```
In[1]:= << "christoffelsymbols.m"
(* First index is upper index Table[FS[cc[[ii,;;,;;]]==T[cc[[ii,;;,;;]]]],{ii,1,4}] *)

In[2]:= showf[assumptions_, simp_ : FullSimplify] := ((Assuming[assumptions, Expand //@ simp@PowerExpand[#]] // MF) &);

In[3]:= (* Show matrix expressions and power expansions*)
show[assumptions_, power_, simp_ : FullSimplify] := ({Assuming[assumptions, simp@PowerExpand[#]] // MF, "\n",
      Assuming[assumptions, simp@PowerExpand[Series[#, {c, Infinity, power}]]] // MF} &);
shows[assumptions_, power_, simp_ : FullSimplify] := ((
      Assuming[assumptions, simp@PowerExpand[Series[#, {c, Infinity, power}]]] // MF) &);
showf[assumptions_, simp_ : FullSimplify] := ((Assuming[assumptions, Expand //@ simp@PowerExpand[#]] // MF) &);
show1[assumptions_, simp_ : Identity] := ((Assuming[assumptions, simp[#]] // MF) &);
show2[assumptions_, power_, simp_ : Identity] := ((
      Assuming[assumptions, simp[Series[#, {c, Infinity, power}]]] // MF) &);

In[8]:= coords = {t, x, y, z}
Out[8]= {t, x, y, z}

In[9]:= (* Flat metric *)
(gg0 = DiagonalMatrix[{-c^2, 1, 1, 1}]) // MF
```

Out[9]//MatrixForm=

$$\begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[10]:= (*D[G*M/Sqrt[x^2+y^2+z^2],{{x,y,z}}] *)

In[11]:= (* -W is the potential gravitational energy: W=GM/r
  that is, F_g(downwards)=grad W
*)

In[12]:= (* Rotating metric from poissonetal *)
(gg = DiagonalMatrix@Diagonal@{{-c^2*(1-2*W/c^2 +O[c, +Infinity]^4(*-2*((*Ψ[t,x,y,z]*)-W[t,x,y,z]^2)/c^4*)),
        -4*Wx/c^2+O[c, +Infinity]^4, -4*Wy/c^2+O[c, +Infinity]^4, -4*Wz/c^2+O[c, +Infinity]^4},
      {-4*Wx/c^2+O[c, +Infinity]^4,
       1+2*W/c^2+O[c, +Infinity]^4, 0, 0},
      {-4*Wy/c^2+O[c, +Infinity]^4, 0, 1+2*W/c^2+O[c, +Infinity]^4, 0},
      {-4*Wz/c^2+O[c, +Infinity]^4, 0, 0, 1+2*W/c^2+O[c, +Infinity]^4}}) // MF
(*(gg=DiagonalMatrix[{-c^2*(1+2*Φ[t,r]/c^2),1+2*Λ[t,r]/c^2,r^2,r^2*Sin[θ]^2}])//MF*)
```

Out[12]//MatrixForm=

$$\begin{pmatrix} -c^2+2W+O[\frac{1}{c}]^2 & 0 & 0 & 0 \\ 0 & 1+\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 & 0 \\ 0 & 0 & 1+\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 \\ 0 & 0 & 0 & 1+\frac{2W}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[13]:= Inverse[gg] // MF
```

Out[13]//MatrixForm=

$$\begin{pmatrix} -(\frac{1}{c})^2-\frac{2W}{c^4}+O[\frac{1}{c}]^6 & 0 & 0 & 0 \\ 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 & 0 \\ 0 & 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 \\ 0 & 0 & 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[14]:= (*(gg=DiagonalMatrix@Diagonal[gg])//MF*)

In[130]:= (* functions to temporarily remove coord-dep *)
tW[xx_] := (xx /. {W → W[t, x, y, z], Wx → Wx[t, x, y, z], Wy → Wy[t, x, y, z], Wz → Wz[t, x, y, z]});
itW[xx_] := (xx /. {W[t, x, y, z] → W, Wx[t, x, y, z] → Wx, Wy[t, x, y, z] → Wy, Wz[t, x, y, z] → Wz});
(ggt = tW[gg]) // MF;

In[133]:= assut = {c > 0, Element[a, Reals], Element[v, Reals], Element[t, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals],
    Element[vx, Reals], Element[vy, Reals], Element[vz, Reals], Element[n, Reals], Element[r, Reals], Element[θ, Reals], Element[ϕ, Reals], Abs[v] < c, -c < vx < c, -c < ux < c, r > 0, 0 < θ < Pi,
    Normal@gg[[1, 1]] / c^2 < 0, Normal@gg[[2, 2]] > 0, Normal@gg[[3, 3]] > 0, Normal@gg[[4, 4]] > 0, n > 0, Element[jx, Reals], Element[jy, Reals], Element[jz, Reals], Element[sxx, Reals], Element[sxy, Reals], Element[sxz, Reals], Element[syy, Reals], Element[syz, Reals], Element[szz, Reals],
    -Normal@Det[gg] > 0, β > 0};

assutt = {c > 0, Element[a, Reals], Element[v, Reals], Element[t, Reals], Element[x, Reals], Element[y, Reals], Element[z, Reals],
    Element[v_x, Reals], Element[v_y, Reals], Element[v_z, Reals], Element[n, Reals], Element[r, Reals], Element[θ, Reals], Element[ϕ, Reals], Abs[v] < c, -c < v_x < c, -c < u_x < c, r > 0, 0 < θ < Pi,
    β > 0, Normal@ggt[[1, 1]] / c^2 < 0, Normal@ggt[[2, 2]] > 0, Normal@ggt[[3, 3]] > 0, Normal@ggt[[4, 4]] > 0,
    -Normal@Det[ggt] > 0};

In[135]:= (igg = Assuming[assut, FullSimplify@PowerExpand[Inverse[gg]]]) // MF
```

Out[135]//MatrixForm=

$$\begin{pmatrix} -(\frac{1}{c})^2-\frac{2W}{c^4}+O[\frac{1}{c}]^6 & 0 & 0 & 0 \\ 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 & 0 \\ 0 & 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 & 0 \\ 0 & 0 & 0 & 1-\frac{2W}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[136]:= (*show[assut,2]@ChristoffelSymbol[gg,coords][[2]]*)

In[137]:= (* volume element *)
(dg = Assuming[assut, FullSimplify@PowerExpand[Sqrt[-Det[gg]] / c]]) // MF
```

Out[137]//MatrixForm=

$$1+\frac{2W}{c^2}+O\left[\frac{1}{c}\right]^4$$

```
In[138]:= (* Christoffel symbols  *)
cc = Assuming[assut, FullSimplify@PowerExpand[itW[ChristoffelSymbol[ggt, coords]]]];

In[139]:=

In[140]:= (* 3-vector of moving surface parallel to yz moving with velocity V   *)
surface = {-(Vx*Ax+Vy*Ay+Vz*Az), Ax, Ay, Az}*Δt;

In[141]:=
```

(* Matter current *)

```
In[142]:= (* matter-current 3-covector *)
NJ = {n, jx, jy, jz};

In[143]:= (* norm of matter 3-covector *)
Assuming[assut, Expand //@ FS@PowerExpand[Sqrt[-NJ.gg.NJ] / c]]
```

Out[143]=

$$n+\frac{-\frac{jx^2}{2n}-\frac{jy^2}{2n}-\frac{jz^2}{2n}-nW}{c^2}+O\left[\frac{1}{c}\right]^4$$

```
In[144]:= (* matter associated 1-vector *)
(NJvec = Assuming[assut, Expand //@ FS@PowerExpand[NJ / dg]]) // MF
```

Out[144]//MatrixForm=

$$\begin{pmatrix} n-\frac{2(nW)}{c^2}+O[\frac{1}{c}]^4 \\ jx-\frac{2(jx\,W)}{c^2}+O[\frac{1}{c}]^4 \\ jy-\frac{2(jy\,W)}{c^2}+O[\frac{1}{c}]^4 \\ jz-\frac{2(jz\,W)}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[145]:= (* matter associated 4-vel vector *)
(uu = Assuming[assut, Expand //@ FS@PowerExpand[c*NJvec / Sqrt[-NJvec.gg.NJvec]]]) // MF
```

Out[145]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{jx^2}{2n^2}+\frac{jy^2}{2n^2}+\frac{jz^2}{2n^2}+W}{c^2}+O[\frac{1}{c}]^4 \\ \frac{jx}{n}+\frac{\frac{jx^3}{2n^3}+\frac{jx\,jy^2}{2n^3}+\frac{jx\,jz^2}{2n^3}+\frac{jx\,W}{n}}{c^2}+O[\frac{1}{c}]^4 \\ \frac{jy}{n}+\frac{\frac{jx^2\,jy}{2n^3}+\frac{jy^3}{2n^3}+\frac{jy\,jz^2}{2n^3}+\frac{jy\,W}{n}}{c^2}+O[\frac{1}{c}]^4 \\ \frac{jz}{n}+\frac{\frac{jx^2\,jz}{2n^3}+\frac{jy^2\,jz}{2n^3}+\frac{jz^3}{2n^3}+\frac{jz\,W}{n}}{c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[146]:= (* it is normalized *)
uu.gg.uu
```

Out[146]=

$$-c^2+O\left[\frac{1}{c}\right]^2$$

```
In[147]:= (* matter associated 1-covector *)
(NJcov = Assuming[assut, Expand //@ FS@PowerExpand[gg.(NJ / dg)]]) // MF
```

Out[147]//MatrixForm=

$$\begin{pmatrix} -n\,c^2+4\,n\,W+O[\frac{1}{c}]^2 \\ jx+O[\frac{1}{c}]^4 \\ jy+O[\frac{1}{c}]^4 \\ jz+O[\frac{1}{c}]^4 \end{pmatrix}$$

```
In[148]:= (* scalar product with de/de_x (for momentum x-component) *)
uu.gg.{0, 1, 0, 0}
```

Out[148]=

$$\frac{jx}{n}+\frac{\frac{jx^3}{2n^3}+\frac{jx\,jy^2}{2n^3}+\frac{jx\,jz^2}{2n^3}+\frac{3\,jx\,W}{n}}{c^2}+O\left[\frac{1}{c}\right]^4$$

In[149]:=
```
(* scalar product with de/de_i (for momentum i-component) *)
uu.gg // MF
```
Out[149]//MatrixForm=

$$\begin{pmatrix} -c^2+\left(-\frac{jx^2}{2\,n^2}-\frac{jy^2}{2\,n^2}-\frac{jz^2}{2\,n^2}+W\right)+O\left[\frac{1}{c}\right]^2 \\ \frac{jx}{n}+\frac{\frac{jx^3}{2\,n^3}+\frac{jx\,jy^2}{2\,n^3}+\frac{jx\,jz^2}{2\,n^3}+\frac{3\,jx\,W}{n}}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n}+\frac{\frac{jx^2\,jy}{2\,n^3}+\frac{jy^3}{2\,n^3}+\frac{jy\,jz^2}{2\,n^3}+\frac{3\,jy\,W}{n}}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n}+\frac{\frac{jx^2\,jz}{2\,n^3}+\frac{jy^2\,jz}{2\,n^3}+\frac{jz^3}{2\,n^3}+\frac{3\,jz\,W}{n}}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[150]:=
```
(* retransform matter 4-vel to matter 3-covector *)
showf[assut][uu * dg / c * Sqrt[-NJvec.gg.NJvec]]
```
Out[150]//MatrixForm=

$$\begin{pmatrix} n+O\left[\frac{1}{c}\right]^5 \\ jx+O\left[\frac{1}{c}\right]^5 \\ jy+O\left[\frac{1}{c}\right]^5 \\ jz+O\left[\frac{1}{c}\right]^5 \end{pmatrix}$$

In[151]:=
```
(* simplification to x-directed matter flux and velocity *)
assutjx = Join[assut, {jy == 0, jz == 0, uy == 0, uz == 0}];
```

In[152]:=
```
(* flux of matter across surface *)
Simplify[surface.NJ / (Δt)]
```
Out[152]=

$$Ax\,(jx-n\,Vx)+Ay\,(jy-n\,Vy)+Az\,(jz-n\,Vz)$$

In[153]:=
```
(* normalized zero-flux velocity is same as U *)
vnoflux = {1, jx / n, jy / n, jz / n};
FS[c * vnoflux / Sqrt[-vnoflux.gg.vnoflux] == uu]
```
Out[154]=

True

In[155]:=
```
(* replace matter flux in terms of velocity*)
replaceJu = {jx → ux * n, jy → uy * n, jz → uz * n}
```
Out[155]=

$$\{jx\to n\,ux,\ jy\to n\,uy,\ jz\to n\,uz\}$$

In[156]:=
```
(* collect velocity magnitude*)
replaceuUnorm = {ux ^ 2 → U ^ 2 - uy ^ 2 - uz ^ 2, ux ^ 3 → ux * (U ^ 2 - uy ^ 2 - uz ^ 2), jx ^ 2 → J ^ 2 - jy ^ 2 - jz ^ 2, jx ^ 3 → jx * (J ^ 2 - jy ^ 2 - jz ^ 2)}
```
Out[156]=

$$\{ux^2\to U^2-uy^2-uz^2,\ ux^3\to ux\,(U^2-uy^2-uz^2),\ jx^2\to J^2-jy^2-jz^2,\ jx^3\to jx\,(J^2-jy^2-jz^2)\}$$

In[157]:=
```
FS[uu /. replaceuUnorm] // MF
```
Out[157]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{J^2}{2\,n^2}+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jx}{n}+\frac{jx\,(J^2+2\,n^2\,W)}{2\,n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n}+\frac{jy\,(J^2+2\,n^2\,W)}{2\,n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n}+\frac{jz\,(J^2+2\,n^2\,W)}{2\,n^3\,c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[158]:=
```
FS[uu /. replaceJu] // MF
```
Out[158]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{1}{2}\,(ux^2+uy^2+uz^2)+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ ux+\frac{ux\,(ux^2+uy^2+uz^2+2\,W)}{2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ uy+\frac{uy\,(ux^2+uy^2+uz^2+2\,W)}{2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ uz+\frac{uz\,(ux^2+uy^2+uz^2+2\,W)}{2\,c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[159]:=
```
FS[(uu /. replaceJu) /. replaceuUnorm] // MF
```
Out[159]//MatrixForm=

$$\begin{pmatrix} 1+\frac{\frac{U^2}{2}+W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ ux+\frac{\frac{U^2\,ux}{2}+ux\,W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ uy+\frac{\frac{U^2\,uy}{2}+uy\,W}{c^2}+O\left[\frac{1}{c}\right]^4 \\ uz+\frac{\frac{U^2\,uz}{2}+uz\,W}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[160]:=

In[161]:=
```
(* Project along u velocity *)
proju = Assuming[assut, Expand //@ FS@PowerExpand[-Outer[Times, uu, gg.uu] / c ^ 2]];
projperpu = Assuming[assut, Expand //@ FS@PowerExpand[IdentityMatrix[4] - proju]];
testproj[ass_, x_] := showf[ass] /@ {Assuming[ass, Expand //@ FS@PowerExpand[proju.x.proju == x]], projperpu.x.proju == x, proju.x.projperpu == x, projperpu.x.projperpu == x}
```

In[164]:=
```
(* Project along u velocity *)
proju = Assuming[assut, FS[-Outer[Times, uu, gg.uu] / c ^ 2]];
projperpu = Assuming[assut, FS[IdentityMatrix[4] - proju]];
testproj[ass_, x_] := showf[ass] /@ {Assuming[ass, Expand //@ FS@PowerExpand[proju.x.proju == x]], projperpu.x.proju == x, proju.x.projperpu == x, projperpu.x.projperpu == x}
```

In[167]:=
```
(* aux 4-velocity *)
auu = {temp, aux, auy, auz};
solu = temp /. Solve[Normal[auu.gg.auu] == -c ^ 2, temp][[2]]
```
Out[168]=

$$\frac{\sqrt{aux^2+auy^2+auz^2+c^2}}{\sqrt{c^2-2\,W}}$$

In[169]:=
```
(auu = Assuming[assut, FS[auu /. {temp → solu}]]) // MF
```
Out[169]//MatrixForm=

$$\begin{pmatrix} \sqrt{\frac{aux^2+auy^2+auz^2+c^2}{c^2-2\,W}} \\ aux \\ auy \\ auz \end{pmatrix}$$

In[170]:=
```
auu.gg.auu
```
Out[170]=

$$-c^2+O\left[\frac{1}{c}\right]^2$$

In[211]:=
```
(* 4-velocity and matter current with explicit coordinate dependence *)
tjv[xx_] := (xx /. {n → n[t, x, y, z], jx → ux[t, x, y, z] * n[t, x, y, z], jy → uy[t, x, y, z] * n[t, x, y, z], jz → uz[t, x, y, z] * n[t, x, y, z]});
tjn[xx_] := (xx /. {n → n[t, x, y, z], jx → jx[t, x, y, z], jy → jy[t, x, y, z], jz → jz[t, x, y, z]});
itjn[xx_] := (xx /. {n[t, x, y, z] → n, jx[t, x, y, z] → jx, jy[t, x, y, z] → jy, jz[t, x, y, z] → jz});
itjv[xx_] := (xx /. {n[t, x, y, z] → n, ux[t, x, y, z] → jx / n, uy[t, x, y, z] → jy / n, uz[t, x, y, z] → jz / n});
repjn = {D[n[t, x, y, z], t] → D[jx[t, x, y, z], x] + D[jy[t, x, y, z], y] + D[jz[t, x, y, z], z]};
MF /@ {uut = Assuming[assut, FS[tjn[tW[uu]]]], uuv = Assuming[assut, FS[tjv[tW[uu]]]]}
```
Out[216]=

$$\left\{\begin{pmatrix} 1+\frac{\frac{jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2}{2\,n[t,x,y,z]^2}+W[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jx[t,x,y,z]}{n[t,x,y,z]}+\frac{jx[t,x,y,z]\,(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z])}{2\,n[t,x,y,z]^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy[t,x,y,z]}{n[t,x,y,z]}+\frac{jy[t,x,y,z]\,(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z])}{2\,n[t,x,y,z]^3\,c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz[t,x,y,z]}{n[t,x,y,z]}+\frac{jz[t,x,y,z]\,(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z])}{2\,n[t,x,y,z]^3\,c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} 1+\frac{\frac{1}{2}\,(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2)+W[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \\ ux[t,x,y,z]+\frac{ux[t,x,y,z]\,(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z])}{2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ uy[t,x,y,z]+\frac{uy[t,x,y,z]\,(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z])}{2\,c^2}+O\left[\frac{1}{c}\right]^4 \\ uz[t,x,y,z]+\frac{uz[t,x,y,z]\,(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z])}{2\,c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[171]:=

In[217]:=

(* Construction of energy–momentum tensor *)

In[218]:=
```
(* definition of heat-flux, orthogonal to matter-current *)
Qtemp = {qt, qx, qy, qz};
```

In[219]:=
```
(proju.Qtemp) // MF
```
Out[219]//MatrixForm=

$$\begin{pmatrix} qt+\frac{\frac{(jx^2+jy^2+jz^2)\,qt}{n^2}-\frac{jx\,qx}{n}-\frac{jy\,qy}{n}-\frac{jz\,qz}{n}}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jx\,qt}{n}+\frac{\frac{jx\,(jx^2+jy^2+jz^2)\,qt}{n^3}-\frac{jx^2\,qx}{n^2}-\frac{jx\,jy\,qy}{n^2}-\frac{jx\,jz\,qz}{n^2}}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jy\,qt}{n}+\frac{\frac{jy\,(jx^2+jy^2+jz^2)\,qt}{n^3}-\frac{jx\,jy\,qx}{n^2}-\frac{jy^2\,qy}{n^2}-\frac{jy\,jz\,qz}{n^2}}{c^2}+O\left[\frac{1}{c}\right]^4 \\ \frac{jz\,qt}{n}+\frac{\frac{jz\,(jx^2+jy^2+jz^2)\,qt}{n^3}-\frac{jx\,jz\,qx}{n^2}-\frac{jy\,jz\,qy}{n^2}-\frac{jz^2\,qz}{n^2}}{c^2}+O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[220]:=
```
qsol = Solve[Normal[proju.Qtemp] == 0, qt][[1]]
```
Out[220]=

$$\left\{qt\to\frac{n\,(jx\,qx+jy\,qy+jz\,qz)}{jx^2+jy^2+jz^2+c^2\,n^2}\right\}$$

In[221]:=
```
(Q = Assuming[assut, FS[Qtemp /. qsol]]) // MF
```
Out[221]//MatrixForm=

$$\begin{pmatrix} \frac{n\,(jx\,qx+jy\,qy+jz\,qz)}{jx^2+jy^2+jz^2+c^2\,n^2} \\ qx \\ qy \\ qz \end{pmatrix}$$

In[222]:=
```
{Normal@Series[Q.NJ, {c, Infinity, 1}] == 0}
```
Out[222]=

$$\{jx\,qx + jy\,qy + jz\,qz == 0\}$$

In[223]:=
```
assutQ = Join[assut, {Normal@Series[Q.NJ, {c, Infinity, 1}] == 0}];
```
In[224]:=
```
Assuming[assutQ, FS@{proju.Q, projperpu.Q == Q}]
```
Out[224]=

$$\left\{\left\{O\left[\frac{1}{c}\right]^6,\ O\left[\frac{1}{c}\right]^6,\ O\left[\frac{1}{c}\right]^6,\ O\left[\frac{1}{c}\right]^6\right\},\ \text{True}\right\}$$

In[225]:=
```
(* non-symmetric heat-tensor *)
Assuming[assutQ, FS[Qtens = Assuming[assut, Expand //@ FS@PowerExpand[Outer[Times, Q, gg.uu / c ^ 2]]]]] // MF
```
Out[225]//MatrixForm=

$$\begin{pmatrix}
O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \\
-qx + \frac{-\frac{(jx^2+jy^2+jz^2)\,qx}{2 n^2} + qx\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{jx\,qx}{n c^2} + \frac{jx\,qx\,(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jy\,qx}{n c^2} + \frac{jy\,qx\,(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jz\,qx}{n c^2} + \frac{jz\,qx\,(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 \\
-qy + \frac{-\frac{(jx^2+jy^2+jz^2)\,qy}{2 n^2} + qy\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{jx\,qy}{n c^2} + \frac{jx\,qy\,(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jy\,qy}{n c^2} + \frac{jy\,qy\,(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jz\,qy}{n c^2} + \frac{jz\,qy\,(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 \\
-qz + \frac{-\frac{(jx^2+jy^2+jz^2)\,qz}{2 n^2} + qz\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{jx\,qz}{n c^2} + \frac{jx\,qz\,(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jy\,qz}{n c^2} + \frac{jy\,qz\,(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jz\,qz}{n c^2} + \frac{jz\,qz\,(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6
\end{pmatrix}$$

In[226]:=
```
Assuming[assutQ, FS[T[Qtens.Inverse[gg]].gg - Qtens]] // MF
```
Out[226]//MatrixForm=

$$\begin{pmatrix}
O[\frac{1}{c}]^6 & \frac{qx}{c^2} + \frac{\frac{(jx^2+jy^2+jz^2)\,qx}{2 n^2} + 3\,qx\,W}{c^4} + O[\frac{1}{c}]^6 & \frac{qy}{c^2} + \frac{\frac{(jx^2+jy^2+jz^2)\,qy}{2 n^2} + 3\,qy\,W}{c^4} + O[\frac{1}{c}]^6 & \frac{qz}{c^2} + \frac{\frac{(jx^2+jy^2+jz^2)\,qz}{2 n^2} + 3\,qz\,W}{c^4} + O[\frac{1}{c}]^6 \\
qx + \frac{\frac{(jx^2+jy^2+jz^2)\,qx}{2 n^2} - qx\,W}{c^2} + O[\frac{1}{c}]^4 & O[\frac{1}{c}]^6 & \frac{-jy\,qx+jx\,qy}{n c^2} + \frac{(-jy\,qx+jx\,qy)(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{-jz\,qx+jx\,qz}{n c^2} + \frac{(-jz\,qx+jx\,qz)(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 \\
qy + \frac{\frac{(jx^2+jy^2+jz^2)\,qy}{2 n^2} - qy\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{jy\,qx-jx\,qy}{n c^2} + \frac{(jy\,qx-jx\,qy)(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & O[\frac{1}{c}]^6 & \frac{-jz\,qy+jy\,qz}{n c^2} + \frac{(-jz\,qy+jy\,qz)(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 \\
qz + \frac{\frac{(jx^2+jy^2+jz^2)\,qz}{2 n^2} - qz\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{jz\,qx-jx\,qz}{n c^2} + \frac{(jz\,qx-jx\,qz)(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jz\,qy-jy\,qz}{n c^2} + \frac{(jz\,qy-jy\,qz)(jx^2+jy^2+jz^2+6 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & O[\frac{1}{c}]^6
\end{pmatrix}$$

In[227]:=
```
(* definition of momentum-flux, orthogonal to matter-current *)
Ptemp = {pt, px, py, pz};
```
In[228]:=
```
Assuming[assut, FS[Ptemp.proju]] // MF
```
Out[228]//MatrixForm=

$$\begin{pmatrix}
\frac{n\,pt+jx\,px+jy\,py+jz\,pz}{n} + \frac{(jx^2+jy^2+jz^2)(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^3 c^2} + O[\frac{1}{c}]^4 \\
-\frac{jx\,(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^2 c^2} - \frac{jx\,(n\,pt+jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+4 n^2 W)}{n^4 c^4} + O[\frac{1}{c}]^6 \\
-\frac{jy\,(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^2 c^2} - \frac{jy\,(n\,pt+jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+4 n^2 W)}{n^4 c^4} + O[\frac{1}{c}]^6 \\
-\frac{jz\,(n\,pt+jx\,px+jy\,py+jz\,pz)}{n^2 c^2} - \frac{jz\,(n\,pt+jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+4 n^2 W)}{n^4 c^4} + O[\frac{1}{c}]^6
\end{pmatrix}$$

In[229]:=
```
psol = Solve[Normal[Ptemp.proju] == 0, pt][[1]]
```
Out[229]=

$$\left\{pt \to -\frac{jx\,px + jy\,py + jz\,pz}{n}\right\}$$

In[230]:=
```
(P = Assuming[assut, FS[Ptemp /. psol]]) // MF
```
Out[230]//MatrixForm=

$$\begin{pmatrix}
-\frac{jx\,px+jy\,py+jz\,pz}{n} \\
px \\
py \\
pz
\end{pmatrix}$$

In[231]:=
```
{FS[Normal@Series[P.uu, {c, Infinity, 1}]] == 0}
```
Out[231]=

{True}

In[232]:=
```
Assuming[assutQ, FS@{P.proju, P.projperpu == P}]
```
Out[232]=

$$\left\{\left\{O\left[\frac{1}{c}\right]^4,\ O\left[\frac{1}{c}\right]^6,\ O\left[\frac{1}{c}\right]^6,\ O\left[\frac{1}{c}\right]^6\right\},\ \text{True}\right\}$$

In[233]:=
```
(* non-symmetric momentum-tensor *)
Assuming[assut, FS[Ptens = Assuming[assut, FS[Outer[Times, uu, P / c ^ 2]]]]] // MF
```
Out[233]//MatrixForm=

$$\begin{pmatrix}
-\frac{jx\,px+jy\,py+jz\,pz}{n c^2} - \frac{(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{px}{c^2} + \frac{\frac{(jx^2+jy^2+jz^2)\,px}{2 n^2} + px\,W}{c^4} + O[\frac{1}{c}]^6 & \frac{py}{c^2} + \frac{\frac{(jx^2+jy^2+jz^2)\,py}{2 n^2} + py\,W}{c^4} + O[\frac{1}{c}]^6 & \frac{pz}{c^2} + \frac{\frac{(jx^2+jy^2+jz^2)\,pz}{2 n^2} + pz\,W}{c^4} + O[\frac{1}{c}]^6 \\
-\frac{jx\,(jx\,px+jy\,py+jz\,pz)}{n^2 c^2} - \frac{jx\,(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^4 c^4} + O[\frac{1}{c}]^6 & \frac{jx\,px}{n c^2} + \frac{jx\,px\,(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jx\,py}{n c^2} + \frac{jx\,py\,(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jx\,pz}{n c^2} + \frac{jx\,pz\,(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 \\
-\frac{jy\,(jx\,px+jy\,py+jz\,pz)}{n^2 c^2} - \frac{jy\,(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^4 c^4} + O[\frac{1}{c}]^6 & \frac{jy\,px}{n c^2} + \frac{jy\,px\,(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jy\,py}{n c^2} + \frac{jy\,py\,(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jy\,pz}{n c^2} + \frac{jy\,pz\,(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 \\
-\frac{jz\,(jx\,px+jy\,py+jz\,pz)}{n^2 c^2} - \frac{jz\,(jx\,px+jy\,py+jz\,pz)(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^4 c^4} + O[\frac{1}{c}]^6 & \frac{jz\,px}{n c^2} + \frac{jz\,px\,(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jz\,py}{n c^2} + \frac{jz\,py\,(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jz\,pz}{n c^2} + \frac{jz\,pz\,(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6
\end{pmatrix}$$

In[234]:=
```
FS[T[Ptens.Inverse[gg]].gg - Ptens] // MF
```
Out[234]//MatrixForm=

$$\begin{pmatrix}
O[\frac{1}{c}]^6 & -\frac{px}{c^2} + \frac{jx^2\,px + 2\,jx\,(jy\,py+jz\,pz) - px\,(jy^2+jz^2+2 n^2 W)}{2 n^2 c^4} + O[\frac{1}{c}]^6 & -\frac{py}{c^2} + \frac{-\frac{(jx^2+jy^2+jz^2)\,py}{2 n^2} + \frac{jy\,(jx\,px+jy\,py+jz\,pz)}{n^2} - py\,W}{c^4} + O[\frac{1}{c}]^6 & -\frac{pz}{c^2} + \frac{-\frac{(jx^2+jy^2+jz^2)\,pz}{2 n^2} + \frac{jz\,(jx\,px+jy\,py+jz\,pz)}{n^2} - pz\,W}{c^4} + O[\frac{1}{c}]^6 \\
-px + \frac{-\frac{(jx^2+jy^2+jz^2)\,px}{2 n^2} + \frac{jx\,(jx\,px+jy\,py+jz\,pz)}{n^2} + 3\,px\,W}{c^2} + O[\frac{1}{c}]^4 & O[\frac{1}{c}]^6 & \frac{jy\,px-jx\,py}{n c^2} + \frac{(jy\,px-jx\,py)(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{jz\,px-jx\,pz}{n c^2} + \frac{(jz\,px-jx\,pz)(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 \\
-py + \frac{-\frac{(jx^2+jy^2+jz^2)\,py}{2 n^2} + \frac{jy\,(jx\,px+jy\,py+jz\,pz)}{n^2} + 3\,py\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{-jy\,px+jx\,py}{n c^2} + \frac{(-jy\,px+jx\,py)(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & O[\frac{1}{c}]^6 & \frac{jz\,py-jy\,pz}{n c^2} + \frac{(jz\,py-jy\,pz)(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 \\
-pz + \frac{-\frac{(jx^2+jy^2+jz^2)\,pz}{2 n^2} + \frac{jz\,(jx\,px+jy\,py+jz\,pz)}{n^2} + 3\,pz\,W}{c^2} + O[\frac{1}{c}]^4 & \frac{-jz\,px+jx\,pz}{n c^2} + \frac{(-jz\,px+jx\,pz)(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & \frac{-jz\,py+jy\,pz}{n c^2} + \frac{(-jz\,py+jy\,pz)(jx^2+jy^2+jz^2+2 n^2 W)}{2 n^3 c^4} + O[\frac{1}{c}]^6 & O[\frac{1}{c}]^6
\end{pmatrix}$$

