

Measuring the influence of prior beliefs on probabilistic estimations.

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Abstract

Previous research has shown that the mental models participants use throughout a task influence the efficiency with which they learn and adapt to changes in their environment (Lee & Johnson-Laird, 2013; Stöttinger et al., 2014). We wanted to measure the influence of different types of mental models participants hold *before* engaging in a task. Using a modified version of the game “Plinko”, participants predicted the likelihood that a ball falling through pegs would land in one of forty slots. Importantly, participants were asked to make likelihood estimations before seeing the first ball drop. This initial probability estimate was used to categorize participants into different groups based on distinct a priori models. Results indicated that participants came into this task with a number of distinct initial models, and that the type of model influenced their ability to accurately represent different distributions of ball drops in Plinko.

Introduction

Humans are proficient at detecting regularities in their environment (Turk-Browne et al., 2005; Griffiths & Tenenbaum, 2006). This ability allows us to compress large volumes of sensory information and build mental models to represent the events we perceive (Tenenbaum et al., 2011). When these models fail to explain certain observations, they must be updated to reflect new environmental contingencies (Danckert et al., 2012; Filipowicz et al., 2013). The ability to build and update models depends in part on the expectations we have when interpreting sensory information (Lee & Johnson-Laird, 2013; Stöttinger et al., 2014). The aim of the present study was to explore the role of prior expectations on model building and updating.

A large body of research has demonstrated the efficiency with which humans detect regularities in their environment (Turk-Browne et al., 2005). These processes can occur automatically (Turk-

Browne et al., 2005; Nissen & Bullemer, 1987) and manifest themselves at an early age (Saffran et al., 1996). Yet despite this seemingly optimal proficiency, studies have found consistent *suboptimal* behavior on certain statistical learning tasks. One classic example is a phenomenon known as *probability matching*: when asked to predict the result of a stochastic event with a specific rate of bias, rather than choose the biased event 100% of the time, participants tend to predict the biased event at the same rate as its underlying probability (e.g., if a biased coin comes up heads 70% of the time, participants will choose heads as the likely next outcome on 70% of their guesses rather than following the optimal prediction strategy of choosing heads 100% of the time; Vulkan, 2000). How do we reconcile findings that show optimality in some forms of learning, yet suboptimal behavior in others?

Mental model theory attempts to explain this discrepancy by implicating prior knowledge in our ability to learn from our environment. One of the primary tenets of this theory describes mental models as being formed by an interaction between our direct perception of events and the knowledge we have accumulated over our lifetime (Johnson-Laird, 2004). This theory suggests that our a priori expectations related to specific events influence our current predictions. Indeed, Green and colleagues (2010) found that probability matching behaviour depended on a participant’s belief regarding the underlying process responsible for generating the events. Participants who believed they had control over the task parameters were much more likely to maximize their selection of the optimal choice than those that revealed uncer-

tainty about the task's underlying generative process (Green et al., 2010). This suggests that our ability to build and update models depends not only on the current sensory information we are attempting to interpret, but also on the beliefs and expectations we use to interpret the information. In support of this notion, studies have found that model building and updating are facilitated when the expectations relating to a task closely match the task's underlying structure (Lee & Johnson-Laird, 2013; Stöttinger et al., 2014).

Most research exploring our ability to exploit event regularities does not take into account a participant's prior belief insofar as they are not measured prior to commencing the task of interest. Clearly, it is challenging to first objectively establish what prior beliefs a participant has before they begin a task in which we have external control of the contingencies. Here we present a novel task that provides a measure of a participant's a priori expectations coming into the task, *before* they have seen or responded to any actual stimulus events. This allows us to then measure the influence of these a priori models on a participant's ability to learn and adapt to task probabilities. Using a computerized version of the game "Plinko", we had participants make estimations of the probability that a ball would land in a series of slots before starting the task. We used these initial estimates to categorize different participant models and measured the efficiency with which participants managed to learn a distribution of events.

Methods

Participants

40 undergraduates from the University of Waterloo participated in our study (27 female, mean age = 19.5 years, SD = 1.6 years). The study protocol was approved by the University of Waterloo's Office of Research Ethics and each participant gave informed consent before participating in the study.

Task environment and instructions

Participants were exposed to a computerized version of the game "Plinko" (a game featured on the American game show *The Price is Right*). The entire task environment was programmed in Py-

thon using the PsychoPy library (Peirce, 2009). In our game, a red ball would fall through a pyramid of pegs and land in one of 40 possible slots located side by side below the pegs. The pyramid consisted of 29 rows of black pegs that increased in number from the top to the bottom of the pyramid (i.e., the top row contained 1 peg and the bottom row contained 29 pegs). A rectangle was located below the 40 slots spanning their width. Participants were instructed to make their responses in this space (Fig. 1).

