Einstein's equations from a Newtonian perspective

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1 Introduction

* If space metric and chronometry are influenced by matter, then all "rates of change" are determined by matter.

* Rate of change of momentum reinterpreted as inertial force (Mach)

* "Motion" is not clear any longer. Topologically we can consider a bulk of matter as static and the metric among its elements as changing, or vice versa.

* The problem is that only the relations of the geometrical objects with respect to one another is observable; but such relations are not all arbitrary, there are constraints among them. In Newtonian mechanics we use a reference frame – a *rigid* body – to find a compact description that gets rid of the constraints [1–5]. In general relativity there is no "rigidity" and we have more choice.

* Do the equations for the metric have constitutive parts?

* If mass is twisted 3-form, then (analogously to displacement current) gravitational potential must be related to a twisted 2-form [6; 7, ch. V, pp. 192–195]. Then is there an equivalent "magnetic field strength" that relates to momentum?

$$\mathrm{d}m = 0, \qquad \partial_t \rho + \mathrm{d}j = 0, \tag{1}$$

$$\mathrm{d}F = \rho, \qquad \partial_t F + \mathrm{d}H = j \tag{2}$$

* Extrinsic curvature can be understood (kinematically) as the time-derivative of 3-metric (no Einstein eqns needed).

Setting lapse $\alpha = 1$ and shift $\beta = 0$:

$$\partial_t \boldsymbol{h} = -2\boldsymbol{K} \cdot \boldsymbol{h} \tag{3}$$

$$\partial_t \boldsymbol{K} = \boldsymbol{R} + \boldsymbol{K} \operatorname{tr} \boldsymbol{K} + 4\pi(\operatorname{tr} \boldsymbol{T} - E - 2\boldsymbol{T}) \tag{4}$$

$$\operatorname{tr} \boldsymbol{R} + (\operatorname{tr} \boldsymbol{K})^2 - \boldsymbol{K} : \boldsymbol{K} = 16\pi E \tag{5}$$

$$\nabla \cdot \boldsymbol{K} - \nabla \operatorname{tr} \boldsymbol{K} = 8\pi \boldsymbol{p} \tag{6}$$

the second should come from

$$\partial_t(\boldsymbol{K} \cdot \boldsymbol{h}) = \boldsymbol{R} \cdot \boldsymbol{h} + \boldsymbol{h} \cdot \boldsymbol{K} \operatorname{tr} \boldsymbol{K} - 2\boldsymbol{h} \cdot \boldsymbol{K} \cdot \boldsymbol{K} + 4\pi[\boldsymbol{h}(\operatorname{tr} \boldsymbol{T} - E) - 2\boldsymbol{T} \cdot \boldsymbol{h}) \tag{7}$$

$$\partial_t \boldsymbol{p} = \tfrac{1}{8\pi}(\nabla \cdot \boldsymbol{R} - \partial_t \nabla \operatorname{tr} \boldsymbol{K} + \operatorname{tr} \boldsymbol{K}\, \nabla \cdot \boldsymbol{K}) + \tfrac{1}{2}\nabla \operatorname{tr} \boldsymbol{T} - \tfrac{1}{2}\nabla E - \nabla \cdot \boldsymbol{T} \tag{8}$$

$$\partial_t \boldsymbol{p} - \boldsymbol{p} \operatorname{tr} \boldsymbol{K} = \tfrac{1}{8\pi}[\nabla \cdot \boldsymbol{R} - \partial_t \nabla \operatorname{tr} \boldsymbol{K} + \tfrac{1}{2}\nabla(\operatorname{tr} \boldsymbol{K})^2] + \tfrac{1}{2}\nabla \operatorname{tr} \boldsymbol{T} - \tfrac{1}{2}\nabla E - \nabla \cdot \boldsymbol{T} \tag{9}$$

$$\partial_t \rho + \nabla \cdot (\rho \boldsymbol{v}) = 0, \tag{10a}$$

$$\partial_t(\rho \boldsymbol{v}) + \nabla \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v}) - \nabla \cdot \boldsymbol{T} - \rho \boldsymbol{b} = 0, \tag{10b}$$

$$\boldsymbol{T}^\top - \boldsymbol{T} = 0, \tag{10c}$$

$$\partial_t(\rho u) + \nabla \cdot (\rho \boldsymbol{v} u) - \boldsymbol{T} : \nabla \boldsymbol{v} + \nabla \cdot \boldsymbol{q} + \rho Q = 0, \tag{10d}$$

$$\partial_t\big(\rho u + \tfrac{1}{2}\rho \boldsymbol{v}^2\big) + \nabla \cdot \big(\rho \boldsymbol{v} u + \tfrac{1}{2}\rho \boldsymbol{v} \boldsymbol{v}^2\big) - \nabla \cdot (\boldsymbol{T} \cdot \boldsymbol{v}) - \rho \boldsymbol{b} \cdot \boldsymbol{v} - \nabla \cdot \boldsymbol{q} - \rho Q = 0, \tag{10e}$$

$$\partial_t(\rho s) + \nabla \cdot (\rho \boldsymbol{v} s) - \nabla \cdot (\boldsymbol{q}/\Theta) - \rho Q/\Theta \geqslant 0. \tag{10f}$$

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Bibliography

[1] H. Zanstra: *Motion relativated by means of a hypothesis of A. Föppl*. Proc. Acad. Sci. Amsterdam (Proc. of the Section of Sciences Koninklijke Nederlandse Akademie van Wetenschappen) **23**$^{\mathrm{II}}$ (1922), 1412–1418.

[2] H. Zanstra: *Die Relativierung der Bewegung mit Hilfe der Hypothese von A. Föppl*. Ann. der Phys. **70**^{2} (1923), 153–160.

[3] H. Zanstra: *A study of relative motion in connection with classical mechanics*. Phys. Rev. **23**^{4} (1924), 528–545.

[4] H. Zanstra: *On the meaning of absolute systems in mechanics and physics*. Physica **12**^{5} (1946), 301–310.

[5] J. Barbour: *The definition of Mach's principle*. Found. Phys. **40**^{9} (2010), 1263–1284. arXiv:1007.3368.

[6] F. Kottler: *Newton'sches Gesetz und Metrik*. Sitzungsber. Akad. Wiss. Wien, Math.-Naturw. Klasse, Abt. IIa **131**^{2} (1922), 1–14.

[7] E. T. Whittaker: *A History of the Theories of Aether and Electricity. Vol. 2: The Modern Theories, 1900–1926*. Thomas Nelson and Sons, London (1953). First publ. 1910.

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