In[235]:=
```
(* definition of stress, orthogonal to matter-current *)
(Stemp = {{stt, stx, sty, stz}, {sxt, sxx, sxy, sxz}, {syt, syx, syy, syz}, {szt, szx, szy, szz}}) // MF
```
Out[235]//MatrixForm=

$$\begin{pmatrix}
stt & stx & sty & stz \\
sxt & sxx & sxy & sxz \\
syt & syx & syy & syz \\
szt & szx & szy & szz
\end{pmatrix}$$

In[236]:=
```
(Stempsym = Assuming[assut, FS[(T[Stemp.Inverse[gg]].gg + Stemp) / 2]]) // MF
```
Out[236]//MatrixForm=

$$\begin{pmatrix}
stt + O[\frac{1}{c}]^4 & \frac{stx}{2} - \frac{sxt}{2 c^2} - \frac{2\,(sxt\,W)}{c^4} + O[\frac{1}{c}]^6 & \frac{sty}{2} - \frac{syt}{2 c^2} - \frac{2\,(syt\,W)}{c^4} + O[\frac{1}{c}]^6 & \frac{stz}{2} - \frac{szt}{2 c^2} - \frac{2\,(szt\,W)}{c^4} + O[\frac{1}{c}]^6 \\
-\frac{stx\,c^2}{2} + \frac{1}{2}\,(sxt + 4\,stx\,W) + O[\frac{1}{c}]^2 & sxx + O[\frac{1}{c}]^4 & \frac{sxy+syx}{2} + O[\frac{1}{c}]^4 & \frac{sxz+szx}{2} + O[\frac{1}{c}]^4 \\
-\frac{sty\,c^2}{2} + \frac{1}{2}\,(syt + 4\,sty\,W) + O[\frac{1}{c}]^2 & \frac{sxy+syx}{2} + O[\frac{1}{c}]^4 & syy + O[\frac{1}{c}]^4 & \frac{syz+szy}{2} + O[\frac{1}{c}]^4 \\
-\frac{stz\,c^2}{2} + \frac{1}{2}\,(szt + 4\,stz\,W) + O[\frac{1}{c}]^2 & \frac{sxz+szx}{2} + O[\frac{1}{c}]^4 & \frac{syz+szy}{2} + O[\frac{1}{c}]^4 & szz + O[\frac{1}{c}]^4
\end{pmatrix}$$

In[237]:=
```
FS[proju.Stemp.proju] // MF
```
Out[237]//MatrixForm=

$$\begin{pmatrix}
\frac{n\,stt+jx\,stx+jy\,sty+jz\,stz}{n} + \frac{2\,jx^3\,stx+2\,jy^3\,sty+jx^2\,(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx)-jy\,n^2\,syt+jy^2\,(2\,n\,stt+2\,jz\,stz-n\,syy)+jx\,(2\,jy^2\,stx-n^2\,sxt-jy\,n\,(sxy+syx)+jz\,(2\,jz\,stx-n\,(sxz+szx)))+jy\,jz\,(2\,jz\,sty-n\,(syz+szy))+jz\,(2\,jz^2\,stz-n^2\,szt+jz\,n\,(2\,stt-szz))}{n^3 c^2} + O[\frac{1}{c}]^4 & -\frac{jx\,(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2 c^2} + \frac{jx\,(-2\,jx^3\,stx-2\,jy^3\,sty+jx^2\,(-2\,(n\,stt+jy\,sty+jz\,stz)+n\,sxx)+jy^2\,(-2\,jz\,stz+n\,(-2\,stt+syy))+jz\,(-2\,jz^2\,stz+n^2\,szt+jz\,n\,(-2\,stt+szz))-4\,n^2\,(n\,stt+jz\,stz)\,W+jx\,(-2 \ldots}{n^4 c^4} \\
\frac{jx\,(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2} + \frac{jx\,(2\,jx^3\,stx+2\,jy^3\,sty+jx^2\,(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx)-jy\,n^2\,syt+jy^2\,(2\,n\,stt+2\,jz\,stz-n\,syy)+jx\,(2\,jy^2\,stx-n^2\,sxt-jy\,n\,(sxy+syx)+jz\,(2\,jz\,stx-n\,(sxz+szx)))+jy\,jz\,(2\,jz\,sty-n\,(syz+szy))+jz\,(2\,jz^2\,stz-n^2\,szt+jz\,n\,(2\,stt-szz)))}{n^4 c^2} + O[\frac{1}{c}]^4 & -\frac{jx^2\,(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^3 c^2} + \frac{jx^2\,(-2\,jx^3\,stx-2\,jy^3\,sty+jx^2\,(-2\,(n\,stt+jy\,sty+jz\,stz)+n\,sxx)+jy^2\,(-2\,jz\,stz+n\,(-2\,stt+syy))+jz\,(-2\,jz^2\,stz+n^2\,szt+jz\,n\,(-2\,stt+szz))-4\,n^2\,(n\,stt+jz\,stz)\,W+jx\,(\ldots}{n^5 c^4} \\
\frac{jy\,(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2} + \frac{jy\,(2\,jx^3\,stx+2\,jy^3\,sty+jx^2\,(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx)-jy\,n^2\,syt+jy^2\,(2\,n\,stt+2\,jz\,stz-n\,syy)+jx\,(2\,jy^2\,stx-n^2\,sxt-jy\,n\,(sxy+syx)+jz\,(2\,jz\,stx-n\,(sxz+szx)))+jy\,jz\,(2\,jz\,sty-n\,(syz+szy))+jz\,(2\,jz^2\,stz-n^2\,szt+jz\,n\,(2\,stt-szz)))}{n^4 c^2} + O[\frac{1}{c}]^4 & -\frac{jx\,jy\,(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^3 c^2} + \frac{jx\,jy\,(-2\,jx^3\,stx-2\,jy^3\,sty+jx^2\,(-2\,(n\,stt+jy\,sty+jz\,stz)+n\,sxx)+jy^2\,(-2\,jz\,stz+n\,(-2\,stt+syy))+jz\,(-2\,jz^2\,stz+n^2\,szt+jz\,n\,(-2\,stt+szz))-4\,n^2\,(n\,stt+jz\,stz)\,W+ \ldots}{n^5 c^4} \\
\frac{jz\,(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^2} + \frac{jz\,(2\,jx^3\,stx+2\,jy^3\,sty+jx^2\,(2\,n\,stt+2\,jy\,sty+2\,jz\,stz-n\,sxx)-jy\,n^2\,syt+jy^2\,(2\,n\,stt+2\,jz\,stz-n\,syy)+jx\,(2\,jy^2\,stx-n^2\,sxt-jy\,n\,(sxy+syx)+jz\,(2\,jz\,stx-n\,(sxz+szx)))+jy\,jz\,(2\,jz\,sty-n\,(syz+szy))+jz\,(2\,jz^2\,stz-n^2\,szt+jz\,n\,(2\,stt-szz)))}{n^4 c^2} + O[\frac{1}{c}]^4 & -\frac{jx\,jz\,(n\,stt+jx\,stx+jy\,sty+jz\,stz)}{n^3 c^2} + \frac{jx\,jz\,(-2\,jx^3\,stx-2\,jy^3\,sty+jx^2\,(-2\,(n\,stt+jy\,sty+jz\,stz)+n\,sxx)+jy^2\,(-2\,jz\,stz+n\,(-2\,stt+syy))+jz\,(-2\,jz^2\,stz+n^2\,szt+jz\,n\,(-2\,stt+szz))-4\,n^2\,(n\,stt+jz\,stz)\,W+ \ldots}{n^5 c^4}
\end{pmatrix}$$

In[238]:=
```
ssol = Solve[{Normal[proju.Stemp.proju] == 0, Normal[projperpu.Stemp.projperpu] == Stemp}, {stt, stx, sty, stz, sxt, syt, szt}][[1]]
```
Out[238]=

$$\left\{stt \to -\frac{jx^2\,sxx + jx\,jy\,sxy + jx\,jz\,sxz + jx\,jy\,syx + jy^2\,syy + jy\,jz\,syz + jx\,jz\,szx + jy\,jz\,szy + jz^2\,szz}{jx^2 + jy^2 + jz^2 + c^2 n^2},\ stx \to \frac{n\,(jx\,sxx + jy\,syx + jz\,szx)}{jx^2 + jy^2 + jz^2 + c^2 n^2},\ sty \to \frac{n\,(jx\,sxy + jy\,syy + jz\,szy)}{jx^2 + jy^2 + jz^2 + c^2 n^2},\ stz \to \frac{n\,(jx\,sxz + jy\,syz + jz\,szz)}{jx^2 + jy^2 + jz^2 + c^2 n^2},\ sxt \to -\frac{jx\,sxx + jy\,sxy + jz\,sxz}{n},\ syt \to -\frac{jx\,syx + jy\,syy + jz\,syz}{n},\ szt \to -\frac{jx\,szx + jy\,szy + jz\,szz}{n}\right\}$$

In[239]:=
```
ssol = Solve[Normal[proju.Stemp] == 0 && Normal[Stemp.proju] == 0, {stt, stx, sty, stz, sxt, syt, szt}][[1]]
```
Out[239]=

$$\left\{stt \to -\frac{jx^2\,sxx + jx\,jy\,sxy + jx\,jz\,sxz + jx\,jy\,syx + jy^2\,syy + jy\,jz\,syz + jx\,jz\,szx + jy\,jz\,szy + jz^2\,szz}{jx^2 + jy^2 + jz^2 + c^2 n^2},\ stx \to \frac{n\,(jx\,sxx + jy\,syx + jz\,szx)}{jx^2 + jy^2 + jz^2 + c^2 n^2},\ sty \to \frac{n\,(jx\,sxy + jy\,syy + jz\,szy)}{jx^2 + jy^2 + jz^2 + c^2 n^2},\ stz \to \frac{n\,(jx\,sxz + jy\,syz + jz\,szz)}{jx^2 + jy^2 + jz^2 + c^2 n^2},\ sxt \to -\frac{jx\,sxx + jy\,sxy + jz\,sxz}{n},\ syt \to -\frac{jx\,syx + jy\,syy + jz\,syz}{n},\ szt \to -\frac{jx\,szx + jy\,szy + jz\,szz}{n}\right\}$$

In[240]:=
```
(S = Assuming[assut, FS[(Stemp /. ssol)]]) // MF
```
Out[240]//MatrixForm=

$$\begin{pmatrix}
-\frac{jx^2\,sxx + jx\,jy\,(sxy+syx) + jy^2\,syy + jx\,jz\,(sxz+szx) + jy\,jz\,(syz+szy) + jz^2\,szz}{jx^2+jy^2+jz^2+c^2 n^2} & \frac{n\,(jx\,sxx+jy\,syx+jz\,szx)}{jx^2+jy^2+jz^2+c^2 n^2} & \frac{n\,(jx\,sxy+jy\,syy+jz\,szy)}{jx^2+jy^2+jz^2+c^2 n^2} & \frac{n\,(jx\,sxz+jy\,syz+jz\,szz)}{jx^2+jy^2+jz^2+c^2 n^2} \\
-\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n} & sxx & sxy & sxz \\
-\frac{jx\,syx+jy\,syy+jz\,syz}{n} & syx & syy & syz \\
-\frac{jx\,szx+jy\,szy+jz\,szz}{n} & szx & szy & szz
\end{pmatrix}$$

In[241]:=
```
MF /@ FS@{proju.S.proju, Assuming[assut, Expand //@ FS@PowerExpand[projperpu.S.projperpu - S]]}
```
Out[241]=

$$\left\{\begin{pmatrix}
O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \\
O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \\
O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 \\
O[\frac{1}{c}]^6 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8 & O[\frac{1}{c}]^8
\end{pmatrix},\ \begin{pmatrix}
-\frac{(jx^2\,sxx + jx\,jy\,(sxy+syx) + jy^2\,syy + jx\,jz\,(sxz+szx) + jy\,jz\,(syz+szy) + jz^2\,szz)(jx^2+jy^2+jz^2+4 n^2 W)}{n^4 c^4} + O[\frac{1}{c}]^6 & \frac{(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4 n^2 W)}{n^3 c^4} + O[\frac{1}{c}]^6 & \frac{(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4 n^2 W)}{n^3 c^4} + O[\frac{1}{c}]^6 & \frac{(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4 n^2 W)}{n^3 c^4} + O[\frac{1}{c}]^6 \\
O[\frac{1}{c}]^4 & \frac{jx\,(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4 n^2 W)}{n^4 c^4} + O[\frac{1}{c}]^6 & \frac{jx\,(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4 n^2 W)}{n^4 c^4} + O[\frac{1}{c}]^6 & \frac{jx\,(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4 n^2 W)}{n^4 c^4} + O[\frac{1}{c}]^6 \\
O[\frac{1}{c}]^4 & \frac{jy\,(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4 n^2 W)}{n^4 c^4} + O[\frac{1}{c}]^6 & \frac{jy\,(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4 n^2 W)}{n^4 c^4} + O[\frac{1}{c}]^6 & \frac{jy\,(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4 n^2 W)}{n^4 c^4} + O[\frac{1}{c}]^6 \\
O[\frac{1}{c}]^4 & \frac{jz\,(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4 n^2 W)}{n^4 c^4} + O[\frac{1}{c}]^6 & \frac{jz\,(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4 n^2 W)}{n^4 c^4} + O[\frac{1}{c}]^6 & \frac{jz\,(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4 n^2 W)}{n^4 c^4} + O[\frac{1}{c}]^6
\end{pmatrix}\right\}$$

In[242]:=
```
FS[T[S.Inverse[gg]].gg - S] // MF
```
Out[242]//MatrixForm=

$$\begin{pmatrix}
O[\frac{1}{c}]^6 & \frac{jy\,sxy+jz\,sxz-jy\,syx-jz\,szx}{n c^2} + \frac{\frac{(jx^2+jy^2+jz^2)(jy\,sxy+jz\,sxz-jy\,syx-jz\,szx)}{n^2} + \frac{4\,(jy\,sxy+jz\,sxz-jy\,syx-jz\,szx)\,W}{n}}{c^4} + O[\frac{1}{c}]^6 & \frac{jx\,(-sxy+syx)+jz\,(syz-szy)}{n c^2} + \frac{\frac{(jx^2+jy^2+jz^2)(jx\,(-sxy+syx)+jz\,(syz-szy))}{n^2} + \frac{4\,(jx\,(-sxy+syx)+jz\,(syz-szy))\,W}{n}}{c^4} + O[\frac{1}{c}]^6 & \frac{jx\,(-sxz+szx)+jy\,(-syz+szy)}{n c^2} + \frac{\frac{(jx^2+jy^2+jz^2)(jx\,(-sxz+szx)+jy\,(-syz+szy))}{n^2} + \frac{4\,(jx\,(-sxz+szx)+jy\,(-syz+szy))\,W}{n}}{c^4} + O[\frac{1}{c}]^6 \\
\frac{jy\,sxy+jz\,sxz-jy\,syx-jz\,szx}{n} + \frac{(jx\,sxx+jy\,syx+jz\,szx)(jx^2+jy^2+jz^2+4 n^2 W)}{n^3 c^2} + O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & (-sxy+syx) + O[\frac{1}{c}]^4 & (-sxz+szx) + O[\frac{1}{c}]^4 \\
\frac{jx\,(-sxy+syx)+jz\,(syz-szy)}{n} + \frac{(jx\,sxy+jy\,syy+jz\,szy)(jx^2+jy^2+jz^2+4 n^2 W)}{n^3 c^2} + O[\frac{1}{c}]^4 & (sxy-syx) + O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & (-syz+szy) + O[\frac{1}{c}]^4 \\
\frac{jx\,(-sxz+szx)+jy\,(-syz+szy)}{n} + \frac{(jx\,sxz+jy\,syz+jz\,szz)(jx^2+jy^2+jz^2+4 n^2 W)}{n^3 c^2} + O[\frac{1}{c}]^4 & (sxz-szx) + O[\frac{1}{c}]^4 & (syz-szy) + O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4
\end{pmatrix}$$

In[243]:=
```
(* define "dust" 4-stress *)
(dust = Assuming[assut, FS[(ρ * c^2 + ϵ) * Outer[Times, NJ, gg.uu / c^2]]]) // MF
```

Out[243]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & jx\,\rho+\frac{\frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2}+jx(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jy\,\rho+\frac{\frac{jy(jx^2+jy^2+jz^2)\rho}{2n^2}+jy(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jz\,\rho+\frac{\frac{jz(jx^2+jy^2+jz^2)\rho}{2n^2}+jz(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -jx\,\rho c^2+\left(-\frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2}+jx(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & \frac{jx^2\rho}{n}+\frac{jx^2((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jx\,jy\,\rho}{n}+\frac{jx\,jy((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jx\,jz\,\rho}{n}+\frac{jx\,jz((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 \\ -jy\,\rho c^2+\left(-\frac{jy(jx^2+jy^2+jz^2)\rho}{2n^2}+jy(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & \frac{jx\,jy\,\rho}{n}+\frac{jx\,jy((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jy^2\rho}{n}+\frac{jy^2((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jy\,jz\,\rho}{n}+\frac{jy\,jz((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 \\ -jz\,\rho c^2+\left(-\frac{jz(jx^2+jy^2+jz^2)\rho}{2n^2}+jz(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & \frac{jx\,jz\,\rho}{n}+\frac{jx\,jz((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jy\,jz\,\rho}{n}+\frac{jy\,jz((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \frac{jz^2\rho}{n}+\frac{jz^2((jx^2+jy^2+jz^2)\rho+2n^2(\epsilon+3W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[244]:=
```
FS[T[dust.Inverse[gg]] == dust.Inverse[gg]]
```

Out[244]=

True

In[245]:=
```
FS[T[dust].gg == gg.dust]
```

Out[245]=

True

In[246]:=
```
MF /@ FS@{proju.dust.proju == dust, projperpu.dust.projperpu}
```

Out[246]=

$$\left\{\text{True}, \begin{pmatrix} O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

In[247]:=
```
showf[assut][dust2 = Assuming[assut, Expand //@ FS@PowerExpand[(ρ * c^2 + ϵ) * Outer[Times, NJ, gg.auu / c^2]]]];
```

In[248]:=
```
(* define generic 4-stress *)
(EPS = Assuming[assut, FS[dust + Qtens + Ptens + S]]) // MF
```

Out[248]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & jx\,\rho+\frac{px+\frac{jx\,sxx+jy\,syx+jz\,szx}{n}+\frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2}+jx(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jy\,\rho+\frac{py+\frac{jx\,sxy+jy\,syy+jz\,szy}{n}+\frac{jy(jx^2+jy^2+jz^2)\rho}{2n^2}+jy(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jz\,\rho+\frac{pz+\frac{jx\,sxz+jy\,syz+jz\,szz}{n}+\frac{jz(jx^2+jy^2+jz^2)\rho}{2n^2}+jz(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -jx\,\rho c^2-\frac{2n(jx\,sxx+jy\,sxy+jz\,sxz+n(qx+jx\,\epsilon))+jx(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}+O[\frac{1}{c}]^2 & \left(sxx+\frac{jx^2\rho}{n}\right)+\frac{jx(jx(jx^2+jy^2+jz^2)\rho+2n^2(px+qx+jx\,\epsilon+3jx\,W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \left(sxy+\frac{jx\,jy\,\rho}{n}\right)+\frac{\frac{jx\,py+jy\,qx+jx\,jy\,\epsilon}{n}+\frac{jx\,jy(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(sxz+\frac{jx\,jz\,\rho}{n}\right)+\frac{\frac{jx\,pz+jz\,qx+jx\,jz\,\epsilon}{n}+\frac{jx\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 \\ -jy\,\rho c^2-\frac{2n(jx\,syx+jy\,syy+jz\,syz+n(qy+jy\,\epsilon))+jy(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}+O[\frac{1}{c}]^2 & \left(syx+\frac{jx\,jy\,\rho}{n}\right)+\frac{\frac{jy\,px+jx\,qy+jx\,jy\,\epsilon}{n}+\frac{jx\,jy(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(syy+\frac{jy^2\rho}{n}\right)+\frac{jy(jy(jx^2+jy^2+jz^2)\rho+2n^2(py+qy+jy\,\epsilon+3jy\,W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 & \left(syz+\frac{jy\,jz\,\rho}{n}\right)+\frac{\frac{jy\,pz+jz\,qy+jy\,jz\,\epsilon}{n}+\frac{jy\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 \\ -jz\,\rho c^2-\frac{2n(jx\,szx+jy\,szy+jz\,szz+n(qz+jz\,\epsilon))+jz(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}+O[\frac{1}{c}]^2 & \left(szx+\frac{jx\,jz\,\rho}{n}\right)+\frac{\frac{jz\,px+jx\,qz+jx\,jz\,\epsilon}{n}+\frac{jx\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(szy+\frac{jy\,jz\,\rho}{n}\right)+\frac{\frac{jz\,py+jy\,qz+jy\,jz\,\epsilon}{n}+\frac{jy\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(szz+\frac{jz^2\rho}{n}\right)+\frac{jz(jz(jx^2+jy^2+jz^2)\rho+2n^2(pz+qz+jz\,\epsilon+3jz\,W\rho))}{2n^3c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[249]:=
```
FS[T[EPS.Inverse[gg]].gg - EPS] // MF
```

Out[249]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^2 & \frac{n(-px+qx)+jy(sxy-syx)+jz(sxz-szx)}{nc^2}+O[\frac{1}{c}]^4 & \frac{n(-py+qy)+jx(-sxy+syx)+jz(syz-szy)}{nc^2}+O[\frac{1}{c}]^4 & \frac{n(-pz+qz)+jx(-sxz+szx)+jy(-syz+szy)}{nc^2}+O[\frac{1}{c}]^4 \\ \frac{n(-px+qx)+jy(sxy-syx)+jz(sxz-szx)}{n}+O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & (-sxy+syx)+\frac{jy\,px-jx\,py-jy\,qx+jx\,qy}{nc^2}+O[\frac{1}{c}]^4 & (-sxz+szx)+\frac{jz\,px-jx\,pz-jz\,qx+jx\,qz}{nc^2}+O[\frac{1}{c}]^4 \\ \frac{n(-py+qy)+jx(-sxy+syx)+jz(syz-szy)}{n}+O[\frac{1}{c}]^2 & (sxy-syx)+\frac{jy(-px+qx)+jx(py-qy)}{nc^2}+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & (-syz+szy)+\frac{jz\,py-jy\,pz-jz\,qy+jy\,qz}{nc^2}+O[\frac{1}{c}]^4 \\ \frac{n(-pz+qz)+jx(-sxz+szx)+jy(-syz+szy)}{n}+O[\frac{1}{c}]^2 & (sxz-szx)+\frac{jz(-px+qx)+jx(pz-qz)}{nc^2}+O[\frac{1}{c}]^4 & (syz-szy)+\frac{jz(-py+qy)+jy(pz-qz)}{nc^2}+O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}$$

In[250]:=
```
(* Conditions for symmetry of total 4-stress tensor *)
subsym = {px → qx, py → qy, pz → qz, syx → sxy, szx → sxz, szy → syz};
```

In[251]:=
```
(EPSsym = Assuming[assut, FS[EPS /. subsym]]) // MF
```

Out[251]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(-\epsilon+W\rho)\right)+O[\frac{1}{c}]^2 & jx\,\rho+\frac{qx+\frac{jx\,sxx+jy\,sxy+jz\,sxz}{n}+\frac{jx(jx^2+jy^2+jz^2)\rho}{2n^2}+jx(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jy\,\rho+\frac{qy+\frac{jx\,sxy+jy\,syy+jz\,syz}{n}+\frac{jy(jx^2+jy^2+jz^2)\rho}{2n^2}+jy(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 & jz\,\rho+\frac{qz+\frac{jx\,sxz+jy\,syz+jz\,szz}{n}+\frac{jz(jx^2+jy^2+jz^2)\rho}{2n^2}+jz(\epsilon+3W\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -jx\,\rho c^2-\frac{2n(jx\,sxx+jy\,sxy+jz\,sxz+n(qx+jx\,\epsilon))+jx(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}+O[\frac{1}{c}]^2 & \left(sxx+\frac{jx^2\rho}{n}\right)+\frac{jx(jx(jx^2+jy^2+jz^2)\rho+2n^2(2qx+jx(\epsilon+3W\rho)))}{2n^3c^2}+O[\frac{1}{c}]^4 & \left(sxy+\frac{jx\,jy\,\rho}{n}\right)+\frac{\frac{jy\,qx+jx\,qy+jx\,jy\,\epsilon}{n}+\frac{jx\,jy(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(sxz+\frac{jx\,jz\,\rho}{n}\right)+\frac{\frac{jz\,qx+jx\,qz+jx\,jz\,\epsilon}{n}+\frac{jx\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 \\ -jy\,\rho c^2-\frac{2n(jx\,sxy+jy\,syy+jz\,syz+n(qy+jy\,\epsilon))+jy(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}+O[\frac{1}{c}]^2 & \left(sxy+\frac{jx\,jy\,\rho}{n}\right)+\frac{\frac{jy\,qx+jx\,qy+jx\,jy\,\epsilon}{n}+\frac{jx\,jy(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(syy+\frac{jy^2\rho}{n}\right)+\frac{jy(jy(jx^2+jy^2+jz^2)\rho+2n^2(2qy+jy(\epsilon+3W\rho)))}{2n^3c^2}+O[\frac{1}{c}]^4 & \left(syz+\frac{jy\,jz\,\rho}{n}\right)+\frac{\frac{jz\,qy+jy\,qz+jy\,jz\,\epsilon}{n}+\frac{jy\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 \\ -jz\,\rho c^2-\frac{2n(jx\,sxz+jy\,syz+jz\,szz+n(qz+jz\,\epsilon))+jz(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}+O[\frac{1}{c}]^2 & \left(sxz+\frac{jx\,jz\,\rho}{n}\right)+\frac{\frac{jz\,qx+jx\,qz+jx\,jz\,\epsilon}{n}+\frac{jx\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(syz+\frac{jy\,jz\,\rho}{n}\right)+\frac{\frac{jz\,qy+jy\,qz+jy\,jz\,\epsilon}{n}+\frac{jy\,jz(jx^2+jy^2+jz^2+6n^2W)\rho}{2n^3}}{c^2}+O[\frac{1}{c}]^4 & \left(szz+\frac{jz^2\rho}{n}\right)+\frac{jz(jz(jx^2+jy^2+jz^2)\rho+2n^2(2qz+jz(\epsilon+3W\rho)))}{2n^3c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[252]:=
```
FS[T[EPSsym.Inverse[gg]].gg - EPSsym] // MF
```

Out[252]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^2 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}$$

In[253]:=
```
(* in terms of matter velocity *)
Assuming[assut, FS[EPS /. replaceJu]] // MF
```

Out[253]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-\frac{1}{2}n(2\epsilon+(ux^2+uy^2+uz^2-2W)\rho)+O[\frac{1}{c}]^2 & n\,ux\,\rho+\frac{px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon+\frac{1}{2}n\,ux(ux^2+uy^2+uz^2+6W)\rho}{c^2}+O[\frac{1}{c}]^4 & n\,uy\,\rho+\frac{py+sxy\,ux+syy\,uy+szy\,uz+n\,uy\,\epsilon+\frac{1}{2}n\,uy(ux^2+uy^2+uz^2+6W)\rho}{c^2}+O[\frac{1}{c}]^4 & n\,uz\,\rho+\frac{pz+sxz\,ux+syz\,uy+szz\,uz+n\,uz\,\epsilon+\frac{1}{2}n\,uz(ux^2+uy^2+uz^2+6W)\rho}{c^2}+O[\frac{1}{c}]^4 \\ -n\,ux\,\rho c^2+\left(-qx-sxy\,uy-sxz\,uz-ux(sxx+n\epsilon)-\frac{1}{2}n\,ux(ux^2+uy^2+uz^2-2W)\rho\right)+O[\frac{1}{c}]^2 & (sxx+n\,ux^2\rho)+\frac{ux(2(px+qx+n\,ux\,\epsilon)+n\,ux(ux^2+uy^2+uz^2+6W)\rho)}{2c^2}+O[\frac{1}{c}]^4 & (sxy+n\,ux\,uy\,\rho)+\frac{py\,ux+qx\,uy+\frac{1}{2}n\,ux\,uy(2\epsilon+(ux^2+uy^2+uz^2+6W)\rho)}{c^2}+O[\frac{1}{c}]^4 & (sxz+n\,ux\,uz\,\rho)+\frac{pz\,ux+qx\,uz+\frac{1}{2}n\,ux\,uz(2\epsilon+(ux^2+uy^2+uz^2+6W)\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -n\,uy\,\rho c^2+\left(-qy-syx\,ux-syz\,uz-uy(syy+n\epsilon)-\frac{1}{2}n\,uy(ux^2+uy^2+uz^2-2W)\rho\right)+O[\frac{1}{c}]^2 & (syx+n\,ux\,uy\,\rho)+\frac{qy\,ux+px\,uy+\frac{1}{2}n\,ux\,uy(2\epsilon+(ux^2+uy^2+uz^2+6W)\rho)}{c^2}+O[\frac{1}{c}]^4 & (syy+n\,uy^2\rho)+\frac{uy(2(py+qy+n\,uy\,\epsilon)+n\,uy(ux^2+uy^2+uz^2+6W)\rho)}{2c^2}+O[\frac{1}{c}]^4 & (syz+n\,uy\,uz\,\rho)+\frac{pz\,uy+qy\,uz+\frac{1}{2}n\,uy\,uz(2\epsilon+(ux^2+uy^2+uz^2+6W)\rho)}{c^2}+O[\frac{1}{c}]^4 \\ -n\,uz\,\rho c^2+\left(-qz-szx\,ux-szy\,uy-uz(szz+n\epsilon)-\frac{1}{2}n\,uz(ux^2+uy^2+uz^2-2W)\rho\right)+O[\frac{1}{c}]^2 & (szx+n\,ux\,uz\,\rho)+\frac{qz\,ux+px\,uz+\frac{1}{2}n\,ux\,uz(2\epsilon+(ux^2+uy^2+uz^2+6W)\rho)}{c^2}+O[\frac{1}{c}]^4 & (szy+n\,uy\,uz\,\rho)+\frac{qz\,uy+py\,uz+\frac{1}{2}n\,uy\,uz(2\epsilon+(ux^2+uy^2+uz^2+6W)\rho)}{c^2}+O[\frac{1}{c}]^4 & (szz+n\,uz^2\rho)+\frac{uz(2(pz+qz+n\,uz\,\epsilon)+n\,uz(ux^2+uy^2+uz^2+6W)\rho)}{2c^2}+O[\frac{1}{c}]^4 \end{pmatrix}$$

