In[1]:=
```
<< "christoffelsymbols.m"
(* First index is upper index Table[FS[cc[[ii,;;,;;]]==T[cc[[ii,;;,;;]]]],{ii,1,4}] *)
```

In[2]:=
```
(*showf[assumptions_,simp_:FullSimplify]:=((Assuming[assumptions,Expand//@simp@PowerExpand[#]]//MF)&);*)
```

In[3]:=
```
(* Show matrix expressions and power expansions*)
(*
show[assumptions_,power_,simp_:FullSimplify]:=({Assuming[assumptions,simp@PowerExpand[#]]//MF,"\n",
    Assuming[assumptions,simp@PowerExpand[Series[#,{c,Infinity,power}]]]//MF}&);
shows[assumptions_,power_,simp_:FullSimplify]:=((
    Assuming[assumptions,simp@PowerExpand[Series[#,{c,Infinity,power}]]]//MF)&);
showf[assumptions_,simp_:FullSimplify]:=((Assuming[assumptions,Expand//@simp@PowerExpand[#]]//MF)&);
show1[assumptions_,simp_:Identity]:=((Assuming[assumptions,simp[#]]//MF)&);
show2[assumptions_,power_,simp_:Identity]:=((
    Assuming[assumptions,simp[Series[#,{c,Infinity,power}]]]//MF)&);
*)
```

In[4]:=
```
coords = {t, x, y, z}
```

Out[4]= $\{t, x, y, z\}$

In[5]:=
```
(* Flat metric *)
(gg0 = DiagonalMatrix[{-c^2, 1, 1, 1}]) // MF
```

Out[5]//MatrixForm=

$$\begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[6]:=
```
(*D[G*M/Sqrt[x^2+y^2+z^2],{{x,y,z}}] *)
```

In[7]:=
```
(* -W is the potential gravitational energy: W=GM/r
   that is, F_g(downwards)=grad W
*)
```

In[8]:=
```
(* Rotating metric from poissonetal *)
(gg = {{-c^2*(1-2*W/c^2 +2*W^2/c^4)+O[c, +Infinity]^3,
     -4*Wx/c^2+O[c, +Infinity]^4, -4*Wy/c^2+O[c, +Infinity]^4, -4*Wz/c^2+O[c, +Infinity]^4},
    {-4*Wx/c^2+O[c, +Infinity]^4,
     1+2*W/c^2+O[c, +Infinity]^4, 0, 0},
    {-4*Wy/c^2+O[c, +Infinity]^4, 0, 1+2*W/c^2+O[c, +Infinity]^4, 0},
    {-4*Wz/c^2+O[c, +Infinity]^4, 0, 0, 1+2*W/c^2+O[c, +Infinity]^4}}) // MF
(*(gg=DiagonalMatrix[{-c^2*(1+2*Φ[t,r]/c^2),1+2*Λ[t,r]/c^2,r^2,r^2*Sin[θ]^2}])//MF*)
```

Out[8]//MatrixForm=

$$\begin{pmatrix} -c^2+2W-\frac{2W^2}{c^2}+O\left[\frac{1}{c}\right]^3 & -\frac{4Wx}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{4Wy}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{4Wz}{c^2}+O\left[\frac{1}{c}\right]^4 \\ -\frac{4Wx}{c^2}+O\left[\frac{1}{c}\right]^4 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 0 \\ -\frac{4Wy}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 \\ -\frac{4Wz}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 0 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[9]:=
```
(ggnot = (gg /. {Wx → O[c, +Infinity]*c, Wy → O[c, +Infinity]*c, Wz → O[c, +Infinity]*c})) // MF
```

Out[9]//MatrixForm=

$$\begin{pmatrix} -c^2+2W-\frac{2W^2}{c^2}+O\left[\frac{1}{c}\right]^3 & O\left[\frac{1}{c}\right]^2 & O\left[\frac{1}{c}\right]^2 & O\left[\frac{1}{c}\right]^2 \\ O\left[\frac{1}{c}\right]^2 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & 0 \\ O\left[\frac{1}{c}\right]^2 & 0 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 \\ O\left[\frac{1}{c}\right]^2 & 0 & 0 & 1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[10]:=
```
Inverse[gg] // MF
```

Out[10]//MatrixForm=

$$\begin{pmatrix} -\left(\frac{1}{c}\right)^2-\frac{2W}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4Wz}{c^4}+O\left[\frac{1}{c}\right]^6 \\ -\frac{4Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{16(Wx\,Wy)}{c^6}+O\left[\frac{1}{c}\right]^8 & -\frac{16(Wx\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 \\ -\frac{4Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{16(Wx\,Wy)}{c^6}+O\left[\frac{1}{c}\right]^8 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{16(Wy\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 \\ -\frac{4Wz}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{16(Wx\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 & -\frac{16(Wy\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[11]:=
```
(*(gg=DiagonalMatrix@Diagonal[gg])//MF*)
```

In[12]:=
```
(* functions to temporarily remove coord-dep *)
tW[xx_] := (xx /. {W → W[t, x, y, z], Wx → Wx[t, x, y, z], Wy → Wy[t, x, y, z], Wz → Wz[t, x, y, z]});
itW[xx_] := (xx /. {W[t, x, y, z] → W, Wx[t, x, y, z] → Wx, Wy[t, x, y, z] → Wy, Wz[t, x, y, z] → Wz});
(ggt = tW[gg]) // MF;
```

In[15]:=
```
assut = {c > 0, Element[a, Reals], Element[v, Reals], Element[t, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals],
    Element[vx, Reals], Element[vy, Reals], Element[vz, Reals], Element[n, Reals], Element[r, Reals], Element[θ, Reals], Element[ϕ, Reals], Abs[v] < c, -c < vx < c, -c < ux < c, r > 0, 0 < θ < Pi,
    Normal@gg[[1, 1]] / c^2 < 0, Normal@gg[[2, 2]] > 0, Normal@gg[[3, 3]] > 0, Normal@gg[[4, 4]] > 0, n > 0, Element[jx, Reals], Element[jy, Reals], Element[jz, Reals], Element[sxx, Reals], Element[sxy, Reals], Element[sxz, Reals], Element[syy, Reals], Element[syz, Reals], Element[szz, Reals],
    -Normal@Det[gg] > 0, β > 0};

assutt = {c > 0, Element[a, Reals], Element[v, Reals], Element[t, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals],
    Element[v_x, Reals], Element[v_y, Reals], Element[v_z, Reals], Element[n, Reals], Element[r, Reals], Element[θ, Reals], Element[ϕ, Reals], Abs[v] < c, -c < v_x < c, -c < u_x < c, r > 0, 0 < θ < Pi,
    β > 0, Normal@ggt[[1, 1]] / c^2 < 0, Normal@ggt[[2, 2]] > 0, Normal@ggt[[3, 3]] > 0, Normal@ggt[[4, 4]] > 0,
    -Normal@Det[ggt] > 0};
```

In[17]:=
```
(igg = Assuming[assut, FullSimplify@PowerExpand[Inverse[gg]]]) // MF
```

Out[17]//MatrixForm=

$$\begin{pmatrix} -\left(\frac{1}{c}\right)^2-\frac{2W}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4Wz}{c^4}+O\left[\frac{1}{c}\right]^6 \\ -\frac{4Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{16(Wx\,Wy)}{c^6}+O\left[\frac{1}{c}\right]^8 & -\frac{16(Wx\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 \\ -\frac{4Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{16(Wx\,Wy)}{c^6}+O\left[\frac{1}{c}\right]^8 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{16(Wy\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 \\ -\frac{4Wz}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{16(Wx\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 & -\frac{16(Wy\,Wz)}{c^6}+O\left[\frac{1}{c}\right]^8 & 1-\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[18]:=
```
(*show[assut,2]@ChristoffelSymbol[gg,coords][[2]]*)
```

In[19]:=
```
(* volume element *)
(dg = Assuming[assut, FullSimplify@PowerExpand[Sqrt[-Det[gg]] / c]]) // MF
```

Out[19]//MatrixForm=

$$1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4$$

In[20]:=
```
(* Christoffel symbols  *)
cc = Assuming[assut, FullSimplify@PowerExpand[itW[ChristoffelSymbol[ggt, coords]]]];
```

In[21]:=
```
(* 3-vector of moving surface parallel to yz moving with velocity V   *)
surface = {-(Vx*Ax+Vy*Ay+Vz*Az), Ax, Ay, Az}*Δt;
```

In[22]:=

(* Matter current *)

In[23]:=
```
(* matter-current 3-covector *)
NJ = {n, jx, jy, jz};
```

In[24]:=
```
(* norm of matter 3-covector *)
Assuming[assut, FS[Sqrt[-NJ.gg.NJ] / c]]
```

Out[24]= $$n-\frac{jx^2+jy^2+jz^2+2n^2W}{2nc^2}-\frac{jx^4+jy^4+jz^4+12jz^2n^2W-4n^4W^2+2jy^2\left(jz^2+6n^2W\right)+2jx^2\left(jy^2+jz^2+6n^2W\right)-32jx\,n^3Wx-32jy\,n^3Wy-32jz\,n^3Wz}{8n^3c^4}+O\left[\frac{1}{c}\right]^5$$

In[25]:=
```
(* matter associated 1-vector *)
(NJvec = Assuming[assut, FS[NJ / dg]]) // MF
```

Out[25]//MatrixForm=

$$\begin{pmatrix} n-\frac{2(nW)}{c^2}+O\left[\frac{1}{c}\right]^4 \\ jx-\frac{2(jx\,W)}{c^2}+O\left[\frac{1}{c}\right]^4 \\ jy-\frac{2(jy\,W)}{c^2}+O\left[\frac{1}{c}\right]^4 \\ jz-\frac{2(jz\,W)}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[26]:=
```
(* matter associated 4-vel vector *)
(uu = Assuming[assut, FS[c*NJvec / Sqrt[-NJvec.gg.NJvec]]]) // MF
```

Out[26]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{jx^2+jy^2+jz^2}{2n^2}+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jx}{n}+\frac{jx\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n}+\frac{jy\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n}+\frac{jz\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[27]:=
```
(* replace matter flux in terms of velocity*)
replaceJu = {jx → ux*n, jy → uy*n, jz → uz*n}
```

Out[27]= $\{jx \to n\,ux,\ jy \to n\,uy,\ jz \to n\,uz\}$

In[28]:=
```
(* collect velocity magnitude*)
replaceuUnorm = {ux^2 → U^2-uy^2-uz^2, ux^3 → ux*(U^2-uy^2-uz^2), jx^2 → J^2-jy^2-jz^2, jx^3 → jx*(J^2-jy^2-jz^2)}
```

Out[28]= $\{ux^2 \to U^2-uy^2-uz^2,\ ux^3 \to ux\left(U^2-uy^2-uz^2\right),\ jx^2 \to J^2-jy^2-jz^2,\ jx^3 \to jx\left(J^2-jy^2-jz^2\right)\}$

In[29]:=
```
Assuming[assut, FS[uu /. replaceJu]] // MF
```
Out[29]//MatrixForm=
$$\begin{pmatrix} 1+\frac{\frac{1}{2}(ux^2+uy^2+uz^2)+W}{c^2}+O[\frac{1}{c}]^4 \\ ux+\frac{ux(ux^2+uy^2+uz^2+2W)}{2c^2}+O[\frac{1}{c}]^4 \\ uy+\frac{uy(ux^2+uy^2+uz^2+2W)}{2c^2}+O[\frac{1}{c}]^4 \\ uz+\frac{uz(ux^2+uy^2+uz^2+2W)}{2c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[30]:=
```
(* it is normalized *)
FS[uu.gg.uu]
```
Out[30]= $-c^2+O[\frac{1}{c}]^2$

In[31]:=
```
(* matter associated 1-covector *)
(NJcov = Assuming[assut, FS[gg.(NJ / dg)]]) // MF
```
Out[31]//MatrixForm=
$$\begin{pmatrix} -n\,c^2+4\,n\,W+O[\frac{1}{c}]^2 \\ jx-\frac{4(n\,Wx)}{c^2}+O[\frac{1}{c}]^4 \\ jy-\frac{4(n\,Wy)}{c^2}+O[\frac{1}{c}]^4 \\ jz-\frac{4(n\,Wz)}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[32]:=
```
(* scalar product with de/de_x (for momentum x-component) *)
Simplify[uu.gg.{0, 1, 0, 0}]
```
Out[32]= $$\frac{jx}{n}+\frac{jx^3+jx\,(jy^2+jz^2+6\,n^2\,W)-8\,n^3\,Wx}{2\,n^3\,c^2}+O[\frac{1}{c}]^4$$

In[33]:=
```
(* scalar product with de/de_i (for momentum i-component) *)
Simplify[uu.gg] // MF
```
Out[33]//MatrixForm=
$$\begin{pmatrix} -c^2+\left(-\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)+O[\frac{1}{c}]^2 \\ \frac{jx}{n}+\frac{jx^3+jx(jy^2+jz^2+6n^2W)-8n^3Wx}{2n^3c^2}+O[\frac{1}{c}]^4 \\ \frac{jy}{n}+\frac{jx^2\,jy+jy^3+jy\,jz^2+6\,jy\,n^2W-8n^3Wy}{2n^3c^2}+O[\frac{1}{c}]^4 \\ \frac{jz}{n}+\frac{jx^2\,jz+jy^2\,jz+jz^3+6\,jz\,n^2W-8n^3Wz}{2n^3c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[34]:=
```
(* retransform matter 4-vel to matter 3-covector *)
Assuming[assut, FS[uu * dg / c * Sqrt[-NJvec.gg.NJvec]]] // MF
```
Out[34]//MatrixForm=
$$\begin{pmatrix} n+O[\frac{1}{c}]^4 \\ jx+O[\frac{1}{c}]^4 \\ jy+O[\frac{1}{c}]^4 \\ jz+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[35]:=
```
(* simplification to x-directed matter flux and velocity *)
assutjx = Join[assut, {jy == 0, jz == 0, uy == 0, uz == 0}];
```

In[36]:=
```
(* flux of matter across surface *)
Simplify[surface.NJ / (Δt)]
```
Out[36]= $Ax\,(jx-n\,Vx)+Ay\,(jy-n\,Vy)+Az\,(jz-n\,Vz)$

In[37]:=
```
(* normalized zero-flux velocity is same as U *)
vnoflux = {1, jx / n, jy / n, jz / n};
FS[c * vnoflux / Sqrt[-vnoflux.gg.vnoflux] == uu]
```
Out[38]= True

In[39]:=
```
FS[uu /. replaceuUnorm] // MF
```
Out[39]//MatrixForm=
$$\begin{pmatrix} 1+\frac{\frac{J^2}{2n^2}+W}{c^2}+O[\frac{1}{c}]^4 \\ \frac{jx}{n}+\frac{jx(J^2+2n^2W)}{2n^3c^2}+O[\frac{1}{c}]^4 \\ \frac{jy}{n}+\frac{jy(J^2+2n^2W)}{2n^3c^2}+O[\frac{1}{c}]^4 \\ \frac{jz}{n}+\frac{jz(J^2+2n^2W)}{2n^3c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[40]:=
```
FS[uu /. replaceJu] // MF
```
Out[40]//MatrixForm=
$$\begin{pmatrix} 1+\frac{\frac{1}{2}(ux^2+uy^2+uz^2)+W}{c^2}+O[\frac{1}{c}]^4 \\ ux+\frac{ux(ux^2+uy^2+uz^2+2W)}{2c^2}+O[\frac{1}{c}]^4 \\ uy+\frac{uy(ux^2+uy^2+uz^2+2W)}{2c^2}+O[\frac{1}{c}]^4 \\ uz+\frac{uz(ux^2+uy^2+uz^2+2W)}{2c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[41]:=

In[42]:=
```
(* Project along u velocity *)
proju = Assuming[assut, Expand //@ FS@PowerExpand[-Outer[Times, uu, gg.uu] / c ^ 2]];
projperpu = Assuming[assut, Expand //@ FS@PowerExpand[IdentityMatrix[4] - proju]];
testproj[ass_, x_] := showf[ass] /@ {Assuming[ass, Expand //@ FS@PowerExpand[proju.x.proju == x]], projperpu.x.proju == x, proju.x.projperpu == x, projperpu.x.projperpu == x}
```

In[45]:=
```
(* Project along u velocity *)
proju = Assuming[assut, FS[-Outer[Times, uu, gg.uu] / c ^ 2]];
projperpu = Assuming[assut, FS[IdentityMatrix[4] - proju]];
testproj[ass_, x_] := showf[ass] /@ {Assuming[ass, Expand //@ FS@PowerExpand[proju.x.proju == x]], projperpu.x.proju == x, proju.x.projperpu == x, projperpu.x.projperpu == x}
```

In[48]:=
```
(* aux 4-velocity *)
auu = {temp, aux, auy, auz};
solu = FS[temp /. Solve[Normal[auu.gg.auu] == -c ^ 2, temp][[2]]]
```
Out[49]= $$-\frac{4\,aux\,Wx+4\,auy\,Wy+4\,auz\,Wz+c^2\sqrt{\frac{(c^2(aux^2+auy^2+auz^2+c^2)+2(aux^2+auy^2+auz^2)W)(c^4-2c^2W+2W^2)+16(aux\,Wx+auy\,Wy+auz\,Wz)^2}{c^4}}}{c^4-2\,c^2\,W+2\,W^2}$$

In[50]:=
```
(auu = Assuming[assut, FS[auu /. {temp → solu}]]) // MF
```
Out[50]//MatrixForm=
$$\begin{pmatrix} -\frac{4\,aux\,Wx+4\,auy\,Wy+4\,auz\,Wz+\sqrt{(c^2(aux^2+auy^2+auz^2+c^2)+2(aux^2+auy^2+auz^2)W)(c^4-2c^2W+2W^2)+16(aux\,Wx+auy\,Wy+auz\,Wz)^2}}{c^4-2c^2W+2W^2} \\ aux \\ auy \\ auz \end{pmatrix}$$

In[51]:=
```
FS[auu.gg.auu]
```
Out[51]= $-c^2+O[\frac{1}{c}]^3$

In[52]:=
```
(* 4-velocity and matter current with explicit coordinate dependence *)
tjv[xx_] := (xx /. {n → n[t, x, y, z], jx → ux[t, x, y, z] * n[t, x, y, z], jy → uy[t, x, y, z] * n[t, x, y, z], jz → uz[t, x, y, z] * n[t, x, y, z]});
tjn[xx_] := (xx /. {n → n[t, x, y, z], jx → jx[t, x, y, z], jy → jy[t, x, y, z], jz → jz[t, x, y, z]});
itjn[xx_] := (xx /. {n[t, x, y, z] → n, jx[t, x, y, z] → jx, jy[t, x, y, z] → jy, jz[t, x, y, z] → jz});
itjv[xx_] := (xx /. {n[t, x, y, z] → n, ux[t, x, y, z] → jx / n, uy[t, x, y, z] → jy / n, uz[t, x, y, z] → jz / n});
repjn = {D[n[t, x, y, z], t] → D[jx[t, x, y, z], x] + D[jy[t, x, y, z], y] + D[jz[t, x, y, z], z]};
MF /@ {uut = Assuming[assut, FS[tjn[tW[uu]]]], uuv = Assuming[assut, FS[tjv[tW[uu]]]]}
```
Out[57]=
$$\left\{\begin{pmatrix} 1+\frac{\frac{jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2}{2\,n[t,x,y,z]^2}+W[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ \frac{jx[t,x,y,z]}{n[t,x,y,z]}+\frac{jx[t,x,y,z](jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z])}{2\,n[t,x,y,z]^3\,c^2}+O[\frac{1}{c}]^4 \\ \frac{jy[t,x,y,z]}{n[t,x,y,z]}+\frac{jy[t,x,y,z](jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z])}{2\,n[t,x,y,z]^3\,c^2}+O[\frac{1}{c}]^4 \\ \frac{jz[t,x,y,z]}{n[t,x,y,z]}+\frac{jz[t,x,y,z](jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z])}{2\,n[t,x,y,z]^3\,c^2}+O[\frac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} 1+\frac{\frac{1}{2}(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2)+W[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ ux[t,x,y,z]+\frac{ux[t,x,y,z](ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z])}{2\,c^2}+O[\frac{1}{c}]^4 \\ uy[t,x,y,z]+\frac{uy[t,x,y,z](ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z])}{2\,c^2}+O[\frac{1}{c}]^4 \\ uz[t,x,y,z]+\frac{uz[t,x,y,z](ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z])}{2\,c^2}+O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

In[58]:=

In[59]:=

(* Construction of energy–momentum tensor *)

In[60]:=
```
(* definition of heat-flux, orthogonal to matter-current *)
Qtemp = {qt, qx, qy, qz};
```

In[61]:=
```
(proju.Qtemp) // MF
```
Out[61]//MatrixForm=
$$\begin{pmatrix} qt+\frac{\frac{(jx^2+jy^2+jz^2)qt}{n^2}-\frac{jx\,qx}{n}-\frac{jy\,qy}{n}-\frac{jz\,qz}{n}}{c^2}+O[\frac{1}{c}]^4 \\ \frac{jx\,qt}{n}+\frac{\frac{jx(jx^2+jy^2+jz^2)qt}{n^3}-\frac{jx^2\,qx}{n^2}-\frac{jx\,jy\,qy}{n^2}-\frac{jx\,jz\,qz}{n^2}}{c^2}+O[\frac{1}{c}]^4 \\ \frac{jy\,qt}{n}+\frac{\frac{jy(jx^2+jy^2+jz^2)qt}{n^3}-\frac{jx\,jy\,qx}{n^2}-\frac{jy^2\,qy}{n^2}-\frac{jy\,jz\,qz}{n^2}}{c^2}+O[\frac{1}{c}]^4 \\ \frac{jz\,qt}{n}+\frac{\frac{jz(jx^2+jy^2+jz^2)qt}{n^3}-\frac{jx\,jz\,qx}{n^2}-\frac{jy\,jz\,qy}{n^2}-\frac{jz^2\,qz}{n^2}}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[62]:=
```
qsol = Solve[Normal[proju.Qtemp] == 0, qt][[1]]
```
Out[62]= $$\left\{qt\to\frac{n\,(jx\,qx+jy\,qy+jz\,qz)}{jx^2+jy^2+jz^2+c^2\,n^2}\right\}$$

In[63]:= `(Q = Assuming[assut, FS[Qtemp /. qsol]]) // MF`

Out[63]//MatrixForm=

$$\begin{pmatrix} \frac{n\left(jx\,qx+jy\,qy+jz\,qz\right)}{jx^2+jy^2+jz^2+c^2\,n^2} \\ qx \\ qy \\ qz \end{pmatrix}$$

In[64]:= `{Normal@Series[Q.NJ, {c, Infinity, 1}] == 0}`

Out[64]= $\{jx\,qx + jy\,qy + jz\,qz == 0\}$

In[65]:= `assutQ = Join[assut, {Normal@Series[Q.NJ, {c, Infinity, 1}] == 0}];`

In[66]:= `Assuming[assutQ, FS@{proju.Q, projperpu.Q == Q}]`

Out[66]=

$$\left\{\left\{\frac{4\left(qx\,Wx+qy\,Wy+qz\,Wz\right)}{c^4}+O\left[\frac{1}{c}\right]^6,\ \frac{4\left(-jy\,qy\,Wx-jz\,qz\,Wx+jx\,qy\,Wy+jx\,qz\,Wz\right)}{n\,c^4}+O\left[\frac{1}{c}\right]^6,\ \frac{4\,jy\left(qx\,Wx+qy\,Wy+qz\,Wz\right)}{n\,c^4}+O\left[\frac{1}{c}\right]^6,\ \frac{4\,jz\left(qx\,Wx+qy\,Wy+qz\,Wz\right)}{n\,c^4}+O\left[\frac{1}{c}\right]^6\right\},\right.$$

$$\left.\left\{-\frac{4\left(qx\,Wx+qy\,Wy+qz\,Wz\right)}{c^4}+O\left[\frac{1}{c}\right]^6,\ \frac{4\left(jy\,qy\,Wx+jz\,qz\,Wx-jx\left(qy\,Wy+qz\,Wz\right)\right)}{n\,c^4}+O\left[\frac{1}{c}\right]^6,\ -\frac{4\left(jy\left(qx\,Wx+qy\,Wy+qz\,Wz\right)\right)}{n\,c^4}+O\left[\frac{1}{c}\right]^6,\ -\frac{4\left(jz\left(qx\,Wx+qy\,Wy+qz\,Wz\right)\right)}{n\,c^4}+O\left[\frac{1}{c}\right]^6\right\}==\{0,0,0,0\}\right\}$$

In[67]:= `(* non-symmetric heat-tensor *)`
`Assuming[assutQ, FS[Qtens = Assuming[assut, Expand //@ FS@PowerExpand[Outer[Times, Q, gg.uu / c ^ 2]]]]] // MF`

Out[67]//MatrixForm=

$$\begin{pmatrix}
O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^8 & O\left[\frac{1}{c}\right]^8 & O\left[\frac{1}{c}\right]^8 \\
-qx+\frac{-\frac{\left(jx^2+jy^2+jz^2\right)qx}{2n^2}+qx\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jx\,qx}{n\,c^2}+\frac{\frac{jx\,qx\left(jx^2+jy^2+jz^2+6n^2W\right)}{2n^3}-4\,qx\,Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jy\,qx}{n\,c^2}+\frac{\frac{jy\,qx\left(jx^2+jy^2+jz^2+6n^2W\right)}{2n^3}-4\,qx\,Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jz\,qx}{n\,c^2}+\frac{\frac{jz\,qx\left(jx^2+jy^2+jz^2+6n^2W\right)}{2n^3}-4\,qx\,Wz}{c^4}+O\left[\frac{1}{c}\right]^6 \\
-qy+\frac{-\frac{\left(jx^2+jy^2+jz^2\right)qy}{2n^2}+qy\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jx\,qy}{n\,c^2}+\frac{\frac{jx\,qy\left(jx^2+jy^2+jz^2+6n^2W\right)}{2n^3}-4\,qy\,Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jy\,qy}{n\,c^2}+\frac{\frac{jy\,qy\left(jx^2+jy^2+jz^2+6n^2W\right)}{2n^3}-4\,qy\,Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jz\,qy}{n\,c^2}+\frac{\frac{jz\,qy\left(jx^2+jy^2+jz^2+6n^2W\right)}{2n^3}-4\,qy\,Wz}{c^4}+O\left[\frac{1}{c}\right]^6 \\
-qz+\frac{-\frac{\left(jx^2+jy^2+jz^2\right)qz}{2n^2}+qz\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jx\,qz}{n\,c^2}+\frac{\frac{jx\,qz\left(jx^2+jy^2+jz^2+6n^2W\right)}{2n^3}-4\,qz\,Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jy\,qz}{n\,c^2}+\frac{\frac{jy\,qz\left(jx^2+jy^2+jz^2+6n^2W\right)}{2n^3}-4\,qz\,Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jz\,qz}{n\,c^2}+\frac{\frac{jz\,qz\left(jx^2+jy^2+jz^2+6n^2W\right)}{2n^3}-4\,qz\,Wz}{c^4}+O\left[\frac{1}{c}\right]^6
\end{pmatrix}$$

