Beware of stochastic explanations

P.G.L. Porta Mana
<piero.mana@ntnu.no>

Draft of 17 February 2019 (first drafted 16 February 2019)

***

Note: Dear Reader & Peer, this manuscript is being peer-reviewed by you. Thank you.

Modern scientific literature abounds with statements and explanations involving the terms 'stochastic', 'random', and similar ones. Here is a recent example from the neuroscientific literature:

> *stochastic spike arrival in the input can lead to a broad interspike interval distribution in the output of a neuron.* (1)

I won't give the source of this statement because my discussion concerns this kind of statements in general, not just this particular one; and I consider its source an otherwise excellent work.

The statement above is grammatically correct. It contains several technical terms and seems to have a meaning. It also suggests the following ideas:

1. stochasticity is a possible physical property of spike arrival;
2. the stochasticity of spike arrival can be the cause of a broad interspike-interval distribution; hence,
3. if a broad interspike-interval distribution is observed, a possible cause is the stochasticity of the spike arrival.

Unfortunately statement (1) and the three ideas it suggests are false or meaningless – 'not even wrong', as Pauli would say (Peierls 1960). Let's analyse it.

We must first ask: what does *stochastic* mean? Scientific works that try to explain this term – and not all do – define it in terms that are just as vague, such as *randomness* or *chance*. I can't recall any work in which the meaning of the latter terms is accurately given. This nonchalant attitude is unexpected in modern science, considering that Einstein (1905 § 1) showed that even an apparently unambiguous term like *simultaneous* can be operationally ill-defined.[1]

[1] The scientific community felt shock and shame at Einstein's *semantic* findings, as can be sensed in this passage by Bridgman:

Since scientific texts don't help us, let us consult a dictionary. The Concise Oxford English Dictionary (2004) defines *stochastic* in terms of *random*, and *random* as 'made, done, or happening without method or conscious decision'.[2] I'd like to refine this definition by saying 'without *apparent* method', since there could be a method but we aren't aware of it.

Statement (1) is therefore equivalent to this:

> *spike arrival that happens without apparent method in the input can lead to a broad interspike interval distribution in the output of a neuron.* (1′)

If you want to argue against the equivalence of the two statements, then you first owe me an accurate, non-circular definition of *stochastic*, *random*, and similar words.

The rephrased statement (1′) is trivial and empty. Sure, spikes arriving without apparent method might lead to a broad interspike-interval distribution. And then again they just might not. Likewise for spikes arriving *with* apparent method: they might or might not. The ideas suggested by the original statement are therefore false:

(1′) the absence of apparent method is not a physical property of spike arrivals, but just our state of knowledge about them;

(2′) there isn't any causal or explanatory relationship between the apparent absence of method in spike arrival and a broad interspike-interval distribution; hence

(3′) if a broad interspike-interval distribution is observed, we can't say whether the was an apparent method in spike arrival or not.

Note in particular that the second idea would amount to believing in telekinesis: our state of knowledge about the input would influence the interspike-interval distribution of the neuron's output.

> It was a great shock to discover that classical concepts, accepted unquestioningly, were inadequate to meet the actual situation, and the shock of this discovery has resulted in a critical attitude toward our whole conceptual structure which must at least in part be permanent. Reflection on the situation after the event shows that it should not have needed the new experimental facts which led to relativity to convince us of the inadequacy of our previous concepts, but that a sufficiently shrewd analysis should have prepared us for at least the possibility of what Einstein did. (Bridgman 1958 p. 1)

[2] It also gives the more specific meaning 'governed by or involving equal chances for each item', but this circular definition (what's *chance*?) is not usually associated with *stochastic*. Originally, *random* roughly meant 'fast', a meaning still visible in 'random-access memory'.