In[254]:=

(* Balanced quantities constructed from energy–momentum tensor, and their supplies *)

In[255]:=
```
(* Symmetrized energy-stress tensor, with explicit dep. on coords *)
(TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]]) // MF; (TTxsym = tW[tjv[(EPSsym + T[EPSsym.Inverse[gg]].gg) / 2]]) // MF
```

Out[255]//MatrixForm=

$$-\rho\, n[t,x,y,z]\,c^2+\frac{1}{2}\left(-\frac{\rho(n[t,x,y,z]^2\,ux[t,x,y,z]^2+n[t,x,y,z]^2\,uy[t,x,y,z]^2+n[t,x,y,z]^2\,uz[t,x,y,z]^2)}{n[t,x,y,z]}+2\,n[t,x,y,z](-\epsilon+\rho\,W[t,x,y,z])\right)+O[\tfrac{1}{c}]^2$$

$$-\rho\, n[t,x,y,z]\times ux[t,x,y,z]\,c^2+\frac{1}{2}\left(-qx-\frac{sxx\,n[t,x,y,z]\times ux[t,x,y,z]+sxy\,n[t,x,y,z]\times uy[t,x,y,z]+sxz\,n[t,x,y,z]\times uz[t,x,y,z]}{n[t,x,y,z]}-\frac{\rho\,ux[t,x,y,z](n[t,x,y,z]^2\,ux[t,x,y,z]^2+n[t,x,y,z]^2\,uy[t,x,y,z]^2+n[t,x,y,z]^2\,uz[t,x,y,z]^2)}{2\,n[t,x,y,z]}+4\rho\, n[t,x,y,z]\times ux[t,x,y,z]\times W[t,x,y,z]-n[t,x,y,z]\times ux[t,x,y,z](\epsilon+3\rho\,W[t,x,y,z])\right)-\frac{2\,n[t,x,y,z](sxx\,n[t,x,y,z]\times ux[t,x,y,z]+n[t,x,y,z](qx+\epsilon\,n[t,x,y,z]\times ux[t,x,y,z])+\ldots}{}$$

$$-\rho\, n[t,x,y,z]\times uy[t,x,y,z]\,c^2+\frac{1}{2}\left(-qy-\frac{sxy\,n[t,x,y,z]\times ux[t,x,y,z]+syy\,n[t,x,y,z]\times uy[t,x,y,z]+syz\,n[t,x,y,z]\times uz[t,x,y,z]}{n[t,x,y,z]}-\frac{\rho\,uy[t,x,y,z](n[t,x,y,z]^2\,ux[t,x,y,z]^2+n[t,x,y,z]^2\,uy[t,x,y,z]^2+n[t,x,y,z]^2\,uz[t,x,y,z]^2)}{2\,n[t,x,y,z]}+4\rho\, n[t,x,y,z]\times uy[t,x,y,z]\times W[t,x,y,z]-n[t,x,y,z]\times uy[t,x,y,z](\epsilon+3\rho\,W[t,x,y,z])\right)-\frac{2\,n[t,x,y,z](sxy\,n[t,x,y,z]\times ux[t,x,y,z]+syy\,n[t,x,y,z]\times uy[t,x,y,z]+n[t,x,y,z](q\ldots}{}$$

$$-\rho\, n[t,x,y,z]\times uz[t,x,y,z]\,c^2+\frac{1}{2}\left(-qz-\frac{sxz\,n[t,x,y,z]\times ux[t,x,y,z]+syz\,n[t,x,y,z]\times uy[t,x,y,z]+szz\,n[t,x,y,z]\times uz[t,x,y,z]}{n[t,x,y,z]}-\frac{\rho\,uz[t,x,y,z](n[t,x,y,z]^2\,ux[t,x,y,z]^2+n[t,x,y,z]^2\,uy[t,x,y,z]^2+n[t,x,y,z]^2\,uz[t,x,y,z]^2)}{2\,n[t,x,y,z]}+4\rho\, n[t,x,y,z]\times uz[t,x,y,z]\times W[t,x,y,z]-n[t,x,y,z]\times uz[t,x,y,z](\epsilon+3\rho\,W[t,x,y,z])\right)-\frac{2\,n[t,x,y,z](sxz\,n[t,x,y,z]\times ux[t,x,y,z]+syz\,n[t,x,y,z]\times uy[t,x,y,z]+szz\,n[t,x,y,\ldots}{}$$

In[256]:=
```
(Duu = Assuming[assut, FS[(D[Normal@uut, {coords}] + Sum[uut[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]) // MF
```

Out[256]//MatrixForm=

$$\frac{n[t,x,y,z](jx[t,x,y,z](-n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+jx^{(1,0,0,0)}[t,x,y,z])+jy[t,x,y,z](-n[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]+jy^{(1,0,0,0)}[t,x,y,z])+jz[t,x,y,z](-n[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]+jz^{(1,0,0,0)}[t,x,y,z]))-(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2)\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^3\,c^2}+O[\tfrac{1}{c}]^4 \quad \frac{n[t,x,y,z](jx[t,x,y,z]\,jx^{(0,1,0,0)}[t,x,y,z]+jy[t,x,y,z]\,jy^{(0,1,0,0)}[t,x,y,z]+jz[t,x,y,z]\,jz^{(0,1,0,0)}[t,x,y,z])-(jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2)\,n^{(0,1,0,0)}[t,x,y,z]}{n[t,x,y,z]^3\,c^2}+O[\tfrac{1}{c}]^4$$

$$\left(-W^{(0,1,0,0)}[t,x,y,z]+\frac{n[t,x,y,z]\,jx^{(1,0,0,0)}[t,x,y,z]-jx[t,x,y,z]\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}\right)+O[\tfrac{1}{c}]^2 \quad \frac{n[t,x,y,z]\,jx^{(0,1,0,0)}[t,x,y,z]-jx[t,x,y,z]\,n^{(0,1,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}+\frac{2\,jy[t,x,y,z](n[t,x,y,z]^3\,W^{(0,0,1,0)}[t,x,y,z]+jx[t,x,y,z]\,n[t,x,y,z]\,jy^{(0,1,0,0)}[t,x,y,z])+2\,jz[t,x,y,z](n[t,x,y,z]^3\,W^{(0,0,0,1)}[t,x,y,z]+\ldots}{}$$

$$\left(-W^{(0,0,1,0)}[t,x,y,z]+\frac{n[t,x,y,z]\,jy^{(1,0,0,0)}[t,x,y,z]-jy[t,x,y,z]\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}\right)+O[\tfrac{1}{c}]^2 \quad \frac{n[t,x,y,z]\,jy^{(0,1,0,0)}[t,x,y,z]-jy[t,x,y,z]\,n^{(0,1,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}+\frac{-2\,jx[t,x,y,z]\,n[t,x,y,z]^3\,W^{(0,0,1,0)}[t,x,y,z]+n[t,x,y,z](jx[t,x,y,z]^2+jy[t,x,y,z]^2+jz[t,x,y,z]^2+2\,n[t,x,y,z]^2\,W[t,x,y,z])\,jy^{(0,1,0,0)}\ldots}{}$$

$$\left(-W^{(0,0,0,1)}[t,x,y,z]+\frac{n[t,x,y,z]\,jz^{(1,0,0,0)}[t,x,y,z]-jz[t,x,y,z]\,n^{(1,0,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}\right)+O[\tfrac{1}{c}]^2 \quad \frac{n[t,x,y,z]\,jz^{(0,1,0,0)}[t,x,y,z]-jz[t,x,y,z]\,n^{(0,1,0,0)}[t,x,y,z]}{n[t,x,y,z]^2}+\frac{jx[t,x,y,z](-2\,n[t,x,y,z]^3\,W^{(0,0,0,1)}[t,x,y,z]+2\,jz[t,x,y,z]\,n[t,x,y,z]\,jx^{(0,1,0,0)}[t,x,y,z])+2\,jy[t,x,y,z]\,jz[t,x,y,z]\,n[t,x,y,z]\,jy^{(0\ldots}{}$$

In[257]:=
```
(Duv = Assuming[assut, FS[(D[Normal@uuv, {coords}] + Sum[uuv[[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]) // MF
```

Out[257]//MatrixForm=

$$\frac{ux[t,x,y,z](-W^{(0,1,0,0)}[t,x,y,z]+ux^{(1,0,0,0)}[t,x,y,z])+uy[t,x,y,z](-W^{(0,0,1,0)}[t,x,y,z]+uy^{(1,0,0,0)}[t,x,y,z])+uz[t,x,y,z](-W^{(0,0,0,1)}[t,x,y,z]+uz^{(1,0,0,0)}[t,x,y,z])}{c^2}+O[\tfrac{1}{c}]^4 \quad \frac{ux[t,x,y,z]\,ux^{(0,1,0,0)}[t,x,y,z]+uy[t,x,y,z]\,uy^{(0,1,0,0)}[t,x,y,z]+uz[t,x,y,z]\,uz^{(0,1,0,0)}[t,x,y,z]}{c^2}+O[\tfrac{1}{c}]^4$$

$$\left(-W^{(0,1,0,0)}[t,x,y,z]+ux^{(1,0,0,0)}[t,x,y,z]\right)+O[\tfrac{1}{c}]^2 \quad ux^{(0,1,0,0)}[t,x,y,z]+\frac{uy[t,x,y,z]^2\,ux^{(0,1,0,0)}[t,x,y,z]+uz[t,x,y,z]^2\,ux^{(0,1,0,0)}[t,x,y,z]+(3\,ux[t,x,y,z]^2+2\,W[t,x,y,z])\,ux^{(0,1,0,0)}[t,x,y,z]+2\,uy[t,x,y,z](W^{(0,0,1,0)}[t,x,y,z]+ux[t,x,y,z]\,uy^{(0,1,0,0)}[t,x,y,z])+2\,uz[t,x,y,z](W^{(0,0,0,1)}[t,x,y,z]+ux[t,x,y,z]\,uz^{(0,1,0,0)}[t,x,y,z])+4\,ux[t,x,y,z]\,W^{(0,1,0,0)}[t,x\ldots}{2c^2}$$

$$\left(-W^{(0,0,1,0)}[t,x,y,z]+uy^{(1,0,0,0)}[t,x,y,z]\right)+O[\tfrac{1}{c}]^2 \quad uy^{(0,1,0,0)}[t,x,y,z]+\frac{-2\,ux[t,x,y,z](W^{(0,0,1,0)}[t,x,y,z]-uy[t,x,y,z]\,ux^{(0,1,0,0)}[t,x,y,z])+ux[t,x,y,z]^2\,uy^{(0,1,0,0)}[t,x,y,z]+3\,uy[t,x,y,z]^2\,uy^{(0,1,0,0)}[t,x,y,z]+(uz[t,x,y,z]^2+2\,W[t,x,y,z])\,uy^{(0,1,0,0)}[t,x,y,z]+2\,uy[t,x,y,z](uz[t,x,y,z]\,uz^{(0,1,0,0)}[t,x,y,z]+2\,W^{(0,1,0,0)}[t,x,y,z])}{2c^2}+O[\tfrac{1}{c}]^4$$

$$\left(-W^{(0,0,0,1)}[t,x,y,z]+uz^{(1,0,0,0)}[t,x,y,z]\right)+O[\tfrac{1}{c}]^2 \quad uz^{(0,1,0,0)}[t,x,y,z]+\frac{-ux[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]+\frac{1}{2}(ux[t,x,y,z]^2+uy[t,x,y,z]^2+uz[t,x,y,z]^2+2\,W[t,x,y,z])\,uz^{(0,1,0,0)}[t,x,y,z]+uz[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+uz[t,x,y,z](ux[t,x,y,z]\,ux^{(0,1,0,0)}[t,x,y,z]+uy[t,x,y,z]\,uy^{(0,1,0,0)}[t,x,y,z]+uz[t,x,y,z]\,uz^{(0,1,0,0)}[t,x,y,z]+W^{(0,1,0,0)}[t,x,y,z])}{c^2}+O[\ldots$$

In[283]:=

```
(* Energy current and supply according to 4-velocity *)
pvec = uu; Dpvec = Duv;
MF@(MF /@ {Efluxuu = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], , Esupplysymuu = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Esupplyuu = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[284]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-n\epsilon+O[\frac{1}{c}]^2 \\ (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2+(-Ax(qx+(jx-n\,Vx)\epsilon)-Ay(qy+jy\,\epsilon-n\,Vy\,\epsilon)-Az(qz+jz\,\epsilon-n\,Vz\,\epsilon))+O[\frac{1}{c}]^2 \end{pmatrix}$$

Null

$$(szz\,uz^{(0,0,0,1)}[t,x,y,z]+syy\,uy^{(0,0,1,0)}[t,x,y,z]+syz(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z])+sxx\,ux^{(0,1,0,0)}[t,x,y,z]+sxy(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z])+sxz(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]))+O[\tfrac{1}{c}]^2$$

Null

$$\frac{1}{2}(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z]+(syz+szy)(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z])+2(syy\,uy^{(0,0,1,0)}[t,x,y,z]+sxx\,ux^{(0,1,0,0)}[t,x,y,z])+(sxy+syx)(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z])+(sxz+szx)(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]))+O[\tfrac{1}{c}]^2$$

In[285]:=
```
MF@(MF /@ (FS[{Efluxuu, , Esupplysymuu, , Esupplyuu} /. replaceJu] /. replaceuUnorm))
```

Out[285]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2-n\epsilon+O[\frac{1}{c}]^2 \\ n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\rho c^2+(-Ax\,qx-Ay\,qy-Az\,qz+n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\epsilon)+O[\frac{1}{c}]^2 \end{pmatrix}$$

Null

$$(szz\,uz^{(0,0,0,1)}[t,x,y,z]+syy\,uy^{(0,0,1,0)}[t,x,y,z]+syz(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z])+sxx\,ux^{(0,1,0,0)}[t,x,y,z]+sxy(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z])+sxz(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]))+O[\tfrac{1}{c}]^2$$

Null

$$\frac{1}{2}(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z]+(syz+szy)(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z])+2(syy\,uy^{(0,0,1,0)}[t,x,y,z]+sxx\,ux^{(0,1,0,0)}[t,x,y,z])+(sxy+syx)(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z])+(sxz+szx)(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]))+O[\tfrac{1}{c}]^2$$

In[286]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[286]=

$$\left\{\begin{pmatrix} \frac{jx}{n}+\frac{qx}{n\rho c^2}+O[\frac{1}{c}]^4 \\ \frac{jy}{n}+\frac{qy}{n\rho c^2}+O[\frac{1}{c}]^4 \\ \frac{jz}{n}+\frac{qz}{n\rho c^2}+O[\frac{1}{c}]^4 \end{pmatrix}, \begin{pmatrix} ux+\frac{qx}{n\rho c^2}+O[\frac{1}{c}]^4 \\ uy+\frac{qy}{n\rho c^2}+O[\frac{1}{c}]^4 \\ uz+\frac{qz}{n\rho c^2}+O[\frac{1}{c}]^4 \end{pmatrix}\right\}$$

In[287]:=
```
(* Energy current and supply according to t-vector *)
pvec = - {1, 0, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec⟦ii⟧ * cc⟦ ;; ,  ;; , ii⟧, {ii, 1, 4}])]];
MF@(MF /@ ({Efluxt = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], , Esupplysymt = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Esupplyt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]}))
```

Out[288]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\,\rho\,c^2 + \left(\frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n} + n\,(\epsilon - W\,\rho)\right) + O\left[\frac{1}{c}\right]^2 \\ (Ay\,jy + Az\,jz + Ax\,(jx - n\,Vx) - n\,(Ay\,Vy + Az\,Vz))\,\rho\,c^2 + \frac{2\,n\,(Ax\,(jx\,sxx+jy\,sxy+jz\,sxz+n\,(qx+(jx-n\,Vx)\,\epsilon))+Ay\,(jx\,syx+jy\,syy+jz\,syz+n\,(qy+jy\,\epsilon-n\,Vy\,\epsilon))+Az\,(jx\,szx+jy\,szy+jz\,szz+n\,(qz+jz\,\epsilon-n\,Vz\,\epsilon)))+(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jx^2+jy^2+jz^2-2\,n^2\,W)\,\rho}{2\,n^2} + O\left[\frac{1}{c}\right]^2 \end{pmatrix} \\ \text{Null} \\ -n\,\rho\,W^{(1,0,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \\ \text{Null} \\ -n\,\rho\,W^{(1,0,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[289]:=
```
MF@(MF /@ (FS[({Efluxt, , Esupplysymt, , Esupplyt} /. replaceJu) /. replaceuUnorm]))
```

Out[289]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\,\rho\,c^2 + n\left(\epsilon + \frac{1}{2}\,(U^2 - 2\,W)\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ n\,(Ax\,(ux - Vx) + Ay\,(uy - Vy) + Az\,(uz - Vz))\,\rho\,c^2 + \left(Ax\,(qx + sxx\,ux + sxy\,uy + sxz\,uz + n\,(ux - Vx)\,\epsilon) + Ay\,(qy + syx\,ux + syy\,uy + syz\,uz + n\,(uy - Vy)\,\epsilon) + Az\,(qz + szx\,ux + szy\,uy + szz\,uz + n\,(uz - Vz)\,\epsilon) + \frac{1}{2}\,n\,(Ax\,(ux - Vx) + Ay\,(uy - Vy) + Az\,(uz - Vz))\,(U^2 - 2\,W)\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix} \\ \text{Null} \\ -n\,\rho\,W^{(1,0,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \\ \text{Null} \\ -n\,\rho\,W^{(1,0,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[290]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp⟦2 ;; 4⟧ / temp⟦1⟧] // MF, FS[temp⟦2 ;; 4⟧ / temp⟦1⟧ /. replaceJu] // MF}
```

Out[290]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[295]:=
```
(* Energy current and supply according to norm. t-vector *)
pvec = - c * {1, 0, 0, 0} / Sqrt[-gg⟦1, 1⟧]; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec⟦ii⟧] * cc⟦ ;; ,  ;; , ii⟧, {ii, 1, 4}])]];
MF@(MF /@ {Efluxnt = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], , Esupplysymnt = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Esupplynt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[296]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\,\rho\,c^2 + \left(n\,\epsilon + \frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n}\right) + O\left[\frac{1}{c}\right]^2 \\ (Ay\,jy + Az\,jz + Ax\,(jx - n\,Vx) - n\,(Ay\,Vy + Az\,Vz))\,\rho\,c^2 + \frac{2\,n\,(Ax\,(jx\,sxx+jy\,sxy+jz\,sxz+n\,(qx+(jx-n\,Vx)\,\epsilon))+Ay\,(jx\,syx+jy\,syy+jz\,syz+n\,(qy+jy\,\epsilon-n\,Vy\,\epsilon))+Az\,(jx\,szx+jy\,szy+jz\,szz+n\,(qz+jz\,\epsilon-n\,Vz\,\epsilon)))+(jx^2+jy^2+jz^2)\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho}{2\,n^2} + O\left[\frac{1}{c}\right]^2 \end{pmatrix} \\ \text{Null} \\ \rho\,(jz\,W^{(0,0,0,1)}[t,x,y,z] + jy\,W^{(0,0,1,0)}[t,x,y,z] + jx\,W^{(0,1,0,0)}[t,x,y,z]) + O\left[\frac{1}{c}\right]^2 \\ \text{Null} \\ \rho\,(jz\,W^{(0,0,0,1)}[t,x,y,z] + jy\,W^{(0,0,1,0)}[t,x,y,z] + jx\,W^{(0,1,0,0)}[t,x,y,z]) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[297]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp⟦2 ;; 4⟧ / temp⟦1⟧] // MF, FS[temp⟦2 ;; 4⟧ / temp⟦1⟧ /. replaceJu] // MF}
```

Out[297]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[298]:=
```
(* Energy current and supply according to cov. t-vector *)
pvec = c^2 * igg.{1, 0, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec⟦ii⟧] * cc⟦ ;; ,  ;; , ii⟧, {ii, 1, 4}])]];
MF@(MF /@ {Efluxcovt = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], , Esupplysymcovt = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Esupplycovt = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[299]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\,\rho\,c^2 + \left(\frac{(jx^2+jy^2+jz^2)\,\rho}{2\,n} + n\,(\epsilon + W\,\rho)\right) + O\left[\frac{1}{c}\right]^2 \\ (Ay\,jy + Az\,jz + Ax\,(jx - n\,Vx) - n\,(Ay\,Vy + Az\,Vz))\,\rho\,c^2 + \frac{2\,n\,(Ax\,(jx\,sxx+jy\,sxy+jz\,sxz+n\,(qx+(jx-n\,Vx)\,\epsilon))+Ay\,(jx\,syx+jy\,syy+jz\,syz+n\,(qy+jy\,\epsilon-n\,Vy\,\epsilon))+Az\,(jx\,szx+jy\,szy+jz\,szz+n\,(qz+jz\,\epsilon-n\,Vz\,\epsilon)))+(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jx^2+jy^2+jz^2+2\,n^2\,W)\,\rho}{2\,n^2} + O\left[\frac{1}{c}\right]^2 \end{pmatrix} \\ \text{Null} \\ \rho\,(2\,jz\,W^{(0,0,0,1)}[t,x,y,z] + 2\,jy\,W^{(0,0,1,0)}[t,x,y,z] + 2\,jx\,W^{(0,1,0,0)}[t,x,y,z] + n\,W^{(1,0,0,0)}[t,x,y,z]) + O\left[\frac{1}{c}\right]^2 \\ \text{Null} \\ \rho\,(2\,jz\,W^{(0,0,0,1)}[t,x,y,z] + 2\,jy\,W^{(0,0,1,0)}[t,x,y,z] + 2\,jx\,W^{(0,1,0,0)}[t,x,y,z] + n\,W^{(1,0,0,0)}[t,x,y,z]) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[300]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp⟦2 ;; 4⟧ / temp⟦1⟧] // MF, FS[temp⟦2 ;; 4⟧ / temp⟦1⟧ /. replaceJu] // MF}
```

Out[300]=

$$\left\{\begin{pmatrix} \frac{jx}{n} + \frac{n\,qx+jx\,sxx+jy\,sxy+jz\,sxz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jy}{n} + \frac{n\,qy+jx\,syx+jy\,syy+jz\,syz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \frac{jz}{n} + \frac{n\,qz+jx\,szx+jy\,szy+jz\,szz}{n^2\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} ux + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uy + \frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \\ uz + \frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\,\rho\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[323]:=
```
(* Momentum current and supply according to x-vector *)
pvec = {0, 1, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec⟦ii⟧ * cc⟦ ;; ,  ;; , ii⟧, {ii, 1, 4}])]];
MF@(MF /@ {Pfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], , Psupplysymx = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Psupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[324]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} jx\,\rho + \frac{px + \frac{jx\,sxx+jy\,syx+jz\,szx}{n} + \frac{jx\,(jx^2+jy^2+jz^2)\,\rho}{2\,n^2} + jx\,(\epsilon + 3\,W\,\rho)}{c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + \frac{jx\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho}{n}\right) + \frac{-n\,(Ax\,Vx+Ay\,Vy+Az\,Vz)\,(2\,n\,(jx\,sxx+jy\,syx+jz\,szx+n\,(px+jx\,\epsilon))+jx\,(jx^2+jy^2+jz^2+6\,n^2\,W)\,\rho)+Ay\,(2\,n^2\,(jx\,qy+jy\,(px+jx\,\epsilon))+jx\,jy\,(jx^2+jy^2+jz^2+6\,n^2\,W)\,\rho)+Az\,(2\,n^2\,(jx\,qz+jz\,(px+jx\,\epsilon))+jx\,jz\,(jx^2+jy^2+jz^2+6\,n^2\,W)\,\rho)+Ax\,jx\,(jx\,(jx^2+jy^2+jz^2)\,\rho+2\,n^2\,(px+qx+jx\,\epsilon+3\,jx\,W\,\rho))}{2\,n^3\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix} \\ \text{Null} \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \\ \text{Null} \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[325]:=
```
MF@(MF /@ (FS[({Pfluxx, , Psupplysymx, , Psupplyx} /. replaceJu) /. replaceuUnorm]))
```

Out[325]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} n\,ux\,\rho + \frac{px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon+\frac{1}{2}\,n\,ux\,(U^2+6\,W)\,\rho}{c^2} + O\left[\frac{1}{c}\right]^4 \\ (Ax\,sxx + Ay\,syx + Az\,szx + n\,ux\,(Ax\,(ux - Vx) + Ay\,(uy - Vy) + Az\,(uz - Vz))\,\rho) + \frac{Ax\,(qx\,ux+px\,(ux-Vx)-(sxx\,ux+syx\,uy+szx\,uz)\,Vx+n\,ux\,(ux-Vx)\,\epsilon)+Ay\,(qy\,ux+px\,(uy-Vy)-(sxx\,ux+syx\,uy+szx\,uz)\,Vy+n\,ux\,(uy-Vy)\,\epsilon)+Az\,(qz\,ux+px\,(uz-Vz)-(sxx\,ux+syx\,uy+szx\,uz)\,Vz+n\,ux\,(uz-Vz)\,\epsilon)+\frac{1}{2}\,Ax\,n\,ux\,(ux-Vx)\,(U^2+6\,W)\,\rho+\frac{1}{2}\,n\,ux\,(Ay\,(uy-Vy)+Az\,(uz-Vz))\,(U^2+6\,W)\,\rho}{c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix} \\ \text{Null} \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \\ \text{Null} \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[328]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp⟦2 ;; 4⟧ / temp⟦1⟧] // MF, FS[temp⟦2 ;; 4⟧ / temp⟦1⟧ /. replaceJu] // MF}
```

Out[328]=

$$\left\{\begin{pmatrix} \left(\frac{jx}{n} + \frac{sxx}{jx\,\rho}\right) - \frac{2\,n\,sxx\,(jx\,sxx+jy\,syx+jz\,szx+n\,(px+jx\,\epsilon))+jx\,(3\,jx^2\,sxx+2\,jx\,(-n\,qx+jy\,syx+jz\,szx)+sxx\,(jy^2+jz^2+6\,n^2\,W))\,\rho}{2\,(jx^2\,n^2\,\rho^2)\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(\frac{jy}{n} + \frac{syx}{jx\,\rho}\right) - \frac{2\,n\,syx\,(jx\,sxx+jy\,syx+jz\,szx+n\,(px+jx\,\epsilon))+jx\,(-2\,jx\,n\,qy+2\,jx\,jy\,sxx+jx^2\,syx+3\,jy^2\,syx+jz^2\,syx+2\,jy\,jz\,szx+6\,n^2\,syx\,W)\,\rho}{2\,(jx^2\,n^2\,\rho^2)\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(\frac{jz}{n} + \frac{szx}{jx\,\rho}\right) - \frac{2\,n\,szx\,(jx\,sxx+jy\,syx+jz\,szx+n\,(px+jx\,\epsilon))+jx\,(-2\,jx\,n\,qz+2\,jx\,jz\,sxx+2\,jy\,jz\,syx+jx^2\,szx+jy^2\,szx+3\,jz^2\,szx+6\,n^2\,szx\,W)\,\rho}{2\,(jx^2\,n^2\,\rho^2)\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} \left(ux + \frac{sxx}{n\,ux\,\rho}\right) - \frac{2\,sxx\,(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon)+n\,ux\,(-2\,qx\,ux+2\,ux\,(syx\,uy+szx\,uz)+sxx\,(3\,ux^2+uy^2+uz^2+6\,W))\,\rho}{2\,(n^2\,ux^2\,\rho^2)\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(uy + \frac{syx}{n\,ux\,\rho}\right) - \frac{2\,syx\,(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon)+n\,ux\,(-2\,qy\,ux+2\,uy\,(sxx\,ux+szx\,uz)+syx\,(ux^2+3\,uy^2+uz^2+6\,W))\,\rho}{2\,(n^2\,ux^2\,\rho^2)\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(uz + \frac{szx}{n\,ux\,\rho}\right) - \frac{2\,szx\,(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon)+n\,ux\,(-2\,qz\,ux+2\,(sxx\,ux+syx\,uy)\,uz+szx\,(ux^2+uy^2+3\,uz^2+6\,W))\,\rho}{2\,(n^2\,ux^2\,\rho^2)\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[305]:=
```
(* Momentum current and supply according to cov. x-vector *)
pvec = igg.{0, 1, 0, 0}; Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec⟦ii⟧] * cc⟦ ;; ,  ;; , ii⟧, {ii, 1, 4}])]];
MF@(MF /@ {Pfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], , Psupplysymx = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Psupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[306]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} jx\,\rho + \frac{px + \frac{jx\,sxx+jy\,syx+jz\,szx}{n} + \frac{jx\,(jx^2+jy^2+jz^2)\,\rho}{2\,n^2} + jx\,(\epsilon + W\,\rho)}{c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(Ax\,sxx + Ay\,syx + Az\,szx + \frac{jx\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,\rho}{n}\right) + \frac{\frac{Ax\,(-((n\,px+jy\,syx+jz\,szx)\,Vx)-2\,n\,sxx\,W+jx^2\,\epsilon+jx\,(px+qx-Vx\,(sxx+n\,\epsilon)))+Ay\,(-((n\,px+jz\,szx)\,Vy)-2\,n\,syx\,W+jy\,(px-syx\,Vy+jx\,\epsilon)+jx\,(qy-Vy\,(sxx+n\,\epsilon)))+Az\,(-((n\,px+jy\,syx)\,Vz)-2\,n\,szx\,W+jz\,(px-szx\,Vz+jx\,\epsilon)+jx\,(qz-Vz\,(sxx+n\,\epsilon)))}{n} + \frac{jx\,(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jx^2+jy^2+jz^2+2\,n^2\,W)\,\rho}{2\,n^3}}{c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix} \\ \text{Null} \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \\ \text{Null} \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z] + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[307]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp⟦2 ;; 4⟧ / temp⟦1⟧] // MF, FS[temp⟦2 ;; 4⟧ / temp⟦1⟧ /. replaceJu] // MF}
```

