In[1]:=
```
<< "christoffelsymbols.m"
(* First index is upper index Table[FS[cc⟦ii,;;,;;⟧==T[cc⟦ii,;;,;;⟧]],{ii,1,4}] *)
```

In[∘]:=
```
showf[assumptions_, simp_ : FullSimplify] := ((Assuming[assumptions, Expand //@ simp@PowerExpand[#]] // MF) &);
```

In[∘]:=
```
(* Show matrix expressions and power expansions*)
show[assumptions_, power_, simp_ : FullSimplify] := ({Assuming[assumptions, simp@PowerExpand[#]] // MF, "\n",
      Assuming[assumptions, simp@PowerExpand[Series[#, {c, Infinity, power}]]] // MF} &);
shows[assumptions_, power_, simp_ : FullSimplify] := ((
      Assuming[assumptions, simp@PowerExpand[Series[#, {c, Infinity, power}]]] // MF) &);
showf[assumptions_, simp_ : FullSimplify] := ((Assuming[assumptions, Expand //@ simp@PowerExpand[#]] // MF) &);
show1[assumptions_, simp_ : Identity] := ((Assuming[assumptions, simp[#]] // MF) &);
show2[assumptions_, power_, simp_ : Identity] := ((
      Assuming[assumptions, simp[Series[#, {c, Infinity, power}]]] // MF) &);
```

In[2]:=
```
coords = {t, x, y, z}
```

Out[2]= $\{t, x, y, z\}$

In[3]:=
```
(* Flat metric *)
(gg0 = DiagonalMatrix[{-c^2, 1, 1, 1}]) // MF
```

Out[3]//MatrixForm=

$$\begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[∘]:=
```
(*D[G*M/Sqrt[x^2+y^2+z^2],{{x,y,z}}] *)
```

In[∘]:=
```
(* -W is the potential gravitational energy: W=GM/r
  that is, F_g(downwards)=grad W
*)
```

In[4]:=
```
(* Rotating metric from poissonetal *)
(gg = {{-c^2*(1-2*W/c^2 +2*W^2/c^4)+O[c, +Infinity]^3,
     -4*Wx/c^2+O[c, +Infinity]^4, -4*Wy/c^2+O[c, +Infinity]^4, -4*Wz/c^2+O[c, +Infinity]^4},
    {-4*Wx/c^2+O[c, +Infinity]^4,
     1+2*W/c^2+O[c, +Infinity]^4, 0, 0},
    {-4*Wy/c^2+O[c, +Infinity]^4, 0, 1+2*W/c^2+O[c, +Infinity]^4, 0},
    {-4*Wz/c^2+O[c, +Infinity]^4, 0, 0, 1+2*W/c^2+O[c, +Infinity]^4}}) // MF
(*(gg=DiagonalMatrix[{-c^2*(1+2*Φ[t,r]/c^2),1+2*Λ[t,r]/c^2,r^2,r^2*Sin[θ]^2}])//MF*)
```

Out[4]//MatrixForm=

$$\begin{pmatrix} -c^2+2W-\frac{2W^2}{c^2}+O\left[\frac{1}{c}\right]^3 & -\frac{4Wx}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{4Wy}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{4Wz}{c^2}+O\left[\frac{1}{c}\right]^4 \\ -\frac{4Wx}{c^2}+O\left[\frac{1}{c}\right]^4 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 0 \\ -\frac{4Wy}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 \\ -\frac{4Wz}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 0 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[5]:=
```
Inverse[gg] // MF
```

Out[5]//MatrixForm=

$$\begin{pmatrix} -\left(\frac{1}{c}\right)^2-\frac{2W}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4Wz}{c^4}+O\left[\frac{1}{c}\right]^6 \\ -\frac{4Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{16(Wx\,Wy)}{c^6}+O\left[\frac{1}{c}\right]^8 & -\frac{16(Wx\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 \\ -\frac{4Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{16(Wx\,Wy)}{c^6}+O\left[\frac{1}{c}\right]^8 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{16(Wy\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 \\ -\frac{4Wz}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{16(Wx\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 & -\frac{16(Wy\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[∘]:=
```
(*(gg=DiagonalMatrix@Diagonal[gg])//MF*)
```

In[6]:=
```
(* functions to temporarily remove coord-dep *)
tW[xx_] := (xx /. {W → W[t, x, y, z], Wx → Wx[t, x, y, z], Wy → Wy[t, x, y, z], Wz → Wz[t, x, y, z]});
itW[xx_] := (xx /. {W[t, x, y, z] → W, Wx[t, x, y, z] → Wx, Wy[t, x, y, z] → Wy, Wz[t, x, y, z] → Wz});
(ggt = tW[gg]) // MF;
```

In[9]:=
```
assut = {c > 0, Element[a, Reals], Element[v, Reals], Element[t, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals],
   Element[vx, Reals], Element[vy, Reals], Element[vz, Reals], Element[n, Reals], Element[r, Reals], Element[θ, Reals], Element[ϕ, Reals], Abs[v] < c, -c < vx < c, -c < ux < c, r > 0, 0 < θ < Pi,
   Normal@gg⟦1, 1⟧/c^2 < 0, Normal@gg⟦2, 2⟧ > 0, Normal@gg⟦3, 3⟧ > 0, Normal@gg⟦4, 4⟧ > 0, n > 0, Element[jx, Reals], Element[jy, Reals], Element[jz, Reals], Element[sxx, Reals], Element[sxy, Reals], Element[sxz, Reals], Element[syy, Reals], Element[syz, Reals], Element[szz, Reals],
   -Normal@Det[gg] > 0, β > 0};

assutt = {c > 0, Element[a, Reals], Element[v, Reals], Element[t, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals],
   Element[vx, Reals], Element[vy, Reals], Element[vz, Reals], Element[n, Reals], Element[r, Reals], Element[θ, Reals], Element[ϕ, Reals], Abs[v] < c, -c < vx < c, -c < ux < c, r > 0, 0 < θ < Pi,
   β > 0, Normal@ggt⟦1, 1⟧/c^2 < 0, Normal@ggt⟦2, 2⟧ > 0, Normal@ggt⟦3, 3⟧ > 0, Normal@ggt⟦4, 4⟧ > 0,
   -Normal@Det[ggt] > 0};
```

In[11]:=
```
(igg = Assuming[assut, FullSimplify@PowerExpand[Inverse[gg]]]) // MF
```

Out[11]//MatrixForm=

$$\begin{pmatrix} -\left(\frac{1}{c}\right)^2-\frac{2W}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4Wz}{c^4}+O\left[\frac{1}{c}\right]^6 \\ -\frac{4Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{16(Wx\,Wy)}{c^6}+O\left[\frac{1}{c}\right]^8 & -\frac{16(Wx\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 \\ -\frac{4Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{16(Wx\,Wy)}{c^6}+O\left[\frac{1}{c}\right]^8 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{16(Wy\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 \\ -\frac{4Wz}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{16(Wx\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 & -\frac{16(Wy\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[∘]:=
```
(*show[assut,2]@ChristoffelSymbol[gg,coords]⟦2⟧*)
```

In[12]:=
```
(* volume element *)
(dg = Assuming[assut, FullSimplify@PowerExpand[Sqrt[-Det[gg]]/c]]) // MF
```

Out[12]//MatrixForm=

$$1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4$$

In[13]:=
```
(* Christoffel symbols  *)
cc = Assuming[assut, FullSimplify@PowerExpand[itW[ChristoffelSymbol[ggt, coords]]]];
```

In[∘]:=

In[14]:=
```
(* 3-vector of moving surface parallel to yz moving with velocity V   *)
surface = {-(Vx*Ax+Vy*Ay+Vz*Az), Ax, Ay, Az}*Δt;
```

In[∘]:=

(* Matter current *)

In[15]:=
```
(* matter-current 3-covector *)
NJ = {n, jx, jy, jz};
```

In[16]:=
```
(* norm of matter 3-covector *)
Assuming[assut, FS[Sqrt[-NJ.gg.NJ]/c]]
```

Out[16]=

$$n-\frac{jx^2+jy^2+jz^2+2n^2W}{2nc^2}-\frac{jx^4+jy^4+jz^4+12jz^2n^2W-4n^4W^2+2jy^2\left(jz^2+6n^2W\right)+2jx^2\left(jy^2+jz^2+6n^2W\right)-32\,jx\,n^3Wx-32\,jy\,n^3Wy-32\,jz\,n^3Wz}{8n^3c^4}+O\left[\frac{1}{c}\right]^5$$

In[17]:=
```
(* matter associated 1-vector *)
(NJvec = Assuming[assut, FS[NJ/dg]]) // MF
```

Out[17]//MatrixForm=

$$\begin{pmatrix} n-\frac{2(nW)}{c^2}+O\left[\frac{1}{c}\right]^4 \\ jx-\frac{2(jx\,W)}{c^2}+O\left[\frac{1}{c}\right]^4 \\ jy-\frac{2(jy\,W)}{c^2}+O\left[\frac{1}{c}\right]^4 \\ jz-\frac{2(jz\,W)}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[18]:=
```
(* matter associated 4-vel vector *)
(uu = Assuming[assut, FS[c*NJvec/Sqrt[-NJvec.gg.NJvec]]]) // MF
```

Out[18]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{jx^2+jy^2+jz^2}{2n^2}+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jx}{n}+\frac{jx\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n}+\frac{jy\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n}+\frac{jz\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[19]:=
```
(* replace matter flux in terms of velocity*)
replaceJu = {jx → ux*n, jy → uy*n, jz → uz*n}
```

Out[19]= $\{jx \to n\,ux,\ jy \to n\,uy,\ jz \to n\,uz\}$

In[20]:=
```
(* collect velocity magnitude*)
replaceuUnorm = {ux^2 → U^2-uy^2-uz^2, ux^3 → ux*(U^2-uy^2-uz^2), jx^2 → J^2-jy^2-jz^2, jx^3 → jx*(J^2-jy^2-jz^2)}
```

Out[20]= $\{ux^2 \to U^2-uy^2-uz^2,\ ux^3 \to ux\left(U^2-uy^2-uz^2\right),\ jx^2 \to J^2-jy^2-jz^2,\ jx^3 \to jx\left(J^2-jy^2-jz^2\right)\}$

In[21]:=
```
Assuming[assut, FS[uu /. replaceJu]] // MF
```

Out[21]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{1}{2}\left(ux^2+uy^2+uz^2\right)+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ ux+\frac{ux\left(ux^2+uy^2+uz^2+2W\right)}{2c^2}+O\left[\frac{1}{c}\right]^4 \\ uy+\frac{uy\left(ux^2+uy^2+uz^2+2W\right)}{2c^2}+O\left[\frac{1}{c}\right]^4 \\ uz+\frac{uz\left(ux^2+uy^2+uz^2+2W\right)}{2c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[23]:=
```
(* it is normalized *)
FS[uu.gg.uu]
```

Out[23]= $-c^2+O\left[\frac{1}{c}\right]^2$

In[24]:=
```
(* matter associated 1-covector *)
(NJcov = Assuming[assut, FS[gg.(NJ / dg)]]) // MF
```
Out[24]//MatrixForm=

$$\begin{pmatrix} -n\,c^2 + 4\,n\,W + O\left[\frac{1}{c}\right]^2 \\ jx - \frac{4\,(n\,Wx)}{c^2} + O\left[\frac{1}{c}\right]^4 \\ jy - \frac{4\,(n\,Wy)}{c^2} + O\left[\frac{1}{c}\right]^4 \\ jz - \frac{4\,(n\,Wz)}{c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[28]:=
```
(* scalar product with de/de_x (for momentum x-component) *)
Simplify[uu.gg.{0, 1, 0, 0}]
```
Out[28]=
$$\frac{jx}{n} + \frac{jx^3 + jx\,(jy^2 + jz^2 + 6\,n^2\,W) - 8\,n^3\,Wx}{2\,n^3\,c^2} + O\left[\frac{1}{c}\right]^4$$

In[27]:=
```
(* scalar product with de/de_i (for momentum i-component) *)
Simplify[uu.gg] // MF
```
Out[27]//MatrixForm=

$$\begin{pmatrix} -c^2 + \left(-\frac{jx^2+jy^2+jz^2}{2\,n^2} + W\right) + O\left[\frac{1}{c}\right]^2 \\ \frac{jx}{n} + \frac{jx^3+jx\,(jy^2+jz^2+6\,n^2\,W)-8\,n^3\,Wx}{2\,n^3\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n} + \frac{jx^2\,jy+jy^3+jy\,jz^2+6\,jy\,n^2\,W-8\,n^3\,Wy}{2\,n^3\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n} + \frac{jx^2\,jz+jy^2\,jz+jz^3+6\,jz\,n^2\,W-8\,n^3\,Wz}{2\,n^3\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[29]:=
```
(* retransform matter 4-vel to matter 3-covector *)
Assuming[assut, FS[uu * dg / c * Sqrt[-NJvec.gg.NJvec]]] // MF
```
Out[29]//MatrixForm=

$$\begin{pmatrix} n + O\left[\frac{1}{c}\right]^4 \\ jx + O\left[\frac{1}{c}\right]^4 \\ jy + O\left[\frac{1}{c}\right]^4 \\ jz + O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[30]:=
```
(* simplification to x-directed matter flux and velocity *)
assutjx = Join[assut, {jy == 0, jz == 0, uy == 0, uz == 0}];
```

In[31]:=
```
(* flux of matter across surface *)
Simplify[surface.NJ / (Δt)]
```
Out[31]= $Ax\,(jx - n\,Vx) + Ay\,(jy - n\,Vy) + Az\,(jz - n\,Vz)$

In[32]:=
```
(* normalized zero-flux velocity is same as U *)
vnoflux = {1, jx / n, jy / n, jz / n};
FS[c * vnoflux / Sqrt[-vnoflux.gg.vnoflux] == uu]
```
Out[33]= True

In[34]:=
```
FS[uu /. replaceuUnorm] // MF
```
Out[34]//MatrixForm=

$$\begin{pmatrix} 1 + \frac{\frac{J^2}{2\,n^2}+W}{c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jx}{n} + \frac{jx\,(J^2+2\,n^2\,W)}{2\,n^3\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n} + \frac{jy\,(J^2+2\,n^2\,W)}{2\,n^3\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n} + \frac{jz\,(J^2+2\,n^2\,W)}{2\,n^3\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[35]:=
```
FS[uu /. replaceJu] // MF
```
Out[35]//MatrixForm=

$$\begin{pmatrix} 1 + \frac{\frac{1}{2}\,(ux^2+uy^2+uz^2)+W}{c^2} + O\left[\frac{1}{c}\right]^4 \\ ux + \frac{ux\,(ux^2+uy^2+uz^2+2\,W)}{2\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uy + \frac{uy\,(ux^2+uy^2+uz^2+2\,W)}{2\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uz + \frac{uz\,(ux^2+uy^2+uz^2+2\,W)}{2\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[•]:=

In[36]:=
```
(* Project along u velocity *)
proju = Assuming[assut, Expand //@ FS@PowerExpand[-Outer[Times, uu, gg.uu] / c ^ 2]];
projperpu = Assuming[assut, Expand //@ FS@PowerExpand[IdentityMatrix[4] - proju]];
testproj[ass_, x_] := showf[ass] /@ {Assuming[ass, Expand //@ FS@PowerExpand[proju.x.proju == x]], projperpu.x.proju == x, proju.x.projperpu == x, projperpu.x.projperpu == x}
```

In[39]:=
```
(* Project along u velocity *)
proju = Assuming[assut, FS[-Outer[Times, uu, gg.uu] / c ^ 2]];
projperpu = Assuming[assut, FS[IdentityMatrix[4] - proju]];
testproj[ass_, x_] := showf[ass] /@ {Assuming[ass, Expand //@ FS@PowerExpand[proju.x.proju == x]], projperpu.x.proju == x, proju.x.projperpu == x, projperpu.x.projperpu == x}
```

In[44]:=
```
(* aux 4-velocity *)
auu = {temp, aux, auy, auz};
solu = FS[temp /. Solve[Normal[auu.gg.auu] == -c ^ 2, temp][[2]]]
```
Out[45]=
$$-\frac{4\,aux\,Wx + 4\,auy\,Wy + 4\,auz\,Wz + c^2\,\sqrt{\frac{\left(c^2\,(aux^2+auy^2+auz^2+c^2)+2\,(aux^2+auy^2+auz^2)\,W\right)\left(c^4-2\,c^2\,W+2\,W^2\right)+16\,\left(aux\,Wx+auy\,Wy+auz\,Wz\right)^2}{c^4}}}{c^4 - 2\,c^2\,W + 2\,W^2}$$

In[46]:=
```
(auu = Assuming[assut, FS[auu /. {temp → solu}]]) // MF
```
Out[46]//MatrixForm=

$$\begin{pmatrix} -\frac{4\,aux\,Wx+4\,auy\,Wy+4\,auz\,Wz+\sqrt{\left(c^2\,(aux^2+auy^2+auz^2+c^2)+2\,(aux^2+auy^2+auz^2)\,W\right)\left(c^4-2\,c^2\,W+2\,W^2\right)+16\,\left(aux\,Wx+auy\,Wy+auz\,Wz\right)^2}}{c^4-2\,c^2\,W+2\,W^2} \\ aux \\ auy \\ auz \end{pmatrix}$$

In[48]:=
```
FS[auu.gg.auu]
```
Out[48]= $-c^2 + O\left[\frac{1}{c}\right]^3$

In[49]:=
```
(* 4-velocity and matter current with explicit coordinate dependence *)
tjv[xx_] := (xx /. {n → n[t, x, y, z], jx → ux[t, x, y, z] * n[t, x, y, z], jy → uy[t, x, y, z] * n[t, x, y, z], jz → uz[t, x, y, z] * n[t, x, y, z]});
tjn[xx_] := (xx /. {n → n[t, x, y, z], jx → jx[t, x, y, z], jy → jy[t, x, y, z], jz → jz[t, x, y, z]});
itjn[xx_] := (xx /. {n[t, x, y, z] → n, jx[t, x, y, z] → jx, jy[t, x, y, z] → jy, jz[t, x, y, z] → jz});
itjv[xx_] := (xx /. {n[t, x, y, z] → n, ux[t, x, y, z] → jx / n, uy[t, x, y, z] → jy / n, uz[t, x, y, z] → jz / n});
repjn = {D[n[t, x, y, z], t] → D[jx[t, x, y, z], x] + D[jy[t, x, y, z], y] + D[jz[t, x, y, z], z]};
MF /@ {uut = Assuming[assut, FS[tjn[tW[uu]]]], uuv = Assuming[assut, FS[tjv[tW[uu]]]]}
```
Out[54]=
$$\left\{\begin{pmatrix} 1 + \frac{\frac{jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2}{2\,n[t,x,y,z]^2}+W[t,x,y,z]}{c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jx[t,x,y,z]}{n[t,x,y,z]} + \frac{jx[t,x,y,z]\left(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z]\right)}{2\,n[t,x,y,z]^3\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jy[t,x,y,z]}{n[t,x,y,z]} + \frac{jy[t,x,y,z]\left(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z]\right)}{2\,n[t,x,y,z]^3\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jz[t,x,y,z]}{n[t,x,y,z]} + \frac{jz[t,x,y,z]\left(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z]\right)}{2\,n[t,x,y,z]^3\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} 1 + \frac{\frac{1}{2}\,(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2)+W[t,x,y,z]}{c^2} + O\left[\frac{1}{c}\right]^4 \\ ux[t,x,y,z] + \frac{ux[t,x,y,z]\left(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z]\right)}{2\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uy[t,x,y,z] + \frac{uy[t,x,y,z]\left(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z]\right)}{2\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uz[t,x,y,z] + \frac{uz[t,x,y,z]\left(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z]\right)}{2\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[•]:=

In[•]:=

(* Construction of energy–momentum tensor *)

In[55]:=
```
(* definition of heat-flux, orthogonal to matter-current *)
Qtemp = {qt, qx, qy, qz};
```

In[56]:=
```
(proju.Qtemp) // MF
```
Out[56]//MatrixForm=

$$\begin{pmatrix} qt + \frac{\frac{(jx^2+jy^2+jz^2)\,qt}{n^2}-\frac{jx\,qx}{n}-\frac{jy\,qy}{n}-\frac{jz\,qz}{n}}{c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jx\,qt}{n} + \frac{\frac{jx\,(jx^2+jy^2+jz^2)\,qt}{n^3}-\frac{jx^2\,qx}{n^2}-\frac{jx\,jy\,qy}{n^2}-\frac{jx\,jz\,qz}{n^2}}{c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jy\,qt}{n} + \frac{\frac{jy\,(jx^2+jy^2+jz^2)\,qt}{n^3}-\frac{jx\,jy\,qx}{n^2}-\frac{jy^2\,qy}{n^2}-\frac{jy\,jz\,qz}{n^2}}{c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jz\,qt}{n} + \frac{\frac{jz\,(jx^2+jy^2+jz^2)\,qt}{n^3}-\frac{jx\,jz\,qx}{n^2}-\frac{jy\,jz\,qy}{n^2}-\frac{jz^2\,qz}{n^2}}{c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[57]:=
```
qsol = Solve[Normal[proju.Qtemp] == 0, qt][[1]]
```
Out[57]=
$$\left\{qt \to \frac{n\,(jx\,qx + jy\,qy + jz\,qz)}{jx^2 + jy^2 + jz^2 + c^2\,n^2}\right\}$$

In[58]:=
```
(Q = Assuming[assut, FS[Qtemp /. qsol]]) // MF
```
Out[58]//MatrixForm=

$$\begin{pmatrix} \frac{n\,(jx\,qx+jy\,qy+jz\,qz)}{jx^2+jy^2+jz^2+c^2\,n^2} \\ qx \\ qy \\ qz \end{pmatrix}$$

In[59]:=
```
{Normal@Series[Q.NJ, {c, Infinity, 1}] == 0}
```
Out[59]= $\{jx\,qx + jy\,qy + jz\,qz == 0\}$

In[60]:=
```
assutQ = Join[assut, {Normal@Series[Q.NJ, {c, Infinity, 1}] == 0}];
```

In[61]:=
```
Assuming[assutQ, FS@{proju.Q, projperpu.Q == Q}]
```

Out[61]=
$$\left\{\left\{\frac{4\left(qx\,Wx+qy\,Wy+qz\,Wz\right)}{c^4}+O\left[\frac{1}{c}\right]^6,\ \frac{4\left(-jy\,qy\,Wx-jz\,qz\,Wx+jx\,qy\,Wy+jx\,qz\,Wz\right)}{n\,c^4}+O\left[\frac{1}{c}\right]^6,\ \frac{4\,jy\left(qx\,Wx+qy\,Wy+qz\,Wz\right)}{n\,c^4}+O\left[\frac{1}{c}\right]^6,\ \frac{4\,jz\left(qx\,Wx+qy\,Wy+qz\,Wz\right)}{n\,c^4}+O\left[\frac{1}{c}\right]^6\right\},\right.$$
$$\left.\left\{-\frac{4\left(qx\,Wx+qy\,Wy+qz\,Wz\right)}{c^4}+O\left[\frac{1}{c}\right]^6,\ \frac{4\left(jy\,qy\,Wx+jz\,qz\,Wx-jx\left(qy\,Wy+qz\,Wz\right)\right)}{n\,c^4}+O\left[\frac{1}{c}\right]^6,\ -\frac{4\left(jy\left(qx\,Wx+qy\,Wy+qz\,Wz\right)\right)}{n\,c^4}+O\left[\frac{1}{c}\right]^6,\ -\frac{4\left(jz\left(qx\,Wx+qy\,Wy+qz\,Wz\right)\right)}{n\,c^4}+O\left[\frac{1}{c}\right]^6\right\}==\{0,0,0,0\}\right\}$$

In[62]:=
```
(* non-symmetric heat-tensor *)
Assuming[assutQ, FS[Qtens = Assuming[assut, Expand //@ FS@PowerExpand[Outer[Times, Q, gg.uu / c^2]]]]] // MF
```

Out[62]//MatrixForm=
$$\begin{pmatrix}
O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \\
-qx+\frac{-\frac{(jx^2+jy^2+jz^2)qx}{2n^2}+qx\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{jx\,qx}{n c^2}+\frac{\frac{jx\,qx(jx^2+jy^2+jz^2+6n^2W)}{2n^3}-4\,qx\,Wx}{c^4}+O[\frac{1}{c}]^6 & \frac{jy\,qx}{n c^2}+\frac{\frac{jy\,qx(jx^2+jy^2+jz^2+6n^2W)}{2n^3}-4\,qx\,Wy}{c^4}+O[\frac{1}{c}]^6 & \frac{jz\,qx}{n c^2}+\frac{\frac{jz\,qx(jx^2+jy^2+jz^2+6n^2W)}{2n^3}-4\,qx\,Wz}{c^4}+O[\frac{1}{c}]^6 \\
-qy+\frac{-\frac{(jx^2+jy^2+jz^2)qy}{2n^2}+qy\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{jx\,qy}{n c^2}+\frac{\frac{jx\,qy(jx^2+jy^2+jz^2+6n^2W)}{2n^3}-4\,qy\,Wx}{c^4}+O[\frac{1}{c}]^6 & \frac{jy\,qy}{n c^2}+\frac{\frac{jy\,qy(jx^2+jy^2+jz^2+6n^2W)}{2n^3}-4\,qy\,Wy}{c^4}+O[\frac{1}{c}]^6 & \frac{jz\,qy}{n c^2}+\frac{\frac{jz\,qy(jx^2+jy^2+jz^2+6n^2W)}{2n^3}-4\,qy\,Wz}{c^4}+O[\frac{1}{c}]^6 \\
-qz+\frac{-\frac{(jx^2+jy^2+jz^2)qz}{2n^2}+qz\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{jx\,qz}{n c^2}+\frac{\frac{jx\,qz(jx^2+jy^2+jz^2+6n^2W)}{2n^3}-4\,qz\,Wx}{c^4}+O[\frac{1}{c}]^6 & \frac{jy\,qz}{n c^2}+\frac{\frac{jy\,qz(jx^2+jy^2+jz^2+6n^2W)}{2n^3}-4\,qz\,Wy}{c^4}+O[\frac{1}{c}]^6 & \frac{jz\,qz}{n c^2}+\frac{\frac{jz\,qz(jx^2+jy^2+jz^2+6n^2W)}{2n^3}-4\,qz\,Wz}{c^4}+O[\frac{1}{c}]^6
\end{pmatrix}$$

