy\,uz+szy\,uy\,x+szz\,uz\,x)+szx\,(t^2\,ux\,(ux^2+uy^2+3\,uz^2+6W)+t\,(ux^2-uy^2-3\,uz^2+2W)\,x-2\,ux\,x^2))\,\rho}{2\,(n^2\,(-t\,ux+x)^2\,\rho^2)\,c^2}+O\left[\frac{1}{c}\right]^4 \end{array}\right)$$

In[392]:=
```
(* Ang.boost-momentum current and supply according to cov. tx-vector *)
pvec = igg.(t*{0, 1, 0, 0} - x*{1, 0, 0, 0}); Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF/@{Bfluxcx = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Bsupplycx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[393]//MatrixForm=

$$\left(\begin{array}{l} (jx\,t-n\,x)\,\rho+\frac{2\,n\,((n\,px+jx\,sxx+jy\,syx+jz\,szx)\,t+n\,(jx\,t-n\,x)\,\epsilon)+(jx^2+jy^2+jz^2+2\,n^2\,W)\,(jx\,t-n\,x)\,\rho}{2\,n^2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \left((Ax\,sxx+Ay\,syx+Az\,szx)\,t+\frac{(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jx\,t-n\,x)\,\rho}{n}\right)+\frac{-2\,Ay\,n^2\,(t\,(-jx\,qy+n\,px\,Vy+jx\,sxx\,Vy+jz\,szx\,Vy+2\,n\,syx\,W)+(n\,qy+jx\,syx+jz\,syz)\,x+jy\,(-px\,t+syx\,t\,Vy+syy\,x))-2\,Az\,n^2\,(t\,(-jx\,qz+n\,px\,Vz+jx\,sxx\,Vz+jy\ldots}{2\,n^3\,c^2} \\ n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \end{array}\right)$$

In[394]:=
```
MF@(MF/@(FS[({Bfluxcx, Bsupplycx} /. replaceJu) /. replaceuUnorm]))
```

Out[394]//MatrixForm=

$$\left(\begin{array}{l} n\,(t\,ux-x)\,\rho+\frac{px\,t+sxx\,t\,ux+syx\,t\,uy+szx\,t\,uz+n\,t\,ux\,\epsilon-n\,x\,\epsilon+\frac{1}{2}\,n\,(U^2+2W)\,(t\,ux-x)\,\rho}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \left((Ax\,sxx+Ay\,syx+Az\,szx)\,t+n\,(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz))\,(t\,ux-x)\,\rho\right)+\frac{\ldots}{c^2} \\ n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \end{array}\right)$$

In[395]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]] /. replaceJu] // MF}
```

Out[395]=

[illegible]

```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg)/2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand//@ FS@PowerExpand[Tr[supply.TTx]], {supply, {Dtxyzvec[[1]], Dtxyzvec[[2]], {0, 0, 0, 0}, Dgtxyzvec[[1]]*c^2, Dgtxyzvec[[2]], {0, 0, 0, 0}, DLxvec, Dxboost/c, {0, 0, 0, 0}, DgLxvec, Dgxboost, {0, 0, 0, 0}, Duu, Dtvecnorm}}]]

(* coordinate/internal/coordinate-proper energy and x-momentum, content and fluxes (TRANSPOSED) *)
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, igg[[1]]*c^2, igg[[2]], {0, 0, 0, 0}, Lxvec, xboost/c, {0, 0, 0, 0}, gLxvec, gxboost, {0, 0, 0, 0}, uu, tvecnorm}])]]]
```

Out[=]//MatrixForm=

$$\left(\begin{array}{ll}
-n\rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(-\epsilon+W\rho)\right)+O\left[\frac{1}{c}\right]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\,\rho c^2+\left(-\frac{Ax\,(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz)+Ay\,(n\,qy+jx\,syx+jy\,syy+jz\,syz)+Az\,(n\,qz+jx\,szx+jy\,szy+jz\,szz)}{n}+(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\,\epsilon-\frac{(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jx^2+jy^2+jz^2-2\,n^2\,W)\,\rho}{2\,n^2}\right)+O\left[\frac{1}{c}\right]^2 \\
jx\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\
0 & 0 \\
n\rho c^2+\left(\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(\epsilon+W\rho)\right)+O\left[\frac{1}{c}\right]^2 & (Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho c^2+\frac{2\,n\,(Ax\,(jx\,sxx+jy\,sxy+jz\,sxz+n\,(qx+(jx-n\,Vx)\,\epsilon))+Ay\,(jx\,syx+jy\,syy+jz\,syz+n\,(qy+jy\,\epsilon-n\,Vy\,\epsilon))+Az\,(jx\,szx+jy\,szy+jz\,szz+n\,(qz+jz\,\epsilon-n\,Vz\,\epsilon)))+(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jx^2+jy^2+jz^2+2\,n^2\,W)\,\rho}{2\,n^2}+O\left[\frac{1}{c}\right]^2 \\
jx\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\
0 & 0 \\
(jz\,y-jy\,z)\,\rho+O\left[\frac{1}{c}\right]^2 & \left((Ax\,sxz+Ay\,syz+Az\,szz)\,y-(Ax\,sxy+Ay\,syy+Az\,szy)\,z+\frac{(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jz\,y-jy\,z)\,\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\
(jx\,t-n\,x)\,\rho+O\left[\frac{1}{c}\right]^2 & \left((Ax\,sxx+Ay\,syx+Az\,szx)\,t+\frac{(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jx\,t-n\,x)\,\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\
0 & 0 \\
(jz\,y-jy\,z)\,\rho+O\left[\frac{1}{c}\right]^2 & \left((Ax\,sxz+Ay\,syz+Az\,szz)\,y-(Ax\,sxy+Ay\,syy+Az\,szy)\,z+\frac{(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jz\,y-jy\,z)\,\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\
(-jx\,t\,\rho+n\,x\,\rho)+O\left[\frac{1}{c}\right]^2 & \left(-((Ax\,sxx+Ay\,syx+Az\,szx)\,t)-\frac{(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jx\,t-n\,x)\,\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\
0 & 0 \\
-n\rho c^2-n\epsilon+O\left[\frac{1}{c}\right]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\,\rho c^2+(-Ax\,(qx+(jx-n\,Vx)\,\epsilon)-Ay\,(qy+jy\,\epsilon-n\,Vy\,\epsilon)-Az\,(qz+jz\,\epsilon-n\,Vz\,\epsilon))+O\left[\frac{1}{c}\right]^2 \\
-n\rho c^2+\left(-n\epsilon-\frac{(jx^2+jy^2+jz^2)\rho}{2n}\right)+O\left[\frac{1}{c}\right]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\,\rho c^2+\left(-\frac{Ax\,(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz)+Ay\,(n\,qy+jx\,syx+jy\,syy+jz\,syz)+Az\,(n\,qz+jx\,szx+jy\,szy+jz\,szz)}{n}+(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\,\epsilon-\frac{(jx^2+jy^2+jz^2)\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho}{2\,n^2}\right)+O\left[\frac{1}{c}\right]^2
\end{array}\right)$$

```
show2[assut, 1][T[variousfluxes = FS[{{{1, 0, 0, 0}, surface / (Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, igg[[1]] * c^2, igg[[2]], {0, 0, 0, 0}, Lxvec, xboost / c, {0, 0, 0, 0}, gLxvec, gxboost, {0, 0, 0, 0}, uu, tvecnorm}]) /. replaceJu]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho\,c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,(ux-Vx)\,\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n\,(uy-Vy)\,\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n\,(uz-Vz)\,\epsilon\right)-\frac{1}{2}\,n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ 0 & 0 \\ n\,\rho\,c^2+\left(n\,\epsilon+\frac{1}{2}\,n\left(ux^2+uy^2+uz^2+2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\rho\,c^2+\left(Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,(ux-Vx)\,\epsilon\right)+Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n\,(uy-Vy)\,\epsilon\right)+Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n\,(uz-Vz)\,\epsilon\right)+\frac{1}{2}\,n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(ux^2+uy^2+uz^2+2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ 0 & 0 \\ n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\left(t\,ux-x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t+n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ 0 & 0 \\ n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\left(-t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ 0 & 0 \\ -n\,\rho\,c^2-n\,\epsilon+O\left[\frac{1}{c}\right]^2 & n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho\,c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\epsilon\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(ux^2+uy^2+uz^2\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho\,c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,(ux-Vx)\,\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n\,(uy-Vy)\,\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n\,(uz-Vz)\,\epsilon\right)-\frac{1}{2}\,n\left(ux^2+uy^2+uz^2\right)\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[supply.TTx]], {supply, {Dtxyzvec[[1]], Dtxyzvec[[2]], {0, 0, 0, 0}, Dgtxyzvec[[1]] * c^2, Dgtxyzvec[[2]], {0, 0, 0, 0}, DLxvec, Dxboost / c, {0, 0, 0, 0}, DgLxvec, Dgxboost, {0, 0, 0, 0}, Duu, Dtvecnorm}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \rho\,n[t,\,x,\,y,\,z]\,W^{(1,0,0,0)}[t,\,x,\,y,\,z]+O\left[\frac{1}{c}\right]^2 \\ \rho\,n[t,\,x,\,y,\,z]\,W^{(0,1,0,0)}[t,\,x,\,y,\,z]+O\left[\frac{1}{c}\right]^2 \\ 0 \\ \left(2\,\rho\,n[t,\,x,\,y,\,z]\times uz[t,\,x,\,y,\,z]\,W^{(0,0,0,1)}[t,\,x,\,y,\,z]+2\,\rho\,n[t,\,x,\,y,\,z]\times uy[t,\,x,\,y,\,z]\,W^{(0,0,1,0)}[t,\,x,\,y,\,z]+2\,\rho\,n[t,\,x,\,y,\,z]\times ux[t,\,x,\,y,\,z]\,W^{(0,1,0,0)}[t,\,x,\,y,\,z]+\rho\,n[t,\,x,\,y,\,z]\,W^{(1,0,0,0)}[t,\,x,\,y,\,z]\right)+O\left[\frac{1}{c}\right]^2 \\ \rho\,n[t,\,x,\,y,\,z]\,W^{(0,1,0,0)}[t,\,x,\,y,\,z]+O\left[\frac{1}{c}\right]^2 \\ 0 \\ \left(y\,\rho\,n[t,\,x,\,y,\,z]\,W^{(0,0,0,1)}[t,\,x,\,y,\,z]-z\,\rho\,n[t,\,x,\,y,\,z]\,W^{(0,0,1,0)}[t,\,x,\,y,\,z]\right)+O\left[\frac{1}{c}\right]^2 \\ t\,\rho\,n[t,\,x,\,y,\,z]\,W^{(0,1,0,0)}[t,\,x,\,y,\,z]+O\left[\frac{1}{c}\right]^2 \\ 0 \\ \left(y\,\rho\,n[t,\,x,\,y,\,z]\,W^{(0,0,0,1)}[t,\,x,\,y,\,z]-z\,\rho\,n[t,\,x,\,y,\,z]\,W^{(0,0,1,0)}[t,\,x,\,y,\,z]\right)+O\left[\frac{1}{c}\right]^2 \\ -t\,\rho\,n[t,\,x,\,y,\,z]\,W^{(0,1,0,0)}[t,\,x,\,y,\,z]+O\left[\frac{1}{c}\right]^2 \\ 0 \\ \left(\frac{sxz\,jx^{(0,0,0,1)}[t,x,y,z]}{2\,n[t,x,y,z]}+\frac{szx\,jx^{(0,0,0,1)}[t,x,y,z]}{2\,n[t,x,y,z]}-\frac{\rho\,jx[t,x,y,z]\cdot uz[t,x,y,z]\,jx^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}+\rho\,ux[t,\,x,\,y,\,z]\times uz[t,\,x,\,y,\,z]\,jx^{(0,0,0,1)}[t,\,x,\,y,\,z]+\frac{syz\,jy^{(0,0,0,1)}[t,x,y,z]}{2\,n[t,x,y,z]}+\frac{szy\,jy^{(0,0,0,1)}[t,x,y,z]}{2\,n[t,x,y,z]}-\frac{\rho\,jy[t,x,y,z]\cdot uz[t,x,y,z]\,jy^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}+\rho\,uy[t,\,x,\,y,\,z]\times uz[t,\,x,\,y,\,z]\,jy^{(0,0,0,1)}[t,\,x,\,y,\,z]+\frac{szz\,jz^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}-\frac{\rho\,jz[t,x,y,z]\cdot uz[t,x,y,z]\,jz^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}+\rho\,uz[t,\,x,\,y,\,z]^2\,jz^{(0,0,0,1)} \\ \left(-\rho\,n[t,\,x,\,y,\,z]\times uz[t,\,x,\,y,\,z]\,W^{(0,0,0,1)}[t,\,x,\,y,\,z]-\rho\,n[t,\,x,\,y,\,z]\times uy[t,\,x,\,y,\,z]\,W^{(0,0,1,0)}[t,\,x,\,y,\,z]-\rho\,n[t,\,x,\,y,\,z]\times ux[t,\,x,\,y,\,z]\,W^{(0,1,0,0)}[t,\,x,\,y,\,z]\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* covariant derivatives of coordinate 4-vectors (equivalent to Christoffel symbols), for later use *)
(Dtxyzvec = Table[ Assuming[assut, Expand //@ FS@PowerExpand[(D[IdentityMatrix[4][[aa]], {coords}] + Sum[IdentityMatrix[4][[aa]][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]], {aa, 1, 4}]);
```

In[•]:=
```
(* normalized coordinate-t 4-vector*)
tvecnorm = Assuming[assut, Expand //@ FS@PowerExpand[c * {1, 0, 0, 0} / Sqrt[-gg[[1, 1]]]]]
```

Out[•]=
$$\left\{1+\frac{W}{c^2}+O\left[\frac{1}{c}\right]^4,\ 0,\ 0,\ 0\right\}$$

