Quantifying uncertainty
about the contents of sequences of urns

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Note: Dear Reader & Peer, this manuscript is being peer-reviewed by you. Thank you.

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Quantifying our uncertainty about the contents of a urn that contains, say, white and black balls in unknown proportions is a standard textbook topic. Our uncertainty depends on our data about the urn – draws – and on our state of knowledge about the urn before we knew those data. Jaynes's book (2003 chs 3 & 6) gives a superb analysis and discussion of this problem and of the formulae that typically arise there; and even of more realistic variants (ibid. § 3.9).

In this note I derive some formulae for a more complex version of the problem above, and some approximations of these formulae. I believe that they can be useful in several scientific questions.

The problem we consider is the following. There is an urn with N balls, which can be white or black. At regular time intervals every ball can change its colour: a white can turn black, or vice versa; or it can keep its colour. We can observe n out of the N balls for T time intervals. The n observed balls are initially chosen in a way unknown to us. Note that we observe the *same n* balls at all times. We ask several questions about this magic urn:

Q1   What was the proportion of white and black balls in the urn at some specific time, among the times we observed?

Q2   What was (or will be) the proportion of white and black balls in the urn at some specific time, among the times we did not observe?

Q3   How frequently did every possible proportion of white and black balls appear, during the times we observed?

Q4   How frequently will every possible proportion of white and black balls appear, during all times the urn exists?

Bibliography

('de *X*' is listed under D, 'van *X*' under V, and so on, regardless of national conventions.)

Jaynes, E. T. (2003): *Probability Theory: The Logic of Science.* (Cambridge University Press, Cambridge). Ed. by G. Larry Bretthorst. First publ. 1994. https://archive.org/detail ls/XQUHIUXHIQUHIQXUIHX2, http://www-biba.inrialpes.fr/Jaynes/prob.html.