Regression and exchangeability

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Exchangeability as a form of regression, regression as a form of exchangeability.

Note: Dear Reader & Peer, this manuscript is being peer-reviewed by you. *Thank you.*

1 Regression

'Regression', as usually intended, is the problem of inferring the value of a quantity x given that of another quantity t; the two quantities having values in two specific sets. It is also assumed that in two or more observations with the same value of t the values of x may be different.

In this note I use 'regression' in a restricted sense: two or more observations with the same value of t have also the same value of x. So the second quantity is given by a function of the first. This is the case in many classification problems and similar inferences. The focus in this kind of problems is to infer x in an observation with a new value of t, given observations of the pair of quantities for different values of t.

If we assume for the moment that t and x assume finite sets of values, then also the set of functions between them is finite. Regression in the restricted sense above is then equivalent to inferring a function $f : t \mapsto x$:

$$\mathrm{p}(x|\,t, I) = \int \mathrm{d}f\; \mathrm{p}(x|\,t, f, I)\; \mathrm{p}(f|\,I), \tag{1}$$

with

$$\mathrm{p}(x|\,t, f, I) = \delta[x - f(t)]. \tag{2}$$

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