In[68]:= `Assuming[assutQ, FS[T[Qtens.Inverse[gg]].gg - Qtens]] // MF`

Out[68]//MatrixForm=

$$\begin{pmatrix}
-\frac{4\left(qx\,Wx+qy\,Wy+qz\,Wz\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{qx}{c^2}+\frac{\frac{\left(jx^2+jy^2+jz^2\right)qx}{2n^2}+3\,qx\,W}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{qy}{c^2}+\frac{\frac{\left(jx^2+jy^2+jz^2\right)qy}{2n^2}+3\,qy\,W}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{qz}{c^2}+\frac{\frac{\left(jx^2+jy^2+jz^2\right)qz}{2n^2}+3\,qz\,W}{c^4}+O\left[\frac{1}{c}\right]^6 \\
qx+\frac{\frac{\left(jx^2+jy^2+jz^2\right)qx}{2n^2}-qx\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{4\,qx\,Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{-jy\,qx+jx\,qy}{n\,c^2}+\frac{-jy^3\,qx+2\,jx\,jy^2\,qy+jx\,qy\left(jx^2+jz^2+6n^2W\right)+jy\left(-jz^2\,qx+jx\,jz\,qz-6n^2\,qx\,W\right)+8n^3\,qx\,Wy}{2n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{-jz\,qx+jx\,qz}{n\,c^2}+\frac{-jz^3\,qx+jx\,jy\,jz\,qy+jx^3\,qz+2\,jx\,jz^2\,qz+jy^2\left(-jz\,qx+jx\,qz\right)-6\,jz\,n^2\,qx\,W+6\,jx\,n^2\,qz\,W+8n^3\,qx\,Wz}{2n^3\,c^4}+O\left[\frac{1}{c}\right]^6 \\
qy+\frac{\frac{\left(jx^2+jy^2+jz^2\right)qy}{2n^2}-qy\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jy\,qx-jx\,qy}{n\,c^2}+\frac{\frac{\left(jy\,qx-jx\,qy\right)\left(jx^2+jy^2+jz^2+6n^2W\right)}{2n^3}+4\,qy\,Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{4\,qy\,Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{-jz\,qy+jy\,qz}{n\,c^2}+\frac{\frac{\left(-jz\,qy+jy\,qz\right)\left(jx^2+jy^2+jz^2+6n^2W\right)}{2n^3}+4\,qy\,Wz}{c^4}+O\left[\frac{1}{c}\right]^6 \\
qz+\frac{\frac{\left(jx^2+jy^2+jz^2\right)qz}{2n^2}-qz\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{jz\,qx-jx\,qz}{n\,c^2}+\frac{\frac{\left(jz\,qx-jx\,qz\right)\left(jx^2+jy^2+jz^2+6n^2W\right)}{2n^3}+4\,qz\,Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jz\,qy-jy\,qz}{n\,c^2}+\frac{\frac{\left(jz\,qy-jy\,qz\right)\left(jx^2+jy^2+jz^2+6n^2W\right)}{2n^3}+4\,qz\,Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{4\,qz\,Wz}{c^4}+O\left[\frac{1}{c}\right]^6
\end{pmatrix}$$

In[69]:= `(* definition of momentum-flux, orthogonal to matter-current *)`
`Ptemp = {pt, px, py, pz};`

In[70]:= `Assuming[assut, FS[Ptemp.proju]] // MF`

Out[70]//MatrixForm=

$$\begin{pmatrix}
\frac{n\,pt+jx\,px+jy\,py+jz\,pz}{n}+\frac{\left(jx^2+jy^2+jz^2\right)\left(n\,pt+jx\,px+jy\,py+jz\,pz\right)}{n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\
-\frac{jx\left(n\,pt+jx\,px+jy\,py+jz\,pz\right)}{n^2\,c^2}-\frac{\left(n\,pt+jx\,px+jy\,py+jz\,pz\right)\left(jx^3+jx\left(jy^2+jz^2+4n^2W\right)-4n^3\,Wx\right)}{n^4\,c^4}+O\left[\frac{1}{c}\right]^6 \\
-\frac{jy\left(n\,pt+jx\,px+jy\,py+jz\,pz\right)}{n^2\,c^2}-\frac{\left(n\,pt+jx\,px+jy\,py+jz\,pz\right)\left(jy\left(jx^2+jy^2+jz^2+4n^2W\right)-4n^3\,Wy\right)}{n^4\,c^4}+O\left[\frac{1}{c}\right]^6 \\
-\frac{jz\left(n\,pt+jx\,px+jy\,py+jz\,pz\right)}{n^2\,c^2}-\frac{\left(n\,pt+jx\,px+jy\,py+jz\,pz\right)\left(jz\left(jx^2+jy^2+jz^2+4n^2W\right)-4n^3\,Wz\right)}{n^4\,c^4}+O\left[\frac{1}{c}\right]^6
\end{pmatrix}$$

In[71]:= `psol = Solve[Normal[Ptemp.proju] == 0, pt][[1]]`

Out[71]= $\left\{pt \to -\frac{jx\,px+jy\,py+jz\,pz}{n}\right\}$

In[72]:= `(P = Assuming[assut, FS[Ptemp /. psol]]) // MF`

Out[72]//MatrixForm=

$$\begin{pmatrix} -\frac{jx\,px+jy\,py+jz\,pz}{n} \\ px \\ py \\ pz \end{pmatrix}$$

In[73]:= `{FS[Normal@Series[P.uu, {c, Infinity, 1}]] == 0}`

Out[73]= {True}

In[74]:= `Assuming[assutQ, FS@{P.proju, P.projperpu == P}]`

Out[74]= $\left\{\left\{O\left[\frac{1}{c}\right]^4,\ O\left[\frac{1}{c}\right]^6,\ O\left[\frac{1}{c}\right]^6,\ O\left[\frac{1}{c}\right]^6\right\},\ True\right\}$

In[75]:= `(* non-symmetric momentum-tensor *)`
`Assuming[assut, FS[Ptens = Assuming[assut, FS[Outer[Times, uu, P / c ^ 2]]]]] // MF`

Out[75]//MatrixForm=

$$\begin{pmatrix}
-\frac{jx\,px+jy\,py+jz\,pz}{n\,c^2}-\frac{\left(jx\,px+jy\,py+jz\,pz\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{px}{c^2}+\frac{px\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{py}{c^2}+\frac{py\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{pz}{c^2}+\frac{pz\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O\left[\frac{1}{c}\right]^6 \\
-\frac{jx\left(jx\,px+jy\,py+jz\,pz\right)}{n^2\,c^2}-\frac{jx\left(jx\,px+jy\,py+jz\,pz\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^4\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jx\,px}{n\,c^2}+\frac{jx\,px\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jx\,py}{n\,c^2}+\frac{jx\,py\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jx\,pz}{n\,c^2}+\frac{jx\,pz\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3\,c^4}+O\left[\frac{1}{c}\right]^6 \\
-\frac{jy\left(jx\,px+jy\,py+jz\,pz\right)}{n^2\,c^2}-\frac{jy\left(jx\,px+jy\,py+jz\,pz\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^4\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jy\,px}{n\,c^2}+\frac{jy\,px\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jy\,py}{n\,c^2}+\frac{jy\,py\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jy\,pz}{n\,c^2}+\frac{jy\,pz\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3\,c^4}+O\left[\frac{1}{c}\right]^6 \\
-\frac{jz\left(jx\,px+jy\,py+jz\,pz\right)}{n^2\,c^2}-\frac{jz\left(jx\,px+jy\,py+jz\,pz\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^4\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jz\,px}{n\,c^2}+\frac{jz\,px\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jz\,py}{n\,c^2}+\frac{jz\,py\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3\,c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jz\,pz}{n\,c^2}+\frac{jz\,pz\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3\,c^4}+O\left[\frac{1}{c}\right]^6
\end{pmatrix}$$

In[76]:= `FS[T[Ptens.Inverse[gg]].gg - Ptens] // MF`

Out[76]//MatrixForm=

$$\begin{pmatrix}
\frac{4\left(px\,Wx+py\,Wy+pz\,Wz\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{px}{c^2}+\frac{\frac{jx\left(jx\,px+jy\,py+jz\,pz\right)}{n^2}-px\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{py}{c^2}+\frac{\frac{jy\left(jx\,px+jy\,py+jz\,pz\right)}{n^2}-py\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{pz}{c^2}+\frac{\frac{jz\left(jx\,px+jy\,py+jz\,pz\right)}{n^2}-pz\left(\frac{jx^2+jy^2+jz^2}{2n^2}+W\right)}{c^4}+O\left[\frac{1}{c}\right]^6 \\
-px+\frac{jx^2\,px+2\,jx\left(jy\,py+jz\,pz\right)-px\left(jy^2+jz^2-6n^2W\right)}{2n^2\,c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{4\left(px\,Wx\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jy\,px-jx\,py}{n\,c^2}+\frac{\frac{\left(jy\,px-jx\,py\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3}-4\,px\,Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jz\,px-jx\,pz}{n\,c^2}+\frac{\frac{\left(jz\,px-jx\,pz\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3}-4\,px\,Wz}{c^4}+O\left[\frac{1}{c}\right]^6 \\
-py+\frac{-\frac{\left(jx^2+jy^2+jz^2\right)py}{2n^2}+\frac{jy\left(jx\,px+jy\,py+jz\,pz\right)}{n^2}+3\,py\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{-jy\,px+jx\,py}{n\,c^2}+\frac{\frac{\left(-jy\,px+jx\,py\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3}-4\,py\,Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4\left(py\,Wy\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{jz\,py-jy\,pz}{n\,c^2}+\frac{\frac{\left(jz\,py-jy\,pz\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3}-4\,py\,Wz}{c^4}+O\left[\frac{1}{c}\right]^6 \\
-pz+\frac{-\frac{\left(jx^2+jy^2+jz^2\right)pz}{2n^2}+\frac{jz\left(jx\,px+jy\,py+jz\,pz\right)}{n^2}+3\,pz\,W}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{-jz\,px+jx\,pz}{n\,c^2}+\frac{\frac{\left(-jz\,px+jx\,pz\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3}-4\,pz\,Wx}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{-jz\,py+jy\,pz}{n\,c^2}+\frac{\frac{\left(-jz\,py+jy\,pz\right)\left(jx^2+jy^2+jz^2+2n^2W\right)}{2n^3}-4\,pz\,Wy}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{4\left(pz\,Wz\right)}{c^4}+O\left[\frac{1}{c}\right]^6
\end{pmatrix}$$

In[77]:= `(* definition of stress, orthogonal to matter-current *)`
`(Stemp = {{stt, stx, sty, stz}, {sxt, sxx, sxy, sxz}, {syt, syx, syy, syz}, {szt, szx, szy, szz}}) // MF`

Out[77]//MatrixForm=

$$\begin{pmatrix} stt & stx & sty & stz \\ sxt & sxx & sxy & sxz \\ syt & syx & syy & syz \\ szt & szx & szy & szz \end{pmatrix}$$

In[78]:= `(Stempsym = Assuming[assut, FS[(T[Stemp.Inverse[gg]].gg + Stemp) / 2]]) // MF`

Out[78]//MatrixForm=

$$\begin{pmatrix}
stt+\frac{2\left(stx\,Wx+sty\,Wy+stz\,Wz\right)}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{stx}{2}-\frac{sxt}{2c^2}-\frac{2\left(sxt\,W-stt\,Wx+sxx\,Wx+sxy\,Wy+sxz\,Wz\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{sty}{2}-\frac{syt}{2c^2}-\frac{2\left(syt\,W+syx\,Wx-stt\,Wy+syy\,Wy+syz\,Wz\right)}{c^4}+O\left[\frac{1}{c}\right]^6 & \frac{stz}{2}-\frac{szt}{2c^2}-\frac{2\left(szt\,W+szx\,Wx+szy\,Wy-stt\,Wz+szz\,Wz\right)}{c^4}+O\left[\frac{1}{c}\right]^6 \\
-\frac{stx\,c^2}{2}+\frac{1}{2}\left(sxt+4\,stx\,W\right)+O\left[\frac{1}{c}\right]^2 & sxx-\frac{2\left(stx\,Wx\right)}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{sxy+syx}{2}-\frac{2\left(stx\,Wy\right)}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{sxz+szx}{2}-\frac{2\left(stx\,Wz\right)}{c^2}+O\left[\frac{1}{c}\right]^4 \\
-\frac{sty\,c^2}{2}+\frac{1}{2}\left(syt+4\,sty\,W\right)+O\left[\frac{1}{c}\right]^2 & \frac{sxy+syx}{2}-\frac{2\left(sty\,Wx\right)}{c^2}+O\left[\frac{1}{c}\right]^4 & syy-\frac{2\left(sty\,Wy\right)}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{syz+szy}{2}-\frac{2\left(sty\,Wz\right)}{c^2}+O\left[\frac{1}{c}\right]^4 \\
-\frac{stz\,c^2}{2}+\frac{1}{2}\left(szt+4\,stz\,W\right)+O\left[\frac{1}{c}\right]^2 & \frac{sxz+szx}{2}-\frac{2\left(stz\,Wx\right)}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{syz+szy}{2}-\frac{2\left(stz\,Wy\right)}{c^2}+O\left[\frac{1}{c}\right]^4 & szz-\frac{2\left(stz\,Wz\right)}{c^2}+O\left[\frac{1}{c}\right]^4
\end{pmatrix}$$

In[79]:= `FS[proju.Stemp.proju] // MF`

Out[79]//MatrixForm=

$$\frac{n\,stt+jx\,stx+jy\,sty+jz\,stz}{n}+\frac{2\,jx^3\,stx+2\,jy^3\,sty+jx^2\left(2n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx\right)-jy\,n^2\,syt+jy^2\left(2n\,stt+2\,jz\,stz-n\,syy\right)+jx\left(2\,jy^2\,stx-n^2\,sxt-jy\,n\left(sxy+syx\right)+jz\left(2\,jz\,stx-n\left(sxz+szx\right)\right)\right)+jy\,jz\left(2\,jz\,sty-n\left(syz+szy\right)\right)+jz\left(2\,jz^2\,stz-n^2\,szt+jz\,n\left(2\,stt-szz\right)\right)}{n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \quad -\frac{jx\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^2\,c^2}+\frac{-2\,jx^4\,stx+jx^3\left(-2\left(n\,stt+jy\,sty+jz\,stz\right)+n\,sxx\right)+jx^2\left(-2\,jy^2\,stx-2\,jz^2\,stx+jy\,n\left(sxy+syx\right)+jz\,n\left(sxz+szx\right)+n^2\left(sxt-4\,stx\,W\right)\right)+4n^3\left(n\,stt+jy\,sty+jz\,stz\right)Wx+jx\left(-2\,j\ldots}{n^4\,c^4}$$

$$\frac{jx\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^2}+\frac{jx\left(2\,jx^3\,stx+2\,jy^3\,sty+jx^2\left(2n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx\right)-jy\,n^2\,syt+jy^2\left(2n\,stt+2\,jz\,stz-n\,syy\right)+jx\left(2\,jy^2\,stx-n^2\,sxt-jy\,n\left(sxy+syx\right)+jz\left(2\,jz\,stx-n\left(sxz+szx\right)\right)\right)+jy\,jz\left(2\,jz\,sty-n\left(syz+szy\right)\right)+jz\left(2\,jz^2\,stz-n^2\,szt+jz\,n\left(2\,stt-szz\right)\right)\right)}{n^4\,c^2}+O\left[\frac{1}{c}\right]^4 \quad -\frac{jx^2\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^3\,c^2}+\frac{jx\left(-2\,jx^4\,stx+jx^3\left(-2\left(n\,stt+jy\,sty+jz\,stz\right)+n\,sxx\right)+jx^2\left(-2\,jy^2\,stx-2\,jz^2\,stx+jy\,n\left(sxy+syx\right)+jz\,n\left(sxz+szx\right)+n^2\left(sxt-4\,stx\,W\right)\right)+4n^3\left(n\,stt+jy\,sty+jz\,stz\right)Wx+jx\left(-2\,j\ldots$$

$$\frac{jy\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^2}+\frac{jy\left(2\,jx^3\,stx+2\,jy^3\,sty+jx^2\left(2n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx\right)-jy\,n^2\,syt+jy^2\left(2n\,stt+2\,jz\,stz-n\,syy\right)+jx\left(2\,jy^2\,stx-n^2\,sxt-jy\,n\left(sxy+syx\right)+jz\left(2\,jz\,stx-n\left(sxz+szx\right)\right)\right)+jy\,jz\left(2\,jz\,sty-n\left(syz+szy\right)\right)+jz\left(2\,jz^2\,stz-n^2\,szt+jz\,n\left(2\,stt-szz\right)\right)\right)}{n^4\,c^2}+O\left[\frac{1}{c}\right]^4 \quad -\frac{jx\,jy\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^3\,c^2}+\frac{jy\left(-2\,jx^4\,stx+jx^3\left(-2\left(n\,stt+jy\,sty+jz\,stz\right)+n\,sxx\right)+jx^2\left(-2\,jy^2\,stx-2\,jz^2\,stx+jy\,n\left(sxy+syx\right)+jz\,n\left(sxz+szx\right)+n^2\left(sxt-4\,stx\,W\right)\right)+4n^3\left(n\,stt+jy\,sty\ldots$$

$$\frac{jz\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^2}+\frac{jz\left(2\,jx^3\,stx+2\,jy^3\,sty+jx^2\left(2n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx\right)-jy\,n^2\,syt+jy^2\left(2n\,stt+2\,jz\,stz-n\,syy\right)+jx\left(2\,jy^2\,stx-n^2\,sxt-jy\,n\left(sxy+syx\right)+jz\left(2\,jz\,stx-n\left(sxz+szx\right)\right)\right)+jy\,jz\left(2\,jz\,sty-n\left(syz+szy\right)\right)+jz\left(2\,jz^2\,stz-n^2\,szt+jz\,n\left(2\,stt-szz\right)\right)\right)}{n^4\,c^2}+O\left[\frac{1}{c}\right]^4 \quad -\frac{jx\,jz\left(n\,stt+jx\,stx+jy\,sty+jz\,stz\right)}{n^3\,c^2}+\frac{jz\left(-2\,jx^4\,stx+jx^3\left(-2\left(n\,stt+jy\,sty+jz\,stz\right)+n\,sxx\right)+jx^2\left(-2\,jy^2\,stx-2\,jz^2\,stx+jy\,n\left(sxy+syx\right)+jz\,n\left(sxz+szx\right)+n^2\left(sxt-4\,stx\,W\right)\right)+4n^3\left(n\,stt+jy\,sty\ldots$$

In[80]:= `ssol = Solve[{Normal[proju.Stemp.proju] == 0, Normal[projperpu.Stemp.projperpu] == Stemp}, {stt, stx, sty, stz, sxt, syt, szt}][[1]]`

Out[80]=

$$\left\{stt \to -\frac{jx^2\,sxx+jx\,jy\,sxy+jx\,jz\,sxz+jx\,jy\,syx+jy^2\,syy+jy\,jz\,syz+jx\,jz\,szx+jy\,jz\,szy+jz^2\,szz}{jx^2+jy^2+jz^2+c^2\,n^2},\ stx \to \frac{n\left(jx\,sxx+jy\,syx+jz\,szx\right)}{jx^2+jy^2+jz^2+c^2\,n^2},\ sty \to \frac{n\left(jx\,sxy+jy\,syy+jz\,szy\right)}{jx^2+jy^2+jz^2+c^2\,n^2},\ stz \to \frac{n\left(jx\,sxz+jy\,syz+jz\,szz\right)}{jx^2+jy^2+jz^2+c^2\,n^2},\ sxt \to -\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n},\ syt \to -\frac{jx\,syx+jy\,syy+jz\,syz}{n},\ szt \to -\frac{jx\,szx+jy\,szy+jz\,szz}{n}\right\}$$

In[81]:= `ssol = Solve[Normal[proju.Stemp] == 0 && Normal[Stemp.proju] == 0, {stt, stx, sty, stz, sxt, syt, szt}][[1]]`

Out[81]=

$$\left\{stt \to -\frac{jx^2\,sxx+jx\,jy\,sxy+jx\,jz\,sxz+jx\,jy\,syx+jy^2\,syy+jy\,jz\,syz+jx\,jz\,szx+jy\,jz\,szy+jz^2\,szz}{jx^2+jy^2+jz^2+c^2\,n^2},\ stx \to \frac{n\left(jx\,sxx+jy\,syx+jz\,szx\right)}{jx^2+jy^2+jz^2+c^2\,n^2},\ sty \to \frac{n\left(jx\,sxy+jy\,syy+jz\,szy\right)}{jx^2+jy^2+jz^2+c^2\,n^2},\ stz \to \frac{n\left(jx\,sxz+jy\,syz+jz\,szz\right)}{jx^2+jy^2+jz^2+c^2\,n^2},\ sxt \to -\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n},\ syt \to -\frac{jx\,syx+jy\,syy+jz\,syz}{n},\ szt \to -\frac{jx\,szx+jy\,szy+jz\,szz}{n}\right\}$$

In[82]:= `(S = Assuming[assut, FS[(Stemp /. ssol)]]) // MF`

Out[82]//MatrixForm=

$$\begin{pmatrix}
-\frac{jx^2\,sxx+jx\,jy\left(sxy+syx\right)+jy^2\,syy+jx\,jz\left(sxz+szx\right)+jy\,jz\left(syz+szy\right)+jz^2\,szz}{jx^2+jy^2+jz^2+c^2\,n^2} & \frac{n\left(jx\,sxx+jy\,syx+jz\,szx\right)}{jx^2+jy^2+jz^2+c^2\,n^2} & \frac{n\left(jx\,sxy+jy\,syy+jz\,szy\right)}{jx^2+jy^2+jz^2+c^2\,n^2} & \frac{n\left(jx\,sxz+jy\,syz+jz\,szz\right)}{jx^2+jy^2+jz^2+c^2\,n^2} \\
-\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n} & sxx & sxy & sxz \\
-\frac{jx\,syx+jy\,syy+jz\,syz}{n} & syx & syy & syz \\
-\frac{jx\,szx+jy\,szy+jz\,szz}{n} & szx & szy & szz
\end{pmatrix}$$

In[83]:= `MF /@ FS@{proju.S.proju, Assuming[assut, FS[projperpu.S.projperpu - S]]}`

Out[83]=

$$\left\{\begin{pmatrix}
O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^8 & O\left[\frac{1}{c}\right]^8 & O\left[\frac{1}{c}\right]^8 \\
O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^8 & O\left[\frac{1}{c}\right]^8 & O\left[\frac{1}{c}\right]^8 \\
O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^8 & O\left[\frac{1}{c}\right]^8 & O\left[\frac{1}{c}\right]^8 \\
O\left[\frac{1}{c}\right]^6 & O\left[\frac{1}{c}\right]^8 & O\left[\frac{1}{c}\right]^8 & O\left[\frac{1}{c}\right]^8
\end{pmatrix},\right.$$

$$-\frac{jx^4\,sxx+jy^4\,syy+jx^3\left(jy\left(sxy+syx\right)+jz\left(sxz+szx\right)\right)+jy^3\,jz\left(syz+szy\right)+jx^2\left(jy^2\left(sxx+syy\right)+jy\,jz\left(syz+szy\right)+jz^2\left(sxx+szz\right)+4n^2\,sxx\,W\right)+jy^2\left(jz^2\left(syy+szz\right)+4n^2\,syy\,W\right)+jx\left(jy^3\left(sxy+syx\right)+jy^2\,jz\left(sxz+szx\right)+jz^3\left(sxz+szx\right)+4\,jz\,n^2\left(sxz+szx\right)W+jy\left(sxy+syx\right)\left(jz^2+4n^2W\right)-4n^3\left(sxx\,Wx+syx\,Wy+szx\,Wz\right)\right)+jy\left(jz^3\left(syz+szy\right)+4\,jz\,n^2\left(syz+szy\right)W-4n^3\left(sxy\,Wx+syy\,Wy+szy\,Wz\right)\right)+jz\left(jz^3\,szz+4\,jz\,n^2\,szz\,W-4n^3\left(sxz\,Wx+syz\,Wy+szz\,Wz\right)\right)}{n^4\,c^4}+O\left[\frac{1}{c}\right]^6 \quad \frac{jx^3\,sxx+jy^3\,syx+j\ldots}{}$$

$$O\left[\frac{1}{c}\right]^4 \quad \frac{jx\left(jx^3\,sxx+jy^3\,sy\ldots\right.}{}$$

$$O\left[\frac{1}{c}\right]^4 \quad \frac{jy\left(jx^3\,sxx+jy^3\,sy\ldots\right.}{}$$

$$O\left[\frac{1}{c}\right]^4 \quad \frac{jz\left(jx^3\,sxx+jy^3\,sy\ldots\right.}{}$$

In[84]:=
```
FS[T[S.Inverse[gg]].gg - S] // MF
```
Out[84]//MatrixForm=

In[85]:=
```
(* define "dust" 4-stress *)
(dust = Assuming[assut, FS[(ρ * c ^ 2 + ϵ) * Outer[Times, NJ, gg.uu / c ^ 2]]]) // MF
```
Out[85]//MatrixForm=

In[86]:=
```
FS[T[dust.Inverse[gg]] == dust.Inverse[gg]]
```
Out[86]= True

In[87]:=
```
FS[T[dust].gg == gg.dust]
```
Out[87]= True

In[88]:=
```
MF /@ FS@{proju.dust.proju == dust, projperpu.dust.projperpu}
```
Out[88]=
$$\left\{\text{True}, \begin{pmatrix} O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

In[89]:=
```
showf[assut][dust2 = Assuming[assut, Expand //@ FS@PowerExpand[(ρ * c ^ 2 + ϵ) * Outer[Times, NJ, gg.auu / c ^ 2]]]];
```

In[90]:=
```
(* define generic 4-stress *)
(EPS = Assuming[assut, FS[dust + Qtens + Ptens + S]]) // MF
```
Out[90]//MatrixForm=

In[91]:=
```
Assuming[assut, FS[(EPS /. replaceJu)]] // MF
```
Out[91]//MatrixForm=

In[92]:=
```
Assuming[assut, FS[(EPS /. replaceJu) /. replaceuUnorm]] // MF
```
Out[92]//MatrixForm=