In[•]:=
```
(* and its covariant derivative *)
(Dtvecnorm = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[tvecnorm], {coords}] + Sum[tW[tvecnorm][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]) // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix} O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 \\ -W^{(0,1,0,0)}[t,\,x,\,y,\,z]+O\left[\frac{1}{c}\right]^2 & \frac{W^{1,0,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 0 \\ -W^{(0,0,1,0)}[t,\,x,\,y,\,z]+O\left[\frac{1}{c}\right]^2 & 0 & \frac{W^{1,0,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 \\ -W^{(0,0,0,1)}[t,\,x,\,y,\,z]+O\left[\frac{1}{c}\right]^2 & 0 & 0 & \frac{W^{1,0,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[•]:=
```
(* "raised" coordinate 4-covectors *)
(gtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[igg.IdentityMatrix[4]]]) // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -\left(\frac{1}{c}\right)^2-\frac{2\,W}{c^4}+O\left[\frac{1}{c}\right]^6 & 0 & 0 & 0 \\ 0 & 1-\frac{2\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 0 \\ 0 & 0 & 1-\frac{2\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 \\ 0 & 0 & 0 & 1-\frac{2\,W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[•]:=
```
(* and their covariant derivatives *)
(Dgtxyzvec = Table[ Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[igg[[aa]]], {coords}] + Sum[tW[igg[[aa]][[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]], {aa, 1, 4}]);
```

In[•]:=
```
(* x-component of rot vector *)
Lxvec = Assuming[assut, Expand //@ FS@PowerExpand[{0, 0, -z, y}]]
```

Out[•]=
$$\{0,\ 0,\ -z,\ y\}$$

In[•]:=
```
(* and its covariant derivative *)
showf[assut][DLxvec = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[Lxvec], {coords}] + Sum[tW[Lxvec][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{-y\,W^{0,0,0,1}[t,x,y,z]+z\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & -\frac{z\,W^{1,0,0,0}[t,x,y,z]}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{y\,W^{1,0,0,0}[t,x,y,z]}{c^4}+O\left[\frac{1}{c}\right]^6 \\ 0 & \frac{y\,W^{0,0,0,1}[t,x,y,z]-z\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{z\,W^{0,1,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{y\,W^{0,1,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \\ -\frac{z\,W^{1,0,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{z\,W^{0,1,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{y\,W^{0,0,0,1}[t,x,y,z]-z\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & -1+\frac{-z\,W^{0,0,0,1}[t,x,y,z]-y\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{y\,W^{1,0,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{y\,W^{0,1,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & 1+\frac{z\,W^{0,0,0,1}[t,x,y,z]+y\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{y\,W^{0,0,0,1}[t,x,y,z]-z\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[•]:=
```
(* "raised" x-component of rot co-vector *)
gLxvec = Assuming[assut, Expand //@ FS@PowerExpand[igg.{0, 0, -z, y}]]
```

Out[•]=
$$\left\{0,\ 0,\ -z+\frac{2\,W\,z}{c^2}+O\left[\frac{1}{c}\right]^4,\ y-\frac{2\left(W\,y\right)}{c^2}+O\left[\frac{1}{c}\right]^4\right\}$$

In[•]:=
```
(* and its covariant derivative *)
showf[assut][DgLxvec = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[gLxvec], {coords}] + Sum[tW[gLxvec][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{-y\,W^{0,0,0,1}[t,x,y,z]+z\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & -\frac{z\,W^{1,0,0,0}[t,x,y,z]}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{y\,W^{1,0,0,0}[t,x,y,z]}{c^4}+O\left[\frac{1}{c}\right]^6 \\ 0 & \frac{y\,W^{0,0,0,1}[t,x,y,z]-z\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{z\,W^{0,1,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{y\,W^{0,1,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{z\,W^{1,0,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{z\,W^{0,1,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{y\,W^{0,0,0,1}[t,x,y,z]+z\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & -1+\frac{2\,W[t,x,y,z]+z\,W^{0,0,0,1}[t,x,y,z]-y\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \\ -\frac{y\,W^{1,0,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{y\,W^{0,1,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & 1+\frac{-2\,W[t,x,y,z]+z\,W^{0,0,0,1}[t,x,y,z]-y\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{-y\,W^{0,0,0,1}[t,x,y,z]-z\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[•]:=
```
(* x-component of boost vector *)
xboost = Assuming[assut, Expand //@ FS@PowerExpand[{x / c, t * c, 0, 0}]]
```

Out[•]=
$$\left\{\frac{x}{c},\ c\,t,\ 0,\ 0\right\}$$

In[•]:=
```
(* and its covariant derivative *)
showf[assut][Dxboost = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[xboost], {coords}] + Sum[tW[xboost][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -\frac{t\,W^{0,1,0,0}[t,x,y,z]}{c}+O\left[\frac{1}{c}\right]^3 & \frac{1}{c}+\frac{-x\,W^{0,1,0,0}[t,x,y,z]+t\,W^{1,0,0,0}[t,x,y,z]}{c^3}+O\left[\frac{1}{c}\right]^5 & -\frac{x\,W^{0,0,1,0}[t,x,y,z]}{c^3}+O\left[\frac{1}{c}\right]^5 & -\frac{x\,W^{0,0,0,1}[t,x,y,z]}{c^3}+O\left[\frac{1}{c}\right]^5 \\ c+\frac{-x\,W^{0,1,0,0}[t,x,y,z]+t\,W^{1,0,0,0}[t,x,y,z]}{c}+O\left[\frac{1}{c}\right]^3 & \frac{t\,W^{0,1,0,0}[t,x,y,z]}{c}+O\left[\frac{1}{c}\right]^3 & \frac{t\,W^{0,0,1,0}[t,x,y,z]}{c}+O\left[\frac{1}{c}\right]^3 & \frac{t\,W^{0,0,0,1}[t,x,y,z]}{c}+O\left[\frac{1}{c}\right]^3 \\ -\frac{x\,W^{0,0,1,0}[t,x,y,z]}{c}+O\left[\frac{1}{c}\right]^3 & -\frac{t\,W^{0,0,1,0}[t,x,y,z]}{c}+O\left[\frac{1}{c}\right]^3 & \frac{t\,W^{0,1,0,0}[t,x,y,z]}{c}+O\left[\frac{1}{c}\right]^3 & 0 \\ -\frac{x\,W^{0,0,0,1}[t,x,y,z]}{c}+O\left[\frac{1}{c}\right]^3 & -\frac{t\,W^{0,0,0,1}[t,x,y,z]}{c}+O\left[\frac{1}{c}\right]^3 & 0 & \frac{t\,W^{0,1,0,0}[t,x,y,z]}{c}+O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:=
```
(* "raised" x-component of boost co-vector *)
gxboost = Assuming[assut, Expand //@ FS@PowerExpand[igg.{x, -t, 0, 0}]]
```

Out[•]=
$$\left\{-\frac{x}{c^2}-\frac{2\,(W\,x)}{c^4}+O\left[\frac{1}{c}\right]^6,\ -t+\frac{2\,t\,W}{c^2}+O\left[\frac{1}{c}\right]^4,\ 0,\ 0\right\}$$

In[•]:=
```
(* and its covariant derivative *)
showf[assut][Dgxboost = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[gxboost], {coords}] + Sum[tW[gxboost][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} \frac{t\,W^{0,1,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & -\left(\frac{1}{c}\right)^2+\frac{-2\,W[t,x,y,z]-x\,W^{0,1,0,0}[t,x,y,z]-t\,W^{1,0,0,0}[t,x,y,z]}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{x\,W^{0,0,1,0}[t,x,y,z]}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{x\,W^{0,0,0,1}[t,x,y,z]}{c^4}+O\left[\frac{1}{c}\right]^6 \\ -1+\frac{2\,W[t,x,y,z]+x\,W^{0,1,0,0}[t,x,y,z]+t\,W^{1,0,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{t\,W^{0,1,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{t\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{t\,W^{0,0,0,1}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{x\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{t\,W^{0,0,1,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{t\,W^{0,1,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 \\ \frac{x\,W^{0,0,0,1}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{t\,W^{0,0,0,1}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & -\frac{t\,W^{0,1,0,0}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

```
(* content and flux of coordinatevector-energy and coordinatevector-momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n}+n\left(-\epsilon+W\,\rho\right)\right)+O\left[\frac{1}{c}\right]^2 & \left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\rho\,c^2+\left(-\frac{Ax\left(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz\right)+Ay\left(n\,qy+jx\,syx+jy\,syy+jz\,syz\right)+Az\left(n\,qz+jx\,szx+jy\,szy+jz\,szz\right)}{n}+\left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\epsilon-\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx^2+jy^2+jz^2-2\,n^2\,W\right)\rho}{2\,n^2}\right)+O\left[\frac{1}{c}\right]^2 \\ jx\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\ jy\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+\frac{jy\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\ jz\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+\frac{jz\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[Dtxyzvec[[aa,  ;; ,  ;;]].TTx]], {aa, 1, 4}]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} \rho\,n[t,\,x,\,y,\,z]\,W^{(1,0,0,0)}[t,\,x,\,y,\,z]+O\left[\frac{1}{c}\right]^2 \\ \rho\,n[t,\,x,\,y,\,z]\,W^{(0,1,0,0)}[t,\,x,\,y,\,z]+O\left[\frac{1}{c}\right]^2 \\ \rho\,n[t,\,x,\,y,\,z]\,W^{(0,0,1,0)}[t,\,x,\,y,\,z]+O\left[\frac{1}{c}\right]^2 \\ \rho\,n[t,\,x,\,y,\,z]\,W^{(0,0,0,1)}[t,\,x,\,y,\,z]+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

Out[•]//MatrixForm=
$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n}+n\left(-\epsilon+W\,\rho\right)\right)+O\left[\frac{1}{c}\right]^2 & \left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\rho\,c^2+\left(-\frac{Ax\left(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz\right)+Ay\left(n\,qy+jx\,syx+jy\,syy+jz\,syz\right)+Az\left(n\,qz+jx\,szx+jy\,szy+jz\,szz\right)}{n}+\left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\epsilon-\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx^2+jy^2+jz^2-2\,n^2\,W\right)\rho}{2\,n^2}\right)+O\left[\frac{1}{c}\right]^2 \\ jx\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\ jy\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+\frac{jy\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\ jz\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+\frac{jz\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

```
(* content and flux of raised coordinatecovector-energy and coordinatecovector-momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

```
(* content and flux of coord-energy and momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxEPS = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n}+n\left(-\epsilon+W\,\rho\right)\right)+O\left[\frac{1}{c}\right]^2 & \left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\rho\,c^2+\left(-\frac{Ax\left(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz\right)+Ay\left(n\,qy+jx\,syx+jy\,syy+jz\,syz\right)+Az\left(n\,qz+jx\,szx+jy\,szy+jz\,szz\right)}{n}+\left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\epsilon-\frac{\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx^2+jy^2+jz^2-2\,n^2\,W\right)\rho}{2\,n^2}\right)+O\left[\frac{1}{c}\right]^2 \\ jx\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\ jy\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+\frac{jy\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\ jz\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+\frac{jz\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

```
(* content and flux of coord-energy and momentum for dust (TRANSPOSED) *)
shows[assut, 1][T[fluxdust = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.dust2]]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(aux^2+auy^2+auz^2-2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & \left(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz\right)\rho\,c^2-\frac{1}{2}\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)\right)\left(2\,\epsilon+\left(aux^2+auy^2+auz^2-2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ aux\,n\,\rho+O\left[\frac{1}{c}\right]^2 & aux\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho+O\left[\frac{1}{c}\right]^2 \\ auy\,n\,\rho+O\left[\frac{1}{c}\right]^2 & auy\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho+O\left[\frac{1}{c}\right]^2 \\ auz\,n\,\rho+O\left[\frac{1}{c}\right]^2 & auz\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
shows[assut, 1][T[fluxEPS /. replaceJu]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho\,c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,(ux-Vx)\,\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n\,(uy-Vy)\,\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n\,(uz-Vz)\,\epsilon\right)-\frac{1}{2}\,n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\,uy\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+n\,uy\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\,uz\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+n\,uz\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* momentum flux = A.σ + P A.(u-V)*)
fluxPS = ({Ax, Ay, Az}.(S[[2 ;; 4, 2 ;; 4]]).{1, 0, 0} + EPS[[1, 2]] * ({Ax, Ay, Az}.({jx, jy, jz} / n - {Vx, Vy, Vz})))
```

Out[•]=
$$\left(Ax\,sxx+Ay\,syx+Az\,szx+jx\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\rho\right)+\frac{1}{c^2}\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\left(px+\frac{jx\,sxx}{n}+\frac{jy\,syx}{n}+\frac{jz\,szx}{n}+jx\,\epsilon+\frac{jx^3\,\rho}{2\,n^2}+\frac{jx\,jy^2\,\rho}{2\,n^2}+\frac{jx\,jz^2\,\rho}{2\,n^2}+3\,jx\,W\,\rho\right)+O\left[\frac{1}{c}\right]^4$$

In[•]:=
```
shows[assut, 1][Expand //@ FS@PowerExpand[fluxEPS[[2, 2]] - fluxPS]]
```

Out[•]//MatrixForm=
$$O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* energy flux = A.q + A.σ.u + E A.(u-V)*)
fluxE = -({Ax, Ay, Az}.{qx, qy, qz} + {Ax, Ay, Az}.(S[[2 ;; 4, 2 ;; 4]] + Qtens[[2 ;; 4, 2 ;; 4]]).{jx, jy, jz} / n + (-EPS[[1, 1]]) * {Ax, Ay, Az}.({jx, jy, jz} / n - {Vx, Vy, Vz}))
```

Out[•]=
$$-n\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\rho\,c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz-\frac{1}{n}\left(jx\left(Ax\,sxx+Ay\,syx+Az\,szx\right)+jy\left(Ax\,sxy+Ay\,syy+Az\,szy\right)+jz\left(Ax\,sxz+Ay\,syz+Az\,szz\right)\right)-\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\left(n\,\epsilon+\frac{jx^2\,\rho}{2\,n}+\frac{jy^2\,\rho}{2\,n}+\frac{jz^2\,\rho}{2\,n}-n\,W\,\rho\right)\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[fluxEPS[[2, 1]] - fluxE]]
```

Out[•]//MatrixForm=
$$O\left[\frac{1}{c}\right]^2$$

