Parameter priors for Ising models

research notes

Y. Roudi
<yasser.roudi@ntnu.no>

P.G.L. Porta Mana
<piero.mana@ntnu.no>

25 June 2018; updated 26 June 2018

Study of uniform priors in parameter space and in constraint space for Ising models

'Flat priors do not exist'
(anonymous)

1 A two-unit model with sufficient statistics

Consider a population of two binary units $\boldsymbol{s} \coloneqq (s_1, s_2)$ with values in $\{0, 1\}$. One observation of this population can thus give four results: $\boldsymbol{s} \in \{00, 01, 10, 11\}$.

Assume that we have m observations $(\boldsymbol{s}^{(1)}, \dots, \boldsymbol{s}^{(m)})$ of this or other populations prepared in similar conditions, so that knowledge of these observations is relevant for our forecast of a new observation $\boldsymbol{s}$, again in similar conditions. Also assume that only the number, the mean, and the second moments of these past observations are relevant to forecast the new one; that is,

$$m, \qquad \tfrac{1}{m}\big(\boldsymbol{s}^{(1)} + \dots + \boldsymbol{s}^{(m)}\big), \qquad \tfrac{1}{m}\big(\boldsymbol{s}^{(1)}\boldsymbol{s}^{(1)\intercal} + \dots + \boldsymbol{s}^{(m)}\boldsymbol{s}^{(m)\intercal}\big) \tag{1}$$

are sufficient statistics; note that the second sum contains the first as its diagonal. These assumptions are collectively denoted I. Then the Koopman-Pitman theorem says that our probabilistic forecast must assume this form:

$$\mathrm{P}(\boldsymbol{s} \mid \boldsymbol{s}^{(1)}, \dots, \boldsymbol{s}^{(m)}, I) = \int \tag{2}$$