Open Science Framework DOI:10.31219/osf.io/***

Foundations of inference under symmetry: A derivation of algorithms for non-parametric density inference

P.G.L. Porta Mana
<pgl@portamana.org>

15 January 2023; updated 5 February 2023 [draft]

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0 A warning on notation

The probability calculus, being a generalization of the propositional calculus, concerns propositions or statements. In the expression "$\mathrm{P}(A \mid B)$", A and B thus stand for statements or compositions of statements. In scientific applications most statements of interest are of the form "Measurement of the quantity X yields outcome x" or "The quantity X is set to the value x", implicitly or explicitly accompanied by other statements expressing contextual information, for instance "The measurement is made at time ... in laboratory ... under conditions ...".

Writing full statements such as these within probability formulae would take an impractical amount of space. This impracticality is solved by an abuse of notation: outside a probability formula, a symbol like X may denote a quantity; but within a probability formula, say $\mathrm{p}(X \mid \dots)$, it instead denotes a full statement such as "Measurement of the quantity X yields outcome x"; sometimes the latter is written "$X = x$" when the value x is not generic or needs to be explicit.

To limit the confusion that can arise from this notation abuse, I use a convention similar to Jaynes's (2003 § 2.5 p. 43): the probability symbol "P" is only used when the symbols in its arguments univocally denote statements; the symbol "p" is used instead to warn that some of the symbols in its arguments are abused. For example, if the quantity N denotes the number of people satisfying some condition, $A \coloneqq$ "Measurement of N yields 101 people", and B denotes some contextual statement, then these three expressions have the same meaning:

$$\mathrm{P}(A \mid B) = 0.3\,, \qquad \mathrm{p}(N \mid B) = 0.3\,, \qquad \mathrm{p}(N = 101 \mid B) = 0.3\,.$$

Bibliography

("de *X*" is listed under D, "van *X*" under V, and so on, regardless of national conventions.)

Jaynes, E. T. (2003): *Probability Theory: The Logic of Science*. (Cambridge University Press, Cambridge). Ed. by G. Larry Bretthorst. First publ. 1994. doi:10.1017/CBO9780511790423, https://archive.org/details/XQUHIUXHIQUHIQXUIHX2, http://www-biba.inrialpes.fr/Jaynes/prob.html.