```
(* matter flux n A.(u-V) *)
shows[assut, 1][fluxNJ = Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.NJ /. replaceJu]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} n \\ n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right) \end{pmatrix}$$

```
(* content and flux of coord-energy and momentum assuming no matter flux (transposed) *)
shows[Join[assut, {((surface / Δt).NJ) == 0} /. replaceJu], 1][T@fluxEPS /. replaceJu]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & \left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz\right)\right)+O\left[\frac{1}{c}\right]^2 \\ n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx\right)+O\left[\frac{1}{c}\right]^2 \\ n\,uy\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy\right)+O\left[\frac{1}{c}\right]^2 \\ n\,uz\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=

```
(* coordinate/internal/coordinate-proper energy and x-momentum, content and fluxes (TRANSPOSED) *)
show2[assut, 1][T[variousfluxes = FS[{{{1, 0, 0, 0}, surface / (Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, Lxvec, Lxvec2, xboost / c, xboost2, uu, ntvec}]) /. replaceJu]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho\,c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,(ux-Vx)\,\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n\,(uy-Vy)\,\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n\,(uz-Vz)\,\epsilon\right)-\frac{1}{2}\,n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\left(t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(t\,ux+x\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\left(-t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,\rho\,c^2-n\,\epsilon+O\left[\frac{1}{c}\right]^2 & n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho\,c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\epsilon\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(ux^2+uy^2+uz^2\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho\,c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,(ux-Vx)\,\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n\,(uy-Vy)\,\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n\,(uz-Vz)\,\epsilon\right)-\frac{1}{2}\,n\left(ux^2+uy^2+uz^2\right)\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:= **show2[assut, 1][T[variousfluxes /. {Vy → 0, Vz → 0, uy → 0, uz → 0}]]**

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(ux^2-2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & Ax\,n\,(-ux+Vx)\,\rho\,c^2+\left(-Ay\left(qy+syx\,ux\right)-Az\left(qz+szx\,ux\right)-Ax\left(qx+sxx\,ux+n\,(ux-Vx)\,\epsilon\right)-\frac{1}{2}\,Ax\,n\,(ux-Vx)\left(ux^2-2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+Ax\,n\,ux\,(ux-Vx)\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z\right)+O\left[\frac{1}{c}\right]^2 \\ O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z\right)+O\left[\frac{1}{c}\right]^2 \\ -n\left(t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-Ax\,n\,(ux-Vx)\left(t\,ux+x\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\left(-t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-Ax\,n\,(ux-Vx)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,\rho\,c^2-n\,\epsilon+O\left[\frac{1}{c}\right]^2 & Ax\,n\,(-ux+Vx)\,\rho\,c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+Ax\,n\,(-ux+Vx)\,\epsilon\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+ux^2\,\rho\right)+O\left[\frac{1}{c}\right]^2 & Ax\,n\,(-ux+Vx)\,\rho\,c^2+\left(-Ay\left(qy+syx\,ux\right)-Az\left(qz+szx\,ux\right)-Ax\left(qx+sxx\,ux+n\,(ux-Vx)\,\epsilon\right)-\frac{1}{2}\,Ax\,n\,ux^2\,(ux-Vx)\,\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:= **(* velocity of energy *)**
shows[assut, 5][(EPS.{1, 0, 0, 0})[[2 ;; 4]] / (EPS.{1, 0, 0, 0})[[1]] /. replaceJu]

Out[•]//MatrixForm=

$$\begin{pmatrix} ux+\frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\,\rho\,c^2}+O\left[\frac{1}{c}\right]^4 \\ uy+\frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\,\rho\,c^2}+O\left[\frac{1}{c}\right]^4 \\ uz+\frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\,\rho\,c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[•]:= **temp = SeriesCoefficient[tt.{1, 0, 0, 0}, {c, Infinity, -2}];**
shows[assut, 5][(tt.{1, 0, 0, 0} - temp * c^2)[[2 ;; 4]] / (tt.{1, 0, 0, 0} - temp * c^2)[[1]] /. j2v]

Out[•]//MatrixForm=

$$\begin{pmatrix} \left(ux+\frac{2\left(qx+sxx\,ux+sxy\,uy+sxz\,uz\right)}{2\,n\,\epsilon+n\left(ux^2+uy^2+uz^2-2\,W\right)\rho}\right)+O\left[\frac{1}{c}\right]^2 \\ \left(uy+\frac{2\left(qy+syx\,ux+syy\,uy+syz\,uz\right)}{2\,n\,\epsilon+n\left(ux^2+uy^2+uz^2-2\,W\right)\rho}\right)+O\left[\frac{1}{c}\right]^2 \\ \left(uz+\frac{2\left(qz+szx\,ux+szy\,uy+szz\,uz\right)}{2\,n\,\epsilon+n\left(ux^2+uy^2+uz^2-2\,W\right)\rho}\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:= **showf[assut][{variousfluxes[[;; , 1]] - variousfluxes[[;; , 7]], variousfluxes[[;; , 1]] - variousfluxes[[;; , 8]], variousfluxes[[;; , 7]] - variousfluxes[[;; , 8]]}]**

Out[•]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{2}\,n\,ux^2\,\rho-\frac{1}{2}\,n\,uy^2\,\rho-\frac{1}{2}\,n\,uz^2\,\rho+n\,W\rho\right)+O\left[\frac{1}{c}\right]^2 & \left(-Ax\,sxx\,ux-Ay\,syx\,ux-Az\,szx\,ux-Ax\,sxy\,uy-Ay\,syy\,uy-Az\,szy\,uy-Ax\,sxz\,uz-Ay\,syz\,uz-Az\,szz\,uz-\frac{1}{2}\,Ax\,n\,ux^3\,\rho-\frac{1}{2}\,Ay\,n\,ux^2\,uy\,\rho-\frac{1}{2}\,Ax\,n\,ux\,uy^2\,\rho-\frac{1}{2}\,Ay\,n\,uy^3\,\rho-\frac{1}{2}\,Az\,n\,ux^2\,uz\,\rho-\frac{1}{2}\,Az\,n\,uy^2\,uz\,\rho-\frac{1}{2}\,Ax\,n\,ux\,uz^2\,\rho-\frac{1}{2}\,Ay\,n\,uy\,uz^2\,\rho-\frac{1}{2}\,Az\,n\,uz^3\,\rho+\frac{1}{2}\,Ax\,n\,ux^2\,Vx\,\rho+\frac{1}{2}\,Ax\,n\,uy^2\,Vx\,\rho+\frac{1}{2}\,Ax\,n\,u \\ n\,W\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,n\,ux\,W\,\rho+Ay\,n\,uy\,W\,\rho+Az\,n\,uz\,W\,\rho-Ax\,n\,Vx\,W\,\rho-Ay\,n\,Vy\,W\,\rho-Az\,n\,Vz\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ \left(\frac{1}{2}\,n\,ux^2\,\rho+\frac{1}{2}\,n\,uy^2\,\rho+\frac{1}{2}\,n\,uz^2\,\rho\right)+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx\,ux+Ay\,syx\,ux+Az\,szx\,ux+Ax\,sxy\,uy+Ay\,syy\,uy+Az\,szy\,uy+Ax\,sxz\,uz+Ay\,syz\,uz+Az\,szz\,uz+\frac{1}{2}\,Ax\,n\,ux^3\,\rho+\frac{1}{2}\,Ay\,n\,ux^2\,uy\,\rho+\frac{1}{2}\,Ax\,n\,ux\,uy^2\,\rho+\frac{1}{2}\,Ay\,n\,uy^3\,\rho+\frac{1}{2}\,Az\,n\,ux^2\,uz\,\rho+\frac{1}{2}\,Az\,n\,uy^2\,uz\,\rho+\frac{1}{2}\,Ax\,n\,ux\,uz^2\,\rho+\frac{1}{2}\,Ay\,n\,uy\,uz^2\,\rho+\frac{1}{2}\,Az\,n\,uz^3\,\rho-\frac{1}{2}\,Ax\,n\,ux^2\,Vx\,\rho-\frac{1}{2}\,Ax\,n\,uy^2\,Vx\,\rho-\frac{1}{2}\,Ax\,n\,u; \end{pmatrix}$$

In[•]:= **(* supply terms *)**
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
show2[assut, 2][FS[(itjv[Tr[#.TTx]]) /. replaceJu] & /@ {Dxyzvec[[1, ;; , ;;]], Dxyzvec[[2, ;; , ;;]], DLxvec, DLxvec2, Dxboost / c, Dxboost2, Duv, Dntvec} // MF

Out[•]//MatrixForm=

$$\begin{pmatrix} n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,\rho\left(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\ \frac{1}{2}\left(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z]+(syz+szy)\left(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\right)+2\left(syy\,uy^{(0,0,1,0)}[t,x,y,z]+sxx\,ux^{(0,1,0,0)}[t,x,y,z]\right)+(sxy+syx)\left(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z]\right)+(sxz+szx)\left(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\right)\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,\rho\left(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

shows[assut, 1][T[Expand //@ FS@PowerExpand[{{{1, 0, 0, 0}, surface / (Δt)}.EPSsym.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, Lxvec, Lxvec2, xboost / c, xboost2, uu, ntvec}]) /. replaceJu]]

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho\,c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,(ux-Vx)\,\epsilon\right)-Ay\left(qy+sxy\,ux+syy\,uy+syz\,uz+n\,(uy-Vy)\,\epsilon\right)-Az\left(qz+sxz\,ux+syz\,uy+szz\,uz+n\,(uz-Vz)\,\epsilon\right)-\frac{1}{2}\,n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(ux^2+uy^2+uz^2-2\,W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,sxy+Az\,sxz+n\,ux\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,syz\,z+n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,syz\,z+n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\left(t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,sxy+Az\,sxz\right)t\right)-n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(t\,ux+x\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ n\left(-t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,sxy+Az\,sxz\right)t\right)-n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,\rho\,c^2-n\,\epsilon+O\left[\frac{1}{c}\right]^2 & n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho\,c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\epsilon\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,\rho\,c^2-\frac{1}{2}\,n\left(2\,\epsilon+\left(ux^2+uy^2+uz^2\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho\,c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,(ux-Vx)\,\epsilon\right)-Ay\left(qy+sxy\,ux+syy\,uy+syz\,uz+n\,(uy-Vy)\,\epsilon\right)-Az\left(qz+sxz\,ux+syz\,uy+szz\,uz+n\,(uz-Vz)\,\epsilon\right)-\frac{1}{2}\,n\left(ux^2+uy^2+uz^2\right)\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:= **TTx = tW[tjv[(EPSsym + T[EPSsym.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)**
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. replaceJu] & /@ {Dxyzvec[[1, ;; , ;;]], Dxyzvec[[2, ;; , ;;]], DLxvec, DLxvec2, Dxboost / c, Dxboost2, Duv, Dntvec} // MF

Out[•]//MatrixForm=

$$\begin{pmatrix} n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\ n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,\rho\left(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\ \left(szz\,uz^{(0,0,0,1)}[t,x,y,z]+syy\,uy^{(0,0,1,0)}[t,x,y,z]+syz\left(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\right)+sxx\,ux^{(0,1,0,0)}[t,x,y,z]+sxy\left(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z]\right)+sxz\left(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\right)\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,\rho\left(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:= **(* 2-vector of surface parallel to yz surfacefx={-Vx*A*Δt,A*Δt,0,0}; *)**
(yzsurface = ((T[{{0, 0, Ly, 0}}].{{0, 0, 0, Lz}} - T[{{0, 0, 0, Lz}}].{{0, 0, Ly, 0}}) /. {Ly → Ayz / Lz})) // MF

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Ayz \\ 0 & 0 & -Ayz & 0 \end{pmatrix}$$

In[•]:= **(* 2-vector of surface parallel to tx surfacefx={-Vx*A*Δt,A*Δt,0,0}; *)**
(txsurface = (-(T[{{Δt, 0, 0, 0}}].{{0, Lx, 0, 0}} - T[{{0, Lx, 0, 0}}].{{Δt, 0, 0, 0}}))) // MF

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -Lx\,\Delta t & 0 & 0 \\ Lx\,\Delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[•]:= **(* 2-vector of surface parallel to ty surfacefx={-Vx*A*Δt,A*Δt,0,0}; *)**
(tysurface = (-(T[{{Δt, 0, 0, 0}}].{{0, 0, Ly, 0}} - T[{{0, 0, Ly, 0}}].{{Δt, 0, 0, 0}}))) // MF

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -Ly\,\Delta t & 0 \\ 0 & 0 & 0 & 0 \\ Ly\,\Delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[•]:= **(* 2-vector of surface parallel to y moving to x surfacefx={-Vx*A*Δt,A*Δt,0,0}; *)**
(yVxsurface = (-(T[{{1, Vx, 0, 0}} * Δt].{{0, 0, Ly, 0}} - T[{{0, 0, Ly, 0}}].{{1, Vx, 0, 0}} * Δt))) // MF

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -Ly\,\Delta t & 0 \\ 0 & 0 & -Ly\,Vx\,\Delta t & 0 \\ Ly\,\Delta t & Ly\,Vx\,\Delta t & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[•]:= **(Tr[T[txsurface].txsurface]) // MF**

Out[•]//MatrixForm=

$$2\,Lx^2\,\Delta t^2$$

In[•]:= **(* Faraday tensor *)**
repE = {Ex → Ex * c * Sqrt[μ * ϵ], Ey → Ey * c * Sqrt[μ * ϵ], Ez → Ez * c * Sqrt[μ * ϵ]};
fftemp = {{0, -Ex, -Ey, -Ez}, {0, 0, Bz, -By}, {0, 0, 0, Bx}, {0, 0, 0, 0}} /. repE;
showf[assut][F = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -c\,Ex\,\sqrt{\epsilon}\,\sqrt{\mu} & -c\,Ey\,\sqrt{\epsilon}\,\sqrt{\mu} & -c\,Ez\,\sqrt{\epsilon}\,\sqrt{\mu} \\ c\,Ex\,\sqrt{\epsilon}\,\sqrt{\mu} & 0 & Bz & -By \\ c\,Ey\,\sqrt{\epsilon}\,\sqrt{\mu} & -Bz & 0 & Bx \\ c\,Ez\,\sqrt{\epsilon}\,\sqrt{\mu} & By & -Bx & 0 \end{pmatrix}$$

