# Notes on general-relativistic continuum electromagneto-thermo-mechanics

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Personal notes on topics in general-relativistic continuum electromagneto-thermo-mechanics.

## 1 Bases of multivector spaces

Take an ordered coordinate system $(t, x, y, z)$, which also defines an orientation. The associated bases for 1-vector and 1-covector fields are $\partial_t, \dots$ and $\mathrm{d}t, \dots$. The bases for 2-vector and 2-covector fields are $\partial^2_{tx}, \dots$ and $\mathrm{d}^2 tx, \dots$; and so on.

A twisted or outer 3-covector such as $\widetilde{\mathrm{d}^3 xyz}$ has an associated outer direction, say positive $t$. This is indicated with a notation such as

$$\mathrm{d}^3_t \coloneqq \widetilde{\mathrm{d}^3 xyz}\ . \tag{1}$$

The determinant of the metric $\boldsymbol{g}$ is denoted shortly

$$\sqrt{g} \coloneqq \sqrt{-\det \boldsymbol{g}}\ . \tag{2}$$

The volume element induced by a metric $\boldsymbol{g}$ is denoted (note the boldface)

$$\boldsymbol{\sqrt{g}} \coloneqq \sqrt{g}\ \widetilde{\mathrm{d}^4 txyz} \tag{3}$$

and its corresponding inverse, a twisted 4-vector:

$$\boldsymbol{\sqrt{g^{-1}}} \coloneqq \frac{1}{\sqrt{g}}\ \widetilde{\partial^4_{txyz}} \tag{4}$$

Contraction with the volume element or its inverse establishes a correspondence between outer $n$-vectors and inner $(4-n)$-covectors. In particular

$$\begin{aligned}
\boldsymbol{\sqrt{g^{-1}}} \cdot \mathrm{d}^3_t &= \frac{1}{\sqrt{g}}\,\partial_t \qquad & \boldsymbol{\sqrt{g^{-1}}} \cdot \mathrm{d}^3_x &= \frac{1}{\sqrt{g}}\,\partial_x \\
\boldsymbol{\sqrt{g}} \cdot \partial_t &= -\sqrt{g}\,\mathrm{d}_t \qquad & \boldsymbol{\sqrt{g}} \cdot \partial_x &= -\sqrt{g}\,\mathrm{d}_x
\end{aligned} \tag{5}$$



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## 2 Four-stress

The stress-energy-momentum tensor, or simply 4-stress, is a covector-valued 3-covector field, the 3-covector being outer-oriented. It has the dimensions of an action, and can be decomposed as

$$\boldsymbol{T} = \epsilon\, \mathrm{d}^3_t \otimes \mathrm{d}t + q^i\, \mathrm{d}^3_i \otimes \mathrm{d}t + p_j\, \mathrm{d}^3_t \otimes \mathrm{d}x^j + \sigma^i_j\, \mathrm{d}^3_i \otimes \mathrm{d}x^j \tag{6}$$

the indices $i, j$ running over $x, y, z$, and where

$$\begin{aligned} \epsilon &= \text{volumic energy} & q^i &= \text{aeric energy flux} \\ p_i &= \text{volumic momentum} & \sigma^i_j &= \text{3-stress} \end{aligned} \tag{7}$$

measured in the coordinate system $txyz$. (The energy $\epsilon$ and momentum $p_i$ are per unit *coordinate* volume $xyz$, not per unit metric volume, or "per unit mass", or per unit mole.)