In[93]:=
```
FS[T[EPS.Inverse[gg]].gg - EPS] // MF
```
Out[93]//MatrixForm=
$$\begin{pmatrix} O[\frac{1}{c}]^2 & \frac{n(-px+qx)+jy(sxy-syx)+jz(sxz-szx)}{n c^2}+O[\frac{1}{c}]^4 & \frac{n(-py+qy)+jx(-sxy+syx)+jz(syz-szy)}{n c^2}+O[\frac{1}{c}]^4 & \frac{n(-pz+qz)+jx(-sxz+szx)+jy(-syz+szy)}{n c^2}+O[\frac{1}{c}]^4 \\ \frac{n(-px+qx)+jy(sxy-syx)+jz(sxz-szx)}{n}+O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & (-sxy+syx)+\frac{jy\,px-jx\,py-jy\,qx+jx\,qy}{n c^2}+O[\frac{1}{c}]^4 & (-sxz+szx)+\frac{jz\,px-jx\,pz-jz\,qx+jx\,qz}{n c^2}+O[\frac{1}{c}]^4 \\ \frac{n(-py+qy)+jx(-sxy+syx)+jz(syz-szy)}{n}+O[\frac{1}{c}]^2 & (sxy-syx)+\frac{jy(-px+qx)+jx(py-qy)}{n c^2}+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & (-syz+szy)+\frac{jz\,py-jy\,pz-jz\,qy+jy\,qz}{n c^2}+O[\frac{1}{c}]^4 \\ \frac{n(-pz+qz)+jx(-sxz+szx)+jy(-syz+szy)}{n}+O[\frac{1}{c}]^2 & (sxz-szx)+\frac{jz(-px+qx)+jx(pz-qz)}{n c^2}+O[\frac{1}{c}]^4 & (syz-szy)+\frac{jz(-py+qy)+jy(pz-qz)}{n c^2}+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}$$

In[94]:=
```
(* Conditions for symmetry of total 4-stress tensor *)
subsym = {px → qx, py → qy, pz → qz, syx → sxy, szx → sxz, szy → syz};
```

In[95]:=
```
(EPSsym = Assuming[assut, FS[EPS /. subsym]]) // MF
```
Out[95]//MatrixForm=

In[96]:=
```
FS[T[EPSsym.Inverse[gg]].gg - EPSsym] // MF
```
Out[96]//MatrixForm=
$$\begin{pmatrix} O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}$$

In[97]:=
```
Assuming[assut, FS[(EPSsym /. replaceJu) /. replaceuUnorm]] // MF
```
Out[97]//MatrixForm=

In[98]:=

(* Balanced quantities constructed from energy–momentum tensor, and their supplies *)

In[99]:=
```
(* Symmetrized energy-stress tensor, with explicit dep. on coords *)
(TTx = Assuming[assut, FS[tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]]]]) // MF
```
Out[99]//MatrixForm=

In[100]:=
```
(Duu = Assuming[assut, FS[(D[Normal@uut, {coords}] + Sum[uut[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]) // MF
```
Out[100]//MatrixForm=

In[101]:=
```
(Duv = Assuming[assut, FS[(D[Normal@uuv, {coords}] + Sum[uuv[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]) // MF
```
Out[101]//MatrixForm=

(* Energy *)

In[102]:=
```
(* Energy current and supply according to 4-velocity *)
pvec = uu; Dpvec = Duv;
MF@(MF /@ {Efluxuu = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Esupplyuu = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[103]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} -n\,\rho\,c^2 - n\,\epsilon + O[\tfrac{1}{c}]^2 \\ \left(-Ax\,jx - Ay\,jy - Az\,jz + Ax\,n\,Vx + Ay\,n\,Vy + Az\,n\,Vz\right)\rho\,c^2 + \left(-Ax\left(qx + (jx - n\,Vx)\,\epsilon\right) - Ay\left(qy + jy\,\epsilon - n\,Vy\,\epsilon\right) - Az\left(qz + jz\,\epsilon - n\,Vz\,\epsilon\right)\right) + O[\tfrac{1}{c}]^2 \end{pmatrix} \\ \tfrac{1}{2}\left(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z] + (syz + szy)\left(uy^{(0,0,0,1)}[t,x,y,z] + uz^{(0,0,1,0)}[t,x,y,z]\right) + 2\left(syy\,uy^{(0,0,1,0)}[t,x,y,z] + sxx\,ux^{(0,1,0,0)}[t,x,y,z]\right) + (sxy + syx)\left(ux^{(0,0,1,0)}[t,x,y,z] + uy^{(0,1,0,0)}[t,x,y,z]\right) + (sxz + szx)\left(ux^{(0,0,0,1)}[t,x,y,z] + uz^{(0,1,0,0)}[t,x,y,z]\right)\right) + \ldots \end{pmatrix}$$

In[104]:=
```
MF@(MF /@ (FS[({Efluxuu, Esupplyuu} /. replaceJu) /. replaceuUnorm]))
```

Out[104]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} -n\,\rho\,c^2 - n\,\epsilon + O[\tfrac{1}{c}]^2 \\ n\left(Ax\,(-ux + Vx) + Ay\,(-uy + Vy) + Az\,(-uz + Vz)\right)\rho\,c^2 + \left(-Ax\,qx - Ay\,qy - Az\,qz + n\left(Ax\,(-ux + Vx) + Ay\,(-uy + Vy) + Az\,(-uz + Vz)\right)\epsilon\right) + O[\tfrac{1}{c}]^2 \end{pmatrix} \\ \tfrac{1}{2}\left(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z] + (syz + szy)\left(uy^{(0,0,0,1)}[t,x,y,z] + uz^{(0,0,1,0)}[t,x,y,z]\right) + 2\left(syy\,uy^{(0,0,1,0)}[t,x,y,z] + sxx\,ux^{(0,1,0,0)}[t,x,y,z]\right) + (sxy + syx)\left(ux^{(0,0,1,0)}[t,x,y,z] + uy^{(0,1,0,0)}[t,x,y,z]\right) + (sxz + szx)\left(ux^{(0,0,0,1)}[t,x,y,z] + uz^{(0,1,0,0)}[t,x,y,z]\right)\right) + \ldots \end{pmatrix}$$

In[105]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[105]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{qx}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ \frac{jy}{n} + \frac{qy}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ \frac{jz}{n} + \frac{qz}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ uy + \frac{qy}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ uz + \frac{qz}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix}\right\}$$

In[106]:=
```
(* Energy current and supply according to t-vector *)
pvec = - {1, 0, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ ({Efluxt = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Esupplyt = Assuming[assut, FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]]}))
```

Out[107]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} n\,\rho\,c^2 + n\left(\epsilon + \frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n^2} - W\,\rho\right) + O[\tfrac{1}{c}]^2 \\ \left(Ay\,jy + Az\,jz + Ax\left(jx - n\,Vx\right) - n\left(Ay\,Vy + Az\,Vz\right)\right)\rho\,c^2 + \frac{2\,n\left(Ax\left(jx\,sxx+jy\,sxy+jz\,sxz+n\left(qx+(jx-n\,Vx)\,\epsilon\right)\right)+Ay\left(jx\,syx+jy\,syy+jz\,syz+n\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)\right)+Az\left(jx\,szx+jy\,szy+jz\,szz+n\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)\right)+\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx^2+jy^2+jz^2-2\,n^2\,W\right)\rho}{2\,n^2} + O[\tfrac{1}{c}]^2 \end{pmatrix} \\ -n\,\rho\,W^{(1,0,0,0)}[t,x,y,z] + \frac{-\frac{\left(3\left(jx^2+jy^2+jz^2\right)\rho+2\,n\left(sxx+syy+szz+n\,\epsilon-n\,W\,\rho\right)\right)W^{(1,0,0,0)}[t,x,y,z]}{2\,n}+4\,\rho\left(jx\,Wx^{(1,0,0,0)}[t,x,y,z]+jy\,Wy^{(1,0,0,0)}[t,x,y,z]+jz\,Wz^{(1,0,0,0)}[t,x,y,z]\right)}{c^2} + O[\tfrac{1}{c}]^3 \end{pmatrix}$$

In[108]:=
```
MF@(MF /@ (FS[({Efluxt, Esupplyt} /. replaceJu) /. replaceuUnorm]))
```

Out[108]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} n\,\rho\,c^2 + n\left(\epsilon + \tfrac{1}{2}\left(U^2 - 2\,W\right)\rho\right) + O[\tfrac{1}{c}]^2 \\ n\left(Ax\,(ux - Vx) + Ay\,(uy - Vy) + Az\,(uz - Vz)\right)\rho\,c^2 + \left(Ax\left(qx + sxx\,ux + sxy\,uy + sxz\,uz + n\,(ux - Vx)\,\epsilon\right) + Ay\left(qy + syx\,ux + syy\,uy + syz\,uz + n\,(uy - Vy)\,\epsilon\right) + Az\left(qz + szx\,ux + szy\,uy + szz\,uz + n\,(uz - Vz)\,\epsilon\right) + \tfrac{1}{2}\,n\left(Ax\,(ux - Vx) + Ay\,(uy - Vy) + Az\,(uz - Vz)\right)\left(U^2 - 2\,W\right)\rho\right) + O[\tfrac{1}{c}]^2 \end{pmatrix} \\ -n\,\rho\,W^{(1,0,0,0)}[t,x,y,z] + \frac{-\frac{1}{2}\left(2\left(sxx+syy+szz+n\,\epsilon\right)+n\left(3\,U^2-2\,W\right)\rho\right)W^{(1,0,0,0)}[t,x,y,z]+4\,n\,\rho\left(ux\,Wx^{(1,0,0,0)}[t,x,y,z]+uy\,Wy^{(1,0,0,0)}[t,x,y,z]+uz\,Wz^{(1,0,0,0)}[t,x,y,z]\right)}{c^2} + O[\tfrac{1}{c}]^3 \end{pmatrix}$$

In[109]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[109]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix}\right\}$$

In[110]:=
```
(* Energy current and supply according to norm. t-vector *)
pvec = - c * {1, 0, 0, 0} / Sqrt[-gg[[1, 1]]]; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Efluxnt = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Esupplynt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[111]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} n\,\rho\,c^2 + \left(n\,\epsilon + \frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n}\right) + O[\tfrac{1}{c}]^2 \\ \left(Ay\,jy + Az\,jz + Ax\left(jx - n\,Vx\right) - n\left(Ay\,Vy + Az\,Vz\right)\right)\rho\,c^2 + \frac{2\,n\left(Ax\left(jx\,sxx+jy\,sxy+jz\,sxz+n\left(qx+(jx-n\,Vx)\,\epsilon\right)\right)+Ay\left(jx\,syx+jy\,syy+jz\,syz+n\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)\right)+Az\left(jx\,szx+jy\,szy+jz\,szz+n\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)\right)+\left(jx^2+jy^2+jz^2\right)\rho\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{2\,n^2} + O[\tfrac{1}{c}]^2 \end{pmatrix} \\ \rho\left(jz\,W^{(0,0,0,1)}[t,x,y,z] + jy\,W^{(0,0,1,0)}[t,x,y,z] + jx\,W^{(0,1,0,0)}[t,x,y,z]\right) + \frac{\left(n\left(jx\,(sxz+szx)+jy\,(syz+szy)+2\,jz\,szz+n\left(pz+qz+2\,jz\,\epsilon\right)\right)+jz\left(jx^2+jy^2+jz^2\right)\rho\right)W^{(0,0,0,1)}[t,x,y,z]+\left(n\left(jx\,(sxy+syx)+2\,jy\,syy+jz\,(syz+szy)+n\left(py+qy+2\,jy\,\epsilon\right)\right)+jy\left(jx^2+jy^2+jz^2\right)\rho\right)W^{(0,0,1,0)}[t,x,y,z]+n^2\,px\,W^{(0,1,0,0)}[t,x,y,z]+n^2\,qx\,W^{(0,1,0,0)}[t,x,y,z]+2\,jx\,n\,sxx\,W^{(0,1,0,0)}[t,x,y,z]+jy\,n\,sxy\,W^{(0,1,0,0)}[t,x,y,z]+jz\,n\,sxz\,W^{(0,1,0,0)}[t,x,y,z]+jy\,n\,syx\,W^{(0,1,0,0)}[t,x,y,z]+jz\,n\,szx\,W^{(0,1,0,0)}[t,x,\ldots}{2\,n^2\,c^2} \end{pmatrix}$$

In[112]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[112]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix}\right\}$$

In[113]:=
```
(* Energy current and supply according to cov. t-vector *)
pvec = c^2 * igg.{1, 0, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Efluxcovt = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Esupplycovt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[114]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} n\,\rho\,c^2 + \left(\frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n} + n\left(\epsilon + W\,\rho\right)\right) + O[\tfrac{1}{c}]^2 \\ \left(Ay\,jy + Az\,jz + Ax\left(jx - n\,Vx\right) - n\left(Ay\,Vy + Az\,Vz\right)\right)\rho\,c^2 + \frac{2\,n\left(Ax\left(jx\,sxx+jy\,sxy+jz\,sxz+n\left(qx+(jx-n\,Vx)\,\epsilon\right)\right)+Ay\left(jx\,syx+jy\,syy+jz\,syz+n\left(qy+jy\,\epsilon-n\,Vy\,\epsilon\right)\right)+Az\left(jx\,szx+jy\,szy+jz\,szz+n\left(qz+jz\,\epsilon-n\,Vz\,\epsilon\right)\right)\right)+\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx^2+jy^2+jz^2+2\,n^2\,W\right)\rho}{2\,n^2} + O[\tfrac{1}{c}]^2 \end{pmatrix} \\ \rho\left(2\,jz\,W^{(0,0,0,1)}[t,x,y,z] + 2\,jy\,W^{(0,0,1,0)}[t,x,y,z] + 2\,jx\,W^{(0,1,0,0)}[t,x,y,z] + n\,W^{(1,0,0,0)}[t,x,y,z]\right) - \frac{-2\left(n\left(jx\,(sxz+szx)+jy\,(syz+szy)+2\,jz\,szz\right)+jz\left(jx^2+jy^2+jz^2\right)\rho-4\,n^3\,Wz\,\rho+n^2\left(pz+qz+2\,jz\,\epsilon-2\,jz\,W\,\rho\right)\right)W^{(0,0,0,1)}[t,x,y,z]-4\,n\left(n\left(sxx+szz\right)+2\,jx\,jz\,\rho\right)Wx^{(0,0,0,1)}[t,x,y,z]+4\,n^2\left(syz+szy\right)Wy^{(0,0,0,1)}[t,x,y,z]+8\,jy\,jz\,n\,\rho\,Wy^{(0,0,0,1)}[t,x,y,z]+8\,n^2\,szz\,Wz^{(0,0,0,1)}[t,x,y,z]+8\,jz^2\,n\,\rho\,Wz^{(0,0,0,1)}[t,x,y,z]-2\,n^2\left(py+qy\right)W^{(0,0,1,0)}[t,x,y,z]-2\,jx\,n\,sxy\,W^{\ldots}}{\ldots} \end{pmatrix}$$

In[115]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[115]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\,\rho\,c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix}\right\}$$

(* Momentum *)

In[116]:=
```
(* Momentum current and supply according to x-vector *)
pvec = {0, 1, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Pfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Psupplyx = Assuming[assut, FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]]})
```

Out[117]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} jx\,\rho + \frac{px+\frac{jx\,sxx+jy\,syx+jz\,szx}{n}+\frac{jx\left(jx^2+jy^2+jz^2\right)\rho}{2\,n^2}-4\,n\,Wx\,\rho+jx\left(\epsilon+3\,W\,\rho\right)}{c^2} + O[\tfrac{1}{c}]^4 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + \frac{jx\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right) + \frac{Ay\left(2\,n^2\left(jx\,qy+jy\left(px+jx\,\epsilon\right)\right)+jy\left(jx^3+jx\left(jy^2+jz^2+6\,n^2\,W\right)-8\,n^3\,Wx\right)\rho\right)+Az\left(2\,n^2\left(jx\,qz+jz\left(px+jx\,\epsilon\right)\right)+jz\left(jx^3+jx\left(jy^2+jz^2+6\,n^2\,W\right)-8\,n^3\,Wx\right)\rho\right)+Ax\,jx\left(jx\left(jx^2+jy^2+jz^2\right)\rho-8\,n^3\,Wx\,\rho+2\,n^2\left(px+qx+jx\,\epsilon+3\,jx\,W\,\rho\right)\right)-n\left(Ax\,Vx+Ay\,Vy+Az\,Vz\right)\left(2\,n\left(jx\,sxx+jy\,syx+jz\,szx\right)+jx\left(jx^2+jy^2+jz^2\right)\rho-8\,n^3\,Wx\,\rho+2\,n^2\left(px+jx\left(\epsilon+3\,W\,\rho\right)\right)\right)}{2\,n^3\,c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix} \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + \frac{\frac{\left(3\left(jx^2+jy^2+jz^2\right)\rho+2\,n\left(sxx+syy+szz+n\,\epsilon-n\,W\,\rho\right)\right)W^{(0,1,0,0)}[t,x,y,z]}{2\,n}-4\,\rho\left(jx\,Wx^{(0,1,0,0)}[t,x,y,z]+jy\,Wy^{(0,1,0,0)}[t,x,y,z]+jz\,Wz^{(0,1,0,0)}[t,x,y,z]\right)}{c^2} + O[\tfrac{1}{c}]^3 \end{pmatrix}$$

In[118]:=
```
MF@(MF /@ (FS[({Pfluxx, Psupplyx} /. replaceJu) /. replaceuUnorm]))
```

Out[118]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} n\,ux\,\rho + \frac{px+sxx\,ux+syx\,uy+szx\,uz-4\,n\,Wx\,\rho+n\,ux\left(\epsilon+\frac{1}{2}\left(U^2+6\,W\right)\rho\right)}{c^2} + O[\tfrac{1}{c}]^4 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + n\,ux\left(Ax\,(ux - Vx) + Ay\,(uy - Vy) + Az\,(uz - Vz)\right)\rho\right) + \frac{Ax\left(qx\,ux+px\,(ux-Vx)-\left(sxx\,ux+syx\,uy+szx\,uz\right)Vx+n\,ux\,(ux-Vx)\,\epsilon\right)+Ay\left(qy\,ux+px\,(uy-Vy)-\left(sxx\,ux+syx\,uy+szx\,uz\right)Vy+n\,ux\,(uy-Vy)\,\epsilon\right)+Az\left(qz\,ux+px\,(uz-Vz)-\left(sxx\,ux+syx\,uy+szx\,uz\right)Vz+n\,ux\,(uz-Vz)\,\epsilon\right)+\frac{1}{2}\,Ax\,n\,(ux-Vx)\left(U^2\,ux+6\,ux\,W-8\,Wx\right)\rho+\frac{1}{2}\,n\left(Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(U^2\,ux+6\,ux\,W-8\,Wx\right)\rho}{c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix} \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + \frac{\left(sxx+syy+szz+n\left(\epsilon+\frac{3\,U^2\,\rho}{2}-W\,\rho\right)\right)W^{(0,1,0,0)}[t,x,y,z]-4\,n\,\rho\left(ux\,Wx^{(0,1,0,0)}[t,x,y,z]+uy\,Wy^{(0,1,0,0)}[t,x,y,z]+uz\,Wz^{(0,1,0,0)}[t,x,y,z]\right)}{c^2} + O[\tfrac{1}{c}]^3 \end{pmatrix}$$

In[119]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[119]=

$$\left\{\begin{pmatrix} \left(\frac{jx}{n} + \frac{sxx}{jx\,\rho}\right) - \frac{2\,n\,sxx\left(jx\,sxx+jy\,syx+jz\,szx+n\left(px+jx\,\epsilon\right)\right)+\left(3\,jx^3\,sxx+2\,jx^2\left(-n\,qx+jy\,syx+jz\,szx\right)+jx\,sxx\left(jy^2+jz^2+6\,n^2\,W\right)-8\,n^3\,sxx\,Wx\right)\rho}{2\left(jx^2\,n^2\,\rho^2\right)c^2} + O[\tfrac{1}{c}]^4 \\ \left(\frac{jy}{n} + \frac{syx}{jx\,\rho}\right) - \frac{2\,n\,syx\left(jx\,sxx+jy\,syx+jz\,szx+n\left(px+jx\,\epsilon\right)\right)+jx\left(-2\,jx\,n\,qy+2\,jx\,jy\,sxx+jx^2\,syx+3\,jy^2\,syx+jz^2\,syx+2\,jy\,jz\,szx+6\,n^2\,syx\,W\right)\rho-8\,n^3\,syx\,Wx\,\rho}{2\left(jx^2\,n^2\,\rho^2\right)c^2} + O[\tfrac{1}{c}]^4 \\ \left(\frac{jz}{n} + \frac{szx}{jx\,\rho}\right) - \frac{2\,n\,szx\left(jx\,sxx+jy\,syx+jz\,szx+n\left(px+jx\,\epsilon\right)\right)+jx\left(-2\,jx\,n\,qz+2\,jx\,jz\,sxx+2\,jy\,jz\,syx+jx^2\,szx+jy^2\,szx+3\,jz^2\,szx+6\,n^2\,szx\,W\right)\rho-8\,n^3\,szx\,Wx\,\rho}{2\left(jx^2\,n^2\,\rho^2\right)c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} \left(ux + \frac{sxx}{n\,ux\,\rho}\right) - \frac{2\,sxx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\left(-2\,qx\,ux^2+2\,ux^2\left(syx\,uy+szx\,uz\right)+sxx\,ux\left(3\,ux^2+uy^2+uz^2+6\,W\right)-8\,sxx\,Wx\right)\rho}{2\left(n^2\,ux^2\,\rho^2\right)c^2} + O[\tfrac{1}{c}]^4 \\ \left(uy + \frac{syx}{n\,ux\,\rho}\right) - \frac{2\,syx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\left(-2\,qy\,ux^2+2\,ux\,uy\left(sxx\,ux+szx\,uz\right)+syx\,ux\left(ux^2+3\,uy^2+uz^2+6\,W\right)-8\,syx\,Wx\right)\rho}{2\left(n^2\,ux^2\,\rho^2\right)c^2} + O[\tfrac{1}{c}]^4 \\ \left(uz + \frac{szx}{n\,ux\,\rho}\right) - \frac{2\,szx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\left(-2\,qz\,ux^2+2\,ux\left(sxx\,ux+syx\,uy\right)uz+szx\,ux\left(ux^2+uy^2+3\,uz^2+6\,W\right)-8\,szx\,Wx\right)\rho}{2\left(n^2\,ux^2\,\rho^2\right)c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix}\right\}$$

In[120]:=
```
(* Momentum current and supply according to cov. x-vector *)
pvec = igg.{0, 1, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Pfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Psupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[121]//MatrixForm=

$$\begin{pmatrix}\begin{pmatrix} jx\,\rho + \frac{px+\frac{jx\,sxx+jy\,syx+jz\,szx}{n}+\frac{jx\left(jx^2+jy^2+jz^2\right)\rho}{2\,n^2}+jx\left(\epsilon+W\,\rho\right)}{c^2} + O[\tfrac{1}{c}]^4 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + \frac{jx\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\rho}{n}\right) + \frac{-2\,n^2\left(Ay\left(-jx\,qy+n\,px\,Vy+jx\,sxx\,Vy+jz\,szx\,Vy+2\,n\,syx\,W+jx\,n\,Vy\,\epsilon-jy\left(px-syx\,Vy+jx\,\epsilon\right)\right)+Az\left(-jx\,qz+n\,px\,Vz+jx\,sxx\,Vz+jy\,syx\,Vz+2\,n\,szx\,W+jx\,n\,Vz\,\epsilon-jz\left(px-szx\,Vz+jx\,\epsilon\right)\right)\right)-2\,Ax\,n^2\left(n\,px\,Vx+jy\,syx\,Vx+jz\,szx\,Vx+2\,n\,sxx\,W-jx^2\,\epsilon-jx\left(px+qx-Vx\left(sxx+n\,\epsilon\right)\right)\right)+jx\left(Ay\,jy+Az\,jz+Ax\left(jx-n\,Vx\right)-n\left(Ay\,Vy+Az\,Vz\right)\right)\left(jx^2+jy^2+jz^2+2\,n^2\,W\right)\rho}{2\,n^3\,c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix} \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + \frac{-\frac{\left(n\,(sxz+szx)+2\,jx\,jz\,\rho\right)W^{(0,0,0,1)}[t,x,y,z]}{n}-(sxy+syx)\,W^{(0,0,1,0)}[t,x,y,z]+(-sxx+syy+szz)\,W^{(0,1,0,0)}[t,x,y,z]+n\,\epsilon\,W^{(0,1,0,0)}[t,x,y,z]-\frac{\rho\left(-8\,jz\,n\,Wx^{(0,0,0,1)}[t,x,y,z]+4\,jx\,jy\,W^{(0,0,1,0)}[t,x,y,z]-8\,jy\,n\,Wx^{(0,0,1,0)}[t,x,y,z]+jx^2\,W^{(0,1,0,0)}[t,x,y,z]-3\,jy^2\,W^{(0,1,0,0)}[t,x,y,z]-3\,jz^2\,W^{(0,1,0,0)}[t,x,y,z]+\ldots\right)}{2\,n}}{c^2} + O[\tfrac{1}{c}]^3 \end{pmatrix}$$

In[122]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[122]=

$$\left\{\begin{pmatrix} \left(\frac{jx}{n} + \frac{sxx}{jx\,\rho}\right) - \frac{2\,n\,sxx\left(jx\,sxx+jy\,syx+jz\,szx+n\left(px+jx\,\epsilon\right)\right)+jx\left(3\,jx^2\,sxx+2\,jx\left(-n\,qx+jy\,syx+jz\,szx\right)+sxx\left(jy^2+jz^2+6\,n^2\,W\right)\right)\rho}{2\left(jx^2\,n^2\,\rho^2\right)c^2} + O[\tfrac{1}{c}]^4 \\ \left(\frac{jy}{n} + \frac{syx}{jx\,\rho}\right) - \frac{2\,n\,syx\left(jx\,sxx+jy\,syx+jz\,szx+n\left(px+jx\,\epsilon\right)\right)+jx\left(-2\,jx\,n\,qy+2\,jx\,jy\,sxx+jx^2\,syx+3\,jy^2\,syx+jz^2\,syx+2\,jy\,jz\,szx+6\,n^2\,syx\,W\right)\rho}{2\left(jx^2\,n^2\,\rho^2\right)c^2} + O[\tfrac{1}{c}]^4 \\ \left(\frac{jz}{n} + \frac{szx}{jx\,\rho}\right) - \frac{2\,n\,szx\left(jx\,sxx+jy\,syx+jz\,szx+n\left(px+jx\,\epsilon\right)\right)+jx\left(-2\,jx\,n\,qz+2\,jx\,jz\,sxx+2\,jy\,jz\,syx+jx^2\,szx+jy^2\,szx+3\,jz^2\,szx+6\,n^2\,szx\,W\right)\rho}{2\left(jx^2\,n^2\,\rho^2\right)c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} \left(ux + \frac{sxx}{n\,ux\,\rho}\right) - \frac{2\,sxx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\,ux\left(-2\,qx\,ux+2\,ux\left(syx\,uy+szx\,uz\right)+sxx\left(3\,ux^2+uy^2+uz^2+6\,W\right)\right)\rho}{2\left(n^2\,ux^2\,\rho^2\right)c^2} + O[\tfrac{1}{c}]^4 \\ \left(uy + \frac{syx}{n\,ux\,\rho}\right) - \frac{2\,syx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\,ux\left(-2\,qy\,ux+2\,uy\left(sxx\,ux+szx\,uz\right)+syx\left(ux^2+3\,uy^2+uz^2+6\,W\right)\right)\rho}{2\left(n^2\,ux^2\,\rho^2\right)c^2} + O[\tfrac{1}{c}]^4 \\ \left(uz + \frac{szx}{n\,ux\,\rho}\right) - \frac{2\,szx\left(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon\right)+n\,ux\left(-2\,qz\,ux+2\left(sxx\,ux+syx\,uy\right)uz+szx\left(ux^2+uy^2+3\,uz^2+6\,W\right)\right)\rho}{2\left(n^2\,ux^2\,\rho^2\right)c^2} + O[\tfrac{1}{c}]^4 \end{pmatrix}\right\}$$