In[•]:= **(FS[{Tr[yzsurface.T[F]], Tr[T[txsurface].F], Tr[T[tysurface].F], Tr[T[yVxsurface].F]} / 2]) // MF**

Out[•]//MatrixForm=

$$\begin{pmatrix} Ayz\,Bx \\ c\,Ex\,Lx\,\Delta t\,\sqrt{\epsilon}\,\sqrt{\mu} \\ c\,Ey\,Ly\,\Delta t\,\sqrt{\epsilon}\,\sqrt{\mu} \\ Ly\,\Delta t\left(-Bz\,Vx+c\,Ey\,\sqrt{\epsilon}\,\sqrt{\mu}\right) \end{pmatrix}$$

In[•]:= **(* charge-current-potential tensor *)**
fftemp = {{0, -Hx, -Hy, -Hz}, {0, 0, Dz, -Dy}, {0, 0, 0, Dx}, {0, 0, 0, 0}};
showf[assut][H = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -Hx & -Hy & -Hz \\ Hx & 0 & Dz & -Dy \\ Hy & -Dz & 0 & Dx \\ Hz & Dy & -Dx & 0 \end{pmatrix}$$

In[∘]:=
```
showf[assut][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
   ]]]
```

Out[∘]//MatrixForm=

(⋯ 1 ⋯)

Full expression not available (original memory size: 0.7 MB)

In[∘]:=
```
shows[assut, 1][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
   ]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & \frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{(-Bz\,Ex+Bx\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{(By\,Ex-Bx\,Ey)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ \frac{(-Bz\,Ey+By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(Bz\,Ey-By\,Ez)\,W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{-Bx^2+By^2+Bz^2+(-Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & -\frac{Bx\,By+Ex\,Ey\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & -\frac{Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 \\ \frac{(Bz\,Ex-Bx\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(-Bz\,Ex+Bx\,Ez)\,W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & -\frac{Bx\,By+Ex\,Ey\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & \frac{Bx^2-By^2+Bz^2+(Ex^2-Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & -\frac{By\,Bz+Ey\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 \\ \frac{(-By\,Ex+Bx\,Ey)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(By\,Ex-Bx\,Ey)\,W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & -\frac{Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & -\frac{By\,Bz+Ey\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & \frac{Bx^2+By^2-Bz^2+(Ex^2+Ey^2-Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[∘]:=
```
showf[assut][T[tte.Inverse[gg]].gg - tte]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^4 & O[\frac{1}{c}]^5 & O[\frac{1}{c}]^5 & O[\frac{1}{c}]^5 \\ O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}$$

In[∘]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.tte.T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```

Out[∘]//MatrixForm=

```
( -(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + O[1/c]²      (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (Ax Vx+Ay Vy+Az Vz)(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + 2 (Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) W √ϵ0/(√μ0 c) + O[1/c]²
  -(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + (Bz Ey ux-By Ez ux-Bz Ex uy+Bx Ez uy+By Ex uz-Bx Ey uz) √ϵ0/(√μ0 c) + O[1/c]²      (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (Ax (-2 Bx (By uy+Bz uz)+Bx² (-ux+Vx)+(By²+Bz²) (ux+Vx)+(-2 Ex (Ey uy+Ez uz)+Ex² (-ux+Vx)+(Ey²+Ez²) (ux+Vx)) ϵ0 μ0)+Ay (-2 Bx By ux-2 By Bz uz+By² (-uy+Vy)+Bx² (uy+Vy)+Bz² (uy+Vy)+(-2 Ex Ey ux-2 Ey Ez uz+Ey² (-uy+Vy)+Ex² (uy+Vy)+Ez² (uy+Vy)) ϵ0 μ0)+Az (-2 Bx Bz ux-2 By Bz uy+Bz² (-uz+Vz)+Bx² (uz+Vz)+By² …)/(2 μ0)
  -(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + O[1/c]²      (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (Ax Vx+Ay Vy+Az Vz)(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + (Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) W √ϵ0/(√μ0 c) + O[1/c]²
  (Bz Ey-By Ez) √ϵ0/(√μ0 c) + O[1/c]²      (-2 (Ay Bx By+Az Bx Bz+Ay Ex Ey ϵ0 μ0+Az Ex Ez ϵ0 μ0)+Ax (-Bx²+By²+Bz²+(-Ex²+Ey²+Ez²) ϵ0 μ0))/(2 μ0) - (Bz Ey-By Ez)(Ax Vx+Ay Vy+Az Vz) √ϵ0/(√μ0 c) + O[1/c]²
  (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²      (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+(-2 (Ax Ex+Ay Ey) Ez y+Az (Ex²+Ey²-Ez²) y+2 Ey (Ax Ex+Az Ez) z-Ay (Ex²-Ey²+Ez²) z) ϵ0 μ0)/(2 μ0) - (Ax Vx+Ay Vy+Az Vz)(By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²
  (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²      (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+(-2 (Ax Ex+Ay Ey) Ez y+Az (Ex²+Ey²-Ez²) y+2 Ey (Ax Ex+Az Ez) z-Ay (Ex²-Ey²+Ez²) z) ϵ0 μ0)/(2 μ0) - (Ax Vx+Ay Vy+Az Vz)(By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²
  (-Bz Ey+By Ez) t √ϵ0/(√μ0 c) + O[1/c]²      t (2 (Ay Bx By+Az Bx Bz+Ay Ex Ey ϵ0 μ0+Az Ex Ez ϵ0 μ0)+Ax (Bx²-By²-Bz²+(Ex²-Ey²-Ez²) ϵ0 μ0))/(2 μ0) + ((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) x) √ϵ0/(√μ0 c) + O[1/c]²
  (-Bz Ey+By Ez) t √ϵ0/(√μ0 c) + O[1/c]²      t (2 (Ay Bx By+Az Bx Bz+Ay Ex Ey ϵ0 μ0+Az Ex Ez ϵ0 μ0)+Ax (Bx²-By²-Bz²+(Ex²-Ey²-Ez²) ϵ0 μ0))/(2 μ0) + ((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) x) √ϵ0/(√μ0 c) + O[1/c]² )
```

In[∘]:=
```
TTx = tW[tjv[(tte + T[tte.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```

Out[∘]//MatrixForm=

```
( (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) W^(1,0,0,0)[t,x,y,z]/(μ0 c²) + O[1/c]³
  -2 (Bx Bz+Ex Ez ϵ0 μ0) ux^(0,0,0,1)[t,x,y,z]-2 (By Bz+Ey Ez ϵ0 μ0) uy^(0,0,0,1)[t,x,y,z]+Bx² uz^(0,0,0,1)[t,x,y,z]+By² uz^(0,0,0,1)[t,x,y,z]-Bz² uz^(0,0,0,1)[t,x,y,z]+Ex² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]+Ey² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-Ez² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-2 Bx By ux^(0,0,1,0)[t,x,y,z]-2 Ex Ey ϵ0 μ0 ux^(0,0,1,0)[t,x,y,z]+Bx² uy^(0,0,1,0)[t,x,y,z]-By² uy^(0,0,1,0)[t,x,y,z]+Bz² uy^(0,0,1,0)[t,x,y,z]+Ex² ϵ0 μ0 uy^(0,0,1,0)[t,x,y,z]-Ey² ϵ0 μ0 uy^(0,0,1,0)[t,x,y,z]+Ez² ϵ0 μ0 uy^(0,0,1,0)[t,x,y,z]-2 By Bz uz^(0,0,1,0)[t,x …  /(2 μ0)
  √ϵ0 ((-By Ex+Bx Ey) W^(0,0,0,1)[t,x,y,z]+(Bz Ex-Bx Ez) W^(0,0,1,0)[t,x,y,z]+(-Bz Ey+By Ez) W^(0,1,0,0)[t,x,y,z])/(√μ0 c) + (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) W^(1,0,0,0)[t,x,y,z]/(2 μ0 c²) + O[1/c]³
  (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) W^(0,1,0,0)[t,x,y,z]/(μ0 c²) + O[1/c]³
  (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) (y W^(0,0,0,1)[t,x,y,z]-z W^(0,0,1,0)[t,x,y,z])/(μ0 c²) + O[1/c]³
  2 ((Bz² y-By Bz z+Ez (Ez y-Ey z) ϵ0 μ0) W^(0,0,0,1)[t,x,y,z]+(By Bz y-By² z+Ey (Ez y-Ey z) ϵ0 μ0) W^(0,0,1,0)[t,x,y,z]+(Bx (Bz y-By z)+Ex (Ez y-Ey z) ϵ0 μ0) W^(0,1,0,0)[t,x,y,z])/(μ0 c²) + O[1/c]³
  2 (-Bz Ey+By Ez) √ϵ0/(√μ0 c) - t (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) W^(0,1,0,0)[t,x,y,z]/(μ0 c²) + O[1/c]³
  -2 (t ((Bx Bz+Ex Ez ϵ0 μ0) W^(0,0,0,1)[t,x,y,z]+(Bx By+Ex Ey ϵ0 μ0) W^(0,0,1,0)[t,x,y,z]+(Bx²+Ex² ϵ0 μ0) W^(0,1,0,0)[t,x,y,z]))/(μ0 c²) + O[1/c]³ )
```

In[∘]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.(tte + ttsym).T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```

Out[∘]//MatrixForm=

```
( -n ρ c² + (-n ϵ - (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) - 1/2 n (ux² + uy² + uz² - 2 W) ρ) + O[1/c]²      n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (-2 Ax (qx+sxx ux+sxy uy+sxz uz+n ux ϵ) μ0-2 Ay (qy+sxy ux+syy uy+syz uz+n uy ϵ) μ0-2 Az (qz+sxz ux+syz uy+szz uz+n uz ϵ) μ0+Ax Vx (Bx²+By²+Bz²+2 n ϵ μ0+(Ex²+Ey²+Ez²) ϵ0 μ0)+Ay Vy (Bx²+By²+Bz²+2 n ϵ μ0+(Ex²+ …)/(2 μ0)
  -n ρ c² - (Bx²+By²+Bz²+2 n ϵ μ0+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + (Bz Ey ux-By Ez ux-Bz Ex uy+Bx Ez uy+By Ex uz-Bx Ey uz) √ϵ0/(√μ0 c) + O[1/c]²      n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (Ay (-2 Bx By ux-2 By Bz uz+By² (-uy+Vy)+Bx² (uy+Vy)+Bz² (uy+Vy)-2 (qy+n (uy-Vy) ϵ) μ0+(-2 Ex Ey ux-2 Ey Ez uz+Ey² (-uy+Vy)+Ex² (uy+Vy)+Ez² (uy+Vy)) ϵ0 μ0)+Ax (-2 Bx (By uy+Bz uz)+Bx² (-ux+Vx)+(By²+Bz²) (ux+Vx)+(- …
  -n ρ c² + (-n ϵ - (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) - 1/2 n (ux² + uy² + uz²) ρ) + O[1/c]²      n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (-2 Ax (qx+sxx ux+sxy uy+sxz uz+n ux ϵ) μ0-2 Ay (qy+sxy ux+syy uy+syz uz+n uy ϵ) μ0-2 Az (qz+sxz ux+syz uy+szz uz+n uz ϵ) μ0+Ax Vx (Bx²+By²+Bz²+2 n ϵ μ0+(Ex²+Ey²+Ez²) ϵ0 μ0)+Ay Vy (Bx²+By²+Bz²+2 n ϵ μ0+(Ex²+ …)/(2 μ0)
  n ux ρ + (Bz Ey-By Ez) √ϵ0/(√μ0 c) + O[1/c]²      (-2 Ay (Bx By-sxy μ0+Ex Ey ϵ0 μ0+n ux (-uy+Vy) μ0 ρ)+Ax (-Bx²+By²+Bz²+μ0 (2 sxx+(-Ex²+Ey²+Ez²) ϵ0+2 n ux (ux-Vx) ρ))+2 Az (-Bx Bz+μ0 (sxz-Ex Ez ϵ0+n ux (uz-Vz) ρ)))/(2 μ0) - (Bz Ey-By Ez)(Ax Vx+Ay Vy+Az Vz) √ϵ0/(√μ0 c) + O[1/c]²
  n (uz y - uy z) ρ + (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²      (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+2 Ax sxz y μ0+2 Ay syz y μ0+2 Az szz y μ0-2 Ax sxy z μ0-2 Ay syy z μ0-2 Az syz z μ0+Az Ex² y ϵ0 μ0+Az Ey² y ϵ0 μ0-2 Ax Ex Ez y ϵ0 μ0-2 Ay Ey Ez y ϵ0 μ0-Az Ez² y ϵ0 μ0-Ay Ex² z ϵ0 μ0+2 Ax Ex Ey z ϵ0 μ0+Ay Ey² z ϵ0 μ0+2 Az Ey Ez z ϵ0 μ0-Ay Ez² z ϵ0 μ0+2 …)/(2 μ0)
  n (uz y - uy z) ρ + (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²      (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+2 Ax sxz y μ0+2 Ay syz y μ0+2 Az szz y μ0-2 Ax sxy z μ0-2 Ay syy z μ0-2 Az syz z μ0+Az Ex² y ϵ0 μ0+Az Ey² y ϵ0 μ0-2 Ax Ex Ez y ϵ0 μ0-2 Ay Ey Ez y ϵ0 μ0-Az Ez² y ϵ0 μ0-Ay Ex² z ϵ0 μ0+2 Ax Ex Ey z ϵ0 μ0+Ay Ey² z ϵ0 μ0+2 Az Ey Ez z ϵ0 μ0-Ay Ez² z ϵ0 μ0+2 …)/(2 μ0)
  -n (t ux + x) ρ + (-Bz Ey+By Ez) t √ϵ0/(√μ0 c) + O[1/c]²      (t (Ax Bx²+2 Ay Bx By-Ax By²+2 Az Bx Bz-Ax Bz²-(2 Ax sxx+2 Ay sxy+2 Az sxz-2 Ex (Ay Ey+Az Ez) ϵ0+Ax (-Ex²+Ey²+Ez²) ϵ0) μ0)/(2 μ0) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux + x) ρ) + ((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) x) √ϵ0/(√μ0 c) + O[1/c]²
  n (-t ux + x) ρ + (-Bz Ey+By Ez) t √ϵ0/(√μ0 c) + O[1/c]²      (t (Ax Bx²+2 Ay Bx By-Ax By²+2 Az Bx Bz-Ax Bz²-(2 Ax sxx+2 Ay sxy+2 Az sxz-2 Ex (Ay Ey+Az Ez) ϵ0+Ax (-Ex²+Ey²+Ez²) ϵ0) μ0)/(2 μ0) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux - x) ρ) + ((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) x) √ϵ0/(√μ0 c) + O[1/c]² )
```

In[∘]:=
```
TTx = tW[tjv[((tte + ttsym) + T[(tte + ttsym).Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```

