Given sample, infer population

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An analysis of the problem of inferring the state of a population of neurons from that of a sample.

Note: Dear Reader & Peer, this manuscript is being peer-reviewed by you. Thank you.

1 The Question

The problem we want to consider is the inference about the state of a population of neurons from the observation of the state of a sample of that population. By inference we mean the numerical evaluation of our degree of belief about the population's state. We consider the state at one instant of time. The state will be taken to be the binarized activity from a time-binned sequence; but our discussion shall apply to more general definitions of 'state'. Denote by N the size of the population, by n that of the sample, by S_i the activity of the ith neuron in the population, by $\boldsymbol{S} \coloneqq (S_1, \dots, S_N)$ the joint activity – the state – and by s_j, $\boldsymbol{s} \coloneqq (s_1, \dots, s_n)$ the corresponding activities and state of the sample.

Let's make some additional remarks to better understand what our inference is about.

First, our inference is not about the dynamics of the population. In the problem of inferring the dynamics, we assume that the state $\boldsymbol{S}(t)$ at time t is determined through a dynamical law by the state $\boldsymbol{S}(t - \triangle t)$ at time $t - \triangle t$ together with some external quantities $Q(t - \triangle t)$:

$$\boldsymbol{S}(t) = F[\boldsymbol{S}(t - \triangle t), Q(t - \triangle t)], \tag{1}$$

but we are uncertain about the mathematical form of the dynamical law F and the values of the external quantities $Q(t - \triangle t)$. For this reason we can express our degree of belief about $\boldsymbol{S}(t)$ given only our knowledge about $\boldsymbol{S}(t - \triangle t)$ and other initial assumptions I:

$$\mathrm{p}[\boldsymbol{S}(t) \mid \boldsymbol{S}(t - \triangle t), I], \tag{2}$$

and, given a sequence of states, our degree of belief about the dynamical law:

$$\mathrm{p}[F \mid \boldsymbol{S}(t_1), \boldsymbol{S}(t_2), \dots, I]. \tag{3}$$

Our present problem doesn't concern this inference of the dynamics, but is very relevant to it, because to infer the dynamics we must infer the population states $\boldsymbol{S}(t_k)$ first, from the observation of a population sample.