(* Angular momentum *)

In[123]:=
```
(* Ang.momentum current and supply according to yz-vector *)
pvec = y*{0, 0, 0, 1} - z*{0, 0, 1, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF/@{Lfluxx = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Lsupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[124]//MatrixForm=

In[125]:=
```
MF@(MF/@(FS[({Lfluxx, Lsupplyx}/. replaceJu)/. replaceuUnorm]))
```

Out[125]//MatrixForm=

In[126]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]]/. replaceJu] // MF}
```

Out[126]=

In[127]:=
```
(* Ang.momentum current and supply according to yz-vector as g_{ab} x^{a} (Kopeinik & al.) *)
pvec = (gg.coords)[[3]]*{0, 0, 0, 1} - (gg.coords)[[4]]*{0, 0, 1, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF/@{Lfluxkx = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Lsupplykx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[128]//MatrixForm=

In[129]:=
```
MF@(MF/@(FS[({Lfluxkx, Lsupplykx}/. replaceJu)/. replaceuUnorm]))
```

Out[129]//MatrixForm=

In[130]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]]/. replaceJu] // MF}
```

Out[130]=

In[131]:=
```
(* Ang.momentum current and supply according to cov. yz-vector *)
pvec = igg.(y*{0, 0, 0, 1} - z*{0, 0, 1, 0}); Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF/@{Lfluxcx = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Lsupplycx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[132]//MatrixForm=

In[133]:=
```
MF@(MF/@(FS[({Lfluxcx, Lsupplycx}/. replaceJu)/. replaceuUnorm]))
```

Out[133]//MatrixForm=

In[134]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]]/. replaceJu] // MF}
```

Out[134]=

(* Boost momentum *)

In[135]:=
```
(* Ang.boost-momentum current and supply according to tx-vector *)
pvec = t*{0, 1, 0, 0} + x*{1, 0, 0, 0}/c^2; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]]*cc[[ ;; , ;; , ii]], {ii, 1, 4}])]];
MF@(MF/@{Bfluxx = FS[({{1, 0, 0, 0}, surface/(Δt)}.EPS.pvec)], Bsupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[136]//MatrixForm=

In[137]:=
```
MF@(MF/@(FS[({Bfluxx, Bsupplyx}/. replaceJu)/. replaceuUnorm]))
```

Out[137]//MatrixForm=

In[138]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]]/temp[[1]]] // MF, FS[temp[[2 ;; 4]]/temp[[1]]/. replaceJu] // MF}
```

Out[138]=

In[139]:=
```
(* Ang.boost-momentum current and supply according to cov. tx-vector as g_{ab} x^{a} (Kopeinik & al.) *)
pvec = -((gg.coords)[[1]] * {0, 1, 0, 0} - (gg.coords)[[2]] * {1, 0, 0, 0}) / c^2; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Bfluxkx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Bsupplykx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[140]//MatrixForm=
```
((jx t - n x) ρ + (2 n ((n px + jx sxx + jy syx + jz szx) t + n (jx t - n x) ϵ) + (jx^2 + jy^2 + jz^2 + 2 n^2 W) (jx t - n x) ρ)/(2 n^2 c^2) + O[1/c]^4
 ((Ax sxx + Ay syx + Az szx) t + ((Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) (jx t - n x) ρ)/n) + (-2 Ay n^2 (t (-jx qy + n px Vy + jx sxx Vy + jz szx Vy + 2 n syx W) + (n qy + jx syx + jz syz) x + jy (-px t + syx t Vy + syy x)) - 2 Az n^2 (t (-jx qz + n px Vz + jx sxx Vz + jy syx Vz + 2 n szx W) + (n qz + jx szx + jy szy) x + jz (-px t + szx t Vz + szz x)) + 2 Ay n^2 (jy - n Vy) (jx t - n x) ϵ + 2 Az n^2 (jz - n Vz) (jx t - n x) ϵ + 2 Ax n^2 (jx t (px + qx - sxx Vx) - t (n px Vx + jy syx Vx + jz szx Vx + 2 n sxx W) - jx sxx x - (n qx + jy sxy + jz sxz) x + ...)/(2 n^3 c^2)
 n t ρ W^(0,1,0,0)[t, x, y, z] - (2 (n (sxz + szx) t + 2 jz (jx t + n x) ρ) W^(0,0,0,1)[t,x,y,z] - 2 n^2 t ϵ W^(0,1,0,0)[t,x,y,z] + jx^2 t ρ W^(0,1,0,0)[t,x,y,z] - 3 jy^2 t ρ W^(0,1,0,0)[t,x,y,z] - 3 jz^2 t ρ W^(0,1,0,0)[t,x,y,z] + 6 n^2 t W ρ W^(0,1,0,0)[t,x,y,z] + 4 jx n x ρ W^(0,1,0,0)[t,x,y,z] + 2 n t ((sxy + syx) W^(0,0,1,0)[t,x,y,z] + (sxx - syy - szz) W^(0,1,0,0)[t,x,y,z]) + 4 jy ρ ((jx t + n x) W^(0,0,1,0)[t,x,y,z] + 2 n t (-Wx^(0,0,1,0)[t,x,y,z] + Wy^(0,1,0,0)[t,x,y,z])) + 8 jz n t ρ (-Wx^(0,0,0,1)[t,x,y,z] + Wz^(0,1,0,0)[t,x,y,z]) + 4 jx n t ρ W^(1,0,0,0)[t,x,y,z] ...)/(2 n c^2))
```

In[141]:=
```
MF@(MF /@ (FS[({Bfluxkx, Bsupplykx} /. replaceJu) /. replaceuUnorm]))
```

Out[141]//MatrixForm=
```
((n (t ux - x) ρ + (px t + sxx t ux + syx t uy + szx t uz + n t ux ϵ - n x ϵ + 1/2 n (U^2 + 2 W) (t ux - x) ρ)/c^2 + O[1/c]^4
 ((Ax sxx + Ay syx + Az szx) t + n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux - x) ρ) + (-Ay (t (-qy ux + (px + sxx ux + szx uz) Vy + 2 syx W) + (qy + syx ux + syz uz) x + uy (-px t + syx t Vy + syy x)) - Az (t (-qz ux + (px + sxx ux + syx uy) Vz + 2 szx W) + (qz + szx ux + szy uy) x + uz (-px t + szx t Vz + szz x)) + Ay n (uy - Vy) (t ux - x) ϵ + Az n (uz - Vz) (t ux - x) ϵ + Ax (t ux (px + qx - sxx Vx) - t ((px + syx uy + szx uz) Vx + 2 sxx W) - sxx ux x - (qx + sxy uy + sxz uz) x + n (ux - Vx) (t ux - x) ϵ) + 1/2 Ax n (ux - Vx) (U^2 ...)/c^2
 n t ρ W^(0,1,0,0)[t, x, y, z] + (-2 ((sxz + szx) t + 2 n uz (t ux + x) ρ) W^(0,0,0,1)[t,x,y,z] + 2 n t ϵ W^(0,1,0,0)[t,x,y,z] + 3 n t uy^2 ρ W^(0,1,0,0)[t,x,y,z] + 3 n t uz^2 ρ W^(0,1,0,0)[t,x,y,z] + n t (-U^2 + uy^2 + uz^2) ρ W^(0,1,0,0)[t,x,y,z] - 6 n t W ρ W^(0,1,0,0)[t,x,y,z] - 4 n ux x ρ W^(0,1,0,0)[t,x,y,z] - 2 t ((sxy + syx) W^(0,0,1,0)[t,x,y,z] + (sxx - syy - szz) W^(0,1,0,0)[t,x,y,z]) - 4 uy ρ (n (t ux + x) W^(0,0,1,0)[t,x,y,z] + 2 n t (-Wx^(0,0,1,0)[t,x,y,z] + Wy^(0,1,0,0)[t,x,y,z])) + 8 n t uz ρ (Wx^(0,0,0,1)[t,x,y,z] - Wz^(0,1,0,0)[t,x,y,z]) - 4 n t ux ρ W^(1,0,0,0) ...)/(2 c^2))
```

In[142]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[142]=
```
{((jx/n + sxx t/(jx t ρ - n x ρ)) - (2 n sxx t ((n px + jx sxx + jy syx + jz szx) t + n (jx t - n x) ϵ) + (jx t - n x) (t (3 jx^2 sxx + 2 jx (-n qx + jy syx + jz szx) + sxx (jy^2 + jz^2 + 6 n^2 W)) + 2 n (n qx + jx sxx + jy sxy + jz sxz) x) ρ)/(2 (n^2 (jx t - n x)^2 ρ^2) c^2) + O[1/c]^4
  (jy/n + syx t/(jx t ρ - n x ρ)) - (2 n syx t ((n px + jx sxx + jy syx + jz szx) t + n (jx t - n x) ϵ) + (jx t - n x) (t (-2 jx n qy + 2 jx jy sxx + jx^2 syx + 3 jy^2 syx + jz^2 syx + 2 jy jz szx + 6 n^2 syx W) + 2 n (n qy + jx syx + jy syy + jz syz) x) ρ)/(2 (n^2 (jx t - n x)^2 ρ^2) c^2) + O[1/c]^4
  (jz/n + szx t/(jx t ρ - n x ρ)) - (2 n szx t ((n px + jx sxx + jy syx + jz szx) t + n (jx t - n x) ϵ) + (jx t - n x) (t (-2 jx n qz + 2 jx jz sxx + 2 jy jz syx + jx^2 szx + jy^2 szx + 3 jz^2 szx + 6 n^2 szx W) + 2 n (n qz + jx szx + jy szy + jz szz) x) ρ)/(2 (n^2 (jx t - n x)^2 ρ^2) c^2) + O[1/c]^4),
 ((ux + sxx t/(n t ux ρ - n x ρ)) - (2 sxx t (t (px + sxx ux + syx uy + szx uz) + n (t ux - x) ϵ) + n (t ux - x) (2 t ux (syx uy + szx uz) + sxx t (3 ux^2 + uy^2 + uz^2 + 6 W) + 2 (sxx ux + sxy uy + sxz uz) x + 2 qx (-t ux + x)) ρ)/(2 (n^2 (-t ux + x)^2 ρ^2) c^2) + O[1/c]^4
  (uy + syx t/(n t ux ρ - n x ρ)) - (2 syx t (t (px + sxx ux + syx uy + szx uz) + n (t ux - x) ϵ) + n (t ux - x) (2 t uy (sxx ux + szx uz) + syx t (ux^2 + 3 uy^2 + uz^2 + 6 W) + 2 (syx ux + syy uy + syz uz) x + 2 qy (-t ux + x)) ρ)/(2 (n^2 (-t ux + x)^2 ρ^2) c^2) + O[1/c]^4
  (uz + szx t/(n t ux ρ - n x ρ)) - (2 szx t (t (px + sxx ux + syx uy + szx uz) + n (t ux - x) ϵ) + n (t ux - x) (2 t (sxx ux + syx uy) uz + szx t (ux^2 + uy^2 + 3 uz^2 + 6 W) + 2 (szx ux + szy uy + szz uz) x + 2 qz (-t ux + x)) ρ)/(2 (n^2 (-t ux + x)^2 ρ^2) c^2) + O[1/c]^4)}
```

In[143]:=
```
(* Ang.boost-momentum current and supply according to cov. tx-vector *)
pvec = igg.(t * {0, 1, 0, 0} - x * {1, 0, 0, 0}); Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Bfluxcx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], Bsupplycx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[144]//MatrixForm=
```
((jx t - n x) ρ + (2 n ((n px + jx sxx + jy syx + jz szx) t + n (jx t - n x) ϵ) + (jx^2 + jy^2 + jz^2 + 2 n^2 W) (jx t - n x) ρ)/(2 n^2 c^2) + O[1/c]^4
 ((Ax sxx + Ay syx + Az szx) t + ((Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) (jx t - n x) ρ)/n) + (-2 Ay n^2 (t (-jx qy + n px Vy + jx sxx Vy + jz szx Vy + 2 n syx W) + (n qy + jx syx + jz syz) x + jy (-px t + syx t Vy + syy x)) - 2 Az n^2 (t (-jx qz + n px Vz + jx sxx Vz + jy syx Vz + 2 n szx W) + (n qz + jx szx + jy szy) x + jz (-px t + szx t Vz + szz x)) + 2 Ay n^2 (jy - n Vy) (jx t - n x) ϵ + 2 Az n^2 (jz - n Vz) (jx t - n x) ϵ + 2 Ax n^2 (jx t (px + qx - sxx Vx) - t (n px Vx + jy syx Vx + jz szx Vx + 2 n sxx W) - jx sxx x - (n qx + jy sxy + jz sxz) x + ...)/(2 n^3 c^2)
 n t ρ W^(0,1,0,0)[t, x, y, z] - (2 (n (sxz + szx) t + 2 jz (jx t + n x) ρ) W^(0,0,0,1)[t,x,y,z] - 2 n^2 t ϵ W^(0,1,0,0)[t,x,y,z] + jx^2 t ρ W^(0,1,0,0)[t,x,y,z] - 3 jy^2 t ρ W^(0,1,0,0)[t,x,y,z] - 3 jz^2 t ρ W^(0,1,0,0)[t,x,y,z] + 6 n^2 t W ρ W^(0,1,0,0)[t,x,y,z] + 4 jx n x ρ W^(0,1,0,0)[t,x,y,z] + 2 n t ((sxy + syx) W^(0,0,1,0)[t,x,y,z] + (sxx - syy - szz) W^(0,1,0,0)[t,x,y,z]) + 4 jy ρ ((jx t + n x) W^(0,0,1,0)[t,x,y,z] + 2 n t (-Wx^(0,0,1,0)[t,x,y,z] + Wy^(0,1,0,0)[t,x,y,z])) + 8 jz n t ρ (-Wx^(0,0,0,1) ...)/(2 n c^2))
```

In[145]:=
```
MF@(MF /@ (FS[({Bfluxcx, Bsupplycx} /. replaceJu) /. replaceuUnorm]))
```

Out[145]//MatrixForm=
```
((n (t ux - x) ρ + (px t + sxx t ux + syx t uy + szx t uz + n t ux ϵ - n x ϵ + 1/2 n (U^2 + 2 W) (t ux - x) ρ)/c^2 + O[1/c]^4
 ((Ax sxx + Ay syx + Az szx) t + n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux - x) ρ) + (-Ay (t (-qy ux + (px + sxx ux + szx uz) Vy + 2 syx W) + (qy + syx ux + syz uz) x + uy (-px t + syx t Vy + syy x)) - Az (t (-qz ux + (px + sxx ux + syx uy) Vz + 2 szx W) + (qz + szx ux + szy uy) x + uz (-px t + szx t Vz + szz x)) + Ay n (uy - Vy) (t ux - x) ϵ + Az n (uz - Vz) (t ux - x) ϵ + Ax (t ux (px + qx - sxx Vx) - t ((px + syx uy + szx uz) Vx + 2 sxx W) - sxx ux x - (qx + sxy uy + sxz uz) x + n (ux - Vx) (t ux - x) ϵ) + 1/2 Ax n (ux - Vx) (U^2 ...)/c^2
 n t ρ W^(0,1,0,0)[t, x, y, z] + (-2 ((sxz + szx) t + 2 n uz (t ux + x) ρ) W^(0,0,0,1)[t,x,y,z] + 2 n t ϵ W^(0,1,0,0)[t,x,y,z] + 3 n t uy^2 ρ W^(0,1,0,0)[t,x,y,z] + 3 n t uz^2 ρ W^(0,1,0,0)[t,x,y,z] + n t (-U^2 + uy^2 + uz^2) ρ W^(0,1,0,0)[t,x,y,z] - 6 n t W ρ W^(0,1,0,0)[t,x,y,z] - 4 n ux x ρ W^(0,1,0,0)[t,x,y,z] - 2 t ((sxy + syx) W^(0,0,1,0)[t,x,y,z] + (sxx - syy - szz) W^(0,1,0,0)[t,x,y,z]) - 4 uy ρ (n (t ux + x) W^(0,0,1,0)[t,x,y,z] + 2 n t (-Wx^(0,0,1,0)[t,x,y,z] + Wy^(0,1,0,0)[t,x,y,z])) + 8 n t uz ρ (Wx^(0,0,0,1)[t,x,y,z] - Wz^(0,1,0,0)[t,x,y,z]) - 4 n t ux ρ W^(1,0,0,0) ...)/(2 c^2))
```

In[146]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[146]=
```
{((jx/n + sxx t/(jx t ρ - n x ρ)) - (2 n sxx t ((n px + jx sxx + jy syx + jz szx) t + n (jx t - n x) ϵ) + (jx t - n x) (t (3 jx^2 sxx + 2 jx (-n qx + jy syx + jz szx) + sxx (jy^2 + jz^2 + 6 n^2 W)) + 2 n (n qx + jx sxx + jy sxy + jz sxz) x) ρ)/(2 (n^2 (jx t - n x)^2 ρ^2) c^2) + O[1/c]^4
  (jy/n + syx t/(jx t ρ - n x ρ)) - (2 n syx t ((n px + jx sxx + jy syx + jz szx) t + n (jx t - n x) ϵ) + (jx t - n x) (t (-2 jx n qy + 2 jx jy sxx + jx^2 syx + 3 jy^2 syx + jz^2 syx + 2 jy jz szx + 6 n^2 syx W) + 2 n (n qy + jx syx + jy syy + jz syz) x) ρ)/(2 (n^2 (jx t - n x)^2 ρ^2) c^2) + O[1/c]^4
  (jz/n + szx t/(jx t ρ - n x ρ)) - (2 n szx t ((n px + jx sxx + jy syx + jz szx) t + n (jx t - n x) ϵ) + (jx t - n x) (t (-2 jx n qz + 2 jx jz sxx + 2 jy jz syx + jx^2 szx + jy^2 szx + 3 jz^2 szx + 6 n^2 szx W) + 2 n (n qz + jx szx + jy szy + jz szz) x) ρ)/(2 (n^2 (jx t - n x)^2 ρ^2) c^2) + O[1/c]^4),
 ((ux + sxx t/(n t ux ρ - n x ρ)) - (2 sxx t (t (px + sxx ux + syx uy + szx uz) + n (t ux - x) ϵ) + n (t ux - x) (2 t ux (syx uy + szx uz) + sxx t (3 ux^2 + uy^2 + uz^2 + 6 W) + 2 (sxx ux + sxy uy + sxz uz) x + 2 qx (-t ux + x)) ρ)/(2 (n^2 (-t ux + x)^2 ρ^2) c^2) + O[1/c]^4
  (uy + syx t/(n t ux ρ - n x ρ)) - (2 syx t (t (px + sxx ux + syx uy + szx uz) + n (t ux - x) ϵ) + n (t ux - x) (2 t uy (sxx ux + szx uz) + syx t (ux^2 + 3 uy^2 + uz^2 + 6 W) + 2 (syx ux + syy uy + syz uz) x + 2 qy (-t ux + x)) ρ)/(2 (n^2 (-t ux + x)^2 ρ^2) c^2) + O[1/c]^4
  (uz + szx t/(n t ux ρ - n x ρ)) - (2 szx t (t (px + sxx ux + syx uy + szx uz) + n (t ux - x) ϵ) + n (t ux - x) (2 t (sxx ux + syx uy) uz + szx t (ux^2 + uy^2 + 3 uz^2 + 6 W) + 2 (szx ux + szy uy + szz uz) x + 2 qz (-t ux + x)) ρ)/(2 (n^2 (-t ux + x)^2 ρ^2) c^2) + O[1/c]^4)}
```

```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[supply.TTx]], {supply, {Dtxyzvec[[1]], Dtxyzvec[[2]], {0, 0, 0, 0}, Dgtxyzvec[[1]] * c^2, Dgtxyzvec[[2]], {0, 0, 0, 0}, DLxvec, Dxboost / c, {0, 0, 0, 0}, DgLxvec, Dgxboost, {0, 0, 0, 0}, Duu, Dtvecnorm}}]]

(* coordinate/internal/coordinate-proper energy and x-momentum, content and fluxes (TRANSPOSED) *)
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, igg[[1]] * c^2, igg[[2]], {0, 0, 0, 0}, Lxvec, xboost / c, {0, 0, 0, 0}, gLxvec, gxboost, {0, 0, 0, 0}, uu, tvecnorm}])]]]
```

Out[∘]//MatrixForm=
```
(-n ρ c^2 + (-((jx^2 + jy^2 + jz^2) ρ)/(2 n) + n (-ϵ + W ρ)) + O[1/c]^2    (-Ax jx - Ay jy - Az jz + Ax n Vx + Ay n Vy + Az n Vz) ρ c^2 + (-(Ax (n qx + jx sxx + jy sxy + jz sxz) + Ay (n qy + jx syx + jy syy + jz syz) + Az (n qz + jx szx + jy szy + jz szz))/n + (-Ax jx - Ay jy - Az jz + Ax n Vx + Ay n Vy + Az n Vz) ϵ - ((Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) (jx^2 + jy^2 + jz^2 - 2 n^2 W) ρ)/(2 n^2)) + O[1/c]^2
 jx ρ + O[1/c]^2    (Ax sxx + Ay syx + Az szx + (jx (Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) ρ)/n) + O[1/c]^2
 0    0
 n ρ c^2 + ((jx^2 + jy^2 + jz^2) ρ/(2 n) + n (ϵ + W ρ)) + O[1/c]^2    (Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) ρ c^2 + (2 n (Ax (jx sxx + jy sxy + jz sxz + n (qx + (jx - n Vx) ϵ)) + Ay (jx syx + jy syy + jz syz + n (qy + jy ϵ - n Vy ϵ)) + Az (jx szx + jy szy + jz szz + n (qz + jz ϵ - n Vz ϵ))) + (Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) (jx^2 + jy^2 + jz^2 + 2 n^2 W) ρ)/(2 n^2) + O[1/c]^2
 jx ρ + O[1/c]^2    (Ax sxx + Ay syx + Az szx + (jx (Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) ρ)/n) + O[1/c]^2
 0    0
 (jz y - jy z) ρ + O[1/c]^2    ((Ax sxz + Ay syz + Az szz) y - (Ax sxy + Ay syy + Az szy) z + ((Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) (jz y - jy z) ρ)/n) + O[1/c]^2
 (jx t - n x) ρ + O[1/c]^2    ((Ax sxx + Ay syx + Az szx) t + ((Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) (jx t - n x) ρ)/n) + O[1/c]^2
 0    0
 (jz y - jy z) ρ + O[1/c]^2    ((Ax sxz + Ay syz + Az szz) y - (Ax sxy + Ay syy + Az szy) z + ((Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) (jz y - jy z) ρ)/n) + O[1/c]^2
 (-jx t ρ + n x ρ) + O[1/c]^2    (-((Ax sxx + Ay syx + Az szx) t) - ((Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) (jx t - n x) ρ)/n) + O[1/c]^2
 0    0
 -n ρ c^2 - n ϵ + O[1/c]^2    (-Ax jx - Ay jy - Az jz + Ax n Vx + Ay n Vy + Az n Vz) ρ c^2 + (-Ax (qx + (jx - n Vx) ϵ) - Ay (qy + jy ϵ - n Vy ϵ) - Az (qz + jz ϵ - n Vz ϵ)) + O[1/c]^2
 -n ρ c^2 + (-n ϵ - ((jx^2 + jy^2 + jz^2) ρ)/(2 n)) + O[1/c]^2    (-Ax jx - Ay jy - Az jz + Ax n Vx + Ay n Vy + Az n Vz) ρ c^2 + (-(Ax (n qx + jx sxx + jy sxy + jz sxz) + Ay (n qy + jx syx + jy syy + jz syz) + Az (n qz + jx szx + jy szy + jz szz))/n + (-Ax jx - Ay jy - Az jz + Ax n Vx + Ay n Vy + Az n Vz) ϵ - ((jx^2 + jy^2 + jz^2) (Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) ρ)/(2 n^2)) + O[1/c]^2)
```

```
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, igg[[1]] * c^2, igg[[2]], {0, 0, 0, 0}, Lxvec, xboost / c, {0, 0, 0, 0}, gLxvec, gxboost, {0, 0, 0, 0}, uu, tvecnorm}]) /. replaceJu]]]
```

Out[∘]//MatrixForm=
```
(-n ρ c^2 - 1/2 n (2 ϵ + (ux^2 + uy^2 + uz^2 - 2 W) ρ) + O[1/c]^2    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c^2 + (-Ax (qx + sxx ux + sxy uy + sxz uz + n (ux - Vx) ϵ) - Ay (qy + syx ux + syy uy + syz uz + n (uy - Vy) ϵ) - Az (qz + szx ux + szy uy + szz uz + n (uz - Vz) ϵ) - 1/2 n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (ux^2 + uy^2 + uz^2 - 2 W) ρ) + O[1/c]^2
 n ux ρ + O[1/c]^2    (Ax sxx + Ay syx + Az szx + n ux (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) ρ) + O[1/c]^2
 0    0
 n ρ c^2 + (n ϵ + 1/2 n (ux^2 + uy^2 + uz^2 + 2 W) ρ) + O[1/c]^2    n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) ρ c^2 + (Ax (qx + sxx ux + sxy uy + sxz uz + n (ux - Vx) ϵ) + Ay (qy + syx ux + syy uy + syz uz + n (uy - Vy) ϵ) + Az (qz + szx ux + szy uy + szz uz + n (uz - Vz) ϵ) + 1/2 n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (ux^2 + uy^2 + uz^2 + 2 W) ρ) + O[1/c]^2
 n ux ρ + O[1/c]^2    (Ax sxx + Ay syx + Az szx + n ux (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) ρ) + O[1/c]^2
 0    0
 n (uz y - uy z) ρ + O[1/c]^2    (Ax sxz y + Ay syz y + Az szz y - Ax sxy z - Ay syy z - Az szy z + n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (uz y - uy z) ρ) + O[1/c]^2
 n (t ux - x) ρ + O[1/c]^2    ((Ax sxx + Ay syx + Az szx) t + n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux - x) ρ) + O[1/c]^2
 0    0
 n (uz y - uy z) ρ + O[1/c]^2    (Ax sxz y + Ay syz y + Az szz y - Ax sxy z - Ay syy z - Az szy z + n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (uz y - uy z) ρ) + O[1/c]^2
 n (-t ux + x) ρ + O[1/c]^2    (-((Ax sxx + Ay syx + Az szx) t) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux - x) ρ) + O[1/c]^2
 0    0
 -n ρ c^2 - n ϵ + O[1/c]^2    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c^2 + (-Ax qx - Ay qy - Az qz + n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ϵ) + O[1/c]^2
 -n ρ c^2 - 1/2 n (2 ϵ + (ux^2 + uy^2 + uz^2) ρ) + O[1/c]^2    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c^2 + (-Ax (qx + sxx ux + sxy uy + sxz uz + n (ux - Vx) ϵ) - Ay (qy + syx ux + syy uy + syz uz + n (uy - Vy) ϵ) - Az (qz + szx ux + szy uy + szz uz + n (uz - Vz) ϵ) - 1/2 n (ux^2 + uy^2 + uz^2) (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) ρ) + O[1/c]^2)
```

