On the difference between statistics and probability

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Note: Dear Reader & Peer, this manuscript is being peer-reviewed by you. Thank you.

I must confess to a sin: I've always used 'statistical' and 'probabilistic' as interchangeable terms in most contexts. I know of the technical difference between 'statistical model' and 'probabilistic model' made by some statisticians (McCullagh 2002); from a Bayesian point of view it's an artificial distinction, but more importantly it doesn't do justice to the original useful distinction between 'statistical' and 'probabilistic' which we're now going to explore.

If we read Maxwell's paper *On the dynamical evidence of the molecular constitution of bodies* (1875) we find this passage on page 374:

> By distributing the molecules into groups according to their velocities, we may substitute for the impossible task of following every individual molecule through all its encounters, that of registering the increase or decrease of the number of molecules in the different groups.
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> By following this method, which is the only one available either experimentally or mathematically, we pass from the methods of strict dynamics *to those of statistics and probability*.

The final emphasis is mine, and stresses the most interesting part of this passage for our present discussion. Maxwell was very scrupulous and precise in his choice of words. Here he doesn't say 'to those of statistics' (full stop), nor does he say 'to those of probability'. He says 'to those of statistics *and* probability'. The two terms must therefore denote for him two distinct notions.

Maxwell conceived probability as a calculus of uncertainty that generalizes logic – quite a modern view (cf. Hailperin 1996; 2011). His view can be glimpsed in this letter of his to L. Campbell (June? 1850: Campbell et al. 1882 ch. V, p. 143):

> They say that Understanding ought to work by the rules of right reason. These rules are, or ought to be, contained in Logic; but the actual science of Logic is conversant at present only with

things either certain, impossible, or *entirely* doubtful, none of which (fortunately) we have to reason on. Therefore the true Logic for this world is the Calculus of Probabilities, which takes account of the magnitude of the probability (which is, or which ought to be in a reasonable man's mind).

What, then, is *statistics* for Maxwell? We find a beautiful explanation on page 440 of his *Molecules* (1873):

> When the working members of Section F[1] get hold of a Report of the Census, or any other document containing the numerical data of Economic and Social Science, they begin by distributing the whole population into groups, according to age, income-tax, education, religious belief, or criminal convicts. The number of individuals is far too great to allow of their tracing the history of each separately, so that, in order to reduce their labour within human limits, they concentrate their attention on small number of artificial groups. The varying number of individuals in each group, and not the varying state of each individual, is the primary datum from which they work.
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> This, of course, is not the only method of studying human nature. We may observe the conduct of individual men and compare it with that conduct which their previous character and their present circumstances, according to the best existing theory, would lead us to expect. Those who practise this method endeavour to improve their knowledge of the elements of human nature, in much the same way as an astronomer corrects the elements of a planet by comparing its actual position with that deduced from the received elements. The study of human nature by parents and schoolmasters, by historians and statesmen, is therefore to be distinguished from that carried on by registrars and tabulators, and by those statesmen who put their faith in figures. The one may be called the historical, and the other the statistical method.
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> The equations of dynamics completely express the laws of the historical method as applied to matter, but the application of these equations implies a perfect knowledge of all the data. But the smallest portion of matter which we can subject to experiment consists of millions of molecules, not one of which ever becomes individually sensible to us. We cannot, therefore, ascertain the actual motion of any one of these molecules, so that we are obliged to

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[1] I believe Maxwell is referring to a Section (Economic Science and Statistics) of the British Association for the Advancement of Science.

abandon the strict historical method, and to adopt the statistical method of dealing with large groups of molecules.

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('de *X*' is listed under D, 'van *X*' under V, and so on, regardless of national conventions.)

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