Given sample, infer population

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An analysis of the problem of inferring the state of a population of neurons from that of a sample.
Note: Dear Reader & Peer, this manuscript is being peer-reviewed by you. Thank you.

1 The Question

The problem we want to consider is the inference about the state of a population of neurons from the observation of the state of a sample of that population. By inference we mean the numerical evaluation of our degree of belief about the population's state. The state is taken to be the binarized activity from a time-binned sequence. Denote by N the size of the population, by n that of the sample, by $S_i(t)$ the activity of the ith neuron in the population at time t, by $\boldsymbol{S}(t) \coloneqq \bigl(S_1(t), \dots, S_N(t)\bigr)$ the joint activity – the state – of the population, and by $s_j(t)$, $\boldsymbol{s}(t) \coloneqq \bigl(s_1(t), \dots, s_n(t)\bigr)$ the corresponding activities and state of the neurons making up the sample. Many points of our discussion, however, apply to more general definitions of 'state'.

In order to better understand what our inference is about, let's also stress what it is *not* about. Our inference is not about the *dynamics* of the population. This latter inference is roughly as follows. We assume that the state $\boldsymbol{S}(t)$ at time t is determined through a dynamical law by the states $\{\boldsymbol{S}(\tau)\}_{\tau<t}$ at some previous times together with some external quantities $Q(t)$ (such as physical states of synapses, inputs from peripheral nervous system, and similar extra-neuronal quantities):

$$\boldsymbol{S}(t) = F[\{\boldsymbol{S}(\tau)\}_{\tau<t}, Q(t)]. \tag{1}$$

We are uncertain about the mathematical form of the dynamical law F and the values of the external quantities $Q(t)$. We can therefore consider various degrees of belief: for example the one about $\boldsymbol{S}(t)$ given only knowledge about some previous states and other initial assumptions I:

$$\mathrm{p}[\boldsymbol{S}(t) \mid \{\boldsymbol{S}(\tau)\}_{\tau<t}, I], \tag{2}$$

or the one about the dynamical law, given a time sequence of states:

$$\mathrm{p}[F \mid \{\boldsymbol{S}(\tau)\}, I]. \tag{3}$$

Our present problem doesn't concern this kind of inferences, but is very relevant to them: to infer the dynamics, eq. (3), we usually must first infer the population states $\boldsymbol{S}(\tau)$ from the observation of a population sample.

Note that if the dynamics is excluded from our problem, then samples at times $\tau < t$ cannot be used for the inference of the population state at time t, because such inferential chain involves the dynamics: schematically, the inference would be $\boldsymbol{s}(\tau) \rightsquigarrow \boldsymbol{S}(\tau) \rightsquigarrow \boldsymbol{S}(t)$, and the latter step involves the degree of belief (2). Our discussion will therefore refer to one time t only, which is conveniently suppressed from our notation.