

The Pitman-Koopman theorem and informationally relevant quantities

research notes

Y. Roudi

<yasser.roudi@ntnu.no>

P.G.L. Porta Mana

<piero.mana@ntnu.no>

23 June 2018; updated 24 June 2018

I believe that the greatest unsolved problem in statistics
is communicating the subject to others

N. Davies (Copas et al. 1995 p. 445)

1 From qualitative to quantitative statements

We often have the need of quantifying a qualitative property or phenomenon, for example to ascertain whether it does or doesn't occur in some situations. This quantification is usually difficult because the property itself is only vaguely defined in our minds.

At times we sweep this difficulty under the carpet by choosing a quantitative measure that 'feels right' and proceeding with the calculations. It's obvious that the quantitative results we obtain this way are mostly smoke: despite the deceiving appearance of numbers, our results are only qualitative, since our starting point was vague and the quantitative measure wasn't unique. Which results would we have obtained if we had started from a slightly different starting point or used a slightly different quantitative measure?

Other times we choose several quantitative measures that 'feel right', proceed with the calculations, and then compare the quantitative results obtained from all these measures. Results obtained this way are more robust – for example if our measures lead to the conclusion that the property is there – but unfortunately still not unequivocally quantitative.

In science there are successful examples of qualitative definitions that lead to unique quantitative measures or theories. The most impressive example is probably that of the probability calculus: simple intuitive desiderata about reasoning in situations of uncertainty, such as those

examined by Pólya (1971; 1949; 1954; 1968), turn out to have a unique mathematical translation: the laws of the probability calculus (Cox 1946; Paris 2006; Jaynes 2003 ch. 2; Snow 1998; 2001; Aczél 2004; Dupré et al. 2009; Terenin et al. 2017).

Thanks

PGLPM thanks Mari & Miri for continuous encouragement and affection; Buster Keaton and Saitama for filling life with awe and inspiration; the developers and maintainers of L^AT_EX, Emacs, AUC_TE_X, Open Science Framework, Python, Inkscape, Sci-Hub for making a free and unfiltered scientific exchange possible.

Bibliography

- (‘de X ’ is listed under D, ‘van X ’ under V, and so on, regardless of national conventions.)
- Aczél, J. (2004): *The associativity equation re-revisited*. Am. Inst. Phys. Conf. Proc. **707**, 195–203.
- Chatfield, C. (1995): *Model uncertainty, data mining and statistical inference*. J. Roy. Stat. Soc. A **158**³, 419–444. See also discussion (Copas, Davies, Hand, Lunneborg, Ehrenberg, Gilmour, Draper, Green, et al. 1995).
- Copas, J. B., Davies, N., Hand, D. J., Lunneborg, C. E., Ehrenberg, A. S., Gilmour, S. G., Draper, D., Green, P. J., et al. (1995): *Discussion of the paper by Chatfield [Model uncertainty, data mining and statistical inference]*. J. Roy. Stat. Soc. A **158**³, 444–466. See (Chatfield 1995).
- Cox, R. T. (1946): *Probability, frequency, and reasonable expectation*. Am. J. Phys. **14**¹, 1–13. <http://algomagic.org/ProbabilityFrequencyReasonableExpectation.pdf>.
- Dupré, M. J., Tipler, F. J. (2009): *New axioms for rigorous Bayesian probability*. Bayesian Anal. **4**³, 599–606.
- Jaynes, E. T. (2003): *Probability Theory: The Logic of Science*. (Cambridge University Press, Cambridge). Edited by G. Larry Bretthorst. First publ. 1994. <https://archive.org/de-tails/XQUHIUXHIQUHIQXUIHX2>, <http://www-biba.inrialpes.fr/Jaynes/prob.html>, <http://omega.albany.edu:8008/JaynesBook.html>.
- Paris, J. B. (2006): *The Uncertain Reasoner’s Companion: A Mathematical Perspective*, reprint. (Cambridge University Press, Cambridge). See also (Snow 1998).
- Pólya, G. (1949): *Preliminary remarks on a logic of plausible inference*. Dialectica **3**^{1–2}, 28–35.
- (1954): *Mathematics and Plausible Reasoning: Vol. I: Induction and Analogy in Mathematics*. (Princeton University Press, Princeton).
- (1968): *Mathematics and Plausible Reasoning: Vol. II: Patterns of Plausible Inference*, 2nd ed. (Princeton University Press, Princeton). First publ. 1954.
- (1971): *How To Solve It: A New Aspect of Mathematical Method*, 2nd ed. (Princeton University Press, Princeton). First publ. 1945.

- Snow, P. (1998): *On the correctness and reasonableness of Cox's theorem for finite domains*. Comput. Intell. **14**³, 452–459.
- (2001): *The reasonableness of possibility from the perspective of Cox*. Comput. Intell. **17**¹, 178–192.
- Terenin, A., Draper, D. (2017): *Cox's theorem and the Jaynesian interpretation of probability*. [arXiv:1507.06597](#). First publ. 2015.