In[63]:=
```
Assuming[assutQ, FS[T[Qtens.Inverse[gg]].gg - Qtens]] // MF
```

Out[63]//MatrixForm=
$$\begin{pmatrix}
-\frac{4(qx\,Wx+qy\,Wy+qz\,Wz)}{c^4}+O[\frac{1}{c}]^6 & \frac{qx}{c^2}+\frac{\frac{(jx^2+jy^2+jz^2)qx}{2n^2}+3\,qx\,W}{c^4}+O[\frac{1}{c}]^6 & \frac{qy}{c^2}+\frac{\frac{(jx^2+jy^2+jz^2)qy}{2n^2}+3\,qy\,W}{c^4}+O[\frac{1}{c}]^6 & \frac{qz}{c^2}+\frac{\frac{(jx^2+jy^2+jz^2)qz}{2n^2}+3\,qz\,W}{c^4}+O[\frac{1}{c}]^6 \\
qx+\frac{\frac{(jx^2+jy^2+jz^2)qx}{2n^2}-qx\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{4\,qx\,Wx}{c^4}+O[\frac{1}{c}]^6 & \frac{-jy\,qx+jx\,qy}{n c^2}+\frac{-jy^3 qx+2\,jx\,jy^2 qy+jx\,qy\left(jx^2+jz^2+6n^2W\right)+jy\left(-jz^2 qx+jx\,jz\,qz-6n^2 qx\,W\right)+8n^3 qx\,Wy}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{-jz\,qx+jx\,qz}{n c^2}+\frac{-jz^3 qx+jx\,jy\,jz\,qy+jx^3 qz+2\,jx\,jz^2 qz+jy^2\left(-jz\,qx+jx\,qz\right)-6\,jz\,n^2 qx\,W+6\,jx\,n^2 qz\,W+8n^3 qx\,Wz}{2n^3c^4}+O[\frac{1}{c}]^6 \\
qy+\frac{\frac{(jx^2+jy^2+jz^2)qy}{2n^2}-qy\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{jy\,qx-jx\,qy}{n c^2}+\frac{\frac{(jy\,qx-jx\,qy)(jx^2+jy^2+jz^2+6n^2W)}{2n^3}+4\,qy\,Wx}{c^4}+O[\frac{1}{c}]^6 & \frac{4\,qy\,Wy}{c^4}+O[\frac{1}{c}]^6 & \frac{-jz\,qy+jy\,qz}{n c^2}+\frac{\frac{(-jz\,qy+jy\,qz)(jx^2+jy^2+jz^2+6n^2W)}{2n^3}+4\,qy\,Wz}{c^4}+O[\frac{1}{c}]^6 \\
qz+\frac{\frac{(jx^2+jy^2+jz^2)qz}{2n^2}-qz\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{jz\,qx-jx\,qz}{n c^2}+\frac{\frac{(jz\,qx-jx\,qz)(jx^2+jy^2+jz^2+6n^2W)}{2n^3}+4\,qz\,Wx}{c^4}+O[\frac{1}{c}]^6 & \frac{jz\,qy-jy\,qz}{n c^2}+\frac{\frac{(jz\,qy-jy\,qz)(jx^2+jy^2+jz^2+6n^2W)}{2n^3}+4\,qz\,Wy}{c^4}+O[\frac{1}{c}]^6 & \frac{4\,qz\,Wz}{c^4}+O[\frac{1}{c}]^6
\end{pmatrix}$$

In[64]:=
```
(* definition of momentum-flux, orthogonal to matter-current *)
Ptemp = {pt, px, py, pz};
```

In[65]:=
```
Assuming[assut, FS[Ptemp.proju]] // MF
```

Out[65]//MatrixForm=
$$\begin{pmatrix}
\frac{n\,pt+jx\,px+jy\,py+jz\,pz}{n}+\frac{(jx^2+jy^2+jz^2)(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^3c^2}+O[\frac{1}{c}]^4 \\
-\frac{jx(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^2c^2}-\frac{(n\,pt+jx\,px+jy\,py+jz\,pz)\left(jx^3+jx\left(jy^2+jz^2+4n^2W\right)-4n^3Wx\right)}{n^4c^4}+O[\frac{1}{c}]^6 \\
-\frac{jy(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^2c^2}-\frac{(n\,pt+jx\,px+jy\,py+jz\,pz)\left(jy\left(jx^2+jy^2+jz^2+4n^2W\right)-4n^3Wy\right)}{n^4c^4}+O[\frac{1}{c}]^6 \\
-\frac{jz(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^2c^2}-\frac{(n\,pt+jx\,px+jy\,py+jz\,pz)\left(jz\left(jx^2+jy^2+jz^2+4n^2W\right)-4n^3Wz\right)}{n^4c^4}+O[\frac{1}{c}]^6
\end{pmatrix}$$

In[66]:=
```
psol = Solve[Normal[Ptemp.proju] == 0, pt][[1]]
```

Out[66]=
$$\left\{pt\to-\frac{jx\,px+jy\,py+jz\,pz}{n}\right\}$$

In[67]:=
```
(P = Assuming[assut, FS[Ptemp /. psol]]) // MF
```

Out[67]//MatrixForm=
$$\begin{pmatrix}-\frac{jx\,px+jy\,py+jz\,pz}{n}\\ px\\ py\\ pz\end{pmatrix}$$

In[68]:=
```
{FS[Normal@Series[P.uu, {c, Infinity, 1}]] == 0}
```

Out[68]= {True}

In[69]:=
```
Assuming[assutQ, FS@{P.proju, P.projperpu == P}]
```

Out[69]=
$$\left\{\left\{O\left[\frac{1}{c}\right]^4,\ O\left[\frac{1}{c}\right]^6,\ O\left[\frac{1}{c}\right]^6,\ O\left[\frac{1}{c}\right]^6\right\},\ True\right\}$$

In[70]:=
```
(* non-symmetric momentum-tensor *)
Assuming[assut, FS[Ptens = Assuming[assut, FS[Outer[Times, uu, P / c^2]]]]] // MF
```

Out[70]//MatrixForm=
$$\begin{pmatrix}
-\frac{jx\,px+jy\,py+jz\,pz}{n c^2}-\frac{(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{px}{c^2}+\frac{px\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O[\frac{1}{c}]^6 & \frac{py}{c^2}+\frac{py\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O[\frac{1}{c}]^6 & \frac{pz}{c^2}+\frac{pz\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O[\frac{1}{c}]^6 \\
-\frac{jx(jx\,px+jy\,py+jz\,pz)}{n^2c^2}-\frac{jx(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^4c^4}+O[\frac{1}{c}]^6 & \frac{jx\,px}{n c^2}+\frac{jx\,px(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jx\,py}{n c^2}+\frac{jx\,py(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jx\,pz}{n c^2}+\frac{jx\,pz(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 \\
-\frac{jy(jx\,px+jy\,py+jz\,pz)}{n^2c^2}-\frac{jy(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^4c^4}+O[\frac{1}{c}]^6 & \frac{jy\,px}{n c^2}+\frac{jy\,px(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jy\,py}{n c^2}+\frac{jy\,py(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jy\,pz}{n c^2}+\frac{jy\,pz(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 \\
-\frac{jz(jx\,px+jy\,py+jz\,pz)}{n^2c^2}-\frac{jz(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^4c^4}+O[\frac{1}{c}]^6 & \frac{jz\,px}{n c^2}+\frac{jz\,px(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jz\,py}{n c^2}+\frac{jz\,py(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6 & \frac{jz\,pz}{n c^2}+\frac{jz\,pz(jx^2+jy^2+jz^2+2n^2W)}{2n^3c^4}+O[\frac{1}{c}]^6
\end{pmatrix}$$

In[71]:=
```
FS[T[Ptens.Inverse[gg]].gg - Ptens] // MF
```

Out[71]//MatrixForm=
$$\begin{pmatrix}
\frac{4(px\,Wx+py\,Wy+pz\,Wz)}{c^4}+O[\frac{1}{c}]^6 & -\frac{px}{c^2}+\frac{\frac{jx(jx\,px+jy\,py+jz\,pz)}{n^2}-px\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O[\frac{1}{c}]^6 & -\frac{py}{c^2}+\frac{\frac{jy(jx\,px+jy\,py+jz\,pz)}{n^2}-py\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O[\frac{1}{c}]^6 & -\frac{pz}{c^2}+\frac{\frac{jz(jx\,px+jy\,py+jz\,pz)}{n^2}-pz\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O[\frac{1}{c}]^6 \\
-px+\frac{jx^2 px+2\,jx(jy\,py+jz\,pz)-px\left(jy^2+jz^2-6n^2W\right)}{2n^2c^2}+O[\frac{1}{c}]^4 & -\frac{4(px\,Wx)}{c^4}+O[\frac{1}{c}]^6 & \frac{jy\,px-jx\,py}{n c^2}+\frac{\frac{(jy\,px-jx\,py)(jx^2+jy^2+jz^2+2n^2W)}{2n^3}-4\,px\,Wy}{c^4}+O[\frac{1}{c}]^6 & \frac{jz\,px-jx\,pz}{n c^2}+\frac{\frac{(jz\,px-jx\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3}-4\,px\,Wz}{c^4}+O[\frac{1}{c}]^6 \\
-py+\frac{-\frac{(jx^2+jy^2+jz^2)py}{2n^2}+\frac{jy(jx\,px+jy\,py+jz\,pz)}{n^2}+3\,py\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{-jy\,px+jx\,py}{n c^2}+\frac{\frac{(-jy\,px+jx\,py)(jx^2+jy^2+jz^2+2n^2W)}{2n^3}-4\,py\,Wx}{c^4}+O[\frac{1}{c}]^6 & -\frac{4(py\,Wy)}{c^4}+O[\frac{1}{c}]^6 & \frac{jz\,py-jy\,pz}{n c^2}+\frac{\frac{(jz\,py-jy\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3}-4\,py\,Wz}{c^4}+O[\frac{1}{c}]^6 \\
-pz+\frac{-\frac{(jx^2+jy^2+jz^2)pz}{2n^2}+\frac{jz(jx\,px+jy\,py+jz\,pz)}{n^2}+3\,pz\,W}{c^2}+O[\frac{1}{c}]^4 & \frac{-jz\,px+jx\,pz}{n c^2}+\frac{\frac{(-jz\,px+jx\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3}-4\,pz\,Wx}{c^4}+O[\frac{1}{c}]^6 & \frac{-jz\,py+jy\,pz}{n c^2}+\frac{\frac{(-jz\,py+jy\,pz)(jx^2+jy^2+jz^2+2n^2W)}{2n^3}-4\,pz\,Wy}{c^4}+O[\frac{1}{c}]^6 & -\frac{4(pz\,Wz)}{c^4}+O[\frac{1}{c}]^6
\end{pmatrix}$$

In[72]:=
```
(* definition of stress, orthogonal to matter-current *)
(Stemp = {{stt, stx, sty, stz}, {sxt, sxx, sxy, sxz}, {syt, syx, syy, syz}, {szt, szx, szy, szz}}) // MF
```

Out[72]//MatrixForm=
$$\begin{pmatrix}stt & stx & sty & stz\\ sxt & sxx & sxy & sxz\\ syt & syx & syy & syz\\ szt & szx & szy & szz\end{pmatrix}$$

In[73]:=
```
(Stempsym = Assuming[assut, FS[(T[Stemp.Inverse[gg]].gg + Stemp) / 2]]) // MF
```

Out[73]//MatrixForm=
$$\begin{pmatrix}
stt+\frac{2(stx\,Wx+sty\,Wy+stz\,Wz)}{c^2}+O[\frac{1}{c}]^4 & \frac{stx}{2}-\frac{sxt}{2c^2}-\frac{2(sxt\,W-stt\,Wx+sxx\,Wx+sxy\,Wy+sxz\,Wz)}{c^4}+O[\frac{1}{c}]^6 & \frac{sty}{2}-\frac{syt}{2c^2}-\frac{2(syt\,W+syx\,Wx-stt\,Wy+syy\,Wy+syz\,Wz)}{c^4}+O[\frac{1}{c}]^6 & \frac{stz}{2}-\frac{szt}{2c^2}-\frac{2(szt\,W+szx\,Wx+szy\,Wy-stt\,Wz+szz\,Wz)}{c^4}+O[\frac{1}{c}]^6 \\
-\frac{stx\,c^2}{2}+\frac{1}{2}(sxt+4\,stx\,W)+O[\frac{1}{c}]^2 & sxx-\frac{2(stx\,Wx)}{c^2}+O[\frac{1}{c}]^4 & \frac{sxy+syx}{2}-\frac{2(stx\,Wy)}{c^2}+O[\frac{1}{c}]^4 & \frac{sxz+szx}{2}-\frac{2(stx\,Wz)}{c^2}+O[\frac{1}{c}]^4 \\
-\frac{sty\,c^2}{2}+\frac{1}{2}(syt+4\,sty\,W)+O[\frac{1}{c}]^2 & \frac{sxy+syx}{2}-\frac{2(sty\,Wx)}{c^2}+O[\frac{1}{c}]^4 & syy-\frac{2(sty\,Wy)}{c^2}+O[\frac{1}{c}]^4 & \frac{syz+szy}{2}-\frac{2(sty\,Wz)}{c^2}+O[\frac{1}{c}]^4 \\
-\frac{stz\,c^2}{2}+\frac{1}{2}(szt+4\,stz\,W)+O[\frac{1}{c}]^2 & \frac{sxz+szx}{2}-\frac{2(stz\,Wx)}{c^2}+O[\frac{1}{c}]^4 & \frac{syz+szy}{2}-\frac{2(stz\,Wy)}{c^2}+O[\frac{1}{c}]^4 & szz-\frac{2(stz\,Wz)}{c^2}+O[\frac{1}{c}]^4
\end{pmatrix}$$

In[74]:=
```
FS[proju.Stemp.proju] // MF
```

Out[74]//MatrixForm= (first column; further columns run beyond the page edge)

$$N = 2\,jx^3 stx+2\,jy^3 sty+jx^2\left(2n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx\right)-jy\,n^2 syt+jy^2\left(2n\,stt+2\,jz\,stz-n\,syy\right)+jx\left(2\,jy^2 stx-n^2 sxt-jy\,n(sxy+syx)+jz\left(2\,jz\,stx-n(sxz+szx)\right)\right)+jy\,jz\left(2\,jz\,sty-n(syz+szy)\right)+jz\left(2\,jz^2 stz-n^2 szt+jz\,n(2\,stt-szz)\right)$$

$$\begin{pmatrix}
\frac{n\,stt+jx\,stx+jy\,sty+jz\,stz}{n}+\frac{N}{n^3c^2}+O[\frac{1}{c}]^4 & -\frac{jx(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2c^2}+\frac{-2\,jx^4 stx+jx^3\left(-2(n\,stt+jy\,sty+jz\,stz)+n\,sxx\right)+jx^2\left(-2\,jy^2 stx-2\,jz^2 stx+jy\,n(sxy+syx)+jz\,n(sxz+szx)+n^2(sxt-4\,stx\,W)\right)+4n^3(n\,stt+jy\,sty+jz\,stz)Wx+\ldots}{n^4c^4} \\
\frac{jx(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2}+\frac{jx\,N}{n^4c^2}+O[\frac{1}{c}]^4 & -\frac{jx^2(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^3c^2}+\frac{jx\left(-2\,jx^4 stx+jx^3\left(-2(n\,stt+jy\,sty+jz\,stz)+n\,sxx\right)+\ldots\right)}{\ldots} \\
\frac{jy(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2}+\frac{jy\,N}{n^4c^2}+O[\frac{1}{c}]^4 & -\frac{jx\,jy(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^3c^2}+\frac{jy\left(-2\,jx^4 stx+jx^3\left(-2(n\,stt+jy\,sty+jz\,stz)+n\,sxx\right)+\ldots\right)}{\ldots} \\
\frac{jz(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2}+\frac{jz\,N}{n^4c^2}+O[\frac{1}{c}]^4 & -\frac{jx\,jz(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^3c^2}+\frac{jz\left(-2\,jx^4 stx+jx^3\left(-2(n\,stt+jy\,sty+jz\,stz)+n\,sxx\right)+\ldots\right)}{\ldots}
\end{pmatrix}$$

In[75]:=
```
ssol = Solve[{Normal[proju.Stemp.proju] == 0, Normal[projperpu.Stemp.projperpu] == Stemp}, {stt, stx, sty, stz, sxt, syt, szt}][[1]]
```

Out[75]=
$$\left\{stt\to-\frac{jx^2 sxx+jx\,jy\,sxy+jx\,jz\,sxz+jx\,jy\,syx+jy^2 syy+jy\,jz\,syz+jx\,jz\,szx+jy\,jz\,szy+jz^2 szz}{jx^2+jy^2+jz^2+c^2n^2},\ stx\to\frac{n\left(jx\,sxx+jy\,syx+jz\,szx\right)}{jx^2+jy^2+jz^2+c^2n^2},\ sty\to\frac{n\left(jx\,sxy+jy\,syy+jz\,szy\right)}{jx^2+jy^2+jz^2+c^2n^2},\ stz\to\frac{n\left(jx\,sxz+jy\,syz+jz\,szz\right)}{jx^2+jy^2+jz^2+c^2n^2},\ sxt\to-\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n},\ syt\to-\frac{jx\,syx+jy\,syy+jz\,syz}{n},\ szt\to-\frac{jx\,szx+jy\,szy+jz\,szz}{n}\right\}$$

In[76]:=
```
ssol = Solve[Normal[proju.Stemp] == 0 && Normal[Stemp.proju] == 0, {stt, stx, sty, stz, sxt, syt, szt}][[1]]
```

Out[76]=
$$\left\{stt\to-\frac{jx^2 sxx+jx\,jy\,sxy+jx\,jz\,sxz+jx\,jy\,syx+jy^2 syy+jy\,jz\,syz+jx\,jz\,szx+jy\,jz\,szy+jz^2 szz}{jx^2+jy^2+jz^2+c^2n^2},\ stx\to\frac{n\left(jx\,sxx+jy\,syx+jz\,szx\right)}{jx^2+jy^2+jz^2+c^2n^2},\ sty\to\frac{n\left(jx\,sxy+jy\,syy+jz\,szy\right)}{jx^2+jy^2+jz^2+c^2n^2},\ stz\to\frac{n\left(jx\,sxz+jy\,syz+jz\,szz\right)}{jx^2+jy^2+jz^2+c^2n^2},\ sxt\to-\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n},\ syt\to-\frac{jx\,syx+jy\,syy+jz\,syz}{n},\ szt\to-\frac{jx\,szx+jy\,szy+jz\,szz}{n}\right\}$$

In[77]:=
```
(S = Assuming[assut, FS[(Stemp /. ssol)]]) // MF
```

Out[77]//MatrixForm=
$$\begin{pmatrix}
-\frac{jx^2 sxx+jx\,jy(sxy+syx)+jy^2 syy+jx\,jz(sxz+szx)+jy\,jz(syz+szy)+jz^2 szz}{jx^2+jy^2+jz^2+c^2n^2} & \frac{n(jx\,sxx+jy\,syx+jz\,szx)}{jx^2+jy^2+jz^2+c^2n^2} & \frac{n(jx\,sxy+jy\,syy+jz\,szy)}{jx^2+jy^2+jz^2+c^2n^2} & \frac{n(jx\,sxz+jy\,syz+jz\,szz)}{jx^2+jy^2+jz^2+c^2n^2} \\
-\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n} & sxx & sxy & sxz \\
-\frac{jx\,syx+jy\,syy+jz\,syz}{n} & syx & syy & syz \\
-\frac{jx\,szx+jy\,szy+jz\,szz}{n} & szx & szy & szz
\end{pmatrix}$$

In[79]:=
```
MF /@ FS@{proju.S.proju, Assuming[assut, FS[projperpu.S.projperpu - S]]}
```

Out[79]= (the second matrix runs beyond the page edge)
$$\left\{\begin{pmatrix}O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8\\ O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8\\ O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8\\ O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8\end{pmatrix},\right.$$
$$\begin{pmatrix}
-\frac{jx^4 sxx+jy^4 syy+jx^3\left(jy(sxy+syx)+jz(sxz+szx)\right)+jy^3 jz(syz+szy)+jx^2\left(jy^2(sxx+syy)+jy\,jz(syz+szy)+jz^2(sxx+szz)+4n^2 sxx\,W\right)+jy^2\left(jz^2(syy+szz)+4n^2 syy\,W\right)+jx\left(jy^3(sxy+syx)+jy^2 jz(sxz+szx)+jz^3(sxz+szx)+4\,jz\,n^2(sxz+szx)W+jy(sxy+syx)\left(jz^2+4n^2W\right)-4n^3\left(sxx\,Wx+syx\,Wy+szx\,Wz\right)\right)+jy\left(jz^3(syz+szy)+4\,jz\,n^2(syz+szy)W-4n^3\left(sxy\,Wx+syy\,Wy+szy\,Wz\right)\right)+jz\left(jz^3 szz+4\,jz\,n^2 szz\,W-4n^3\left(sxz\,Wx+syz\,Wy+szz\,Wz\right)\right)}{n^4c^4}+O[\frac{1}{c}]^6 & \frac{jx^3 sxx+jy^3 syx+\ldots}{\ldots} \\
O[\frac{1}{c}]^4 & \frac{jx\left(jx^3 sxx+jy^3 \ldots\right)}{\ldots} \\
O[\frac{1}{c}]^4 & \frac{jy\left(jx^3 sxx+jy^3 \ldots\right)}{\ldots} \\
O[\frac{1}{c}]^4 & \frac{jz\left(jx^3 sxx+jy^3 \ldots\right)}{\ldots}
\end{pmatrix}$$

In[80]:=
```
FS[T[S.Inverse[gg]].gg - S] // MF
```

Out[80]//MatrixForm=
$$\begin{pmatrix}
\frac{4\left(jx(sxy-syx)Wy+jz(-sxz\,Wx+szx\,Wx-syz\,Wy+szy\,Wy)+jx(sxz-szx)Wz+jy(-sxy\,Wx+syx\,Wx+syz\,Wz-szy\,Wz)\right)}{n\,c^4}+O[\frac{1}{c}]^6 & \frac{jy\,sxy+jz\,sxz-jy\,syx-jz\,szx}{n c^2}+\frac{\frac{(jx^2+jy^2+jz^2)(jx\,sxx+jy\,syx+jz\,szx)}{n^3}+\frac{4(jx\,sxx+jy\,sxy+jz\,sxz)W}{n}-4\,sxx\,Wx-4\,sxy\,Wy-4\,sxz\,Wz}{c^4}+O[\frac{1}{c}]^6 & \frac{jx(-sxy+syx)+jz(syz-szy)}{n c^2}+\frac{\frac{(jx^2+jy^2+jz^2)(jx\,sxy+jy\,syy+jz\,szy)}{n^3}+\frac{4(jx\,syx+jy\,syy+jz\,syz)W}{n}-4\,syx\,Wx-4\,syy\,Wy-4\,syz\,Wz}{c^4}+O[\frac{1}{c}]^6 & \frac{jx(-sxz+szx)+jy(-syz+szy)}{n c^2}+\frac{\frac{(jx^2+jy^2+jz^2)(jx\,sxz+jy\,syz+jz\,szz)}{n^3}+\frac{4(jx\,szx+jy\,szy+jz\,szz)W}{n}-4\,szx\,Wx-4\,szy\,Wy-4\,szz\,Wz}{c^4}+O[\ldots \\
\frac{jy\,sxy+jz\,sxz-jy\,syx-jz\,szx}{n}+\frac{\frac{2(jx\,sxx+jy\,syx+jz\,szx)W}{n}+\frac{(jx\,sxx+jy\,sxy+jz\,sxz)(jx^2+jy^2+jz^2+2n^2W)}{n^3}-4\,sxx\,Wx-4\,syx\,Wy-4\,szx\,Wz}{c^2}+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & (-sxy+syx)+O[\frac{1}{c}]^4 & (-sxz+szx)+O[\frac{1}{c}]^4 \\
\frac{jx(-sxy+syx)+jz(syz-szy)}{n}+\frac{\frac{2(jx\,sxy+jy\,syy+jz\,szy)W}{n}+\frac{(jx\,syx+jy\,syy+jz\,syz)(jx^2+jy^2+jz^2+2n^2W)}{n^3}-4\,sxy\,Wx-4\,syy\,Wy-4\,szy\,Wz}{c^2}+O[\frac{1}{c}]^4 & (sxy-syx)+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & (-syz+szy)+O[\frac{1}{c}]^4 \\
\frac{jx(-sxz+szx)+jy(-syz+szy)}{n}+\frac{\frac{2(jx\,sxz+jy\,syz+jz\,szz)W}{n}+\frac{(jx\,szx+jy\,szy+jz\,szz)(jx^2+jy^2+jz^2+2n^2W)}{n^3}-4\,sxz\,Wx-4\,syz\,Wy-4\,szz\,Wz}{c^2}+O[\frac{1}{c}]^4 & (sxz-szx)+O[\frac{1}{c}]^4 & (syz-szy)+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4
\end{pmatrix}$$

In[81]:=
```
(* define "dust" 4-stress *)
(dust = Assuming[assut, FS[(ρ * c ^ 2 + ϵ) * Outer[Times, NJ, gg.uu / c ^ 2]]]) // MF
```

Out[81]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2+\left(-n\epsilon+n\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac{1}{c}]^2 & jx\,\rho+\frac{jx^3\rho+jx(jy^2+jz^2)\rho-8n^3Wx\,\rho+2jx\,n^2(\epsilon+3W\rho)}{2n^2c^2}+O[\tfrac{1}{c}]^4 & jy\,\rho+\frac{jy\,\epsilon+\frac{jy(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^2}-4nWy\,\rho}{c^2}+O[\tfrac{1}{c}]^4 & jz\,\rho+\frac{jz\,\epsilon+\frac{jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^2}-4nWz\,\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-jx\,\rho c^2+\left(-jx\,\epsilon+jx\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac{1}{c}]^2 & \frac{jx^2\rho}{n}+\frac{jx\left(jx^3\rho+jx(jy^2+jz^2)\rho-8n^3Wx\,\rho+2jx\,n^2(\epsilon+3W\rho)\right)}{2n^3c^2}+O[\tfrac{1}{c}]^4 & \frac{jx\,jy\,\rho}{n}+\frac{jx\left(jy^3\rho+jy(jx^2+jz^2)\rho-8n^3Wy\,\rho+2jy\,n^2(\epsilon+3W\rho)\right)}{2n^3c^2}+O[\tfrac{1}{c}]^4 & \frac{jx\,jz\,\rho}{n}+\frac{jx\left((jx^2+jy^2)jz\,\rho+jz^3\rho-8n^3Wz\,\rho+2jz\,n^2(\epsilon+3W\rho)\right)}{2n^3c^2}+O[\tfrac{1}{c}]^4 \\
-jy\,\rho c^2+\left(-jy\,\epsilon+jy\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac{1}{c}]^2 & \frac{jx\,jy\,\rho}{n}+\frac{jy\left(jx^3\rho+jx(jy^2+jz^2)\rho-8n^3Wx\,\rho+2jx\,n^2(\epsilon+3W\rho)\right)}{2n^3c^2}+O[\tfrac{1}{c}]^4 & \frac{jy^2\rho}{n}+\frac{jy\left(2jy\,n^2\epsilon+jy(jx^2+jy^2+jz^2+6n^2W)\rho-8n^3Wy\,\rho\right)}{2n^3c^2}+O[\tfrac{1}{c}]^4 & \frac{jy\,jz\,\rho}{n}+\frac{jy\left((jx^2+jy^2)jz\,\rho+jz^3\rho-8n^3Wz\,\rho+2jz\,n^2(\epsilon+3W\rho)\right)}{2n^3c^2}+O[\tfrac{1}{c}]^4 \\
-jz\,\rho c^2+\left(-jz\,\epsilon+jz\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac{1}{c}]^2 & \frac{jx\,jz\,\rho}{n}+\frac{jz\left(jx^3\rho+jx(jy^2+jz^2)\rho-8n^3Wx\,\rho+2jx\,n^2(\epsilon+3W\rho)\right)}{2n^3c^2}+O[\tfrac{1}{c}]^4 & \frac{jy\,jz\,\rho}{n}+\frac{jz\left(jy^3\rho+jy(jx^2+jz^2)\rho-8n^3Wy\,\rho+2jy\,n^2(\epsilon+3W\rho)\right)}{2n^3c^2}+O[\tfrac{1}{c}]^4 & \frac{jz^2\rho}{n}+\frac{jz\left(2jz\,n^2\epsilon+jz(jx^2+jy^2+jz^2+6n^2W)\rho-8n^3Wz\,\rho\right)}{2n^3c^2}+O[\tfrac{1}{c}]^4
\end{pmatrix}$$