You may think that in the end I'm just quibbling, and there's no real problem out there. I beg you to reconsider the importance of my complaints, considering the following facts:

a. If you search the literature for works containing words like *stochastic* or *random*, and you do a careful exegesis of them, you'll find plenty of statements similar to (1) that on a deeper analysis turn out to be empty or trivial. Even if their presence doesn't affect the overall correctness of the work, they're often so numerous that they become a nuisance and make you wonder whether the authors are thinking about what they're saying, or whether they're just writing memorized linguistic constructions. (Again, I don't want to give explicit examples.)

b. My impression is that scientific education is becoming more and more just a high-level kind of memorization. Students are *trained* in a vastly more subtle and complex version of this: 'whenever you see this formula, say blah blah stochastic process blah blah. If they ask you about the stochastic process, reply blah blah random blah blah. That should settle it. But if they ask about random, hide your ignorance by smirking and implying that they are too naive or ignorant about this subject'. Note that Feynman complained about this state of affairs already in 1985. Referring to freshmen and possibly to a research assistant, he complains:

> I don't know what's the matter with people: they don't learn by understanding; they learn by some other way – by rote, or something. Their knowledge is so fragile!
>
> (Feynman 1989 p. 23)

We don't help our students and young researchers to learn by understanding, rather than by rote, if we riddle our scientific works with technically worded statements that are seemingly meaningful but actually empty.

c. *Semantics* has always been important in science. The preceding example with Einstein and *simultaneity* is the best known, but there are many others. Modern science has grown from the unrelenting analysis and refinement of notions like force, momentum, stress, energy; and this analysis goes on (think about *entropy*). Euler (1757 esp. § XIII) gave new, fruitful directions to mechanics by modifying our concept of *pressure* (Truesdell 1954a see the full account in). Born (1926a p. 865) gave a new understanding of quantum mechanics by reinterpreting the meaning of the wave function.

Before I close with a hope and a recommendation, let me point out a curious and significant fact. James Clerk Maxwell was one of the main creators of kinetic theory and statistical mechanics. If we search for the word 'random' in the whole two volumes of his *Scientific Papers* (1965a,b), his *Theory of Heat* (1875), and his *Matter and Motion* (1920), we *will not find any occurrences* except for one article (Maxwell 1855 p. 281) where that word is only colloquially used, outside of technical statements. If we search for 'chance', we find only three instances (Maxwell 1965a p. 488; 1965b p. 374; 1875 p. 30) – again colloquial uses outside of technical statements. Why didn't Maxwell use this word, even when dealing with kinetic theory and statistical mechanics? It wasn't because of lack of sophistication with technical terms – in fact he widely uses actions and Lagrangians, and even presents his electromagnetic theory using quaternions (Maxwell 1881a e.g. ch. IV; 1881b e.g. ch. IX). Anyone who's read some of his works knows that he always strove for precision and simplicity (Campbell et al. 1882). I contend that he avoided 'random' and 'chance' in technical statements because he was aware of their ambiguity and their lack of physical consequences. Also significant is that Gibbs, in his *Elementary Principles in Statistical Mechanics* (1902), uses only the expression 'at random' (that is, 'with equal probabilities'), and only a dozen times; he never uses 'chance'. Readers of Gibbs know that he also held precision very highly (see the colourful account in Wightman 1990), although his prose occasionally lacks clarity. (The term 'stochastic' appeared only later: see Shafer 2018).

Bibliography

('de *X*' is listed under D, 'van *X*' under V, and so on, regardless of national conventions.)

Concise Oxford English Dictionary (2004), 11th ed. Oxford University Press. First publ. 1911.

Born, M. (1926a): *Zur Quantenmechanik der Stoßvorgänge*. Z. Phys. **37**[12], 863–867. http://www.physics.princeton.edu/~mcdonald/examples/EM/. See also Born (1926b,c).

— (1926b): *Quantenmechanik der Stoßvorgänge*. Z. Phys. **38**[11–12], 803–827. http://www.physics.princeton.edu/~mcdonald/examples/EM/. See also Born (1926a,c).

— (1926c): *Zur Wellenmechanik der Stoßvorgänge*. Ges. d. Wiss. Nachrichten (Gött.) Math.-Phys. Kl. **1926**, 146–160.

Bridgman, P. W. (1958): *The Logic of Modern Physics*, eight printing. (Macmillan, New York). First publ. 1927.

Caldi, D. G., Mostow, G. D., eds. (1990): *Proceedings of the Gibbs Symposium: Yale University, May 15–17, 1989*. (American Mathematical Society, Providence, USA).

Campbell, L., Garnett, W. (1882): *The Life of James Clerk Maxwell: With a Selection from His Correspondence and Occasional Writings and a Sketch of His Contributions to Science*. (Macmillan, London).