Out[307]=

$$\left\{\begin{pmatrix} \left(\frac{jx}{n} + \frac{sxx}{jx\,\rho}\right) - \frac{2\,n\,sxx\,(jx\,sxx+jy\,syx+jz\,szx+n\,(px+jx\,\epsilon))+jx\,(3\,jx^2\,sxx+2\,jx\,(-n\,qx+jy\,syx+jz\,szx)+sxx\,(jy^2+jz^2+6\,n^2\,W))\,\rho}{2\,(jx^2\,n^2\,\rho^2)\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(\frac{jy}{n} + \frac{syx}{jx\,\rho}\right) - \frac{2\,n\,syx\,(jx\,sxx+jy\,syx+jz\,szx+n\,(px+jx\,\epsilon))+jx\,(-2\,jx\,n\,qy+2\,jx\,jy\,sxx+jx^2\,syx+3\,jy^2\,syx+jz^2\,syx+2\,jy\,jz\,szx+6\,n^2\,syx\,W)\,\rho}{2\,(jx^2\,n^2\,\rho^2)\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(\frac{jz}{n} + \frac{szx}{jx\,\rho}\right) - \frac{2\,n\,szx\,(jx\,sxx+jy\,syx+jz\,szx+n\,(px+jx\,\epsilon))+jx\,(-2\,jx\,n\,qz+2\,jx\,jz\,sxx+2\,jy\,jz\,syx+jx^2\,szx+jy^2\,szx+3\,jz^2\,szx+6\,n^2\,szx\,W)\,\rho}{2\,(jx^2\,n^2\,\rho^2)\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}, \begin{pmatrix} \left(ux + \frac{sxx}{n\,ux\,\rho}\right) - \frac{2\,sxx\,(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon)+n\,ux\,(-2\,qx\,ux+2\,ux\,(syx\,uy+szx\,uz)+sxx\,(3\,ux^2+uy^2+uz^2+6\,W))\,\rho}{2\,(n^2\,ux^2\,\rho^2)\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(uy + \frac{syx}{n\,ux\,\rho}\right) - \frac{2\,syx\,(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon)+n\,ux\,(-2\,qy\,ux+2\,uy\,(sxx\,ux+szx\,uz)+syx\,(ux^2+3\,uy^2+uz^2+6\,W))\,\rho}{2\,(n^2\,ux^2\,\rho^2)\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(uz + \frac{szx}{n\,ux\,\rho}\right) - \frac{2\,szx\,(px+sxx\,ux+syx\,uy+szx\,uz+n\,ux\,\epsilon)+n\,ux\,(-2\,qz\,ux+2\,(sxx\,ux+syx\,uy)\,uz+szx\,(ux^2+uy^2+3\,uz^2+6\,W))\,\rho}{2\,(n^2\,ux^2\,\rho^2)\,c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}\right\}$$

In[308]:=
```
(* Ang.momentum current and supply according to yz-vector *)
pvec = y * {0, 0, 0, 1} - z * {0, 0, 1, 0}; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec⟦ii⟧ * cc⟦ ;; ,  ;; , ii⟧, {ii, 1, 4}])]];
MF@(MF /@ {Lfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], , Lsupplysymx = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Lsupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[309]//MatrixForm=

$$\begin{pmatrix} \begin{pmatrix} (jz\,y - jy\,z)\,\rho + \frac{2\,n\,((jx\,sxz+jy\,syz+jz\,szz)\,y-(jx\,sxy+jy\,syy+jz\,szy)\,z+n\,(pz\,y+jz\,y\,\epsilon-z\,(py+jy\,\epsilon)))+(jx^2+jy^2+jz^2+6\,n^2\,W)\,(jz\,y-jy\,z)\,\rho}{2\,n^2\,c^2} + O\left[\frac{1}{c}\right]^4 \\ \left((Ax\,sxz + Ay\,syz + Az\,szz)\,y - (Ax\,sxy + Ay\,syy + Az\,szy)\,z + \frac{(Ay\,jy+Az\,jz+Ax\,(jx-n\,Vx)-n\,(Ay\,Vy+Az\,Vz))\,(jz\,y-jy\,z)\,\rho}{n}\right) + \frac{-z\,(-n\,(Ax\,Vx+Ay\,Vy+Az\,Vz)\,(2\,n\,(jx\,sxy+jy\,syy+jz\,szy+n\,(py+jy\,\epsilon))+jy\,(jx^2+jy^2+jz^2+6\,n^2\,W)\,\rho)+Ax\,(2\,n^2\,(jy\,qx+jx\,(py+jy\,\epsilon))+jx\,jy\,(jx^2+jy^2+jz^2+6\,n^2\,W)\,\rho)+Az\,(2\,n^2\,(jy\,qz+jz\,(py+jy\,\epsilon))+jy\,jz\,(jx^2+jy^2+jz^2+6\,n^2\,W)\,\rho)+Ay\,jy\,(jy\,(jx^2+jy^2+jz^2)\,\rho+2\,n^2\,(py+qy+jy\,\epsilon+3\,jy\,W\,\rho)))+y\,(-n\,(Ax\,Vx+Ay\,Vy+Az\,Vz)\,(2\,n\,(jx\,sxz+jy\,syz+jz\,szz+n\,(pz+jz\,\epsilon))+jz\,(jx^2+jy^2}{2\,n^3\,c^2} \end{pmatrix} \\ \text{Null} \\ n\,\rho\,(y\,W^{(0,0,0,1)}[t,x,y,z] - z\,W^{(0,0,1,0)}[t,x,y,z]) + O\left[\frac{1}{c}\right]^2 \\ \text{Null} \\ n\,\rho\,(y\,W^{(0,0,0,1)}[t,x,y,z] - z\,W^{(0,0,1,0)}[t,x,y,z]) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[310]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[310]=
```
{( (jx/n + (sxz y-sxy z)/(jz y ρ-jy z ρ)) - (2 n (sxz y-sxy z) ((jx sxz+jy syz+jz szz) y-(jx sxy+jy syy+jz szy) z)-2 n qx (jz y-jy z)^2 ρ+(jz y-jy z) (3 jx^2 (sxz y-sxy z)+(jy^2+jz^2) (sxz y-sxy z)+2 jx (jy syz y+jz szz y-jy syy z-jz szy z)) ρ+2 n^2 (sxz y-sxy z) (pz y-py z+(jz y-jy z) (ϵ+3 W ρ)))/(2 (n^2 (jz y-jy z)^2 ρ^2) c^2) + O[1/c]^4
   (jy/n + (syz y-syy z)/(jz y ρ-jy z ρ)) + (-2 n (syz y-syy z) ((jx sxz+jy syz+jz szz) y-(jx sxy+jy syy+jz szy) z)+2 n qy (jz y-jy z)^2 ρ+(-jz y+jy z) ((2 jx jy sxz+jx^2 syz+3 jy^2 syz+jz^2 syz+2 jy jz szz) y-(2 jx jy sxy+jx^2 syy+3 jy^2 syy+jz^2 syy+2 jy jz szy) z) ρ-2 n^2 (syz y-syy z) (pz y-py z+(jz y-jy z) (ϵ+3 W ρ)))/(2 n^2 (jz y-jy z)^2 ρ^2 c^2) + O[1/c]^4
   (jz/n + (szz y-szy z)/(jz y ρ-jy z ρ)) - (2 n (szz y-szy z) ((jx sxz+jy syz+jz szz) y-(jx sxy+jy syy+jz szy) z)-2 n qz (jz y-jy z)^2 ρ+(jz y-jy z) (2 jz (jx sxz+jy syz) y+(jx^2+jy^2+3 jz^2) szz y-2 jz (jx sxy+jy syy) z-(jx^2+jy^2+3 jz^2) szy z) ρ+2 n^2 (szz y-szy z) (pz y-py z+(jz y-jy z) (ϵ+3 W ρ)))/(2 (n^2 (jz y-jy z)^2 ρ^2) c^2) + O[1/c]^4 ),

 ( (ux + (sxz y-sxy z)/(n uz y ρ-n uy z ρ)) + (-((2 (pz y+(sxz ux+syz uy+szz uz) y-(sxy ux+syy uy+szy uz) z+n uz y ϵ-z (py+n uy ϵ))+n (ux^2+uy^2+uz^2+6 W) (uz y-uy z) ρ) (sxz y+n ux uz y ρ-z (sxy+n ux uy ρ)))+n (uz y-uy z) ρ (2 pz ux y-2 py ux z+(uz y-uy z) (2 qx+n ux (2 ϵ+(ux^2+uy^2+uz^2+6 W) ρ))))/(2 n^2 (uz y-uy z)^2 ρ^2 c^2) + O[1/c]^4
   (uy + (syz y-syy z)/(n uz y ρ-n uy z ρ)) - (2 pz y (syz y-syy z)+2 (syz y-syy z) (sxz ux y+syz uy y+szz uz y-(py+sxy ux+syy uy) z-szy uz z+n uz y ϵ-n uy z ϵ)+n (uz y-uy z) (2 (sxz ux uy-qy uz+szz uy uz) y+syz (ux^2+3 uy^2+uz^2+6 W) y-(2 uy (-qy+sxy ux+szy uz)+syy (ux^2+3 uy^2+uz^2+6 W)) z) ρ)/(2 (n^2 (uz y-uy z)^2 ρ^2) c^2) + O[1/c]^4
   (uz + (szz y-szy z)/(n uz y ρ-n uy z ρ)) - (2 pz y (szz y-szy z)+2 (szz y-szy z) (sxz ux y+syz uy y+szz uz y-(py+sxy ux+syy uy) z-szy uz z+n uz y ϵ-n uy z ϵ)+n (uz y-uy z) (2 (-qz+sxz ux+syz uy) uz y+szz (ux^2+uy^2+3 uz^2+6 W) y-(-2 qz uy+2 sxy ux uz+2 syy uy uz+szy (ux^2+uy^2+3 uz^2+6 W)) z) ρ)/(2 (n^2 (uz y-uy z)^2 ρ^2) c^2) + O[1/c]^4 )}
```

In[314]:=
```
(* Ang.momentum current and supply according to cov. yz-vector *)
pvec = igg.(y*{0, 0, 0, 1} - z*{0, 0, 1, 0}); Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Lfluxcx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], , Lsupplysymcx = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Lsupplycx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[315]//MatrixForm=
```
(jz y - jy z) ρ + (2 n ((jx sxz+jy syz+jz szz) y-(jx sxy+jy syy+jz szy) z+n (pz y+jz y ϵ-z (py+jy ϵ)))+(jx^2+jy^2+jz^2+2 n^2 W) (jz y-jy z) ρ)/(2 n^2 c^2) + O[1/c]^4
((Ax sxz + Ay syz + Az szz) y - (Ax sxy + Ay syy + Az szy) z + ((Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jz y-jy z) ρ)/n) + (4 n^2 W y (-n (Ax sxz+Ay syz+Az szz)+jz (-Ax jx-Ay jy-Az jz+Ax n Vx+Ay n Vy+Az n Vz) ρ)+4 n^2 W z (n (Ax sxy+Ay syy+Az szy)+jy (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) ρ)-z (-n (Ax Vx+Ay Vy+Az Vz) (2 n (jx sxy+jy syy+jz szy+n (py+jy ϵ))+jy (jx^2+jy^2+jz^2+6 n^2 W) ρ)+Ax (2 n^2 (jy qx+jx (py+jy ϵ))+jx jy (jx^2+jy^2+jz^2+6 n^2 W) ρ)+Az (2 n^2 (jy qz+jz (p
Null
n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]^2
Null
n ρ (y W^(0,0,0,1)[t, x, y, z] - z W^(0,0,1,0)[t, x, y, z]) + O[1/c]^2
```

In[316]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[316]=
```
{( (jx/n + (sxz y-sxy z)/(jz y ρ-jy z ρ)) - (2 n (sxz y-sxy z) ((jx sxz+jy syz+jz szz) y-(jx sxy+jy syy+jz szy) z)-2 n qx (jz y-jy z)^2 ρ+(jz y-jy z) (3 jx^2 (sxz y-sxy z)+(jy^2+jz^2) (sxz y-sxy z)+2 jx (jy syz y+jz szz y-jy syy z-jz szy z)) ρ+2 n^2 (sxz y-sxy z) (pz y-py z+(jz y-jy z) (ϵ+3 W ρ)))/(2 (n^2 (jz y-jy z)^2 ρ^2) c^2) + O[1/c]^4
   (jy/n + (syz y-syy z)/(jz y ρ-jy z ρ)) + (-2 n (syz y-syy z) ((jx sxz+jy syz+jz szz) y-(jx sxy+jy syy+jz szy) z)+2 n qy (jz y-jy z)^2 ρ+(-jz y+jy z) ((2 jx jy sxz+jx^2 syz+3 jy^2 syz+jz^2 syz+2 jy jz szz) y-(2 jx jy sxy+jx^2 syy+3 jy^2 syy+jz^2 syy+2 jy jz szy) z) ρ-2 n^2 (syz y-syy z) (pz y-py z+(jz y-jy z) (ϵ+3 W ρ)))/(2 n^2 (jz y-jy z)^2 ρ^2 c^2) + O[1/c]^4
   (jz/n + (szz y-szy z)/(jz y ρ-jy z ρ)) - (2 n (szz y-szy z) ((jx sxz+jy syz+jz szz) y-(jx sxy+jy syy+jz szy) z)-2 n qz (jz y-jy z)^2 ρ+(jz y-jy z) (2 jz (jx sxz+jy syz) y+(jx^2+jy^2+3 jz^2) szz y-2 jz (jx sxy+jy syy) z-(jx^2+jy^2+3 jz^2) szy z) ρ+2 n^2 (szz y-szy z) (pz y-py z+(jz y-jy z) (ϵ+3 W ρ)))/(2 (n^2 (jz y-jy z)^2 ρ^2) c^2) + O[1/c]^4 ),

 ( (ux + (sxz y-sxy z)/(n uz y ρ-n uy z ρ)) - (2 pz y (sxz y-sxy z)+2 (sxz y-sxy z) (sxz ux y+syz uy y+szz uz y-(py+sxy ux+syy uy) z-szy uz z+n uz y ϵ-n uy z ϵ)+n (uz y-uy z) (2 (syz ux uy-qx uz+szz ux uz) y+sxz (3 ux^2+uy^2+uz^2+6 W) y-(-2 qx uy+2 syy ux uy+2 szy ux uz+sxy (3 ux^2+uy^2+uz^2+6 W)) z) ρ)/(2 (n^2 (uz y-uy z)^2 ρ^2) c^2) + O[1/c]^4
   (uy + (syz y-syy z)/(n uz y ρ-n uy z ρ)) - (2 pz y (syz y-syy z)+2 (syz y-syy z) (sxz ux y+syz uy y+szz uz y-(py+sxy ux+syy uy) z-szy uz z+n uz y ϵ-n uy z ϵ)+n (uz y-uy z) (2 (sxz ux uy-qy uz+szz uy uz) y+syz (ux^2+3 uy^2+uz^2+6 W) y-(2 uy (-qy+sxy ux+szy uz)+syy (ux^2+3 uy^2+uz^2+6 W)) z) ρ)/(2 (n^2 (uz y-uy z)^2 ρ^2) c^2) + O[1/c]^4
   (uz + (szz y-szy z)/(n uz y ρ-n uy z ρ)) - (2 pz y (szz y-szy z)+2 (szz y-szy z) (sxz ux y+syz uy y+szz uz y-(py+sxy ux+syy uy) z-szy uz z+n uz y ϵ-n uy z ϵ)+n (uz y-uy z) (2 (-qz+sxz ux+syz uy) uz y+szz (ux^2+uy^2+3 uz^2+6 W) y-(-2 qz uy+2 sxy ux uz+2 syy uy uz+szy (ux^2+uy^2+3 uz^2+6 W)) z) ρ)/(2 (n^2 (uz y-uy z)^2 ρ^2) c^2) + O[1/c]^4 )}
```

In[317]:=
```
(* Ang.boost-momentum current and supply according to tx-vector *)
pvec = t*{0, 1, 0, 0} + x*{1, 0, 0, 0} / c^2; Dpvec = Assuming[assut, FS[(D[Normal@pvec, {coords}] + Sum[pvec[[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Bfluxx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], , Bsupplysymx = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Bsupplyx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[318]//MatrixForm=
```
(jx t - n x) ρ + (x (-((jx^2+jy^2+jz^2) ρ)/(2 n) + n (-ϵ+W ρ)) + t (px + (jx sxx+jy syx+jz szx)/n + (jx (jx^2+jy^2+jz^2) ρ)/(2 n^2) + jx (ϵ+3 W ρ)))/c^2 + O[1/c]^4
((Ax sxx + Ay syx + Az szx) t + ((Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jx t-n x) ρ)/n) + (-n x (2 n (Ax (jx sxx+jy sxy+jz sxz+n (qx+(jx-n Vx) ϵ))+Ay (jx syx+jy syy+jz syz+n (qy+jy ϵ-n Vy ϵ))+Az (jx szx+jy szy+jz szz+n (qz+jz ϵ-n Vz ϵ)))+(Ay jy+Az jz+Ax (jx-n Vx)-
Null
n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
Null
n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
```

In[319]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[319]=
```
{( (jx/n + (sxx t)/(jx t ρ-n x ρ)) + (-2 n sxx t ((n px+jx sxx+jy syx+jz szx) t+n (jx t-n x) ϵ)+(-jx t^2 (3 jx^2 sxx+2 jx (-n qx+jy syx+jz szx)+sxx (jy^2+jz^2+6 n^2 W))+n t (jx^2 sxx-2 jx (2 n qx+jy sxy+jz sxz-jy syx-jz szx)+sxx (jy^2+jz^2-2 n^2 W)) x+2 n^2 (n qx+jx sxx+jy sxy+jz sxz) x^2) ρ)/(2 n^2 (jx t-n x)^2 ρ^2 c^2) + O[1/c]^4
   (jy/n + (syx t)/(jx t ρ-n x ρ)) - (2 n syx t ((n px+jx sxx+jy syx+jz szx) t+n (jx t-n x) ϵ)+(jx t^2 (-2 jx n qy+2 jx jy sxx+jx^2 syx+3 jy^2 syx+jz^2 syx+2 jy jz szx+6 n^2 syx W)+n t (jx^2 syx-3 jy^2 syx-jz^2 syx+2 jx (2 n qy-jy sxx+jy syy+jz syz)-2 jy jz szx+2 n^2 syx W) x-2 n^2 (n qy+jx syx+jy syy+jz syz) x^2) ρ)/(2 (n^2 (jx t-n x)^2 ρ^2) c^2) + O[1/c]^4
   (jz/n + (szx t)/(jx t ρ-n x ρ)) - (2 n szx t ((n px+jx sxx+jy syx+jz szx) t+n (jx t-n x) ϵ)+(jx t^2 (-2 jx n qz+2 jx jz sxx+2 jy jz syx+jx^2 szx+jy^2 szx+3 jz^2 szx+6 n^2 szx W)+n t (-2 jy jz syx+jx^2 szx-jy^2 szx-3 jz^2 szx+2 jx (2 n qz-jz sxx+jy szy+jz szz)+2 n^2 szx W) x-2 n^2 (n qz+jx szx+jy szy+jz szz) x^2) ρ)/(2 (n^2 (jx t-n x)^2 ρ^2) c^2) + O[1/c]^4 ),

 ( (ux + (sxx t)/(n t ux ρ-n x ρ)) + (-2 sxx t (t (px+sxx ux+syx uy+szx uz)+n (t ux-x) ϵ)+n (2 qx (-t ux+x)^2-2 (t ux-x) (syx t ux uy+szx t ux uz+sxy uy x+sxz uz x)+sxx (-t^2 ux (3 ux^2+uy^2+uz^2+6 W)+t (ux^2+uy^2+uz^2-2 W) x+2 ux x^2)) ρ)/(2 n^2 (-t ux+x)^2 ρ^2 c^2) + O[1/c]^4
   (uy + (syx t)/(n t ux ρ-n x ρ)) - (2 syx t (t (px+sxx ux+syx uy+szx uz)+n (t ux-x) ϵ)+n (-2 qy (-t ux+x)^2+2 (t ux-x) (sxx t ux uy+szx t uy uz+syy uy x+syz uz x)+syx (t^2 ux (ux^2+3 uy^2+uz^2+6 W)+t (ux^2-3 uy^2-uz^2+2 W) x-2 ux x^2)) ρ)/(2 (n^2 (-t ux+x)^2 ρ^2) c^2) + O[1/c]^4
   (uz + (szx t)/(n t ux ρ-n x ρ)) - (2 szx t (t (px+sxx ux+syx uy+szx uz)+n (t ux-x) ϵ)+n (-2 qz (-t ux+x)^2+2 (t ux-x) (sxx t ux uz+syx t uy uz+szy uy x+szz uz x)+szx (t^2 ux (ux^2+uy^2+3 uz^2+6 W)+t (ux^2-uy^2-3 uz^2+2 W) x-2 ux x^2)) ρ)/(2 (n^2 (-t ux+x)^2 ρ^2) c^2) + O[1/c]^4 )}
```

In[320]:=
```
(* Ang.boost-momentum current and supply according to cov. tx-vector *)
pvec = igg.(t*{0, 1, 0, 0} - x*{1, 0, 0, 0}); Dpvec = Assuming[assut, FS[(D[Normal@tW[pvec], {coords}] + Sum[tW[pvec[[ii]]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]];
MF@(MF /@ {Bfluxcx = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.pvec)], , Bsupplysymcx = FS[itjv[itW[FS[Tr[Dpvec.TTxsym]]]]], , Bsupplycx = FS[itjv[itW[FS[Tr[Dpvec.TTx]]]]]})
```

Out[321]//MatrixForm=
```
(jx t - n x) ρ + (2 n ((n px+jx sxx+jy syx+jz szx) t+n (jx t-n x) ϵ)+(jx^2+jy^2+jz^2+2 n^2 W) (jx t-n x) ρ)/(2 n^2 c^2) + O[1/c]^4
((Ax sxx + Ay syx + Az szx) t + ((Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jx t-n x) ρ)/n) + (-2 Ay n^2 (t (-jx qy+n px Vy+jx sxx Vy+jz szx Vy+2 n syx W)+(n qy+jx syx+jz syz) x+jy (-px t+syx t Vy+syy x))-2 Az n^2 (t (-jx qz+n px Vz+jx sxx Vz+jy syx Vz+2 n szx W)+(n qz
Null
n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
Null
n t ρ W^(0,1,0,0)[t, x, y, z] + O[1/c]^2
```

In[322]:=
```
(* "velocity" *)
temp = FS[EPS.pvec]; {FS[temp[[2 ;; 4]] / temp[[1]]] // MF, FS[temp[[2 ;; 4]] / temp[[1]] /. replaceJu] // MF}
```

Out[322]=
```
{( (jx/n + (sxx t)/(jx t ρ-n x ρ)) - (2 n sxx t ((n px+jx sxx+jy syx+jz szx) t+n (jx t-n x) ϵ)+(jx t-n x) (t (3 jx^2 sxx+2 jx (-n qx+jy syx+jz szx)+sxx (jy^2+jz^2+6 n^2 W))+2 n (n qx+jx sxx+jy sxy+jz sxz) x) ρ)/(2 (n^2 (jx t-n x)^2 ρ^2) c^2) + O[1/c]^4
   (jy/n + (syx t)/(jx t ρ-n x ρ)) - (2 n syx t ((n px+jx sxx+jy syx+jz szx) t+n (jx t-n x) ϵ)+(jx t-n x) (t (-2 jx n qy+2 jx jy sxx+jx^2 syx+3 jy^2 syx+jz^2 syx+2 jy jz szx+6 n^2 syx W)+2 n (n qy+jx syx+jy syy+jz syz) x) ρ)/(2 (n^2 (jx t-n x)^2 ρ^2) c^2) + O[1/c]^4
   (jz/n + (szx t)/(jx t ρ-n x ρ)) - (2 n szx t ((n px+jx sxx+jy syx+jz szx) t+n (jx t-n x) ϵ)+(jx t-n x) (t (-2 jx n qz+2 jx jz sxx+2 jy jz syx+jx^2 szx+jy^2 szx+3 jz^2 szx+6 n^2 szx W)+2 n (n qz+jx szx+jy szy+jz szz) x) ρ)/(2 (n^2 (jx t-n x)^2 ρ^2) c^2) + O[1/c]^4 ),
 ( (ux + (sxx t)/(n t ux ρ-n x ρ)) - (2 sxx t (t (px+sxx ux+syx uy+szx uz)+n (t ux-x) ϵ)+n (t ux-x) (2 t ux (syx uy+szx uz)+sxx t (3 ux^2+uy^2+uz^2+6 W)+2 (sxx ux+sxy uy+sxz uz) x+2 qx (-t ux+x)) ρ)/(2 (n^2 (-t ux+x)^2 ρ^2) c^2) + O[1/c]^4
   (uy + (syx t)/(n t ux ρ-n x ρ)) - (2 syx t (t (px+sxx ux+syx uy+szx uz)+n (t ux-x) ϵ)+n (t ux-x) (2 t uy (sxx ux+szx uz)+syx t (ux^2+3 uy^2+uz^2+6 W)+2 (syx ux+syy uy+syz uz) x+2 qy (-t ux+x)) ρ)/(2 (n^2 (-t ux+x)^2 ρ^2) c^2) + O[1/c]^4
   (uz + (szx t)/(n t ux ρ-n x ρ)) - (2 szx t (t (px+sxx ux+syx uy+szx uz)+n (t ux-x) ϵ)+n (t ux-x) (2 t (sxx ux+syx uy) uz+szx t (ux^2+uy^2+3 uz^2+6 W)+2 (szx ux+szy uy+szz uz) x+2 qz (-t ux+x)) ρ)/(2 (n^2 (-t ux+x)^2 ρ^2) c^2) + O[1/c]^4 )}
```

```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[supply.TTx]], {supply, {Dtxyzvec[[1]], Dtxyzvec[[2]], {0, 0, 0, 0}, Dgtxyzvec[[1]] * c^2, Dgtxyzvec[[2]], {0, 0, 0, 0}, DLxvec, Dxboost / c, {0, 0, 0, 0}, DgLxvec, Dgxboost, {0, 0, 0, 0}, Duu, Dtvecnorm}}]]

(* coordinate/internal/coordinate-proper energy and x-momentum, content and fluxes (TRANSPOSED) *)
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, igg[[1]] * c^2, igg[[2]], {0, 0, 0, 0}, Lxvec, xboost / c, {0, 0, 0, 0}, gLxvec, gxboost, {0, 0, 0, 0}, uu, tvecnorm}])]]]
```

Out[•]//MatrixForm=
```
-n ρ c^2 + (-((jx^2+jy^2+jz^2) ρ)/(2 n) + n (-ϵ + W ρ)) + O[1/c]^2    (-Ax jx - Ay jy - Az jz + Ax n Vx + Ay n Vy + Az n Vz) ρ c^2 + (-(Ax (n qx+jx sxx+jy sxy+jz sxz)+Ay (n qy+jx syx+jy syy+jz syz)+Az (n qz+jx szx+jy szy+jz szz))/n + (-Ax jx - Ay jy - Az jz + Ax n Vx + Ay n Vy + Az n Vz) ϵ - ((Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jx^2+jy^2+jz^2-2 n^2 W) ρ)/(2 n^2)) + O[1/c]^2
jx ρ + O[1/c]^2    (Ax sxx + Ay syx + Az szx + (jx (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) ρ)/n) + O[1/c]^2
0    0
n ρ c^2 + (((jx^2+jy^2+jz^2) ρ)/(2 n) + n (ϵ + W ρ)) + O[1/c]^2    (Ay jy + Az jz + Ax (jx - n Vx) - n (Ay Vy + Az Vz)) ρ c^2 + (2 n (Ax (jx sxx+jy sxy+jz sxz+n (qx+(jx-n Vx) ϵ))+Ay (jx syx+jy syy+jz syz+n (qy+jy ϵ-n Vy ϵ))+Az (jx szx+jy szy+jz szz+n (qz+jz ϵ-n Vz ϵ)))+(Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jx^2+jy^2+jz^2+2 n^2 W) ρ)/(2 n^2) + O[1/c]^2
jx ρ + O[1/c]^2    (Ax sxx + Ay syx + Az szx + (jx (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) ρ)/n) + O[1/c]^2
0    0
(jz y - jy z) ρ + O[1/c]^2    ((Ax sxz + Ay syz + Az szz) y - (Ax sxy + Ay syy + Az szy) z + ((Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jz y-jy z) ρ)/n) + O[1/c]^2
(jx t - n x) ρ + O[1/c]^2    ((Ax sxx + Ay syx + Az szx) t + ((Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jx t-n x) ρ)/n) + O[1/c]^2
0    0
(jz y - jy z) ρ + O[1/c]^2    ((Ax sxz + Ay syz + Az szz) y - (Ax sxy + Ay syy + Az szy) z + ((Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jz y-jy z) ρ)/n) + O[1/c]^2
(-jx t ρ + n x ρ) + O[1/c]^2    (-((Ax sxx + Ay syx + Az szx) t) - ((Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) (jx t-n x) ρ)/n) + O[1/c]^2
0    0
-n ρ c^2 - n ϵ + O[1/c]^2    (-Ax jx - Ay jy - Az jz + Ax n Vx + Ay n Vy + Az n Vz) ρ c^2 + (-Ax (qx + (jx - n Vx) ϵ) - Ay (qy + jy ϵ - n Vy ϵ) - Az (qz + jz ϵ - n Vz ϵ)) + O[1/c]^2
-n ρ c^2 + (-n ϵ - ((jx^2+jy^2+jz^2) ρ)/(2 n)) + O[1/c]^2    (-Ax jx - Ay jy - Az jz + Ax n Vx + Ay n Vy + Az n Vz) ρ c^2 + (-(Ax (n qx+jx sxx+jy sxy+jz sxz)+Ay (n qy+jx syx+jy syy+jz syz)+Az (n qz+jx szx+jy szy+jz szz))/n + (-Ax jx - Ay jy - Az jz + Ax n Vx + Ay n Vy + Az n Vz) ϵ - ((jx^2+jy^2+jz^2) (Ay jy+Az jz+Ax (jx-n Vx)-n (Ay Vy+Az Vz)) ρ)/(2 n^2)) + O[1/c]^2
```