Out[∘]//MatrixForm=

```
( n ρ W^(1,0,0,0)[t, x, y, z] + O[1/c]²
  -2 (Bx Bz-sxz μ0+Ex Ez ϵ0 μ0) ux^(0,0,0,1)[t,x,y,z]-2 (By Bz-syz μ0+Ey Ez ϵ0 μ0) uy^(0,0,0,1)[t,x,y,z]+Bx² uz^(0,0,0,1)[t,x,y,z]+By² uz^(0,0,0,1)[t,x,y,z]-Bz² uz^(0,0,0,1)[t,x,y,z]+2 szz μ0 uz^(0,0,0,1)[t,x,y,z]+Ex² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]+Ey² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-Ez² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-2 Bx By ux^(0,0,1,0)[t,x,y,z]+2 sxy μ0 ux^(0,0,1,0)[t,x,y,z]-2 Ex Ey ϵ0 μ0 ux^(0,0,1,0)[t,x,y,z]+Bx² uy^(0,0,1,0)[t,x,y,z]-By² uy^(0,0,1,0)[t,x,y,z]+Bz² uy^(0,0,1,0)[t,x,y,z]+2 syy μ0 uy^(0,0,1,0)[t,x,y,z]+Ex² ϵ0 μ …
  -n ρ (uz W^(0,0,0,1)[t, x, y, z] + uy W^(0,0,1,0)[t, x, y, z] + ux W^(0,1,0,0)[t, x, y, z]) + √ϵ0 ((-By Ex+Bx Ey) W^(0,0,0,1)[t,x,y,z]+(Bz Ex-Bx Ez) W^(0,0,1,0)[t,x,y,z]+(-Bz Ey+By Ez) W^(0,1,0,0)[t,x,y,z])/(√μ0 c) + O[1/c]²
  n ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]²
  n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]²
  n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]²
  -n ρ (2 ux + t W^(0,1,0,0)[t, x, y, z]) + 2 (-Bz Ey+By Ez) √ϵ0/(√μ0 c) + O[1/c]²
  -n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]² )
```

In[∘]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.(tt + tte).T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```

Out[∘]//MatrixForm=

```
( -n ρ c² + (-(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) - 1/2 n (ux² + uy² + uz² - 2 W + 2 ϵ) ρ) + O[1/c]²      n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (Ax (-2 (qx+sxx ux+sxy uy+sxz uz) μ0+Vx (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)+n (-ux+Vx) (ux²+uy²+uz²-2 W+2 ϵ) μ0 ρ)+Ay (-2 (qy+syx ux+syy uy+syz uz) μ0+Vy (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)+n (-uy+Vy) (ux²+uy²+u …)/(2 μ0)
  -n ρ c² - (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0+2 n ϵ μ0 ρ)/(2 μ0) + (Bz Ey ux-By Ez ux-Bz Ex uy+Bx Ez uy+By Ex uz-Bx Ey uz) √ϵ0/(√μ0 c) + O[1/c]²      n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (Ax (Bz² ux-2 Bx (By uy+Bz uz)+Bz² Vx+Bx² (-ux+Vx)+By² (ux+Vx)-2 qx μ0-Ex² ux ϵ0 μ0+Ey² ux ϵ0 μ0+Ez² ux ϵ0 μ0-2 Ex Ey uy ϵ0 μ0-2 Ex Ez uz ϵ0 μ0+Ex² Vx ϵ0 μ0+Ey² Vx ϵ0 μ0+Ez² Vx ϵ0 μ0+2 n (-ux+Vx) ϵ μ0 ρ)+Ay (-2 Bx …
  -n ρ c² + (-(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) - 1/2 n (ux² + uy² + uz² + 2 ϵ) ρ) + O[1/c]²      n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (Ax (-2 (qx+sxx ux+sxy uy+sxz uz) μ0+Vx (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)+n (-ux+Vx) (ux²+uy²+uz²+2 ϵ) μ0 ρ)+Ay (-2 (qy+syx ux+syy uy+syz uz) μ0+Vy (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)+n (-uy+Vy) (ux²+uy²+uz²+ …)/(2 μ0)
  n ux ρ + (Bz Ey-By Ez) √ϵ0/(√μ0 c) + O[1/c]²      (-2 Ay (Bx By-syx μ0+Ex Ey ϵ0 μ0+n ux (-uy+Vy) μ0 ρ)+Ax (-Bx²+By²+Bz²+μ0 (2 sxx+(-Ex²+Ey²+Ez²) ϵ0+2 n ux (ux-Vx) ρ))+2 Az (-Bx Bz+μ0 (szx-Ex Ez ϵ0+n ux (uz-Vz) ρ)))/(2 μ0) - (Bz Ey-By Ez)(Ax Vx+Ay Vy+Az Vz) √ϵ0/(√μ0 c) + O[1/c]²
  n (uz y - uy z) ρ + (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²      (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+2 Ax sxz y μ0+2 Ay syz y μ0+2 Az szz y μ0-2 Ax sxy z μ0-2 Ay syy z μ0-2 Az szy z μ0+Az Ex² y ϵ0 μ0+Az Ey² y ϵ0 μ0-2 Ax Ex Ez y ϵ0 μ0-2 Ay Ey Ez y ϵ0 μ0-Az Ez² y ϵ0 μ0-Ay Ex² z ϵ0 μ0+2 Ax Ex Ey z ϵ0 μ0+Ay Ey² z ϵ0 μ0+2 Az Ey Ez z ϵ0 μ0-Ay Ez² z ϵ0 μ0+ …)/(2 μ0)
  n (uz y - uy z) ρ + (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²      (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+2 Ax sxz y μ0+2 Ay syz y μ0+2 Az szz y μ0-2 Ax sxy z μ0-2 Ay syy z μ0-2 Az szy z μ0+Az Ex² y ϵ0 μ0+Az Ey² y ϵ0 μ0-2 Ax Ex Ez y ϵ0 μ0-2 Ay Ey Ez y ϵ0 μ0-Az Ez² y ϵ0 μ0-Ay Ex² z ϵ0 μ0+2 Ax Ex Ey z ϵ0 μ0+Ay Ey² z ϵ0 μ0+2 Az Ey Ez z ϵ0 μ0-Ay Ez² z ϵ0 μ0+ …)/(2 μ0)
  -n (t ux + x) ρ + (-Bz Ey+By Ez) t √ϵ0/(√μ0 c) + O[1/c]²      (t (Ax Bx²+2 Ay Bx By-Ax By²+2 Az Bx Bz-Ax Bz²-(2 Ax sxx+2 Ay syx+2 Az szx-2 Ex (Ay Ey+Az Ez) ϵ0+Ax (-Ex²+Ey²+Ez²) ϵ0) μ0)/(2 μ0) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux + x) ρ) + ((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) x) √ϵ0/(√μ0 c) + O[1/c]²
  n (-t ux + x) ρ + (-Bz Ey+By Ez) t √ϵ0/(√μ0 c) + O[1/c]²      (t (Ax Bx²+2 Ay Bx By-Ax By²+2 Az Bx Bz-Ax Bz²-(2 Ax sxx+2 Ay syx+2 Az szx-2 Ex (Ay Ey+Az Ez) ϵ0+Ax (-Ex²+Ey²+Ez²) ϵ0) μ0)/(2 μ0) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux - x) ρ) + ((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) x) √ϵ0/(√μ0 c) + O[1/c]² )
```

In[∘]:=
```
TTx = tW[tjv[((tt + tte) + T[(tt + tte).Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 4][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```

Out[∘]//MatrixForm=

```
( n ρ W^(1,0,0,0)[t, x, y, z] + O[1/c]²
  (-2 Bx Bz+(sxz+szx-2 Ex Ez ϵ0) μ0) ux^(0,0,0,1)[t,x,y,z]+(-2 By Bz+(syz+szy-2 Ey Ez ϵ0) μ0) uy^(0,0,0,1)[t,x,y,z]+Bx² uz^(0,0,0,1)[t,x,y,z]+By² uz^(0,0,0,1)[t,x,y,z]-Bz² uz^(0,0,0,1)[t,x,y,z]+2 szz μ0 uz^(0,0,0,1)[t,x,y,z]+Ex² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]+Ey² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-Ez² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-2 Bx By ux^(0,0,1,0)[t,x,y,z]+sxy μ0 ux^(0,0,1,0)[t,x,y,z]+syx μ0 ux^(0,0,1,0)[t,x,y,z]-2 Ex Ey ϵ0 μ0 ux^(0,0,1,0)[t,x,y,z]+Bx² uy^(0,0,1,0)[t,x,y,z]-By² uy^(0,0,1,0)[t,x,y,z]+Bz² uy^(0,0,1,0)[t,x,y,z]+ …
  -n ρ (uz W^(0,0,0,1)[t, x, y, z] + uy W^(0,0,1,0)[t, x, y, z] + ux W^(0,1,0,0)[t, x, y, z]) + √ϵ0 ((-By Ex+Bx Ey) W^(0,0,0,1)[t,x,y,z]+(Bz Ex-Bx Ez) W^(0,0,1,0)[t,x,y,z]+(-Bz Ey+By Ez) W^(0,1,0,0)[t,x,y,z])/(√μ0 c) + O[1/c]²
  n ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]²
  n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]²
  n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]²
  -n ρ (2 ux + t W^(0,1,0,0)[t, x, y, z]) + 2 (-Bz Ey+By Ez) √ϵ0/(√μ0 c) + O[1/c]²
  -n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]² )
```