In[82]:=
```
FS[T[dust.Inverse[gg]] == dust.Inverse[gg]]
```

Out[82]= True

In[83]:=
```
FS[T[dust].gg == gg.dust]
```

Out[83]= True

In[84]:=
```
MF /@ FS@{proju.dust.proju == dust, projperpu.dust.projperpu}
```

Out[84]=

$$\left\{\text{True},\begin{pmatrix}
O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 \\
O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 \\
O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 \\
O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4
\end{pmatrix}\right\}$$

In[∘]:=
```
showf[assut][dust2 = Assuming[assut, Expand //@ FS@PowerExpand[(ρ * c ^ 2 + ϵ) * Outer[Times, NJ, gg.auu / c ^ 2]]]];
```

In[85]:=
```
(* define generic 4-stress *)
(EPS = Assuming[assut, FS[dust + Qtens + Ptens + S]]) // MF
```

Out[85]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2+\left(-n\epsilon+n\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac{1}{c}]^2 & jx\,\rho+\frac{px+\frac{jx\,sxx+jy\,syx+jz\,szx}{n}+\frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2}-4nWx\,\rho+jx(\epsilon+3W\rho)}{c^2}+O[\tfrac{1}{c}]^4 & jy\,\rho+\frac{py+\frac{jx\,sxy+jy\,syy+jz\,szy}{n}+jy\,\epsilon+\frac{jy(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^2}-4nWy\,\rho}{c^2}+O[\tfrac{1}{c}]^4 & jz\,\rho+\frac{pz+\frac{jx\,sxz+jy\,syz+jz\,szz}{n}+jz\,\epsilon+\frac{jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^2}-4nWz\,\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-jx\,\rho c^2+\left(-qx-\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n}-jx\,\epsilon+jx\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac{1}{c}]^2 & \left(sxx+\frac{jx^2\rho}{n}\right)+\frac{jx\left(\frac{jx(jx^2+jy^2+jz^2)\rho}{n^3}-8Wx\,\rho+\frac{2(px+qx+jx\,\epsilon+3jx\,W\rho)}{n}\right)}{2c^2}+O[\tfrac{1}{c}]^4 & \left(sxy+\frac{jx\,jy\,\rho}{n}\right)+\frac{\frac{jy\,qx}{n}+\frac{jx(py+jy\,\epsilon)}{n}+\frac{jx\,jy(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}-4jx\,Wy\,\rho}{c^2}+O[\tfrac{1}{c}]^4 & \left(sxz+\frac{jx\,jz\,\rho}{n}\right)+\frac{\frac{jz\,qx}{n}+\frac{jx(pz+jz\,\epsilon)}{n}+\frac{jx\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}-4jx\,Wz\,\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-jy\,\rho c^2+\left(-qy-\frac{jx\,syx+jy\,syy+jz\,syz}{n}-jy\,\epsilon+jy\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac{1}{c}]^2 & \left(syx+\frac{jx\,jy\,\rho}{n}\right)+\frac{\frac{jy\,px+jx\,qy+jx\,jy\,\epsilon}{n}+\frac{jy(jx^3+jx(jy^2+jz^2+6n^2W)-8n^3Wx)\rho}{2n^3}}{c^2}+O[\tfrac{1}{c}]^4 & \left(syy+\frac{jy^2\rho}{n}\right)+\frac{jy\left(\frac{jy(jx^2+jy^2+jz^2)\rho}{n^3}-8Wy\,\rho+\frac{2(py+qy+jy\,\epsilon+3jy\,W\rho)}{n}\right)}{2c^2}+O[\tfrac{1}{c}]^4 & \left(syz+\frac{jy\,jz\,\rho}{n}\right)+\frac{\frac{jz\,qy}{n}+\frac{jy(pz+jz\,\epsilon)}{n}+\frac{jy\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}-4jy\,Wz\,\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-jz\,\rho c^2+\left(-qz-\frac{jx\,szx+jy\,szy+jz\,szz}{n}-jz\,\epsilon+jz\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac{1}{c}]^2 & \left(szx+\frac{jx\,jz\,\rho}{n}\right)+\frac{\frac{jz\,px+jx\,qz+jx\,jz\,\epsilon}{n}+\frac{jz(jx^3+jx(jy^2+jz^2+6n^2W)-8n^3Wx)\rho}{2n^3}}{c^2}+O[\tfrac{1}{c}]^4 & \left(szy+\frac{jy\,jz\,\rho}{n}\right)+\frac{\frac{jy\,qz}{n}+\frac{jz(py+jy\,\epsilon)}{n}+\frac{jy\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}-4jz\,Wy\,\rho}{c^2}+O[\tfrac{1}{c}]^4 & \left(szz+\frac{jz^2\rho}{n}\right)+\frac{jz\left(\frac{jz(jx^2+jy^2+jz^2)\rho}{n^3}-8Wz\,\rho+\frac{2(pz+qz+jz\,\epsilon+3jz\,W\rho)}{n}\right)}{2c^2}+O[\tfrac{1}{c}]^4
\end{pmatrix}$$

In[86]:=
```
Assuming[assut, FS[(EPS /. replaceJu)]] // MF
```

Out[86]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+(ux^2+uy^2+uz^2-2W)\rho\right)+O[\tfrac{1}{c}]^2 & n\,ux\,\rho+\frac{px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon+\frac{1}{2}n\left(ux(ux^2+uy^2+uz^2+6W)-8Wx\right)\rho}{c^2}+O[\tfrac{1}{c}]^4 & n\,uy\,\rho+\frac{py+sxy\,ux+syy\,uy+szy\,uz+n\,uy\,\epsilon+\frac{1}{2}n\left(uy(ux^2+uy^2+uz^2+6W)-8Wy\right)\rho}{c^2}+O[\tfrac{1}{c}]^4 & n\,uz\,\rho+\frac{pz+sxz\,ux+syz\,uy+szz\,uz+n\,uz\,\epsilon+\frac{1}{2}n\left(uz(ux^2+uy^2+uz^2+6W)-8Wz\right)\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-n\,ux\,\rho c^2+\left(-qx-sxy\,uy-sxz\,uz-ux(sxx+n\epsilon)-\frac{1}{2}n\,ux(ux^2+uy^2+uz^2-2W)\rho\right)+O[\tfrac{1}{c}]^2 & (sxx+n\,ux^2\rho)+\frac{ux(px+qx+n\,ux\,\epsilon)+\frac{1}{2}n\,ux\left(ux^3+ux(uy^2+uz^2+6W)-8Wx\right)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (sxy+n\,ux\,uy\,\rho)+\frac{py\,ux+qx\,uy+n\,ux\,uy\,\epsilon+\frac{1}{2}n\,ux\left(uy(ux^2+uy^2+uz^2+6W)-8Wy\right)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (sxz+n\,ux\,uz\,\rho)+\frac{pz\,ux+qx\,uz+n\,ux\,uz\,\epsilon+\frac{1}{2}n\,ux\left(uz(ux^2+uy^2+uz^2+6W)-8Wz\right)\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-n\,uy\,\rho c^2+\left(-qy-syx\,ux-syz\,uz-uy(syy+n\epsilon)-\frac{1}{2}n\,uy(ux^2+uy^2+uz^2-2W)\rho\right)+O[\tfrac{1}{c}]^2 & (syx+n\,ux\,uy\,\rho)+\frac{qy\,ux+px\,uy+n\,ux\,uy\,\epsilon+\frac{1}{2}n\,uy\left(ux(ux^2+uy^2+uz^2+6W)-8Wx\right)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (syy+n\,uy^2\rho)+\frac{uy(py+qy+n\,uy\,\epsilon)+\frac{1}{2}n\,uy\left(uy(ux^2+uy^2+uz^2+6W)-8Wy\right)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (syz+n\,uy\,uz\,\rho)+\frac{pz\,uy+qy\,uz+n\,uy\,uz\,\epsilon+\frac{1}{2}n\,uy\left(uz(ux^2+uy^2+uz^2+6W)-8Wz\right)\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-n\,uz\,\rho c^2+\left(-qz-szx\,ux-szy\,uy-uz(szz+n\epsilon)-\frac{1}{2}n\,uz(ux^2+uy^2+uz^2-2W)\rho\right)+O[\tfrac{1}{c}]^2 & (szx+n\,ux\,uz\,\rho)+\frac{qz\,ux+px\,uz+n\,ux\,uz\,\epsilon+\frac{1}{2}n\,uz\left(ux(ux^2+uy^2+uz^2+6W)-8Wx\right)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (szy+n\,uy\,uz\,\rho)+\frac{qz\,uy+py\,uz+n\,uy\,uz\,\epsilon+\frac{1}{2}n\,uz\left(uy(ux^2+uy^2+uz^2+6W)-8Wy\right)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (szz+n\,uz^2\rho)+\frac{uz(pz+qz+n\,uz\,\epsilon)+\frac{1}{2}n\,uz\left(uz(ux^2+uy^2+uz^2+6W)-8Wz\right)\rho}{c^2}+O[\tfrac{1}{c}]^4
\end{pmatrix}$$

In[87]:=
```
Assuming[assut, FS[(EPS /. replaceJu) /. replaceuUnorm]] // MF
```

Out[87]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2+\left(-n\epsilon-\frac{1}{2}nU^2\rho+nW\rho\right)+O[\tfrac{1}{c}]^2 & n\,ux\,\rho+\frac{px+sxx\,ux+syx\,uy+szx\,uz-4nWx\,\rho+n\,ux\left(\epsilon+\frac{1}{2}(U^2+6W)\rho\right)}{c^2}+O[\tfrac{1}{c}]^4 & n\,uy\,\rho+\frac{py+sxy\,ux+syy\,uy+szy\,uz-4nWy\,\rho+n\,uy\left(\epsilon+\frac{1}{2}(U^2+6W)\rho\right)}{c^2}+O[\tfrac{1}{c}]^4 & n\,uz\,\rho+\frac{pz+sxz\,ux+syz\,uy+szz\,uz-4nWz\,\rho+n\,uz\left(\epsilon+\frac{1}{2}(U^2+6W)\rho\right)}{c^2}+O[\tfrac{1}{c}]^4 \\
-n\,ux\,\rho c^2+\left(-qx-sxy\,uy-sxz\,uz-\frac{1}{2}ux\left(2(sxx+n\epsilon)+n(U^2-2W)\rho\right)\right)+O[\tfrac{1}{c}]^2 & \left(sxx+n(U^2-uy^2-uz^2)\rho\right)+\frac{ux(px+qx+n\,ux\,\epsilon)+\frac{1}{2}n\,ux(U^2ux+6ux\,W-8Wx)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (sxy+n\,ux\,uy\,\rho)+\frac{py\,ux+uy(qx+n\,ux\,\epsilon)+\frac{1}{2}n\,ux(U^2uy+6uy\,W-8Wy)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (sxz+n\,ux\,uz\,\rho)+\frac{pz\,ux+uz(qx+n\,ux\,\epsilon)+\frac{1}{2}n\,ux(U^2uz+6uz\,W-8Wz)\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-n\,uy\,\rho c^2+\left(-qy-syx\,ux-syz\,uz-\frac{1}{2}uy\left(2(syy+n\epsilon)+n(U^2-2W)\rho\right)\right)+O[\tfrac{1}{c}]^2 & (syx+n\,ux\,uy\,\rho)+\frac{qy\,ux+uy(px+n\,ux\,\epsilon)+\frac{1}{2}n\,uy(U^2ux+6ux\,W-8Wx)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (syy+n\,uy^2\rho)+\frac{uy(py+qy+n\,uy\,\epsilon)+\frac{1}{2}n\,uy(U^2uy+6uy\,W-8Wy)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (syz+n\,uy\,uz\,\rho)+\frac{pz\,uy+uz(qy+n\,uy\,\epsilon)+\frac{1}{2}n\,uy(U^2uz+6uz\,W-8Wz)\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-n\,uz\,\rho c^2+\left(-qz-szx\,ux-szy\,uy-\frac{1}{2}uz\left(2(szz+n\epsilon)+n(U^2-2W)\rho\right)\right)+O[\tfrac{1}{c}]^2 & (szx+n\,ux\,uz\,\rho)+\frac{qz\,ux+uz(px+n\,ux\,\epsilon)+\frac{1}{2}n\,uz(U^2ux+6ux\,W-8Wx)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (szy+n\,uy\,uz\,\rho)+\frac{qz\,uy+uz(py+n\,uy\,\epsilon)+\frac{1}{2}n\,uz(U^2uy+6uy\,W-8Wy)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (szz+n\,uz^2\rho)+\frac{uz(pz+qz+n\,uz\,\epsilon)+\frac{1}{2}n\,uz(U^2uz+6uz\,W-8Wz)\rho}{c^2}+O[\tfrac{1}{c}]^4
\end{pmatrix}$$

In[88]:=
```
FS[T[EPS.Inverse[gg]].gg - EPS] // MF
```

Out[88]//MatrixForm=

$$\begin{pmatrix}
O[\tfrac{1}{c}]^2 & \frac{n(-px+qx)+jy(sxy-syx)+jz(sxz-szx)}{nc^2}+O[\tfrac{1}{c}]^4 & \frac{n(-py+qy)+jx(-sxy+syx)+jz(syz-szy)}{nc^2}+O[\tfrac{1}{c}]^4 & \frac{n(-pz+qz)+jx(-sxz+szx)+jy(-syz+szy)}{nc^2}+O[\tfrac{1}{c}]^4 \\
\frac{n(-px+qx)+jy(sxy-syx)+jz(sxz-szx)}{n}+O[\tfrac{1}{c}]^2 & O[\tfrac{1}{c}]^4 & (-sxy+syx)+\frac{jy\,px-jx\,py-jy\,qx+jx\,qy}{nc^2}+O[\tfrac{1}{c}]^4 & (-sxz+szx)+\frac{jz\,px-jx\,pz-jz\,qx+jx\,qz}{nc^2}+O[\tfrac{1}{c}]^4 \\
\frac{n(-py+qy)+jx(-sxy+syx)+jz(syz-szy)}{n}+O[\tfrac{1}{c}]^2 & (sxy-syx)+\frac{jy(-px+qx)+jx(py-qy)}{nc^2}+O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & (-syz+szy)+\frac{jz\,py-jy\,pz-jz\,qy+jy\,qz}{nc^2}+O[\tfrac{1}{c}]^4 \\
\frac{n(-pz+qz)+jx(-sxz+szx)+jy(-syz+szy)}{n}+O[\tfrac{1}{c}]^2 & (sxz-szx)+\frac{jz(-px+qx)+jx(pz-qz)}{nc^2}+O[\tfrac{1}{c}]^4 & (syz-szy)+\frac{jz(-py+qy)+jy(pz-qz)}{nc^2}+O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4
\end{pmatrix}$$

In[89]:=
```
(* Conditions for symmetry of total 4-stress tensor *)
subsym = {px → qx, py → qy, pz → qz, syx → sxy, szx → sxz, szy → syz};
```

In[90]:=
```
(EPSsym = Assuming[assut, FS[EPS /. subsym]]) // MF
```

Out[90]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2+\left(-n\epsilon+n\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac{1}{c}]^2 & jx\,\rho+\frac{qx+\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n}+\frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2}-4nWx\,\rho+jx(\epsilon+3W\rho)}{c^2}+O[\tfrac{1}{c}]^4 & jy\,\rho+\frac{qy+\frac{jx\,sxy+jy\,syy+jz\,syz}{n}+jy\,\epsilon+\frac{jy(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^2}-4nWy\,\rho}{c^2}+O[\tfrac{1}{c}]^4 & jz\,\rho+\frac{qz+\frac{jx\,sxz+jy\,syz+jz\,szz}{n}+jz\,\epsilon+\frac{jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^2}-4nWz\,\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-jx\,\rho c^2+\left(-qx-\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n}-jx\,\epsilon+jx\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac{1}{c}]^2 & \left(sxx+\frac{jx^2\rho}{n}\right)+\frac{jx\left(\frac{jx(jx^2+jy^2+jz^2)\rho}{n^3}-8Wx\,\rho+\frac{4qx+2jx\,\epsilon+6jx\,W\rho}{n}\right)}{2c^2}+O[\tfrac{1}{c}]^4 & \left(sxy+\frac{jx\,jy\,\rho}{n}\right)+\frac{\frac{jy\,qx}{n}+\frac{jx(qy+jy\,\epsilon)}{n}+\frac{jx\,jy(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}-4jx\,Wy\,\rho}{c^2}+O[\tfrac{1}{c}]^4 & \left(sxz+\frac{jx\,jz\,\rho}{n}\right)+\frac{\frac{jz\,qx}{n}+\frac{jx(qz+jz\,\epsilon)}{n}+\frac{jx\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}-4jx\,Wz\,\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-jy\,\rho c^2+\left(-qy-\frac{jx\,sxy+jy\,syy+jz\,syz}{n}-jy\,\epsilon+jy\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac{1}{c}]^2 & \left(sxy+\frac{jx\,jy\,\rho}{n}\right)+\frac{\frac{jy\,qx+jx\,qy+jx\,jy\,\epsilon}{n}+\frac{jy(jx^3+jx(jy^2+jz^2+6n^2W)-8n^3Wx)\rho}{2n^3}}{c^2}+O[\tfrac{1}{c}]^4 & \left(syy+\frac{jy^2\rho}{n}\right)+\frac{jy\left(\frac{jy(jx^2+jy^2+jz^2)\rho}{n^3}-8Wy\,\rho+\frac{4qy+2jy\,\epsilon+6jy\,W\rho}{n}\right)}{2c^2}+O[\tfrac{1}{c}]^4 & \left(syz+\frac{jy\,jz\,\rho}{n}\right)+\frac{\frac{jz\,qy}{n}+\frac{jy(qz+jz\,\epsilon)}{n}+\frac{jy\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}-4jy\,Wz\,\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-jz\,\rho c^2+\left(-qz-\frac{jx\,sxz+jy\,syz+jz\,szz}{n}-jz\,\epsilon+jz\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)\rho\right)+O[\tfrac{1}{c}]^2 & \left(sxz+\frac{jx\,jz\,\rho}{n}\right)+\frac{\frac{jz\,qx+jx\,qz+jx\,jz\,\epsilon}{n}+\frac{jz(jx^3+jx(jy^2+jz^2+6n^2W)-8n^3Wx)\rho}{2n^3}}{c^2}+O[\tfrac{1}{c}]^4 & \left(syz+\frac{jy\,jz\,\rho}{n}\right)+\frac{\frac{jy\,qz}{n}+\frac{jz(qy+jy\,\epsilon)}{n}+\frac{jy\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}-4jz\,Wy\,\rho}{c^2}+O[\tfrac{1}{c}]^4 & \left(szz+\frac{jz^2\rho}{n}\right)+\frac{jz\left(\frac{jz(jx^2+jy^2+jz^2)\rho}{n^3}-8Wz\,\rho+\frac{4qz+2jz\,\epsilon+6jz\,W\rho}{n}\right)}{2c^2}+O[\tfrac{1}{c}]^4
\end{pmatrix}$$

In[91]:=
```
FS[T[EPSsym.Inverse[gg]].gg - EPSsym] // MF
```

Out[91]//MatrixForm=

$$\begin{pmatrix}
O[\tfrac{1}{c}]^2 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 \\
O[\tfrac{1}{c}]^2 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 \\
O[\tfrac{1}{c}]^2 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 \\
O[\tfrac{1}{c}]^2 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4 & O[\tfrac{1}{c}]^4
\end{pmatrix}$$

In[92]:=
```
Assuming[assut, FS[(EPSsym /. replaceJu) /. replaceuUnorm]] // MF
```

Out[92]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2+\left(-n\epsilon-\frac{1}{2}nU^2\rho+nW\rho\right)+O[\tfrac{1}{c}]^2 & n\,ux\,\rho+\frac{qx+sxx\,ux+sxy\,uy+sxz\,uz-4nWx\,\rho+n\,ux\left(\epsilon+\frac{1}{2}(U^2+6W)\rho\right)}{c^2}+O[\tfrac{1}{c}]^4 & n\,uy\,\rho+\frac{qy+sxy\,ux+syy\,uy+syz\,uz-4nWy\,\rho+n\,uy\left(\epsilon+\frac{1}{2}(U^2+6W)\rho\right)}{c^2}+O[\tfrac{1}{c}]^4 & n\,uz\,\rho+\frac{qz+sxz\,ux+syz\,uy+szz\,uz-4nWz\,\rho+n\,uz\left(\epsilon+\frac{1}{2}(U^2+6W)\rho\right)}{c^2}+O[\tfrac{1}{c}]^4 \\
-n\,ux\,\rho c^2+\left(-qx-sxy\,uy-sxz\,uz-\frac{1}{2}ux\left(2(sxx+n\epsilon)+n(U^2-2W)\rho\right)\right)+O[\tfrac{1}{c}]^2 & \left(sxx+n(U^2-uy^2-uz^2)\rho\right)+\frac{ux\left(4qx-8nWx\,\rho+n\,ux\left(2\epsilon+(U^2+6W)\rho\right)\right)}{2c^2}+O[\tfrac{1}{c}]^4 & (sxy+n\,ux\,uy\,\rho)+\frac{qy\,ux+uy(qx+n\,ux\,\epsilon)+\frac{1}{2}n\,ux(U^2uy+6uy\,W-8Wy)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (sxz+n\,ux\,uz\,\rho)+\frac{qz\,ux+uz(qx+n\,ux\,\epsilon)+\frac{1}{2}n\,ux(U^2uz+6uz\,W-8Wz)\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-n\,uy\,\rho c^2+\left(-qy-sxy\,ux-syz\,uz-\frac{1}{2}uy\left(2(syy+n\epsilon)+n(U^2-2W)\rho\right)\right)+O[\tfrac{1}{c}]^2 & (sxy+n\,ux\,uy\,\rho)+\frac{qy\,ux+uy(qx+n\,ux\,\epsilon)+\frac{1}{2}n\,uy(U^2ux+6ux\,W-8Wx)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (syy+n\,uy^2\rho)+\frac{uy\left(4qy-8nWy\,\rho+n\,uy\left(2\epsilon+(U^2+6W)\rho\right)\right)}{2c^2}+O[\tfrac{1}{c}]^4 & (syz+n\,uy\,uz\,\rho)+\frac{qz\,uy+uz(qy+n\,uy\,\epsilon)+\frac{1}{2}n\,uy(U^2uz+6uz\,W-8Wz)\rho}{c^2}+O[\tfrac{1}{c}]^4 \\
-n\,uz\,\rho c^2+\left(-qz-sxz\,ux-syz\,uy-\frac{1}{2}uz\left(2(szz+n\epsilon)+n(U^2-2W)\rho\right)\right)+O[\tfrac{1}{c}]^2 & (sxz+n\,ux\,uz\,\rho)+\frac{qz\,ux+uz(qx+n\,ux\,\epsilon)+\frac{1}{2}n\,uz(U^2ux+6ux\,W-8Wx)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (syz+n\,uy\,uz\,\rho)+\frac{qz\,uy+uz(qy+n\,uy\,\epsilon)+\frac{1}{2}n\,uz(U^2uy+6uy\,W-8Wy)\rho}{c^2}+O[\tfrac{1}{c}]^4 & (szz+n\,uz^2\rho)+\frac{uz\left(4qz-8nWz\,\rho+n\,uz\left(2\epsilon+(U^2+6W)\rho\right)\right)}{2c^2}+O[\tfrac{1}{c}]^4
\end{pmatrix}$$

In[∘]:=

(* Balanced quantities constructed from energy–momentum tensor, and their supplies *)

In[97]:=
```
(* Symmetrized energy-stress tensor, with explicit dep. on coords *)
(TTx = Assuming[assut, FS[tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]]]]) // MF
```

Out[97]//MatrixForm=

$$-\rho\,n[t,x,y,z]\,c^2-\frac{1}{2}n[t,x,y,z]\left(2\epsilon+\rho\left(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2-2W[t,x,y,z]\right)\right)+O[\tfrac{1}{c}]^2 \quad \rho\,n[t,x,y,z]\,ux[t,x,y,z]+\ldots$$

$$-\rho\,n[t,x,y,z]\times ux[t,x,y,z]\,c^2+\frac{1}{2}\Big(-px-qx-(sxy+syx)\,uy[t,x,y,z]-(sxz+szx)\,uz[t,x,y,z]-ux[t,x,y,z]\Big(2\,sxx+n[t,x,y,z]\Big(2\epsilon+\rho\big(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2-2W[t,x,y,z]\big)\Big)\Big)\Big)+O[\tfrac{1}{c}]^2 \quad \left(sxx+\rho\,n[t,x,y,z]\,ux[t,x,y,z]^2\right)+\ldots$$

$$-\rho\,n[t,x,y,z]\times uy[t,x,y,z]\,c^2+\frac{1}{2}\Big(-py-qy-(sxy+syx)\,ux[t,x,y,z]-\rho\,n[t,x,y,z]\,ux[t,x,y,z]^2\,uy[t,x,y,z]-(syz+szy)\,uz[t,x,y,z]-uy[t,x,y,z]\Big(2\,syy+n[t,x,y,z]\Big(2\epsilon+\rho\big(uy[t,x,y,z]^2+uz[t,x,y,z]^2-2W[t,x,y,z]\big)\Big)\Big)\Big)+O[\tfrac{1}{c}]^2 \quad \frac{1}{2}\left(sxy+syx+2\rho\,n[t,x,y,z]\times ux[t,x,y,z]\times uy[t,x,\ldots\right.$$

$$-\rho\,n[t,x,y,z]\times uz[t,x,y,z]\,c^2+\frac{1}{2}\Big(-pz-qz-(sxz+szx)\,ux[t,x,y,z]-(syz+szy)\,uy[t,x,y,z]-2\big(szz+\epsilon\,n[t,x,y,z]\big)\,uz[t,x,y,z]-\rho\,n[t,x,y,z]\,ux[t,x,y,z]^2\,uz[t,x,y,z]-\rho\,n[t,x,y,z]\times uz[t,x,y,z]\big(uy[t,x,y,z]^2+uz[t,x,y,z]^2-2W[t,x,y,z]\big)\Big)+O[\tfrac{1}{c}]^2 \quad \frac{1}{2}\left(sxz+szx+2\rho\,n[t,x,y,z]\times ux[t,x,y,z]\times uz[t,x,\ldots\right.$$

In[98]:=
```
(Duu = Assuming[assut, FS[(D[Normal@uut, {coords}] + Sum[uut[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]) // MF
```

Out[98]//MatrixForm=

$$\frac{n[t,x,y,z]\Big(jx[t,x,y,z]\big(-n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+jx^{(1,0,0,0)}[t,x,y,z]\big)+jy[t,x,y,z]\big(-n[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]+jy^{(1,0,0,0)}[t,x,y,z]\big)+jz[t,x,y,z]\big(-n[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]+jz^{(1,0,0,0)}[t,x,y,z]\big)\Big)-\big(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2\big)\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^3\,c^2}-\ldots$$

$$\left(-W^{(0,1,0,0)}[t,x,y,z]+\frac{n[t,x,y,z]\,jx^{(1,0,0,0)}[t,x,y,z]-jx[t,x,y,z]\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}\right)+\frac{4\,W\,W^{(0,1,0,0)}[t,x,y,z]-\ldots}{\ldots}$$

$$\left(-W^{(0,0,1,0)}[t,x,y,z]+\frac{n[t,x,y,z]\,jy^{(1,0,0,0)}[t,x,y,z]-jy[t,x,y,z]\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}\right)+\frac{4\,W\,W^{(0,0,1,0)}[t,x,y,z]-\ldots}{\ldots}$$

$$\left(-W^{(0,0,0,1)}[t,x,y,z]+\frac{n[t,x,y,z]\,jz^{(1,0,0,0)}[t,x,y,z]-jz[t,x,y,z]\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}\right)+\frac{4\,W\,W^{(0,0,0,1)}[t,x,y,z]-\ldots}{\ldots}$$