```
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}, igg[[1]] * c^2, igg[[2]], {0, 0, 0, 0}, Lxvec, xboost / c, {0, 0, 0, 0}, gLxvec, gxboost, {0, 0, 0, 0}, uu, tvecnorm}]) /. replaceJu]]]
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
-n\rho c^2 - \frac{1}{2} n\left(2\epsilon + \left(ux^2+uy^2+uz^2-2W\right)\rho\right) + O[\frac{1}{c}]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2 + \left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right) - Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right) - Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon\right) - \frac{1}{2} n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(ux^2+uy^2+uz^2-2W\right)\rho\right) + O[\frac{1}{c}]^2 \\
n\,ux\,\rho + O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right) + O[\frac{1}{c}]^2 \\
0 & 0 \\
n\rho c^2 + \left(n\epsilon + \frac{1}{2} n\left(ux^2+uy^2+uz^2+2W\right)\rho\right) + O[\frac{1}{c}]^2 & n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho c^2 + \left(Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right) + Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right) + Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon\right) + \frac{1}{2} n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(ux^2+uy^2+uz^2+2W\right)\rho\right) + O[\frac{1}{c}]^2 \\
n\,ux\,\rho + O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right) + O[\frac{1}{c}]^2 \\
0 & 0 \\
n\left(uz\,y-uy\,z\right)\rho + O[\frac{1}{c}]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right) + O[\frac{1}{c}]^2 \\
n\left(t\,ux-x\right)\rho + O[\frac{1}{c}]^2 & \left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t + n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(t\,ux-x\right)\rho\right) + O[\frac{1}{c}]^2 \\
0 & 0 \\
n\left(uz\,y-uy\,z\right)\rho + O[\frac{1}{c}]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right) + O[\frac{1}{c}]^2 \\
n\left(-t\,ux+x\right)\rho + O[\frac{1}{c}]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right) - n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(t\,ux-x\right)\rho\right) + O[\frac{1}{c}]^2 \\
0 & 0 \\
-n\rho c^2 - n\epsilon + O[\frac{1}{c}]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2 + \left(-Ax\,qx-Ay\,qy-Az\,qz+n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\epsilon\right) + O[\frac{1}{c}]^2 \\
-n\rho c^2 - \frac{1}{2} n\left(2\epsilon + \left(ux^2+uy^2+uz^2\right)\rho\right) + O[\frac{1}{c}]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2 + \left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right) - Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right) - Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon\right) - \frac{1}{2} n\left(ux^2+uy^2+uz^2\right)\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right) + O[\frac{1}{c}]^2
\end{pmatrix}
$$

```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[supply.TTx]], {supply, {Dtxyzvec[[1]], Dtxyzvec[[2]], {0, 0, 0, 0}, Dgtxyzvec[[1]] * c^2, Dgtxyzvec[[2]], {0, 0, 0, 0}, DLxvec, Dxboost / c, {0, 0, 0, 0}, DgLxvec, Dgxboost, {0, 0, 0, 0}, Duu, Dtvecnorm}}]]
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
\rho\, n[t,x,y,z]\, W^{(1,0,0,0)}[t,x,y,z] + O[\frac{1}{c}]^2 \\
\rho\, n[t,x,y,z]\, W^{(0,1,0,0)}[t,x,y,z] + O[\frac{1}{c}]^2 \\
0 \\
\left(2\rho\, n[t,x,y,z] \times uz[t,x,y,z]\, W^{(0,0,0,1)}[t,x,y,z] + 2\rho\, n[t,x,y,z] \times uy[t,x,y,z]\, W^{(0,0,1,0)}[t,x,y,z] + 2\rho\, n[t,x,y,z] \times ux[t,x,y,z]\, W^{(0,1,0,0)}[t,x,y,z] + \rho\, n[t,x,y,z]\, W^{(1,0,0,0)}[t,x,y,z]\right) + O[\frac{1}{c}]^2 \\
\rho\, n[t,x,y,z]\, W^{(0,1,0,0)}[t,x,y,z] + O[\frac{1}{c}]^2 \\
0 \\
\left(y\rho\, n[t,x,y,z]\, W^{(0,0,0,1)}[t,x,y,z] - z\rho\, n[t,x,y,z]\, W^{(0,0,1,0)}[t,x,y,z]\right) + O[\frac{1}{c}]^2 \\
t\rho\, n[t,x,y,z]\, W^{(0,1,0,0)}[t,x,y,z] + O[\frac{1}{c}]^2 \\
0 \\
\left(y\rho\, n[t,x,y,z]\, W^{(0,0,0,1)}[t,x,y,z] - z\rho\, n[t,x,y,z]\, W^{(0,0,1,0)}[t,x,y,z]\right) + O[\frac{1}{c}]^2 \\
-t\rho\, n[t,x,y,z]\, W^{(0,1,0,0)}[t,x,y,z] + O[\frac{1}{c}]^2 \\
0 \\
\left(\frac{sxz\,jx^{(0,0,0,1)}[t,x,y,z]}{2\,n[t,x,y,z]} + \frac{szx\,jx^{(0,0,0,1)}[t,x,y,z]}{2\,n[t,x,y,z]} - \frac{\rho\,jx[t,x,y,z]\times uz[t,x,y,z]\,jx^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]} + \rho\,ux[t,x,y,z] \times uz[t,x,y,z]\,jx^{(0,0,0,1)}[t,x,y,z] + \frac{syz\,jy^{(0,0,0,1)}[t,x,y,z]}{2\,n[t,x,y,z]} + \frac{szy\,jy^{(0,0,0,1)}[t,x,y,z]}{2\,n[t,x,y,z]} - \frac{\rho\,jy[t,x,y,z]\times uz[t,x,y,z]\,jy^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]} + \rho\,uy[t,x,y,z] \times uz[t,x,y,z]\,jy^{(0,0,0,1)}[t,x,y,z] + \frac{szz\,jz^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]} - \frac{\rho\,jz[t,x,y,z]\times uz[t,x,y,z]\,jz^{(0,0,0,1)}[t,x,y,z]}{n[t,x,y,z]} + \rho\,uz[t,x,y,z]^2\,jz^{(0,0,0,1)} \right. \\
\left(-\rho\, n[t,x,y,z] \times uz[t,x,y,z]\, W^{(0,0,0,1)}[t,x,y,z] - \rho\, n[t,x,y,z] \times uy[t,x,y,z]\, W^{(0,0,1,0)}[t,x,y,z] - \rho\, n[t,x,y,z] \times ux[t,x,y,z]\, W^{(0,1,0,0)}[t,x,y,z]\right) + O[\frac{1}{c}]^2
\end{pmatrix}
$$

```
(* covariant derivatives of coordinate 4-vectors (equivalent to Christoffel symbols), for later use *)
(Dtxyzvec = Table[ Assuming[assut, Expand //@ FS@PowerExpand[(D[IdentityMatrix[4][[aa]], {coords}] + Sum[IdentityMatrix[4][[aa]][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]], {aa, 1, 4}]);
```

```
(* normalized coordinate-t 4-vector*)
tvecnorm = Assuming[assut, Expand //@ FS@PowerExpand[c * {1, 0, 0, 0} / Sqrt[-gg[[1, 1]]]]]
```

Out[•]= $\left\{1 + \frac{W}{c^2} + O[\frac{1}{c}]^4, 0, 0, 0\right\}$

```
(* and its covariant derivative *)
(Dtvecnorm = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[tvecnorm], {coords}] + Sum[tW[tvecnorm][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]) // MF
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\
-W^{(0,1,0,0)}[t,x,y,z] + O[\frac{1}{c}]^2 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & 0 & 0 \\
-W^{(0,0,1,0)}[t,x,y,z] + O[\frac{1}{c}]^2 & 0 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & 0 \\
-W^{(0,0,0,1)}[t,x,y,z] + O[\frac{1}{c}]^2 & 0 & 0 & \frac{W^{(1,0,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4
\end{pmatrix}
$$

```
(* "raised" coordinate 4-covectors *)
(gtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[igg.IdentityMatrix[4]]]) // MF
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
-\left(\frac{1}{c}\right)^2 - \frac{2W}{c^4} + O[\frac{1}{c}]^6 & 0 & 0 & 0 \\
0 & 1 - \frac{2W}{c^2} + O[\frac{1}{c}]^4 & 0 & 0 \\
0 & 0 & 1 - \frac{2W}{c^2} + O[\frac{1}{c}]^4 & 0 \\
0 & 0 & 0 & 1 - \frac{2W}{c^2} + O[\frac{1}{c}]^4
\end{pmatrix}
$$

```
(* and their covariant derivatives *)
(Dgtxyzvec = Table[ Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[igg[[aa]]], {coords}] + Sum[tW[igg[[aa]]][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]], {aa, 1, 4}]);
```

```
(* x-component of rot vector *)
Lxvec = Assuming[assut, Expand //@ FS@PowerExpand[{0, 0, -z, y}]]
```

Out[•]= $\{0, 0, -z, y\}$

```
(* and its covariant derivative *)
showf[assut][DLxvec = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[Lxvec], {coords}] + Sum[tW[Lxvec][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
\frac{-y W^{(0,0,0,1)}[t,x,y,z] + z W^{(0,0,1,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & 0 & -\frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^4} + O[\frac{1}{c}]^6 & \frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^4} + O[\frac{1}{c}]^6 \\
0 & \frac{y W^{(0,0,0,1)}[t,x,y,z] - z W^{(0,0,1,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & \frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & -\frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 \\
-\frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & -\frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & \frac{y W^{(0,0,0,1)}[t,x,y,z] - z W^{(0,0,1,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & -1 + \frac{-z W^{(0,0,0,1)}[t,x,y,z] - y W^{(0,0,1,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 \\
\frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & \frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & 1 + \frac{z W^{(0,0,0,1)}[t,x,y,z] + y W^{(0,0,1,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & \frac{y W^{(0,0,0,1)}[t,x,y,z] - z W^{(0,0,1,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4
\end{pmatrix}
$$

```
(* "raised" x-component of rot co-vector *)
gLxvec = Assuming[assut, Expand //@ FS@PowerExpand[igg.{0, 0, -z, y}]]
```

Out[•]= $\left\{0, 0, -z + \frac{2Wz}{c^2} + O[\frac{1}{c}]^4, y - \frac{2(Wy)}{c^2} + O[\frac{1}{c}]^4\right\}$

```
(* and its covariant derivative *)
showf[assut][DgLxvec = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[gLxvec], {coords}] + Sum[tW[gLxvec][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
\frac{-y W^{(0,0,0,1)}[t,x,y,z] + z W^{(0,0,1,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & 0 & -\frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^4} + O[\frac{1}{c}]^6 & \frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^4} + O[\frac{1}{c}]^6 \\
0 & \frac{y W^{(0,0,0,1)}[t,x,y,z] - z W^{(0,0,1,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & \frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & -\frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 \\
\frac{z W^{(1,0,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & \frac{z W^{(0,1,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & \frac{y W^{(0,0,0,1)}[t,x,y,z] + z W^{(0,0,1,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & -1 + \frac{2 W[t,x,y,z] + z W^{(0,0,0,1)}[t,x,y,z] - y W^{(0,0,1,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 \\
-\frac{y W^{(1,0,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & -\frac{y W^{(0,1,0,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & 1 + \frac{-2 W[t,x,y,z] + z W^{(0,0,0,1)}[t,x,y,z] - y W^{(0,0,1,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4 & \frac{-y W^{(0,0,0,1)}[t,x,y,z] - z W^{(0,0,1,0)}[t,x,y,z]}{c^2} + O[\frac{1}{c}]^4
\end{pmatrix}
$$

```
(* x-component of boost vector *)
xboost = Assuming[assut, Expand //@ FS@PowerExpand[{x / c, t * c, 0, 0}]]
```

Out[•]= $\left\{\frac{x}{c}, c\,t, 0, 0\right\}$

```
(* and its covariant derivative *)
showf[assut][Dxboost = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[xboost], {coords}] + Sum[tW[xboost][[ii]] * cc[[ ;; ,  ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=

$$
\begin{pmatrix}
-\frac{t W^{(0,1,0,0)}[t,x,y,z]}{c} + O[\frac{1}{c}]^3 & \frac{1}{c} + \frac{-x W^{(0,1,0,0)}[t,x,y,z] + t W^{(1,0,0,0)}[t,x,y,z]}{c^3} + O[\frac{1}{c}]^5 & -\frac{x W^{(0,0,1,0)}[t,x,y,z]}{c^3} + O[\frac{1}{c}]^5 & -\frac{x W^{(0,0,0,1)}[t,x,y,z]}{c^3} + O[\frac{1}{c}]^5 \\
c + \frac{-x W^{(0,1,0,0)}[t,x,y,z] + t W^{(1,0,0,0)}[t,x,y,z]}{c} + O[\frac{1}{c}]^3 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c} + O[\frac{1}{c}]^3 & \frac{t W^{(0,0,1,0)}[t,x,y,z]}{c} + O[\frac{1}{c}]^3 & \frac{t W^{(0,0,0,1)}[t,x,y,z]}{c} + O[\frac{1}{c}]^3 \\
-\frac{x W^{(0,0,1,0)}[t,x,y,z]}{c} + O[\frac{1}{c}]^3 & -\frac{t W^{(0,0,1,0)}[t,x,y,z]}{c} + O[\frac{1}{c}]^3 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c} + O[\frac{1}{c}]^3 & 0 \\
-\frac{x W^{(0,0,0,1)}[t,x,y,z]}{c} + O[\frac{1}{c}]^3 & -\frac{t W^{(0,0,0,1)}[t,x,y,z]}{c} + O[\frac{1}{c}]^3 & 0 & \frac{t W^{(0,1,0,0)}[t,x,y,z]}{c} + O[\frac{1}{c}]^3
\end{pmatrix}
$$

In[•]:=
```
(* "raised" x-component of boost co-vector *)
gxboost = Assuming[assut, Expand //@ FS@PowerExpand[igg.{x, -t, 0, 0}]]
```

Out[•]=
$$\left\{-\frac{x}{c^2}-\frac{2(W x)}{c^4}+O\left[\frac{1}{c}\right]^6,\ -t+\frac{2tW}{c^2}+O\left[\frac{1}{c}\right]^4,\ 0,\ 0\right\}$$

In[•]:=
```
(* and its covariant derivative *)
showf[assut][Dgxboost = Assuming[assut, Expand //@ FS@PowerExpand[(D[Normal@tW[gxboost], {coords}] + Sum[tW[gxboost][[ii]] * cc[[ ;; , ;; , ii]], {ii, 1, 4}])]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix}
\frac{t W^{(0,1,0,0)}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & -\left(\frac{1}{c}\right)^2+\frac{-2W[t,x,y,z]-xW^{(0,1,0,0)}[t,x,y,z]-tW^{(1,0,0,0)}[t,x,y,z]}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{xW^{(0,0,1,0)}[t,x,y,z]}{c^4}+O\left[\frac{1}{c}\right]^6 & -\frac{xW^{(0,0,0,1)}[t,x,y,z]}{c^4}+O\left[\frac{1}{c}\right]^6 \\
-1+\frac{2W[t,x,y,z]+xW^{(0,1,0,0)}[t,x,y,z]+tW^{(1,0,0,0)}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{tW^{(0,1,0,0)}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{tW^{(0,0,1,0)}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{tW^{(0,0,0,1)}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 \\
\frac{xW^{(0,0,1,0)}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{tW^{(0,0,1,0)}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & -\frac{tW^{(0,1,0,0)}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 \\
\frac{xW^{(0,0,0,1)}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & \frac{tW^{(0,0,0,1)}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4 & 0 & -\frac{tW^{(0,1,0,0)}[t,x,y,z]}{c^2}+O\left[\frac{1}{c}\right]^4
\end{pmatrix}$$

```
(* content and flux of coordinatevector-energy and coordinatevector-momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix}
-n\rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(-\epsilon+W\rho)\right)+O\left[\frac{1}{c}\right]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2+\left(-\frac{Ax(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz)+Ay(n\,qy+jx\,syx+jy\,syy+jz\,syz)+Az(n\,qz+jx\,szx+jy\,szy+jz\,szz)}{n}+(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\epsilon-\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}\right)+O\left[\frac{1}{c}\right]^2 \\
jx\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\
jy\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+\frac{jy(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\
jz\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+\frac{jz(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[•]:=
```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
showf[assut][Table[Expand //@ FS@PowerExpand[Tr[Dtxyzvec[[aa, ;; , ;;]].TTx]], {aa, 1, 4}]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix}
\rho\,n[t,x,y,z]\,W^{(1,0,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\
\rho\,n[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\
\rho\,n[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2 \\
\rho\,n[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z]+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

```
(* content and flux of raised coordinatecovector-energy and coordinatecovector-momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxtxyzvec = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

```
(* content and flux of coord-energy and momentum (TRANSPOSED) *)
shows[assut, 1][T[fluxEPS = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.EPS]]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix}
-n\rho c^2+\left(-\frac{(jx^2+jy^2+jz^2)\rho}{2n}+n(-\epsilon+W\rho)\right)+O\left[\frac{1}{c}\right]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2+\left(-\frac{Ax(n\,qx+jx\,sxx+jy\,sxy+jz\,sxz)+Ay(n\,qy+jx\,syx+jy\,syy+jz\,syz)+Az(n\,qz+jx\,szx+jy\,szy+jz\,szz)}{n}+(-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\epsilon-\frac{(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))(jx^2+jy^2+jz^2-2n^2W)\rho}{2n^2}\right)+O\left[\frac{1}{c}\right]^2 \\
jx\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+\frac{jx(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\
jy\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+\frac{jy(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O\left[\frac{1}{c}\right]^2 \\
jz\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+\frac{jz(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz))\rho}{n}\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

```
(* content and flux of coord-energy and momentum for dust (TRANSPOSED) *)
shows[assut, 1][T[fluxdust = Assuming[assut, Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.dust2]]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+(aux^2+auy^2+auz^2-2W)\rho\right)+O\left[\frac{1}{c}\right]^2 & (-Ax\,jx-Ay\,jy-Az\,jz+Ax\,n\,Vx+Ay\,n\,Vy+Az\,n\,Vz)\rho c^2-\frac{1}{2}\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)\left(2\epsilon+(aux^2+auy^2+auz^2-2W)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
aux\,n\,\rho+O\left[\frac{1}{c}\right]^2 & aux\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)\rho+O\left[\frac{1}{c}\right]^2 \\
auy\,n\,\rho+O\left[\frac{1}{c}\right]^2 & auy\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)\rho+O\left[\frac{1}{c}\right]^2 \\
auz\,n\,\rho+O\left[\frac{1}{c}\right]^2 & auz\left(Ay\,jy+Az\,jz+Ax(jx-n\,Vx)-n(Ay\,Vy+Az\,Vz)\right)\rho+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[•]:=
```
(* in terms of matter velocity *)
shows[assut, 1][T[fluxEPS /. replaceJu]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+(ux^2+uy^2+uz^2-2W)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac{1}{2}n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
n\,uy\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxy+Ay\,syy+Az\,szy+n\,uy\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
n\,uz\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz+Ay\,syz+Az\,szz+n\,uz\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[•]:=
```
(* momentum flux = A.σ + P A.(u-V)*)
fluxPS = ({Ax, Ay, Az}.(S[[2 ;; 4, 2 ;; 4]]).{1, 0, 0} + EPS[[1, 2]] * ({Ax, Ay, Az}.({jx, jy, jz} / n - {Vx, Vy, Vz})))
```

Out[•]=
$$\left(Ax\,sxx+Ay\,syx+Az\,szx+jx\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\rho\right)+\frac{1}{c^2}\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\left(px+\frac{jx\,sxx}{n}+\frac{jy\,syx}{n}+\frac{jz\,szx}{n}+jx\,\epsilon+\frac{jx^3\rho}{2n^2}+\frac{jx\,jy^2\rho}{2n^2}+\frac{jx\,jz^2\rho}{2n^2}+3\,jx\,W\rho\right)+O\left[\frac{1}{c}\right]^4$$

In[•]:=
```
shows[assut, 1][Expand //@ FS@PowerExpand[fluxEPS[[2, 2]] - fluxPS]]
```

Out[•]//MatrixForm=
$$O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
(* energy flux = A.q + A.σ.u + E A.(u-V)*)
fluxE = -({Ax, Ay, Az}.{qx, qy, qz} + {Ax, Ay, Az}.(S[[2 ;; 4, 2 ;; 4]] + Qtens[[2 ;; 4, 2 ;; 4]]).{jx, jy, jz} / n + (-EPS[[1, 1]]) * {Ax, Ay, Az}.({jx, jy, jz} / n - {Vx, Vy, Vz}))
```

Out[•]=
$$-n\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz-\frac{1}{n}\left(jx(Ax\,sxx+Ay\,syx+Az\,szx)+jy(Ax\,sxy+Ay\,syy+Az\,szy)+jz(Ax\,sxz+Ay\,syz+Az\,szz)\right)-\left(Ax\left(\frac{jx}{n}-Vx\right)+Ay\left(\frac{jy}{n}-Vy\right)+Az\left(\frac{jz}{n}-Vz\right)\right)\left(n\epsilon+\frac{jx^2\rho}{2n}+\frac{jy^2\rho}{2n}+\frac{jz^2\rho}{2n}-nW\rho\right)\right)+O\left[\frac{1}{c}\right]^2$$

In[•]:=
```
showf[assut][Expand //@ FS@PowerExpand[fluxEPS[[2, 1]] - fluxE]]
```

Out[•]//MatrixForm=
$$O\left[\frac{1}{c}\right]^2$$

```
(* matter flux n A.(u-V) *)
shows[assut, 1][fluxNJ = Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.NJ /. replaceJu]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix}
n \\
n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)
\end{pmatrix}$$

```
(* content and flux of coord-energy and momentum assuming no matter flux (transposed) *)
shows[Join[assut, {((surface / Δt).NJ) == 0} /. replaceJu], 1][T@fluxEPS /. replaceJu]
```

Out[•]//MatrixForm=
$$\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+(ux^2+uy^2+uz^2-2W)\rho\right)+O\left[\frac{1}{c}\right]^2 & \left(-Ax(qx+sxx\,ux+sxy\,uy+sxz\,uz)-Ay(qy+syx\,ux+syy\,uy+syz\,uz)-Az(qz+szx\,ux+szy\,uy+szz\,uz)\right)+O\left[\frac{1}{c}\right]^2 \\
n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & (Ax\,sxx+Ay\,syx+Az\,szx)+O\left[\frac{1}{c}\right]^2 \\
n\,uy\,\rho+O\left[\frac{1}{c}\right]^2 & (Ax\,sxy+Ay\,syy+Az\,szy)+O\left[\frac{1}{c}\right]^2 \\
n\,uz\,\rho+O\left[\frac{1}{c}\right]^2 & (Ax\,sxz+Ay\,syz+Az\,szz)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[•]:=

```
(* coordinate/internal/coordinate-proper energy and x-momentum, content and fluxes (TRANSPOSED) *)
show2[assut, 1][T[variousfluxes = FS[({{1, 0, 0, 0}, surface / (Δt)}.EPS.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, Lxvec, Lxvec2, xboost / c, xboost2, uu, ntvec}]) /. replaceJu]]]
```

Out[•]//MatrixForm=
$$\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+(ux^2+uy^2+uz^2-2W)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac{1}{2}n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
n\,ux\,\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+n\,ux\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
n(uz\,y-uy\,z)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)(uz\,y-uy\,z)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
n(uz\,y-uy\,z)\rho+O\left[\frac{1}{c}\right]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z+n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)(uz\,y-uy\,z)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
-n(t\,ux+x)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left((Ax\,sxx+Ay\,syx+Az\,szx)t\right)-n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)(t\,ux+x)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
n(-t\,ux+x)\rho+O\left[\frac{1}{c}\right]^2 & \left(-\left((Ax\,sxx+Ay\,syx+Az\,szx)t\right)-n\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)(t\,ux-x)\rho\right)+O\left[\frac{1}{c}\right]^2 \\
-n\rho c^2-n\epsilon+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\epsilon\right)+O\left[\frac{1}{c}\right]^2 \\
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+(ux^2+uy^2+uz^2)\rho\right)+O\left[\frac{1}{c}\right]^2 & n\left(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n(ux-Vx)\epsilon\right)-Ay\left(qy+syx\,ux+syy\,uy+syz\,uz+n(uy-Vy)\epsilon\right)-Az\left(qz+szx\,ux+szy\,uy+szz\,uz+n(uz-Vz)\epsilon\right)-\frac{1}{2}n\left(ux^2+uy^2+uz^2\right)\left(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz)\right)\rho\right)+O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[ ]:=
```
show2[assut, 1][T[variousfluxes /. {Vy → 0, Vz → 0, uy → 0, uz → 0}]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2-2W\right)\rho\right)+O[\frac{1}{c}]^2 & Ax\,n\,(-ux+Vx)\,\rho c^2+\left(-Ay\left(qy+syx\,ux\right)-Az\left(qz+szx\,ux\right)-Ax\left(qx+sxx\,ux+n\,(ux-Vx)\,\epsilon\right)-\frac{1}{2}Ax\,n\,(ux-Vx)\left(ux^2-2W\right)\rho\right)+O[\frac{1}{c}]^2 \\
n\,ux\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,syx+Az\,szx+Ax\,n\,ux\,(ux-Vx)\,\rho\right)+O[\frac{1}{c}]^2 \\
O[\frac{1}{c}]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z\right)+O[\frac{1}{c}]^2 \\
O[\frac{1}{c}]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,szy\,z\right)+O[\frac{1}{c}]^2 \\
-n\left(t\,ux+x\right)\rho+O[\frac{1}{c}]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-Ax\,n\,(ux-Vx)\left(t\,ux+x\right)\rho\right)+O[\frac{1}{c}]^2 \\
n\left(-t\,ux+x\right)\rho+O[\frac{1}{c}]^2 & \left(-\left(\left(Ax\,sxx+Ay\,syx+Az\,szx\right)t\right)-Ax\,n\,(ux-Vx)\left(t\,ux-x\right)\rho\right)+O[\frac{1}{c}]^2 \\
-n\rho c^2-n\epsilon+O[\frac{1}{c}]^2 & Ax\,n\,(-ux+Vx)\,\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+Ax\,n\,(-ux+Vx)\,\epsilon\right)+O[\frac{1}{c}]^2 \\
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+ux^2\rho\right)+O[\frac{1}{c}]^2 & Ax\,n\,(-ux+Vx)\,\rho c^2+\left(-Ay\left(qy+syx\,ux\right)-Az\left(qz+szx\,ux\right)-Ax\left(qx+sxx\,ux+n\,(ux-Vx)\,\epsilon\right)-\frac{1}{2}Ax\,n\,ux^2\,(ux-Vx)\,\rho\right)+O[\frac{1}{c}]^2
\end{pmatrix}$$

In[ ]:=
```
(* velocity of energy *)
shows[assut, 5][(EPS.{1, 0, 0, 0})[[2 ;; 4]] / (EPS.{1, 0, 0, 0})[[1]] /. replaceJu]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
ux+\frac{qx+sxx\,ux+sxy\,uy+sxz\,uz}{n\rho c^2}+O[\frac{1}{c}]^4 \\
uy+\frac{qy+syx\,ux+syy\,uy+syz\,uz}{n\rho c^2}+O[\frac{1}{c}]^4 \\
uz+\frac{qz+szx\,ux+szy\,uy+szz\,uz}{n\rho c^2}+O[\frac{1}{c}]^4
\end{pmatrix}$$

In[ ]:=
```
temp = SeriesCoefficient[tt.{1, 0, 0, 0}, {c, Infinity, -2}];
shows[assut, 5][(tt.{1, 0, 0, 0} - temp * c^2)[[2 ;; 4]] / (tt.{1, 0, 0, 0} - temp * c^2)[[1]] /. j2v]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
\left(ux+\frac{2\left(qx+sxx\,ux+sxy\,uy+sxz\,uz\right)}{2n\epsilon+n\left(ux^2+uy^2+uz^2-2W\right)\rho}\right)+O[\frac{1}{c}]^2 \\
\left(uy+\frac{2\left(qy+syx\,ux+syy\,uy+syz\,uz\right)}{2n\epsilon+n\left(ux^2+uy^2+uz^2-2W\right)\rho}\right)+O[\frac{1}{c}]^2 \\
\left(uz+\frac{2\left(qz+szx\,ux+szy\,uy+szz\,uz\right)}{2n\epsilon+n\left(ux^2+uy^2+uz^2-2W\right)\rho}\right)+O[\frac{1}{c}]^2
\end{pmatrix}$$