```
(* Flux of energy is given by: *)
```

In[•]:= **shows[assut, 1][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.tt /. {Ax → 0, Ay → 0, Vz → 0, jz → 0} /. j2v]]**

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(ux^2+uy^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & n\,uy\,\rho+O\left[\frac{1}{c}\right]^2 & O\left[\frac{1}{c}\right]^2 \\ -Az\left(qz+sxz\,ux+syz\,uy\right)+O\left[\frac{1}{c}\right]^2 & Az\,sxz+O\left[\frac{1}{c}\right]^2 & Az\,syz+O\left[\frac{1}{c}\right]^2 & Az\,szz+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:= **shows[assut, 1][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → 0, Vx → 0} /. j2v]]**

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(uy^2+uz^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & O\left[\frac{1}{c}\right]^2 & n\,uy\,\rho+O\left[\frac{1}{c}\right]^2 & n\,uz\,\rho+O\left[\frac{1}{c}\right]^2 \\ \left(-qx-sxy\,uy-sxz\,uz\right)+O\left[\frac{1}{c}\right]^2 & sxx+O\left[\frac{1}{c}\right]^2 & sxy+O\left[\frac{1}{c}\right]^2 & sxz+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:= **shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v]]**

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(ux^2+uy^2+uz^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & n\,ux\,\rho+\frac{qx+sxx\,ux+sxy\,uy+sxz\,uz+\frac{1}{2}\,n\left(-8\,Wx+ux\left(ux^2+uy^2+uz^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & n\,uy\,\rho+\frac{qy+sxy\,ux+syy\,uy+syz\,uz+\frac{1}{2}\,n\left(-8\,Wy+uy\left(ux^2+uy^2+uz^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & n\,uz\,\rho+\frac{qz+sxz\,ux+syz\,uy+szz\,uz+\frac{1}{2}\,n\left(-8\,Wz+uz\left(ux^2+uy^2+uz^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+ \\ n\left(-ux+Vx\right)\rho\,c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\left(ux-Vx\right)\left(ux^2+uy^2+uz^2-2\,W+2\,\epsilon\right)\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(sxx+n\,ux\left(ux-Vx\right)\rho\right)+\frac{qx\left(2\,ux-Vx\right)-\left(sxx\,ux+sxy\,uy+sxz\,uz\right)Vx+\frac{1}{2}\,n\left(ux-Vx\right)\left(ux^3-8\,Wx+ux\left(uy^2+uz^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \left(sxy+n\,uy\left(ux-Vx\right)\rho\right)+\frac{qx\,uy+qy\left(ux-Vx\right)-\left(sxy\,ux+syy\,uy+syz\,uz\right)Vx+\frac{1}{2}\,n\left(ux-Vx\right)\left(-8\,Wy+uy\left(ux^2+uy^2+uz^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \left(sxz+n\,uz\left(ux-Vx\right)\rho\right)+\frac{qx\,uz+qz\left(ux-Vx\right)-\left(sxz\,ux+syz\,uy+szz\,uz\right)}{} \end{pmatrix}$$

In[•]:= **shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v /. {sxx → SXX − n * ux * (ux − Vx) * ρ, sxy → SXY − n * uy * (ux − Vx) * ρ, sxz → SXZ − n * uz * (ux − Vx) * ρ}]]**

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(ux^2+uy^2+uz^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & n\,ux\,\rho+\frac{qx+SXX\,ux+SXY\,uy+SXZ\,uz-\frac{1}{2}\,n\left(ux^3-2\,ux^2\,Vx-2\left(uy^2+uz^2\right)Vx+8\,Wx+ux\left(uy^2+uz^2-6\,W-2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & n\,uy\,\rho+\frac{qy+SXY\,ux+syy\,uy+syz\,uz+\frac{1}{2}\,n\left(-8\,Wy+uy\left(-ux^2+uy^2+uz^2+2\,ux\,Vx+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & n\,uz\,\rho+\frac{qz+SXZ\,ux+syz\,uy+szz\,uz+\frac{1}{2}\,n\left(-8\,Wz+uz\left(-ux^2+uy^2+uz^2+2\,ux\,Vx+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right] \\ n\left(-ux+Vx\right)\rho\,c^2+\left(-qx-SXX\,ux-SXY\,uy-SXZ\,uz+\frac{1}{2}\,n\left(ux-Vx\right)\left(ux^2+uy^2+uz^2+2\,W-2\,\epsilon\right)\right)\rho+O\left[\frac{1}{c}\right]^2 & SXX+\frac{qx\left(2\,ux-Vx\right)-\left(SXX\,ux+SXY\,uy+SXZ\,uz\right)Vx+\frac{1}{2}\,n\left(ux-Vx\right)\left(ux^3+2\,ux^2\,Vx+2\left(uy^2+uz^2\right)Vx-8\,Wx+ux\left(uy^2+uz^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & SXY+\frac{qx\,uy+qy\left(ux-Vx\right)-\left(SXY\,ux+syy\,uy+syz\,uz\right)Vx+\frac{1}{2}\,n\left(ux-Vx\right)\left(-8\,Wy+uy\left(ux^2+uy^2+uz^2+2\,ux\,Vx+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & SXZ+\frac{qx\,uz+qz\left(ux-Vx\right)-\left(SXZ\,ux+syz\,uy+szz\,uz\right)Vx+\frac{1}{2}\,n\left(ux-Vx\right)\left(-8\,Wz+uz\left(ux^2+uy^2+uz^2+2\,ux\right.\right.} \end{pmatrix}$$

In[•]:= **(* matter flux in same direction as imaginary moving surface, different velocity *)**
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jy → 0, jz → 0} /. j2v]]

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(ux^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & n\,ux\,\rho+\frac{qx+sxx\,ux+\frac{1}{2}\,n\left(ux^3+6\,ux\,W-8\,Wx+2\,ux\,\epsilon\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \frac{qy+sxy\,ux-4\,n\,Wy\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \frac{qz+sxz\,ux-4\,n\,Wz\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 \\ n\left(-ux+Vx\right)\rho\,c^2+\left(-qx-sxx\,ux-\frac{1}{2}\,n\left(ux-Vx\right)\left(ux^2-2\,W+2\,\epsilon\right)\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(sxx+n\,ux\left(ux-Vx\right)\rho\right)+\frac{2\,qx\,ux-\left(qx+sxx\,ux\right)Vx+\frac{1}{2}\,n\left(ux-Vx\right)\left(ux^3-8\,Wx+2\,ux\left(3\,W+\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & sxy+\frac{qy\left(ux-Vx\right)-sxy\,ux\,Vx+4\,n\left(-ux+Vx\right)Wy\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & sxz+\frac{qz\left(ux-Vx\right)-sxz\,ux\,Vx+4\,n\left(-ux+Vx\right)Wz\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:= **(* imaginary moving surface, no matter flux through it *)**
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v /. {ux → Vx}]]

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(uy^2+uz^2+Vx^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & n\,Vx\,\rho+\frac{qx+sxy\,uy+sxz\,uz+sxx\,Vx+\frac{1}{2}\,n\left(-8\,Wx+Vx\left(uy^2+uz^2+Vx^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & n\,uy\,\rho+\frac{qy+syy\,uy+syz\,uz+sxy\,Vx+\frac{1}{2}\,n\left(-8\,Wy+uy\left(uy^2+uz^2+Vx^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & n\,uz\,\rho+\frac{qz+syz\,uy+szz\,uz+sxz\,Vx+\frac{1}{2}\,n\left(-8\,Wz+uz\left(uy^2+uz^2+Vx^2+6\,W+2\,\epsilon\right)\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 \\ \left(-qx-sxy\,uy-sxz\,uz-sxx\,Vx\right)+O\left[\frac{1}{c}\right]^2 & sxx-\frac{Vx\left(-qx+sxy\,uy+sxz\,uz+sxx\,Vx\right)}{c^2}+O\left[\frac{1}{c}\right]^3 & sxy+\frac{qx\,uy-Vx\left(syy\,uy+syz\,uz+sxy\,Vx\right)}{c^2}+O\left[\frac{1}{c}\right]^3 & sxz+\frac{qx\,uz-Vx\left(syz\,uy+szz\,uz+sxz\,Vx\right)}{c^2}+O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:= **(* imaginary moving surface, no matter flux through it and no transversal matter motion *)**
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jy → 0, jz → 0} /. j2v /. {ux → Vx}]]

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\left(Vx^2-2\,W+2\,\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & n\,Vx\,\rho+\frac{qx+sxx\,Vx+\frac{1}{2}\,n\left(Vx^3+6\,Vx\,W-8\,Wx+2\,Vx\,\epsilon\right)\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \frac{qy+sxy\,Vx-4\,n\,Wy\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \frac{qz+sxz\,Vx-4\,n\,Wz\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 \\ \left(-qx-sxx\,Vx\right)+O\left[\frac{1}{c}\right]^2 & sxx+\frac{Vx\left(qx-sxx\,Vx\right)}{c^2}+O\left[\frac{1}{c}\right]^3 & sxy-\frac{sxy\,Vx^2}{c^2}+O\left[\frac{1}{c}\right]^3 & sxz-\frac{sxz\,Vx^2}{c^2}+O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:= **(* imaginary moving surface, matter at rest in coordinates *)**
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → 0, jy → 0, jz → 0}]]

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+n\left(W-\epsilon\right)\rho+O\left[\frac{1}{c}\right]^2 & \frac{qx-4\,n\,Wx\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \frac{qy-4\,n\,Wy\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & \frac{qz-4\,n\,Wz\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 \\ n\,Vx\,\rho\,c^2+\left(-qx+n\,Vx\left(-W+\epsilon\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & sxx+\frac{-qx\,Vx+4\,n\,Vx\,Wx\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & sxy+\frac{-qy\,Vx+4\,n\,Vx\,Wy\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 & sxz+\frac{-qz\,Vx+4\,n\,Vx\,Wz\,\rho}{c^2}+O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:= **showf[assut][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → n * Vx, jy → 0, jz → 0}]]**

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{1}{2}\,n\,Vx^2\,\rho+n\,W\,\rho-n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 & n\,Vx\,\rho+\frac{qx+sxx\,Vx+\frac{1}{2}\,n\,Vx^3\,\rho+3\,n\,Vx\,W\,\rho-4\,n\,Wx\,\rho+n\,Vx\,\epsilon\,\rho}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{qy+sxy\,Vx-4\,n\,Wy\,\rho}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{qz+sxz\,Vx-4\,n\,Wz\,\rho}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \left(-qx-sxx\,Vx\right)+O\left[\frac{1}{c}\right]^2 & sxx+\frac{qx\,Vx-sxx\,Vx^2}{c^2}+O\left[\frac{1}{c}\right]^4 & sxy-\frac{sxy\,Vx^2}{c^2}+O\left[\frac{1}{c}\right]^4 & sxz-\frac{sxz\,Vx^2}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[•]:= **(* COORDINATE ENERGY *)**

In[•]:= **(* energy 3-form when projected along coord. axes *)**
showf[assut][Expand //@ FS@PowerExpand[tt.{1, 0, 0, 0}]]

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{jx^2\,\rho}{2\,n}-\frac{jy^2\,\rho}{2\,n}-\frac{jz^2\,\rho}{2\,n}+n\,W\,\rho-n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -jx\,\rho\,c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\,\rho}{2\,n^2}-\frac{jx\,jy^2\,\rho}{2\,n^2}-\frac{jx\,jz^2\,\rho}{2\,n^2}+jx\,W\,\rho-jx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -jy\,\rho\,c^2+\left(-qy-\frac{jx\,sxy}{n}-\frac{jy\,syy}{n}-\frac{jz\,syz}{n}-\frac{jx^2\,jy\,\rho}{2\,n^2}-\frac{jy^3\,\rho}{2\,n^2}-\frac{jy\,jz^2\,\rho}{2\,n^2}+jy\,W\,\rho-jy\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -jz\,\rho\,c^2+\left(-qz-\frac{jx\,sxz}{n}-\frac{jy\,syz}{n}-\frac{jz\,szz}{n}-\frac{jx^2\,jz\,\rho}{2\,n^2}-\frac{jy^2\,jz\,\rho}{2\,n^2}-\frac{jz^3\,\rho}{2\,n^2}+jz\,W\,\rho-jz\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:= **(* in terms of matter velocity *)**
showf[assut][Expand //@ FS@PowerExpand[tt.{1, 0, 0, 0} /. j2v]]

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{1}{2}\,n\,ux^2\,\rho-\frac{1}{2}\,n\,uy^2\,\rho-\frac{1}{2}\,n\,uz^2\,\rho+n\,W\,\rho-n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,ux\,\rho\,c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^3\,\rho-\frac{1}{2}\,n\,ux\,uy^2\,\rho-\frac{1}{2}\,n\,ux\,uz^2\,\rho+n\,ux\,W\,\rho-n\,ux\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,uy\,\rho\,c^2+\left(-qy-sxy\,ux-syy\,uy-syz\,uz-\frac{1}{2}\,n\,ux^2\,uy\,\rho-\frac{1}{2}\,n\,uy^3\,\rho-\frac{1}{2}\,n\,uy\,uz^2\,\rho+n\,uy\,W\,\rho-n\,uy\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -n\,uz\,\rho\,c^2+\left(-qz-sxz\,ux-syz\,uy-szz\,uz-\frac{1}{2}\,n\,ux^2\,uz\,\rho-\frac{1}{2}\,n\,uy^2\,uz\,\rho-\frac{1}{2}\,n\,uz^3\,\rho+n\,uz\,W\,\rho-n\,uz\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:= **(* flux of coord. energy across surface *)**
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt)]]

Out[•]//MatrixForm=

$$\left(-jx\,\rho+n\,Vx\,\rho\right)c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\,\rho}{2\,n^2}-\frac{jx\,jy^2\,\rho}{2\,n^2}-\frac{jx\,jz^2\,\rho}{2\,n^2}+\frac{jx^2\,Vx\,\rho}{2\,n}+\frac{jy^2\,Vx\,\rho}{2\,n}+\frac{jz^2\,Vx\,\rho}{2\,n}+jx\,W\,\rho-n\,Vx\,W\,\rho-jx\,\epsilon\,\rho+n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:= **showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr]]**

Out[•]//MatrixForm=

$$-Lx\,n\,\rho\,c^2+\left(-qx-sxx\,ux-\frac{1}{2}\,Lx\,n\,ux^2\,\rho+Lx\,n\,W\,\rho-Lx\,n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:= **showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. {jx → 0, jy → 0, jz → 0}]]**

Out[•]//MatrixForm=

$$n\,Vx\,\rho\,c^2+\left(-qx-n\,Vx\,W\,\rho+n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:= **(* in terms of matter flux*)**
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf]]

Out[•]//MatrixForm=

$$-JX\,\rho\,c^2+\left(-qx-\frac{JX\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-sxx\,Vx-\frac{JX^3\,\rho}{2\,n^2}-\frac{JX\,jy^2\,\rho}{2\,n^2}-\frac{JX\,jz^2\,\rho}{2\,n^2}-\frac{JX^2\,Vx\,\rho}{n}-\frac{1}{2}\,JX\,Vx^2\,\rho+JX\,W\,\rho-JX\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:= **showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf]]**

Out[•]//MatrixForm=

$$-JX\,\rho\,c^2+\left(-qx-\frac{JX\,sxx}{n}-sxx\,Vx-\frac{JX^3\,\rho}{2\,n^2}-\frac{JX^2\,Vx\,\rho}{n}-\frac{1}{2}\,JX\,Vx^2\,\rho+JX\,W\,\rho-JX\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:= **(* in terms of matter flux & matter velocity*)**
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf /. j2v]]

Out[•]//MatrixForm=

$$-JX\,\rho\,c^2+\left(-qx-\frac{JX\,sxx}{n}-sxy\,uy-sxz\,uz-sxx\,Vx-\frac{JX^3\,\rho}{2\,n^2}-\frac{1}{2}\,JX\,uy^2\,\rho-\frac{1}{2}\,JX\,uz^2\,\rho-\frac{JX^2\,Vx\,\rho}{n}-\frac{1}{2}\,JX\,Vx^2\,\rho+JX\,W\,\rho-JX\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:= **showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf /. {JX → 0, jy → 0, jz → 0}]]**

Out[•]//MatrixForm=

$$\left(-qx-sxx\,Vx\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:= **(* in terms of relative velocity*)**
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. relv]]

Out[•]//MatrixForm=

$$-n\,Vx\,\rho\,c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^2\,Vx\,\rho}{2\,n}-\frac{jy^2\,Vx\,\rho}{2\,n}-\frac{jz^2\,Vx\,\rho}{2\,n}+n\,Vx\,W\,\rho-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:= **(* in terms of relative velocity and matter velocity*)**
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr]]

Out[•]//MatrixForm=

$$-n\,Vx\,\rho\,c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^2\,Vx\,\rho-\frac{1}{2}\,n\,uy^2\,Vx\,\rho-\frac{1}{2}\,n\,uz^2\,Vx\,\rho+n\,Vx\,W\,\rho-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:= **(* with zero rel. velocity*)**
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr /. {Vx → 0}]]

Out[•]//MatrixForm=

$$\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:= **(* supply term for coord. energy *)**
TTx = tW[Normal[tt]]; shows[assut, 2][Expand //@ FS@PowerExpand[Tr[1 / 2 * Normal@(Inverse[gg].T[Dcoords[[1, ;; , ;;]]].gg + Dcoords[[1, ;; , ;;]]).TTx]]]

Out[•]//MatrixForm=

$$n\,\rho\,W^{(1,0,0,0)}[t,\,x,\,y,\,z]+\frac{\left(3\left(jx^2+jy^2+jz^2\right)\rho+2\,n\left(sxx+syy+szz+n\,\epsilon\,\rho\right)-2\,n^2\,\rho\,W[t,\,x,\,y,\,z]\right)W^{(1,0,0,0)}[t,\,x,\,y,\,z]}{2\,n\,c^2}+O\left[\frac{1}{c}\right]^3$$

In[•]:= **(* INTERNAL ENERGY *)**

In[•]:= **(* energy 3-form when projected along matter 4-velocity, "internal energy" *)**
showf[assut][Expand //@ FS@PowerExpand[tt.uu]]

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-n\,\epsilon\,\rho+O\left[\frac{1}{c}\right]^2 \\ -jx\,\rho\,c^2+\left(-qx-jx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -jy\,\rho\,c^2+\left(-qy-jy\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \\ -jz\,\rho\,c^2+\left(-qz-jz\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.uu /. j2vr]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2 - n\,\epsilon\,\rho + O\left[\frac{1}{c}\right]^2 \\ -n\,ux\,\rho\,c^2 + \left(-qx - n\,ux\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uy\,\rho\,c^2 + \left(-qy - n\,uy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uz\,\rho\,c^2 + \left(-qz - n\,uz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* flux of internal energy across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(-j\,x\,\rho + n\,vx\,\rho\right)c^2 + \left(-qx - j\,x\,\epsilon\,\rho + n\,vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. relv]]
```
Out[•]//MatrixForm=