In[∘]:=
```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[supply.TTx]], {supply, {Dtxyzvec[[1]], Dtxyzvec[[2]], {0, 0, 0, 0}, Dgtxyzvec[[1]] * c^2, Dgtxyzvec[[2]], {0, 0, 0, 0}, DLxvec, Dxboost / c, {0, 0, 0, 0}, DgLxvec, Dgxboost, {0, 0, 0, 0}, Duu, Dtvecnorm}}]]
```

Out[∘]//MatrixForm=
```
(ρ n[t, x, y, z] W^(1,0,0,0)[t, x, y, z] + O[1/c]^2
 ρ n[t, x, y, z] W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
 0
 (2 ρ n[t, x, y, z] × uz[t, x, y, z] W^(0,0,0,1)[t, x, y, z] + 2 ρ n[t, x, y, z] × uy[t, x, y, z] W^(0,0,1,0)[t, x, y, z] + 2 ρ n[t, x, y, z] × ux[t, x, y, z] W^(0,1,0,0)[t, x, y, z] + ρ n[t, x, y, z] W^(1,0,0,0)[t, x, y, z]) + O[1/c]^2
 ρ n[t, x, y, z] W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
 0
 (y ρ n[t, x, y, z] W^(0,0,0,1)[t, x, y, z] - z ρ n[t, x, y, z] W^(0,0,1,0)[t, x, y, z]) + O[1/c]^2
 t ρ n[t, x, y, z] W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
 0
 (y ρ n[t, x, y, z] W^(0,0,0,1)[t, x, y, z] - z ρ n[t, x, y, z] W^(0,0,1,0)[t, x, y, z]) + O[1/c]^2
 -t ρ n[t, x, y, z] W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
 0
 (sxz jx^(0,0,0,1)[t,x,y,z]/(2 n[t,x,y,z]) + szx jx^(0,0,0,1)[t,x,y,z]/(2 n[t,x,y,z]) - ρ jx[t,x,y,z] × uz[t,x,y,z] jx^(0,0,0,1)[t,x,y,z]/n[t,x,y,z] + ρ ux[t, x, y, z] × uz[t, x, y, z] jx^(0,0,0,1)[t, x, y, z] + syz jy^(0,0,0,1)[t,x,y,z]/(2 n[t,x,y,z]) + szy jy^(0,0,0,1)[t,x,y,z]/(2 n[t,x,y,z]) - ρ jy[t,x,y,z] × uz[t,x,y,z] jy^(0,0,0,1)[t,x,y,z]/n[t,x,y,z] + ρ uy[t, x, y, z] × uz[t, x, y, z] jy^(0,0,0,1)[t, x, y, z] + szz jz^(0,0,0,1)[t,x,y,z]/n[t,x,y,z] - ρ jz[t,x,y,z] × uz[t,x,y,z] jz^(0,0,0,1)[t,x,y,z]/n[t,x,y,z] + ρ uz[t, x, y, z]^2 jz^(0,0,0,1) ...
 (-ρ n[t, x, y, z] × uz[t, x, y, z] W^(0,0,0,1)[t, x, y, z] - ρ n[t, x, y, z] × uy[t, x, y, z] W^(0,0,1,0)[t, x, y, z] - ρ n[t, x, y, z] × ux[t, x, y, z] W^(0,1,0,0)[t, x, y, z]) + O[1/c]^2)
```

```
In[•]:= (* covariant derivatives of coordinate 4-vectors (equivalent to Christoffel symbols), for later use *)
(Dtxyzvec = Table[ Assuming[assut, Expand //@ FS@PowerExpand[(D[IdentityMatrix[4][[aa]], {coords}] + Sum[IdentityMatrix[4][[aa]][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]], {aa, 1, 4}]);
```

```
In[•]:= (* normalized coordinate-t 4-vector*)
tvecnorm = Assuming[assut, Expand //@ FS@PowerExpand[c * {1, 0, 0, 0} / Sqrt[-gg[[1, 1]]]]]
```

Out[•]= $\left\{1+\frac{W}{c^2}+O\left[\frac{1}{c}\right]^4, 0, 0, 0\right\}$

```
In[•]:= (* and its covariant derivative *)
(Dtvecnorm = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[tvecnorm], {coords}] + Sum[tW[tvecnorm][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]) // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ -W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 & 0 \\ -W^{(0,0,1,0)}[t,x,y,z]+O[\frac{1}{c}]^2 & 0 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 \\ -W^{(0,0,0,1)}[t,x,y,z]+O[\frac{1}{c}]^2 & 0 & 0 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[•]:= (* "raised" coordinate 4-covectors *)
(gtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[igg.IdentityMatrix[4]]]) // MF
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -(\frac{1}{c})^2-\frac{2W}{c^4}+O[\frac{1}{c}]^6 & 0 & 0 & 0 \\ 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 & 0 \\ 0 & 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 \\ 0 & 0 & 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[•]:= (* and their covariant derivatives *)
(Dgtxyzvec = Table[ Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[igg[[aa]]], {coords}] + Sum[tW[igg[[aa]][[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]], {aa, 1, 4}]);
```

```
In[•]:= (* x-component of rot vector *)
Lxvec = Assuming[assut, Expand //@ FS@PowerExpand[{0, 0, -z, y}]]
```

Out[•]= $\{0, 0, -z, y\}$

```
In[•]:= (* and its covariant derivative *)
showf[assut][DLxvec = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[Lxvec], {coords}] + Sum[tW[Lxvec][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{-y W^{(0,0,0,1)}[t,x,y,z]+z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 & -\frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 & \frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 \\ 0 & \frac{y W^{(0,0,0,1)}[t,x,y,z]-z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ -\frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{y W^{(0,0,0,1)}[t,x,y,z]-z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -1+\frac{-z W^{(0,0,0,1)}[t,x,y,z]-y W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ \frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 1+\frac{z W^{(0,0,0,1)}[t,x,y,z]+y W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{y W^{(0,0,0,1)}[t,x,y,z]-z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[•]:= (* "raised" x-component of rot co-vector *)
gLxvec = Assuming[assut, Expand //@ FS@PowerExpand[igg.{0, 0, -z, y}]]
```

Out[•]= $\left\{0, 0, -z+\frac{2 W z}{c^2}+O\left[\frac{1}{c}\right]^4, y-\frac{2 (W y)}{c^2}+O\left[\frac{1}{c}\right]^4\right\}$

```
In[•]:= (* and its covariant derivative *)
showf[assut][DgLxvec = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[gLxvec], {coords}] + Sum[tW[gLxvec][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{-y W^{(0,0,0,1)}[t,x,y,z]+z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 & -\frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 & \frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 \\ 0 & \frac{y W^{(0,0,0,1)}[t,x,y,z]-z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ \frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{y W^{(0,0,0,1)}[t,x,y,z]+z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -1+\frac{2 W[t,x,y,z]+z W^{(0,0,0,1)}[t,x,y,z]-y W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ -\frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 1+\frac{-2 W[t,x,y,z]+z W^{(0,0,0,1)}[t,x,y,z]-y W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{-y W^{(0,0,0,1)}[t,x,y,z]-z W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[•]:= (* x-component of boost vector *)
xboost = Assuming[assut, Expand //@ FS@PowerExpand[{x / c, t * c, 0, 0}]]
```

Out[•]= $\left\{\frac{x}{c}, c\,t, 0, 0\right\}$

```
In[•]:= (* and its covariant derivative *)
showf[assut][Dxboost = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[xboost], {coords}] + Sum[tW[xboost][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -\frac{t W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{1}{c}+\frac{-x W^{(0,1,0,0)}[t,x,y,z]+t W^{(1,0,0,0)}[t,x,y,z]}{c^3}+O[\frac{1}{c}]^5 & -\frac{x W^{(0,0,1,0)}[t,x,y,z]}{c^3}+O[\frac{1}{c}]^5 & -\frac{x W^{(0,0,0,1)}[t,x,y,z]}{c^3}+O[\frac{1}{c}]^5 \\ c+\frac{-x W^{(0,1,0,0)}[t,x,y,z]+t W^{(1,0,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{t W^{(0,0,1,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{t W^{(0,0,0,1)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 \\ -\frac{x W^{(0,0,1,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & -\frac{t W^{(0,0,1,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & 0 \\ -\frac{x W^{(0,0,0,1)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & -\frac{t W^{(0,0,0,1)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 & 0 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c}+O[\frac{1}{c}]^3 \end{pmatrix}$$

```
In[•]:= (* "raised" x-component of boost co-vector *)
gxboost = Assuming[assut, Expand //@ FS@PowerExpand[igg.{x, -t, 0, 0}]]
```

Out[•]= $\left\{-\frac{x}{c^2}-\frac{2 (W x)}{c^4}+O\left[\frac{1}{c}\right]^6, -t+\frac{2 t W}{c^2}+O\left[\frac{1}{c}\right]^4, 0, 0\right\}$

```
In[•]:= (* and its covariant derivative *)
showf[assut][Dgxboost = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[gxboost], {coords}] + Sum[tW[gxboost][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -(\frac{1}{c})^2+\frac{-2 W[t,x,y,z]-x W^{(0,1,0,0)}[t,x,y,z]-t W^{(1,0,0,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 & -\frac{x W^{(0,0,1,0)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 & -\frac{x W^{(0,0,0,1)}[t,x,y,z]}{c^4}+O[\frac{1}{c}]^6 \\ -1+\frac{2 W[t,x,y,z]+x W^{(0,1,0,0)}[t,x,y,z]+t W^{(1,0,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t W^{(0,0,0,1)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \\ \frac{x W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t W^{(0,0,1,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & -\frac{t W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 \\ \frac{x W^{(0,0,0,1)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & \frac{t W^{(0,0,0,1)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 & 0 & -\frac{t W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
(* content and flux of coordinatevector-energy and coordinatevector-momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2+\left(-\frac{Ax(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz)+Ay(n\,qy+jx\,syx+jy\,syy+jz\,syz)+Az(n\,qz+jx\,szx+jy\,szy+jz\,szz)}{n}+(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\epsilon-\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}\right)+O[\frac{1}{c}]^2 \\ jx\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jy\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+\frac{jy(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O[\frac{1}{c}]^2 \\ jz\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+\frac{jz(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O[\frac{1}{c}]^2 \end{pmatrix}$$

```
In[•]:= (* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[Dtxyzvec[[aa,  ;; ,  ;;]].TTx]], {aa, 1, 4}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \rho\,n[t,x,y,z]\,W^{(1,0,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ \rho\,n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ \rho\,n[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ \rho\,n[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]+O[\frac{1}{c}]^2 \end{pmatrix}$$

```
(* content and flux of raised coordinatecovector-energy and coordinatecovector-momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

```
(* content and flux of coord-energy and momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxEPS = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 + \left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n} + n(-\epsilon + W\rho)\right) + O\left[\frac{1}{c}\right]^2 & (-Ax\,jx - Ay\,jy - Az\,jz + Ax\,n\,Vx + Ay\,n\,Vy + Az\,n\,Vz)\rho c^2 + \left(-\frac{Ax(n\,qx + jx\,sxx + jy\,sxy + jz\,sxz) + Ay(n\,qy + jx\,syx + jy\,syy + jz\,syz) + Az(n\,qz + jx\,szx + jy\,szy + jz\,szz)}{n} + (-Ax\,jx - Ay\,jy - Az\,jz + Ax\,n\,Vx + Ay\,n\,Vy + Az\,n\,Vz)\epsilon - \frac{(Ay\,jy + Az\,jz + Ax(jx - n\,Vx) - n(Ay\,Vy + Az\,Vz))(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}\right) + O\left[\frac{1}{c}\right]^2 \\ jx\,\rho + O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx + Ay\,syx + Az\,szx + \frac{jx(Ay\,jy + Az\,jz + Ax(jx - n\,Vx) - n(Ay\,Vy + Az\,Vz))\rho}{n}\right) + O\left[\frac{1}{c}\right]^2 \\ jy\,\rho + O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxy + Ay\,syy + Az\,szy + \frac{jy(Ay\,jy + Az\,jz + Ax(jx - n\,Vx) - n(Ay\,Vy + Az\,Vz))\rho}{n}\right) + O\left[\frac{1}{c}\right]^2 \\ jz\,\rho + O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz + Ay\,syz + Az\,szz + \frac{jz(Ay\,jy + Az\,jz + Ax(jx - n\,Vx) - n(Ay\,Vy + Az\,Vz))\rho}{n}\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

```
(* content and flux of coord-energy and momentum for dust (TRANSPOSED) *)
shows[assut, 1][T[fluxdust = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.dust2]]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2}n\left(2\epsilon + (aux^2 + auy^2 + auz^2 - 2W)\rho\right) + O\left[\frac{1}{c}\right]^2 & (-Ax\,jx - Ay\,jy - Az\,jz + Ax\,n\,Vx + Ay\,n\,Vy + Az\,n\,Vz)\rho c^2 - \frac{1}{2}(Ay\,jy + Az\,jz + Ax(jx - n\,Vx) - n(Ay\,Vy + Az\,Vz))\left(2\epsilon + (aux^2 + auy^2 + auz^2 - 2W)\rho\right) + O\left[\frac{1}{c}\right]^2 \\ aux\,n\,\rho + O\left[\frac{1}{c}\right]^2 & aux(Ay\,jy + Az\,jz + Ax(jx - n\,Vx) - n(Ay\,Vy + Az\,Vz))\rho + O\left[\frac{1}{c}\right]^2 \\ auy\,n\,\rho + O\left[\frac{1}{c}\right]^2 & auy(Ay\,jy + Az\,jz + Ax(jx - n\,Vx) - n(Ay\,Vy + Az\,Vz))\rho + O\left[\frac{1}{c}\right]^2 \\ auz\,n\,\rho + O\left[\frac{1}{c}\right]^2 & auz(Ay\,jy + Az\,jz + Ax(jx - n\,Vx) - n(Ay\,Vy + Az\,Vz))\rho + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
shows[assut, 1][T[fluxEPS /. replaceJu]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2}n\left(2\epsilon + (ux^2 + uy^2 + uz^2 - 2W)\rho\right) + O\left[\frac{1}{c}\right]^2 & n(Ax(-ux + Vx) + Ay(-uy + Vy) + Az(-uz + Vz))\rho c^2 + \left(-Ax(qx + sxx\,ux + sxy\,uy + sxz\,uz + n(ux - Vx)\epsilon) - Ay(qy + syx\,ux + syy\,uy + syz\,uz + n(uy - Vy)\epsilon) - Az(qz + szx\,ux + szy\,uy + szz\,uz + n(uz - Vz)\epsilon) - \frac{1}{2}n(Ax(ux - Vx) + Ay(uy - Vy) + Az(uz - Vz))(ux^2 + uy^2 + uz^2 - 2W)\rho\right) + O\left[\frac{1}{c}\right]^2 \\ n\,ux\,\rho + O\left[\frac{1}{c}\right]^2 & (Ax\,sxx + Ay\,syx + Az\,szx + n\,ux(Ax(ux - Vx) + Ay(uy - Vy) + Az(uz - Vz))\rho) + O\left[\frac{1}{c}\right]^2 \\ n\,uy\,\rho + O\left[\frac{1}{c}\right]^2 & (Ax\,sxy + Ay\,syy + Az\,szy + n\,uy(Ax(ux - Vx) + Ay(uy - Vy) + Az(uz - Vz))\rho) + O\left[\frac{1}{c}\right]^2 \\ n\,uz\,\rho + O\left[\frac{1}{c}\right]^2 & (Ax\,sxz + Ay\,syz + Az\,szz + n\,uz(Ax(ux - Vx) + Ay(uy - Vy) + Az(uz - Vz))\rho) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* momentum flux = A.σ + P A.(u-V)*)
fluxPS = ({Ax, Ay, Az}.(S[[2 ;; 4, 2 ;; 4]]).{1, 0, 0} + EPS[[1, 2]] * ({Ax, Ay, Az}.({jx, jy, jz} / n - {Vx, Vy, Vz})))
```

Out[∘]=

$$\left(Ax\,sxx + Ay\,syx + Az\,szx + jx\left(Ax\left(\frac{jx}{n} - Vx\right) + Ay\left(\frac{jy}{n} - Vy\right) + Az\left(\frac{jz}{n} - Vz\right)\right)\rho\right) + \frac{1}{c^2}\left(Ax\left(\frac{jx}{n} - Vx\right) + Ay\left(\frac{jy}{n} - Vy\right) + Az\left(\frac{jz}{n} - Vz\right)\right)\left(px + \frac{jx\,sxx}{n} + \frac{jy\,syx}{n} + \frac{jz\,szx}{n} + jx\,\epsilon + \frac{jx^3\rho}{2n^2} + \frac{jx\,jy^2\rho}{2n^2} + \frac{jx\,jz^2\rho}{2n^2} + 3\,jx\,W\rho\right) + O\left[\frac{1}{c}\right]^4$$

In[∘]:=
```
shows[assut, 1][Expand //@ FS@PowerExpand[fluxEPS[[2, 2]] - fluxPS]]
```

Out[∘]//MatrixForm=

$$O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* energy flux = A.q + A.σ.u + E A.(u-V)*)
fluxE = -({Ax, Ay, Az}.{qx, qy, qz} + {Ax, Ay, Az}.(S[[2 ;; 4, 2 ;; 4]] + Qtens[[2 ;; 4, 2 ;; 4]]).{jx, jy, jz} / n + (-EPS[[1, 1]]) * {Ax, Ay, Az}.({jx, jy, jz} / n - {Vx, Vy, Vz}))
```

Out[∘]=

$$-n\left(Ax\left(\frac{jx}{n} - Vx\right) + Ay\left(\frac{jy}{n} - Vy\right) + Az\left(\frac{jz}{n} - Vz\right)\right)\rho c^2 + \left(-Ax\,qx - Ay\,qy - Az\,qz - \frac{1}{n}\left(jx(Ax\,sxx + Ay\,syx + Az\,szx) + jy(Ax\,sxy + Ay\,syy + Az\,szy) + jz(Ax\,sxz + Ay\,syz + Az\,szz)\right) - \left(Ax\left(\frac{jx}{n} - Vx\right) + Ay\left(\frac{jy}{n} - Vy\right) + Az\left(\frac{jz}{n} - Vz\right)\right)\left(n\epsilon + \frac{jx^2\rho}{2n} + \frac{jy^2\rho}{2n} + \frac{jz^2\rho}{2n} - n\,W\rho\right)\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
showf[assut][Expand //@ FS@PowerExpand[fluxEPS[[2, 1]] - fluxE]]
```

Out[∘]//MatrixForm=

$$O\left[\frac{1}{c}\right]^2$$

```
(* matter flux n A.(u-V) *)
shows[assut, 1][fluxNJ = Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.NJ /. replaceJu]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} n \\ n(Ax(ux - Vx) + Ay(uy - Vy) + Az(uz - Vz)) \end{pmatrix}$$

```
(* content and flux of coord-energy and momentum assuming no matter flux (transposed) *)
shows[Join[assut, {((surface / Δt).NJ) == 0} /. replaceJu], 1][T@fluxEPS /. replaceJu]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2}n\left(2\epsilon + (ux^2 + uy^2 + uz^2 - 2W)\rho\right) + O\left[\frac{1}{c}\right]^2 & \left(-Ax(qx + sxx\,ux + sxy\,uy + sxz\,uz) - Ay(qy + syx\,ux + syy\,uy + syz\,uz) - Az(qz + szx\,ux + szy\,uy + szz\,uz)\right) + O\left[\frac{1}{c}\right]^2 \\ n\,ux\,\rho + O\left[\frac{1}{c}\right]^2 & (Ax\,sxx + Ay\,syx + Az\,szx) + O\left[\frac{1}{c}\right]^2 \\ n\,uy\,\rho + O\left[\frac{1}{c}\right]^2 & (Ax\,sxy + Ay\,syy + Az\,szy) + O\left[\frac{1}{c}\right]^2 \\ n\,uz\,\rho + O\left[\frac{1}{c}\right]^2 & (Ax\,sxz + Ay\,syz + Az\,szz) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=