Einstein, A. (1905): *Zur Elektrodynamik bewegter Körper*. Ann. der Phys. **17**, 891–921. Transl. in Einstein (1989a), Doc. 23, pp. 140–171.

— (1989a): *The Collected Papers of Albert Einstein. Vol. 2: The Swiss Years: Writings, 1900–1909. (English translation)*. (Princeton University Press, Princeton). Transl. of Einstein (1989b) by Anna Beck and Peter Havas.

— (1989b): *The Collected Papers of Albert Einstein. Vol. 2: The Swiss Years: Writings, 1900–1909*. (Princeton University Press, Princeton). Ed. by John Stachel. Transl. in Einstein (1989a).

Euler, L. (1757): *Principes généraux de l'état d'équilibre des fluides*. Mémoires de l'académie des sciences de Berlin **11**, 217–273, 402(plates). http://eulerarchive.maa.org/; presented 1753; repr. in *Opera Omnia* series 2, vol. 12, pp. 2–53.

Feynman, R. P. (1989): *"Surely You're Joking, Mr. Feynman!": Adventures of a Curious Character*, reprint. (Bantam, New York). 'As told to Ralph Leighton', ed. by Edward Hutchings. First publ. 1985.

Gibbs, J. W. (1902): *Elementary Principles in Statistical Mechanics: Developed with Especial Reference to the Rational Foundation of Thermodynamics*. (Charles Scribner's Sons, New York). https://archive.org/details/ElementaryPrinciplesInStatisticalMechanics; repr. in Gibbs (1928), Part One.

— (1928): *The Collected Works of J. Willard Gibbs. Vol. II: Statistical Mechanics and Dynamics*. (Longmans, Green and Co., New York). First publ. 1906.

Maxwell, J. C. (1855): *Experiments on colour, as perceived by the eye, with remarks on colour-blindness*. Trans. Roy. Soc. Edinb. **XXI**[II], 275–298. Repr. in Maxwell (1965a), pp. 126–154, Plate I. http://www.jimworthey.com/archive/.

— (1875): *Theory of Heat*, 4th ed. (Longmans, Green, & Co., London). https://books.google.com/books/about/Theory_of_Heat.html?id=p_lYAAAAYAAJ. Last edition published during Maxwell's life. First publ. 1871.

— (1881a): *A Treatise on Electricity and Magnetism. Vol. I*, 2nd ed. (Oxford University Press, Oxford). Last edition worked upon by Maxwell. First publ. 1873. https://archive.org/details/treatiseonelectr01maxwrich.

Maxwell, J. C. (1881b): *A Treatise on Electricity and Magnetism. Vol. II*, 2nd ed. (Oxford University Press, Oxford). Last edition worked upon by Maxwell. First publ. 1873. https://archive.org/details/treatiseonelect02maxwrich.

— (1920): *Matter and Motion*. (Macmillan, New York). 'With notes and appendices by Sir Joseph Larmor'. First publ. 1877.
— (1965a): *The Scientific Papers of James Clerk Maxwell. Vol. One*, reprint. (Dover, New York). Ed. by W. D. Niven. https://archive.org/details/scientificpapers01maxwuoft. First publ. 1890.
— (1965b): *The Scientific Papers of James Clerk Maxwell. Vol. Two*, reprint. (Dover, New York). Ed. by W. D. Niven. https://archive.org/details/scientificpapers02maxwuoft. First publ. 1890.
Peierls, R. E. (1960): *Wolfgang Ernst Pauli: 1900–1958*. Biogr. Mem. Fellows Royal Soc. **5**, 174–192.
Shafer, G. (2018): *When to call a variable random*. Tech. rep. 41. (The Game-Theoretic Probability and Finance Project). http://www.probabilityandfinance.com/articles/41.pdf. First publ. 2015.
Truesdell III, C. A. (1954a): *Rational Fluid Mechanics, 1687–1765*. In: Truesdell (1954b), VII–CXXV. Editor's introduction to Euleri Opera omnia II 12.
— ed. (1954b): *Leonhardi Euleri commentationes mechanicae ad theoriam corporum fluidorum pertinentes. Vol. I*. (Orell Füssli, Zurich).
Wightman, A. S. (1990): *On the prescience of J. Willard Gibbs*. In: Caldi, Mostow (1990), 23–38.