$$-n\,Vx\,\rho\,c^2 + \left(-qx - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=

$$-n\,Vx\,\rho\,c^2 + \left(-qx - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=

$$-qx + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* supply term for internal energy (should be reversed in sign; remember that stress is compressive, not tensile) *)
TTx = tW[tjv@tt]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Duv].gg + Duv).TTx]]]
```
Out[•]//MatrixForm=

$$\left(sxz\,vx^{(0,0,0,1)}[t,x,y,z] + syz\,vy^{(0,0,0,1)}[t,x,y,z] + szz\,vz^{(0,0,0,1)}[t,x,y,z] + sxy\,vx^{(0,0,1,0)}[t,x,y,z] + syy\,vy^{(0,0,1,0)}[t,x,y,z] + syz\,vz^{(0,0,1,0)}[t,x,y,z] + sxx\,vx^{(0,1,0,0)}[t,x,y,z] + sxy\,vy^{(0,1,0,0)}[t,x,y,z] + sxz\,vz^{(0,1,0,0)}[t,x,y,z]\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* difference between "coord. energy" and "internal energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - uu)]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{jx^2\rho}{2n} - \frac{jy^2\rho}{2n} - \frac{jz^2\rho}{2n} + n\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^3\rho}{2n^2} - \frac{jx\,jy^2\rho}{2n^2} - \frac{jx\,jz^2\rho}{2n^2} + j\,x\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\,sxy}{n} - \frac{jy\,syy}{n} - \frac{jz\,syz}{n} - \frac{jx^2\,jy\,\rho}{2n^2} - \frac{jy^3\rho}{2n^2} - \frac{jy\,jz^2\rho}{2n^2} + j\,y\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\,sxz}{n} - \frac{jy\,syz}{n} - \frac{jz\,szz}{n} - \frac{jx^2\,jz\,\rho}{2n^2} - \frac{jy^2\,jz\,\rho}{2n^2} - \frac{jz^3\rho}{2n^2} + j\,z\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - uu) /. j2vr]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{1}{2}\,n\,ux^2\,\rho - \frac{1}{2}\,n\,uy^2\,\rho - \frac{1}{2}\,n\,uz^2\,\rho + n\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2}\,n\,ux^3\,\rho - \frac{1}{2}\,n\,ux\,uy^2\,\rho - \frac{1}{2}\,n\,ux\,uz^2\,\rho + n\,ux\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-sxy\,ux - syy\,uy - syz\,uz - \frac{1}{2}\,n\,ux^2\,uy\,\rho - \frac{1}{2}\,n\,uy^3\,\rho - \frac{1}{2}\,n\,uy\,uz^2\,\rho + n\,uy\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-sxz\,ux - syz\,uy - szz\,uz - \frac{1}{2}\,n\,ux^2\,uz\,\rho - \frac{1}{2}\,n\,uy^2\,uz\,\rho - \frac{1}{2}\,n\,uz^3\,\rho + n\,uz\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(-\frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^3\rho}{2n^2} - \frac{jx\,jy^2\rho}{2n^2} - \frac{jx\,jz^2\rho}{2n^2} + \frac{jx^2\,vx\,\rho}{2n} + \frac{jy^2\,vx\,\rho}{2n} + \frac{jz^2\,vx\,\rho}{2n} + j\,x\,W\,\rho - n\,vx\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. relv]]
```
Out[•]//MatrixForm=

$$\left(-\frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^2\,Vx\,\rho}{2n} - \frac{jy^2\,Vx\,\rho}{2n} - \frac{jz^2\,Vx\,\rho}{2n} + n\,Vx\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=

$$\left(-sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2}\,n\,ux^2\,Vx\,\rho - \frac{1}{2}\,n\,uy^2\,Vx\,\rho - \frac{1}{2}\,n\,uz^2\,Vx\,\rho + n\,Vx\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=

$$\left(-sxx\,ux - sxy\,uy - sxz\,uz\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* PROPER-TIME COORD ENERGY*)
```

In[•]:=
```
(* energy 3-form when projected along normalized coord-t
   note how the gravitational term is missing *)
showf[assut][Expand //@ FS@PowerExpand[tt.vtn]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2 + \left(-\frac{jx^2\rho}{2n} - \frac{jy^2\rho}{2n} - \frac{jz^2\rho}{2n} - n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -j\,x\,\rho\,c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^3\rho}{2n^2} - \frac{jx\,jy^2\rho}{2n^2} - \frac{jx\,jz^2\rho}{2n^2} - j\,x\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -j\,y\,\rho\,c^2 + \left(-qy - \frac{jx\,sxy}{n} - \frac{jy\,syy}{n} - \frac{jz\,syz}{n} - \frac{jx^2\,jy\,\rho}{2n^2} - \frac{jy^3\rho}{2n^2} - \frac{jy\,jz^2\rho}{2n^2} - j\,y\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -j\,z\,\rho\,c^2 + \left(-qz - \frac{jx\,sxz}{n} - \frac{jy\,syz}{n} - \frac{jz\,szz}{n} - \frac{jx^2\,jz\,\rho}{2n^2} - \frac{jy^2\,jz\,\rho}{2n^2} - \frac{jz^3\rho}{2n^2} - j\,z\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.vtn /. j2vr]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2 + \left(-\frac{1}{2}\,n\,ux^2\,\rho - \frac{1}{2}\,n\,uy^2\,\rho - \frac{1}{2}\,n\,uz^2\,\rho - n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,ux\,\rho\,c^2 + \left(-qx - sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2}\,n\,ux^3\,\rho - \frac{1}{2}\,n\,ux\,uy^2\,\rho - \frac{1}{2}\,n\,ux\,uz^2\,\rho - n\,ux\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uy\,\rho\,c^2 + \left(-qy - sxy\,ux - syy\,uy - syz\,uz - \frac{1}{2}\,n\,ux^2\,uy\,\rho - \frac{1}{2}\,n\,uy^3\,\rho - \frac{1}{2}\,n\,uy\,uz^2\,\rho - n\,uy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uz\,\rho\,c^2 + \left(-qz - sxz\,ux - syz\,uy - szz\,uz - \frac{1}{2}\,n\,ux^2\,uz\,\rho - \frac{1}{2}\,n\,uy^2\,uz\,\rho - \frac{1}{2}\,n\,uz^3\,\rho - n\,uz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* flux of normalized-coord-t energy across surface *)
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(-j\,x\,\rho + n\,vx\,\rho\right)c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jx^3\rho}{2n^2} + \frac{jx^2\,vx\,\rho}{2n} - j\,x\,\epsilon\,\rho + n\,vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(*in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. relv]]
```
Out[•]//MatrixForm=

$$-n\,Vx\,\rho\,c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^2\,Vx\,\rho}{2n} - \frac{jy^2\,Vx\,\rho}{2n} - \frac{jz^2\,Vx\,\rho}{2n} - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=

$$-n\,Vx\,\rho\,c^2 + \left(-qx - sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2}\,n\,ux^2\,Vx\,\rho - \frac{1}{2}\,n\,uy^2\,Vx\,\rho - \frac{1}{2}\,n\,uz^2\,Vx\,\rho - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=

$$\left(-qx - sxx\,ux - sxy\,uy - sxz\,uz\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* supply term for normalized-coord-t energy
   we obtain the "power generated by the gravity field" *)
TTx = tW[tjv@tt]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Dvtn].gg + Dvtn).TTx]]]
```
Out[•]//MatrixForm=

$$\left(-\rho\,n[t,x,y,z] \times vz[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z] - \rho\,n[t,x,y,z] \times vy[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z] - \rho\,n[t,x,y,z] \times vx[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=

In[•]:=
```
(* difference between "coord. energy" and "proper-time coord. energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - vtn)]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} n\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ j\,x\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ j\,y\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ j\,z\,W\,\rho + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - vtn) /. j2vr]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} n\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ n\,ux\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ n\,uy\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ n\,uz\,W\,\rho + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(\text{jx W } \rho - \text{n vx W } \rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. relv]]
```
Out[•]//MatrixForm=

$$\text{n Vx W } \rho + O\left[\frac{1}{c}\right]^2$$

In[•]:
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=

$$\text{n Vx W } \rho + O\left[\frac{1}{c}\right]^2$$

In[•]:
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=

$$O\left[\frac{1}{c}\right]^2$$

In[•]:
```
(* difference between "internal energy" and "proper-time coord. energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.(uu - vtn)]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} \left(\frac{\text{jx}^2\,\rho}{2\,\text{n}} + \frac{\text{jy}^2\,\rho}{2\,\text{n}} + \frac{\text{jz}^2\,\rho}{2\,\text{n}}\right) + O\left[\frac{1}{c}\right]^2 \\ \left(\frac{\text{jx sxx}}{\text{n}} + \frac{\text{jy sxy}}{\text{n}} + \frac{\text{jz sxz}}{\text{n}} + \frac{\text{jx}^3\,\rho}{2\,\text{n}^2} + \frac{\text{jx jy}^2\,\rho}{2\,\text{n}^2} + \frac{\text{jx jz}^2\,\rho}{2\,\text{n}^2}\right) + O\left[\frac{1}{c}\right]^2 \\ \left(\frac{\text{jx sxy}}{\text{n}} + \frac{\text{jy syy}}{\text{n}} + \frac{\text{jz syz}}{\text{n}} + \frac{\text{jx}^2\,\text{jy}\,\rho}{2\,\text{n}^2} + \frac{\text{jy}^3\,\rho}{2\,\text{n}^2} + \frac{\text{jy jz}^2\,\rho}{2\,\text{n}^2}\right) + O\left[\frac{1}{c}\right]^2 \\ \left(\frac{\text{jx sxz}}{\text{n}} + \frac{\text{jy syz}}{\text{n}} + \frac{\text{jz szz}}{\text{n}} + \frac{\text{jx}^2\,\text{jz}\,\rho}{2\,\text{n}^2} + \frac{\text{jy}^2\,\text{jz}\,\rho}{2\,\text{n}^2} + \frac{\text{jz}^3\,\rho}{2\,\text{n}^2}\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.(uu - vtn) /. j2vr]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} \left(\frac{1}{2}\,\text{n ux}^2\,\rho + \frac{1}{2}\,\text{n uy}^2\,\rho + \frac{1}{2}\,\text{n uz}^2\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(\text{sxx ux} + \text{sxy uy} + \text{sxz uz} + \frac{1}{2}\,\text{n ux}^3\,\rho + \frac{1}{2}\,\text{n ux uy}^2\,\rho + \frac{1}{2}\,\text{n ux uz}^2\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(\text{sxy ux} + \text{syy uy} + \text{syz uz} + \frac{1}{2}\,\text{n ux}^2\,\text{uy}\,\rho + \frac{1}{2}\,\text{n uy}^3\,\rho + \frac{1}{2}\,\text{n uy uz}^2\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(\text{sxz ux} + \text{syz uy} + \text{szz uz} + \frac{1}{2}\,\text{n ux}^2\,\text{uz}\,\rho + \frac{1}{2}\,\text{n uy}^2\,\text{uz}\,\rho + \frac{1}{2}\,\text{n uz}^3\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - vtn) / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(\frac{\text{jx sxx}}{\text{n}} + \frac{\text{jy sxy}}{\text{n}} + \frac{\text{jz sxz}}{\text{n}} + \frac{\text{jx}^3\,\rho}{2\,\text{n}^2} + \frac{\text{jx jy}^2\,\rho}{2\,\text{n}^2} + \frac{\text{jx jz}^2\,\rho}{2\,\text{n}^2} - \frac{\text{jx}^2\,\text{vx}\,\rho}{2\,\text{n}} - \frac{\text{jy}^2\,\text{vx}\,\rho}{2\,\text{n}} - \frac{\text{jz}^2\,\text{vx}\,\rho}{2\,\text{n}}\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - vtn) / (A * Δt) /. relv]]
```
Out[•]//MatrixForm=

$$\left(\frac{\text{jx sxx}}{\text{n}} + \frac{\text{jy sxy}}{\text{n}} + \frac{\text{jz sxz}}{\text{n}} + \frac{\text{jx}^2\,\text{Vx}\,\rho}{2\,\text{n}} + \frac{\text{jy}^2\,\text{Vx}\,\rho}{2\,\text{n}} + \frac{\text{jz}^2\,\text{Vx}\,\rho}{2\,\text{n}}\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - uu) / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=

$$O\left[\frac{1}{c}\right]^2$$

In[•]:
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - uu) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=