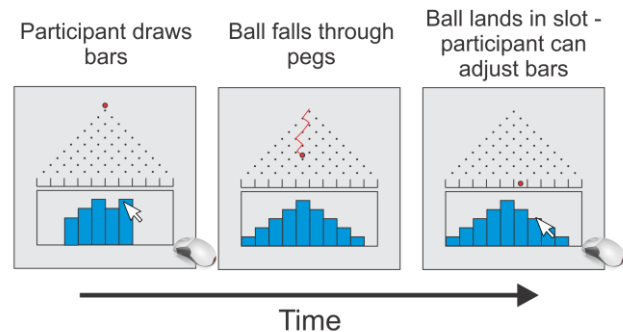


Figure 1. Example of a single trial.

Participants were instructed that a ball would fall through a series of pegs and that their goal was to accurately predict the likelihood that a ball would fall in any of the 40 slots on *future* trials. Participants adjusted bars under each slot in the space below the pyramid to represent their likelihood estimations. Bars were drawn using the computer mouse: the height of the bars could be adjusted by holding down the left mouse button and changing the position of the cursor. The height of the bar would match the position of the cursor within the limits of the rectangle below the slots. Participants could also erase a single bar by right clicking with the cursor on the bar they wished to delete, or by clicking the backspace key to delete all bars on the screen. The bars were not assigned any value; participants were simply told that taller bars represented a higher probability that a ball would fall in a slot, shorter bars a lower probability, and no bars represented zero probability. Participants were informed that they had the option of adjusting their bars at the start of every trial and that they had to have bars on screen before proceeding with the trial. Crucially, this instruction was applied at the start of the task – that is, participants had to indicate their likeli-

hood estimates *before* seeing the first ball drop. Once participants had indicated their likelihood estimates, they pressed the spacebar to proceed with the trial (Fig. 1).

Ball distributions

Participants were exposed to one of two distributions of ball drops. Both distributions were generated by randomly sampling 100 integers from Gaussian distributions with a mean of 17, but different standard deviations (6.0 and 1.9 respectively). The resulting sequences of 100 integers determined the slot in which the ball fell on each trial, with slot 1 representing the slot farthest to the left of the screen and slot 40 representing the slot farthest to the right. 20 participants were exposed to the distribution with a wider variance, while 20 participants were exposed to the distribution with a narrow variance. Participants in each of the two conditions were exposed to the same respective sequences of ball drops.

Results

Accuracy measurements

In order to measure participant estimates, participant bars were normalized on every trial. The height of each bar could have one of 100 equal height increments (a height of 1 being the shortest bar possible and 100 being the tallest bar possible). Each bar was normalized by taking its height and dividing it by the summed height of all drawn bars for that trial. This normalization provided a probability distribution of a participant's slot estimates on every trial. An accuracy score was generated for each participant on each trial by comparing the overlap between the participant's distribution of estimates and the computer's distribution of ball drops. Accuracy scores could range between 0 and 1, with 1 indicating perfect overlap between participant and computer distributions.

Participant initial distributions

We began by categorizing participants based on the shape of their initial probability distributions prior to seeing a single ball drop. We did this by plotting slot estimates on the first trial for each individual participant, and categorizing them

based on similarity in shape (Fig. 2). Initially we expected two primary types of distributions. Those potentially familiar with Sir Francis Galton's work may know that the expected distribution of ball drops in a Plinko game should approximate a normal distribution, with its mean centered on the ball's initial starting position (Galton, 1889). Those unfamiliar with the task may choose to take an approach of extreme uncertainty, and report a uniform distribution, similar to uniform priors used in Bayesian learning algorithms (e.g., Nassar et al., 2010). Of the 40 participants who completed the task, 7 participants reported a Gaussian-like shape as their first distribution, and 5 reported uniform distributions. Of the remaining 28 participants, 12 participants reported a bimodal distribution, 2 participants reported negatively skewed distributions, and 14 participants reported what we termed as "jagged" distributions, where participants only drew a few interspersed bars on the screen (in some cases only one bar; Fig.2).