In[99]:=
```
(Duv = Assuming[assut, FS[(D[Normal@uuv, {coords}] + Sum[uuv[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]) // MF
```

Out[99]//MatrixForm=

$$\frac{ux[t,x,y,z]\big(-W^{(0,1,0,0)}[t,x,y,z]+ux^{(1,0,0,0)}[t,x,y,z]\big)+uy[t,x,y,z]\big(-W^{(0,0,1,0)}[t,x,y,z]+uy^{(1,0,0,0)}[t,x,y,z]\big)+uz[t,x,y,z]\big(-W^{(0,0,0,1)}[t,x,y,z]+uz^{(1,0,0,0)}[t,x,y,z]\big)}{c^2}+\frac{\ldots}{2c^4}+O[\tfrac{1}{c}]^5$$

$$\left(-W^{(0,1,0,0)}[t,x,y,z]+ux^{(1,0,0,0)}[t,x,y,z]\right)+\frac{4\,W\,W^{(0,1,0,0)}[t,x,y,z]-\ldots}{c^2}$$

$$\left(-W^{(0,0,1,0)}[t,x,y,z]+uy^{(1,0,0,0)}[t,x,y,z]\right)+\frac{4\,W\,W^{(0,0,1,0)}[t,x,y,z]-\ldots}{c^2}$$

$$\left(-W^{(0,0,0,1)}[t,x,y,z]+uz^{(1,0,0,0)}[t,x,y,z]\right)+\frac{4\,W\,W^{(0,0,0,1)}[t,x,y,z]-\ldots}{c^2}$$

(* Energy *)

```
(* Energy current and supply according to 4-velocity *)
pvec = uu; Dpvec = Duv;
MF@(MF /@ {Efluxuu = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Esupplyuu = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[101]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2-n\epsilon+O[\tfrac{1}{c}]^2 \\
(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2+\left(-Ax\left(qx+(jx-n\,Vx)\epsilon\right)-Ay\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)-Az\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)+O[\tfrac{1}{c}]^2
\end{pmatrix}$$

$$\frac{1}{2}\Big(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z]+(syz+szy)\big(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\big)+2\big(syy\,uy^{(0,0,1,0)}[t,x,y,z]+sxx\,ux^{(0,1,0,0)}[t,x,y,z]\big)+(sxy+syx)\big(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z]\big)+(sxz+szx)\big(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\big)+\ldots$$

In[102]:=
```
MF@(MF /@ (FS[({Efluxuu, Esupplyuu} /. replaceJu) /. replaceuUnorm]))
```
Out[102]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} -n\rho c^2 - n\epsilon + O[\frac{1}{c}]^2 \\ n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2 + \left(-Ax\,qx - Ay\,qy - Az\,qz + n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\epsilon\right) + O[\frac{1}{c}]^2 \end{pmatrix} \\ \frac{1}{2}\left(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z] + (syz+szy)\left(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\right) + 2\left(syy\,uy^{(0,0,1,0)}[t,x,y,z] + sxx\,ux^{(0,1,0,0)}[t,x,y,z]\right) + (sxy+syx)\left(ux^{(0,0,1,0)}[t,x,y,z] + uy^{(0,1,0,0)}[t,x,y,z]\right) + (sxz+szx)\left(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\right)\right) + \ldots \end{pmatrix}$$

In[103]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```
Out[103]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{qx}{n\rho c^2} + O[\frac{1}{c}]^4 \\ \frac{jy}{n} + \frac{qy}{n\rho c^2} + O[\frac{1}{c}]^4 \\ \frac{jz}{n} + \frac{qz}{n\rho c^2} + O[\frac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx}{n\rho c^2} + O[\frac{1}{c}]^4 \\ uy + \frac{qy}{n\rho c^2} + O[\frac{1}{c}]^4 \\ uz + \frac{qz}{n\rho c^2} + O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

In[106]:=
```
(* Energy current and supply according to t-vector *)
pvec = - {1, 0, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ ({Efluxt = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Esupplyt = Assuming[assut, FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]]}))
```
Out[107]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\rho c^2 + n\left(\epsilon + \frac{(jx^2+jy^2+jz^2)\rho}{2n^2} - W\rho\right) + O[\frac{1}{c}]^2 \\ \left(Ay\,jy + Az\,jz + Ax(jx - n\,Vx) - n(Ay\,Vy + Az\,Vz)\right)\rho c^2 + \frac{2n\left(Ax\left(jx\,sxx+jy\,sxy+jz\,sxz+n\left(qx+(jx-n\,Vx)\epsilon\right)\right)+Ay\left(jx\,syx+jy\,syy+jz\,syz+n\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)\right)+Az\left(jx\,szx+jy\,szy+jz\,szz+n\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)\right)+\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)\left(jx^2+jy^2+jz^2-2n^2W\right)\rho}{2n^2} + O[\frac{1}{c}]^2 \end{pmatrix} \\ -n\rho W^{(1,0,0,0)}[t,x,y,z] + \frac{-\frac{\left(3(jx^2+jy^2+jz^2)\rho+2n\left(sxx+syy+szz+n\epsilon-nW\rho\right)\right)W^{(1,0,0,0)}[t,x,y,z]}{2n}+4\rho\left(jx\,Wx^{(1,0,0,0)}[t,x,y,z]+jy\,Wy^{(1,0,0,0)}[t,x,y,z]+jz\,Wz^{(1,0,0,0)}[t,x,y,z]\right)}{c^2} + O[\frac{1}{c}]^3 \end{pmatrix}$$

In[108]:=
```
MF@(MF /@ (FS[({Efluxt, Esupplyt} /. replaceJu) /. replaceuUnorm]))
```
Out[108]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\rho c^2 + n\left(\epsilon + \frac{1}{2}\left(U^2 - 2W\right)\rho\right) + O[\frac{1}{c}]^2 \\ n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho c^2 + \left(Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)+Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)+Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon\right)+\frac{1}{2}n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(U^2-2W\right)\rho\right) + O[\frac{1}{c}]^2 \end{pmatrix} \\ -n\rho W^{(1,0,0,0)}[t,x,y,z] + \frac{-\frac{1}{2}\left(2(sxx+syy+szz+n\epsilon)+n\left(3U^2-2W\right)\rho\right)W^{(1,0,0,0)}[t,x,y,z]+4n\rho\left(ux\,Wx^{(1,0,0,0)}[t,x,y,z]+uy\,Wy^{(1,0,0,0)}[t,x,y,z]+uz\,Wz^{(1,0,0,0)}[t,x,y,z]\right)}{c^2} + O[\frac{1}{c}]^3 \end{pmatrix}$$

In[109]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```
Out[109]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\rho c^2} + O[\frac{1}{c}]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\rho c^2} + O[\frac{1}{c}]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\rho c^2} + O[\frac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\rho c^2} + O[\frac{1}{c}]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\rho c^2} + O[\frac{1}{c}]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\rho c^2} + O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

In[110]:=
```
(* Energy current and supply according to norm. t-vector *)
pvec = - c * {1, 0, 0, 0} / Sqrt[-gg[[1, 1]]]; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ ({Efluxnt = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Esupplynt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]}))
```
Out[111]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\rho c^2 + \left(n\epsilon + \frac{(jx^2+jy^2+jz^2)\rho}{2n}\right) + O[\frac{1}{c}]^2 \\ \left(Ay\,jy + Az\,jz + Ax(jx - n\,Vx) - n(Ay\,Vy + Az\,Vz)\right)\rho c^2 + \frac{2n\left(Ax\left(jx\,sxx+jy\,sxy+jz\,sxz+n\left(qx+(jx-n\,Vx)\epsilon\right)\right)+Ay\left(jx\,syx+jy\,syy+jz\,syz+n\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)\right)+Az\left(jx\,szx+jy\,szy+jz\,szz+n\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)\right)+(jx^2+jy^2+jz^2)\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)\rho}{2n^2} + O[\frac{1}{c}]^2 \end{pmatrix} \\ \rho\left(jz\,W^{(0,0,0,1)}[t,x,y,z] + jy\,W^{(0,0,1,0)}[t,x,y,z] + jx\,W^{(0,1,0,0)}[t,x,y,z]\right) + \ldots \end{pmatrix}$$

In[112]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```
Out[112]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\rho c^2} + O[\frac{1}{c}]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\rho c^2} + O[\frac{1}{c}]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\rho c^2} + O[\frac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\rho c^2} + O[\frac{1}{c}]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\rho c^2} + O[\frac{1}{c}]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\rho c^2} + O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

In[113]:=
```
(* Energy current and supply according to cov. t-vector *)
pvec = c^2 * igg.{1, 0, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Efluxcovt = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Esupplycovt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```
Out[114]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\rho c^2 + \left(\frac{(jx^2+jy^2+jz^2)\rho}{2n} + n(\epsilon + W\rho)\right) + O[\frac{1}{c}]^2 \\ \left(Ay\,jy + Az\,jz + Ax(jx - n\,Vx) - n(Ay\,Vy + Az\,Vz)\right)\rho c^2 + \frac{2n\left(Ax\left(jx\,sxx+jy\,sxy+jz\,sxz+n\left(qx+(jx-n\,Vx)\epsilon\right)\right)+Ay\left(jx\,syx+jy\,syy+jz\,syz+n\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)\right)+Az\left(jx\,szx+jy\,szy+jz\,szz+n\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)\right)+\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)\left(jx^2+jy^2+jz^2+2n^2W\right)\rho}{2n^2} + O[\frac{1}{c}]^2 \end{pmatrix} \\ \rho\left(2\,jz\,W^{(0,0,0,1)}[t,x,y,z] + 2\,jy\,W^{(0,0,1,0)}[t,x,y,z] + 2\,jx\,W^{(0,1,0,0)}[t,x,y,z] + n\,W^{(1,0,0,0)}[t,x,y,z]\right) - \ldots \end{pmatrix}$$

In[115]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```
Out[115]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\rho c^2} + O[\frac{1}{c}]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\rho c^2} + O[\frac{1}{c}]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\rho c^2} + O[\frac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\rho c^2} + O[\frac{1}{c}]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\rho c^2} + O[\frac{1}{c}]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\rho c^2} + O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

(* Momentum *)

In[123]:=
```
(* Momentum current and supply according to x-vector *)
pvec = {0, 1, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Pfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Psupplyx = Assuming[assut, FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]]})
```
Out[124]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} jx\,\rho + \frac{px + \frac{jx\,sxx+jy\,syx+jz\,szx}{n} + \frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2} - 4n\,Wx\,\rho + jx(\epsilon + 3W\rho)}{c^2} + O[\frac{1}{c}]^4 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + \frac{jx\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)\rho}{n}\right) + \frac{Ay\left(2n^2\left(jx\,qy+jy(px+jx\,\epsilon)\right)+jy\left(jx^3+jx(jy^2+jz^2+6n^2W)-8n^3Wx\right)\rho\right)+Az\left(2n^2\left(jx\,qz+jz(px+jx\,\epsilon)\right)+jz\left(jx^3+jx(jy^2+jz^2+6n^2W)-8n^3Wx\right)\rho\right)+Ax\,jx\left(jx(jx^2+jy^2+jz^2)\rho-8n^3Wx\ldots\right)\ldots}{2n^3c^2} + \ldots \end{pmatrix} \\ n\rho W^{(0,1,0,0)}[t,x,y,z] + \frac{\frac{\left(3(jx^2+jy^2+jz^2)\rho+2n\left(sxx+syy+szz+n\epsilon-nW\rho\right)\right)W^{(0,1,0,0)}[t,x,y,z]}{2n} - 4\rho\left(jx\,Wx^{(0,1,0,0)}[t,x,y,z]+jy\,Wy^{(0,1,0,0)}[t,x,y,z]+jz\,Wz^{(0,1,0,0)}[t,x,y,z]\right)}{c^2} + O[\frac{1}{c}]^3 \end{pmatrix}$$

In[125]:=
```
MF@(MF /@ (FS[({Pfluxx, Psupplyx} /. replaceJu) /. replaceuUnorm]))
```
Out[125]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\,ux\,\rho + \frac{px+sxx\,ux+syx\,uy+szx\,uz-4n\,Wx\,\rho+n\,ux\left(\epsilon+\frac{1}{2}(U^2+6W)\rho\right)}{c^2} + O[\frac{1}{c}]^4 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + n\,ux\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right) + \frac{Ax\left(qx\,ux+px(ux-Vx)-(sxx\,ux+syx\,uy+szx\,uz)Vx+n\,ux(ux-Vx)\epsilon\right)+Ay\left(qy\,ux+px(uy-Vy)-(sxx\,ux+syx\,uy+szx\,uz)Vy+n\,ux(uy-Vy)\epsilon\right)+Az\left(qz\,ux+px(uz-Vz)-\ldots\right)+\frac{1}{2}Ax\,n(ux-Vx)(U^2ux+6ux\,W-8Wx)\rho+\frac{1}{2}n\left(Ay(uy-Vy)+Az(uz-Vz)\right)(U^2ux+6ux\,W-8Wx)\rho}{c^2} + O[\frac{1}{c}]^4 \end{pmatrix} \\ n\rho W^{(0,1,0,0)}[t,x,y,z] + \frac{\left(sxx+syy+szz+n\left(\epsilon+\frac{3U^2\rho}{2}-W\rho\right)\right)W^{(0,1,0,0)}[t,x,y,z]-4n\rho\left(ux\,Wx^{(0,1,0,0)}[t,x,y,z]+uy\,Wy^{(0,1,0,0)}[t,x,y,z]+uz\,Wz^{(0,1,0,0)}[t,x,y,z]\right)}{c^2} + O[\frac{1}{c}]^3 \end{pmatrix}$$

In[126]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```
Out[126]=

$$\left\{\begin{pmatrix} \left(\frac{jx}{n} + \frac{sxx}{jx\,\rho}\right) - \frac{2n\,sxx\left(jx\,sxx+jy\,syx+jz\,szx+n(px+jx\,\epsilon)\right)+\left(3jx^3sxx+2jx^2(-n\,qx+jy\,syx+jz\,szx)+jx\,sxx(jy^2+jz^2+6n^2W)-8n^3sxx\,Wx\right)\rho}{2(jx^2n^2\rho^2)c^2} + O[\frac{1}{c}]^4 \\ \left(\frac{jy}{n} + \frac{syx}{jx\,\rho}\right) - \frac{2n\,syx\left(jx\,sxx+jy\,syx+jz\,szx+n(px+jx\,\epsilon)\right)+jx\left(-2jx\,n\,qy+2jx\,jy\,sxx+jx^2syx+3jy^2syx+jz^2syx+2jy\,jz\,szx+6n^2syx\,W\right)\rho-8n^3syx\,Wx\,\rho}{2(jx^2n^2\rho^2)c^2} + O[\frac{1}{c}]^4 \\ \left(\frac{jz}{n} + \frac{szx}{jx\,\rho}\right) - \frac{2n\,szx\left(jx\,sxx+jy\,syx+jz\,szx+n(px+jx\,\epsilon)\right)+jx\left(-2jx\,n\,qz+2jx\,jz\,sxx+2jy\,jz\,syx+jx^2szx+jy^2szx+3jz^2szx+6n^2szx\,W\right)\rho-8n^3szx\,Wx\,\rho}{2(jx^2n^2\rho^2)c^2} + O[\frac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} \left(ux + \frac{sxx}{n\,ux\,\rho}\right) - \frac{2sxx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\left(-2qx\,ux^2+2ux^2(syx\,uy+szx\,uz)+sxx\,ux(3ux^2+uy^2+uz^2+6W)-8sxx\,Wx\right)\rho}{2(n^2ux^2\rho^2)c^2} + O[\frac{1}{c}]^4 \\ \left(uy + \frac{syx}{n\,ux\,\rho}\right) - \frac{2syx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\left(-2qy\,ux^2+2ux\,uy(sxx\,ux+szx\,uz)+syx\,ux(ux^2+3uy^2+uz^2+6W)-8syx\,Wx\right)\rho}{2(n^2ux^2\rho^2)c^2} + O[\frac{1}{c}]^4 \\ \left(uz + \frac{szx}{n\,ux\,\rho}\right) - \frac{2szx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\left(-2qz\,ux^2+2ux(sxx\,ux+syx\,uy)uz+szx\,ux(ux^2+uy^2+3uz^2+6W)-8szx\,Wx\right)\rho}{2(n^2ux^2\rho^2)c^2} + O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

In[127]:=
```
(* Momentum current and supply according to cov. x-vector *)
pvec = igg.{0, 1, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Pfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Psupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```
Out[128]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} jx\,\rho + \frac{px + \frac{jx\,sxx+jy\,syx+jz\,szx}{n} + \frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2} + jx(\epsilon + W\rho)}{c^2} + O[\frac{1}{c}]^4 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + \frac{jx\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)\rho}{n}\right) + \frac{\ldots}{2n^3c^2} + O[\frac{1}{c}]^4 \end{pmatrix} \\ n\rho W^{(0,1,0,0)}[t,x,y,z] + \frac{\ldots}{c^2} + O[\frac{1}{c}]^3 \end{pmatrix}$$

In[129]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```
Out[129]=

$$\left\{\begin{pmatrix} \left(\frac{jx}{n} + \frac{sxx}{jx\,\rho}\right) - \frac{2n\,sxx\left(jx\,sxx+jy\,syx+jz\,szx+n(px+jx\,\epsilon)\right)+jx\left(3jx^2sxx+2jx(-n\,qx+jy\,syx+jz\,szx)+sxx(jy^2+jz^2+6n^2W)\right)\rho}{2(jx^2n^2\rho^2)c^2} + O[\frac{1}{c}]^4 \\ \left(\frac{jy}{n} + \frac{syx}{jx\,\rho}\right) - \frac{2n\,syx\left(jx\,sxx+jy\,syx+jz\,szx+n(px+jx\,\epsilon)\right)+jx\left(-2jx\,n\,qy+2jx\,jy\,sxx+jx^2syx+3jy^2syx+jz^2syx+2jy\,jz\,szx+6n^2syx\,W\right)\rho}{2(jx^2n^2\rho^2)c^2} + O[\frac{1}{c}]^4 \\ \left(\frac{jz}{n} + \frac{szx}{jx\,\rho}\right) - \frac{2n\,szx\left(jx\,sxx+jy\,syx+jz\,szx+n(px+jx\,\epsilon)\right)+jx\left(-2jx\,n\,qz+2jx\,jz\,sxx+2jy\,jz\,syx+jx^2szx+jy^2szx+3jz^2szx+6n^2szx\,W\right)\rho}{2(jx^2n^2\rho^2)c^2} + O[\frac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} \left(ux + \frac{sxx}{n\,ux\,\rho}\right) - \frac{2sxx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\,ux\left(-2qx\,ux+2ux(syx\,uy+szx\,uz)+sxx(3ux^2+uy^2+uz^2+6W)\right)\rho}{2(n^2ux^2\rho^2)c^2} + O[\frac{1}{c}]^4 \\ \left(uy + \frac{syx}{n\,ux\,\rho}\right) - \frac{2syx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\,ux\left(-2qy\,ux+2uy(sxx\,ux+szx\,uz)+syx(ux^2+3uy^2+uz^2+6W)\right)\rho}{2(n^2ux^2\rho^2)c^2} + O[\frac{1}{c}]^4 \\ \left(uz + \frac{szx}{n\,ux\,\rho}\right) - \frac{2szx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\,ux\left(-2qz\,ux+2(sxx\,ux+syx\,uy)uz+szx(ux^2+uy^2+3uz^2+6W)\right)\rho}{2(n^2ux^2\rho^2)c^2} + O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

(* Angular momentum *)

In[130]:=
```
(* Ang.momentum current and supply according to yz-vector *)
pvec = y * {0, 0, 0, 1} - z * {0, 0, 1, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Lfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Lsupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```
Out[131]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} (jz\,y - jy\,z)\rho + \frac{pz\,y - py\,z + \ldots + 4n(-Wz\,y + Wy\,z)\rho + (jz\,y - jy\,z)(\epsilon + 3W\rho)}{c^2} + O[\frac{1}{c}]^4 \\ \left((Ax\,sxz + Ay\,syz + Az\,szz)y - (Ax\,sxy + Ay\,syy + Az\,szy)z + \frac{\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)(jz\,y-jy\,z)\rho}{n}\right) + \frac{-Ax\,z\left(2jy\,n^2qx+2jx\,n^2(py+jy\,\epsilon)+jx\,jy(jx^2+jy^2+jz^2+6n^2W)\rho-8jx\,n^3Wy\,\rho\right)-Az\,z\left(2jy\,n^2qz+2jz\,n^2(py+jy\,\epsilon)+\ldots\right)\ldots}{\ldots} \end{pmatrix} \\ n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z] - z\,W^{(0,0,1,0)}[t,x,y,z]\right) - \frac{-8jz\,n\,Wy\,\rho+8jy\,n\,Wz\,\rho-y\left(3(jx^2+jy^2+jz^2)\rho+2n(sxx+syy+szz+n\epsilon-nW\rho)\right)W^{(0,0,0,1)}[t,x,y,z]+8jx\,n\,y\,\rho\,Wx^{(0,0,0,1)}[t,x,y,z]+8jy\,n\,y\,\rho\,Wy^{(0,0,0,1)}[t,x,y,z]+8jz\,n\,y\,\rho\,Wz^{(0,0,0,1)}[t,x,y,z]+\ldots}{2n\,c^2} \end{pmatrix}$$

In[132]:=
```
MF@(MF /@ (FS[({Lfluxx, Lsupplyx} /. replaceJu) /. replaceuUnorm]))
```
Out[132]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n(uz\,y - uy\,z)\rho + \frac{pz\,y+sxz\,ux\,y+syz\,uy\,y+szz\,uz\,y-(py+sxy\,ux)z-syy\,uy\,z-szy\,uz\,z+n\,uz\,y\,\epsilon-n\,uy\,z\,\epsilon+\frac{1}{2}n\left(6uz\,W\,y-8Wz\,y-6uy\,W\,z+8Wy\,z+U^2(uz\,y-uy\,z)\right)\rho}{c^2} + O[\frac{1}{c}]^4 \\ \left((Ax\,sxz + Ay\,syz + Az\,szz)y - (Ax\,sxy + Ay\,syy + Az\,szy)z + n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)(uz\,y - uy\,z)\rho\right) + \frac{Ax\left(qx\,uz\,y+pz(ux-Vx)y-sxz\,ux\,Vx\,y-syz\,uy\,Vx\,y-szz\,uz\,Vx\,y-py\,ux\,z-qx\,uy\,z+py\,Vx\,z+sxy\,ux\ldots\right)\ldots}{c^2} \end{pmatrix} \\ n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z] - z\,W^{(0,0,1,0)}[t,x,y,z]\right) + \frac{8n(uz\,Wy-uy\,Wz)\rho+y\left(2(sxx+syy+szz+n\epsilon)+n(3U^2-2W)\rho\right)W^{(0,0,0,1)}[t,x,y,z]-2(sxx+syy+szz)z\,W^{(0,0,1,0)}[t,x,y,z]-2n\,z\,\epsilon\,W^{(0,0,1,0)}[t,x,y,z]+n\rho\left(-8ux\,y\,Wx^{(0,0,0,1)}[t,x,y,z]-8uy\,y\,Wy^{(0,0,0,1)}[t,x,y,z]-\ldots\right)}{2c^2} + O[\frac{1}{c}]^3 \end{pmatrix}$$

```
In[133]:= (* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[133]=

```
In[134]:= (* Ang.momentum current and supply according to cov. yz-vector *)
pvec = igg.(y * {0, 0, 0, 1} - z * {0, 0, 1, 0}); Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Lfluxcx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Lsupplycx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[135]//MatrixForm=

$$(jz\,y - jy\,z)\,\rho + \frac{2\,n\left(\left(jx\,sxz+jy\,syz+jz\,szz\right)y-\left(jx\,sxy+jy\,syy+jz\,szy\right)z+n\left(pz\,y+jz\,y\,\epsilon-z\left(py+jy\,\epsilon\right)\right)\right)+\left(jx^2+jy^2+jz^2+2\,n^2\,W\right)\left(jz\,y-jy\,z\right)\rho}{2\,n^2\,c^2} + O\left[\frac{1}{c}\right]^4$$

```
In[136]:= MF@(MF /@ (FS[({Lfluxcx, Lsupplycx} /. replaceJu) /. replaceuUnorm]))
```

Out[136]//MatrixForm=

$$n\,(uz\,y - uy\,z)\,\rho + \frac{pz\,y+sxz\,ux\,y+syz\,uy\,y+szz\,uz\,y-\left(py+sxy\,ux\right)z-syy\,uy\,z-szy\,uz\,z+n\,uz\,y\,\epsilon-n\,uy\,z\,\epsilon+\frac{1}{2}\,n\left(U^2+2\,W\right)\left(uz\,y-uy\,z\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^4$$

```
In[137]:= (* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[137]=

(* Boost momentum *)

```
In[138]:= (* Ang.boost-momentum current and supply according to tx-vector *)
pvec = t * {0, 1, 0, 0} + x * {1, 0, 0, 0} / c ^ 2; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Bfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Bsupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[139]//MatrixForm=

```
In[140]:= MF@(MF /@ (FS[({Bfluxx, Bsupplyx} /. replaceJu) /. replaceuUnorm]))
```

Out[140]//MatrixForm=

```
In[141]:= (* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[141]=

```
In[142]:= (* Ang.boost-momentum current and supply according to cov. tx-vector *)
pvec = igg.(t * {0, 1, 0, 0} - x * {1, 0, 0, 0}); Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Bfluxcx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Bsupplycx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[143]//MatrixForm=

$$(jx\,t - n\,x)\,\rho + \frac{2\,n\left(\left(n\,px+jx\,sxx+jy\,syx+jz\,szx\right)t+n\left(jx\,t-n\,x\right)\epsilon\right)+\left(jx^2+jy^2+jz^2+2\,n^2\,W\right)\left(jx\,t-n\,x\right)\rho}{2\,n^2\,c^2} + O\left[\frac{1}{c}\right]^4$$

```
In[144]:= MF@(MF /@ (FS[({Bfluxcx, Bsupplycx} /. replaceJu) /. replaceuUnorm]))
```

Out[144]//MatrixForm=

```
In[145]:= (* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[145]=

```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[supply.TTx]], {supply, {Dtxyzvec[[1]], Dtxyzvec[[2]], {0, 0, 0, 0}, Dgtxyzvec[[1]] * c ^ 2, Dgtxyzvec[[2]], {0, 0, 0, 0}, DLxvec, Dxboost / c, {0, 0, 0, 0}, DgLxvec, Dgxboost, {0, 0, 0, 0}, Duu, Dtvecnorm}}]]
```