In[ ]:=
```
showf[assut][{variousfluxes[[ ;; , 1]] - variousfluxes[[ ;; , 7]], variousfluxes[[ ;; , 1]] - variousfluxes[[ ;; , 8]], variousfluxes[[ ;; , 7]] - variousfluxes[[ ;; , 8]]}]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
\left(-\frac{1}{2}n\,ux^2\rho-\frac{1}{2}n\,uy^2\rho-\frac{1}{2}n\,uz^2\rho+n\,W\rho\right)+O[\frac{1}{c}]^2 & \left(-Ax\,sxx\,ux-Ay\,syx\,ux-Az\,szx\,ux-Ax\,sxy\,uy-Ay\,syy\,uy-Az\,szy\,uy-Ax\,sxz\,uz-Ay\,syz\,uz-Az\,szz\,uz-\frac{1}{2}Ax\,n\,ux^3\rho-\frac{1}{2}Ay\,n\,ux^2\,uy\,\rho-\frac{1}{2}Ax\,n\,ux\,uy^2\rho-\frac{1}{2}Ay\,n\,uy^3\rho-\frac{1}{2}Az\,n\,ux^2\,uz\,\rho-\frac{1}{2}Az\,n\,uy^2\,uz\,\rho-\frac{1}{2}Ax\,n\,ux\,uz^2\rho-\frac{1}{2}Ay\,n\,uy\,uz^2\rho-\frac{1}{2}Az\,n\,uz^3\rho+\frac{1}{2}Ax\,n\,ux^2\,Vx\,\rho+\frac{1}{2}Ax\,n\,uy^2\,Vx\,\rho+\frac{1}{2}Ax\,n\ldots \\
n\,W\rho+O[\frac{1}{c}]^2 & \left(Ax\,n\,ux\,W\rho+Ay\,n\,uy\,W\rho+Az\,n\,uz\,W\rho-Ax\,n\,Vx\,W\rho-Ay\,n\,Vy\,W\rho-Az\,n\,Vz\,W\rho\right)+O[\frac{1}{c}]^2 \\
\left(\frac{1}{2}n\,ux^2\rho+\frac{1}{2}n\,uy^2\rho+\frac{1}{2}n\,uz^2\rho\right)+O[\frac{1}{c}]^2 & \left(Ax\,sxx\,ux+Ay\,syx\,ux+Az\,szx\,ux+Ax\,sxy\,uy+Ay\,syy\,uy+Az\,szy\,uy+Ax\,sxz\,uz+Ay\,syz\,uz+Az\,szz\,uz+\frac{1}{2}Ax\,n\,ux^3\rho+\frac{1}{2}Ay\,n\,ux^2\,uy\,\rho+\frac{1}{2}Ax\,n\,ux\,uy^2\rho+\frac{1}{2}Ay\,n\,uy^3\rho+\frac{1}{2}Az\,n\,ux^2\,uz\,\rho+\frac{1}{2}Az\,n\,uy^2\,uz\,\rho+\frac{1}{2}Ax\,n\,ux\,uz^2\rho+\frac{1}{2}Ay\,n\,uy\,uz^2\rho+\frac{1}{2}Az\,n\,uz^3\rho-\frac{1}{2}Ax\,n\,ux^2\,Vx\,\rho-\frac{1}{2}Ax\,n\,uy^2\,Vx\,\rho-\frac{1}{2}Ax\,n\,u\ldots
\end{pmatrix}$$

In[ ]:=
```
(* supply terms *)
TTx = tW[tjv[(EPS + T[EPS.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
show2[assut, 2][FS[(itjv[Tr[#.TTx]]) /. replaceJu]] & /@ {Dxyzvec[[1,  ;; ,  ;;]], Dxyzvec[[2,  ;; ,  ;;]], DLxvec, DLxvec2, Dxboost / c, Dxboost2, Duv, Dntvec} // MF
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
n\rho W^{(1,0,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
n\rho W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O[\frac{1}{c}]^2 \\
n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O[\frac{1}{c}]^2 \\
-n\rho\left(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z]\right)+O[\frac{1}{c}]^2 \\
-n\,t\,\rho W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
\frac{1}{2}\left(2\,szz\,uz^{(0,0,0,1)}[t,x,y,z]+(syz+szy)\left(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\right)+2\left(syy\,uy^{(0,0,1,0)}[t,x,y,z]+sxx\,ux^{(0,1,0,0)}[t,x,y,z]\right)+(sxy+syx)\left(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z]\right)+(sxz+szx)\left(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\right)\right)+O[\frac{1}{c}]^2 \\
-n\rho\left(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z]\right)+O[\frac{1}{c}]^2
\end{pmatrix}$$

```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.EPSsym.T[{{1, 0, 0, 0}, {0, 1, 0, 0}, Lxvec, Lxvec2, xboost / c, xboost2, uu, ntvec}]) /. replaceJu]]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O[\frac{1}{c}]^2 & n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,(ux-Vx)\,\epsilon\right)-Ay\left(qy+sxy\,ux+syy\,uy+syz\,uz+n\,(uy-Vy)\,\epsilon\right)-Az\left(qz+sxz\,ux+syz\,uy+szz\,uz+n\,(uz-Vz)\,\epsilon\right)-\frac{1}{2}n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(ux^2+uy^2+uz^2-2W\right)\rho\right)+O[\frac{1}{c}]^2 \\
n\,ux\,\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxx+Ay\,sxy+Az\,sxz+n\,ux\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\rho\right)+O[\frac{1}{c}]^2 \\
n\left(uz\,y-uy\,z\right)\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,syz\,z+n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O[\frac{1}{c}]^2 \\
n\left(uz\,y-uy\,z\right)\rho+O[\frac{1}{c}]^2 & \left(Ax\,sxz\,y+Ay\,syz\,y+Az\,szz\,y-Ax\,sxy\,z-Ay\,syy\,z-Az\,syz\,z+n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(uz\,y-uy\,z\right)\rho\right)+O[\frac{1}{c}]^2 \\
-n\left(t\,ux+x\right)\rho+O[\frac{1}{c}]^2 & \left(-\left(\left(Ax\,sxx+Ay\,sxy+Az\,sxz\right)t\right)-n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(t\,ux+x\right)\rho\right)+O[\frac{1}{c}]^2 \\
n\left(-t\,ux+x\right)\rho+O[\frac{1}{c}]^2 & \left(-\left(\left(Ax\,sxx+Ay\,sxy+Az\,sxz\right)t\right)-n\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\left(t\,ux-x\right)\rho\right)+O[\frac{1}{c}]^2 \\
-n\rho c^2-n\epsilon+O[\frac{1}{c}]^2 & n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho c^2+\left(-Ax\,qx-Ay\,qy-Az\,qz+n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\epsilon\right)+O[\frac{1}{c}]^2 \\
-n\rho c^2-\frac{1}{2}n\left(2\epsilon+\left(ux^2+uy^2+uz^2\right)\rho\right)+O[\frac{1}{c}]^2 & n\left(Ax\,(-ux+Vx)+Ay\,(-uy+Vy)+Az\,(-uz+Vz)\right)\rho c^2+\left(-Ax\left(qx+sxx\,ux+sxy\,uy+sxz\,uz+n\,(ux-Vx)\,\epsilon\right)-Ay\left(qy+sxy\,ux+syy\,uy+syz\,uz+n\,(uy-Vy)\,\epsilon\right)-Az\left(qz+sxz\,ux+syz\,uy+szz\,uz+n\,(uz-Vz)\,\epsilon\right)-\frac{1}{2}n\left(ux^2+uy^2+uz^2\right)\left(Ax\,(ux-Vx)+Ay\,(uy-Vy)+Az\,(uz-Vz)\right)\rho\right)+O[\frac{1}{c}]^2
\end{pmatrix}$$

In[ ]:=
```
TTx = tW[tjv[(EPSsym + T[EPSsym.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. replaceJu] & /@ {Dxyzvec[[1,  ;; ,  ;;]], Dxyzvec[[2,  ;; ,  ;;]], DLxvec, DLxvec2, Dxboost / c, Dxboost2, Duv, Dntvec} // MF
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
n\rho W^{(1,0,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
n\rho W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O[\frac{1}{c}]^2 \\
n\rho\left(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z]\right)+O[\frac{1}{c}]^2 \\
-n\rho\left(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z]\right)+O[\frac{1}{c}]^2 \\
-n\,t\,\rho W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\
\left(szz\,uz^{(0,0,0,1)}[t,x,y,z]+syy\,uy^{(0,0,1,0)}[t,x,y,z]+syz\left(uy^{(0,0,0,1)}[t,x,y,z]+uz^{(0,0,1,0)}[t,x,y,z]\right)+sxx\,ux^{(0,1,0,0)}[t,x,y,z]+sxy\left(ux^{(0,0,1,0)}[t,x,y,z]+uy^{(0,1,0,0)}[t,x,y,z]\right)+sxz\left(ux^{(0,0,0,1)}[t,x,y,z]+uz^{(0,1,0,0)}[t,x,y,z]\right)\right)+O[\frac{1}{c}]^2 \\
-n\rho\left(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z]\right)+O[\frac{1}{c}]^2
\end{pmatrix}$$

In[ ]:=
```
(* 2-vector of surface parallel to yz  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(yzsurface = ((T[{{0, 0, Ly, 0}}].{{0, 0, 0, Lz}} - T[{{0, 0, 0, Lz}}].{{0, 0, Ly, 0}}) /. {Ly → Ayz / Lz})) // MF
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & Ayz \\
0 & 0 & -Ayz & 0
\end{pmatrix}$$

In[ ]:=
```
(* 2-vector of surface parallel to tx  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(txsurface = (-(T[{{Δt, 0, 0, 0}}].{{0, Lx, 0, 0}} - T[{{0, Lx, 0, 0}}].{{Δt, 0, 0, 0}}))) // MF
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & -Lx\,\Delta t & 0 & 0 \\
Lx\,\Delta t & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$$

In[ ]:=
```
(* 2-vector of surface parallel to ty  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(tysurface = (-(T[{{Δt, 0, 0, 0}}].{{0, 0, Ly, 0}} - T[{{0, 0, Ly, 0}}].{{Δt, 0, 0, 0}}))) // MF
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & -Ly\,\Delta t & 0 \\
0 & 0 & 0 & 0 \\
Ly\,\Delta t & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$$

In[ ]:=
```
(* 2-vector of surface parallel to y moving to x  surfacefx={-Vx*A*Δt,A*Δt,0,0};   *)
(yVxsurface = (-(T[{{1, Vx, 0, 0}} * Δt].{{0, 0, Ly, 0}} - T[{{0, 0, Ly, 0}}].{{1, Vx, 0, 0}} * Δt))) // MF
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & 0 & -Ly\,\Delta t & 0 \\
0 & 0 & -Ly\,Vx\,\Delta t & 0 \\
Ly\,\Delta t & Ly\,Vx\,\Delta t & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$$

In[ ]:=
```
(Tr[T[txsurface].txsurface]) // MF
```

Out[ ]//MatrixForm=

$$2\,Lx^2\,\Delta t^2$$

In[ ]:=
```
(* Faraday tensor *)
repE = {Ex → Ex * c * Sqrt[μ * ϵ], Ey → Ey * c * Sqrt[μ * ϵ], Ez → Ez * c * Sqrt[μ * ϵ]};
fftemp = {{0, -Ex, -Ey, -Ez}, {0, 0, Bz, -By}, {0, 0, 0, Bx}, {0, 0, 0, 0}} /. repE;
showf[assut][F = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & -c\,Ex\sqrt{\epsilon}\sqrt{\mu} & -c\,Ey\sqrt{\epsilon}\sqrt{\mu} & -c\,Ez\sqrt{\epsilon}\sqrt{\mu} \\
c\,Ex\sqrt{\epsilon}\sqrt{\mu} & 0 & Bz & -By \\
c\,Ey\sqrt{\epsilon}\sqrt{\mu} & -Bz & 0 & Bx \\
c\,Ez\sqrt{\epsilon}\sqrt{\mu} & By & -Bx & 0
\end{pmatrix}$$

In[ ]:=
```
(FS[{Tr[yzsurface.T[F]], Tr[T[txsurface].F], Tr[T[tysurface].F], Tr[T[yVxsurface].F]} / 2]) // MF
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
Ayz\,Bx \\
c\,Ex\,Lx\,\Delta t\sqrt{\epsilon}\sqrt{\mu} \\
c\,Ey\,Ly\,\Delta t\sqrt{\epsilon}\sqrt{\mu} \\
Ly\,\Delta t\left(-Bz\,Vx+c\,Ey\sqrt{\epsilon}\sqrt{\mu}\right)
\end{pmatrix}$$

In[ ]:=
```
(* charge-current-potential tensor *)
fftemp = {{0, -Hx, -Hy, -Hz}, {0, 0, Dz, -Dy}, {0, 0, 0, Dx}, {0, 0, 0, 0}};
showf[assut][H = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & -Hx & -Hy & -Hz \\
Hx & 0 & Dz & -Dy \\
Hy & -Dz & 0 & Dx \\
Hz & Dy & -Dx & 0
\end{pmatrix}$$

In[∘]:=
```
showf[assut][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
  ]]]
```

Out[∘]//MatrixForm=

(··· 1 ···)

Full expression not available (original memory size: 0.7 MB)

In[∘]:=
```
shows[assut, 1][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
  ]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & \frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{(-Bz\,Ex+Bx\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{(By\,Ex-Bx\,Ey)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ \frac{(-Bz\,Ey+By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(Bz\,Ey-By\,Ez)\,W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{-Bx^2+By^2+Bz^2+(-Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & -\frac{Bx\,By+Ex\,Ey\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & -\frac{Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 \\ \frac{(Bz\,Ex-Bx\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(-Bz\,Ex+Bx\,Ez)\,W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & -\frac{Bx\,By+Ex\,Ey\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & \frac{Bx^2-By^2+Bz^2+(Ex^2-Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & -\frac{By\,Bz+Ey\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 \\ \frac{(-By\,Ex+Bx\,Ey)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{2(By\,Ex-Bx\,Ey)\,W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & -\frac{Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & -\frac{By\,Bz+Ey\,Ez\,\epsilon0\,\mu0}{\mu0}+O[\frac{1}{c}]^2 & \frac{Bx^2+By^2-Bz^2+(Ex^2+Ey^2-Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[∘]:=
```
showf[assut][T[tte.Inverse[gg]].gg - tte]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} O[\frac{1}{c}]^4 & O[\frac{1}{c}]^5 & O[\frac{1}{c}]^5 & O[\frac{1}{c}]^5 \\ O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \\ O[\frac{1}{c}]^3 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 & O[\frac{1}{c}]^4 \end{pmatrix}$$

In[∘]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.tte.T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & \frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{(Ax\,Vx+Ay\,Vy+Az\,Vz)(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)}{2\,\mu0}+\frac{2(Az\,By\,Ex-Ay\,Bz\,Ex-Az\,Bx\,Ey+Ax\,Bz\,Ey+Ay\,Bx\,Ez-Ax\,By\,Ez)\,W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ -\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+\frac{(Bz\,Ey\,ux-By\,Ez\,ux-Bz\,Ex\,uy+Bx\,Ez\,uy+By\,Ex\,uz-Bx\,Ey\,uz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{Ax(-2\,Bx(By\,uy+Bz\,uz)+Bx^2(-ux+Vx)+(By^2+Bz^2)(ux+Vx)+(-2\,Ex(Ey\,uy+Ez\,uz)+Ex^2(-ux+Vx)+(Ey^2+Ez^2)(ux+Vx))\,\epsilon0\,\mu0)+Ay(-2\,Bx\,By\,ux-2\,By\,Bz\,uz+By^2(-uy+Vy)+Bx^2(uy+Vy)+Bz^2(uy+Vy)+(-2\,Ex\,Ey\,ux-2\,Ey\,Ez\,uz+Ey^2(-uy+Vy)+Ex^2(uy+Vy)+Ez^2(uy+Vy))\,\epsilon0\,\mu0)+Az(-2\,Bx\,Bz\,ux-2\,By\,Bz\,uy+Bz^2(-uz+Vz)+Bx^2 \cdots}{2\,\mu0} \\ -\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+O[\frac{1}{c}]^2 & \frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\frac{(Ax\,Vx+Ay\,Vy+Az\,Vz)(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)}{2\,\mu0}+\frac{(Az\,By\,Ex-Ay\,Bz\,Ex-Az\,Bx\,Ey+Ax\,Bz\,Ey+Ay\,Bx\,Ez-Ax\,By\,Ez)\,W\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ \frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{-2(Ay\,Bx\,By+Az\,Bx\,Bz+Ay\,Ex\,Ey\,\epsilon0\,\mu0+Az\,Ex\,Ez\,\epsilon0\,\mu0)+Ax(-Bx^2+By^2+Bz^2+(-Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)}{2\,\mu0}-\frac{(Bz\,Ey-By\,Ez)(Ax\,Vx+Ay\,Vy+Az\,Vz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ \frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+(-2(Ax\,Ex+Ay\,Ey)\,Ez\,y+Az(Ex^2+Ey^2-Ez^2)\,y+2\,Ey(Ax\,Ex+Az\,Ez)\,z-Ay(Ex^2-Ey^2+Ez^2)\,z)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{(Ax\,Vx+Ay\,Vy+Az\,Vz)(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ \frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+(-2(Ax\,Ex+Ay\,Ey)\,Ez\,y+Az(Ex^2+Ey^2-Ez^2)\,y+2\,Ey(Ax\,Ex+Az\,Ez)\,z-Ay(Ex^2-Ey^2+Ez^2)\,z)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{(Ax\,Vx+Ay\,Vy+Az\,Vz)(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ \frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{t(2(Ay\,Bx\,By+Az\,Bx\,Bz+Ay\,Ex\,Ey\,\epsilon0\,\mu0+Az\,Ex\,Ez\,\epsilon0\,\mu0)+Ax(Bx^2-By^2-Bz^2+(Ex^2-Ey^2-Ez^2)\,\epsilon0\,\mu0))}{2\,\mu0}+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\,x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ \frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{t(2(Ay\,Bx\,By+Az\,Bx\,Bz+Ay\,Ex\,Ey\,\epsilon0\,\mu0+Az\,Ex\,Ez\,\epsilon0\,\mu0)+Ax(Bx^2-By^2-Bz^2+(Ex^2-Ey^2-Ez^2)\,\epsilon0\,\mu0))}{2\,\mu0}+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(Az\,By\,Ex-Ay\,Bz\,Ex-Az\,Bx\,Ey+Ax\,Bz\,Ey+Ay\,Bx\,Ez-Ax\,By\,Ez)\,x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[∘]:=
```
TTx = tW[tjv[(tte + T[tte.Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)\,W^{(1,0,0,0)}[t,x,y,z]}{\mu0\,c^2}+O[\frac{1}{c}]^3 \\ \frac{-2(Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0)\,ux^{(0,0,0,1)}[t,x,y,z]-2(By\,Bz+Ey\,Ez\,\epsilon0\,\mu0)\,uy^{(0,0,0,1)}[t,x,y,z]+Bx^2\,uz^{(0,0,0,1)}[t,x,y,z]+By^2\,uz^{(0,0,0,1)}[t,x,y,z]-Bz^2\,uz^{(0,0,0,1)}[t,x,y,z]+Ex^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ey^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-Ez^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-2\,Bx\,By\,ux^{(0,0,1,0)}[t,x,y,z]-2\,Ex\,Ey\,\epsilon0\,\mu0\,ux^{(0,0,1,0)}[t,x,y,z]+Bx^2\,uy^{(0,0,1,0)}[t,x,y,z]-By^2\,uy^{(0,0,1,0)}[t,x,y,z]+Bz^2\,uy^{(0,0,1,0)}[t,x,y,z]+Ex^2\,\epsilon0\,\mu0\,uy^{(0,0,1,0)}[t,x,y,z]-Ey^2\,\epsilon0\,\mu0\,uy^{(0,0,1,0)}[t,x,y,z]+Ez^2\,\epsilon0\,\mu0\,uy^{(0,0,1,0)}[t,x,y,z]-2\,By\,Bz\,uz^{(0,0,1,0)}[t,x \cdots}{2\,\mu0} \\ \frac{\sqrt{\epsilon0}((-By\,Ex+Bx\,Ey)\,W^{(0,0,0,1)}[t,x,y,z]+(Bz\,Ex-Bx\,Ez)\,W^{(0,0,1,0)}[t,x,y,z]+(-Bz\,Ey+By\,Ez)\,W^{(0,1,0,0)}[t,x,y,z])}{\sqrt{\mu0}\,c}+\frac{(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)\,W^{(1,0,0,0)}[t,x,y,z]}{2\,\mu0\,c^2}+O[\frac{1}{c}]^3 \\ \frac{(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)\,W^{(0,1,0,0)}[t,x,y,z]}{\mu0\,c^2}+O[\frac{1}{c}]^3 \\ \frac{(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z])}{\mu0\,c^2}+O[\frac{1}{c}]^3 \\ \frac{2((Bz^2\,y-By\,Bz\,z+Ez(Ez\,y-Ey\,z)\,\epsilon0\,\mu0)\,W^{(0,0,0,1)}[t,x,y,z]+(By\,Bz\,y-By^2\,z+Ey(Ez\,y-Ey\,z)\,\epsilon0\,\mu0)\,W^{(0,0,1,0)}[t,x,y,z]+(Bx(Bz\,y-By\,z)+Ex(Ez\,y-Ey\,z)\,\epsilon0\,\mu0)\,W^{(0,1,0,0)}[t,x,y,z])}{\mu0\,c^2}+O[\frac{1}{c}]^3 \\ \frac{2(-Bz\,Ey+By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}-\frac{t(Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0)\,W^{(0,1,0,0)}[t,x,y,z]}{\mu0\,c^2}+O[\frac{1}{c}]^3 \\ -\frac{2(t((Bx\,Bz+Ex\,Ez\,\epsilon0\,\mu0)\,W^{(0,0,0,1)}[t,x,y,z]+(Bx\,By+Ex\,Ey\,\epsilon0\,\mu0)\,W^{(0,0,1,0)}[t,x,y,z]+(Bx^2+Ex^2\,\epsilon0\,\mu0)\,W^{(0,1,0,0)}[t,x,y,z]))}{\mu0\,c^2}+O[\frac{1}{c}]^3 \end{pmatrix}$$

In[∘]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.(tte + ttsym).T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+\left(-n\,\epsilon-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{1}{2}\,n\,(ux^2+uy^2+uz^2-2\,W)\,\rho\right)+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\,\rho\,c^2+\frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\cdots \\ -n\,\rho\,c^2-\frac{Bx^2+By^2+Bz^2+2\,n\,\epsilon\,\mu0+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}+\frac{(Bz\,Ey\,ux-By\,Ez\,ux-Bz\,Ex\,uy+Bx\,Ez\,uy+By\,Ex\,uz-Bx\,Ey\,uz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\,\rho\,c^2+\frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\cdots \\ -n\,\rho\,c^2+\left(-n\,\epsilon-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{1}{2}\,n\,(ux^2+uy^2+uz^2)\,\rho\right)+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\,\rho\,c^2+\frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\cdots \\ n\,ux\,\rho+\frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{-2\,Ay(Bx\,By-sxy\,\mu0+Ex\,Ey\,\epsilon0\,\mu0+n\,ux(-uy+Vy)\,\mu0\,\rho)+Ax(-Bx^2+By^2+Bz^2+\mu0(2\,sxx+(-Ex^2+Ey^2+Ez^2)\,\epsilon0+2\,n\,ux(ux-Vx)\,\rho))+2\,Az(-Bx\,Bz+\mu0(sxz-Ex\,Ez\,\epsilon0+n\,ux(uz-Vz)\,\rho))}{2\,\mu0}-\frac{(Bz\,Ey-By\,Ez)(Ax\,Vx+Ay\,Vy+Az\,Vz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ n(uz\,y-uy\,z)\,\rho+\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+2\,Ax\,sxz\,y\,\mu0+2\,Ay\,syz\,y\,\mu0+2\,Az\,szz\,y\,\mu0-2\,Ax\,sxy\,z\,\mu0-2\,Ay\,syy\,z\,\mu0-2\,Az\,syz\,z\,\mu0+Az\,Ex^2\,y\,\epsilon0\,\mu0+Az\,Ey^2\,y\,\epsilon0\,\mu0-2\,Ax\,Ex\,Ez\,y\,\epsilon0\,\mu0-2\,Ay\,Ey\,Ez\,y\,\epsilon0\,\mu0-Az\,Ez^2\,y\,\epsilon0\,\mu0-Ay\,Ex^2\,z\,\epsilon0\,\mu0+2\,Ax\,Ex\,Ey\,z\,\epsilon0\,\mu0+Ay\,Ey^2\,z\,\epsilon0\,\mu0+2\,Az\,Ey\,Ez\,z\,\epsilon0\,\mu0-Ay\,Ez^2\,z\,\epsilon0\cdots}{2\,\mu0} \\ n(uz\,y-uy\,z)\,\rho+\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+2\,Ax\,sxz\,y\,\mu0+2\,Ay\,syz\,y\,\mu0+2\,Az\,szz\,y\,\mu0-2\,Ax\,sxy\,z\,\mu0-2\,Ay\,syy\,z\,\mu0-2\,Az\,syz\,z\,\mu0+Az\,Ex^2\,y\,\epsilon0\,\mu0+Az\,Ey^2\,y\,\epsilon0\,\mu0-2\,Ax\,Ex\,Ez\,y\,\epsilon0\,\mu0-2\,Ay\,Ey\,Ez\,y\,\epsilon0\,\mu0-Az\,Ez^2\,y\,\epsilon0\,\mu0-Ay\,Ex^2\,z\,\epsilon0\,\mu0+2\,Ax\,Ex\,Ey\,z\,\epsilon0\,\mu0+Ay\,Ey^2\,z\,\epsilon0\,\mu0+2\,Az\,Ey\,Ez\,z\,\epsilon0\,\mu0-Ay\,Ez^2\,z\,\epsilon0\cdots}{2\,\mu0} \\ -n(t\,ux+x)\,\rho+\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \left(\frac{t(Ax\,Bx^2+2\,Ay\,Bx\,By-Ax\,By^2+2\,Az\,Bx\,Bz-Ax\,Bz^2-(2\,Ax\,sxx+2\,Ay\,sxy+2\,Az\,sxz-2\,Ex(Ay\,Ey+Az\,Ez)\,\epsilon0+Ax(-Ex^2+Ey^2+Ez^2)\,\epsilon0)\,\mu0)}{2\,\mu0}-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux+x)\,\rho\right)+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\,x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ n(-t\,ux+x)\,\rho+\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \left(\frac{t(Ax\,Bx^2+2\,Ay\,Bx\,By-Ax\,By^2+2\,Az\,Bx\,Bz-Ax\,Bz^2-(2\,Ax\,sxx+2\,Ay\,sxy+2\,Az\,sxz-2\,Ex(Ay\,Ey+Az\,Ez)\,\epsilon0+Ax(-Ex^2+Ey^2+Ez^2)\,\epsilon0)\,\mu0)}{2\,\mu0}-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux-x)\,\rho\right)+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(Az\,By\,Ex-Ay\,Bz\,Ex-Az\,Bx\,Ey+Ax\,Bz\,Ey+Ay\,Bx\,Ez-Ax\,By\,Ez)\,x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[∘]:=
```
TTx = tW[tjv[((tte + ttsym) + T[(tte + ttsym).Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 2][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ \frac{-2(Bx\,Bz-sxz\,\mu0+Ex\,Ez\,\epsilon0\,\mu0)\,ux^{(0,0,0,1)}[t,x,y,z]-2(By\,Bz-syz\,\mu0+Ey\,Ez\,\epsilon0\,\mu0)\,uy^{(0,0,0,1)}[t,x,y,z]+Bx^2\,uz^{(0,0,0,1)}[t,x,y,z]+By^2\,uz^{(0,0,0,1)}[t,x,y,z]-Bz^2\,uz^{(0,0,0,1)}[t,x,y,z]+2\,szz\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ex^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ey^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-Ez^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-2\,Bx\,By\,ux^{(0,0,1,0)}[t,x,y,z]+2\,sxy\,\mu0\,ux^{(0,0,1,0)}[t,x,y,z]-2\,Ex\,Ey\,\epsilon0\,\mu0\,ux^{(0,0,1,0)}[t,x,y,z]+Bx^2\,uy^{(0,0,1,0)}[t,x,y,z]-By^2\,uy^{(0,0,1,0)}[t,x,y,z]+Bz^2\,uy^{(0,0,1,0)}[t,x,y,z]+2\,syy\,\mu0\,uy^{(0,0,1,0)}[t,x,y,z]+Ex^2\,\epsilon0 \cdots}{2\,\mu0} \\ -n\,\rho(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z])+\frac{\sqrt{\epsilon0}((-By\,Ex+Bx\,Ey)\,W^{(0,0,0,1)}[t,x,y,z]+(Bz\,Ex-Bx\,Ez)\,W^{(0,0,1,0)}[t,x,y,z]+(-Bz\,Ey+By\,Ez)\,W^{(0,1,0,0)}[t,x,y,z])}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ n\,\rho(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z])+O[\frac{1}{c}]^2 \\ n\,\rho(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z])+O[\frac{1}{c}]^2 \\ -n\,\rho(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z])+\frac{2(-Bz\,Ey+By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ -n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[∘]:=
```
shows[assut, 1][T[Expand //@ FS@PowerExpand[({{1, 0, 0, 0}, surface / (Δt)}.(tt + tte).T[{{1, 0, 0, 0}, uu, vtn, {0, 1, 0, 0}, Lx, L2x, box / c, bo2x}]) /. j2v]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2+\left(-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{1}{2}\,n\,(ux^2+uy^2+uz^2-2\,W+2\,\epsilon)\,\rho\right)+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\,\rho\,c^2+\frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\cdots \\ -n\,\rho\,c^2-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0+2\,n\,\epsilon\,\mu0\,\rho}{2\,\mu0}+\frac{(Bz\,Ey\,ux-By\,Ez\,ux-Bz\,Ex\,uy+Bx\,Ez\,uy+By\,Ex\,uz-Bx\,Ey\,uz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\,\rho\,c^2+\frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\cdots \\ -n\,\rho\,c^2+\left(-\frac{Bx^2+By^2+Bz^2+(Ex^2+Ey^2+Ez^2)\,\epsilon0\,\mu0}{2\,\mu0}-\frac{1}{2}\,n\,(ux^2+uy^2+uz^2+2\,\epsilon)\,\rho\right)+O[\frac{1}{c}]^2 & n(Ax(-ux+Vx)+Ay(-uy+Vy)+Az(-uz+Vz))\,\rho\,c^2+\frac{(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\sqrt{\epsilon0}\,c}{\sqrt{\mu0}}+\cdots \\ n\,ux\,\rho+\frac{(Bz\,Ey-By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{-2\,Ay(Bx\,By-syx\,\mu0+Ex\,Ey\,\epsilon0\,\mu0+n\,ux(-uy+Vy)\,\mu0\,\rho)+Ax(-Bx^2+By^2+Bz^2+\mu0(2\,sxx+(-Ex^2+Ey^2+Ez^2)\,\epsilon0+2\,n\,ux(ux-Vx)\,\rho))+2\,Az(-Bx\,Bz+\mu0(szx-Ex\,Ez\,\epsilon0+n\,ux(uz-Vz)\,\rho))}{2\,\mu0}-\frac{(Bz\,Ey-By\,Ez)(Ax\,Vx+Ay\,Vy+Az\,Vz)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ n(uz\,y-uy\,z)\,\rho+\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+2\,Ax\,sxz\,y\,\mu0+2\,Ay\,syz\,y\,\mu0+2\,Az\,szz\,y\,\mu0-2\,Ax\,sxy\,z\,\mu0-2\,Ay\,syy\,z\,\mu0-2\,Az\,syz\,z\,\mu0+Az\,Ex^2\,y\,\epsilon0\,\mu0+Az\,Ey^2\,y\,\epsilon0\,\mu0-2\,Ax\,Ex\,Ez\,y\,\epsilon0\,\mu0-2\,Ay\,Ey\,Ez\,y\,\epsilon0\,\mu0-Az\,Ez^2\,y\,\epsilon0\,\mu0-Ay\,Ex^2\,z\,\epsilon0\,\mu0+2\,Ax\,Ex\,Ey\,z\,\epsilon0\,\mu0+Ay\,Ey^2\,z\,\epsilon0\,\mu0+2\,Az\,Ey\,Ez\,z\,\epsilon0\,\mu0-Ay\,Ez^2\,z\,\epsilon0\cdots}{2\,\mu0} \\ n(uz\,y-uy\,z)\,\rho+\frac{(By\,Ex\,y+Bz\,Ex\,z-Bx(Ey\,y+Ez\,z))\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \frac{Az\,Bx^2\,y+Az\,By^2\,y-2\,Ax\,Bx\,Bz\,y-2\,Ay\,By\,Bz\,y-Az\,Bz^2\,y-Ay\,Bx^2\,z+2\,Ax\,Bx\,By\,z+Ay\,By^2\,z+2\,Az\,By\,Bz\,z-Ay\,Bz^2\,z+2\,Ax\,sxz\,y\,\mu0+2\,Ay\,syz\,y\,\mu0+2\,Az\,szz\,y\,\mu0-2\,Ax\,sxy\,z\,\mu0-2\,Ay\,syy\,z\,\mu0-2\,Az\,syz\,z\,\mu0+Az\,Ex^2\,y\,\epsilon0\,\mu0+Az\,Ey^2\,y\,\epsilon0\,\mu0-2\,Ax\,Ex\,Ez\,y\,\epsilon0\,\mu0-2\,Ay\,Ey\,Ez\,y\,\epsilon0\,\mu0-Az\,Ez^2\,y\,\epsilon0\,\mu0-Ay\,Ex^2\,z\,\epsilon0\,\mu0+2\,Ax\,Ex\,Ey\,z\,\epsilon0\,\mu0+Ay\,Ey^2\,z\,\epsilon0\,\mu0+2\,Az\,Ey\,Ez\,z\,\epsilon0\,\mu0-Ay\,Ez^2\,z\,\epsilon0\cdots}{2\,\mu0} \\ -n(t\,ux+x)\,\rho+\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \left(\frac{t(Ax\,Bx^2+2\,Ay\,Bx\,By-Ax\,By^2+2\,Az\,Bx\,Bz-Ax\,Bz^2-(2\,Ax\,sxx+2\,Ay\,syx+2\,Az\,szx-2\,Ex(Ay\,Ey+Az\,Ez)\,\epsilon0+Ax(-Ex^2+Ey^2+Ez^2)\,\epsilon0)\,\mu0)}{2\,\mu0}-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux+x)\,\rho\right)+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(-Az\,By\,Ex+Ay\,Bz\,Ex+Az\,Bx\,Ey-Ax\,Bz\,Ey-Ay\,Bx\,Ez+Ax\,By\,Ez)\,x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ n(-t\,ux+x)\,\rho+\frac{(-Bz\,Ey+By\,Ez)\,t\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 & \left(\frac{t(Ax\,Bx^2+2\,Ay\,Bx\,By-Ax\,By^2+2\,Az\,Bx\,Bz-Ax\,Bz^2-(2\,Ax\,sxx+2\,Ay\,syx+2\,Az\,szx-2\,Ex(Ay\,Ey+Az\,Ez)\,\epsilon0+Ax(-Ex^2+Ey^2+Ez^2)\,\epsilon0)\,\mu0)}{2\,\mu0}-n(Ax(ux-Vx)+Ay(uy-Vy)+Az(uz-Vz))(t\,ux-x)\,\rho\right)+\frac{((Bz\,Ey-By\,Ez)\,t(Ax\,Vx+Ay\,Vy+Az\,Vz)+(Az\,By\,Ex-Ay\,Bz\,Ex-Az\,Bx\,Ey+Ax\,Bz\,Ey+Ay\,Bx\,Ez-Ax\,By\,Ez)\,x)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \end{pmatrix}$$