```
(* coordinate/internal/coordinate-proper energy and x-momentum, content and fluxes (TRANSPOSED) *)
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, Lxvec, Lxvec2, xboost / c, xboost2, uu, ntvec}]) /. replaceJu]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2}n\left(2\epsilon + (ux^2 + uy^2 + uz^2 - 2W)\rho\right) + O\left[\frac{1}{c}\right]^2 & n(Ax(-ux + Vx) + Ay(-uy + Vy) + Az(-uz + Vz))\rho c^2 + \left(-Ax(qx + sxx\,ux + sxy\,uy + sxz\,uz + n(ux - Vx)\epsilon) - Ay(qy + syx\,ux + syy\,uy + syz\,uz + n(uy - Vy)\epsilon) - Az(qz + szx\,ux + szy\,uy + szz\,uz + n(uz - Vz)\epsilon) - \frac{1}{2}n(Ax(ux - Vx) + Ay(uy - Vy) + Az(uz - Vz))(ux^2 + uy^2 + uz^2 - 2W)\rho\right) + O\left[\frac{1}{c}\right]^2 \\ n\,ux\,\rho + O\left[\frac{1}{c}\right]^2 & (Ax\,sxx + Ay\,syx + Az\,szx + n\,ux(Ax(ux - Vx) + Ay(uy - Vy) + Az(uz - Vz))\rho) + O\left[\frac{1}{c}\right]^2 \\ n(uz\,y - uy\,z)\rho + O\left[\frac{1}{c}\right]^2 & (Ax\,sxz\,y + Ay\,syz\,y + Az\,szz\,y - Ax\,sxy\,z - Ay\,syy\,z - Az\,szy\,z + n(Ax(ux - Vx) + Ay(uy - Vy) + Az(uz - Vz))(uz\,y - uy\,z)\rho) + O\left[\frac{1}{c}\right]^2 \\ n(uz\,y - uy\,z)\rho + O\left[\frac{1}{c}\right]^2 & (Ax\,sxz\,y + Ay\,syz\,y + Az\,szz\,y - Ax\,sxy\,z - Ay\,syy\,z - Az\,szy\,z + n(Ax(ux - Vx) + Ay(uy - Vy) + Az(uz - Vz))(uz\,y - uy\,z)\rho) + O\left[\frac{1}{c}\right]^2 \\ -n(t\,ux + x)\rho + O\left[\frac{1}{c}\right]^2 & (-((Ax\,sxx + Ay\,syx + Az\,szx)t) - n(Ax(ux - Vx) + Ay(uy - Vy) + Az(uz - Vz))(t\,ux + x)\rho) + O\left[\frac{1}{c}\right]^2 \\ n(-t\,ux + x)\rho + O\left[\frac{1}{c}\right]^2 & (-((Ax\,sxx + Ay\,syx + Az\,szx)t) - n(Ax(ux - Vx) + Ay(uy - Vy) + Az(uz - Vz))(t\,ux - x)\rho) + O\left[\frac{1}{c}\right]^2 \\ -n\rho c^2 - n\epsilon + O\left[\frac{1}{c}\right]^2 & n(Ax(-ux + Vx) + Ay(-uy + Vy) + Az(-uz + Vz))\rho c^2 + (-Ax\,qx - Ay\,qy - Az\,qz + n(Ax(-ux + Vx) + Ay(-uy + Vy) + Az(-uz + Vz))\epsilon) + O\left[\frac{1}{c}\right]^2 \\ -n\rho c^2 - \frac{1}{2}n\left(2\epsilon + (ux^2 + uy^2 + uz^2)\rho\right) + O\left[\frac{1}{c}\right]^2 & n(Ax(-ux + Vx) + Ay(-uy + Vy) + Az(-uz + Vz))\rho c^2 + \left(-Ax(qx + sxx\,ux + sxy\,uy + sxz\,uz + n(ux - Vx)\epsilon) - Ay(qy + syx\,ux + syy\,uy + syz\,uz + n(uy - Vy)\epsilon) - Az(qz + szx\,ux + szy\,uy + szz\,uz + n(uz - Vz)\epsilon) - \frac{1}{2}n(ux^2 + uy^2 + uz^2)(Ax(ux - Vx) + Ay(uy - Vy) + Az(uz - Vz))\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
show2[assut, 1][T[variousfluxes /. {Vy → 0, Vz → 0, uy → 0, uz → 0}]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2}n\left(2\epsilon + (ux^2 - 2W)\rho\right) + O\left[\frac{1}{c}\right]^2 & Ax\,n(-ux + Vx)\rho c^2 + \left(-Ay(qy + syx\,ux) - Az(qz + szx\,ux) - Ax(qx + sxx\,ux + n(ux - Vx)\epsilon) - \frac{1}{2}Ax\,n(ux - Vx)(ux^2 - 2W)\rho\right) + O\left[\frac{1}{c}\right]^2 \\ n\,ux\,\rho + O\left[\frac{1}{c}\right]^2 & (Ax\,sxx + Ay\,syx + Az\,szx + Ax\,n\,ux(ux - Vx)\rho) + O\left[\frac{1}{c}\right]^2 \\ O\left[\frac{1}{c}\right]^2 & (Ax\,sxz\,y + Ay\,syz\,y + Az\,szz\,y - Ax\,sxy\,z - Ay\,syy\,z - Az\,szy\,z) + O\left[\frac{1}{c}\right]^2 \\ O\left[\frac{1}{c}\right]^2 & (Ax\,sxz\,y + Ay\,syz\,y + Az\,szz\,y - Ax\,sxy\,z - Ay\,syy\,z - Az\,szy\,z) + O\left[\frac{1}{c}\right]^2 \\ -n(t\,ux + x)\rho + O\left[\frac{1}{c}\right]^2 & (-((Ax\,sxx + Ay\,syx + Az\,szx)t) - Ax\,n(ux - Vx)(t\,ux + x)\rho) + O\left[\frac{1}{c}\right]^2 \\ n(-t\,ux + x)\rho + O\left[\frac{1}{c}\right]^2 & (-((Ax\,sxx + Ay\,syx + Az\,szx)t) - Ax\,n(ux - Vx)(t\,ux - x)\rho) + O\left[\frac{1}{c}\right]^2 \\ -n\rho c^2 - n\epsilon + O\left[\frac{1}{c}\right]^2 & Ax\,n(-ux + Vx)\rho c^2 + (-Ax\,qx - Ay\,qy - Az\,qz + Ax\,n(-ux + Vx)\epsilon) + O\left[\frac{1}{c}\right]^2 \\ -n\rho c^2 - \frac{1}{2}n\left(2\epsilon + ux^2\rho\right) + O\left[\frac{1}{c}\right]^2 & Ax\,n(-ux + Vx)\rho c^2 + \left(-Ay(qy + syx\,ux) - Az(qz + szx\,ux) - Ax(qx + sxx\,ux + n(ux - Vx)\epsilon) - \frac{1}{2}Ax\,n\,ux^2(ux - Vx)\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* velocity of energy *)
shows[assut, 5][(EPS.{1, 0, 0, 0})[[2 ;; 4]] / (EPS.{1, 0, 0, 0})[[1]] /. replaceJu]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} ux + \frac{qx + sxx\,ux + sxy\,uy + sxz\,uz}{n\rho c^2} + O\left[\frac{1}{c}\right]^4 \\ uy + \frac{qy + syx\,ux + syy\,uy + syz\,uz}{n\rho c^2} + O\left[\frac{1}{c}\right]^4 \\ uz + \frac{qz + szx\,ux + szy\,uy + szz\,uz}{n\rho c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[∘]:=
```
temp = SeriesCoefficient[tt.{1, 0, 0, 0}, {c, Infinity, -2}];
shows[assut, 5][(tt.{1, 0, 0, 0} - temp * c ^ 2)[[2 ;; 4]] / (tt.{1, 0, 0, 0} - temp * c ^ 2)[[1]] /. j2v]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \left(ux + \frac{2(qx + sxx\,ux + sxy\,uy + sxz\,uz)}{2n\epsilon + n(ux^2 + uy^2 + uz^2 - 2W)\rho}\right) + O\left[\frac{1}{c}\right]^2 \\ \left(uy + \frac{2(qy + syx\,ux + syy\,uy + syz\,uz)}{2n\epsilon + n(ux^2 + uy^2 + uz^2 - 2W)\rho}\right) + O\left[\frac{1}{c}\right]^2 \\ \left(uz + \frac{2(qz + szx\,ux + szy\,uy + szz\,uz)}{2n\epsilon + n(ux^2 + uy^2 + uz^2 - 2W)\rho}\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
showf[assut][{variousfluxes[[ ;; , 1]] - variousfluxes[[ ;; , 7]], variousfluxes[[ ;; , 1]] - variousfluxes[[ ;; , 8]], variousfluxes[[ ;; , 7]] - variousfluxes[[ ;; , 8]]}]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{1}{2}n\,ux^2\rho - \frac{1}{2}n\,uy^2\rho - \frac{1}{2}n\,uz^2\rho + n\,W\rho\right) + O\left[\frac{1}{c}\right]^2 & (-Ax\,sxx\,ux - Ay\,syx\,ux - Az\,szx\,ux - Ax\,sxy\,uy - Ay\,syy\,uy - Az\,szy\,uy - Ax\,sxz\,uz - Ay\,syz\,uz - Az\,szz\,uz - \frac{1}{2}Ax\,n\,ux^3\rho - \frac{1}{2}Ay\,n\,ux^2\,uy\,\rho - \frac{1}{2}Ax\,n\,ux\,uy^2\rho - \frac{1}{2}Ay\,n\,uy^3\rho - \frac{1}{2}Az\,n\,ux^2\,uz\,\rho - \frac{1}{2}Az\,n\,uy^2\,uz\,\rho - \frac{1}{2}Ax\,n\,ux\,uz^2\rho - \frac{1}{2}Ay\,n\,uy\,uz^2\rho - \frac{1}{2}Az\,n\,uz^3\rho + \frac{1}{2}Ax\,n\,ux^2\,Vx\,\rho + \frac{1}{2}Ax\,n\,uy^2\,Vx\,\rho + \frac{1}{2}Ax\,n \\ n\,W\rho + O\left[\frac{1}{c}\right]^2 & (Ax\,n\,ux\,W\rho + Ay\,n\,uy\,W\rho + Az\,n\,uz\,W\rho - Ax\,n\,Vx\,W\rho - Ay\,n\,Vy\,W\rho - Az\,n\,Vz\,W\rho) + O\left[\frac{1}{c}\right]^2 \\ \left(\frac{1}{2}n\,ux^2\rho + \frac{1}{2}n\,uy^2\rho + \frac{1}{2}n\,uz^2\rho\right) + O\left[\frac{1}{c}\right]^2 & (Ax\,sxx\,ux + Ay\,syx\,ux + Az\,szx\,ux + Ax\,sxy\,uy + Ay\,syy\,uy + Az\,szy\,uy + Ax\,sxz\,uz + Ay\,syz\,uz + Az\,szz\,uz + \frac{1}{2}Ax\,n\,ux^3\rho + \frac{1}{2}Ay\,n\,ux^2\,uy\,\rho + \frac{1}{2}Ax\,n\,ux\,uy^2\rho + \frac{1}{2}Ay\,n\,uy^3\rho + \frac{1}{2}Az\,n\,ux^2\,uz\,\rho + \frac{1}{2}Az\,n\,uy^2\,uz\,\rho + \frac{1}{2}Ax\,n\,ux\,uz^2\rho + \frac{1}{2}Ay\,n\,uy\,uz^2\rho + \frac{1}{2}Az\,n\,uz^3\rho - \frac{1}{2}Ax\,n\,ux^2\,Vx\,\rho - \frac{1}{2}Ax\,n\,uy^2\,Vx\,\rho - \frac{1}{2}Ax\,n\,u \end{pmatrix}$$

In[∘]:=
```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords⟦aa,;;,;;⟧].gg+Dcoords⟦aa,;;,;;⟧).TTx]],{aa,1,4}]]*)
show2[assut, 2][FS[(itjv[Tr[#.TTx]]) /. replaceJu]] & /@ {Dxyzvec⟦1, ;; , ;;⟧, Dxyzvec⟦2, ;; , ;;⟧, DLxvec, DLxvec2, Dxboost / c, Dxboost2, Duv, Dntvec} // MF
```
Out[∘]//MatrixForm=

$$\begin{pmatrix}
n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\\
n\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\\
n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\\
n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\\
-n\,\rho\left(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\\
-n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\\
\frac{1}{2}\left(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z]+(syz+szy)\left(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\right)+2\left(syy\,uy^{(0,0,1,0)}[t,x,y,z]+sxx\,ux^{(0,1,0,0)}[t,x,y,z]\right)+(sxy+syx)\left(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z]\right)+(sxz+szx)\left(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\right)\right)+O\left[\frac{1}{c}\right]^2\\
-n\,\rho\left(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.EPSsym.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, Lxvec, Lxvec2, xboost / c, xboost2, uu, ntvec}]) /. replaceJu]]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+sxy\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+sxz\,ux+syz\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac{1}{2}n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,sxy+Az\,sxz+n\,ux\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,syz\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\left(uz\,y-uy\,z\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,syz\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
-n\left(t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,sxy+Az\,sxz\right)t\right)-n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(t\,ux+x\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
n\left(-t\,ux+x\right)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left(\left(Ax\,sxx+Ay\,sxy+Az\,sxz\right)t\right)-n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O\left[\frac{1}{c}\right]^2\\
-n\rho c^2-n\epsilon+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\epsilon\right)+O\left[\frac{1}{c}\right]^2\\
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2+uy^2+uz^2\right)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+sxy\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+sxz\,ux+syz\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac{1}{2}n\left(ux^2+uy^2+uz^2\right)\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[∘]:=
```
TTx = tW[tjv[(EPSsym + T[EPSsym.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords⟦aa,;;,;;⟧].gg+Dcoords⟦aa,;;,;;⟧).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. replaceJu] & /@ {Dxyzvec⟦1, ;; , ;;⟧, Dxyzvec⟦2, ;; , ;;⟧, DLxvec, DLxvec2, Dxboost / c, Dxboost2, Duv, Dntvec} // MF
```
Out[∘]//MatrixForm=

$$\begin{pmatrix}
n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\\
n\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\\
n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\\
n\,\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\\
-n\,\rho\left(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2\\
-n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2\\
\left(szz\,uz^{(0,0,0,1)}[t,x,y,z]+syy\,uy^{(0,0,1,0)}[t,x,y,z]+syz\left(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\right)+sxx\,ux^{(0,1,0,0)}[t,x,y,z]+sxy\left(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z]\right)+sxz\left(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\right)\right)+O\left[\frac{1}{c}\right]^2\\
-n\,\rho\left(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z]\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[∘]:=
```
(* 2-vector of surface parallel to yz  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(yzsurface = ((T[{{0, 0, Ly, 0}}].{{0, 0, 0, Lz}} - T[{{0, 0, 0, Lz}}].{{0, 0, Ly, 0}}) /. {Ly → Ayz / Lz})) // MF
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & Ayz\\ 0 & 0 & -Ayz & 0 \end{pmatrix}$$

In[∘]:=
```
(* 2-vector of surface parallel to tx  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(txsurface = (-(T[{{Δt, 0, 0, 0}}].{{0, Lx, 0, 0}} - T[{{0, Lx, 0, 0}}].{{Δt, 0, 0, 0}}))) // MF
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & -Lx\,\Delta t & 0 & 0\\ Lx\,\Delta t & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[∘]:=
```
(* 2-vector of surface parallel to ty  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(tysurface = (-(T[{{Δt, 0, 0, 0}}].{{0, 0, Ly, 0}} - T[{{0, 0, Ly, 0}}].{{Δt, 0, 0, 0}}))) // MF
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -Ly\,\Delta t & 0\\ 0 & 0 & 0 & 0\\ Ly\,\Delta t & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[∘]:=
```
(* 2-vector of surface parallel to y moving to x  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(yVxsurface = (-(T[{{1, Vx, 0, 0}} * Δt].{{0, 0, Ly, 0}} - T[{{0, 0, Ly, 0}}].{{1, Vx, 0, 0}} * Δt))) // MF
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & -Ly\,\Delta t & 0\\ 0 & 0 & -Ly\,Vx\,\Delta t & 0\\ Ly\,\Delta t & Ly\,Vx\,\Delta t & 0 & 0\\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In[∘]:=
```
(Tr[T[txsurface].txsurface]) // MF
```
Out[∘]//MatrixForm=

$$2\,Lx^2\,\Delta t^2$$

In[∘]:=
```
(* Faraday tensor *)
repE = {Ex → Ex * c * Sqrt[μ * ϵ], Ey → Ey * c * Sqrt[μ * ϵ], Ez → Ez * c * Sqrt[μ * ϵ]};
fftemp = {{0, -Ex, -Ey, -Ez}, {0, 0, Bz, -By}, {0, 0, 0, Bx}, {0, 0, 0, 0}} /. repE;
showf[assut][F = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & -c\,Ex\sqrt{\epsilon}\sqrt{\mu} & -c\,Ey\sqrt{\epsilon}\sqrt{\mu} & -c\,Ez\sqrt{\epsilon}\sqrt{\mu}\\ c\,Ex\sqrt{\epsilon}\sqrt{\mu} & 0 & Bz & -By\\ c\,Ey\sqrt{\epsilon}\sqrt{\mu} & -Bz & 0 & Bx\\ c\,Ez\sqrt{\epsilon}\sqrt{\mu} & By & -Bx & 0 \end{pmatrix}$$

In[∘]:=
```
(FS[{Tr[yzsurface.T[F]], Tr[T[txsurface].F], Tr[T[tysurface].F], Tr[T[yVxsurface].F]} / 2]) // MF
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} Ayz\,Bx\\ c\,Ex\,Lx\,\Delta t\sqrt{\epsilon}\sqrt{\mu}\\ c\,Ey\,Ly\,\Delta t\sqrt{\epsilon}\sqrt{\mu}\\ Ly\,\Delta t\left(-Bz\,Vx+c\,Ey\sqrt{\epsilon}\sqrt{\mu}\right) \end{pmatrix}$$

In[∘]:=
```
(* charge-current-potential tensor *)
fftemp = {{0, -Hx, -Hy, -Hz}, {0, 0, Dz, -Dy}, {0, 0, 0, Dx}, {0, 0, 0, 0}};
showf[assut][H = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 0 & -Hx & -Hy & -Hz\\ Hx & 0 & Dz & -Dy\\ Hy & -Dz & 0 & Dx\\ Hz & Dy & -Dx & 0 \end{pmatrix}$$

In[∘]:=
```
showf[assut][tte = Assuming[assut, Expand //@ FS@PowerExpand[
      (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
    ]]]
```
Out[∘]//MatrixForm=

(⋯ 1 ⋯)

Full expression not available (original memory size: 0.7 MB)

In[∘]:=
```
shows[assut, 1][tte = Assuming[assut, Expand //@ FS@PowerExpand[
      (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
    ]]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix}
-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O\left[\frac{1}{c}\right]^2 & \frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O\left[\frac{1}{c}\right]^2 & \frac{(-Bz\,Ex+Bx\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O\left[\frac{1}{c}\right]^2 & \frac{(By\,Ex-Bx\,Ey)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O\left[\frac{1}{c}\right]^2\\
\frac{(-Bz\,Ey+By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(Bz\,Ey-By\,Ez)W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O\left[\frac{1}{c}\right]^2 & \frac{-Bx^2+By^2+Bz^2+(-Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O\left[\frac{1}{c}\right]^2 & -\frac{Bx\,By+Ex\,Ey\,\epsilon0\,\mu0}{\mu0}+O\left[\frac{1}{c}\right]^2 & -\frac{Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0}{\mu0}+O\left[\frac{1}{c}\right]^2\\
\frac{(Bz\,Ex-Bx\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(-Bz\,Ex+Bx\,Ez)W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O\left[\frac{1}{c}\right]^2 & -\frac{Bx\,By+Ex\,Ey\,\epsilon0\,\mu0}{\mu0}+O\left[\frac{1}{c}\right]^2 & \frac{Bx^2-By^2+Bz^2+(Ex^2-Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O\left[\frac{1}{c}\right]^2 & -\frac{By\,Bz+Ey\,Ez\,\epsilon0\,\mu0}{\mu0}+O\left[\frac{1}{c}\right]^2\\
\frac{(-By\,Ex+Bx\,Ey)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(By\,Ex-Bx\,Ey)W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O\left[\frac{1}{c}\right]^2 & -\frac{Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0}{\mu0}+O\left[\frac{1}{c}\right]^2 & -\frac{By\,Bz+Ey\,Ez\,\epsilon0\,\mu0}{\mu0}+O\left[\frac{1}{c}\right]^2 & \frac{Bx^2+By^2-Bz^2+(Ex^2+Ey^2-Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[∘]:=
```
showf[assut][T[tte.Inverse[gg]].gg - tte]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix}
O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^5 & O\left[\frac{1}{c}\right]^5 & O\left[\frac{1}{c}\right]^5\\
O\left[\frac{1}{c}\right]^3 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4\\
O\left[\frac{1}{c}\right]^3 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4\\
O\left[\frac{1}{c}\right]^3 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4 & O\left[\frac{1}{c}\right]^4
\end{pmatrix}$$

```
In[•]:= shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.tte.T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
Out[•]//MatrixForm=
-(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + O[1/c]²    ((-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c)/√μ0 + ((Ax Vx+Ay Vy+Az Vz)(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0))/(2 μ0) + (2 (Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) W √ϵ0)/(√μ0 c) + O[1/c]²
-(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + ((Bz Ey ux-By Ez ux-Bz Ex uy+Bx Ez uy+By Ex uz-Bx Ey uz) √ϵ0)/(√μ0 c) + O[1/c]²    ((-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c)/√μ0 + (Ax (-2 Bx (By uy+Bz uz)+Bx² (-ux+Vx)+(By²+Bz²)(ux+Vx)+(-2 Ex (Ey uy+Ez uz)+Ex² ...
-(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + O[1/c]²    ((-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c)/√μ0 + ((Ax Vx+Ay Vy+Az Vz)(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0))/(2 μ0) + ((Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) W √ϵ0)/(√μ0 c) + O[1/c]²
((Bz Ey-By Ez) √ϵ0)/(√μ0 c) + O[1/c]²    (-2 (Ay Bx By+Az Bx Bz+Ay Ex Ey ϵ0 μ0+Az Ex Ez ϵ0 μ0)+Ax (-Bx²+By²+Bz²+(-Ex²+Ey²+Ez²) ϵ0 μ0))/(2 μ0) - ((Bz Ey-By Ez)(Ax Vx+Ay Vy+Az Vz) √ϵ0)/(√μ0 c) + O[1/c]²
((By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0)/(√μ0 c) + O[1/c]²    (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+(-2 (Ax Ex+Ay Ey) Ez y+Az (Ex²+Ey²-Ez²) y+2 Ey (Ax Ex+Az Ez) z-Ay (Ex²-Ey²+Ez²) z) ϵ0 μ0)/(2 μ0) - ((Ax Vx+Ay Vy+Az Vz)(By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0)/(√μ0 c) + O[1/c]²
((By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0)/(√μ0 c) + O[1/c]²    (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+(-2 (Ax Ex+Ay Ey) Ez y+Az (Ex²+Ey²-Ez²) y+2 Ey (Ax Ex+Az Ez) z-Ay (Ex²-Ey²+Ez²) z) ϵ0 μ0)/(2 μ0) - ((Ax Vx+Ay Vy+Az Vz)(By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0)/(√μ0 c) + O[1/c]²
((-Bz Ey+By Ez) t √ϵ0)/(√μ0 c) + O[1/c]²    (t (2 (Ay Bx By+Az Bx Bz+Ay Ex Ey ϵ0 μ0+Az Ex Ez ϵ0 μ0)+Ax (Bx²-By²-Bz²+(Ex²-Ey²-Ez²) ϵ0 μ0)))/(2 μ0) + (((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) x) √ϵ0)/(√μ0 c) + O[1/c]²
((-Bz Ey+By Ez) t √ϵ0)/(√μ0 c) + O[1/c]²    (t (2 (Ay Bx By+Az Bx Bz+Ay Ex Ey ϵ0 μ0+Az Ex Ez ϵ0 μ0)+Ax (Bx²-By²-Bz²+(Ex²-Ey²-Ez²) ϵ0 μ0)))/(2 μ0) + (((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) x) √ϵ0)/(√μ0 c) + O[1/c]²
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In[•]:= TTx = tW[tjv[(tte + T[tte.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords⟦aa,;;,;;⟧].gg+Dcoords⟦aa,;;,;;⟧).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords⟦1, ;; , ;;⟧, Duv, Dvtn, Dcoords⟦2, ;; , ;;⟧, DLx, DL2x, Dbox / c, Dbo2x} // MF
Out[•]//MatrixForm=
((Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) W^(1,0,0,0)[t,x,y,z])/(μ0 c²) + O[1/c]³
(-2 (Bx Bz+Ex Ez ϵ0 μ0) ux^(0,0,0,1)[t,x,y,z]-2 (By Bz+Ey Ez ϵ0 μ0) uy^(0,0,0,1)[t,x,y,z]+Bx² uz^(0,0,0,1)[t,x,y,z]+By² uz^(0,0,0,1)[t,x,y,z]-Bz² uz^(0,0,0,1)[t,x,y,z]+Ex² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]+Ey² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-Ez² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-2 Bx By ux^(0,0,1,0)[t,x,y,z]-2 Ex Ey ϵ0 μ0 ux^(0,0,1,0)[t,x,y,z]+Bx² ...)/(2 μ0)
(√ϵ0 ((-By Ex+Bx Ey) W^(0,0,0,1)[t,x,y,z]+(Bz Ex-Bx Ez) W^(0,0,1,0)[t,x,y,z]+(-Bz Ey+By Ez) W^(0,1,0,0)[t,x,y,z]))/(√μ0 c) + ((Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) W^(1,0,0,0)[t,x,y,z])/(2 μ0 c²) + O[1/c]³
((Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) W^(0,1,0,0)[t,x,y,z])/(μ0 c²) + O[1/c]³
((Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) (y W^(0,0,0,1)[t,x,y,z]-z W^(0,0,1,0)[t,x,y,z]))/(μ0 c²) + O[1/c]³
(2 ((Bz y-By z+Ez (Ez y-Ey z) ϵ0 μ0) W^(0,0,0,1)[t,x,y,z]+(Bz y-By² z+Ey (Ez y-Ey z) ϵ0 μ0) W^(0,0,1,0)[t,x,y,z]+(Bx (Bz y-By z)+Ex (Ez y-Ey z) ϵ0 μ0) W^(0,1,0,0)[t,x,y,z]))/(μ0 c²) + O[1/c]³
(2 (-Bz Ey+By Ez) √ϵ0)/(√μ0 c) - (t (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0) W^(0,1,0,0)[t,x,y,z])/(μ0 c²) + O[1/c]³
-(2 (t ((Bx Bz+Ex Ez ϵ0 μ0) W^(0,0,0,1)[t,x,y,z]+(Bx By+Ex Ey ϵ0 μ0) W^(0,0,1,0)[t,x,y,z]+(Bx²+Ex² ϵ0 μ0) W^(0,1,0,0)[t,x,y,z])))/(μ0 c²) + O[1/c]³
```