```
(* coordinate/internal/coordinate-proper energy and x-momentum, content and fluxes (TRANSPOSED) *)
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, igg[[1]]*c^2, igg[[2]], {0, 0, 0, 0}, Lxvec, xboost/c, {0, 0, 0, 0}, gLxvec, gxboost, {0, 0, 0, 0}, uu, tvecnorm}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2+\left(-\frac{Ax(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz)+Ay(n\,qy+jx\,syx+jy\,syy+jz\,syz)+Az(n\,qz+jx\,szx+jy\,szy+jz\,szz)}{n}+(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\epsilon-\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}\right)+O[\frac{1}{c}]^2 \\
jx\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O[\frac{1}{c}]^2 \\
0 & 0 \\
n\rho c^2+\left(\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & (Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho c^2+\frac{2n(Ax(jx\,sxx+jy\,sxy+jz\,sxz+n(qx+(jx-n\,Vx)\epsilon))+Ay(jx\,syx+jy\,syy+jz\,syz+n(qy+jy\,\epsilon-n\,Vy\,\epsilon))+Az(jx\,szx+jy\,szy+jz\,szz+n(qz+jz\,\epsilon-n\,Vz\,\epsilon)))+(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jx^2+jy^2+jz^2+2n^2W)\rho}{2n^2}+O[\frac{1}{c}]^2 \\
jx\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O[\frac{1}{c}]^2 \\
0 & 0 \\
(jz\,y-jy\,z)\rho+O[\frac{1}{c}]^2 & \left((Ax\,sxz+Ay\,syz+Az\,szz)y-(Ax\,sxy+Ay\,syy+Az\,szy)z+\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jz\,y-jy\,z)\rho}{n}\right)+O[\frac{1}{c}]^2 \\
(jx\,t-n\,x)\rho+O[\frac{1}{c}]^2 & \left((Ax\,sxx+Ay\,syx+Az\,szx)t+\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jx\,t-n\,x)\rho}{n}\right)+O[\frac{1}{c}]^2 \\
0 & 0 \\
(jz\,y-jy\,z)\rho+O[\frac{1}{c}]^2 & \left((Ax\,sxz+Ay\,syz+Az\,szz)y-(Ax\,sxy+Ay\,syy+Az\,szy)z+\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jz\,y-jy\,z)\rho}{n}\right)+O[\frac{1}{c}]^2 \\
(-jx\,t\,\rho+n\,x\,\rho)+O[\frac{1}{c}]^2 & \left(-((Ax\,sxx+Ay\,syx+Az\,szx)t)-\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jx\,t-n\,x)\rho}{n}\right)+O[\frac{1}{c}]^2 \\
0 & 0 \\
-n\rho c^2-n\epsilon+O[\frac{1}{c}]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2+(-Ax(qx+(jx-n\,Vx)\epsilon)-Ay(qy+jy\,\epsilon-n\,Vy\,\epsilon)-Az(qz+jz\,\epsilon-n\,Vz\,\epsilon))+O[\frac{1}{c}]^2 \\
-n\rho c^2+\left(-n\epsilon-\frac{(jx^2+jy^2+jz^2)\rho}{2n}\right)+O[\frac{1}{c}]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2+\left(-\frac{Ax(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz)+Ay(n\,qy+jx\,syx+jy\,syy+jz\,syz)+Az(n\,qz+jx\,szx+jy\,szy+jz\,szz)}{n}+(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\epsilon-\frac{(jx^2+jy^2+jz^2)(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{2n^2}\right)+O[\frac{1}{c}]^2
\end{pmatrix}$$

```
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, igg[[1]]*c^2, igg[[2]], {0, 0, 0, 0}, Lxvec, xboost/c, {0, 0, 0, 0}, gLxvec, gxboost, {0, 0, 0, 0}, uu, tvecnorm}]) /. replaceJu]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n(2\epsilon+(ux^2+uy^2+uz^2-2W)\rho)+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\rho c^2+(-Ax(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon)-Ay(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon)-Az(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon)-\frac{1}{2}n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(ux^2+uy^2+uz^2-2W)\rho)+O[\frac{1}{c}]^2 \\
n\,ux\,\rho+O[\frac{1}{c}]^2 & (Ax\,sxx+Ay\,syx+Az\,szx+n\,ux(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))\rho)+O[\frac{1}{c}]^2 \\
0 & 0 \\
n\rho c^2+(n\epsilon+\frac{1}{2}n(ux^2+uy^2+uz^2+2W)\rho)+O[\frac{1}{c}]^2 & n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))\rho c^2+(Ax(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon)+Ay(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon)+Az(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon)+\frac{1}{2}n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(ux^2+uy^2+uz^2+2W)\rho)+O[\frac{1}{c}]^2 \\
n\,ux\,\rho+O[\frac{1}{c}]^2 & (Ax\,sxx+Ay\,syx+Az\,szx+n\,ux(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))\rho)+O[\frac{1}{c}]^2 \\
0 & 0 \\
n(uz\,y-uy\,z)\rho+O[\frac{1}{c}]^2 & (Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(uz\,y-uy\,z)\rho)+O[\frac{1}{c}]^2 \\
n(t\,ux-x)\rho+O[\frac{1}{c}]^2 & ((Ax\,sxx+Ay\,syx+Az\,szx)t+n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux-x)\rho)+O[\frac{1}{c}]^2 \\
0 & 0 \\
n(uz\,y-uy\,z)\rho+O[\frac{1}{c}]^2 & (Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(uz\,y-uy\,z)\rho)+O[\frac{1}{c}]^2 \\
n(-t\,ux+x)\rho+O[\frac{1}{c}]^2 & (-((Ax\,sxx+Ay\,syx+Az\,szx)t)-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux-x)\rho)+O[\frac{1}{c}]^2 \\
0 & 0 \\
-n\rho c^2-n\epsilon+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\rho c^2+(-Ax\,qx-Ay\,qy-Az\,qz+n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\epsilon)+O[\frac{1}{c}]^2 \\
-n\rho c^2-\frac{1}{2}n(2\epsilon+(ux^2+uy^2+uz^2)\rho)+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\rho c^2+(-Ax(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon)-Ay(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon)-Az(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon)-\frac{1}{2}n(ux^2+uy^2+uz^2)(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))\rho)+O[\frac{1}{c}]^2
\end{pmatrix}$$

In[•]:=
```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;,;;]]].gg+Dcoords[[aa,;;,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[supply.TTx]], {supply, {Dtxyzvec[[1]], Dtxyzvec[[2]], {0, 0, 0, 0}, Dgtxyzvec[[1]]*c^2, Dgtxyzvec[[2]], {0, 0, 0, 0}, DLxvec, Dxboost/c, {0, 0, 0, 0}, DgLxvec, Dgxboost, {0, 0, 0, 0}, Duu, Dtvecnorm}}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
\rho\,n[t,x,y,z]\,W^{(1,0,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
\rho\,n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
0 \\
(2\rho\,n[t,x,y,z]\times uz[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]+2\rho\,n[t,x,y,z]\times uy[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]+2\rho\,n[t,x,y,z]\times ux[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+\rho\,n[t,x,y,z]\,W^{(1,0,0,0)}[t,x,y,z])+O[\frac{1}{c}]^2 \\
\rho\,n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
0 \\
(y\rho\,n[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]-z\rho\,n[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z])+O[\frac{1}{c}]^2 \\
t\rho\,n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
0 \\
(y\rho\,n[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]-z\rho\,n[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z])+O[\frac{1}{c}]^2 \\
-t\rho\,n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
0 \\
\left(\frac{sxz\,jx^{(0,0,0,1)}[t,x,y,z]}{2n[t,x,y,z]}+\frac{szx\,jx^{(0,0,0,1)}[t,x,y,z]}{2n[t,x,y,z]}-\frac{\rho\,jx[t,x,y,z]\cdot uz[t,x,y,z]\,jx^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}+\rho\,ux[t,x,y,z]\times uz[t,x,y,z]\,jx^{(0,0,0,1)}[t,x,y,z]+\frac{syz\,jy^{(0,0,0,1)}[t,x,y,z]}{2n[t,x,y,z]}+\frac{szy\,jy^{(0,0,0,1)}[t,x,y,z]}{2n[t,x,y,z]}-\frac{\rho\,jy[t,x,y,z]\cdot uz[t,x,y,z]\,jy^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}+\rho\,uy[t,x,y,z]\times uz[t,x,y,z]\,jy^{(0,0,0,1)}[t,x,y,z]+\frac{szz\,jz^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}-\frac{\rho\,jz[t,x,y,z]\cdot uz[t,x,y,z]\,jz^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]}+\rho\,uz[t,x,y,z]^2\,jz^{(0,0,0,1)}\right. \\
(-\rho\,n[t,x,y,z]\times uz[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]-\rho\,n[t,x,y,z]\times uy[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]-\rho\,n[t,x,y,z]\times ux[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z])+O[\frac{1}{c}]^2
\end{pmatrix}$$

In[•]:=
```
(* covariant derivatives of coordinate 4-vectors (equivalent to Christoffel symbols), for later use *)
(Dtxyzvec = Table[ Assuming[assut, Expand //@ FS@PowerExpand[(D[IdentityMatrix[4][[aa]], {coords}] + Sum[IdentityMatrix[4][[aa]][[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]], {aa, 1, 4}]);
```

In[•]:=
```
(* normalized coordinate-t 4-vector*)
tvecnorm = Assuming[assut, Expand //@ FS@PowerExpand[c * {1, 0, 0, 0} / Sqrt[-gg[[1, 1]]]]]
```

Out[•]=

$$\left\{1+\frac{W}{c^2}+O\left[\frac{1}{c}\right]^4,\ 0,\ 0,\ 0\right\}$$

In[•]:=
```
(* and its covariant derivative *)
(Dtvecnorm = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[tvecnorm], {coords}] + Sum[tW[tvecnorm][[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]) // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\
-W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 & 0 \\
-W^{(0,0,1,0)}[t,x,y,z]+O[\frac{1}{c}]^2 & 0 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 \\
-W^{(0,0,0,1)}[t,x,y,z]+O[\frac{1}{c}]^2 & 0 & 0 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4
\end{pmatrix}$$

In[•]:=
```
(* "raised" coordinate 4-covectors *)
(gtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[igg.IdentityMatrix[4]]]) // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
-\left(\frac{1}{c}\right)^2-\frac{2W}{c^4}+O[\frac{1}{c}]^6 & 0 & 0 & 0 \\
0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 & 0 \\
0 & 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 \\
0 & 0 & 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4
\end{pmatrix}$$

In[•]:=
```
(* and their covariant derivatives *)
(Dgtxyzvec = Table[ Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[igg[[aa]]], {coords}] + Sum[tW[igg[[aa]][[ii]]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]], {aa, 1, 4}]);
```

In[•]:=
```
(* x-component of rot vector *)
Lxvec = Assuming[assut, Expand //@ FS@PowerExpand[{0, 0, -z, y}]]
```

Out[•]= {0, 0, -z, y}

In[•]:=
```
(* and its covariant derivative *)
showf[assut][DLxvec = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[Lxvec], {coords}] + Sum[tW[Lxvec][[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix}
\frac{-y\,W^{(0,0,0,1)}[t,x,y,z]+z\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 & -\frac{z\,W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 & \frac{y\,W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 \\
0 & \frac{y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{z\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{y\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\
-\frac{z\,W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{z\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -1+\frac{-z\,W^{(0,0,0,1)}[t,x,y,z]-y\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\
\frac{y\,W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{y\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 1+\frac{z\,W^{(0,0,0,1)}[t,x,y,z]+y\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4
\end{pmatrix}$$

In[•]:=
```
(* "raised" x-component of rot co-vector *)
gLxvec = Assuming[assut, Expand //@ FS@PowerExpand[igg.{0, 0, -z, y}]]
```

Out[•]=

$$\left\{0,\ 0,\ -z+\frac{2Wz}{c^2}+O\left[\frac{1}{c}\right]^4,\ y-\frac{2(W\,y)}{c^2}+O\left[\frac{1}{c}\right]^4\right\}$$

In[•]:=
```
(* and its covariant derivative *)
showf[assut][DgLxvec = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[gLxvec], {coords}] + Sum[tW[gLxvec][[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{-y\,W^{(0,0,0,1)}[t,x,y,z]+z\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 & -\frac{z\,W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 & \frac{y\,W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 \\ 0 & \frac{y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{z\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{y\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ \frac{z\,W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{z\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{y\,W^{(0,0,0,1)}[t,x,y,z]+z\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -1+\frac{2\,W[t,x,y,z]+z\,W^{(0,0,0,1)}[t,x,y,z]-y\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ -\frac{y\,W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{y\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 1+\frac{-2\,W[t,x,y,z]+z\,W^{(0,0,0,1)}[t,x,y,z]-y\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{-y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[•]:=
```
(* x-component of boost vector *)
xboost = Assuming[assut, Expand //@ FS@PowerExpand[{x / c, t * c, 0, 0}]]
```

Out[•]= $\left\{\frac{x}{c},\ c\,t,\ 0,\ 0\right\}$

In[•]:=
```
(* and its covariant derivative *)
showf[assut][Dxboost = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[xboost], {coords}] + Sum[tW[xboost][[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -\frac{t\,W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{1}{c}+\frac{-x\,W^{(0,1,0,0)}[t,x,y,z]+t\,W^{(1,0,0,0)}[t,x,y,z]}{c^3}+O[\frac{1}{c}]^5 & -\frac{x\,W^{(0,0,1,0)}[t,x,y,z]}{c^3}+O[\frac{1}{c}]^5 & -\frac{x\,W^{(0,0,0,1)}[t,x,y,z]}{c^3}+O[\frac{1}{c}]^5 \\ c+\frac{-x\,W^{(0,1,0,0)}[t,x,y,z]+t\,W^{(1,0,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{t\,W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{t\,W^{(0,0,1,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{t\,W^{(0,0,0,1)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 \\ -\frac{x\,W^{(0,0,1,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & -\frac{t\,W^{(0,0,1,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{t\,W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & 0 \\ -\frac{x\,W^{(0,0,0,1)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & -\frac{t\,W^{(0,0,0,1)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & 0 & \frac{t\,W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 \end{pmatrix}$$

In[•]:=
```
(* "raised" x-component of boost co-vector *)
gxboost = Assuming[assut, Expand //@ FS@PowerExpand[igg.{x, -t, 0, 0}]]
```

Out[•]= $\left\{-\frac{x}{c^2}-\frac{2\,(W\,x)}{c^4}+O[\frac{1}{c}]^6,\ -t+\frac{2\,t\,W}{c^2}+O[\frac{1}{c}]^4,\ 0,\ 0\right\}$

In[•]:=
```
(* and its covariant derivative *)
showf[assut][Dgxboost = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[gxboost], {coords}] + Sum[tW[gxboost][[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{t\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -(\frac{1}{c})^2+\frac{-2\,W[t,x,y,z]-x\,W^{(0,1,0,0)}[t,x,y,z]-t\,W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 & -\frac{x\,W^{(0,0,1,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 & -\frac{x\,W^{(0,0,0,1)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 \\ -1+\frac{2\,W[t,x,y,z]+x\,W^{(0,1,0,0)}[t,x,y,z]+t\,W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t\,W^{(0,0,0,1)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ \frac{x\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t\,W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{t\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 \\ \frac{x\,W^{(0,0,0,1)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t\,W^{(0,0,0,1)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 & -\frac{t\,W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
(* content and flux of coordinatevector-energy and coordinatevector-momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n}+n\,(-\epsilon+W\,\rho)\right)+O[\frac{1}{c}]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\,\rho\,c^2+\left(-\frac{Ax\,(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz)+Ay\,(n\,qy+jx\,syx+jy\,syy+jz\,syz)+Az\,(n\,qz+jx\,szx+jy\,szy+jz\,szz)}{n}+(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\,\epsilon-\frac{(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jx^2+jy^2+jz^2-2\,n^2\,W)\,\rho}{2\,n^2}\right)+O[\frac{1}{c}]^2 \\ jx\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jy\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+\frac{jy\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jz\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+\frac{jz\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho}{n}\right)+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[Dtxyzvec[[aa, ;; , ;;]].TTx]], {aa, 1, 4}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \rho\,n[t,x,y,z]\,W^{(1,0,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ \rho\,n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ \rho\,n[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ \rho\,n[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]+O[\frac{1}{c}]^2 \end{pmatrix}$$

```
(* content and flux of raised coordinatecovector-energy and coordinatecovector-momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

```
(* content and flux of coord-energy and momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxEPS = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n}+n\,(-\epsilon+W\,\rho)\right)+O[\frac{1}{c}]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\,\rho\,c^2+\left(-\frac{Ax\,(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz)+Ay\,(n\,qy+jx\,syx+jy\,syy+jz\,syz)+Az\,(n\,qz+jx\,szx+jy\,szy+jz\,szz)}{n}+(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\,\epsilon-\frac{(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jx^2+jy^2+jz^2-2\,n^2\,W)\,\rho}{2\,n^2}\right)+O[\frac{1}{c}]^2 \\ jx\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jy\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+\frac{jy\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jz\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+\frac{jz\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho}{n}\right)+O[\frac{1}{c}]^2 \end{pmatrix}$$

```
(* content and flux of coord-energy and momentum for dust (TRANSPOSED) *)
shows[assut, 1][T[fluxdust = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.dust2]]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\,(2\,\epsilon+(aux^2+auy^2+auz^2-2\,W)\,\rho)+O[\frac{1}{c}]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\,\rho\,c^2-\frac{1}{2}\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(2\,\epsilon+(aux^2+auy^2+auz^2-2\,W)\,\rho)+O[\frac{1}{c}]^2 \\ aux\,n\,\rho+O[\frac{1}{c}]^2 & aux\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho+O[\frac{1}{c}]^2 \\ auy\,n\,\rho+O[\frac{1}{c}]^2 & auy\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho+O[\frac{1}{c}]^2 \\ auz\,n\,\rho+O[\frac{1}{c}]^2 & auz\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
shows[assut, 1][T[fluxEPS /. replaceJu]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\,(2\,\epsilon+(ux^2+uy^2+uz^2-2\,W)\,\rho)+O[\frac{1}{c}]^2 & n\,(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz))\,\rho\,c^2+\left(-Ax\,(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,(ux-Vx)\,\epsilon)-Ay\,(qy+syx\,ux+syy\,uy+syz\,uz+n\,(uy-Vy)\,\epsilon)-Az\,(qz+szx\,ux+szy\,uy+szz\,uz+n\,(uz-Vz)\,\epsilon)-\frac{1}{2}\,n\,(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz))\,(ux^2+uy^2+uz^2-2\,W)\,\rho\right)+O[\frac{1}{c}]^2 \\ n\,ux\,\rho+O[\frac{1}{c}]^2 & (Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\,(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz))\,\rho)+O[\frac{1}{c}]^2 \\ n\,uy\,\rho+O[\frac{1}{c}]^2 & (Ax\,sxy+Ay\,syy+Az\,szy+n\,uy\,(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz))\,\rho)+O[\frac{1}{c}]^2 \\ n\,uz\,\rho+O[\frac{1}{c}]^2 & (Ax\,sxz+Ay\,syz+Az\,szz+n\,uz\,(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz))\,\rho)+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* momentum flux = A.σ + P A.(u-V)*)
fluxPS = ({Ax, Ay, Az}.(S[[2 ;; 4, 2 ;; 4]]).{1, 0, 0} + EPS[[1, 2]] * ({Ax, Ay, Az}.({jx, jy, jz} / n - {Vx, Vy, Vz})))
```

Out[•]= $\left(Ax\,sxx+Ay\,syx+Az\,szx+jx\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\rho\right)+\frac{1}{c^2}\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\left(px+\frac{jx\,sxx}{n}+\frac{jy\,syx}{n}+\frac{jz\,szx}{n}+jx\,\epsilon+\frac{jx^3\,\rho}{2\,n^2}+\frac{jx\,jy^2\,\rho}{2\,n^2}+\frac{jx\,jz^2\,\rho}{2\,n^2}+3\,jx\,W\,\rho\right)+O\left[\frac{1}{c}\right]^4$

In[•]:=
```
shows[assut, 1][Expand //@ FS@PowerExpand[fluxEPS[[2, 2]] - fluxPS]]
```

Out[•]//MatrixForm=

$O\left[\frac{1}{c}\right]^2$

In[•]:=
```
(* energy flux = A.q + A.σ.u + E A.(u-V)*)
fluxE = -({Ax, Ay, Az}.{qx, qy, qz} + {Ax, Ay, Az}.(S[[2 ;; 4, 2 ;; 4]] + Qtens[[2 ;; 4, 2 ;; 4]]).{jx, jy, jz} / n + (-EPS[[1, 1]]) * {Ax, Ay, Az}.({jx, jy, jz} / n - {Vx, Vy, Vz}))
```

Out[•]= $-n\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\rho\,c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz-\frac{1}{n}\left(jx\,(Ax\,sxx+Ay\,syx+Az\,szx)+jy\,(Ax\,sxy+Ay\,syy+Az\,szy)+jz\,(Ax\,sxz+Ay\,syz+Az\,szz)\right)-\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\left(n\,\epsilon+\frac{jx^2\,\rho}{2\,n}+\frac{jy^2\,\rho}{2\,n}+\frac{jz^2\,\rho}{2\,n}-n\,W\,\rho\right)\right)+O\left[\frac{1}{c}\right]^2$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[fluxEPS[[2, 1]] - fluxE]]
```

Out[•]//MatrixForm=

$O\left[\frac{1}{c}\right]^2$