$$O\left[\frac{1}{c}\right]^2$$

In[•]:
```
(* Faraday tensor *)
repE = {Ex → Ex * c * Sqrt[μ0 * ϵ0], Ey → Ey * c * Sqrt[μ0 * ϵ0], Ez → Ez * c * Sqrt[μ0 * ϵ0]};
fftemp = {{0, -Ex, -Ey, -Ez}, {0, 0, Bz, -By}, {0, 0, 0, Bx}, {0, 0, 0, 0}} /. repE;
showf[assut][ffdd = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -\text{c Ex}\,\sqrt{\epsilon 0}\,\sqrt{\mu 0} & -\text{c Ey}\,\sqrt{\epsilon 0}\,\sqrt{\mu 0} & -\text{c Ez}\,\sqrt{\epsilon 0}\,\sqrt{\mu 0} \\ \text{c Ex}\,\sqrt{\epsilon 0}\,\sqrt{\mu 0} & 0 & \text{Bz} & -\text{By} \\ \text{c Ey}\,\sqrt{\epsilon 0}\,\sqrt{\mu 0} & -\text{Bz} & 0 & \text{Bx} \\ \text{c Ez}\,\sqrt{\epsilon 0}\,\sqrt{\mu 0} & \text{By} & -\text{Bx} & 0 \end{pmatrix}$$

In[•]:
```
showf[assut][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
  ]]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{\text{Ex}^2\,\epsilon 0}{2} - \frac{\text{Ey}^2\,\epsilon 0}{2} - \frac{\text{Ez}^2\,\epsilon 0}{2} - \frac{\text{Bx}^2}{2\,\mu 0} - \frac{\text{By}^2}{2\,\mu 0} - \frac{\text{Bz}^2}{2\,\mu 0}\right) + \frac{-\text{Ex}^2\,\text{W}\,\epsilon 0 - \text{Ey}^2\,\text{W}\,\epsilon 0 - \text{Ez}^2\,\text{W}\,\epsilon 0 + \frac{\text{Bx}^2\,\text{W}}{\mu 0} + \frac{\text{By}^2\,\text{W}}{\mu 0} + \frac{\text{Bz}^2\,\text{W}}{\mu 0}}{\text{c}^2} + O\left[\frac{1}{c}\right]^4 & \frac{\frac{\text{Bz Ey}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{\text{By Ez}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}} + \frac{\frac{2\,\text{Bz Ey W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2\,\text{By Ez W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}^3} + \frac{-\frac{4\,\text{By}^2\,\text{Wx}}{\mu 0} - \frac{4\,\text{Bz}^2\,\text{Wx}}{\mu 0} + \frac{4\,\text{Bx By Wy}}{\mu 0} + \frac{4\,\text{Bx Bz Wz}}{\mu 0}}{\text{c}^4} + O\left[\frac{1}{c}\right]^5 & \frac{-\frac{\text{Bz Ex}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\text{Bx Ez}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}} + \frac{-\frac{2\,\text{Bz Ex W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{2\,\text{Bx Ez W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}^3} + \frac{\frac{4\,\text{Bx By Wx}}{\mu 0} - \frac{4\,\text{Bx}^2\,\text{Wy}}{\mu 0} - \frac{4\,\text{By Bz Wz}}{\mu 0}}{\text{c}^4} + O\left[\frac{1}{c}\right]^5 & \frac{\frac{\text{By Ex}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{\text{Bx Ey}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}} + \\ \left(-\frac{\text{Bz Ey}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\text{By Ez}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right)\text{c} + \frac{\frac{2\,\text{Bz Ey W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2\,\text{By Ez W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}} + \frac{-4\,\text{Ey}^2\,\text{Wx}\,\epsilon 0 - 4\,\text{Ez}^2\,\text{Wx}\,\epsilon 0 + 4\,\text{Ex Ey Wy}\,\epsilon 0 + 4\,\text{Ex Ez Wz}\,\epsilon 0}{\text{c}^2} + O\left[\frac{1}{c}\right]^3 & \left(-\frac{\text{Ex}^2\,\epsilon 0}{2} + \frac{\text{Ey}^2\,\epsilon 0}{2} + \frac{\text{Ez}^2\,\epsilon 0}{2} - \frac{\text{Bx}^2}{2\,\mu 0} + \frac{\text{By}^2}{2\,\mu 0} + \frac{\text{Bz}^2}{2\,\mu 0}\right) + \frac{-\text{Ex}^2\,\text{W}\,\epsilon 0 + \text{Ey}^2\,\text{W}\,\epsilon 0 + \text{Ez}^2\,\text{W}\,\epsilon 0 + \frac{\text{Bx}^2\,\text{W}}{\mu 0} - \frac{\text{By}^2\,\text{W}}{\mu 0} - \frac{\text{Bz}^2\,\text{W}}{\mu 0}}{\text{c}^2} + \frac{-\frac{4\,\text{Bz Ex Wy}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\,\text{Bx Ez Wy}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\,\text{By Ex Wz}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\,\text{Bx By Wz}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}^3} + O\left[\frac{1}{c}\right]^4 & \left(-\text{Ex Ey}\,\epsilon 0 - \frac{\text{Bx By}}{\mu 0}\right) + \frac{-2\,\text{Ex Ey W}\,\epsilon 0 + \frac{2\,\text{Bx By}}{\mu 0}}{\text{c}^2} + \frac{\frac{4\,\text{Bx Ex Wx}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\,\text{By Ex Wx}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\,\text{Bx Ey Wy}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\,\text{Bx Ex Wz}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\,\text{By Ex Wz}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\,\text{Bx Ey Wz}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}^3} + O\left[\frac{1}{c}\right]^4 & \left(-\text{Ex Ez}\,\epsilon 0 - \frac{\text{Bx I}}{\mu} \right. \\ \left(\frac{\text{Bz Ex}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{\text{Bx Ez}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right)\text{c} + \frac{\frac{2\,\text{Bz Ex W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{2\,\text{Bx Ez W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}} + \frac{4\,\text{Ex Ey Wx}\,\epsilon 0 - 4\,\text{Ex}^2\,\text{Wy}\,\epsilon 0 - 4\,\text{Ez}^2\,\text{Wy}\,\epsilon 0 + 4\,\text{Ey Ez Wz}\,\epsilon 0}{\text{c}^2} + O\left[\frac{1}{c}\right]^3 & \left(-\text{Ex Ey}\,\epsilon 0 - \frac{\text{Bx By}}{\mu 0}\right) + \frac{-2\,\text{Ex Ey W}\,\epsilon 0 + \frac{2\,\text{Bx By W}}{\mu 0}}{\text{c}^2} + \frac{\frac{4\,\text{Bz Ex Wx}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\,\text{By Ez Wy}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\,\text{Bx Ex Wz}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\,\text{By Ey Wz}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}^3} + O\left[\frac{1}{c}\right]^4 & \left(\frac{\text{Ex}^2\,\epsilon 0}{2} - \frac{\text{Ey}^2\,\epsilon 0}{2} + \frac{\text{Ez}^2\,\epsilon 0}{2} + \frac{\text{Bx}^2}{2\,\mu 0} - \frac{\text{By}^2}{2\,\mu 0} + \frac{\text{Bz}^2}{2\,\mu 0}\right) + \frac{\text{Ex}^2\,\text{W}\,\epsilon 0 - \text{Ey}^2\,\text{W}\,\epsilon 0 + \text{Ez}^2\,\text{W}\,\epsilon 0 - \frac{\text{Bx}^2\,\text{W}}{\mu 0} + \frac{\text{By}^2\,\text{W}}{\mu 0} - \frac{\text{Bz}^2\,\text{W}}{\mu 0}}{\text{c}^2} + \frac{\frac{4\,\text{Bz Ey Wx}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\,\text{By Ez Wx}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\,\text{Bz Ex Wy}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\,\text{Bx Ez Wy}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\,\text{Bz Ey Wz}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\,\text{By Ex Wz}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}^3} + O\left[\frac{1}{c}\right]^4 & \left(-\text{Ex Ez}\,\epsilon 0 - \frac{\text{By I}}{\mu} \right. \\ \left(-\frac{\text{By Ex}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\text{Bx Ey}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right)\text{c} + \frac{\frac{2\,\text{By Ex W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2\,\text{Bx Ey W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}} + \frac{4\,\text{Ex Ez Wx}\,\epsilon 0 + 4\,\text{Ey Ez Wy}\,\epsilon 0 - 4\,\text{Ex}^2\,\text{Wz}\,\epsilon 0 - 4\,\text{Ey}^2\,\text{Wz}\,\epsilon 0}{\text{c}^2} + O\left[\frac{1}{c}\right]^3 & \left(-\text{Ex Ez}\,\epsilon 0 - \frac{\text{Bx Bz}}{\mu 0}\right) + \frac{-2\,\text{Ex Ez W}\,\epsilon 0 + \frac{2\,\text{Bx Bz W}}{\mu 0}}{\text{c}^2} + \frac{-\frac{4\,\text{By Ex Wx}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\,\text{Bx Ez Wy}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\,\text{Bz Ex Wy}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\,\text{Bz Ey Wz}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}^3} + O\left[\frac{1}{c}\right]^4 & \left(-\text{Ey Ez}\,\epsilon 0 - \frac{\text{By Bz}}{\mu 0}\right) + \frac{-2\,\text{Ey Ez W}\,\epsilon 0 + \frac{2\,\text{By Bz W}}{\mu 0}}{\text{c}^2} + \frac{-\frac{4\,\text{By Ey Wx}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\,\text{Bz Ex Wx}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4\,\text{Bx Ey Wy}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4\,\text{Bx Ex Wz}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}^3} + O\left[\frac{1}{c}\right]^4 & \left(\frac{\text{Ex}^2\,\epsilon 0}{2} + \frac{\text{Ey}^2\,\epsilon 0}{2} - \right. \end{pmatrix}$$

In[•]:
```
(* COORDINATE EM ENERGY *)
```

In[•]:
```
(* energy 3-form when projected along coord. axes *)
showf[assut][Expand //@ FS@PowerExpand[tte.{1, 0, 0, 0}]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{\text{Ex}^2\,\epsilon 0}{2} - \frac{\text{Ey}^2\,\epsilon 0}{2} - \frac{\text{Ez}^2\,\epsilon 0}{2} - \frac{\text{Bx}^2}{2\,\mu 0} - \frac{\text{By}^2}{2\,\mu 0} - \frac{\text{Bz}^2}{2\,\mu 0}\right) + \frac{-\text{Ex}^2\,\text{W}\,\epsilon 0 - \text{Ey}^2\,\text{W}\,\epsilon 0 - \text{Ez}^2\,\text{W}\,\epsilon 0 + \frac{\text{Bx}^2\,\text{W}}{\mu 0} + \frac{\text{By}^2\,\text{W}}{\mu 0} + \frac{\text{Bz}^2\,\text{W}}{\mu 0}}{\text{c}^2} + O\left[\frac{1}{c}\right]^4 \\ \left(-\frac{\text{Bz Ey}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\text{By Ez}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right)\text{c} + \frac{\frac{2\,\text{Bz Ey W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2\,\text{By Ez W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}} + \frac{-4\,\text{Ey}^2\,\text{Wx}\,\epsilon 0 - 4\,\text{Ez}^2\,\text{Wx}\,\epsilon 0 + 4\,\text{Ex Ey Wy}\,\epsilon 0 + 4\,\text{Ex Ez Wz}\,\epsilon 0}{\text{c}^2} + O\left[\frac{1}{c}\right]^3 \\ \left(\frac{\text{Bz Ex}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{\text{Bx Ez}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right)\text{c} + \frac{-\frac{2\,\text{Bz Ex W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{2\,\text{Bx Ez W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}} + \frac{4\,\text{Ex Ey Wx}\,\epsilon 0 - 4\,\text{Ex}^2\,\text{Wy}\,\epsilon 0 - 4\,\text{Ez}^2\,\text{Wy}\,\epsilon 0 + 4\,\text{Ey Ez Wz}\,\epsilon 0}{\text{c}^2} + O\left[\frac{1}{c}\right]^3 \\ \left(-\frac{\text{By Ex}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\text{Bx Ey}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right)\text{c} + \frac{\frac{2\,\text{By Ex W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2\,\text{Bx Ey W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}} + \frac{4\,\text{Ex Ez Wx}\,\epsilon 0 + 4\,\text{Ey Ez Wy}\,\epsilon 0 - 4\,\text{Ex}^2\,\text{Wz}\,\epsilon 0 - 4\,\text{Ey}^2\,\text{Wz}\,\epsilon 0}{\text{c}^2} + O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:
```
(* flux of coord. energy across surface *)
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tte.{1, 0, 0, 0} / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(-\frac{\text{Bz Ey}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\text{By Ez}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right)\text{c} + \left(\frac{1}{2}\,\text{Ex}^2\,\text{vx}\,\epsilon 0 + \frac{1}{2}\,\text{Ey}^2\,\text{vx}\,\epsilon 0 + \frac{1}{2}\,\text{Ez}^2\,\text{vx}\,\epsilon 0 + \frac{\text{Bx}^2\,\text{vx}}{2\,\mu 0} + \frac{\text{By}^2\,\text{vx}}{2\,\mu 0} + \frac{\text{Bz}^2\,\text{vx}}{2\,\mu 0}\right) + \frac{\frac{2\,\text{Bz Ey W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2\,\text{By Ez W}\,\sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{\text{c}} + \frac{\text{Ex}^2\,\text{vx W}\,\epsilon 0 + \text{Ey}^2\,\text{vx W}\,\epsilon 0 + \text{Ez}^2\,\text{vx W}\,\epsilon 0 - 4\,\text{Ey}^2\,\text{Wx}\,\epsilon 0 - 4\,\text{Ez}^2\,\text{Wx}\,\epsilon 0 + 4\,\text{Ex Ey Wy}\,\epsilon 0 + 4\,\text{Ex Ez Wz}\,\epsilon 0 - \frac{\text{Bx}^2\,\text{vx W}}{\mu 0} - \frac{\text{By}^2\,\text{vx W}}{\mu 0} - \frac{\text{Bz}^2\,\text{vx W}}{\mu 0}}{\text{c}^2} + O\left[\frac{1}{c}\right]^3$$

In[•]:
```
(* supply term for coord. energy *)
TTx = tW[tjv@tte]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Dcoords[[1, ;; , ;;]]].gg + Dcoords[[1, ;; , ;;]]).TTx]]]
```
Out[•]//MatrixForm=

$$\frac{1}{\text{c}^2}\left(\text{Ex}^2\,\epsilon 0\,\text{W}^{(1,0,0,0)}[\text{t, x, y, z}] + \text{Ey}^2\,\epsilon 0\,\text{W}^{(1,0,0,0)}[\text{t, x, y, z}] + \text{Ez}^2\,\epsilon 0\,\text{W}^{(1,0,0,0)}[\text{t, x, y, z}] + \frac{\text{Bx}^2\,\text{W}^{(1,0,0,0)}[\text{t, x, y, z}]}{\mu 0} + \frac{\text{By}^2\,\text{W}^{(1,0,0,0)}[\text{t, x, y, z}]}{\mu 0} + \frac{\text{Bz}^2\,\text{W}^{(1,0,0,0)}[\text{t, x, y, z}]}{\mu 0}\right) + O\left[\frac{1}{c}\right]^3$$