In[∘]:=
```
TTx = tW[tjv[((tt + tte) + T[(tt + tte).Inverse[gg]].gg) / 2]];(*showf[assut][Table[Expand//@FS@PowerExpand[Tr[1/2*(Inverse[gg].T[Dcoords[[aa,;;,;;]]].gg+Dcoords[[aa,;;,;;]]).TTx]],{aa,1,4}]]*)
shows[assut, 4][Expand //@ FS@PowerExpand[itjv[Tr[#.TTx]]] /. j2v] & /@ {Dcoords[[1, ;; , ;;]], Duv, Dvtn, Dcoords[[2, ;; , ;;]], DLx, DL2x, Dbox / c, Dbo2x} // MF
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} n\,\rho\,W^{(1,0,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ \frac{(-2\,Bx\,Bz+(sxz+szx-2\,Ex\,Ez\,\epsilon0)\,\mu0)\,ux^{(0,0,0,1)}[t,x,y,z]+(-2\,By\,Bz+(syz+szy-2\,Ey\,Ez\,\epsilon0)\,\mu0)\,uy^{(0,0,0,1)}[t,x,y,z]+Bx^2\,uz^{(0,0,0,1)}[t,x,y,z]+By^2\,uz^{(0,0,0,1)}[t,x,y,z]-Bz^2\,uz^{(0,0,0,1)}[t,x,y,z]+2\,szz\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ex^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]+Ey^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-Ez^2\,\epsilon0\,\mu0\,uz^{(0,0,0,1)}[t,x,y,z]-2\,Bx\,By\,ux^{(0,0,1,0)}[t,x,y,z]+sxy\,\mu0\,ux^{(0,0,1,0)}[t,x,y,z]+syx\,\mu0\,ux^{(0,0,1,0)}[t,x,y,z]-2\,Ex\,Ey\,\epsilon0\,\mu0\,ux^{(0,0,1,0)}[t,x,y,z]+Bx^2\,uy^{(0,0,1,0)}[t,x,y,z]-By^2\,uy^{(0,0,1,0)}[t,x,y,z]+Bz^2\,uy^{(0,0,1,0)}[t,x,y,z]\cdots}{2\,\mu0} \\ -n\,\rho(uz\,W^{(0,0,0,1)}[t,x,y,z]+uy\,W^{(0,0,1,0)}[t,x,y,z]+ux\,W^{(0,1,0,0)}[t,x,y,z])+\frac{\sqrt{\epsilon0}((-By\,Ex+Bx\,Ey)\,W^{(0,0,0,1)}[t,x,y,z]+(Bz\,Ex-Bx\,Ez)\,W^{(0,0,1,0)}[t,x,y,z]+(-Bz\,Ey+By\,Ez)\,W^{(0,1,0,0)}[t,x,y,z])}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ n\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \\ n\,\rho(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z])+O[\frac{1}{c}]^2 \\ n\,\rho(y\,W^{(0,0,0,1)}[t,x,y,z]-z\,W^{(0,0,1,0)}[t,x,y,z])+O[\frac{1}{c}]^2 \\ -n\,\rho(2\,ux+t\,W^{(0,1,0,0)}[t,x,y,z])+\frac{2(-Bz\,Ey+By\,Ez)\sqrt{\epsilon0}}{\sqrt{\mu0}\,c}+O[\frac{1}{c}]^2 \\ -n\,t\,\rho\,W^{(0,1,0,0)}[t,x,y,z]+O[\frac{1}{c}]^2 \end{pmatrix}$$

```
(* Flux of energy is given by: *)
```

In[•]:= `shows[assut, 1][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surface / (Δt)}.tt /. {Ax → 0, Ay → 0, Vz → 0, jz → 0} /. j2v]]`

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2} n\left(ux^2+uy^2-2W+2\epsilon\right)\rho + O\left[\frac{1}{c}\right]^2 & n\,ux\,\rho + O\left[\frac{1}{c}\right]^2 & n\,uy\,\rho + O\left[\frac{1}{c}\right]^2 & O\left[\frac{1}{c}\right]^2 \\ -Az\left(qz + sxz\,ux + syz\,uy\right) + O\left[\frac{1}{c}\right]^2 & Az\,sxz + O\left[\frac{1}{c}\right]^2 & Az\,syz + O\left[\frac{1}{c}\right]^2 & Az\,szz + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:= `shows[assut, 1][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → 0, Vx → 0} /. j2v]]`

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2} n\left(uy^2+uz^2-2W+2\epsilon\right)\rho + O\left[\frac{1}{c}\right]^2 & O\left[\frac{1}{c}\right]^2 & n\,uy\,\rho + O\left[\frac{1}{c}\right]^2 & n\,uz\,\rho + O\left[\frac{1}{c}\right]^2 \\ \left(-qx - sxy\,uy - sxz\,uz\right) + O\left[\frac{1}{c}\right]^2 & sxx + O\left[\frac{1}{c}\right]^2 & sxy + O\left[\frac{1}{c}\right]^2 & sxz + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:= `shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v]]`

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2} n\left(ux^2+uy^2+uz^2-2W+2\epsilon\right)\rho + O\left[\frac{1}{c}\right]^2 & n\,ux\,\rho + \frac{qx+sxx\,ux+sxy\,uy+sxz\,uz+\frac{1}{2}n\left(-8\,Wx+ux\left(ux^2+uy^2+uz^2+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & n\,uy\,\rho + \frac{qy+sxy\,ux+syy\,uy+syz\,uz+\frac{1}{2}n\left(-8\,Wy+uy\left(ux^2+uy^2+uz^2+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & n\,uz\,\rho + \frac{qz+sxz\,ux+syz\,uy+szz\,uz+\frac{1}{2}n\left(-8\,Wz+uz\left(ux^2+uy^2+uz^2+6W+2\epsilon\right)\right)\rho \ldots}{} \\ n(-ux+Vx)\rho c^2 + \left(-qx - sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2} n(ux-Vx)\left(ux^2+uy^2+uz^2-2W+2\epsilon\right)\rho\right) + O\left[\frac{1}{c}\right]^2 & \left(sxx + n\,ux(ux-Vx)\rho\right) + \frac{qx(2\,ux-Vx)-\left(sxx\,ux+sxy\,uy+sxz\,uz\right)Vx+\frac{1}{2}n(ux-Vx)\left(ux^3-8\,Wx+ux\left(uy^2+uz^2+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & \left(sxy + n\,uy(ux-Vx)\rho\right) + \frac{qx\,uy+qy(ux-Vx)-\left(sxy\,ux+syy\,uy+syz\,uz\right)Vx+\frac{1}{2}n(ux-Vx)\left(-8\,Wy+uy\left(ux^2+uy^2+uz^2+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & \left(sxz + n\,uz(ux-Vx)\rho\right) + \frac{qx\,uz+qz(ux-Vx)-\left(sxz\,ux+syz\,uy+szz\,uz\right) \ldots}{} \end{pmatrix}$$

In[•]:= `shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v /. {sxx → SXX - n * ux * (ux - Vx) * ρ, sxy → SXY - n * uy * (ux - Vx) * ρ, sxz → SXZ - n * uz * (ux - Vx) * ρ}]]`

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2} n\left(ux^2+uy^2+uz^2-2W+2\epsilon\right)\rho + O\left[\frac{1}{c}\right]^2 & n\,ux\,\rho + \frac{qx+SXX\,ux+SXY\,uy+SXZ\,uz-\frac{1}{2}n\left(ux^3-2\,ux^2\,Vx-2\left(uy^2+uz^2\right)Vx+8\,Wx+ux\left(uy^2+uz^2-6W-2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & n\,uy\,\rho + \frac{qy+SXY\,ux+syy\,uy+syz\,uz+\frac{1}{2}n\left(-8\,Wy+uy\left(-ux^2+uy^2+uz^2+2\,ux\,Vx+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & n\,uz\,\rho + \frac{qz+SXZ\,ux+syz\,uy+szz\,uz+\frac{1}{2}n\left(-8\,Wz+uz\left(-ux^2+uy^2+uz^2+2\,ux\,Vx+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\ldots\right. \\ n(-ux+Vx)\rho c^2 + \left(-qx - SXX\,ux - SXY\,uy - SXZ\,uz + \frac{1}{2} n(ux-Vx)\left(ux^2+uy^2+uz^2+2W-2\epsilon\right)\rho\right) + O\left[\frac{1}{c}\right]^2 & SXX + \frac{qx(2\,ux-Vx)-\left(SXX\,ux+SXY\,uy+SXZ\,uz\right)Vx+\frac{1}{2}n(ux-Vx)\left(ux^3+2\,ux^2\,Vx+2\left(uy^2+uz^2\right)Vx-8\,Wx+ux\left(uy^2+uz^2+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & SXY + \frac{qx\,uy+qy(ux-Vx)-\left(SXY\,ux+syy\,uy+syz\,uz\right)Vx+\frac{1}{2}n(ux-Vx)\left(-8\,Wy+uy\left(ux^2+uy^2+uz^2+2\,ux\,Vx+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & SXZ + \frac{qx\,uz+qz(ux-Vx)-\left(SXZ\,ux+syz\,uy+szz\,uz\right)Vx+\frac{1}{2}n(ux-Vx)\left(-8\,Wz+uz\left(ux^2+uy^2+uz^2+2\,ux \ldots\right.\right.}{c^2} \end{pmatrix}$$

In[•]:= `(* matter flux in same direction as imaginary moving surface, different velocity *)`
`shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jy → 0, jz → 0} /. j2v]]`

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2} n\left(ux^2-2W+2\epsilon\right)\rho + O\left[\frac{1}{c}\right]^2 & n\,ux\,\rho + \frac{qx+sxx\,ux+\frac{1}{2}n\left(ux^3+6\,ux\,W-8\,Wx+2\,ux\,\epsilon\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & \frac{qy+sxy\,ux-4\,n\,Wy\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & \frac{qz+sxz\,ux-4\,n\,Wz\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 \\ n(-ux+Vx)\rho c^2 + \left(-qx - sxx\,ux - \frac{1}{2} n(ux-Vx)\left(ux^2-2W+2\epsilon\right)\rho\right) + O\left[\frac{1}{c}\right]^2 & \left(sxx + n\,ux(ux-Vx)\rho\right) + \frac{2\,qx\,ux-\left(qx+sxx\,ux\right)Vx+\frac{1}{2}n(ux-Vx)\left(ux^3-8\,Wx+2\,ux(3W+\epsilon)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & sxy + \frac{qy(ux-Vx)-sxy\,ux\,Vx+4\,n(-ux+Vx)\,Wy\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & sxz + \frac{qz(ux-Vx)-sxz\,ux\,Vx+4\,n(-ux+Vx)\,Wz\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:= `(* imaginary moving surface, no matter flux through it *)`
`shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. j2v /. {ux → Vx}]]`

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2} n\left(uy^2+uz^2+Vx^2-2W+2\epsilon\right)\rho + O\left[\frac{1}{c}\right]^2 & n\,Vx\,\rho + \frac{qx+sxy\,uy+sxz\,uz+sxx\,Vx+\frac{1}{2}n\left(-8\,Wx+Vx\left(uy^2+uz^2+Vx^2+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & n\,uy\,\rho + \frac{qy+syy\,uy+syz\,uz+sxy\,Vx+\frac{1}{2}n\left(-8\,Wy+uy\left(uy^2+uz^2+Vx^2+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & n\,uz\,\rho + \frac{qz+syz\,uy+szz\,uz+sxz\,Vx+\frac{1}{2}n\left(-8\,Wz+uz\left(uy^2+uz^2+Vx^2+6W+2\epsilon\right)\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 \\ \left(-qx - sxy\,uy - sxz\,uz - sxx\,Vx\right) + O\left[\frac{1}{c}\right]^2 & sxx - \frac{Vx\left(-qx+sxy\,uy+sxz\,uz+sxx\,Vx\right)}{c^2} + O\left[\frac{1}{c}\right]^3 & sxy + \frac{qx\,uy-Vx\left(syy\,uy+syz\,uz+sxy\,Vx\right)}{c^2} + O\left[\frac{1}{c}\right]^3 & sxz + \frac{qx\,uz-Vx\left(syz\,uy+szz\,uz+sxz\,Vx\right)}{c^2} + O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

`(* imaginary moving surface, no matter flux through it and no transversal matter motion *)`
`shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jy → 0, jz → 0} /. j2v /. {ux → Vx}]]`

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - \frac{1}{2} n\left(Vx^2-2W+2\epsilon\right)\rho + O\left[\frac{1}{c}\right]^2 & n\,Vx\,\rho + \frac{qx+sxx\,Vx+\frac{1}{2}n\left(Vx^3+6\,Vx\,W-8\,Wx+2\,Vx\,\epsilon\right)\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & \frac{qy+sxy\,Vx-4\,n\,Wy\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & \frac{qz+sxz\,Vx-4\,n\,Wz\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 \\ \left(-qx - sxx\,Vx\right) + O\left[\frac{1}{c}\right]^2 & sxx + \frac{Vx\left(qx-sxx\,Vx\right)}{c^2} + O\left[\frac{1}{c}\right]^3 & sxy - \frac{sxy\,Vx^2}{c^2} + O\left[\frac{1}{c}\right]^3 & sxz - \frac{sxz\,Vx^2}{c^2} + O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:= `(* imaginary moving surface, matter at rest in coordinates *)`
`shows[assut, 2][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → 0, jy → 0, jz → 0}]]`

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 + n(W-\epsilon)\rho + O\left[\frac{1}{c}\right]^2 & \frac{qx-4\,n\,Wx\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & \frac{qy-4\,n\,Wy\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & \frac{qz-4\,n\,Wz\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 \\ n\,Vx\,\rho c^2 + \left(-qx + n\,Vx(-W+\epsilon)\rho\right) + O\left[\frac{1}{c}\right]^2 & sxx + \frac{-qx\,Vx+4\,n\,Vx\,Wx\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & sxy + \frac{-qy\,Vx+4\,n\,Vx\,Wy\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 & sxz + \frac{-qz\,Vx+4\,n\,Vx\,Wz\,\rho}{c^2} + O\left[\frac{1}{c}\right]^3 \end{pmatrix}$$

In[•]:= `showf[assut][Expand //@ FS@PowerExpand[{{1, 0, 0, 0}, surfacefx / (A * Δt)}.tt /. {jx → n * Vx, jy → 0, jz → 0}]]`

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 + \left(-\frac{1}{2} n\,Vx^2\rho + n\,W\rho - n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 & n\,Vx\,\rho + \frac{qx+sxx\,Vx+\frac{1}{2}n\,Vx^3\rho+3\,n\,Vx\,W\rho-4\,n\,Wx\,\rho+n\,Vx\,\epsilon\,\rho}{c^2} + O\left[\frac{1}{c}\right]^4 & \frac{qy+sxy\,Vx-4\,n\,Wy\,\rho}{c^2} + O\left[\frac{1}{c}\right]^4 & \frac{qz+sxz\,Vx-4\,n\,Wz\,\rho}{c^2} + O\left[\frac{1}{c}\right]^4 \\ \left(-qx - sxx\,Vx\right) + O\left[\frac{1}{c}\right]^2 & sxx + \frac{qx\,Vx-sxx\,Vx^2}{c^2} + O\left[\frac{1}{c}\right]^4 & sxy - \frac{sxy\,Vx^2}{c^2} + O\left[\frac{1}{c}\right]^4 & sxz - \frac{sxz\,Vx^2}{c^2} + O\left[\frac{1}{c}\right]^4 \end{pmatrix}$$

In[•]:= `(* COORDINATE ENERGY *)`

In[•]:= `(* energy 3-form when projected along coord. axes *)`
`showf[assut][Expand //@ FS@PowerExpand[tt.{1, 0, 0, 0}]]`

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 + \left(-\frac{jx^2\rho}{2n} - \frac{jy^2\rho}{2n} - \frac{jz^2\rho}{2n} + n\,W\rho - n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jx\,\rho c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^3\rho}{2n^2} - \frac{jx\,jy^2\rho}{2n^2} - \frac{jx\,jz^2\rho}{2n^2} + jx\,W\rho - jx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jy\,\rho c^2 + \left(-qy - \frac{jx\,sxy}{n} - \frac{jy\,syy}{n} - \frac{jz\,syz}{n} - \frac{jx^2\,jy\,\rho}{2n^2} - \frac{jy^3\rho}{2n^2} - \frac{jy\,jz^2\rho}{2n^2} + jy\,W\rho - jy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jz\,\rho c^2 + \left(-qz - \frac{jx\,sxz}{n} - \frac{jy\,syz}{n} - \frac{jz\,szz}{n} - \frac{jx^2\,jz\,\rho}{2n^2} - \frac{jy^2\,jz\,\rho}{2n^2} - \frac{jz^3\rho}{2n^2} + jz\,W\rho - jz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:= `(* in terms of matter velocity *)`
`showf[assut][Expand //@ FS@PowerExpand[tt.{1, 0, 0, 0} /. j2v]]`

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 + \left(-\frac{1}{2} n\,ux^2\rho - \frac{1}{2} n\,uy^2\rho - \frac{1}{2} n\,uz^2\rho + n\,W\rho - n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,ux\,\rho c^2 + \left(-qx - sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2} n\,ux^3\rho - \frac{1}{2} n\,ux\,uy^2\rho - \frac{1}{2} n\,ux\,uz^2\rho + n\,ux\,W\rho - n\,ux\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uy\,\rho c^2 + \left(-qy - sxy\,ux - syy\,uy - syz\,uz - \frac{1}{2} n\,ux^2\,uy\,\rho - \frac{1}{2} n\,uy^3\rho - \frac{1}{2} n\,uy\,uz^2\rho + n\,uy\,W\rho - n\,uy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uz\,\rho c^2 + \left(-qz - sxz\,ux - syz\,uy - szz\,uz - \frac{1}{2} n\,ux^2\,uz\,\rho - \frac{1}{2} n\,uy^2\,uz\,\rho - \frac{1}{2} n\,uz^3\rho + n\,uz\,W\rho - n\,uz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[•]:= `(* flux of coord. energy across surface *)`
`showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt)]]`

Out[•]//MatrixForm=

$$\left(-jx\,\rho + n\,Vx\,\rho\right)c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^3\rho}{2n^2} - \frac{jx\,jy^2\rho}{2n^2} - \frac{jx\,jz^2\rho}{2n^2} + \frac{jx^2\,Vx\,\rho}{2n} + \frac{jy^2\,Vx\,\rho}{2n} + \frac{jz^2\,Vx\,\rho}{2n} + jx\,W\rho - n\,Vx\,W\rho - jx\,\epsilon\,\rho + n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:= `showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr]]`

Out[•]//MatrixForm=

$$-Lx\,n\,\rho c^2 + \left(-qx - sxx\,ux - \frac{1}{2} Lx\,n\,ux^2\rho + Lx\,n\,W\rho - Lx\,n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:= `showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. {jx → 0, jy → 0, jz → 0}]]`

Out[•]//MatrixForm=

$$n\,Vx\,\rho c^2 + \left(-qx - n\,Vx\,W\rho + n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:= `(* in terms of matter flux*)`
`showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf]]`

Out[•]//MatrixForm=

$$-JX\,\rho c^2 + \left(-qx - \frac{JX\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - sxx\,Vx - \frac{JX^3\rho}{2n^2} - \frac{JX\,jy^2\rho}{2n^2} - \frac{JX\,jz^2\rho}{2n^2} - \frac{JX^2\,Vx\,\rho}{n} - \frac{1}{2} JX\,Vx^2\rho + JX\,W\rho - JX\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:= `showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf]]`

Out[•]//MatrixForm=

$$-JX\,\rho c^2 + \left(-qx - \frac{JX\,sxx}{n} - sxx\,Vx - \frac{JX^3\rho}{2n^2} - \frac{JX^2\,Vx\,\rho}{n} - \frac{1}{2} JX\,Vx^2\rho + JX\,W\rho - JX\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:= `(* in terms of matter flux & matter velocity*)`
`showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf /. j2v]]`

Out[•]//MatrixForm=

$$-JX\,\rho c^2 + \left(-qx - \frac{JX\,sxx}{n} - sxy\,uy - sxz\,uz - sxx\,Vx - \frac{JX^3\rho}{2n^2} - \frac{1}{2} JX\,uy^2\rho - \frac{1}{2} JX\,uz^2\rho - \frac{JX^2\,Vx\,\rho}{n} - \frac{1}{2} JX\,Vx^2\rho + JX\,W\rho - JX\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:= `showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. repjf /. {JX → 0, jy → 0, jz → 0}]]`

Out[•]//MatrixForm=

$$\left(-qx - sxx\,Vx\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:= `(* in terms of relative velocity*)`
`showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. relv]]`

Out[•]//MatrixForm=

$$-n\,Vx\,\rho c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^2\,Vx\,\rho}{2n} - \frac{jy^2\,Vx\,\rho}{2n} - \frac{jz^2\,Vx\,\rho}{2n} + n\,Vx\,W\rho - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:= `(* in terms of relative velocity and matter velocity*)`
`showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr]]`

Out[•]//MatrixForm=

$$-n\,Vx\,\rho c^2 + \left(-qx - sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2} n\,ux^2\,Vx\,\rho - \frac{1}{2} n\,uy^2\,Vx\,\rho - \frac{1}{2} n\,uz^2\,Vx\,\rho + n\,Vx\,W\rho - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:= `(* with zero rel. velocity*)`
`showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.{1, 0, 0, 0} / (A * Δt) /. j2vr /. {Vx → 0}]]`

Out[•]//MatrixForm=

$$\left(-qx - sxx\,ux - sxy\,uy - sxz\,uz\right) + O\left[\frac{1}{c}\right]^2$$

In[•]:= `(* supply term for coord. energy *)`
`TTx = tW[Normal[tt]]; shows[assut, 2][Expand //@ FS@PowerExpand[Tr[1 / 2 * Normal@(Inverse[gg].T[Dcoords[[1, ;; , ;;]]].gg + Dcoords[[1, ;; , ;;]]).TTx]]]`

Out[•]//MatrixForm=

$$n\,\rho\,W^{(1,0,0,0)}[t, x, y, z] + \frac{\left(3\left(jx^2+jy^2+jz^2\right)\rho + 2n\left(sxx+syy+szz+n\,\epsilon\,\rho\right) - 2n^2\rho\,W[t, x, y, z]\right)W^{(1,0,0,0)}[t, x, y, z]}{2\,n\,c^2} + O\left[\frac{1}{c}\right]^3$$

In[•]:= `(* INTERNAL ENERGY *)`

In[•]:= `(* energy 3-form when projected along matter 4-velocity, "internal energy" *)`
`showf[assut][Expand //@ FS@PowerExpand[tt.uu]]`

Out[•]//MatrixForm=

$$\begin{pmatrix} -n\rho c^2 - n\,\epsilon\,\rho + O\left[\frac{1}{c}\right]^2 \\ -jx\,\rho c^2 + \left(-qx - jx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jy\,\rho c^2 + \left(-qy - jy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jz\,\rho c^2 + \left(-qz - jz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.uu /. j2vr]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2 - n\,\epsilon\,\rho + O\left[\frac{1}{c}\right]^2 \\ -n\,ux\,\rho\,c^2 + \left(-qx - n\,ux\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uy\,\rho\,c^2 + \left(-qy - n\,uy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uz\,\rho\,c^2 + \left(-qz - n\,uz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* flux of internal energy across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt)]]
```
Out[∘]//MatrixForm=

$$\left(-jx\,\rho + n\,vx\,\rho\right) c^2 + \left(-qx - jx\,\epsilon\,\rho + n\,vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. relv]]
```
Out[∘]//MatrixForm=

$$-n\,Vx\,\rho\,c^2 + \left(-qx - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. j2vr]]
```
Out[∘]//MatrixForm=

$$-n\,Vx\,\rho\,c^2 + \left(-qx - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.uu / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[∘]//MatrixForm=

$$-qx + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* supply term for internal energy (should be reversed in sign; remember that stress is compressive, not tensile) *)
TTx = tW[tjv@tt]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Duv].gg + Duv).TTx]]]
```
Out[∘]//MatrixForm=

$$\left(sxz\,vx^{(0,0,0,1)}[t,x,y,z] + syz\,vy^{(0,0,0,1)}[t,x,y,z] + szz\,vz^{(0,0,0,1)}[t,x,y,z] + sxy\,vx^{(0,0,1,0)}[t,x,y,z] + syy\,vy^{(0,0,1,0)}[t,x,y,z] + syz\,vz^{(0,0,1,0)}[t,x,y,z] + sxx\,vx^{(0,1,0,0)}[t,x,y,z] + sxy\,vy^{(0,1,0,0)}[t,x,y,z] + sxz\,vz^{(0,1,0,0)}[t,x,y,z]\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* difference between "coord. energy" and "internal energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - uu)]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{jx^2\rho}{2n} - \frac{jy^2\rho}{2n} - \frac{jz^2\rho}{2n} + n\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^3\rho}{2n^2} - \frac{jx\,jy^2\rho}{2n^2} - \frac{jx\,jz^2\rho}{2n^2} + jx\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\,sxy}{n} - \frac{jy\,syy}{n} - \frac{jz\,syz}{n} - \frac{jx^2\,jy\,\rho}{2n^2} - \frac{jy^3\rho}{2n^2} - \frac{jy\,jz^2\rho}{2n^2} + jy\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-\frac{jx\,sxz}{n} - \frac{jy\,syz}{n} - \frac{jz\,szz}{n} - \frac{jx^2\,jz\,\rho}{2n^2} - \frac{jy^2\,jz\,\rho}{2n^2} - \frac{jz^3\rho}{2n^2} + jz\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - uu) /. j2vr]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{1}{2}\,n\,ux^2\rho - \frac{1}{2}\,n\,uy^2\rho - \frac{1}{2}\,n\,uz^2\rho + n\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2}\,n\,ux^3\rho - \frac{1}{2}\,n\,ux\,uy^2\rho - \frac{1}{2}\,n\,ux\,uz^2\rho + n\,ux\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-sxy\,ux - syy\,uy - syz\,uz - \frac{1}{2}\,n\,ux^2\,uy\,\rho - \frac{1}{2}\,n\,uy^3\rho - \frac{1}{2}\,n\,uy\,uz^2\rho + n\,uy\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ \left(-sxz\,ux - syz\,uy - szz\,uz - \frac{1}{2}\,n\,ux^2\,uz\,\rho - \frac{1}{2}\,n\,uy^2\,uz\,\rho - \frac{1}{2}\,n\,uz^3\rho + n\,uz\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt)]]
```
Out[∘]//MatrixForm=

$$\left(-\frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^3\rho}{2n^2} - \frac{jx\,jy^2\rho}{2n^2} - \frac{jx\,jz^2\rho}{2n^2} + \frac{jx^2\,vx\,\rho}{2n} + \frac{jy^2\,vx\,\rho}{2n} + \frac{jz^2\,vx\,\rho}{2n} + jx\,W\,\rho - n\,vx\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. relv]]
```
Out[∘]//MatrixForm=

$$\left(-\frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^2\,Vx\,\rho}{2n} - \frac{jy^2\,Vx\,\rho}{2n} - \frac{jz^2\,Vx\,\rho}{2n} + n\,Vx\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. j2vr]]
```
Out[∘]//MatrixForm=

$$\left(-sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2}\,n\,ux^2\,Vx\,\rho - \frac{1}{2}\,n\,uy^2\,Vx\,\rho - \frac{1}{2}\,n\,uz^2\,Vx\,\rho + n\,Vx\,W\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - uu) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[∘]//MatrixForm=

$$\left(-sxx\,ux - sxy\,uy - sxz\,uz\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* PROPER-TIME COORD ENERGY*)
```

In[∘]:=
```
(* energy 3-form when projected along normalized coord-t
   note how the gravitational term is missing *)
showf[assut][Expand //@ FS@PowerExpand[tt.vtn]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2 + \left(-\frac{jx^2\rho}{2n} - \frac{jy^2\rho}{2n} - \frac{jz^2\rho}{2n} - n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jx\,\rho\,c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^3\rho}{2n^2} - \frac{jx\,jy^2\rho}{2n^2} - \frac{jx\,jz^2\rho}{2n^2} - jx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jy\,\rho\,c^2 + \left(-qy - \frac{jx\,sxy}{n} - \frac{jy\,syy}{n} - \frac{jz\,syz}{n} - \frac{jx^2\,jy\,\rho}{2n^2} - \frac{jy^3\rho}{2n^2} - \frac{jy\,jz^2\rho}{2n^2} - jy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -jz\,\rho\,c^2 + \left(-qz - \frac{jx\,sxz}{n} - \frac{jy\,syz}{n} - \frac{jz\,szz}{n} - \frac{jx^2\,jz\,\rho}{2n^2} - \frac{jy^2\,jz\,\rho}{2n^2} - \frac{jz^3\rho}{2n^2} - jz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.vtn /. j2vr]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} -n\,\rho\,c^2 + \left(-\frac{1}{2}\,n\,ux^2\rho - \frac{1}{2}\,n\,uy^2\rho - \frac{1}{2}\,n\,uz^2\rho - n\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,ux\,\rho\,c^2 + \left(-qx - sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2}\,n\,ux^3\rho - \frac{1}{2}\,n\,ux\,uy^2\rho - \frac{1}{2}\,n\,ux\,uz^2\rho - n\,ux\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uy\,\rho\,c^2 + \left(-qy - sxy\,ux - syy\,uy - syz\,uz - \frac{1}{2}\,n\,ux^2\,uy\,\rho - \frac{1}{2}\,n\,uy^3\rho - \frac{1}{2}\,n\,uy\,uz^2\rho - n\,uy\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \\ -n\,uz\,\rho\,c^2 + \left(-qz - sxz\,ux - syz\,uy - szz\,uz - \frac{1}{2}\,n\,ux^2\,uz\,\rho - \frac{1}{2}\,n\,uy^2\,uz\,\rho - \frac{1}{2}\,n\,uz^3\rho - n\,uz\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* flux of normalized-coord-t energy across surface *)
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt)]]
```
Out[∘]//MatrixForm=

$$\left(-jx\,\rho + n\,vx\,\rho\right) c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jx^3\rho}{2n^2} + \frac{jx^2\,vx\,\rho}{2n} - jx\,\epsilon\,\rho + n\,vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(*in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. relv]]
```
Out[∘]//MatrixForm=

$$-n\,Vx\,\rho\,c^2 + \left(-qx - \frac{jx\,sxx}{n} - \frac{jy\,sxy}{n} - \frac{jz\,sxz}{n} - \frac{jx^2\,Vx\,\rho}{2n} - \frac{jy^2\,Vx\,\rho}{2n} - \frac{jz^2\,Vx\,\rho}{2n} - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. j2vr]]
```
Out[∘]//MatrixForm=

$$-n\,Vx\,\rho\,c^2 + \left(-qx - sxx\,ux - sxy\,uy - sxz\,uz - \frac{1}{2}\,n\,ux^2\,Vx\,\rho - \frac{1}{2}\,n\,uy^2\,Vx\,\rho - \frac{1}{2}\,n\,uz^2\,Vx\,\rho - n\,Vx\,\epsilon\,\rho\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.vtn / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[∘]//MatrixForm=

$$\left(-qx - sxx\,ux - sxy\,uy - sxz\,uz\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* supply term for normalized-coord-t energy
   we obtain the "power generated by the gravity field" *)
TTx = tW[tjv@tt]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Dvtn].gg + Dvtn).TTx]]]
```
Out[∘]//MatrixForm=

$$\left(-\rho\,n[t,x,y,z] \times vz[t,x,y,z]\,W^{(0,0,0,1)}[t,x,y,z] - \rho\,n[t,x,y,z] \times vy[t,x,y,z]\,W^{(0,0,1,0)}[t,x,y,z] - \rho\,n[t,x,y,z] \times vx[t,x,y,z]\,W^{(0,1,0,0)}[t,x,y,z]\right) + O\left[\frac{1}{c}\right]^2$$