```
(* matter flux n A.(u-V) *)
shows[assut, 1][fluxNJ = Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.NJ /. replaceJu]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} n \\ n\,(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)) \end{pmatrix}$$

```
(* content and flux of coord-energy and momentum assuming no matter flux (transposed) *)
shows[Join[assut, {((surface / Δt).NJ) == 0} /. replaceJu], 1][T@fluxEPS /. replaceJu]
Out[•]//MatrixForm=
( -n ρ c^2 - 1/2 n (2 ϵ + (ux^2 + uy^2 + uz^2 - 2 W) ρ) + O[1/c]^2    (-Ax (qx + sxx ux + sxy uy + sxz uz) - Ay (qy + syx ux + syy uy + syz uz) - Az (qz + szx ux + szy uy + szz uz)) + O[1/c]^2
  n ux ρ + O[1/c]^2    (Ax sxx + Ay syx + Az szx) + O[1/c]^2
  n uy ρ + O[1/c]^2    (Ax sxy + Ay syy + Az szy) + O[1/c]^2
  n uz ρ + O[1/c]^2    (Ax sxz + Ay syz + Az szz) + O[1/c]^2 )

In[•]:=

(* coordinate/internal/coordinate-proper energy and x-momentum, content and fluxes (TRANSPOSED) *)
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, Lxvec, Lxvec2, xboost / c, xboost2, uu, ntvec}]) /. replaceJu]]]
Out[•]//MatrixForm=
( -n ρ c^2 - 1/2 n (2 ϵ + (ux^2 + uy^2 + uz^2 - 2 W) ρ) + O[1/c]^2    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c^2 + (-Ax (qx + sxx ux + sxy uy + sxz uz + n (ux - Vx) ϵ) - Ay (qy + syx ux + syy uy + syz uz + n (uy - Vy) ϵ) - Az (qz + szx ux + szy uy + szz uz + n (uz - Vz) ϵ) - 1/2 n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (ux^2 + uy^2 + uz^2 - 2 W) ρ) + O[1/c]^2
  n ux ρ + O[1/c]^2    (Ax sxx + Ay syx + Az szx + n ux (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) ρ) + O[1/c]^2
  n (uz y - uy z) ρ + O[1/c]^2    (Ax sxz y + Ay syz y + Az szz y - Ax sxy z - Ay syy z - Az szy z + n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (uz y - uy z) ρ) + O[1/c]^2
  n (uz y - uy z) ρ + O[1/c]^2    (Ax sxz y + Ay syz y + Az szz y - Ax sxy z - Ay syy z - Az szy z + n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (uz y - uy z) ρ) + O[1/c]^2
  -n (t ux + x) ρ + O[1/c]^2    (-((Ax sxx + Ay syx + Az szx) t) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux + x) ρ) + O[1/c]^2
  n (-t ux + x) ρ + O[1/c]^2    (-((Ax sxx + Ay syx + Az szx) t) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux - x) ρ) + O[1/c]^2
  -n ρ c^2 - n ϵ + O[1/c]^2    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c^2 + (-Ax qx - Ay qy - Az qz + n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ϵ) + O[1/c]^2
  -n ρ c^2 - 1/2 n (2 ϵ + (ux^2 + uy^2 + uz^2) ρ) + O[1/c]^2    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c^2 + (-Ax (qx + sxx ux + sxy uy + sxz uz + n (ux - Vx) ϵ) - Ay (qy + syx ux + syy uy + syz uz + n (uy - Vy) ϵ) - Az (qz + szx ux + szy uy + szz uz + n (uz - Vz) ϵ) - 1/2 n (ux^2 + uy^2 + uz^2) (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) ρ) + O[1/c]^2 )

In[•]:= show2[assut, 1][T[variousfluxes /. {Vy → 0, Vz → 0, uy → 0, uz → 0}]]
Out[•]//MatrixForm=
( -n ρ c^2 - 1/2 n (2 ϵ + (ux^2 - 2 W) ρ) + O[1/c]^2    Ax n (-ux + Vx) ρ c^2 + (-Ay (qy + syx ux) - Az (qz + szx ux) - Ax (qx + sxx ux + n (ux - Vx) ϵ) - 1/2 Ax n (ux - Vx) (ux^2 - 2 W) ρ) + O[1/c]^2
  n ux ρ + O[1/c]^2    (Ax sxx + Ay syx + Az szx + Ax n ux (ux - Vx) ρ) + O[1/c]^2
  O[1/c]^2    (Ax sxz y + Ay syz y + Az szz y - Ax sxy z - Ay syy z - Az szy z) + O[1/c]^2
  O[1/c]^2    (Ax sxz y + Ay syz y + Az szz y - Ax sxy z - Ay syy z - Az szy z) + O[1/c]^2
  -n (t ux + x) ρ + O[1/c]^2    (-((Ax sxx + Ay syx + Az szx) t) - Ax n (ux - Vx) (t ux + x) ρ) + O[1/c]^2
  n (-t ux + x) ρ + O[1/c]^2    (-((Ax sxx + Ay syx + Az szx) t) - Ax n (ux - Vx) (t ux - x) ρ) + O[1/c]^2
  -n ρ c^2 - n ϵ + O[1/c]^2    Ax n (-ux + Vx) ρ c^2 + (-Ax qx - Ay qy - Az qz + Ax n (-ux + Vx) ϵ) + O[1/c]^2
  -n ρ c^2 - 1/2 n (2 ϵ + ux^2 ρ) + O[1/c]^2    Ax n (-ux + Vx) ρ c^2 + (-Ay (qy + syx ux) - Az (qz + szx ux) - Ax (qx + sxx ux + n (ux - Vx) ϵ) - 1/2 Ax n ux^2 (ux - Vx) ρ) + O[1/c]^2 )

In[•]:= (* velocity of energy *)
shows[assut, 5][(EPS.{1, 0, 0, 0})[[2 ;; 4]] / (EPS.{1, 0, 0, 0})[[1]] /. replaceJu]
Out[•]//MatrixForm=
( ux + (qx+sxx ux+sxy uy+sxz uz)/(n ρ c^2) + O[1/c]^4
  uy + (qy+syx ux+syy uy+syz uz)/(n ρ c^2) + O[1/c]^4
  uz + (qz+szx ux+szy uy+szz uz)/(n ρ c^2) + O[1/c]^4 )

In[•]:= temp = SeriesCoefficient[tt.{1, 0, 0, 0}, {c, Infinity, -2}];
shows[assut, 5][(tt.{1, 0, 0, 0} - temp * c ^ 2)[[2 ;; 4]] / (tt.{1, 0, 0, 0} - temp * c ^ 2)[[1]] /. j2v]
Out[•]//MatrixForm=
( (ux + 2 (qx+sxx ux+sxy uy+sxz uz)/(2 n ϵ+n (ux^2+uy^2+uz^2-2 W) ρ)) + O[1/c]^2
  (uy + 2 (qy+syx ux+syy uy+syz uz)/(2 n ϵ+n (ux^2+uy^2+uz^2-2 W) ρ)) + O[1/c]^2
  (uz + 2 (qz+szx ux+szy uy+szz uz)/(2 n ϵ+n (ux^2+uy^2+uz^2-2 W) ρ)) + O[1/c]^2 )

In[•]:= showf[assut][{variousfluxes[[ ;; , 1]] - variousfluxes[[ ;; , 7]], variousfluxes[[ ;; , 1]] - variousfluxes[[ ;; , 8]], variousfluxes[[ ;; , 7]] - variousfluxes[[ ;; , 8]]}]
Out[•]//MatrixForm=
( (-1/2 n ux^2 ρ - 1/2 n uy^2 ρ - 1/2 n uz^2 ρ + n W ρ) + O[1/c]^2    (-Ax sxx ux - Ay syx ux - Az szx ux - Ax sxy uy - Ay syy uy - Az szy uy - Ax sxz uz - Ay syz uz - Az szz uz - 1/2 Ax n ux^3 ρ - 1/2 Ay n ux^2 uy ρ - 1/2 Ax n ux uy^2 ρ - 1/2 Ay n uy^3 ρ - 1/2 Az n ux^2 uz ρ - 1/2 Az n uy^2 uz ρ - 1/2 Ax n ux uz^2 ρ - 1/2 Ay n uy uz^2 ρ - 1/2 Az n uz^3 ρ + 1/2 Ax n ux^2 Vx ρ + 1/2 Ax n uy^2 Vx ρ + 1/2 Ax n
  n W ρ + O[1/c]^2    (Ax n ux W ρ + Ay n uy W ρ + Az n uz W ρ - Ax n Vx W ρ - Ay n Vy W ρ - Az n Vz W ρ) + O[1/c]^2
  (1/2 n ux^2 ρ + 1/2 n uy^2 ρ + 1/2 n uz^2 ρ) + O[1/c]^2    (Ax sxx ux + Ay syx ux + Az szx ux + Ax sxy uy + Ay syy uy + Az szy uy + Ax sxz uz + Ay syz uz + Az szz uz + 1/2 Ax n ux^3 ρ + 1/2 Ay n ux^2 uy ρ + 1/2 Ax n ux uy^2 ρ + 1/2 Ay n uy^3 ρ + 1/2 Az n ux^2 uz ρ + 1/2 Az n uy^2 uz ρ + 1/2 Ax n ux uz^2 ρ + 1/2 Ay n uy uz^2 ρ + 1/2 Az n uz^3 ρ - 1/2 Ax n ux^2 Vx ρ - 1/2 Ax n uy^2 Vx ρ - 1/2 Ax n u

In[•]:= (* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
show2[assut, 2][FS[(itjv[Tr[#.TTx]]) /. replaceJu]] & /@ {Dxyzvec[[1, ;; , ;;]], Dxyzvec[[2, ;; , ;;]], DLxvec, DLxvec2, Dxboost / c, Dxboost2, Duv, Dntvec} // MF
Out[•]//MatrixForm=
( n ρ W^(1,0,0,0)[t, x, y, z] + O[1/c]^2
  n ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
  n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]^2
  n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]^2
  -n ρ (2 ux + t W^(0,1,0,0)[t, x, y, z]) + O[1/c]^2
  -n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
  1/2 (2 szz uz^(0,0,0,1)[t, x, y, z] + (syz + szy) (uy^(0,0,0,1)[t, x, y, z] + uz^(0,0,1,0)[t, x, y, z]) + 2 (syy uy^(0,0,1,0)[t, x, y, z] + sxx ux^(0,1,0,0)[t, x, y, z]) + (sxy + syx) (ux^(0,0,1,0)[t, x, y, z] + uy^(0,1,0,0)[t, x, y, z]) + (sxz + szx) (ux^(0,0,0,1)[t, x, y, z] + uz^(0,1,0,0)[t, x, y, z])) + O[1/c]^2
  -n ρ (uz W^(0,0,0,1)[t, x, y, z] + uy W^(0,0,1,0)[t, x, y, z] + ux W^(0,1,0,0)[t, x, y, z]) + O[1/c]^2 )

shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.EPSsym.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, Lxvec, Lxvec2, xboost / c, xboost2, uu, ntvec}]) /. replaceJu]]]
Out[•]//MatrixForm=
( -n ρ c^2 - 1/2 n (2 ϵ + (ux^2 + uy^2 + uz^2 - 2 W) ρ) + O[1/c]^2    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c^2 + (-Ax (qx + sxx ux + sxy uy + sxz uz + n (ux - Vx) ϵ) - Ay (qy + sxy ux + syy uy + syz uz + n (uy - Vy) ϵ) - Az (qz + sxz ux + syz uy + szz uz + n (uz - Vz) ϵ) - 1/2 n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (ux^2 + uy^2 + uz^2 - 2 W) ρ) + O[1/c]^2
  n ux ρ + O[1/c]^2    (Ax sxx + Ay sxy + Az sxz + n ux (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) ρ) + O[1/c]^2
  n (uz y - uy z) ρ + O[1/c]^2    (Ax sxz y + Ay syz y + Az szz y - Ax sxy z - Ay syy z - Az syz z + n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (uz y - uy z) ρ) + O[1/c]^2
  n (uz y - uy z) ρ + O[1/c]^2    (Ax sxz y + Ay syz y + Az szz y - Ax sxy z - Ay syy z - Az syz z + n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (uz y - uy z) ρ) + O[1/c]^2
  -n (t ux + x) ρ + O[1/c]^2    (-((Ax sxx + Ay sxy + Az sxz) t) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux + x) ρ) + O[1/c]^2
  n (-t ux + x) ρ + O[1/c]^2    (-((Ax sxx + Ay sxy + Az sxz) t) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux - x) ρ) + O[1/c]^2
  -n ρ c^2 - n ϵ + O[1/c]^2    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c^2 + (-Ax qx - Ay qy - Az qz + n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ϵ) + O[1/c]^2
  -n ρ c^2 - 1/2 n (2 ϵ + (ux^2 + uy^2 + uz^2) ρ) + O[1/c]^2    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c^2 + (-Ax (qx + sxx ux + sxy uy + sxz uz + n (ux - Vx) ϵ) - Ay (qy + sxy ux + syy uy + syz uz + n (uy - Vy) ϵ) - Az (qz + sxz ux + syz uy + szz uz + n (uz - Vz) ϵ) - 1/2 n (ux^2 + uy^2 + uz^2) (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) ρ) + O[1/c]^2 )

In[•]:= TTx = tW[tjv[(EPSsym + T[EPSsym.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. replaceJu] & /@ {Dxyzvec[[1, ;; , ;;]], Dxyzvec[[2, ;; , ;;]], DLxvec, DLxvec2, Dxboost / c, Dxboost2, Duv, Dntvec} // MF
Out[•]//MatrixForm=
( n ρ W^(1,0,0,0)[t, x, y, z] + O[1/c]^2
  n ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
  n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]^2
  n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]^2
  -n ρ (2 ux + t W^(0,1,0,0)[t, x, y, z]) + O[1/c]^2
  -n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
  (szz uz^(0,0,0,1)[t, x, y, z] + syy uy^(0,0,1,0)[t, x, y, z] + syz (uy^(0,0,0,1)[t, x, y, z] + uz^(0,0,1,0)[t, x, y, z]) + sxx ux^(0,1,0,0)[t, x, y, z] + sxy (ux^(0,0,1,0)[t, x, y, z] + uy^(0,1,0,0)[t, x, y, z]) + sxz (ux^(0,0,0,1)[t, x, y, z] + uz^(0,1,0,0)[t, x, y, z])) + O[1/c]^2
  -n ρ (uz W^(0,0,0,1)[t, x, y, z] + uy W^(0,0,1,0)[t, x, y, z] + ux W^(0,1,0,0)[t, x, y, z]) + O[1/c]^2 )

In[•]:= (* 2-vector of surface parallel to yz  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(yzsurface = ((T[{{0, 0, Ly, 0}}].{{0, 0, 0, Lz}} - T[{{0, 0, 0, Lz}}].{{0, 0, Ly, 0}}) /. {Ly → Ayz / Lz})) // MF
Out[•]//MatrixForm=
( 0 0 0    0
  0 0 0    0
  0 0 0    Ayz
  0 0 -Ayz 0 )

In[•]:= (* 2-vector of surface parallel to tx  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(txsurface = (-(T[{{Δt, 0, 0, 0}}].{{0, Lx, 0, 0}} - T[{{0, Lx, 0, 0}}].{{Δt, 0, 0, 0}}))) // MF
Out[•]//MatrixForm=
( 0     -Lx Δt 0 0
  Lx Δt 0      0 0
  0     0      0 0
  0     0      0 0 )

In[•]:= (* 2-vector of surface parallel to ty  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(tysurface = (-(T[{{Δt, 0, 0, 0}}].{{0, 0, Ly, 0}} - T[{{0, 0, Ly, 0}}].{{Δt, 0, 0, 0}}))) // MF
Out[•]//MatrixForm=
( 0     0 -Ly Δt 0
  0     0 0      0
  Ly Δt 0 0      0
  0     0 0      0 )

In[•]:= (* 2-vector of surface parallel to y moving to x  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(yVxsurface = (-(T[{{1, Vx, 0, 0}} * Δt].{{0, 0, Ly, 0}} - T[{{0, 0, Ly, 0}}].{{1, Vx, 0, 0}} * Δt))) // MF
Out[•]//MatrixForm=
( 0     0        -Ly Δt    0
  0     0        -Ly Vx Δt 0
  Ly Δt Ly Vx Δt 0         0
  0     0        0         0 )

In[•]:= (Tr[T[txsurface].txsurface]) // MF
Out[•]//MatrixForm=
2 Lx^2 Δt^2
```

In[•]:=
```
(* Faraday tensor *)
repE = {Ex → Ex * c * Sqrt[μ * ϵ], Ey → Ey * c * Sqrt[μ * ϵ], Ez → Ez * c * Sqrt[μ * ϵ]};
fftemp = {{0, -Ex, -Ey, -Ez}, {0, 0, Bz, -By}, {0, 0, 0, Bx}, {0, 0, 0, 0}} /. repE;
showf[assut][F = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -c\,Ex\sqrt{\epsilon}\sqrt{\mu} & -c\,Ey\sqrt{\epsilon}\sqrt{\mu} & -c\,Ez\sqrt{\epsilon}\sqrt{\mu} \\ c\,Ex\sqrt{\epsilon}\sqrt{\mu} & 0 & Bz & -By \\ c\,Ey\sqrt{\epsilon}\sqrt{\mu} & -Bz & 0 & Bx \\ c\,Ez\sqrt{\epsilon}\sqrt{\mu} & By & -Bx & 0 \end{pmatrix}$$

In[•]:=
```
{FS[{Tr[yzsurface.T[F]], Tr[T[txsurface].F], Tr[T[tysurface].F], Tr[T[yVxsurface].F]} / 2]} // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix} Ayz\,Bx \\ c\,Ex\,Lx\,\Delta t\sqrt{\epsilon}\sqrt{\mu} \\ c\,Ey\,Ly\,\Delta t\sqrt{\epsilon}\sqrt{\mu} \\ Ly\,\Delta t\left(-Bz\,Vx + c\,Ey\sqrt{\epsilon}\sqrt{\mu}\right) \end{pmatrix}$$

In[•]:=
```
(* charge-current-potential tensor *)
fftemp = {{0, -Hx, -Hy, -Hz}, {0, 0, Dz, -Dy}, {0, 0, 0, Dx}, {0, 0, 0, 0}};
showf[assut][H = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -Hx & -Hy & -Hz \\ Hx & 0 & Dz & -Dy \\ Hy & -Dz & 0 & Dx \\ Hz & Dy & -Dx & 0 \end{pmatrix}$$

In[•]:=
```
showf[assut][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
   ]]]
```

Out[•]//MatrixForm=

(⋯1⋯)

Full expression not available (original memory size: 0.7 MB)

In[•]:=
```
shows[assut, 1][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
   ]]]
```

Out[•]//MatrixForm=

```
( -(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + O[1/c]²   |   (Bz Ey-By Ez) √ϵ0/(√μ0 c) + O[1/c]²   |   (-Bz Ex+Bx Ez) √ϵ0/(√μ0 c) + O[1/c]²   |   (By Ex-Bx Ey) √ϵ0/(√μ0 c) + O[1/c]²
  (-Bz Ey+By Ez) √ϵ0 c/√μ0 + 2 (Bz Ey-By Ez) W √ϵ0/(√μ0 c) + O[1/c]²   |   (-Bx²+By²+Bz²+(-Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + O[1/c]²   |   -(Bx By+Ex Ey ϵ0 μ0)/μ0 + O[1/c]²   |   -(Bx Bz+Ex Ez ϵ0 μ0)/μ0 + O[1/c]²
  (Bz Ex-Bx Ez) √ϵ0 c/√μ0 + 2 (-Bz Ex+Bx Ez) W √ϵ0/(√μ0 c) + O[1/c]²   |   -(Bx By+Ex Ey ϵ0 μ0)/μ0 + O[1/c]²   |   (Bx²-By²+Bz²+(Ex²-Ey²+Ez²) ϵ0 μ0)/(2 μ0) + O[1/c]²   |   -(By Bz+Ey Ez ϵ0 μ0)/μ0 + O[1/c]²
  (-By Ex+Bx Ey) √ϵ0 c/√μ0 + 2 (By Ex-Bx Ey) W √ϵ0/(√μ0 c) + O[1/c]²   |   -(Bx Bz+Ex Ez ϵ0 μ0)/μ0 + O[1/c]²   |   -(By Bz+Ey Ez ϵ0 μ0)/μ0 + O[1/c]²   |   (Bx²+By²-Bz²+(Ex²+Ey²-Ez²) ϵ0 μ0)/(2 μ0) + O[1/c]² )
```

In[•]:=
```
showf[assut][T[tte.Inverse[gg]].gg - tte]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^4 & O[\frac{1}{c}]^5 & O[\frac{1}{c}]^5 & O[\frac{1}{c}]^5 \\ O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}$$

In[•]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.tte.T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```

Out[•]//MatrixForm=

```
( -(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + O[1/c]²   |   (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (Ax Vx+Ay Vy+Az Vz) (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + 2 (Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) W √ϵ0/(√μ0 c) + O[1/c]²
  -(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + (Bz Ey ux-By Ez ux-Bz Ex uy+Bx Ez uy+By Ex uz-Bx Ey uz) √ϵ0/(√μ0 c) + O[1/c]²   |   (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (Ax (-2 Bx (By uy+Bz uz)+Bx² (-ux+Vx)+(By²+Bz²) (ux+Vx)+(-2 Ex (Ey uy+Ez uz)+Ex² (-ux+Vx)+(Ey²+Ez²) (ux+Vx)) ϵ0 μ0)+Ay (-2 Bx By ux-2 By Bz uz+By² (-uy+Vy)+Bx² ⋯
  -(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + O[1/c]²   |   (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (Ax Vx+Ay Vy+Az Vz) (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + (Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) W √ϵ0/(√μ0 c) + O[1/c]²
  (Bz Ey-By Ez) √ϵ0/(√μ0 c) + O[1/c]²   |   (-2 (Ay Bx By+Az Bx Bz+Ay Ex Ey ϵ0 μ0+Az Ex Ez ϵ0 μ0)+Ax (-Bx²+By²+Bz²+(-Ex²+Ey²+Ez²) ϵ0 μ0))/(2 μ0) - (Bz Ey-By Ez) (Ax Vx+Ay Vy+Az Vz) √ϵ0/(√μ0 c) + O[1/c]²
  (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²   |   (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+(-2 (Ax Ex+Ay Ey) Ez y+Az (Ex²+Ey²-Ez²) y+2 Ey (Ax Ex+Az Ez) z-Ay (Ex²-Ey²+Ez²) z) ϵ0 μ0)/(2 μ0) - (Ax Vx+Ay Vy+Az Vz) (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²
  (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²   |   (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+(-2 (Ax Ex+Ay Ey) Ez y+Az (Ex²+Ey²-Ez²) y+2 Ey (Ax Ex+Az Ez) z-Ay (Ex²-Ey²+Ez²) z) ϵ0 μ0)/(2 μ0) - (Ax Vx+Ay Vy+Az Vz) (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²
  (-Bz Ey+By Ez) t √ϵ0/(√μ0 c) + O[1/c]²   |   t (2 (Ay Bx By+Az Bx Bz+Ay Ex Ey ϵ0 μ0+Az Ex Ez ϵ0 μ0)+Ax (Bx²-By²-Bz²+(Ex²-Ey²-Ez²) ϵ0 μ0))/(2 μ0) + ((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) x) √ϵ0/(√μ0 c) + O[1/c]²
  (-Bz Ey+By Ez) t √ϵ0/(√μ0 c) + O[1/c]²   |   t (2 (Ay Bx By+Az Bx Bz+Ay Ex Ey ϵ0 μ0+Az Ex Ez ϵ0 μ0)+Ax (Bx²-By²-Bz²+(Ex²-Ey²-Ez²) ϵ0 μ0))/(2 μ0) + ((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) x) √ϵ0/(√μ0 c) + O[1/c]² )
```

In[•]:=
```
TTx = tW[tjv[(tte + T[tte.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```

Out[•]//MatrixForm=

```
( (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) W^(1,0,0,0)[t,x,y,z]/(μ0 c²) + O[1/c]³
  (-2 (Bx Bz+Ex Ez ϵ0 μ0) ux^(0,0,0,1)[t,x,y,z]-2 (By Bz+Ey Ez ϵ0 μ0) uy^(0,0,0,1)[t,x,y,z]+Bx² uz^(0,0,0,1)[t,x,y,z]+By² uz^(0,0,0,1)[t,x,y,z]-Bz² uz^(0,0,0,1)[t,x,y,z]+Ex² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]+Ey² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-Ez² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-2 Bx By ux^(0,0,1,0)[t,x,y,z]-2 Ex Ey ϵ0 μ0 ux^(0,0,1,0)[t,x,y,z]+Bx² uy^(0,0,1,0)[t,x,y,z]-By² uy^(0,0,1,0)[t,x,y,z]+Bz² uy^(0,0,1,0)[t,x,y,z]+Ex² ϵ0 μ0 uy^(0,0,1,0)[t,x,y,z]-Ey² ϵ0 μ0 uy^(0,0,1,0)[t,x,y,z]+Ez² ϵ0 μ0 uy^(0,0,1,0)[t,x,y,z]-2 Bz By uz^(0,0,1,0)[t,x ⋯)/(2 μ0)
  √ϵ0 ((-By Ex+Bx Ey) W^(0,0,0,1)[t,x,y,z]+(Bz Ex-Bx Ez) W^(0,0,1,0)[t,x,y,z]+(-Bz Ey+By Ez) W^(0,1,0,0)[t,x,y,z])/(√μ0 c) + (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) W^(1,0,0,0)[t,x,y,z]/(2 μ0 c²) + O[1/c]³
  (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) W^(0,1,0,0)[t,x,y,z]/(μ0 c²) + O[1/c]³
  (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) (y W^(0,0,0,1)[t,x,y,z]-z W^(0,0,1,0)[t,x,y,z])/(μ0 c²) + O[1/c]³
  2 ((Bz² y-By Bz z+Ez (Ez y-Ey z) ϵ0 μ0) W^(0,0,0,1)[t,x,y,z]+(By Bz y-By² z+Ey (Ez y-Ey z) ϵ0 μ0) W^(0,0,1,0)[t,x,y,z]+(Bx (Bz y-By z)+Ex (Ez y-Ey z) ϵ0 μ0) W^(0,1,0,0)[t,x,y,z])/(μ0 c²) + O[1/c]³
  2 (-Bz Ey+By Ez) √ϵ0/(√μ0 c) - t (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) W^(0,1,0,0)[t,x,y,z]/(μ0 c²) + O[1/c]³
  -2 (t ((Bx Bz+Ex Ez ϵ0 μ0) W^(0,0,0,1)[t,x,y,z]+(Bx By+Ex Ey ϵ0 μ0) W^(0,0,1,0)[t,x,y,z]+(Bx²+Ex² ϵ0 μ0) W^(0,1,0,0)[t,x,y,z]))/(μ0 c²) + O[1/c]³ )
```

In[•]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.(tte + ttsym).T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```

Out[•]//MatrixForm=

```
( -n ρ c² + (-n ϵ - (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) - 1/2 n (ux² + uy² + uz² - 2 W) ρ) + O[1/c]²   |   n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (-2 Ax (qx+sxx ux+sxy uy+sxz uz+n ux ϵ) μ0-2 Ay (qy+sxy ux+syy uy+syz uz+n uy ϵ) μ0-2 Az (qz+sxz ux+syz uy+szz uz+n uz ϵ) μ0+Ax Vx (Bx²+By²+Bz²+2 n ϵ μ0+(Ex²+Ey²+Ez²) ϵ0 μ0)+Ay Vy (Bx²+By²+Bz²+2 n ϵ μ0+(Ex²+ ⋯)/(2 μ0)
  -n ρ c² - (Bx²+By²+Bz²+2 n ϵ μ0+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + (Bz Ey ux-By Ez ux-Bz Ex uy+Bx Ez uy+By Ex uz-Bx Ey uz) √ϵ0/(√μ0 c) + O[1/c]²   |   n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + Ay (-2 Bx By ux-2 By Bz uz+By² (-uy+Vy)+Bx² (uy+Vy)+Bz² (uy+Vy)-2 (qy+n (uy-Vy) ϵ) μ0+(-2 Ex Ey ux-2 Ey Ez uz+Ey² (-uy+Vy)+Ex² (uy+Vy)+Ez² (uy+Vy)) ϵ0 μ0)+Ax (-2 Bx (By uy+Bz uz)+Bx² (-ux+Vx)+(By²+Bz²) (ux+Vx)+( ⋯
  -n ρ c² + (-n ϵ - (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) - 1/2 n (ux² + uy² + uz²) ρ) + O[1/c]²   |   n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (-2 Ax (qx+sxx ux+sxy uy+sxz uz+n ux ϵ) μ0-2 Ay (qy+sxy ux+syy uy+syz uz+n uy ϵ) μ0-2 Az (qz+sxz ux+syz uy+szz uz+n uz ϵ) μ0+Ax Vx (Bx²+By²+Bz²+2 n ϵ μ0+(Ex²+Ey²+Ez²) ϵ0 μ0)+Ay Vy (Bx²+By²+Bz²+2 n ϵ μ0+(Ex²+ ⋯)/(2 μ0)
  n ux ρ + (Bz Ey-By Ez) √ϵ0/(√μ0 c) + O[1/c]²   |   (-2 Ay (Bx By-sxy μ0+Ex Ey ϵ0 μ0+n ux (-uy+Vy) μ0 ρ)+Ax (-Bx²+By²+Bz²+μ0 (2 sxx+(-Ex²+Ey²+Ez²) ϵ0+2 n ux (ux-Vx) ρ))+2 Az (-Bx Bz+μ0 (sxz-Ex Ez ϵ0+n ux (uz-Vz) ρ)))/(2 μ0) - (Bz Ey-By Ez) (Ax Vx+Ay Vy+Az Vz) √ϵ0/(√μ0 c) + O[1/c]²
  n (uz y - uy z) ρ + (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²   |   (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+2 Ax sxz y μ0+2 Ay syz y μ0+2 Az szz y μ0-2 Ax sxy z μ0-2 Ay syy z μ0-2 Az syz z μ0+Az Ex² y ϵ0 μ0+Az Ey² y ϵ0 μ0-2 Ax Ex Ez y ϵ0 μ0-2 Ay Ey Ez y ϵ0 μ0-Az Ez² y ϵ0 μ0-Ay Ex² z ϵ0 μ0+2 Ax Ex Ey z ϵ0 μ0+Ay Ey² z ϵ0 μ0+2 Az Ey Ez z ϵ0 μ0-Ay Ez² z ϵ0 μ0+ ⋯)/(2 μ0)
  n (uz y - uy z) ρ + (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²   |   (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+2 Ax sxz y μ0+2 Ay syz y μ0+2 Az szz y μ0-2 Ax sxy z μ0-2 Ay syy z μ0-2 Az syz z μ0+Az Ex² y ϵ0 μ0+Az Ey² y ϵ0 μ0-2 Ax Ex Ez y ϵ0 μ0-2 Ay Ey Ez y ϵ0 μ0-Az Ez² y ϵ0 μ0-Ay Ex² z ϵ0 μ0+2 Ax Ex Ey z ϵ0 μ0+Ay Ey² z ϵ0 μ0+2 Az Ey Ez z ϵ0 μ0-Ay Ez² z ϵ0 μ0+ ⋯)/(2 μ0)
  -n (t ux + x) ρ + (-Bz Ey+By Ez) t √ϵ0/(√μ0 c) + O[1/c]²   |   (t (Ax Bx²+2 Ay Bx By-Ax By²+2 Az Bx Bz-Ax Bz²-(2 Ax sxx+2 Ay sxy+2 Az sxz-2 Ex (Ay Ey+Az Ez) ϵ0+Ax (-Ex²+Ey²+Ez²) ϵ0) μ0)/(2 μ0) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux + x)) ρ + ((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) x) √ϵ0/(√μ0 c) + O[1/c]²
  n (-t ux + x) ρ + (-Bz Ey+By Ez) t √ϵ0/(√μ0 c) + O[1/c]²   |   (t (Ax Bx²+2 Ay Bx By-Ax By²+2 Az Bx Bz-Ax Bz²-(2 Ax sxx+2 Ay sxy+2 Az sxz-2 Ex (Ay Ey+Az Ez) ϵ0+Ax (-Ex²+Ey²+Ez²) ϵ0) μ0)/(2 μ0) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux - x)) ρ + ((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) x) √ϵ0/(√μ0 c) + O[1/c]² )
```

In[•]:=
```
TTx = tW[tjv[((tte + ttsym) + T[(tte + ttsym).Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```

Out[•]//MatrixForm=

```
( n ρ W^(1,0,0,0)[t, x, y, z] + O[1/c]²
  (-2 (Bx Bz-sxz μ0+Ex Ez ϵ0 μ0) ux^(0,0,0,1)[t,x,y,z]-2 (By Bz-syz μ0+Ey Ez ϵ0 μ0) uy^(0,0,0,1)[t,x,y,z]+Bx² uz^(0,0,0,1)[t,x,y,z]+By² uz^(0,0,0,1)[t,x,y,z]-Bz² uz^(0,0,0,1)[t,x,y,z]+2 szz μ0 uz^(0,0,0,1)[t,x,y,z]+Ex² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]+Ey² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-Ez² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-2 Bx By ux^(0,0,1,0)[t,x,y,z]+2 sxy μ0 ux^(0,0,1,0)[t,x,y,z]-2 Ex Ey ϵ0 μ0 ux^(0,0,1,0)[t,x,y,z]+Bx² uy^(0,0,1,0)[t,x,y,z]-By² uy^(0,0,1,0)[t,x,y,z]+Bz² uy^(0,0,1,0)[t,x,y,z]+2 syy μ0 uy^(0,0,1,0)[t,x,y,z]+Ex² ϵ0 ⋯)/(2 μ0)
  -n ρ (uz W^(0,0,0,1)[t, x, y, z] + uy W^(0,0,1,0)[t, x, y, z] + ux W^(0,1,0,0)[t, x, y, z]) + √ϵ0 ((-By Ex+Bx Ey) W^(0,0,0,1)[t,x,y,z]+(Bz Ex-Bx Ez) W^(0,0,1,0)[t,x,y,z]+(-Bz Ey+By Ez) W^(0,1,0,0)[t,x,y,z])/(√μ0 c) + O[1/c]²
  n ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]²
  n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]²
  n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]²
  -n ρ (2 ux + t W^(0,1,0,0)[t, x, y, z]) + 2 (-Bz Ey+By Ez) √ϵ0/(√μ0 c) + O[1/c]²
  -n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]² )
```