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In[•]:= shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.(tte + ttsym).T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
Out[•]//MatrixForm=
-n ρ c² + (-n ϵ - (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) - 1/2 n (ux² + uy² + uz² - 2 W) ρ) + O[1/c]²    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + ((-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c)/√μ0 + (-2 Ax (qx+sxx ux+sxy uy+sxz uz+n ux ϵ) μ0-2 Ay (qy+sxy ux+syy uy+syz uz+n uy ϵ) μ0-2 Az (qz+sxz ux+syz uy+szz uz+n uz ϵ) μ0+Ax Vx (Bx²+By²+Bz²+2 n ϵ μ0+(Ex²+Ey²+Ez²) ϵ0 μ0)+Ay Vy (Bx²+By²+Bz²+2 n ϵ μ0+(Ex² ...)/(2 μ0)
-n ρ c² - (Bx²+By²+Bz²+2 n ϵ μ0+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) + ((Bz Ey ux-By Ez ux-Bz Ex uy+Bx Ez uy+By Ex uz-Bx Ey uz) √ϵ0)/(√μ0 c) + O[1/c]²    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + ((-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c)/√μ0 + (Ay (-2 Bx By ux-2 By Bz uz+Bx² (-uy+Vy)+Bz² (uy+Vy)-2 (qy+n (uy-Vy) ϵ) μ0+(-2 Ex Ey ux-2 Ey Ez uz+Ey² (-uy+Vy)+Ex² (uy+Vy)+Ez² (uy+Vy)) ϵ0 μ0)+Ax (-2 Bx (By uy+Bz uz)+Bx² (-ux+Vx)+(By²+Bz²)(ux+Vx)+(- ...
-n ρ c² + (-n ϵ - (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) - 1/2 n (ux² + uy² + uz²) ρ) + O[1/c]²    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + ((-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c)/√μ0 + (-2 Ax (qx+sxx ux+sxy uy+sxz uz+n ux ϵ) μ0-2 Ay (qy+sxy ux+syy uy+syz uz+n uy ϵ) μ0-2 Az (qz+sxz ux+syz uy+szz uz+n uz ϵ) μ0+Ax Vx (Bx²+By²+Bz²+2 n ϵ μ0+(Ex²+Ey²+Ez²) ϵ0 μ0)+Ay Vy (Bx²+By²+Bz²+2 n ϵ μ0+(Ex² ...)/(2 μ0)
n ux ρ + ((Bz Ey-By Ez) √ϵ0)/(√μ0 c) + O[1/c]²    (-2 Ay (Bx By-sxy μ0+Ex Ey ϵ0 μ0+n ux (-uy+Vy) μ0)+Ax (-Bx²+By²+Bz²-μ0 (2 sxx+(-Ex²+Ey²+Ez²) ϵ0+2 n ux (ux-Vx)))+2 Az (-Bx Bz+μ0 (sxz-Ex Ez ϵ0+n ux (uz-Vz))))/(2 μ0) - ((Bz Ey-By Ez)(Ax Vx+Ay Vy+Az Vz) √ϵ0)/(√μ0 c) + O[1/c]²
n (uz y - uy z) ρ + ((By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0)/(√μ0 c) + O[1/c]²    (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+2 Ax sxz y μ0+2 Ay syz y μ0+2 Az szz y μ0-2 Ax sxy z μ0-2 Ay syy z μ0-2 Az syz z μ0+Az Ex² y ϵ0 μ0-2 Ax Ex Ez y ϵ0 μ0-2 Ay Ey Ez y ϵ0 μ0+Az Ey² y ϵ0 μ0-Az Ez² y ϵ0 μ0-Ay Ex² z ϵ0 μ0+2 Ax Ex Ey z ϵ0 μ0+Ay Ey² z ϵ0 μ0+2 Az Ey Ez z ϵ0 μ0-Ay Ez² z ϵ0 μ0 ...)/(2 μ0)
n (uz y - uy z) ρ + ((By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0)/(√μ0 c) + O[1/c]²    (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+2 Ax sxz y μ0+2 Ay syz y μ0+2 Az szz y μ0-2 Ax sxy z μ0-2 Ay syy z μ0-2 Az syz z μ0+Az Ex² y ϵ0 μ0-2 Ax Ex Ez y ϵ0 μ0-2 Ay Ey Ez y ϵ0 μ0+Az Ey² y ϵ0 μ0-Az Ez² y ϵ0 μ0-Ay Ex² z ϵ0 μ0+2 Ax Ex Ey z ϵ0 μ0+Ay Ey² z ϵ0 μ0+2 Az Ey Ez z ϵ0 μ0-Ay Ez² z ϵ0 μ0 ...)/(2 μ0)
-n (t ux + x) ρ + ((-Bz Ey+By Ez) t √ϵ0)/(√μ0 c) + O[1/c]²    ((t (Ax Bx²+2 Ay Bx By-Ax By²+2 Az Bx Bz-Ax Bz²-(2 Ax sxx+2 Ay sxy+2 Az sxz-2 Ex (Ay Ey+Az Ez) ϵ0+Ax (-Ex²+Ey²+Ez²) ϵ0) μ0))/(2 μ0) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux + x) ρ) + (((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) x) √ϵ0)/(√μ0 c) + O[1/c]²
n (-t ux + x) ρ + ((-Bz Ey+By Ez) t √ϵ0)/(√μ0 c) + O[1/c]²    ((t (Ax Bx²+2 Ay Bx By-Ax By²+2 Az Bx Bz-Ax Bz²-(2 Ax sxx+2 Ay sxy+2 Az sxz-2 Ex (Ay Ey+Az Ez) ϵ0+Ax (-Ex²+Ey²+Ez²) ϵ0) μ0))/(2 μ0) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux - x) ρ) + (((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) x) √ϵ0)/(√μ0 c) + O[1/c]²
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In[•]:= TTx = tW[tjv[((tte + ttsym) + T[(tte + ttsym).Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords⟦aa,;;,;;⟧].gg+Dcoords⟦aa,;;,;;⟧).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords⟦1, ;; , ;;⟧, Duv, Dvtn, Dcoords⟦2, ;; , ;;⟧, DLx, DL2x, Dbox / c, Dbo2x} // MF
Out[•]//MatrixForm=
n ρ W^(1,0,0,0)[t, x, y, z] + O[1/c]²
(-2 (Bx Bz-sxz μ0+Ex Ez ϵ0 μ0) ux^(0,0,0,1)[t,x,y,z]-2 (By Bz-syz μ0+Ey Ez ϵ0 μ0) uy^(0,0,0,1)[t,x,y,z]+Bx² uz^(0,0,0,1)[t,x,y,z]+By² uz^(0,0,0,1)[t,x,y,z]-Bz² uz^(0,0,0,1)[t,x,y,z]+2 szz μ0 uz^(0,0,0,1)[t,x,y,z]+Ex² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]+Ey² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-Ez² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-2 Bx By ux^(0,0,1,0)[t,x,y,z]+2 sxy μ0 ux^(0,0,1,0)[t,x,y,z]-2 Ex Ey ϵ0 μ0 ux^(0,0,1,0)[t,x,y,z]+Bx² uy^(0,0,1,0)[t,x,y,z]-By² uy^(0,0,1,0)[t,x,y,z]+2 syy μ0 uy^(0,0,1,0)[t,x,y,z]+Bz² uy^(0,0,1,0)[t,x,y,z]-Ex² ϵ0 μ ...)/(2 μ0)
-n ρ (uz W^(0,0,0,1)[t, x, y, z] + uy W^(0,0,1,0)[t, x, y, z] + ux W^(0,1,0,0)[t, x, y, z]) + (√ϵ0 ((-By Ex+Bx Ey) W^(0,0,0,1)[t,x,y,z]+(Bz Ex-Bx Ez) W^(0,0,1,0)[t,x,y,z]+(-Bz Ey+By Ez) W^(0,1,0,0)[t,x,y,z]))/(√μ0 c) + O[1/c]²
n ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]²
n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]²
n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]²
-n ρ (2 ux + t W^(0,1,0,0)[t, x, y, z]) + (2 (-Bz Ey+By Ez) √ϵ0)/(√μ0 c) + O[1/c]²
-n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]²
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In[•]:= shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.(tt + tte).T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
Out[•]//MatrixForm=
-n ρ c² + (-(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) - 1/2 n (ux² + uy² + uz² - 2 W + 2 ϵ) ρ) + O[1/c]²    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + ((-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c)/√μ0 + (Ax (-2 (qx+sxx ux+sxy uy+sxz uz) μ0+Vx (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)+n (-ux+Vx) (ux²+uy²+uz²-2 W+2 ϵ) μ0 ρ)+Ay (-2 (qy+sxy ux+syy uy+syz uz) μ0+Vy (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)+n (-uy+Vy) (ux²+uy²+ ...)/(2 μ0)
-n ρ c² - (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0+2 n ϵ μ0 ρ)/(2 μ0) + ((Bz Ey ux-By Ez ux-Bz Ex uy+Bx Ez uy+By Ex uz-Bx Ey uz) √ϵ0)/(√μ0 c) + O[1/c]²    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + ((-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c)/√μ0 + (Ax (Bz² ux-2 Bx (By uy+Bz uz)+By² Vx+Bx² (-ux+Vx)+By² (ux+Vx)-2 qx μ0-Ex² ux ϵ0 μ0+Ey² ux ϵ0 μ0+Ez² ux ϵ0 μ0-2 Ex Ey uy ϵ0 μ0-2 Ex Ez uz ϵ0 μ0+Ex² Vx ϵ0 μ0+Ey² Vx ϵ0 μ0+Ez² Vx ϵ0 μ0+2 n (-ux+Vx) ϵ μ0 ρ)+Ay (-2 B ...)/(2 μ0)
-n ρ c² + (-(Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)/(2 μ0) - 1/2 n (ux² + uy² + uz² + 2 ϵ) ρ) + O[1/c]²    n (Ax (-ux + Vx) + Ay (-uy + Vy) + Az (-uz + Vz)) ρ c² + ((-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) √ϵ0 c)/√μ0 + (Ax (-2 (qx+sxx ux+sxy uy+sxz uz) μ0+Vx (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)+n (-ux+Vx) (ux²+uy²+uz²+2 ϵ) μ0 ρ)+Ay (-2 (qy+sxy ux+syy uy+syz uz) μ0+Vy (Bx²+By²+Bz²+(Ex²+Ey²+Ez²) ϵ0 μ0)+n (-uy+Vy) (ux²+uy²+ ...)/(2 μ0)
n ux ρ + ((Bz Ey-By Ez) √ϵ0)/(√μ0 c) + O[1/c]²    (-2 Ay (Bx By-sxy μ0+Ex Ey ϵ0 μ0+n ux (-uy+Vy) μ0 ρ)+Ax (-Bx²+By²+Bz²-μ0 (2 sxx+(-Ex²+Ey²+Ez²) ϵ0+2 n ux (ux-Vx) ρ))+2 Az (-Bx Bz+μ0 (sxz-Ex Ez ϵ0+n ux (uz-Vz) ρ)))/(2 μ0) - ((Bz Ey-By Ez)(Ax Vx+Ay Vy+Az Vz) √ϵ0)/(√μ0 c) + O[1/c]²
n (uz y - uy z) ρ + ((By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0)/(√μ0 c) + O[1/c]²    (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+2 Ax sxz y μ0+2 Ay syz y μ0+2 Az szz y μ0-2 Ax sxy z μ0-2 Ay syy z μ0-2 Az syz z μ0+Az Ex² y ϵ0 μ0-2 Ax Ex Ez y ϵ0 μ0-2 Ay Ey Ez y ϵ0 μ0+Az Ey² y ϵ0 μ0-Az Ez² y ϵ0 μ0-Ay Ex² z ϵ0 μ0+2 Ax Ex Ey z ϵ0 μ0+Ay Ey² z ϵ0 μ0+2 Az Ey Ez z ϵ0 μ0-Ay Ez² z ϵ0 μ0 ...)/(2 μ0)
n (uz y - uy z) ρ + ((By Ex y+Bz Ex z-Bx (Ey y+Ez z)) √ϵ0)/(√μ0 c) + O[1/c]²    (Az Bx² y+Az By² y-2 Ax Bx Bz y-2 Ay By Bz y-Az Bz² y-Ay Bx² z+2 Ax Bx By z+Ay By² z+2 Az By Bz z-Ay Bz² z+2 Ax sxz y μ0+2 Ay syz y μ0+2 Az szz y μ0-2 Ax sxy z μ0-2 Ay syy z μ0-2 Az syz z μ0+Az Ex² y ϵ0 μ0-2 Ax Ex Ez y ϵ0 μ0-2 Ay Ey Ez y ϵ0 μ0+Az Ey² y ϵ0 μ0-Az Ez² y ϵ0 μ0-Ay Ex² z ϵ0 μ0+2 Ax Ex Ey z ϵ0 μ0+Ay Ey² z ϵ0 μ0+2 Az Ey Ez z ϵ0 μ0-Ay Ez² z ϵ0 μ0 ...)/(2 μ0)
-n (t ux + x) ρ + ((-Bz Ey+By Ez) t √ϵ0)/(√μ0 c) + O[1/c]²    ((t (Ax Bx²+2 Ay Bx By-Ax By²+2 Az Bx Bz-Ax Bz²-(2 Ax sxx+2 Ay syx+2 Az szx-2 Ex (Ay Ey+Az Ez) ϵ0+Ax (-Ex²+Ey²+Ez²) ϵ0) μ0))/(2 μ0) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux + x) ρ) + (((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(-Az By Ex+Ay Bz Ex+Az Bx Ey-Ax Bz Ey-Ay Bx Ez+Ax By Ez) x) √ϵ0)/(√μ0 c) + O[1/c]²
n (-t ux + x) ρ + ((-Bz Ey+By Ez) t √ϵ0)/(√μ0 c) + O[1/c]²    ((t (Ax Bx²+2 Ay Bx By-Ax By²+2 Az Bx Bz-Ax Bz²-(2 Ax sxx+2 Ay syx+2 Az szx-2 Ex (Ay Ey+Az Ez) ϵ0+Ax (-Ex²+Ey²+Ez²) ϵ0) μ0))/(2 μ0) - n (Ax (ux - Vx) + Ay (uy - Vy) + Az (uz - Vz)) (t ux - x) ρ) + (((Bz Ey-By Ez) t (Ax Vx+Ay Vy+Az Vz)+(Az By Ex-Ay Bz Ex-Az Bx Ey+Ax Bz Ey+Ay Bx Ez-Ax By Ez) x) √ϵ0)/(√μ0 c) + O[1/c]²
```

```
In[•]:= TTx = tW[tjv[((tt + tte) + T[(tt + tte).Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords⟦aa,;;,;;⟧].gg+Dcoords⟦aa,;;,;;⟧).TTx]],{aa,1,4}]]*)
shows[assut, 4][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords⟦1, ;; , ;;⟧, Duv, Dvtn, Dcoords⟦2, ;; , ;;⟧, DLx, DL2x, Dbox / c, Dbo2x} // MF
Out[•]//MatrixForm=
n ρ W^(1,0,0,0)[t, x, y, z] + O[1/c]²
((-2 Bx Bz+(sxz+szx-2 Ex Ez ϵ0) μ0) ux^(0,0,0,1)[t,x,y,z]+(-2 By Bz+(syz+szy-2 Ey Ez ϵ0) μ0) uy^(0,0,0,1)[t,x,y,z]+Bx² uz^(0,0,0,1)[t,x,y,z]+By² uz^(0,0,0,1)[t,x,y,z]-Bz² uz^(0,0,0,1)[t,x,y,z]+2 szz μ0 uz^(0,0,0,1)[t,x,y,z]+Ex² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]+Ey² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-Ez² ϵ0 μ0 uz^(0,0,0,1)[t,x,y,z]-2 Bx By ux^(0,0,1,0)[t,x,y,z]+sxy μ0 ux^(0,0,1,0)[t,x,y,z]+syx μ0 ux^(0,0,1,0)[t,x,y,z]-2 Ex Ey ϵ0 μ0 ux^(0,0,1,0)[t,x,y,z]+Bx² uy^(0,0,1,0)[t,x,y,z]-By² uy^(0,0,1,0)[t,x,y,z] ...)/(2 μ0)
-n ρ (uz W^(0,0,0,1)[t, x, y, z] + uy W^(0,0,1,0)[t, x, y, z] + ux W^(0,1,0,0)[t, x, y, z]) + (√ϵ0 ((-By Ex+Bx Ey) W^(0,0,0,1)[t,x,y,z]+(Bz Ex-Bx Ez) W^(0,0,1,0)[t,x,y,z]+(-Bz Ey+By Ez) W^(0,1,0,0)[t,x,y,z]))/(√μ0 c) + O[1/c]²
n ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]²
n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]²
n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]²
-n ρ (2 ux + t W^(0,1,0,0)[t, x, y, z]) + (2 (-Bz Ey+By Ez) √ϵ0)/(√μ0 c) + O[1/c]²
-n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]²
```

```
(* Flux of energy is given by: *)
In[•]:= shows[assut, 1][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.tt /. {Ax → 0, Ay → 0, Vz → 0, jz → 0} /. j2v]]
Out[•]//MatrixForm=
( -n ρ c² - 1/2 n (ux² + uy² - 2 W + 2 ϵ) ρ + O[1/c]²    n ux ρ + O[1/c]²    n uy ρ + O[1/c]²    O[1/c]²
  -Az (qz + sxz ux + syz uy) + O[1/c]²    Az sxz + O[1/c]²    Az syz + O[1/c]²    Az szz + O[1/c]² )

In[•]:= shows[assut, 1][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → 0, Vx → 0} /. j2v]]
Out[•]//MatrixForm=
( -n ρ c² - 1/2 n (uy² + uz² - 2 W + 2 ϵ) ρ + O[1/c]²    O[1/c]²    n uy ρ + O[1/c]²    n uz ρ + O[1/c]²
  (-qx - sxy uy - sxz uz) + O[1/c]²    sxx + O[1/c]²    sxy + O[1/c]²    sxz + O[1/c]² )

In[•]:= shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v]]
Out[•]//MatrixForm=
( -n ρ c² - 1/2 n (ux² + uy² + uz² - 2 W + 2 ϵ) ρ + O[1/c]²    n ux ρ + (qx+sxx ux+sxy uy+sxz uz+1/2 n (-8 Wx+ux (ux²+uy²+uz²+6 W+2 ϵ)) ρ)/c² + O[1/c]³    n uy ρ + (qy+sxy ux+syy uy+syz uz+1/2 n (-8 Wy+uy (ux²+uy²+uz²+6 W+2 ϵ)) ρ)/c² + O[1/c]³    n uz ρ + (qz+sxz ux+syz uy+szz uz+1/2 n (-8 Wz+uz (ux²+uy²+uz²+6 W+2 ϵ)) ρ ...
  n (-ux + Vx) ρ c² + (-qx - sxx ux - sxy uy - sxz uz - 1/2 n (ux - Vx) (ux² + uy² + uz² - 2 W + 2 ϵ) ρ) + O[1/c]²    (sxx + n ux (ux - Vx) ρ) + (qx (2 ux-Vx)-(sxx ux+sxy uy+sxz uz) Vx+1/2 n (ux-Vx) (ux³-8 Wx+ux (uy²+uz²+6 W+2 ϵ)) ρ)/c² + O[1/c]³    (sxy + n uy (ux - Vx) ρ) + (qx uy+qy (ux-Vx)-(sxy ux+syy uy+syz uz) Vx+1/2 n (ux-Vx) (-8 Wy+uy (ux²+uy²+uz²+6 W+2 ϵ)) ρ)/c² + O[1/c]³    (sxz + n uz (ux - Vx) ρ) + (qx uz+qz (ux-Vx)-(sxz ux+syz uy+szz uz) ...

In[•]:= shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v /. {sxx → SXX - n * ux * (ux - Vx) * ρ, sxy → SXY - n * uy * (ux - Vx) * ρ, sxz → SXZ - n * uz * (ux - Vx) * ρ}]]
Out[•]//MatrixForm=
( -n ρ c² - 1/2 n (ux² + uy² + uz² - 2 W + 2 ϵ) ρ + O[1/c]²    n ux ρ + (qx+SXX ux+SXY uy+SXZ uz-1/2 n (ux³-2 ux² Vx-2 (uy²+uz²) Vx+8 Wx+ux (uy²+uz²-6 W-2 ϵ)) ρ)/c² + O[1/c]³    n uy ρ + (qy+SXY ux+syy uy+syz uz+1/2 n (-8 Wy+uy (ux²-2 ux Vx+uy²+uz²+6 W+2 ϵ)) ρ)/c² + O[1/c]³    n uz ρ + (qz+SXZ ux+syz uy+szz uz+1/2 n (-8 Wz+uz (ux²-2 ux Vx+uy²+uz²+6 W+2 ϵ)) ρ)/c² + O[1/c]² ...
  n (-ux + Vx) ρ c² + (-qx - SXX ux - SXY uy - SXZ uz + 1/2 n (ux - Vx) (ux² + uy² + uz² + 2 W - 2 ϵ) ρ) + O[1/c]²    SXX + (qx (2 ux-Vx)-(SXX ux+SXY uy+SXZ uz) Vx+1/2 n (ux-Vx) (ux³+2 ux² Vx+2 (uy²+uz²) Vx-8 Wx+ux (uy²+uz²+6 W+2 ϵ)) ρ)/c² + O[1/c]³    SXY + (qx uy+qy (ux-Vx)-(SXY ux+syy uy+syz uz) Vx+1/2 n (ux-Vx) (-8 Wy+uy (ux²+uy²+uz²+2 ux Vx+6 W+2 ϵ)) ρ)/c² + O[1/c]³    SXZ + (qx uz+qz (ux-Vx)-(SXZ ux+syz uy+szz uz) Vx+1/2 n (ux-Vx) (-8 Wz+uz (ux²+uy²+uz²+2 ux Vx+6 W+2 ϵ)) ρ ...

In[•]:= (* matter flux in same direction as imaginary moving surface, different velocity *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jy → 0, jz → 0} /. j2v]]
Out[•]//MatrixForm=
( -n ρ c² - 1/2 n (ux² - 2 W + 2 ϵ) ρ + O[1/c]²    n ux ρ + (qx+sxx ux+1/2 n (ux³+6 ux W-8 Wx+2 ux ϵ) ρ)/c² + O[1/c]³    (qy+sxy ux-4 n Wy ρ)/c² + O[1/c]³    (qz+sxz ux-4 n Wz ρ)/c² + O[1/c]³
  n (-ux + Vx) ρ c² + (-qx - sxx ux - 1/2 n (ux - Vx) (ux² - 2 W + 2 ϵ) ρ) + O[1/c]²    (sxx + n ux (ux - Vx) ρ) + (2 qx ux-(qx+sxx ux) Vx+1/2 n (ux-Vx) (ux³-8 Wx+2 ux (3 W+ϵ)) ρ)/c² + O[1/c]³    sxy + (qy (ux-Vx)-sxy ux Vx+4 n (-ux+Vx) Wy ρ)/c² + O[1/c]³    sxz + (qz (ux-Vx)-sxz ux Vx+4 n (-ux+Vx) Wz ρ)/c² + O[1/c]³ )
```

In[•]:=
```
(* imaginary moving surface, no matter flux through it *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v /. {ux → Vx}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2} n\left(uy^2 + uz^2 + Vx^2 - 2W + 2\epsilon\right)\rho + O\left[\frac{1}{c}\right]^2 & n\,Vx\,\rho + \frac{qx + sxy\,uy + sxz\,uz + sxx\,Vx + \frac{1}{2} n\left(-8\,Wx + Vx\left(uy^2+uz^2+Vx^2+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & n\,uy\,\rho + \frac{qy + syy\,uy + syz\,uz + sxy\,Vx + \frac{1}{2} n\left(-8\,Wy + uy\left(uy^2+uz^2+Vx^2+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & n\,uz\,\rho + \frac{qz + syz\,uy + szz\,uz + sxz\,Vx + \frac{1}{2} n\left(-8\,Wz + uz\left(uy^2+uz^2+Vx^2+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 \\ \left(-qx - sxy\,uy - sxz\,uz - sxx\,Vx\right) + O\left[\frac{1}{c}\right]^2 & sxx - \frac{Vx\left(-qx + sxy\,uy + sxz\,uz + sxx\,Vx\right)}{c^2} + O\left[\frac{1}{c}\right]^3 & sxy + \frac{qx\,uy - Vx\left(syy\,uy + syz\,uz + sxy\,Vx\right)}{c^2} + O\left[\frac{1}{c}\right]^3 & sxz + \frac{qx\,uz - Vx\left(syz\,uy + szz\,uz + sxz\,Vx\right)}{c^2} + O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

```
(* imaginary moving surface, no matter flux through it and no transversal matter motion *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jy → 0, jz → 0} /. j2v /. {ux → Vx}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2} n\left(Vx^2 - 2W + 2\epsilon\right)\rho + O\left[\frac{1}{c}\right]^2 & n\,Vx\,\rho + \frac{qx + sxx\,Vx + \frac{1}{2} n\left(Vx^3 + 6\,Vx\,W - 8\,Wx + 2\,Vx\,\epsilon\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & \frac{qy + sxy\,Vx - 4\,n\,Wy\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & \frac{qz + sxz\,Vx - 4\,n\,Wz\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 \\ \left(-qx - sxx\,Vx\right) + O\left[\frac{1}{c}\right]^2 & sxx + \frac{Vx\left(qx - sxx\,Vx\right)}{c^2} + O\left[\frac{1}{c}\right]^3 & sxy - \frac{sxy\,Vx^2}{c^2} + O\left[\frac{1}{c}\right]^3 & sxz - \frac{sxz\,Vx^2}{c^2} + O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:=
```
(* imaginary moving surface, matter at rest in coordinates *)
shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → 0, jy → 0, jz → 0}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 + n\left(W - \epsilon\right)\rho + O\left[\frac{1}{c}\right]^2 & \frac{qx - 4\,n\,Wx\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & \frac{qy - 4\,n\,Wy\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & \frac{qz - 4\,n\,Wz\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 \\ n\,Vx\,\rho c^2 + \left(-qx + n\,Vx\left(-W + \epsilon\right)\rho\right) + O\left[\frac{1}{c}\right]^2 & sxx + \frac{-qx\,Vx + 4\,n\,Vx\,Wx\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & sxy + \frac{-qy\,Vx + 4\,n\,Vx\,Wy\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & sxz + \frac{-qz\,Vx + 4\,n\,Vx\,Wz\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → n * Vx, jy → 0, jz → 0}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 + \left(-\frac{1}{2} n\,Vx^2\rho + n\,W\rho - n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 & n\,Vx\,\rho + \frac{qx + sxx\,Vx + \frac{1}{2} n\,Vx^3\rho + 3\,n\,Vx\,W\rho - 4\,n\,Wx\,\rho + n\,Vx\,\epsilon\,\rho}{c^2} + O\left[\frac{1}{c}\right]^4 & \frac{qy + sxy\,Vx - 4\,n\,Wy\,\rho}{c^2} + O\left[\frac{1}{c}\right]^4 & \frac{qz + sxz\,Vx - 4\,n\,Wz\,\rho}{c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(-qx - sxx\,Vx\right) + O\left[\frac{1}{c}\right]^2 & sxx + \frac{qx\,Vx - sxx\,Vx^2}{c^2} + O\left[\frac{1}{c}\right]^4 & sxy - \frac{sxy\,Vx^2}{c^2} + O\left[\frac{1}{c}\right]^4 & sxz - \frac{sxz\,Vx^2}{c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[•]:=
```
(* COORDINATE ENERGY *)
```

In[•]:=
```
(* energy 3-form when projected along coord. axes *)
showf[assut][Expand //@ FS@PowerExpand[tt.{1, 0, 0, 0}]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 + \left(-\frac{jx^2\rho}{2n} - \frac{jy^2\rho}{2n} - \frac{jz^2\rho}{2n} + n\,W\rho - n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jx\,\rho c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^3\rho}{2n^2} - \frac{jx\,jy^2\rho}{2n^2} - \frac{jx\,jz^2\rho}{2n^2} + jx\,W\rho - jx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jy\,\rho c^2 + \left(-qy - \frac{jx\,sxy}{n} - \frac{jy\,syy}{n} - \frac{jz\,syz}{n} - \frac{jx^2\,jy\,\rho}{2n^2} - \frac{jy^3\rho}{2n^2} - \frac{jy\,jz^2\rho}{2n^2} + jy\,W\rho - jy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jz\,\rho c^2 + \left(-qz - \frac{jx\,sxz}{n} - \frac{jy\,syz}{n} - \frac{jz\,szz}{n} - \frac{jx^2\,jz\,\rho}{2n^2} - \frac{jy^2\,jz\,\rho}{2n^2} - \frac{jz^3\rho}{2n^2} + jz\,W\rho - jz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.{1, 0, 0, 0} /. j2v]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 + \left(-\frac{1}{2} n\,ux^2\rho - \frac{1}{2} n\,uy^2\rho - \frac{1}{2} n\,uz^2\rho + n\,W\rho - n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,ux\,\rho c^2 + \left(-qx - sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2} n\,ux^3\rho - \frac{1}{2} n\,ux\,uy^2\rho - \frac{1}{2} n\,ux\,uz^2\rho + n\,ux\,W\rho - n\,ux\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uy\,\rho c^2 + \left(-qy - sxy\,ux - syy\,uy - syz\,uz - \frac{1}{2} n\,ux^2\,uy\,\rho - \frac{1}{2} n\,uy^3\rho - \frac{1}{2} n\,uy\,uz^2\rho + n\,uy\,W\rho - n\,uy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uz\,\rho c^2 + \left(-qz - sxz\,ux - syz\,uy - szz\,uz - \frac{1}{2} n\,ux^2\,uz\,\rho - \frac{1}{2} n\,uy^2\,uz\,\rho - \frac{1}{2} n\,uz^3\rho + n\,uz\,W\rho - n\,uz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* flux of coord. energy across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt)]]
```

Out[•]//MatrixForm=

$$\left(-jx\,\rho + n\,Vx\,\rho\right)c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^3\rho}{2n^2} - \frac{jx\,jy^2\rho}{2n^2} - \frac{jx\,jz^2\rho}{2n^2} + \frac{jx^2\,Vx\,\rho}{2n} + \frac{jy^2\,Vx\,\rho}{2n} + \frac{jz^2\,Vx\,\rho}{2n} + jx\,W\rho - n\,Vx\,W\rho - jx\,\epsilon\,\rho + n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr]]
```

Out[•]//MatrixForm=

$$-Lx\,n\,\rho c^2 + \left(-qx - sxx\,ux - \frac{1}{2} Lx\,n\,ux^2\rho + Lx\,n\,W\rho - Lx\,n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. {jx → 0, jy → 0, jz → 0}]]
```

Out[•]//MatrixForm=

$$n\,Vx\,\rho c^2 + \left(-qx - n\,Vx\,W\rho + n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of matter flux*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf]]
```

Out[•]//MatrixForm=

$$-JX\,\rho c^2 + \left(-qx - \frac{JX\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - sxx\,Vx - \frac{JX^3\rho}{2n^2} - \frac{JX\,jy^2\rho}{2n^2} - \frac{JX\,jz^2\rho}{2n^2} - \frac{JX^2\,Vx\,\rho}{n} - \frac{1}{2} JX\,Vx^2\rho + JX\,W\rho - JX\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf]]
```

Out[•]//MatrixForm=

$$-JX\,\rho c^2 + \left(-qx - \frac{JX\,sxx}{n} - sxx\,Vx - \frac{JX^3\rho}{2n^2} - \frac{JX^2\,Vx\,\rho}{n} - \frac{1}{2} JX\,Vx^2\rho + JX\,W\rho - JX\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of matter flux & matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf /. j2v]]
```

Out[•]//MatrixForm=

$$-JX\,\rho c^2 + \left(-qx - \frac{JX\,sxx}{n} - sxy\,uy - sxz\,uz - sxx\,Vx - \frac{JX^3\rho}{2n^2} - \frac{1}{2} JX\,uy^2\rho - \frac{1}{2} JX\,uz^2\rho - \frac{JX^2\,Vx\,\rho}{n} - \frac{1}{2} JX\,Vx^2\rho + JX\,W\rho - JX\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf /. {JX → 0, jy → 0, jz → 0}]]
```

Out[•]//MatrixForm=

$$\left(-qx - sxx\,Vx\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. relv]]
```

Out[•]//MatrixForm=

$$-n\,Vx\,\rho c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^2\,Vx\,\rho}{2n} - \frac{jy^2\,Vx\,\rho}{2n} - \frac{jz^2\,Vx\,\rho}{2n} + n\,Vx\,W\rho - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr]]
```

Out[•]//MatrixForm=

$$-n\,Vx\,\rho c^2 + \left(-qx - sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2} n\,ux^2\,Vx\,\rho - \frac{1}{2} n\,uy^2\,Vx\,\rho - \frac{1}{2} n\,uz^2\,Vx\,\rho + n\,Vx\,W\rho - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr /. {Vx → 0}]]
```

Out[•]//MatrixForm=

$$\left(-qx - sxx\,ux - sxy\,uy - sxz\,uz\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* supply term for coord. energy *)
TTx = tW[Normal[tt]]; shows[assut, 2][Expand //@ FS@PowerExpand[Tr[1 / 2 * Normal@(Inverse[gg].T[Dcoords⟦1, ;; , ;;⟧].gg + Dcoords⟦1, ;; , ;;⟧).TTx]]]
```