In[∘]:=
```
(* difference between "coord. energy" and "proper-time coord. energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - vtn)]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} n\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ jx\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ jy\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ jz\,W\,\rho + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[∘]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.({1, 0, 0, 0} - vtn) /. j2vr]]
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} n\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ n\,ux\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ n\,uy\,W\,\rho + O\left[\frac{1}{c}\right]^2 \\ n\,uz\,W\,\rho + O\left[\frac{1}{c}\right]^2 \end{pmatrix}$$

In[-]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt)]]
```
Out[-]//MatrixForm=
$$\left(\mathrm{jx}\, W \rho - n\, \mathrm{vx}\, W \rho\right) + O\left[\frac{1}{c}\right]^2$$

In[-]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. relv]]
```
Out[-]//MatrixForm=
$$n\, \mathrm{Vx}\, W \rho + O\left[\frac{1}{c}\right]^2$$

In[-]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. j2vr]]
```
Out[-]//MatrixForm=
$$n\, \mathrm{Vx}\, W \rho + O\left[\frac{1}{c}\right]^2$$

In[-]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.({1, 0, 0, 0} - vtn) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[-]//MatrixForm=
$$O\left[\frac{1}{c}\right]^2$$

In[-]:=
```
(* difference between "internal energy" and "proper-time coord. energy" *)
showf[assut][Expand //@ FS@PowerExpand[tt.(uu - vtn)]]
```
Out[-]//MatrixForm=
$$\begin{pmatrix}
\left(\frac{\mathrm{jx}^2 \rho}{2 n} + \frac{\mathrm{jy}^2 \rho}{2 n} + \frac{\mathrm{jz}^2 \rho}{2 n}\right) + O\left[\frac{1}{c}\right]^2 \\
\left(\frac{\mathrm{jx}\, \mathrm{sxx}}{n} + \frac{\mathrm{jy}\, \mathrm{sxy}}{n} + \frac{\mathrm{jz}\, \mathrm{sxz}}{n} + \frac{\mathrm{jx}^3 \rho}{2 n^2} + \frac{\mathrm{jx}\, \mathrm{jy}^2 \rho}{2 n^2} + \frac{\mathrm{jx}\, \mathrm{jz}^2 \rho}{2 n^2}\right) + O\left[\frac{1}{c}\right]^2 \\
\left(\frac{\mathrm{jx}\, \mathrm{sxy}}{n} + \frac{\mathrm{jy}\, \mathrm{syy}}{n} + \frac{\mathrm{jz}\, \mathrm{syz}}{n} + \frac{\mathrm{jx}^2\, \mathrm{jy} \rho}{2 n^2} + \frac{\mathrm{jy}^3 \rho}{2 n^2} + \frac{\mathrm{jy}\, \mathrm{jz}^2 \rho}{2 n^2}\right) + O\left[\frac{1}{c}\right]^2 \\
\left(\frac{\mathrm{jx}\, \mathrm{sxz}}{n} + \frac{\mathrm{jy}\, \mathrm{syz}}{n} + \frac{\mathrm{jz}\, \mathrm{szz}}{n} + \frac{\mathrm{jx}^2\, \mathrm{jz} \rho}{2 n^2} + \frac{\mathrm{jy}^2\, \mathrm{jz} \rho}{2 n^2} + \frac{\mathrm{jz}^3 \rho}{2 n^2}\right) + O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[-]:=
```
(* in terms of matter velocity *)
showf[assut][Expand //@ FS@PowerExpand[tt.(uu - vtn) /. j2vr]]
```
Out[-]//MatrixForm=
$$\begin{pmatrix}
\left(\frac{1}{2} n\, \mathrm{ux}^2 \rho + \frac{1}{2} n\, \mathrm{uy}^2 \rho + \frac{1}{2} n\, \mathrm{uz}^2 \rho\right) + O\left[\frac{1}{c}\right]^2 \\
\left(\mathrm{sxx}\, \mathrm{ux} + \mathrm{sxy}\, \mathrm{uy} + \mathrm{sxz}\, \mathrm{uz} + \frac{1}{2} n\, \mathrm{ux}^3 \rho + \frac{1}{2} n\, \mathrm{ux}\, \mathrm{uy}^2 \rho + \frac{1}{2} n\, \mathrm{ux}\, \mathrm{uz}^2 \rho\right) + O\left[\frac{1}{c}\right]^2 \\
\left(\mathrm{sxy}\, \mathrm{ux} + \mathrm{syy}\, \mathrm{uy} + \mathrm{syz}\, \mathrm{uz} + \frac{1}{2} n\, \mathrm{ux}^2\, \mathrm{uy} \rho + \frac{1}{2} n\, \mathrm{uy}^3 \rho + \frac{1}{2} n\, \mathrm{uy}\, \mathrm{uz}^2 \rho\right) + O\left[\frac{1}{c}\right]^2 \\
\left(\mathrm{sxz}\, \mathrm{ux} + \mathrm{syz}\, \mathrm{uy} + \mathrm{szz}\, \mathrm{uz} + \frac{1}{2} n\, \mathrm{ux}^2\, \mathrm{uz} \rho + \frac{1}{2} n\, \mathrm{uy}^2\, \mathrm{uz} \rho + \frac{1}{2} n\, \mathrm{uz}^3 \rho\right) + O\left[\frac{1}{c}\right]^2
\end{pmatrix}$$

In[-]:=
```
(* flux of difference across surface *)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - vtn) / (A * Δt)]]
```
Out[-]//MatrixForm=
$$\left(\frac{\mathrm{jx}\, \mathrm{sxx}}{n} + \frac{\mathrm{jy}\, \mathrm{sxy}}{n} + \frac{\mathrm{jz}\, \mathrm{sxz}}{n} + \frac{\mathrm{jx}^3 \rho}{2 n^2} + \frac{\mathrm{jx}\, \mathrm{jy}^2 \rho}{2 n^2} + \frac{\mathrm{jx}\, \mathrm{jz}^2 \rho}{2 n^2} - \frac{\mathrm{jx}^2\, \mathrm{vx} \rho}{2 n} - \frac{\mathrm{jy}^2\, \mathrm{vx} \rho}{2 n} - \frac{\mathrm{jz}^2\, \mathrm{vx} \rho}{2 n}\right) + O\left[\frac{1}{c}\right]^2$$

In[-]:=
```
(* in terms of relative velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - vtn) / (A * Δt) /. relv]]
```
Out[-]//MatrixForm=
$$\left(\frac{\mathrm{jx}\, \mathrm{sxx}}{n} + \frac{\mathrm{jy}\, \mathrm{sxy}}{n} + \frac{\mathrm{jz}\, \mathrm{sxz}}{n} + \frac{\mathrm{jx}^2\, \mathrm{Vx} \rho}{2 n} + \frac{\mathrm{jy}^2\, \mathrm{Vx} \rho}{2 n} + \frac{\mathrm{jz}^2\, \mathrm{Vx} \rho}{2 n}\right) + O\left[\frac{1}{c}\right]^2$$

In[-]:=
```
(* in terms of relative velocity and matter velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - uu) / (A * Δt) /. j2vr]]
```
Out[-]//MatrixForm=
$$O\left[\frac{1}{c}\right]^2$$

In[-]:=
```
(* with zero rel. velocity*)
showf[assut][Expand //@ FS@PowerExpand[surfacefx.tt.(uu - uu) / (A * Δt) /. j2vr /. {Vx → 0}]]
```
Out[-]//MatrixForm=
$$O\left[\frac{1}{c}\right]^2$$

In[-]:=
```
(* Faraday tensor *)
repE = {Ex → Ex * c * Sqrt[μ0 * ϵ0], Ey → Ey * c * Sqrt[μ0 * ϵ0], Ez → Ez * c * Sqrt[μ0 * ϵ0]};
fftemp = {{0, -Ex, -Ey, -Ez}, {0, 0, Bz, -By}, {0, 0, 0, Bx}, {0, 0, 0, 0}} /. repE;
showf[assut][ffdd = Assuming[assut, Expand //@ FS@PowerExpand[fftemp - T[fftemp]]]]
```
Out[-]//MatrixForm=
$$\begin{pmatrix}
0 & -c\, \mathrm{Ex} \sqrt{\epsilon 0} \sqrt{\mu 0} & -c\, \mathrm{Ey} \sqrt{\epsilon 0} \sqrt{\mu 0} & -c\, \mathrm{Ez} \sqrt{\epsilon 0} \sqrt{\mu 0} \\
c\, \mathrm{Ex} \sqrt{\epsilon 0} \sqrt{\mu 0} & 0 & \mathrm{Bz} & -\mathrm{By} \\
c\, \mathrm{Ey} \sqrt{\epsilon 0} \sqrt{\mu 0} & -\mathrm{Bz} & 0 & \mathrm{Bx} \\
c\, \mathrm{Ez} \sqrt{\epsilon 0} \sqrt{\mu 0} & \mathrm{By} & -\mathrm{Bx} & 0
\end{pmatrix}$$

In[-]:=
```
showf[assut][tte = Assuming[assut, Expand //@ FS@PowerExpand[
    (1 / μ0 * (Inverse[gg].ffdd.Inverse[gg].T[ffdd].Inverse[gg] - 1 / 4 * Inverse[gg] * Tr[ffdd.Inverse[gg].T[ffdd].Inverse[gg]]).gg * dg)
   ]]]
```
Out[-]//MatrixForm=

$$\begin{pmatrix}
\left(-\frac{\mathrm{Ex}^2 \epsilon 0}{2} - \frac{\mathrm{Ey}^2 \epsilon 0}{2} - \frac{\mathrm{Ez}^2 \epsilon 0}{2} - \frac{\mathrm{Bx}^2}{2 \mu 0} - \frac{\mathrm{By}^2}{2 \mu 0} - \frac{\mathrm{Bz}^2}{2 \mu 0}\right) + \frac{-\mathrm{Ex}^2 W \epsilon 0 - \mathrm{Ey}^2 W \epsilon 0 - \mathrm{Ez}^2 W \epsilon 0 + \frac{\mathrm{Bx}^2 W}{\mu 0} + \frac{\mathrm{By}^2 W}{\mu 0} + \frac{\mathrm{Bz}^2 W}{\mu 0}}{c^2} + O\left[\frac{1}{c}\right]^4 \\
\left(-\frac{\mathrm{Bz}\, \mathrm{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\mathrm{By}\, \mathrm{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{\frac{2 \mathrm{Bz}\, \mathrm{Ey} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2 \mathrm{By}\, \mathrm{Ez} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{-4 \mathrm{Ey}^2 \mathrm{Wx} \epsilon 0 - 4 \mathrm{Ez}^2 \mathrm{Wx} \epsilon 0 + 4 \mathrm{Ex}\, \mathrm{Ey}\, \mathrm{Wy} \epsilon 0 + 4 \mathrm{Ex}\, \mathrm{Ez}\, \mathrm{Wz} \epsilon 0}{c^2} + O\left[\frac{1}{c}\right]^3 \\
\left(\frac{\mathrm{Bz}\, \mathrm{Ex} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{\mathrm{Bx}\, \mathrm{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{-\frac{2 \mathrm{Bz}\, \mathrm{Ex} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{2 \mathrm{Bx}\, \mathrm{Ez} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{4 \mathrm{Ex}\, \mathrm{Ey}\, \mathrm{Wx} \epsilon 0 - 4 \mathrm{Ex}^2 \mathrm{Wy} \epsilon 0 - 4 \mathrm{Ez}^2 \mathrm{Wy} \epsilon 0 + 4 \mathrm{Ey}\, \mathrm{Ez}\, \mathrm{Wz} \epsilon 0}{c^2} + O\left[\frac{1}{c}\right]^3 \\
\left(-\frac{\mathrm{By}\, \mathrm{Ex} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\mathrm{Bx}\, \mathrm{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{\frac{2 \mathrm{By}\, \mathrm{Ex} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2 \mathrm{Bx}\, \mathrm{Ey} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{4 \mathrm{Ex}\, \mathrm{Ez}\, \mathrm{Wx} \epsilon 0 + 4 \mathrm{Ey}\, \mathrm{Ez}\, \mathrm{Wy} \epsilon 0 - 4 \mathrm{Ex}^2 \mathrm{Wz} \epsilon 0 - 4 \mathrm{Ey}^2 \mathrm{Wz} \epsilon 0}{c^2} + O\left[\frac{1}{c}\right]^3
\end{pmatrix}$$

$$\begin{pmatrix}
\frac{\frac{\mathrm{Bz}\, \mathrm{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{\mathrm{By}\, \mathrm{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{\frac{2 \mathrm{Bz}\, \mathrm{Ey} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2 \mathrm{By}\, \mathrm{Ez} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + \frac{-\frac{4 \mathrm{By}^2 \mathrm{Wx}}{\mu 0} - \frac{4 \mathrm{Bz}^2 \mathrm{Wx}}{\mu 0} + \frac{4 \mathrm{Bx}\, \mathrm{By}\, \mathrm{Wy}}{\mu 0} + \frac{4 \mathrm{Bx}\, \mathrm{Bz}\, \mathrm{Wz}}{\mu 0}}{c^4} + O\left[\frac{1}{c}\right]^5 \\
\left(-\frac{\mathrm{Ex}^2 \epsilon 0}{2} + \frac{\mathrm{Ey}^2 \epsilon 0}{2} + \frac{\mathrm{Ez}^2 \epsilon 0}{2} - \frac{\mathrm{Bx}^2}{2 \mu 0} + \frac{\mathrm{By}^2}{2 \mu 0} + \frac{\mathrm{Bz}^2}{2 \mu 0}\right) + \frac{-\mathrm{Ex}^2 W \epsilon 0 + \mathrm{Ey}^2 W \epsilon 0 + \mathrm{Ez}^2 W \epsilon 0 + \frac{\mathrm{Bx}^2 W}{\mu 0} - \frac{\mathrm{By}^2 W}{\mu 0} - \frac{\mathrm{Bz}^2 W}{\mu 0}}{c^2} + \frac{-\frac{4 \mathrm{Bz}\, \mathrm{Ex}\, \mathrm{Wy} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 \mathrm{Bx}\, \mathrm{Ez}\, \mathrm{Wy} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 \mathrm{By}\, \mathrm{Ex}\, \mathrm{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 \mathrm{Bx}\, \mathrm{Ey}\, \mathrm{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O\left[\frac{1}{c}\right]^4 \\
\left(-\mathrm{Ex}\, \mathrm{Ey} \epsilon 0 - \frac{\mathrm{Bx}\, \mathrm{By}}{\mu 0}\right) + \frac{-2 \mathrm{Ex}\, \mathrm{Ey} W \epsilon 0 + \frac{2 \mathrm{Bx}\, \mathrm{By} W}{\mu 0}}{c^2} + \frac{\frac{4 \mathrm{Bz}\, \mathrm{Ex}\, \mathrm{Wx} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 \mathrm{By}\, \mathrm{Ez}\, \mathrm{Wy} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 \mathrm{Bx}\, \mathrm{Ex}\, \mathrm{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 \mathrm{By}\, \mathrm{Ey}\, \mathrm{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O\left[\frac{1}{c}\right]^4 \\
\left(-\mathrm{Ex}\, \mathrm{Ez} \epsilon 0 - \frac{\mathrm{Bx}\, \mathrm{Bz}}{\mu 0}\right) + \frac{-2 \mathrm{Ex}\, \mathrm{Ez} W \epsilon 0 + \frac{2 \mathrm{Bx}\, \mathrm{Bz} W}{\mu 0}}{c^2} + \frac{-\frac{4 \mathrm{By}\, \mathrm{Ex}\, \mathrm{Wx} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 \mathrm{Bx}\, \mathrm{Ex}\, \mathrm{Wy} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 \mathrm{Bz}\, \mathrm{Ez}\, \mathrm{Wy} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 \mathrm{Bz}\, \mathrm{Ey}\, \mathrm{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O\left[\frac{1}{c}\right]^4
\end{pmatrix}$$

$$\begin{pmatrix}
\frac{-\frac{\mathrm{Bz}\, \mathrm{Ex} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\mathrm{Bx}\, \mathrm{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{-\frac{2 \mathrm{Bz}\, \mathrm{Ex} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{2 \mathrm{Bx}\, \mathrm{Ez} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + \frac{\frac{4 \mathrm{Bx}\, \mathrm{By}\, \mathrm{Wx}}{\mu 0} - \frac{4 \mathrm{Bx}^2 \mathrm{Wy}}{\mu 0} - \frac{4 \mathrm{Bz}^2 \mathrm{Wy}}{\mu 0} + \frac{4 \mathrm{By}\, \mathrm{Bz}\, \mathrm{Wz}}{\mu 0}}{c^4} + O\left[\frac{1}{c}\right]^5 \\
\left(-\mathrm{Ex}\, \mathrm{Ey} \epsilon 0 - \frac{\mathrm{Bx}\, \mathrm{By}}{\mu 0}\right) + \frac{-2 \mathrm{Ex}\, \mathrm{Ey} W \epsilon 0 + \frac{2 \mathrm{Bx}\, \mathrm{By} W}{\mu 0}}{c^2} + \frac{\frac{4 \mathrm{Bx}\, \mathrm{Ez}\, \mathrm{Wx} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 \mathrm{Bz}\, \mathrm{Ey}\, \mathrm{Wy} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 \mathrm{Bx}\, \mathrm{Ex}\, \mathrm{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 \mathrm{By}\, \mathrm{Ey}\, \mathrm{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O\left[\frac{1}{c}\right]^4 \\
\left(\frac{\mathrm{Ex}^2 \epsilon 0}{2} - \frac{\mathrm{Ey}^2 \epsilon 0}{2} + \frac{\mathrm{Ez}^2 \epsilon 0}{2} + \frac{\mathrm{Bx}^2}{2 \mu 0} - \frac{\mathrm{By}^2}{2 \mu 0} + \frac{\mathrm{Bz}^2}{2 \mu 0}\right) + \frac{\mathrm{Ex}^2 W \epsilon 0 - \mathrm{Ey}^2 W \epsilon 0 + \mathrm{Ez}^2 W \epsilon 0 - \frac{\mathrm{Bx}^2 W}{\mu 0} + \frac{\mathrm{By}^2 W}{\mu 0} - \frac{\mathrm{Bz}^2 W}{\mu 0}}{c^2} + \frac{\frac{4 \mathrm{Bz}\, \mathrm{Ey}\, \mathrm{Wx} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 \mathrm{By}\, \mathrm{Ez}\, \mathrm{Wx} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 \mathrm{By}\, \mathrm{Ex}\, \mathrm{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 \mathrm{Bx}\, \mathrm{Ey}\, \mathrm{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O\left[\frac{1}{c}\right]^4 \\
\left(-\mathrm{Ey}\, \mathrm{Ez} \epsilon 0 - \frac{\mathrm{By}\, \mathrm{Bz}}{\mu 0}\right) + \frac{-2 \mathrm{Ey}\, \mathrm{Ez} W \epsilon 0 + \frac{2 \mathrm{By}\, \mathrm{Bz} W}{\mu 0}}{c^2} + \frac{-\frac{4 \mathrm{By}\, \mathrm{Ey}\, \mathrm{Wx} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 \mathrm{Bz}\, \mathrm{Ez}\, \mathrm{Wx} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{4 \mathrm{Bx}\, \mathrm{Ey}\, \mathrm{Wy} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{4 \mathrm{Bz}\, \mathrm{Ex}\, \mathrm{Wz} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c^3} + O\left[\frac{1}{c}\right]^4
\end{pmatrix}$$

$$\begin{pmatrix}
\frac{\frac{\mathrm{By}\, \mathrm{Ex} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{\mathrm{Bx}\, \mathrm{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \ldots \\
\left(-\mathrm{Ex}\, \mathrm{Ez} \epsilon 0 - \frac{\mathrm{Bx} \ldots}{\mu 0} \ldots \right. \\
\left(-\mathrm{Ey}\, \mathrm{Ez} \epsilon 0 - \frac{\mathrm{By} \ldots}{\mu 0} \ldots \right. \\
\left(\frac{\mathrm{Ex}^2 \epsilon 0}{2} + \frac{\mathrm{Ey}^2 \epsilon 0}{2} - \ldots \right.
\end{pmatrix}$$

In[-]:=
```
(* COORDINATE EM ENERGY *)
```

In[-]:=
```
(* energy 3-form when projected along coord. axes *)
showf[assut][Expand //@ FS@PowerExpand[tte.{1, 0, 0, 0}]]
```
Out[-]//MatrixForm=
$$\begin{pmatrix}
\left(-\frac{\mathrm{Ex}^2 \epsilon 0}{2} - \frac{\mathrm{Ey}^2 \epsilon 0}{2} - \frac{\mathrm{Ez}^2 \epsilon 0}{2} - \frac{\mathrm{Bx}^2}{2 \mu 0} - \frac{\mathrm{By}^2}{2 \mu 0} - \frac{\mathrm{Bz}^2}{2 \mu 0}\right) + \frac{-\mathrm{Ex}^2 W \epsilon 0 - \mathrm{Ey}^2 W \epsilon 0 - \mathrm{Ez}^2 W \epsilon 0 + \frac{\mathrm{Bx}^2 W}{\mu 0} + \frac{\mathrm{By}^2 W}{\mu 0} + \frac{\mathrm{Bz}^2 W}{\mu 0}}{c^2} + O\left[\frac{1}{c}\right]^4 \\
\left(-\frac{\mathrm{Bz}\, \mathrm{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\mathrm{By}\, \mathrm{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{\frac{2 \mathrm{Bz}\, \mathrm{Ey} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2 \mathrm{By}\, \mathrm{Ez} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{-4 \mathrm{Ey}^2 \mathrm{Wx} \epsilon 0 - 4 \mathrm{Ez}^2 \mathrm{Wx} \epsilon 0 + 4 \mathrm{Ex}\, \mathrm{Ey}\, \mathrm{Wy} \epsilon 0 + 4 \mathrm{Ex}\, \mathrm{Ez}\, \mathrm{Wz} \epsilon 0}{c^2} + O\left[\frac{1}{c}\right]^3 \\
\left(\frac{\mathrm{Bz}\, \mathrm{Ex} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{\mathrm{Bx}\, \mathrm{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{-\frac{2 \mathrm{Bz}\, \mathrm{Ex} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{2 \mathrm{Bx}\, \mathrm{Ez} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{4 \mathrm{Ex}\, \mathrm{Ey}\, \mathrm{Wx} \epsilon 0 - 4 \mathrm{Ex}^2 \mathrm{Wy} \epsilon 0 - 4 \mathrm{Ez}^2 \mathrm{Wy} \epsilon 0 + 4 \mathrm{Ey}\, \mathrm{Ez}\, \mathrm{Wz} \epsilon 0}{c^2} + O\left[\frac{1}{c}\right]^3 \\
\left(-\frac{\mathrm{By}\, \mathrm{Ex} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\mathrm{Bx}\, \mathrm{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \frac{\frac{2 \mathrm{By}\, \mathrm{Ex} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2 \mathrm{Bx}\, \mathrm{Ey} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{4 \mathrm{Ex}\, \mathrm{Ez}\, \mathrm{Wx} \epsilon 0 + 4 \mathrm{Ey}\, \mathrm{Ez}\, \mathrm{Wy} \epsilon 0 - 4 \mathrm{Ex}^2 \mathrm{Wz} \epsilon 0 - 4 \mathrm{Ey}^2 \mathrm{Wz} \epsilon 0}{c^2} + O\left[\frac{1}{c}\right]^3
\end{pmatrix}$$

In[-]:=
```
(* flux of coord. energy across surface *)
showf[assutjx][Expand //@ FS@PowerExpand[surfacefx.tte.{1, 0, 0, 0} / (A * Δt)]]
```
Out[-]//MatrixForm=
$$\left(-\frac{\mathrm{Bz}\, \mathrm{Ey} \sqrt{\epsilon 0}}{\sqrt{\mu 0}} + \frac{\mathrm{By}\, \mathrm{Ez} \sqrt{\epsilon 0}}{\sqrt{\mu 0}}\right) c + \left(\frac{1}{2} \mathrm{Ex}^2\, \mathrm{vx} \epsilon 0 + \frac{1}{2} \mathrm{Ey}^2\, \mathrm{vx} \epsilon 0 + \frac{1}{2} \mathrm{Ez}^2\, \mathrm{vx} \epsilon 0 + \frac{\mathrm{Bx}^2\, \mathrm{vx}}{2 \mu 0} + \frac{\mathrm{By}^2\, \mathrm{vx}}{2 \mu 0} + \frac{\mathrm{Bz}^2\, \mathrm{vx}}{2 \mu 0}\right) + \frac{\frac{2 \mathrm{Bz}\, \mathrm{Ey} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}} - \frac{2 \mathrm{By}\, \mathrm{Ez} W \sqrt{\epsilon 0}}{\sqrt{\mu 0}}}{c} + \frac{\mathrm{Ex}^2\, \mathrm{vx} W \epsilon 0 + \mathrm{Ey}^2\, \mathrm{vx} W \epsilon 0 + \mathrm{Ez}^2\, \mathrm{vx} W \epsilon 0 - 4 \mathrm{Ey}^2 \mathrm{Wx} \epsilon 0 - 4 \mathrm{Ez}^2 \mathrm{Wx} \epsilon 0 + 4 \mathrm{Ex}\, \mathrm{Ey}\, \mathrm{Wy} \epsilon 0 + 4 \mathrm{Ex}\, \mathrm{Ez}\, \mathrm{Wz} \epsilon 0 - \frac{\mathrm{Bx}^2\, \mathrm{vx} W}{\mu 0} - \frac{\mathrm{By}^2\, \mathrm{vx} W}{\mu 0} - \frac{\mathrm{Bz}^2\, \mathrm{vx} W}{\mu 0}}{c^2} + O\left[\frac{1}{c}\right]^3$$

In[-]:=
```
(* supply term for coord. energy *)
TTx = tW[tjv@tte]; showf[assut][Expand //@ FS@PowerExpand[Tr[1 / 2 * (Inverse[gg].T[Dcoords[[1, ;; , ;;]]].gg + Dcoords[[1, ;; , ;;]]).TTx]]]
```
Out[-]//MatrixForm=
$$\frac{\mathrm{Ex}^2 \epsilon 0\, W^{(1,0,0,0)}[t, x, y, z] + \mathrm{Ey}^2 \epsilon 0\, W^{(1,0,0,0)}[t, x, y, z] + \mathrm{Ez}^2 \epsilon 0\, W^{(1,0,0,0)}[t, x, y, z] + \frac{\mathrm{Bx}^2 W^{(1,0,0,0)}[t,x,y,z]}{\mu 0} + \frac{\mathrm{By}^2 W^{(1,0,0,0)}[t,x,y,z]}{\mu 0} + \frac{\mathrm{Bz}^2 W^{(1,0,0,0)}[t,x,y,z]}{\mu 0}}{c^2} + O\left[\frac{1}{c}\right]^3$$