In[•]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.(tt + tte).T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```

Out[•]//MatrixForm=

```
( -n ρ c² + (-(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) - 1/2 n (ux² + uy² + uz² - 2 W + 2 ϵ) ρ) + O[1/c]²   |   n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (Ax (-2 (qx+sxx ux+sxy uy+sxz uz) μ0+Vx (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)+n (-ux+Vx) (ux²+uy²+uz²-2 W+2 ϵ) μ0 ρ)+Ay (-2 (qy+sxy ux+syy uy+syz uz) μ0+Vy (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)+n (-uy+Vy) (ux²+uy²+ ⋯)/(2 μ0)
  -n ρ c² - (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0+2 n ϵ μ0 ρ)/(2 μ0) + (Bz Ey ux-By Ez ux-Bz Ex uy+Bx Ez uy+By Ex uz-Bx Ey uz) √ϵ0/(√μ0 c) + O[1/c]²   |   n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + Ax (-2 Bx (By uy+Bz uz)-Bx² ux-2 qx μ0-2 sxx ux μ0-2 sxy uy μ0-2 sxz uz μ0+Bx² Vx+By² ux+By² Vx+ ⋯
  -n ρ c² + (-(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) - 1/2 n (ux² + uy² + uz² + 2 ϵ) ρ) + O[1/c]²   |   n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + (-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c/√μ0 + (Ax (-2 (qx+sxx ux+sxy uy+sxz uz) μ0+Vx (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)+n (-ux+Vx) (ux²+uy²+uz²+2 ϵ) μ0 ρ)+Ay (-2 (qy+sxy ux+syy uy+syz uz) μ0+ ⋯)/(2 μ0)
  n ux ρ + (Bz Ey-By Ez) √ϵ0/(√μ0 c) + O[1/c]²   |   (-2 Ay (Bx By-syx μ0+Ex Ey ϵ0 μ0+n ux (-uy+Vy) μ0 ρ)+Ax (-Bx²+By²+Bz²+μ0 (2 sxx+(-Ex²+Ey²+Ez²) ϵ0+2 n ux (ux-Vx) ρ))+2 Az (-Bx Bz+μ0 (szx-Ex Ez ϵ0+n ux (uz-Vz) ρ)))/(2 μ0) - (Bz Ey-By Ez) (Ax Vx+Ay Vy+Az Vz) √ϵ0/(√μ0 c) + O[1/c]²
  n (uz y - uy z) ρ + (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²   |   (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+2 Ax sxz y μ0+2 Ay syz y μ0+2 Az szz y μ0-2 Ax sxy z μ0-2 Ay syy z μ0-2 Az syz z μ0+Az Ex² y ϵ0 μ0+Az Ey² y ϵ0 μ0-2 Ax Ex Ez y ϵ0 μ0-2 Ay Ey Ez y ϵ0 μ0-Az Ez² y ϵ0 μ0-Ay Ex² z ϵ0 μ0+2 Ax Ex Ey z ϵ0 μ0+Ay Ey² z ϵ0 μ0+2 Az Ey Ez z ϵ0 μ0-Ay Ez² z ϵ0 μ0+ ⋯)/(2 μ0)
  n (uz y - uy z) ρ + (By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0/(√μ0 c) + O[1/c]²   |   (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+2 Ax sxz y μ0+2 Ay syz y μ0+2 Az szz y μ0-2 Ax sxy z μ0-2 Ay syy z μ0-2 Az syz z μ0+Az Ex² y ϵ0 μ0+Az Ey² y ϵ0 μ0-2 Ax Ex Ez y ϵ0 μ0-2 Ay Ey Ez y ϵ0 μ0-Az Ez² y ϵ0 μ0-Ay Ex² z ϵ0 μ0+2 Ax Ex Ey z ϵ0 μ0+Ay Ey² z ϵ0 μ0+2 Az Ey Ez z ϵ0 μ0-Ay Ez² z ϵ0 μ0+ ⋯)/(2 μ0)
  -n (t ux + x) ρ + (-Bz Ey+By Ez) t √ϵ0/(√μ0 c) + O[1/c]²   |   (t (Ax Bx²+2 Ay Bx By-Ax By²+2 Az Bx Bz-Ax Bz²-(2 Ax sxx+2 Ay syx+2 Az szx-2 Ex (Ay Ey+Az Ez) ϵ0+Ax (-Ex²+Ey²+Ez²) ϵ0) μ0)/(2 μ0) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux + x)) ρ + ((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) x) √ϵ0/(√μ0 c) + O[1/c]²
  n (-t ux + x) ρ + (-Bz Ey+By Ez) t √ϵ0/(√μ0 c) + O[1/c]²   |   (t (Ax Bx²+2 Ay Bx By-Ax By²+2 Az Bx Bz-Ax Bz²-(2 Ax sxx+2 Ay syx+2 Az szx-2 Ex (Ay Ey+Az Ez) ϵ0+Ax (-Ex²+Ey²+Ez²) ϵ0) μ0)/(2 μ0) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux - x)) ρ + ((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) x) √ϵ0/(√μ0 c) + O[1/c]² )
```

In[•]:=
```
TTx = tW[tjv[((tt + tte) + T[(tt + tte).Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 4][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix} n\,\rho\,W^{(1,0,0,0)}[t,x,y,z] + O[\frac{1}{c}]^2 \\ \frac{(-2\,Bx\,Bz+(sxz+szx-2\,Ex\,Ez\,\epsilon0)\,\mu0)\,ux^{(0,0,0,1)}[t,x,y,z]+(-2\,By\,Bz+(syz+szy-2\,Ey\,Ez\,\epsilon0)\,\mu0)\,uy^{(0,0,0,1)}[t,x,y,z]+Bx^2\,uz^{(0,0,0,1)}[t,x,y,z]+By^2\,uz^{(0,0,0,1)}[t,x,y,z]-Bz^2\,uz^{(0,0,0,1)}[t,x,y,z]+2\,szz\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ex^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ey^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-Ez^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-2\,Bx\,By\,ux^{(0,0,1,0)}[t,x,y,z]+sxy\,\mu0\,ux^{(0,0,1,0)}[t,x,y,z]+syx\,\mu0\,ux^{(0,0,1,0)}[t,x,y,z]-2\,Ex\,Ey\,\epsilon0\,\mu0\,ux^{(0,0,1,0)}[t,x,y,z]+Bx^2\,uy^{(0,0,1,0)}[t,x,y,z]-By^2\,uy^{(0,0,1,0)}[t,x,y,z]+Bz^2\,uy^{(0,0,1,0)}[t,x,y,z]+\ldots}{} \\ -n\,\rho\,(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z])+\frac{\sqrt{\epsilon0}\,((-By\,Ex+Bx\,Ey)\,W^{(0,0,0,1)}[t,x,y,z]+(Bz\,Ex-Bx\,Ez)\,W^{(0,0,1,0)}[t,x,y,z]+(-Bz\,Ey+By\,Ez)\,W^{(0,1,0,0)}[t,x,y,z])}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ n\,\rho\,(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z])+O[\frac{1}{c}]^2 \\ n\,\rho\,(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z])+O[\frac{1}{c}]^2 \\ -n\,\rho\,(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z])+\frac{2\,(-Bz\,Ey+By\,Ez)\,\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ -n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \end{pmatrix}$$

```
(* Flux of energy is given by: *)
```

In[•]:=
```
shows[assut, 1][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.tt /. {Ax → 0, Ay → 0, Vz → 0, jz → 0} /. j2v]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\,(ux^2+uy^2-2\,W+2\,\epsilon)\,\rho+O[\frac{1}{c}]^2 & n\,ux\,\rho+O[\frac{1}{c}]^2 & n\,uy\,\rho+O[\frac{1}{c}]^2 & O[\frac{1}{c}]^2 \\ -Az\,(qz+sxz\,ux+syz\,uy)+O[\frac{1}{c}]^2 & Az\,sxz+O[\frac{1}{c}]^2 & Az\,syz+O[\frac{1}{c}]^2 & Az\,szz+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
shows[assut, 1][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → 0, Vx → 0} /. j2v]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\,(uy^2+uz^2-2\,W+2\,\epsilon)\,\rho+O[\frac{1}{c}]^2 & O[\frac{1}{c}]^2 & n\,uy\,\rho+O[\frac{1}{c}]^2 & n\,uz\,\rho+O[\frac{1}{c}]^2 \\ (-qx-sxy\,uy-sxz\,uz)+O[\frac{1}{c}]^2 & sxx+O[\frac{1}{c}]^2 & sxy+O[\frac{1}{c}]^2 & sxz+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\,(ux^2+uy^2+uz^2-2\,W+2\,\epsilon)\,\rho+O[\frac{1}{c}]^2 & n\,ux\,\rho+\frac{qx+sxx\,ux+sxy\,uy+sxz\,uz+\frac{1}{2}\,n\,(-8\,Wx+ux\,(ux^2+uy^2+uz^2+6\,W+2\,\epsilon))\,\rho}{c^2}+O[\frac{1}{c}]^3 & n\,uy\,\rho+\frac{qy+sxy\,ux+syy\,uy+syz\,uz+\frac{1}{2}\,n\,(-8\,Wy+uy\,(ux^2+uy^2+uz^2+6\,W+2\,\epsilon))\,\rho}{c^2}+O[\frac{1}{c}]^3 & n\,uz\,\rho+\frac{qz+sxz\,ux+syz\,uy+szz\,uz+\frac{1}{2}\,n\,(-8\,Wz+uz\,(ux^2+uy^2+uz^2+6\,W+2\,\epsilon))\,\rho}{c^2}\ldots \\ n\,(-ux+Vx)\,\rho\,c^2+(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,(ux-Vx)\,(ux^2+uy^2+uz^2-2\,W+2\,\epsilon)\,\rho)+O[\frac{1}{c}]^2 & (sxx+n\,ux\,(ux-Vx)\,\rho)+\frac{qx\,(2\,ux-Vx)-(sxx\,ux+sxy\,uy+sxz\,uz)\,Vx+\frac{1}{2}\,n\,(ux-Vx)\,(ux^3-8\,Wx+ux\,(uy^2+uz^2+6\,W+2\,\epsilon))\,\rho}{c^2}+O[\frac{1}{c}]^3 & (sxy+n\,uy\,(ux-Vx)\,\rho)+\frac{qx\,uy+qy\,(ux-Vx)-(sxy\,ux+syy\,uy+syz\,uz)\,Vx+\frac{1}{2}\,n\,(ux-Vx)\,(-8\,Wy+uy\,(ux^2+uy^2+uz^2+6\,W+2\,\epsilon))\,\rho}{c^2}+O[\frac{1}{c}]^3 & (sxz+n\,uz\,(ux-Vx)\,\rho)+\frac{qx\,uz+qz\,(ux-Vx)-(sxz\,ux+syz\,uy+szz\,uz)\ldots}{} \end{pmatrix}$$

In[•]:=
```
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v /. {sxx → SXX - n * ux * (ux - Vx) * ρ, sxy → SXY - n * uy * (ux - Vx) * ρ, sxz → SXZ - n * uz * (ux - Vx) * ρ}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\,(ux^2+uy^2+uz^2-2\,W+2\,\epsilon)\,\rho+O[\frac{1}{c}]^2 & n\,ux\,\rho+\frac{qx+SXX\,ux+SXY\,uy+SXZ\,uz-\frac{1}{2}\,n\,(ux^3-2\,ux^2\,Vx-2\,(uy^2+uz^2)\,Vx+8\,Wx+ux\,(uy^2+uz^2-6\,W-2\,\epsilon))\,\rho}{c^2}+O[\frac{1}{c}]^3 & n\,uy\,\rho+\frac{qy+SXY\,ux+syy\,uy+syz\,uz+\frac{1}{2}\,n\,(-8\,Wy+uy\,(-ux^2+uy^2+uz^2+2\,ux\,Vx+6\,W+2\,\epsilon))\,\rho}{c^2}+O[\frac{1}{c}]^3 & n\,uz\,\rho+\frac{qz+SXZ\,ux+syz\,uy+szz\,uz+\frac{1}{2}\,n\,(-8\,Wz+uz\,(-ux^2+uy^2+uz^2+2\,ux\,Vx+6\,W+2\,\epsilon))\,\rho}{c^2}+O[\frac{1}{c}]\ldots \\ n\,(-ux+Vx)\,\rho\,c^2+(-qx-SXX\,ux-SXY\,uy-SXZ\,uz+\frac{1}{2}\,n\,(ux-Vx)\,(ux^2+uy^2+uz^2+2\,W-2\,\epsilon)\,\rho)+O[\frac{1}{c}]^2 & SXX+\frac{qx\,(2\,ux-Vx)-(SXX\,ux+SXY\,uy+SXZ\,uz)\,Vx+\frac{1}{2}\,n\,(ux-Vx)\,(ux^3+2\,ux^2\,Vx+2\,(uy^2+uz^2)\,Vx-8\,Wx+ux\,(uy^2+uz^2+6\,W+2\,\epsilon))\,\rho}{c^2}+O[\frac{1}{c}]^3 & SXY+\frac{qx\,uy+qy\,(ux-Vx)-(SXY\,ux+syy\,uy+syz\,uz)\,Vx+\frac{1}{2}\,n\,(ux-Vx)\,(-8\,Wy+uy\,(-ux^2+uy^2+uz^2+2\,ux\,Vx+6\,W+2\,\epsilon))\,\rho}{c^2}+O[\frac{1}{c}]^3 & SXZ+\frac{qx\,uz+qz\,(ux-Vx)-(SXZ\,ux+syz\,uy+szz\,uz)\,Vx+\frac{1}{2}\,n\,(ux-Vx)\,(-8\,Wz+uz\,(-ux^2+uy^2+uz^2+2\,ux\ldots}{c^2} \end{pmatrix}$$

In[•]:=
```
(* matter flux in same direction as imaginary moving surface, different velocity *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jy → 0, jz → 0} /. j2v]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\,(ux^2-2\,W+2\,\epsilon)\,\rho+O[\frac{1}{c}]^2 & n\,ux\,\rho+\frac{qx+sxx\,ux+\frac{1}{2}\,n\,(ux^3+6\,ux\,W-8\,Wx+2\,ux\,\epsilon)\,\rho}{c^2}+O[\frac{1}{c}]^3 & \frac{qy+sxy\,ux-4\,n\,Wy\,\rho}{c^2}+O[\frac{1}{c}]^3 & \frac{qz+sxz\,ux-4\,n\,Wz\,\rho}{c^2}+O[\frac{1}{c}]^3 \\ n\,(-ux+Vx)\,\rho\,c^2+(-qx-sxx\,ux-\frac{1}{2}\,n\,(ux-Vx)\,(ux^2-2\,W+2\,\epsilon)\,\rho)+O[\frac{1}{c}]^2 & (sxx+n\,ux\,(ux-Vx)\,\rho)+\frac{2\,qx\,ux-(qx+sxx\,ux)\,Vx+\frac{1}{2}\,n\,(ux-Vx)\,(ux^3-8\,Wx+2\,ux\,(3\,W+\epsilon))\,\rho}{c^2}+O[\frac{1}{c}]^3 & sxy+\frac{qy\,(ux-Vx)-sxy\,ux\,Vx+4\,n\,(-ux+Vx)\,Wy\,\rho}{c^2}+O[\frac{1}{c}]^3 & sxz+\frac{qz\,(ux-Vx)-sxz\,ux\,Vx+4\,n\,(-ux+Vx)\,Wz\,\rho}{c^2}+O[\frac{1}{c}]^3 \end{pmatrix}$$

In[•]:=
```
(* imaginary moving surface, no matter flux through it *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v /. {ux → Vx}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\,(uy^2+uz^2+Vx^2-2\,W+2\,\epsilon)\,\rho+O[\frac{1}{c}]^2 & n\,Vx\,\rho+\frac{qx+sxy\,uy+sxz\,uz+sxx\,Vx+\frac{1}{2}\,n\,(-8\,Wx+Vx\,(uy^2+uz^2+Vx^2+6\,W+2\,\epsilon))\,\rho}{c^2}+O[\frac{1}{c}]^3 & n\,uy\,\rho+\frac{qy+syy\,uy+syz\,uz+sxy\,Vx+\frac{1}{2}\,n\,(-8\,Wy+uy\,(uy^2+uz^2+Vx^2+6\,W+2\,\epsilon))\,\rho}{c^2}+O[\frac{1}{c}]^3 & n\,uz\,\rho+\frac{qz+syz\,uy+szz\,uz+sxz\,Vx+\frac{1}{2}\,n\,(-8\,Wz+uz\,(uy^2+uz^2+Vx^2+6\,W+2\,\epsilon))\,\rho}{c^2}+O[\frac{1}{c}]^3 \\ (-qx-sxy\,uy-sxz\,uz-sxx\,Vx)+O[\frac{1}{c}]^2 & sxx-\frac{Vx\,(-qx+sxy\,uy+sxz\,uz+sxx\,Vx)}{c^2}+O[\frac{1}{c}]^3 & sxy+\frac{qx\,uy-Vx\,(syy\,uy+syz\,uz+sxy\,Vx)}{c^2}+O[\frac{1}{c}]^3 & sxz+\frac{qx\,uz-Vx\,(syz\,uy+szz\,uz+sxz\,Vx)}{c^2}+O[\frac{1}{c}]^3 \end{pmatrix}$$

```
(* imaginary moving surface, no matter flux through it and no transversal matter motion *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jy → 0, jz → 0} /. j2v /. {ux → Vx}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2-\frac{1}{2}\,n\,(Vx^2-2\,W+2\,\epsilon)\,\rho+O[\frac{1}{c}]^2 & n\,Vx\,\rho+\frac{qx+sxx\,Vx+\frac{1}{2}\,n\,(Vx^3+6\,Vx\,W-8\,Wx+2\,Vx\,\epsilon)\,\rho}{c^2}+O[\frac{1}{c}]^3 & \frac{qy+sxy\,Vx-4\,n\,Wy\,\rho}{c^2}+O[\frac{1}{c}]^3 & \frac{qz+sxz\,Vx-4\,n\,Wz\,\rho}{c^2}+O[\frac{1}{c}]^3 \\ (-qx-sxx\,Vx)+O[\frac{1}{c}]^2 & sxx+\frac{Vx\,(qx-sxx\,Vx)}{c^2}+O[\frac{1}{c}]^3 & sxy-\frac{sxy\,Vx^2}{c^2}+O[\frac{1}{c}]^3 & sxz-\frac{sxz\,Vx^2}{c^2}+O[\frac{1}{c}]^3 \end{pmatrix}$$

In[•]:=
```
(* imaginary moving surface, matter at rest in coordinates *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → 0, jy → 0, jz → 0}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+n\,(W-\epsilon)\,\rho+O[\frac{1}{c}]^2 & \frac{qx-4\,n\,Wx\,\rho}{c^2}+O[\frac{1}{c}]^3 & \frac{qy-4\,n\,Wy\,\rho}{c^2}+O[\frac{1}{c}]^3 & \frac{qz-4\,n\,Wz\,\rho}{c^2}+O[\frac{1}{c}]^3 \\ n\,Vx\,\rho\,c^2+(-qx+n\,Vx\,(-W+\epsilon)\,\rho)+O[\frac{1}{c}]^2 & sxx+\frac{-qx\,Vx+4\,n\,Vx\,Wx\,\rho}{c^2}+O[\frac{1}{c}]^3 & sxy+\frac{-qy\,Vx+4\,n\,Vx\,Wy\,\rho}{c^2}+O[\frac{1}{c}]^3 & sxz+\frac{-qz\,Vx+4\,n\,Vx\,Wz\,\rho}{c^2}+O[\frac{1}{c}]^3 \end{pmatrix}$$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → n * Vx, jy → 0, jz → 0}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+(-\frac{1}{2}\,n\,Vx^2\,\rho+n\,W\,\rho-n\,\epsilon\,\rho)+O[\frac{1}{c}]^2 & n\,Vx\,\rho+\frac{qx+sxx\,Vx+\frac{1}{2}\,n\,Vx^3\,\rho+3\,n\,Vx\,W\,\rho-4\,n\,Wx\,\rho+n\,Vx\,\epsilon\,\rho}{c^2}+O[\frac{1}{c}]^4 & \frac{qy+sxy\,Vx-4\,n\,Wy\,\rho}{c^2}+O[\frac{1}{c}]^4 & \frac{qz+sxz\,Vx-4\,n\,Wz\,\rho}{c^2}+O[\frac{1}{c}]^4 \\ (-qx-sxx\,Vx)+O[\frac{1}{c}]^2 & sxx+\frac{qx\,Vx-sxx\,Vx^2}{c^2}+O[\frac{1}{c}]^4 & sxy-\frac{sxy\,Vx^2}{c^2}+O[\frac{1}{c}]^4 & sxz-\frac{sxz\,Vx^2}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[•]:=
```
(* COORDINATE ENERGY *)
```

In[•]:=
```
(* energy 3-form when projected along coord. axes *)
showf[assut][Expand //@ FS@PowerExpand[tt.{1, 0, 0, 0}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+(-\frac{jx^2\,\rho}{2\,n}-\frac{jy^2\,\rho}{2\,n}-\frac{jz^2\,\rho}{2\,n}+n\,W\,\rho-n\,\epsilon\,\rho)+O[\frac{1}{c}]^2 \\ -jx\,\rho\,c^2+(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\,\rho}{2\,n^2}-\frac{jx\,jy^2\,\rho}{2\,n^2}-\frac{jx\,jz^2\,\rho}{2\,n^2}+jx\,W\,\rho-jx\,\epsilon\,\rho)+O[\frac{1}{c}]^2 \\ -jy\,\rho\,c^2+(-qy-\frac{jx\,sxy}{n}-\frac{jy\,syy}{n}-\frac{jz\,syz}{n}-\frac{jx^2\,jy\,\rho}{2\,n^2}-\frac{jy^3\,\rho}{2\,n^2}-\frac{jy\,jz^2\,\rho}{2\,n^2}+jy\,W\,\rho-jy\,\epsilon\,\rho)+O[\frac{1}{c}]^2 \\ -jz\,\rho\,c^2+(-qz-\frac{jx\,sxz}{n}-\frac{jy\,syz}{n}-\frac{jz\,szz}{n}-\frac{jx^2\,jz\,\rho}{2\,n^2}-\frac{jy^2\,jz\,\rho}{2\,n^2}-\frac{jz^3\,\rho}{2\,n^2}+jz\,W\,\rho-jz\,\epsilon\,\rho)+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.{1, 0, 0, 0} /. j2v]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+(-\frac{1}{2}\,n\,ux^2\,\rho-\frac{1}{2}\,n\,uy^2\,\rho-\frac{1}{2}\,n\,uz^2\,\rho+n\,W\,\rho-n\,\epsilon\,\rho)+O[\frac{1}{c}]^2 \\ -n\,ux\,\rho\,c^2+(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^3\,\rho-\frac{1}{2}\,n\,ux\,uy^2\,\rho-\frac{1}{2}\,n\,ux\,uz^2\,\rho+n\,ux\,W\,\rho-n\,ux\,\epsilon\,\rho)+O[\frac{1}{c}]^2 \\ -n\,uy\,\rho\,c^2+(-qy-sxy\,ux-syy\,uy-syz\,uz-\frac{1}{2}\,n\,ux^2\,uy\,\rho-\frac{1}{2}\,n\,uy^3\,\rho-\frac{1}{2}\,n\,uy\,uz^2\,\rho+n\,uy\,W\,\rho-n\,uy\,\epsilon\,\rho)+O[\frac{1}{c}]^2 \\ -n\,uz\,\rho\,c^2+(-qz-sxz\,ux-syz\,uy-szz\,uz-\frac{1}{2}\,n\,ux^2\,uz\,\rho-\frac{1}{2}\,n\,uy^2\,uz\,\rho-\frac{1}{2}\,n\,uz^3\,\rho+n\,uz\,W\,\rho-n\,uz\,\epsilon\,\rho)+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* flux of coord. energy across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt)]]
```

Out[•]//MatrixForm=

$$(-jx\,\rho+n\,Vx\,\rho)\,c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\,\rho}{2\,n^2}-\frac{jx\,jy^2\,\rho}{2\,n^2}-\frac{jx\,jz^2\,\rho}{2\,n^2}+\frac{jx^2\,Vx\,\rho}{2\,n}+\frac{jy^2\,Vx\,\rho}{2\,n}+\frac{jz^2\,Vx\,\rho}{2\,n}+jx\,W\,\rho-n\,Vx\,W\,\rho-jx\,\epsilon\,\rho+n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr]]
```

Out[•]//MatrixForm=

$$-Lx\,n\,\rho\,c^2+\left(-qx-sxx\,ux-\frac{1}{2}\,Lx\,n\,ux^2\,\rho+Lx\,n\,W\,\rho-Lx\,n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. {jx → 0, jy → 0, jz → 0}]]
```

Out[•]//MatrixForm=

$$n\,Vx\,\rho\,c^2+(-qx-n\,Vx\,W\,\rho+n\,Vx\,\epsilon\,\rho)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of matter flux*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf]]
```

Out[•]//MatrixForm=

$$-JX\,\rho\,c^2+\left(-qx-\frac{JX\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-sxx\,Vx-\frac{JX^3\,\rho}{2\,n^2}-\frac{JX\,jy^2\,\rho}{2\,n^2}-\frac{JX\,jz^2\,\rho}{2\,n^2}-\frac{JX^2\,Vx\,\rho}{n}-\frac{1}{2}\,JX\,Vx^2\,\rho+JX\,W\,\rho-JX\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf]]
```

Out[•]//MatrixForm=

$$-JX\,\rho\,c^2+\left(-qx-\frac{JX\,sxx}{n}-sxx\,Vx-\frac{JX^3\,\rho}{2\,n^2}-\frac{JX^2\,Vx\,\rho}{n}-\frac{1}{2}\,JX\,Vx^2\,\rho+JX\,W\,\rho-JX\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of matter flux & matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf /. j2v]]
```

Out[•]//MatrixForm=

$$-JX\,\rho\,c^2+\left(-qx-\frac{JX\,sxx}{n}-sxy\,uy-sxz\,uz-sxx\,Vx-\frac{JX^3\,\rho}{2\,n^2}-\frac{1}{2}\,JX\,uy^2\,\rho-\frac{1}{2}\,JX\,uz^2\,\rho-\frac{JX^2\,Vx\,\rho}{n}-\frac{1}{2}\,JX\,Vx^2\,\rho+JX\,W\,\rho-JX\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf /. {JX → 0, jy → 0, jz → 0}]]
```

Out[•]//MatrixForm=

$$(-qx-sxx\,Vx)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. relv]]
```
Out[∘]//MatrixForm=
$$-n\,Vx\,\rho\,c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^2\,Vx\,\rho}{2\,n}-\frac{jy^2\,Vx\,\rho}{2\,n}-\frac{jz^2\,Vx\,\rho}{2\,n}+n\,Vx\,W\,\rho-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr]]
```
Out[∘]//MatrixForm=
$$-n\,Vx\,\rho\,c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^2\,Vx\,\rho-\frac{1}{2}\,n\,uy^2\,Vx\,\rho-\frac{1}{2}\,n\,uz^2\,Vx\,\rho+n\,Vx\,W\,\rho-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[∘]//MatrixForm=
$$\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* supply term for coord. energy *)
TTx = tW[Normal[tt]]; shows[assut, 2][Expand //@ FS@PowerExpand[Tr[1 / 2 * Normal@(Inverse[gg].T[Dcoords[[1, ;; , ;;]]].gg + Dcoords[[1, ;; , ;;]]).TTx]]]
```
Out[∘]//MatrixForm=
$$n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]+\frac{\left(3\left(jx^2+jy^2+jz^2\right)\rho+2\,n\left(sxx+syy+szz+n\,\epsilon\,\rho\right)-2\,n^2\,\rho\,W[t,x,y,z]\right)W^{(1,0,0,0)}[t,x,y,z]}{2\,n\,c^2}+O\left[\frac{1}{c}\right]^3$$

