The relation between leave-one-out log-scores and log-evidence

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Note: Dear Reader & Peer, this manuscript is being peer-reviewed by you. Thank you.

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The probability calculus tells us unequivocally how our degree of belief in a hypothesis H_h given data D and knowledge K is related to our degree of belief in observing those data when we entertain that hypothesis as true:

$$\mathrm{P}(H_h \mid DK) = \frac{\mathrm{P}(D \mid H_h K)\,\mathrm{P}(H \mid K)}{\mathrm{P}(D \mid K)}. \tag{1}$$

The probability in the denominator is usually calculated by considering a set of mutually exclusive and exhaustive hypotheses $\{H_h\}$ comprising H_h and using the law of total probability:

$$\mathrm{P}(D \mid K) = \sum_h \mathrm{P}(D \mid H_h K)\,\mathrm{P}(H_h \mid K). \tag{2}$$

Let's call $\mathrm{P}(H_h \mid DK)$ the post-data probability or belief in H_h. If our pre-data beliefs $\mathrm{P}(H_h \mid K)$ in the hypotheses under consideration are all equal, then the ratio of the post-data probabilities for the various hypotheses is equal to the ratio of the beliefs in the data given the hypotheses:

$$\frac{\mathrm{P}(H_h \mid DK)}{\mathrm{P}(H_{h'} \mid DK)} = \frac{\mathrm{P}(D \mid H_h K)}{\mathrm{P}(D \mid H_{h'} DK)} \quad \text{(if } \mathrm{P}(H_h \mid K) = \mathrm{P}(H_{h'} \mid K)\text{)}. \tag{3}$$

Such ratio or its logarithm is variously called *weight of evidence* and *Bayes factor* (**good1950**; **osteyeeetal1974**; **mackay1992**; **kassetal1995jeffreys1939_r1983**).

'It is historically interesting that the expression "weight of evidence", in its technical sense, anticipated the term "likelihood" by over forty years.' (**osteyeeetal1974**)