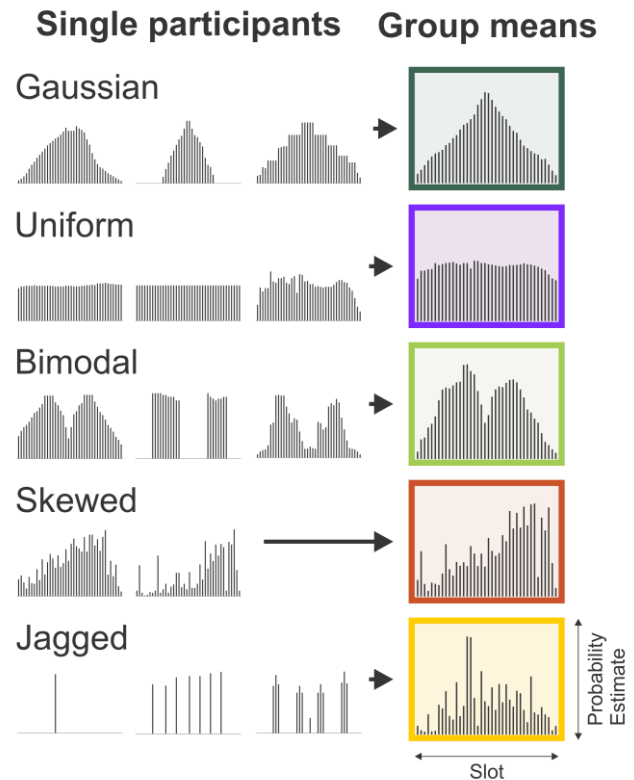


Figure 2. Participant estimates on the first trial. Participants were grouped into categories based on the shape of their initial distribution.

Measuring participant performance

In order to measure how well groups managed to learn each of the distributions, we fit a standard exponential learning curve to the changes in accuracy for each group (e.g., Estes, 1950; Heathcote, Brown, & Mewhort, 2000; Ritter & Schooler, 2001):

$$a_n = a_\infty - (a_\infty - a_0)e^{-\alpha n}$$

where n denotes the trial number, a_n a participant's estimated accuracy on trial n , a_0 initial accuracy, a_∞ asymptotic accuracy, and α a constant rate coefficient to capture how quickly participants reach their asymptote from their initial accuracy. We fit this function to each participant's accuracy scores using a nonlinear least squares function in the R statistical package (nls function; R Core Team, 2014). Given that participant accuracy could only range between 0 and 1, we set the upper and lower limits for both the asymptote and initial accuracy to 1 and 0 respectively. Each fit provided initial, asymptotic, and learning values for each participant. This function fit every participant except one in the uniform group who did not make any changes to their bars throughout the task. We excluded this participant's performance from our subsequent analyses, resulting in a sample of 19 participants exposed to the wide Gaussian distribution and 20 exposed to the narrow Gaussian.

Group initial accuracy

We began by comparing group performance in each of the two conditions of ball drops. Many of the initial participant distributions had high variances, spanning a large number of slots (average standard deviation for initial distribution = 8.83 slots). We therefore predicted that participant initial accuracy values would be lower for participants exposed to a distribution with a smaller variance. An independent t-test confirmed that initial accuracy values in the wide condition were higher than those in the narrow condition (Mean initial accuracy: wide = 0.45, narrow = 0.32; $t(37) = 2.106$, $p < 0.05$). Participants who reported initial estimates with high variance were primarily in the Gaussian, uniform, skew, and bimodal groups, particularly when compared to participants in the jagged condition. We expected that these first groups would have higher initial accuracy in the

wide condition given that their distributions would have more total overlap with the computer's wide distribution. Initial accuracy values ranged between 0.58 and 0.65 for non-jagged groups, with the Gaussian group having the highest initial accuracy value, while participants in the jagged group started with a mean initial accuracy value of only 0.18 (Fig. 3a,c).

These differences were much smaller in participant groups exposed to the narrow distribution given that the computer's distribution covered fewer slots than in the wide condition. Initial accuracy in all groups other than the skewed condition ranged between 0.30 and 0.39 (Fig. 3b,d). In the narrow condition there was only one participant that reported a skewed distribution. This participant's initial expectation was that the majority of balls would fall to the right of center. The computer's distribution fell slightly to the left of center, a stark difference from this participant's initial estimate. Despite having the lowest initial accuracy value, this participant's α value was second highest among participants in the narrow condition, indicating that this participant's accuracy rapidly reached its maximum value from its initial value. When examining the raw accuracy data, this participant's accuracy jumped from .17 to .75 within 8 trials, and stayed in this range for the remainder of the task.

Learning of each distribution

Next we examined the learning rates within the groups that were exposed to wide vs. narrow Gaussian distributions. Of particular interest was the comparison between participant asymptote values in the Gaussian group. Of the 7 participants categorized in the Gaussian group, 3 participants were exposed to the wide distribution, while 4 participants were exposed to the narrow distribution. In both cases, initial estimates for these participants had large variances (mean SD for wide group = 7.63 slots, narrow group = 9.20 slots). This does not present any major consequences for participants exposed to the wide distribution of ball drops, as their initial beliefs match the computer's variance. However, participants exposed to the narrow distribution would need to change the variance in their estimates to reflect the computer's distribution. In total, partic-