In[∘]:=
```
(* INTERNAL ENERGY *)
```

In[∘]:=
```
(* energy 3-form when projected along matter 4-velocity, "internal energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.uu]]
```
Out[∘]//MatrixForm=
$$\begin{pmatrix}-n\,\rho\,c^2-n\,\epsilon\,\rho+O\left[\frac{1}{c}\right]^2\\ -jx\,\rho\,c^2+\left(-qx-jx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ -jy\,\rho\,c^2+\left(-qy-jy\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ -jz\,\rho\,c^2+\left(-qz-jz\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.uu /. j2vr]]
```
Out[∘]//MatrixForm=
$$\begin{pmatrix}-n\,\rho\,c^2-n\,\epsilon\,\rho+O\left[\frac{1}{c}\right]^2\\ -n\,ux\,\rho\,c^2+\left(-qx-n\,ux\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ -n\,uy\,\rho\,c^2+\left(-qy-n\,uy\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ -n\,uz\,\rho\,c^2+\left(-qz-n\,uz\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[∘]:=
```
(* flux of internal energy across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt)]]
```
Out[∘]//MatrixForm=
$$\left(-jx\,\rho+n\,vx\,\rho\right)c^2+\left(-qx-jx\,\epsilon\,\rho+n\,vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. relv]]
```
Out[∘]//MatrixForm=
$$-n\,Vx\,\rho\,c^2+\left(-qx-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. j2vr]]
```
Out[∘]//MatrixForm=
$$-n\,Vx\,\rho\,c^2+\left(-qx-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[∘]//MatrixForm=
$$-qx+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* supply term for internal energy (should be reversed in sign; remember that stress is compressive, not tensile) *)
TTx = tW[tjv@tt]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Duv].gg + Duv).TTx]]]
```
Out[∘]//MatrixForm=
$$\left(sxz\,vx^{(0,0,0,1)}[t,x,y,z]+syz\,vy^{(0,0,0,1)}[t,x,y,z]+szz\,vz^{(0,0,0,1)}[t,x,y,z]+sxy\,vx^{(0,0,1,0)}[t,x,y,z]+syy\,vy^{(0,0,1,0)}[t,x,y,z]+syz\,vz^{(0,0,1,0)}[t,x,y,z]+sxx\,vx^{(0,1,0,0)}[t,x,y,z]+sxy\,vy^{(0,1,0,0)}[t,x,y,z]+sxz\,vz^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* difference between "coord. energy" and "internal energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - uu)]]
```
Out[∘]//MatrixForm=
$$\begin{pmatrix}\left(-\frac{jx^2\rho}{2n}-\frac{jy^2\rho}{2n}-\frac{jz^2\rho}{2n}+n\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ \left(-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\rho}{2n^2}-\frac{jx\,jy^2\rho}{2n^2}-\frac{jx\,jz^2\rho}{2n^2}+jx\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ \left(-\frac{jx\,sxy}{n}-\frac{jy\,syy}{n}-\frac{jz\,syz}{n}-\frac{jx^2\,jy\,\rho}{2n^2}-\frac{jy^3\rho}{2n^2}-\frac{jy\,jz^2\rho}{2n^2}+jy\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ \left(-\frac{jx\,sxz}{n}-\frac{jy\,syz}{n}-\frac{jz\,szz}{n}-\frac{jx^2\,jz\,\rho}{2n^2}-\frac{jy^2\,jz\,\rho}{2n^2}-\frac{jz^3\rho}{2n^2}+jz\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - uu) /. j2vr]]
```
Out[∘]//MatrixForm=
$$\begin{pmatrix}\left(-\frac{1}{2}\,n\,ux^2\,\rho-\frac{1}{2}\,n\,uy^2\,\rho-\frac{1}{2}\,n\,uz^2\,\rho+n\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ \left(-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^3\,\rho-\frac{1}{2}\,n\,ux\,uy^2\,\rho-\frac{1}{2}\,n\,ux\,uz^2\,\rho+n\,ux\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ \left(-sxy\,ux-syy\,uy-syz\,uz-\frac{1}{2}\,n\,ux^2\,uy\,\rho-\frac{1}{2}\,n\,uy^3\,\rho-\frac{1}{2}\,n\,uy\,uz^2\,\rho+n\,uy\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ \left(-sxz\,ux-syz\,uy-szz\,uz-\frac{1}{2}\,n\,ux^2\,uz\,\rho-\frac{1}{2}\,n\,uy^2\,uz\,\rho-\frac{1}{2}\,n\,uz^3\,\rho+n\,uz\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[∘]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt)]]
```
Out[∘]//MatrixForm=
$$\left(-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\rho}{2n^2}-\frac{jx\,jy^2\rho}{2n^2}-\frac{jx\,jz^2\rho}{2n^2}+\frac{jx^2\,vx\,\rho}{2n}+\frac{jy^2\,vx\,\rho}{2n}+\frac{jz^2\,vx\,\rho}{2n}+jx\,W\,\rho-n\,vx\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. relv]]
```
Out[∘]//MatrixForm=
$$\left(-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^2\,Vx\,\rho}{2\,n}-\frac{jy^2\,Vx\,\rho}{2\,n}-\frac{jz^2\,Vx\,\rho}{2\,n}+n\,Vx\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. j2vr]]
```
Out[∘]//MatrixForm=
$$\left(-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^2\,Vx\,\rho-\frac{1}{2}\,n\,uy^2\,Vx\,\rho-\frac{1}{2}\,n\,uz^2\,Vx\,\rho+n\,Vx\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[∘]//MatrixForm=
$$\left(-sxx\,ux-sxy\,uy-sxz\,uz\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* PROPER-TIME COORD ENERGY*)
```

In[∘]:=
```
(* energy 3-form when projected along normalized coord-t
   note how the gravitational term is missing *)
showf[assut][Expand //@ FS@PowerExpand[tt.vtn]]
```
Out[∘]//MatrixForm=
$$\begin{pmatrix}-n\,\rho\,c^2+\left(-\frac{jx^2\rho}{2n}-\frac{jy^2\rho}{2n}-\frac{jz^2\rho}{2n}-n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ -jx\,\rho\,c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^3\rho}{2n^2}-\frac{jx\,jy^2\rho}{2n^2}-\frac{jx\,jz^2\rho}{2n^2}-jx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ -jy\,\rho\,c^2+\left(-qy-\frac{jx\,sxy}{n}-\frac{jy\,syy}{n}-\frac{jz\,syz}{n}-\frac{jx^2\,jy\,\rho}{2n^2}-\frac{jy^3\rho}{2n^2}-\frac{jy\,jz^2\rho}{2n^2}-jy\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ -jz\,\rho\,c^2+\left(-qz-\frac{jx\,sxz}{n}-\frac{jy\,syz}{n}-\frac{jz\,szz}{n}-\frac{jx^2\,jz\,\rho}{2n^2}-\frac{jy^2\,jz\,\rho}{2n^2}-\frac{jz^3\rho}{2n^2}-jz\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.vtn /. j2vr]]
```
Out[∘]//MatrixForm=
$$\begin{pmatrix}-n\,\rho\,c^2+\left(-\frac{1}{2}\,n\,ux^2\,\rho-\frac{1}{2}\,n\,uy^2\,\rho-\frac{1}{2}\,n\,uz^2\,\rho-n\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ -n\,ux\,\rho\,c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^3\,\rho-\frac{1}{2}\,n\,ux\,uy^2\,\rho-\frac{1}{2}\,n\,ux\,uz^2\,\rho-n\,ux\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ -n\,uy\,\rho\,c^2+\left(-qy-sxy\,ux-syy\,uy-syz\,uz-\frac{1}{2}\,n\,ux^2\,uy\,\rho-\frac{1}{2}\,n\,uy^3\,\rho-\frac{1}{2}\,n\,uy\,uz^2\,\rho-n\,uy\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\\ -n\,uz\,\rho\,c^2+\left(-qz-sxz\,ux-syz\,uy-szz\,uz-\frac{1}{2}\,n\,ux^2\,uz\,\rho-\frac{1}{2}\,n\,uy^2\,uz\,\rho-\frac{1}{2}\,n\,uz^3\,\rho-n\,uz\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2\end{pmatrix}$$

In[∘]:=
```
(* flux of normalized-coord-t energy across surface *)
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt)]]
```
Out[∘]//MatrixForm=
$$\left(-jx\,\rho+n\,vx\,\rho\right)c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jx^3\rho}{2\,n^2}+\frac{jx^2\,vx\,\rho}{2\,n}-jx\,\epsilon\,\rho+n\,vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(*in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. relv]]
```
Out[∘]//MatrixForm=
$$-n\,Vx\,\rho\,c^2+\left(-qx-\frac{jx\,sxx}{n}-\frac{jy\,sxy}{n}-\frac{jz\,sxz}{n}-\frac{jx^2\,Vx\,\rho}{2\,n}-\frac{jy^2\,Vx\,\rho}{2\,n}-\frac{jz^2\,Vx\,\rho}{2\,n}-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=
$$-n\,Vx\,\rho\,c^2+\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz-\frac{1}{2}\,n\,ux^2\,Vx\,\rho-\frac{1}{2}\,n\,uy^2\,Vx\,\rho-\frac{1}{2}\,n\,uz^2\,Vx\,\rho-n\,Vx\,\epsilon\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=
$$\left(-qx-sxx\,ux-sxy\,uy-sxz\,uz\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* supply term for normalized-coord-t energy
   we obtain the "power generated by the gravity field" *)
TTx = tW[tjv@tt]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Dvtn].gg + Dvtn).TTx]]]
```
Out[•]//MatrixForm=
$$\left(-\rho\,n[t,x,y,z]\circ vz[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]-\rho\,n[t,x,y,z]\circ vy[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]-\rho\,n[t,x,y,z]\circ vx[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* difference between "coord. energy" and "proper-time coord. energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - vtn)]]
```
Out[•]//MatrixForm=
$$\begin{pmatrix} n\,W\,\rho+O[\frac{1}{c}]^2\\ jx\,W\,\rho+O[\frac{1}{c}]^2\\ jy\,W\,\rho+O[\frac{1}{c}]^2\\ jz\,W\,\rho+O[\frac{1}{c}]^2\end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - vtn) /. j2vr]]
```
Out[•]//MatrixForm=
$$\begin{pmatrix} n\,W\,\rho+O[\frac{1}{c}]^2\\ n\,ux\,W\,\rho+O[\frac{1}{c}]^2\\ n\,uy\,W\,\rho+O[\frac{1}{c}]^2\\ n\,uz\,W\,\rho+O[\frac{1}{c}]^2\end{pmatrix}$$

In[•]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt)]]
```
Out[•]//MatrixForm=
$$\left(jx\,W\,\rho-n\,vx\,W\,\rho\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. relv]]
```
Out[•]//MatrixForm=
$$n\,Vx\,W\,\rho+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=
$$n\,Vx\,W\,\rho+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=
$$O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* difference between "internal energy" and "proper-time coord. energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.(uu - vtn)]]
```
Out[•]//MatrixForm=
$$\begin{pmatrix} \left(\frac{jx^2\rho}{2n}+\frac{jy^2\rho}{2n}+\frac{jz^2\rho}{2n}\right)+O[\frac{1}{c}]^2\\ \left(\frac{jx\,sxx}{n}+\frac{jy\,sxy}{n}+\frac{jz\,sxz}{n}+\frac{jx^3\rho}{2n^2}+\frac{jx\,jy^2\rho}{2n^2}+\frac{jx\,jz^2\rho}{2n^2}\right)+O[\frac{1}{c}]^2\\ \left(\frac{jx\,sxy}{n}+\frac{jy\,syy}{n}+\frac{jz\,syz}{n}+\frac{jx^2\,jy\,\rho}{2n^2}+\frac{jy^3\rho}{2n^2}+\frac{jy\,jz^2\rho}{2n^2}\right)+O[\frac{1}{c}]^2\\ \left(\frac{jx\,sxz}{n}+\frac{jy\,syz}{n}+\frac{jz\,szz}{n}+\frac{jx^2\,jz\,\rho}{2n^2}+\frac{jy^2\,jz\,\rho}{2n^2}+\frac{jz^3\rho}{2n^2}\right)+O[\frac{1}{c}]^2\end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.(uu - vtn) /. j2vr]]
```
Out[•]//MatrixForm=
$$\begin{pmatrix} \left(\frac{1}{2}\,n\,ux^2\rho+\frac{1}{2}\,n\,uy^2\rho+\frac{1}{2}\,n\,uz^2\rho\right)+O[\frac{1}{c}]^2\\ \left(sxx\,ux+sxy\,uy+sxz\,uz+\frac{1}{2}\,n\,ux^3\rho+\frac{1}{2}\,n\,ux\,uy^2\rho+\frac{1}{2}\,n\,ux\,uz^2\rho\right)+O[\frac{1}{c}]^2\\ \left(sxy\,ux+syy\,uy+syz\,uz+\frac{1}{2}\,n\,ux^2\,uy\,\rho+\frac{1}{2}\,n\,uy^3\rho+\frac{1}{2}\,n\,uy\,uz^2\rho\right)+O[\frac{1}{c}]^2\\ \left(sxz\,ux+syz\,uy+szz\,uz+\frac{1}{2}\,n\,ux^2\,uz\,\rho+\frac{1}{2}\,n\,uy^2\,uz\,\rho+\frac{1}{2}\,n\,uz^3\rho\right)+O[\frac{1}{c}]^2\end{pmatrix}$$

In[•]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - vtn) / (A * Δt)]]
```
Out[•]//MatrixForm=
$$\left(\frac{jx\,sxx}{n}+\frac{jy\,sxy}{n}+\frac{jz\,sxz}{n}+\frac{jx^3\rho}{2n^2}+\frac{jx\,jy^2\rho}{2n^2}+\frac{jx\,jz^2\rho}{2n^2}-\frac{jx^2\,vx\,\rho}{2n}-\frac{jy^2\,vx\,\rho}{2n}-\frac{jz^2\,vx\,\rho}{2n}\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - vtn) / (A * Δt) /. relv]]
```
Out[•]//MatrixForm=
$$\left(\frac{jx\,sxx}{n}+\frac{jy\,sxy}{n}+\frac{jz\,sxz}{n}+\frac{jx^2\,Vx\,\rho}{2n}+\frac{jy^2\,Vx\,\rho}{2n}+\frac{jz^2\,Vx\,\rho}{2n}\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - uu) / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=
$$O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - uu) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=
$$O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* Faraday tensor *)
repE = {Ex → Ex * c * Sqrt[μ0 * ϵ0], Ey → Ey * c * Sqrt[μ0 * ϵ0], Ez → Ez * c * Sqrt[μ0 * ϵ0]};
fftemp = {{0, -Ex, -Ey, -Ez}, {0, 0, Bz, -By}, {0, 0, 0, Bx}, {0, 0, 0, 0}} /. repE;
showf[assut][ffdd = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```
Out[•]//MatrixForm=
$$\begin{pmatrix} 0 & -c\,Ex\sqrt{\epsilon0}\sqrt{\mu0} & -c\,Ey\sqrt{\epsilon0}\sqrt{\mu0} & -c\,Ez\sqrt{\epsilon0}\sqrt{\mu0}\\ c\,Ex\sqrt{\epsilon0}\sqrt{\mu0} & 0 & Bz & -By\\ c\,Ey\sqrt{\epsilon0}\sqrt{\mu0} & -Bz & 0 & Bx\\ c\,Ez\sqrt{\epsilon0}\sqrt{\mu0} & By & -Bx & 0\end{pmatrix}$$

In[•]:=
```
showf[assut][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
  ]]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix}
\left(-\frac{Ex^2\epsilon0}{2}-\frac{Ey^2\epsilon0}{2}-\frac{Ez^2\epsilon0}{2}-\frac{Bx^2}{2\mu0}-\frac{By^2}{2\mu0}-\frac{Bz^2}{2\mu0}\right)+\frac{-Ex^2W\epsilon0-Ey^2W\epsilon0-Ez^2W\epsilon0+\frac{Bx^2W}{\mu0}+\frac{By^2W}{\mu0}+\frac{Bz^2W}{\mu0}}{c^2}+O[\frac{1}{c}]^4\\
\left(-\frac{Bz\,Ey\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{By\,Ez\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\frac{-\frac{2Bz\,Ey\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{2By\,Ez\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{-4Ey^2Wx\,\epsilon0-4Ez^2Wx\,\epsilon0+4Ex\,Ey\,Wy\,\epsilon0+4Ex\,Ez\,Wz\,\epsilon0}{c^2}+O[\frac{1}{c}]^3\\
\left(\frac{Bz\,Ex\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{Bx\,Ez\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\frac{\frac{2Bz\,Ex\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{2Bx\,Ez\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{4Ex\,Ey\,Wx\,\epsilon0-4Ex^2Wy\,\epsilon0-4Ez^2Wy\,\epsilon0+4Ey\,Ez\,Wz\,\epsilon0}{c^2}+O[\frac{1}{c}]^3\\
\left(-\frac{By\,Ex\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{Bx\,Ey\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\frac{-\frac{2By\,Ex\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{2Bx\,Ey\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{4Ex\,Ez\,Wx\,\epsilon0+4Ey\,Ez\,Wy\,\epsilon0-4Ex^2Wz\,\epsilon0-4Ey^2Wz\,\epsilon0}{c^2}+O[\frac{1}{c}]^3
\end{pmatrix}$$

$$\begin{pmatrix}
\frac{\frac{Bz\,Ey\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{By\,Ez\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{\frac{2Bz\,Ey\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{2By\,Ez\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c^3}+\frac{-\frac{4By^2Wx}{\mu0}-\frac{4Bz^2Wx}{\mu0}+\frac{4Bx\,By\,Wy}{\mu0}+\frac{4Bx\,Bz\,Wz}{\mu0}}{c^4}+O[\frac{1}{c}]^5\\
\left(-\frac{Ex^2\epsilon0}{2}+\frac{Ey^2\epsilon0}{2}+\frac{Ez^2\epsilon0}{2}-\frac{Bx^2}{2\mu0}+\frac{By^2}{2\mu0}+\frac{Bz^2}{2\mu0}\right)+\frac{-Ex^2W\epsilon0+Ey^2W\epsilon0+Ez^2W\epsilon0+\frac{Bx^2W}{\mu0}-\frac{By^2W}{\mu0}-\frac{Bz^2W}{\mu0}}{c^2}+\frac{-\frac{4Bz\,Ex\,Wy\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{4Bx\,Ez\,Wy\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{4By\,Ex\,Wz\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{4Bx\,Ey\,Wz\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c^3}+O[\frac{1}{c}]^4\\
\left(-Ex\,Ey\,\epsilon0-\frac{Bx\,By}{\mu0}\right)+\frac{-2Ex\,Ey\,W\epsilon0+\frac{2Bx\,By\,W}{\mu0}}{c^2}+\frac{\frac{4Bz\,Ex\,Wx\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{4By\,Ez\,Wy\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{4Bx\,Ez\,Wz\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{4By\,Ey\,Wz\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c^3}+O[\frac{1}{c}]^4\\
\left(-Ex\,Ez\,\epsilon0-\frac{Bx\,Bz}{\mu0}\right)+\frac{-2Ex\,Ez\,W\epsilon0+\frac{2Bx\,Bz\,W}{\mu0}}{c^2}+\frac{-\frac{4By\,Ex\,Wx\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{4Bx\,Ex\,Wy\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{4Bz\,Ez\,Wy\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{4Bz\,Ey\,Wz\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c^3}+O[\frac{1}{c}]^4
\end{pmatrix}$$

$$\begin{pmatrix}
\frac{-\frac{Bz\,Ex\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{Bx\,Ez\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{-\frac{2Bz\,Ex\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{2Bx\,Ez\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c^3}+\frac{\frac{4Bx\,By\,Wx}{\mu0}-\frac{4Bx^2Wy}{\mu0}-\frac{4Bz^2Wy}{\mu0}+\frac{4By\,Bz\,Wz}{\mu0}}{c^4}+O[\frac{1}{c}]^5\\
\left(-Ex\,Ey\,\epsilon0-\frac{Bx\,By}{\mu0}\right)+\frac{-2Ex\,Ey\,W\epsilon0+\frac{2Bx\,By\,W}{\mu0}}{c^2}+\frac{\frac{4Bx\,Ez\,Wx\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{4Bz\,Ey\,Wy\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{4Bx\,Ex\,Wz\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{4By\,Ey\,Wz\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c^3}+O[\frac{1}{c}]^4\\
\left(\frac{Ex^2\epsilon0}{2}-\frac{Ey^2\epsilon0}{2}+\frac{Ez^2\epsilon0}{2}+\frac{Bx^2}{2\mu0}-\frac{By^2}{2\mu0}+\frac{Bz^2}{2\mu0}\right)+\frac{Ex^2W\epsilon0-Ey^2W\epsilon0+Ez^2W\epsilon0-\frac{Bx^2W}{\mu0}+\frac{By^2W}{\mu0}-\frac{Bz^2W}{\mu0}}{c^2}+\frac{\frac{4Bz\,Ey\,Wx\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{4By\,Ez\,Wx\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{4By\,Ex\,Wz\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{4Bx\,Ey\,Wz\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c^3}+O[\frac{1}{c}]^4\\
\left(-Ey\,Ez\,\epsilon0-\frac{By\,Bz}{\mu0}\right)+\frac{-2Ey\,Ez\,W\epsilon0+\frac{2By\,Bz\,W}{\mu0}}{c^2}+\frac{-\frac{4By\,Ey\,Wx\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{4Bz\,Ez\,Wx\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{4Bx\,Ey\,Wy\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{4Bz\,Ex\,Wz\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c^3}+O[\frac{1}{c}]^4
\end{pmatrix}$$

$$\begin{pmatrix}
\frac{\frac{By\,Ex\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{Bx\,Ey\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\ldots\\
\left(-Ex\,Ez\,\epsilon0-\frac{Bx\ldots}{\mu0}\right.\ldots\\
\left(-Ey\,Ez\,\epsilon0-\frac{By\ldots}{\mu0}\right.\ldots\\
\left(\frac{Ex^2\epsilon0}{2}+\frac{Ey^2\epsilon0}{2}-\ldots\right.
\end{pmatrix}$$

In[•]:=
```
(* COORDINATE EM ENERGY *)
```

In[•]:=
```
(* energy 3-form when projected along coord. axes *)
showf[assut][Expand //@ FS@PowerExpand[tte.{1, 0, 0, 0}]]
```
Out[•]//MatrixForm=
$$\begin{pmatrix}
\left(-\frac{Ex^2\epsilon0}{2}-\frac{Ey^2\epsilon0}{2}-\frac{Ez^2\epsilon0}{2}-\frac{Bx^2}{2\mu0}-\frac{By^2}{2\mu0}-\frac{Bz^2}{2\mu0}\right)+\frac{-Ex^2W\epsilon0-Ey^2W\epsilon0-Ez^2W\epsilon0+\frac{Bx^2W}{\mu0}+\frac{By^2W}{\mu0}+\frac{Bz^2W}{\mu0}}{c^2}+O[\frac{1}{c}]^4\\
\left(-\frac{Bz\,Ey\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{By\,Ez\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\frac{-\frac{2Bz\,Ey\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{2By\,Ez\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{-4Ey^2Wx\,\epsilon0-4Ez^2Wx\,\epsilon0+4Ex\,Ey\,Wy\,\epsilon0+4Ex\,Ez\,Wz\,\epsilon0}{c^2}+O[\frac{1}{c}]^3\\
\left(\frac{Bz\,Ex\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{Bx\,Ez\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\frac{\frac{2Bz\,Ex\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{2Bx\,Ez\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{4Ex\,Ey\,Wx\,\epsilon0-4Ex^2Wy\,\epsilon0-4Ez^2Wy\,\epsilon0+4Ey\,Ez\,Wz\,\epsilon0}{c^2}+O[\frac{1}{c}]^3\\
\left(-\frac{By\,Ex\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{Bx\,Ey\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\frac{-\frac{2By\,Ex\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{2Bx\,Ey\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{4Ex\,Ez\,Wx\,\epsilon0+4Ey\,Ez\,Wy\,\epsilon0-4Ex^2Wz\,\epsilon0-4Ey^2Wz\,\epsilon0}{c^2}+O[\frac{1}{c}]^3
\end{pmatrix}$$

In[•]:=
```
(* flux of coord. energy across surface *)
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tte.{1, 0, 0, 0} / (A * Δt)]]
```
Out[•]//MatrixForm=
$$\left(-\frac{Bz\,Ey\sqrt{\epsilon0}}{\sqrt{\mu0}}+\frac{By\,Ez\sqrt{\epsilon0}}{\sqrt{\mu0}}\right)c+\left(\frac{1}{2}\,Ex^2\,vx\,\epsilon0+\frac{1}{2}\,Ey^2\,vx\,\epsilon0+\frac{1}{2}\,Ez^2\,vx\,\epsilon0+\frac{Bx^2\,vx}{2\mu0}+\frac{By^2\,vx}{2\mu0}+\frac{Bz^2\,vx}{2\mu0}\right)+\frac{\frac{2Bz\,Ey\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}-\frac{2By\,Ez\,W\sqrt{\epsilon0}}{\sqrt{\mu0}}}{c}+\frac{Ex^2\,vx\,W\epsilon0+Ey^2\,vx\,W\epsilon0+Ez^2\,vx\,W\epsilon0-4Ey^2Wx\,\epsilon0-4Ez^2Wx\,\epsilon0+4Ex\,Ey\,Wy\,\epsilon0+4Ex\,Ez\,Wz\,\epsilon0-\frac{Bx^2\,vx\,W}{\mu0}-\frac{By^2\,vx\,W}{\mu0}-\frac{Bz^2\,vx\,W}{\mu0}}{c^2}+O\left[\frac{1}{c}\right]^3$$

In[•]:=
```
(* supply term for coord. energy *)
TTx = tW[tjv@tte]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Dcoords[[1, ;; , ;;]]].gg + Dcoords[[1, ;; , ;;]]).TTx]]]
```

Out[•]//MatrixForm=

$$\frac{Ex^2\,\epsilon 0\,W^{(1,0,0,0)}[t,x,y,z] + Ey^2\,\epsilon 0\,W^{(1,0,0,0)}[t,x,y,z] + Ez^2\,\epsilon 0\,W^{(1,0,0,0)}[t,x,y,z] + \frac{Bx^2\,W^{(1,0,0,0)}[t,x,y,z]}{\mu 0} + \frac{By^2\,W^{(1,0,0,0)}[t,x,y,z]}{\mu 0} + \frac{Bz^2\,W^{(1,0,0,0)}[t,x,y,z]}{\mu 0}}{c^2} + O\left[\frac{1}{c}\right]^3$$