Out[•]//MatrixForm=

$$n\,\rho\,W^{(1,0,0,0)}[t,x,y,z] + \frac{\left(3\left(jx^2 + jy^2 + jz^2\right)\rho + 2n\left(sxx + syy + szz + n\,\epsilon\,\rho\right) - 2n^2\rho\,W[t,x,y,z]\right)W^{(1,0,0,0)}[t,x,y,z]}{2\,n\,c^2} + O\left[\frac{1}{c}\right]^3$$

In[•]:=
```
(* INTERNAL ENERGY *)
```

In[•]:=
```
(* energy 3-form when projected along matter 4-velocity, "internal energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.uu]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - n\,\epsilon\,\rho + O\left[\frac{1}{c}\right]^2 \\ -jx\,\rho c^2 + \left(-qx - jx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jy\,\rho c^2 + \left(-qy - jy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jz\,\rho c^2 + \left(-qz - jz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.uu /. j2vr]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - n\,\epsilon\,\rho + O\left[\frac{1}{c}\right]^2 \\ -n\,ux\,\rho c^2 + \left(-qx - n\,ux\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uy\,\rho c^2 + \left(-qy - n\,uy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uz\,\rho c^2 + \left(-qz - n\,uz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* flux of internal energy across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt)]]
```

Out[•]//MatrixForm=

$$\left(-jx\,\rho + n\,vx\,\rho\right)c^2 + \left(-qx - jx\,\epsilon\,\rho + n\,vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. relv]]
```

Out[•]//MatrixForm=

$$-n\,Vx\,\rho c^2 + \left(-qx - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. j2vr]]
```

Out[•]//MatrixForm=

$$-n\,Vx\,\rho c^2 + \left(-qx - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. j2vr /. {Vx → 0}]]
```

Out[•]//MatrixForm=

$$-qx + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* supply term for internal energy (should be reversed in sign; remember that stress is compressive, not tensile) *)
TTx = tW[tjv@tt]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Duv].gg + Duv).TTx]]]
```
Out[•]//MatrixForm=

$$\left(sxz\, vx^{(0,0,0,1)}[t,x,y,z] + syz\, vy^{(0,0,0,1)}[t,x,y,z] + szz\, vz^{(0,0,0,1)}[t,x,y,z] + sxy\, vx^{(0,0,1,0)}[t,x,y,z] + syy\, vy^{(0,0,1,0)}[t,x,y,z] + syz\, vz^{(0,0,1,0)}[t,x,y,z] + sxx\, vx^{(0,1,0,0)}[t,x,y,z] + sxy\, vy^{(0,1,0,0)}[t,x,y,z] + sxz\, vz^{(0,1,0,0)}[t,x,y,z]\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* difference between "coord. energy" and "internal energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - uu)]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{jx^2 \rho}{2n} - \frac{jy^2 \rho}{2n} - \frac{jz^2 \rho}{2n} + n\, W \rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\, sxx}{n} - \frac{jy\, sxy}{n} - \frac{jz\, sxz}{n} - \frac{jx^3 \rho}{2n^2} - \frac{jx\, jy^2 \rho}{2n^2} - \frac{jx\, jz^2 \rho}{2n^2} + jx\, W \rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\, sxy}{n} - \frac{jy\, syy}{n} - \frac{jz\, syz}{n} - \frac{jx^2\, jy\, \rho}{2n^2} - \frac{jy^3 \rho}{2n^2} - \frac{jy\, jz^2 \rho}{2n^2} + jy\, W \rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\, sxz}{n} - \frac{jy\, syz}{n} - \frac{jz\, szz}{n} - \frac{jx^2\, jz\, \rho}{2n^2} - \frac{jy^2\, jz\, \rho}{2n^2} - \frac{jz^3 \rho}{2n^2} + jz\, W \rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - uu) /. j2vr]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{1}{2} n\, ux^2 \rho - \frac{1}{2} n\, uy^2 \rho - \frac{1}{2} n\, uz^2 \rho + n\, W \rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-sxx\, ux - sxy\, uy - sxz\, uz - \frac{1}{2} n\, ux^3 \rho - \frac{1}{2} n\, ux\, uy^2 \rho - \frac{1}{2} n\, ux\, uz^2 \rho + n\, ux\, W \rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-sxy\, ux - syy\, uy - syz\, uz - \frac{1}{2} n\, ux^2\, uy\, \rho - \frac{1}{2} n\, uy^3 \rho - \frac{1}{2} n\, uy\, uz^2 \rho + n\, uy\, W \rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-sxz\, ux - syz\, uy - szz\, uz - \frac{1}{2} n\, ux^2\, uz\, \rho - \frac{1}{2} n\, uy^2\, uz\, \rho - \frac{1}{2} n\, uz^3 \rho + n\, uz\, W \rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(-\frac{jx\, sxx}{n} - \frac{jy\, sxy}{n} - \frac{jz\, sxz}{n} - \frac{jx^3 \rho}{2n^2} - \frac{jx\, jy^2 \rho}{2n^2} - \frac{jx\, jz^2 \rho}{2n^2} + \frac{jx^2\, vx\, \rho}{2n} + \frac{jy^2\, vx\, \rho}{2n} + \frac{jz^2\, vx\, \rho}{2n} + jx\, W \rho - n\, vx\, W \rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. relv]]
```
Out[•]//MatrixForm=

$$\left(-\frac{jx\, sxx}{n} - \frac{jy\, sxy}{n} - \frac{jz\, sxz}{n} - \frac{jx^2\, Vx\, \rho}{2n} - \frac{jy^2\, Vx\, \rho}{2n} - \frac{jz^2\, Vx\, \rho}{2n} + n\, Vx\, W \rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=

$$\left(-sxx\, ux - sxy\, uy - sxz\, uz - \frac{1}{2} n\, ux^2\, Vx\, \rho - \frac{1}{2} n\, uy^2\, Vx\, \rho - \frac{1}{2} n\, uz^2\, Vx\, \rho + n\, Vx\, W \rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=

$$\left(-sxx\, ux - sxy\, uy - sxz\, uz\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* PROPER-TIME COORD ENERGY*)
```

In[•]:=
```
(* energy 3-form when projected along normalized coord-t
   note how the gravitational term is missing *)
showf[assut][Expand //@ FS@PowerExpand[tt.vtn]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n \rho c^2 + \left(-\frac{jx^2 \rho}{2n} - \frac{jy^2 \rho}{2n} - \frac{jz^2 \rho}{2n} - n \epsilon \rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jx\, \rho c^2 + \left(-qx - \frac{jx\, sxx}{n} - \frac{jy\, sxy}{n} - \frac{jz\, sxz}{n} - \frac{jx^3 \rho}{2n^2} - \frac{jx\, jy^2 \rho}{2n^2} - \frac{jx\, jz^2 \rho}{2n^2} - jx\, \epsilon \rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jy\, \rho c^2 + \left(-qy - \frac{jx\, sxy}{n} - \frac{jy\, syy}{n} - \frac{jz\, syz}{n} - \frac{jx^2\, jy\, \rho}{2n^2} - \frac{jy^3 \rho}{2n^2} - \frac{jy\, jz^2 \rho}{2n^2} - jy\, \epsilon \rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jz\, \rho c^2 + \left(-qz - \frac{jx\, sxz}{n} - \frac{jy\, syz}{n} - \frac{jz\, szz}{n} - \frac{jx^2\, jz\, \rho}{2n^2} - \frac{jy^2\, jz\, \rho}{2n^2} - \frac{jz^3 \rho}{2n^2} - jz\, \epsilon \rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.vtn /. j2vr]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} -n \rho c^2 + \left(-\frac{1}{2} n\, ux^2 \rho - \frac{1}{2} n\, uy^2 \rho - \frac{1}{2} n\, uz^2 \rho - n \epsilon \rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\, ux\, \rho c^2 + \left(-qx - sxx\, ux - sxy\, uy - sxz\, uz - \frac{1}{2} n\, ux^3 \rho - \frac{1}{2} n\, ux\, uy^2 \rho - \frac{1}{2} n\, ux\, uz^2 \rho - n\, ux\, \epsilon \rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\, uy\, \rho c^2 + \left(-qy - sxy\, ux - syy\, uy - syz\, uz - \frac{1}{2} n\, ux^2\, uy\, \rho - \frac{1}{2} n\, uy^3 \rho - \frac{1}{2} n\, uy\, uz^2 \rho - n\, uy\, \epsilon \rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\, uz\, \rho c^2 + \left(-qz - sxz\, ux - syz\, uy - szz\, uz - \frac{1}{2} n\, ux^2\, uz\, \rho - \frac{1}{2} n\, uy^2\, uz\, \rho - \frac{1}{2} n\, uz^3 \rho - n\, uz\, \epsilon \rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* flux of normalized-coord-t energy across surface *)
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(-jx\, \rho + n\, vx\, \rho\right) c^2 + \left(-qx - \frac{jx\, sxx}{n} - \frac{jx^3 \rho}{2n^2} + \frac{jx^2\, vx\, \rho}{2n} - jx\, \epsilon \rho + n\, vx\, \epsilon \rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(*in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. relv]]
```
Out[•]//MatrixForm=

$$-n\, Vx\, \rho c^2 + \left(-qx - \frac{jx\, sxx}{n} - \frac{jy\, sxy}{n} - \frac{jz\, sxz}{n} - \frac{jx^2\, Vx\, \rho}{2n} - \frac{jy^2\, Vx\, \rho}{2n} - \frac{jz^2\, Vx\, \rho}{2n} - n\, Vx\, \epsilon \rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=

$$-n\, Vx\, \rho c^2 + \left(-qx - sxx\, ux - sxy\, uy - sxz\, uz - \frac{1}{2} n\, ux^2\, Vx\, \rho - \frac{1}{2} n\, uy^2\, Vx\, \rho - \frac{1}{2} n\, uz^2\, Vx\, \rho - n\, Vx\, \epsilon \rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=

$$\left(-qx - sxx\, ux - sxy\, uy - sxz\, uz\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* supply term for normalized-coord-t energy
   we obtain the "power generated by the gravity field" *)
TTx = tW[tjv@tt]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Dvtn].gg + Dvtn).TTx]]]
```
Out[•]//MatrixForm=

$$\left(-\rho\, n[t,x,y,z] \times vz[t,x,y,z]\, W^{(0,0,0,1)}[t,x,y,z] - \rho\, n[t,x,y,z] \times vy[t,x,y,z]\, W^{(0,0,1,0)}[t,x,y,z] - \rho\, n[t,x,y,z] \times vx[t,x,y,z]\, W^{(0,1,0,0)}[t,x,y,z]\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* difference between "coord. energy" and "proper-time coord. energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - vtn)]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} n\, W \rho + O\left[\frac{1}{c}\right]^2 \\ jx\, W \rho + O\left[\frac{1}{c}\right]^2 \\ jy\, W \rho + O\left[\frac{1}{c}\right]^2 \\ jz\, W \rho + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - vtn) /. j2vr]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} n\, W \rho + O\left[\frac{1}{c}\right]^2 \\ n\, ux\, W \rho + O\left[\frac{1}{c}\right]^2 \\ n\, uy\, W \rho + O\left[\frac{1}{c}\right]^2 \\ n\, uz\, W \rho + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(jx\, W \rho - n\, vx\, W \rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. relv]]
```
Out[•]//MatrixForm=

$$n\, Vx\, W \rho + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=

$$n\, Vx\, W \rho + O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=

$$O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* difference between "internal energy" and "proper-time coord. energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.(uu - vtn)]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} \left(\frac{jx^2 \rho}{2 n} + \frac{jy^2 \rho}{2 n} + \frac{jz^2 \rho}{2 n}\right) + O[\frac{1}{c}]^2 \\ \left(\frac{jx\, sxx}{n} + \frac{jy\, sxy}{n} + \frac{jz\, sxz}{n} + \frac{jx^3 \rho}{2 n^2} + \frac{jx\, jy^2 \rho}{2 n^2} + \frac{jx\, jz^2 \rho}{2 n^2}\right) + O[\frac{1}{c}]^2 \\ \left(\frac{jx\, sxy}{n} + \frac{jy\, syy}{n} + \frac{jz\, syz}{n} + \frac{jx^2\, jy \rho}{2 n^2} + \frac{jy^3 \rho}{2 n^2} + \frac{jy\, jz^2 \rho}{2 n^2}\right) + O[\frac{1}{c}]^2 \\ \left(\frac{jx\, sxz}{n} + \frac{jy\, syz}{n} + \frac{jz\, szz}{n} + \frac{jx^2\, jz \rho}{2 n^2} + \frac{jy^2\, jz \rho}{2 n^2} + \frac{jz^3 \rho}{2 n^2}\right) + O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.(uu - vtn) /. j2vr]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} \left(\frac{1}{2} n\, ux^2 \rho + \frac{1}{2} n\, uy^2 \rho + \frac{1}{2} n\, uz^2 \rho\right) + O[\frac{1}{c}]^2 \\ \left(sxx\, ux + sxy\, uy + sxz\, uz + \frac{1}{2} n\, ux^3 \rho + \frac{1}{2} n\, ux\, uy^2 \rho + \frac{1}{2} n\, ux\, uz^2 \rho\right) + O[\frac{1}{c}]^2 \\ \left(sxy\, ux + syy\, uy + syz\, uz + \frac{1}{2} n\, ux^2\, uy \rho + \frac{1}{2} n\, uy^3 \rho + \frac{1}{2} n\, uy\, uz^2 \rho\right) + O[\frac{1}{c}]^2 \\ \left(sxz\, ux + syz\, uy + szz\, uz + \frac{1}{2} n\, ux^2\, uz \rho + \frac{1}{2} n\, uy^2\, uz \rho + \frac{1}{2} n\, uz^3 \rho\right) + O[\frac{1}{c}]^2 \end{pmatrix}$$

In[•]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - vtn) / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(\frac{jx\, sxx}{n} + \frac{jy\, sxy}{n} + \frac{jz\, sxz}{n} + \frac{jx^3 \rho}{2 n^2} + \frac{jx\, jy^2 \rho}{2 n^2} + \frac{jx\, jz^2 \rho}{2 n^2} - \frac{jx^2\, vx \rho}{2 n} - \frac{jy^2\, vx \rho}{2 n} - \frac{jz^2\, vx \rho}{2 n}\right) + O[\frac{1}{c}]^2$$

In[•]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - vtn) / (A * Δt) /. relv]]
```
Out[•]//MatrixForm=

$$\left(\frac{jx\, sxx}{n} + \frac{jy\, sxy}{n} + \frac{jz\, sxz}{n} + \frac{jx^2\, Vx \rho}{2 n} + \frac{jy^2\, Vx \rho}{2 n} + \frac{jz^2\, Vx \rho}{2 n}\right) + O[\frac{1}{c}]^2$$

In[•]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - uu) / (A * Δt) /. j2vr]]
```
Out[•]//MatrixForm=

$$O[\frac{1}{c}]^2$$

In[•]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - uu) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[•]//MatrixForm=

$$O[\frac{1}{c}]^2$$

In[•]:=
```
(* Faraday tensor *)
repE = {Ex → Ex * c * Sqrt[μ0 * ϵ0], Ey → Ey * c * Sqrt[μ0 * ϵ0], Ez → Ez * c * Sqrt[μ0 * ϵ0]};
fftemp = {{0, -Ex, -Ey, -Ez}, {0, 0, Bz, -By}, {0, 0, 0, Bx}, {0, 0, 0, 0}} /. repE;
showf[assut][ffdd = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -c\, Ex \sqrt{\epsilon 0} \sqrt{\mu 0} & -c\, Ey \sqrt{\epsilon 0} \sqrt{\mu 0} & -c\, Ez \sqrt{\epsilon 0} \sqrt{\mu 0} \\ c\, Ex \sqrt{\epsilon 0} \sqrt{\mu 0} & 0 & Bz & -By \\ c\, Ey \sqrt{\epsilon 0} \sqrt{\mu 0} & -Bz & 0 & Bx \\ c\, Ez \sqrt{\epsilon 0} \sqrt{\mu 0} & By & -Bx & 0 \end{pmatrix}$$

In[•]:=
```
showf[assut][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
   ]]]
```
Out[•]//MatrixForm=

$$\left(\begin{array}{llll}
\left(-\frac{Ex^2 \epsilon 0}{2} - \frac{Ey^2 \epsilon 0}{2} - \frac{Ez^2 \epsilon 0}{2} - \frac{Bx^2}{2 \mu 0} - \frac{By^2}{2 \mu 0} - \frac{Bz^2}{2 \mu 0}\right) + \frac{-Ex^2 W \epsilon 0 - Ey^2 W \epsilon 0 - Ez^2 W \epsilon 0 + \frac{Bx^2 W}{\mu 0} + \frac{By^2 W}{\mu 0} + \frac{Bz^2 W}{\mu 0}}{c^2} + O[\frac{1}{c}]^4 &
\frac{\frac{Bz\, Ey \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{By\, Ez \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{\frac{2 Bz\, Ey\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2 By\, Ez\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + \frac{-\frac{4 By^2 Wx}{\mu 0} - \frac{4 Bz^2 Wx}{\mu 0} + \frac{4 Bx\, By\, Wy}{\mu 0} + \frac{4 Bx\, Bz\, Wz}{\mu 0}}{c^4} + O[\frac{1}{c}]^5 &
\frac{-\frac{Bz\, Ex \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{Bx\, Ez \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{-\frac{2 Bz\, Ex\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{2 Bx\, Ez\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + \frac{\frac{4 Bx\, By\, Wx}{\mu 0} - \frac{4 Bx^2 Wy}{\mu 0} - \frac{4 Bz^2 Wy}{\mu 0} + \frac{4 By\, Bz\, Wz}{\mu 0}}{c^4} + O[\frac{1}{c}]^5 &
\frac{\frac{By\, Ex \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{Bx\, Ey \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \ldots \\
\left(-\frac{Bz\, Ey \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{By\, Ez \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{\frac{2 Bz\, Ey\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2 By\, Ez\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{-4 Ey^2 Wx \epsilon 0 - 4 Ez^2 Wx \epsilon 0 + 4 Ex\, Ey\, Wy \epsilon 0 + 4 Ex\, Ez\, Wz \epsilon 0}{c^2} + O[\frac{1}{c}]^3 &
\left(-\frac{Ex^2 \epsilon 0}{2} + \frac{Ey^2 \epsilon 0}{2} + \frac{Ez^2 \epsilon 0}{2} - \frac{Bx^2}{2 \mu 0} + \frac{By^2}{2 \mu 0} + \frac{Bz^2}{2 \mu 0}\right) + \frac{-Ex^2 W \epsilon 0 + Ey^2 W \epsilon 0 + Ez^2 W \epsilon 0 + \frac{Bx^2 W}{\mu 0} - \frac{By^2 W}{\mu 0} - \frac{Bz^2 W}{\mu 0}}{c^2} + \frac{-\frac{4 Bz\, Ex\, Wy \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 Bx\, Ez\, Wy \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 By\, Ex\, Wz \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 Bx\, Ey\, Wz \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O[\frac{1}{c}]^4 &
\left(-Ex\, Ey \epsilon 0 - \frac{Bx\, By}{\mu 0}\right) + \frac{-2 Ex\, Ey\, W \epsilon 0 + \frac{2 Bx\, By\, W}{\mu 0}}{c^2} + \frac{\frac{4 Bx\, Ez\, Wx \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 Bz\, Ey\, Wy \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 Bx\, Ex\, Wz \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 By\, Ey\, Wz \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O[\frac{1}{c}]^4 &
\left(-Ex\, Ez \epsilon 0 - \frac{Bx \ldots}{\mu 0} \ldots\right. \\
\left(\frac{Bz\, Ex \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{Bx\, Ez \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{-\frac{2 Bz\, Ex\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{2 Bx\, Ez\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{4 Ex\, Ey\, Wx \epsilon 0 - 4 Ex^2 Wy \epsilon 0 - 4 Ez^2 Wy \epsilon 0 + 4 Ey\, Ez\, Wz \epsilon 0}{c^2} + O[\frac{1}{c}]^3 &
\left(-Ex\, Ey \epsilon 0 - \frac{Bx\, By}{\mu 0}\right) + \frac{-2 Ex\, Ey\, W \epsilon 0 + \frac{2 Bx\, By\, W}{\mu 0}}{c^2} + \frac{\frac{4 Bz\, Ex\, Wx \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 By\, Ez\, Wy \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 Bx\, Ex\, Wz \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 By\, Ey\, Wz \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O[\frac{1}{c}]^4 &
\left(\frac{Ex^2 \epsilon 0}{2} - \frac{Ey^2 \epsilon 0}{2} + \frac{Ez^2 \epsilon 0}{2} + \frac{Bx^2}{2 \mu 0} - \frac{By^2}{2 \mu 0} + \frac{Bz^2}{2 \mu 0}\right) + \frac{Ex^2 W \epsilon 0 - Ey^2 W \epsilon 0 + Ez^2 W \epsilon 0 - \frac{Bx^2 W}{\mu 0} + \frac{By^2 W}{\mu 0} - \frac{Bz^2 W}{\mu 0}}{c^2} + \frac{\frac{4 Bz\, Ey\, Wx \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 By\, Ez\, Wx \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 By\, Ex\, Wz \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 Bx\, Ey\, Wz \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O[\frac{1}{c}]^4 &
\left(-Ey\, Ez \epsilon 0 - \frac{By \ldots}{\mu 0} \ldots\right. \\
\left(-\frac{By\, Ex \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{Bx\, Ey \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{\frac{2 By\, Ex\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2 Bx\, Ey\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{4 Ex\, Ez\, Wx \epsilon 0 + 4 Ey\, Ez\, Wy \epsilon 0 - 4 Ex^2 Wz \epsilon 0 - 4 Ey^2 Wz \epsilon 0}{c^2} + O[\frac{1}{c}]^3 &
\left(-Ex\, Ez \epsilon 0 - \frac{Bx\, Bz}{\mu 0}\right) + \frac{-2 Ex\, Ez\, W \epsilon 0 + \frac{2 Bx\, Bz\, W}{\mu 0}}{c^2} + \frac{-\frac{4 By\, Ex\, Wx \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 Bx\, Ex\, Wy \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 Bz\, Ez\, Wy \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 Bz\, Ey\, Wz \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O[\frac{1}{c}]^4 &
\left(-Ey\, Ez \epsilon 0 - \frac{By\, Bz}{\mu 0}\right) + \frac{-2 Ey\, Ez\, W \epsilon 0 + \frac{2 By\, Bz\, W}{\mu 0}}{c^2} + \frac{-\frac{4 By\, Ey\, Wx \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 Bz\, Ez\, Wx \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 Bx\, Ey\, Wy \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 Bz\, Ex\, Wz \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O[\frac{1}{c}]^4 &
\left(\frac{Ex^2 \epsilon 0}{2} + \frac{Ey^2 \epsilon 0}{2} - \ldots\right.
\end{array}\right)$$

In[•]:=
```
(* COORDINATE EM ENERGY *)
```

In[•]:=
```
(* energy 3-form when projected along coord. axes *)
showf[assut][Expand //@ FS@PowerExpand[tte.{1, 0, 0, 0}]]
```
Out[•]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{Ex^2 \epsilon 0}{2} - \frac{Ey^2 \epsilon 0}{2} - \frac{Ez^2 \epsilon 0}{2} - \frac{Bx^2}{2 \mu 0} - \frac{By^2}{2 \mu 0} - \frac{Bz^2}{2 \mu 0}\right) + \frac{-Ex^2 W \epsilon 0 - Ey^2 W \epsilon 0 - Ez^2 W \epsilon 0 + \frac{Bx^2 W}{\mu 0} + \frac{By^2 W}{\mu 0} + \frac{Bz^2 W}{\mu 0}}{c^2} + O[\frac{1}{c}]^4 \\ \left(-\frac{Bz\, Ey \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{By\, Ez \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{\frac{2 Bz\, Ey\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2 By\, Ez\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{-4 Ey^2 Wx \epsilon 0 - 4 Ez^2 Wx \epsilon 0 + 4 Ex\, Ey\, Wy \epsilon 0 + 4 Ex\, Ez\, Wz \epsilon 0}{c^2} + O[\frac{1}{c}]^3 \\ \left(\frac{Bz\, Ex \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{Bx\, Ez \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{-\frac{2 Bz\, Ex\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{2 Bx\, Ez\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{4 Ex\, Ey\, Wx \epsilon 0 - 4 Ex^2 Wy \epsilon 0 - 4 Ez^2 Wy \epsilon 0 + 4 Ey\, Ez\, Wz \epsilon 0}{c^2} + O[\frac{1}{c}]^3 \\ \left(-\frac{By\, Ex \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{Bx\, Ey \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{\frac{2 By\, Ex\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2 Bx\, Ey\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{4 Ex\, Ez\, Wx \epsilon 0 + 4 Ey\, Ez\, Wy \epsilon 0 - 4 Ex^2 Wz \epsilon 0 - 4 Ey^2 Wz \epsilon 0}{c^2} + O[\frac{1}{c}]^3 \end{pmatrix}$$

In[•]:=
```
(* flux of coord. energy across surface *)
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tte.{1, 0, 0, 0} / (A * Δt)]]
```
Out[•]//MatrixForm=

$$\left(-\frac{Bz\, Ey \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{By\, Ez \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \left(\frac{1}{2} Ex^2 vx \epsilon 0 + \frac{1}{2} Ey^2 vx \epsilon 0 + \frac{1}{2} Ez^2 vx \epsilon 0 + \frac{Bx^2 vx}{2 \mu 0} + \frac{By^2 vx}{2 \mu 0} + \frac{Bz^2 vx}{2 \mu 0}\right) + \frac{\frac{2 Bz\, Ey\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2 By\, Ez\, W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{Ex^2 vx\, W \epsilon 0 + Ey^2 vx\, W \epsilon 0 + Ez^2 vx\, W \epsilon 0 - 4 Ey^2 Wx \epsilon 0 - 4 Ez^2 Wx \epsilon 0 + 4 Ex\, Ey\, Wy \epsilon 0 + 4 Ex\, Ez\, Wz \epsilon 0 - \frac{Bx^2 vx\, W}{\mu 0} - \frac{By^2 vx\, W}{\mu 0} - \frac{Bz^2 vx\, W}{\mu 0}}{c^2} + O[\frac{1}{c}]^3$$

In[•]:=
```
(* supply term for coord. energy *)
TTx = tW[tjv@tte]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Dcoords[[1, ;; , ;;]]].gg + Dcoords[[1, ;; , ;;]]).TTx]]]
```
Out[•]//MatrixForm=

$$\frac{Ex^2 \epsilon 0\, W^{(1,0,0,0)}[t, x, y, z] + Ey^2 \epsilon 0\, W^{(1,0,0,0)}[t, x, y, z] + Ez^2 \epsilon 0\, W^{(1,0,0,0)}[t, x, y, z] + \frac{Bx^2 W^{(1,0,0,0)}[t,x,y,z]}{\mu 0} + \frac{By^2 W^{(1,0,0,0)}[t,x,y,z]}{\mu 0} + \frac{Bz^2 W^{(1,0,0,0)}[t,x,y,z]}{\mu 0}}{c^2} + O[\frac{1}{c}]^3$$