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Kinematics, dynamics, force, inertia, metric in Newtonian relativity and in general relativity [draft]

P.G.L. Porta Mana
<piero.mana@ntnu.no>

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This note is an exploration of some notions around kinematics, dynamics, matter, inertia, and force in Newtonian- and general-relativistic continuum mechanics. The conceptual pivots of this exploration are these: (i) kinematics should not allow us to distinguish between Newtonian relativity and general relativity; (ii) in neither theory should there be any distinction between inertial and non-inertial motion, or between forced and unforced motion.
Note: Dear Reader & Peer, this manuscript is being peer-reviewed by you. Thank you.

1 From kinematics to dynamics

Our first primitive is the notion of spacetime, as the seats of all events. It's represented by a 4D differential manifold. In the following I misuse 'topological' as a shorthand for 'differential-geometric', to denote objects that only require a differential structure – so no connections, metrics, volume elements, et similar – for their definition. In spacetime there's no predefined notion of simultaneity nor of 'identity through time'.

Our second primitive is the notion of matter, of things. How to represent it? Matter is characterized by our ability to mark a small part of it – as when we draw a '◯' sign on an body, or throw some leaves in flowing water – and then to follow it as it moves. It relates to notions of identity. Two topological objects can represent this: a inner-oriented vector field or an outer-oriented, closed 3-covector field[1]. The first can be depicted as a collection of 1D curves; the second as a collection of 3D tubes[2]. Either has the property that, given one event in spacetime, a unique 1D curve through it is selected.

An outer-oriented, closed 3-covector field has an additional feature: it can be integrated over an outer-oriented 3D surface (count the 3D tubes which cross that surface). Later on, such a surface could be characterized as spacelike or timelike. This field would therefore allow us to speak of the 'quantity' of matter within a 3D region at some time, or the

[1] Schouten 1989. [2] Misner et al. 2003; Schouten 1989.

quantity of matter that passes through a 2D surface during a given time. Its closedness would say that the quantity of matter passing through a closed 2D surface must be zero. The mathematics show that this conservation property is tightly connected with the notion of 'identity' mentioned above.

So the representation of matter as an outer-oriented, closed 3-covector field gives us its 'identity' properties and also its conservation. In general relativity it's the conservation of baryonic number[3]. In Newtonian relativity traditionally it's the conservation of mass, but it's easy to show that this theory could be formulated with a baryon conservation instead; a similar reformulation in fact takes place within the subtheory of mixtures[4]. We call this outer-oriented, closed 3-covector 'matter form'.

A spacetime and one or more matter forms give us a first, topological kinematics. Such kinematics doesn't allow us to say that parts of matters are at rest or in motion, or approaching or diverging; but it allows us to follow their trajectory through spacetime.

[3] Eckart 1940; Gourgoulhon 2012; Alcubierre 2008; Misner et al. 1973 § 22.2. [4] Samohýl 1975; Bowen 1976; Samohýl et al. 2014; Faria 2003.

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(‘de *X*’ is listed under D, ‘van *X*’ under V, and so on, regardless of national conventions.)

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