Open Science Framework DOI:10.31219/osf.io/***

Memos on ideal gases

P.G.L. Porta Mana
Kavli Institute, Trondheim
<piero.mana@ntnu.no>

17 February 2020; updated 19 February 2020

Memos on ideal gases, from a thermomechanical viewpoint.

1 Notation

t: time instant

$\boldsymbol{r}$: position

n: mole density (a 3-form)

$\boldsymbol{v}$: velocity

M: molar mass

p: pressure

T: temperature

u: internal energy density

$\boldsymbol{q}$: heat flow

For an ideal gas, the fields $\bigl(n(t,\boldsymbol{r}), \boldsymbol{v}(t,\boldsymbol{r}), T(t,\boldsymbol{r})\bigr)$ constitute the state.

We also define

$$\nabla\boldsymbol{v}^{+} := \tfrac{1}{2}(\nabla\boldsymbol{v} + \nabla\boldsymbol{v}^{\intercal}), \quad \nabla\boldsymbol{v}^{\perp} := \nabla\boldsymbol{v}^{+} - 1/3\nabla\cdot\boldsymbol{v}\,\boldsymbol{I} \tag{1}$$

and note that from the balance of mass below

$$\nabla\cdot\boldsymbol{v} \equiv \operatorname{tr}(\nabla\boldsymbol{v}^{+}) = -\frac{\partial_t n + \boldsymbol{v}\cdot\nabla n}{n} \tag{2}$$

The equations below are derived for example in Samohýl & Pekař[1]; compare also Truesdell & Muncaster[2].

[1] Samohýl & Pekař 2014 §§ 3.5–3.7.

[2] Truesdell & Muncaster 1980 ch. I.

2 Balance laws

Quantity of matter, force (in an inertial frame), energy:

$$\partial_t n + \nabla \cdot (n\boldsymbol{v}) = 0 \tag{3}$$

$$M\partial_t(n\boldsymbol{v}) + M\nabla \cdot (n\boldsymbol{v} \otimes \boldsymbol{v}) - \nabla \cdot \boldsymbol{\tau} = 0 \tag{4}$$

$$\partial_t u + \nabla \cdot (u\boldsymbol{v}) + \nabla \cdot \boldsymbol{q} - \boldsymbol{\tau} : \nabla\boldsymbol{v}^{+} = 0 \tag{5}$$

Constitutive equations for heat flow, stress, internal energy:

$$\boldsymbol{q} = -k(n, T)\, \nabla T \tag{6}$$

$$\boldsymbol{\tau} = -[RnT - \zeta(n, T)\, \nabla \cdot \boldsymbol{v}]\boldsymbol{I} + \eta(n, T)\, \nabla\boldsymbol{v}^{\perp} \tag{7}$$

$$u = cT \tag{8}$$

$$k, \eta, \zeta \geqslant 0 \tag{9}$$

k is the thermal conductivity, η the shear viscosity, ζ the bulk or volume viscosity. Equations for these are given e.g. in Kannuluik & Carman[3], Sutherland[4], Cramer or Sharma & Kumar[5].

Bibliography

('de X' is listed under D, 'van X' under V, and so on, regardless of national conventions.)

Cramer, M. S. (2012): *Numerical estimates for the bulk viscosity of ideal gases*. Phys. Fluids **24**[6], 066102.

Kannuluik, W. G., Carman, E. H. (1951): *The temperature dependence of the thermal conductivity of air*. **4**[3], 305–314.

Samohýl, I. (1987): *Thermodynamics of Irreversible Processes in Fluid Mixtures (Approached by Rational Thermodynamics)*. (Teubner, Leipzig). Partially rewritten as Samohýl, Pekař (2014).

Samohýl, I., Pekař, M. (2014): *The Thermodynamics of Linear Fluids and Fluid Mixtures*. (Springer, Cham). First published as Samohýl (1987).

Sharma, B., Kumar, R. (2019): *Estimation of bulk viscosity of dilute gases using a nonequilibrium molecular dynamics approach*. Phys. Rev. E **100**[1], 013309.

Sutherland, W. (1893): *The viscosity of gases and molecular force*. Phil. Mag. **36**[223], 507–531.

Truesdell III, C. A., Muncaster, R. G. (1980): *Fundamentals of Maxwell's Kinetic Theory of a Simple Monatomic Gas: Treated as a Branch of Rational Mechanics*. (Academic Press, New York).

[3] Kannuluik & Carman 1951. [4] Sutherland 1893. [5] Cramer 2012; Sharma & Kumar 2019.