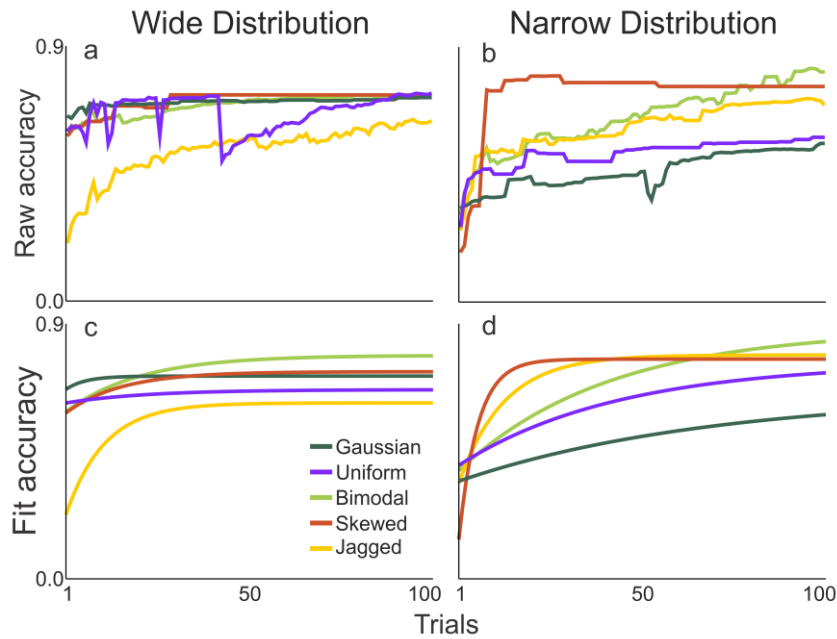


Figure 3. Participant accuracy performance in the narrow and wide distribution conditions. Raw accuracy is depicted in graphs (a) and (b) while fit accuracy is depicted in graphs (c) and (d).

ipants exposed to the wide distribution tended to have higher mean asymptote values than those in the narrow condition (mean asymptote values: wide = 0.72, narrow = 0.65). Of the 4 participants in the narrow group, 2 participants managed to adapt their distributions to reflect a narrow variance by the end of the task, resulting in a mean asymptote of 0.85, while the other 2 participants made fewer changes to the variance of their distributions, resulting in a mean asymptote of 0.45.

Online task performance

As a last step we were interested in tracking changes to participant distributions over the course of the experiment. One of the aspects of participant performance that is not fully captured by the exponential learning curve is some of the participant strategies used throughout the task. Of particular note, there was a large dip in accuracy in the uniform group at trial 43 (Fig. 3a). A closer look at individual performances showed that this dip is the result of one participant who deleted all bars on screen save one, and continued through the experiment by drawing bars trial by trial in slots that received a ball (ultimately leading to a final raw accuracy score of .81, second highest among participants exposed to the wide distribution).

A look at the trends from participants estimating the narrow distribution shows at least one par-

ticipant that followed a similar strategy. As indicated earlier, 2 of the 4 participants in the Gaussian group managed to adjust their distributions to reflect the tighter variance in ball drops. One of these participants deleted most of their bars on trial 53 and followed a strategy similar to that of the participant in the uniform condition (leading this participant to finish with the highest raw accuracy among those in the Gaussian group; Fig. 3b).

Discussion

The aim of the current study was to explore the influence of a priori mental models on our ability to learn from the regularity of events in our environment. Previous research has demonstrated that models we build *during* a task influence our ability to adapt to later changes in incoming information (Lee & Johnson-Laird, 2013; Stöttinger et al., 2014). We expanded on this research by measuring the influence of a participant's a priori expectations on their ability to learn the probability distribution of certain events occurring.

We demonstrated that far from being uniform, participant expectations coming into our task varied widely. In addition to the distributions we had predicted (i.e., Gaussian and uniform), 30% of the participants reported initially expecting a bimodal distribution of ball drops, while another 35% of participants followed a 'jagged' strategy. In the

groups that we did expect to find, performance varied, with participants in the Gaussian condition performing well on distributions that matched their initial estimate of variance, but having more difficulty representing distributions with narrower variance. Our task also provided us with the opportunity to see how these initial models changed over time. We found two examples in which participants completely abandoned their initial strategies and adopted new ones, eventually leading them to high levels of accuracy. We were able to locate when this shift in strategy occurred, and track the participants' new strategy as they progressed on future trials. Finally, we saw one example of a highly erroneous a priori model leading a participant to rapidly and efficiently adapt to the correct task contingencies. This example suggests that the level of mismatch between an expected model and observations can influence the efficiency with which we detect and adjust to prediction errors.

We acknowledge that our observations are based on small groups of participants, and that larger samples are needed to make more conclusive statements about the influence of specific a priori expectations. Nevertheless, our results provide evidence to support the notion that our mental models coming into a task are not always uniform, and can affect the way we learn and adapt to task contingencies.

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