Bayesian Plinko

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Draft of 14 April 2018 (first drafted 14 April 2018)

How does a Bayesian robot do at Plinko, using an infinitely exchangeable model and borrowing a human participant's prior?

1 The Bayesian robot

In the context of these notes we call 'model' any set of assumptions that allows us to assign a probability to a new observation, given a number of observations of a similar kind. Denote such assumptions by a proposition M – surely a proposition very difficult to express in writing – and the ith observation by $D_i \coloneqq$ 'The ith observation is d_i', with $d_i \in \{1, \dots N\}$. Then M allows us to give a numeric value to

$$\mathrm{P}(D_{m+1} | D_1 \wedge D_2 \wedge \dots \wedge D_m \wedge M), \tag{1}$$

with

The proposition D_i In the following we abbreviate logical conjunction '$\wedge$' with a comma, for simplicity. Our statistical terminology and notation follow ISO standards (ISO 2009; 2006) otherwise.

We shall consider a robot who uses an *infinitely exchangeable* model. This model, introduced by de Finetti (1930; 1937; Heath et al. 1976) and described in detail in Bernardo et al. (2000 § 4.2), is determined by the following assumption of *infinite exchangeability*: the joint distribution for any number of observations is symmetric with respect to their order; that is, the order of the observations is irrelevant for inferential purposes. Distributions for different number of observations must of course be consistent with one another through marginalization. Infinite exchangeability may in turn be motivated by other specific assumptions, but the details of these are irrelevant for the mathematical form of this model.

Infinite exchangeability determines this form of the probability above:

$$\mathrm{P}(D_1, D_2, \dots, D_m | M) = \int_\Delta \left(\prod_{i=1}^{m} q_{d_i} \right) \mathrm{p}(\boldsymbol{q} | M) \, \mathrm{d}\boldsymbol{q}. \tag{2}$$

Bibliography

('de *X*' is listed under D, 'van *X*' under V, and so on, regardless of national conventions.)

Bernardo, J.-M., Smith, A. F. (2000): *Bayesian Theory*, reprint. (Wiley, New York). First publ. 1994.

de Finetti, B. (1930): *Funzione caratteristica di un fenomeno aleatorio*. Atti Accad. Lincei: Sc. Fis. Mat. Nat. **IV**^{5}, 86–133. http://www.brunodefinetti.it/Opere.htm.

— (1937): *La prévision : ses lois logiques, ses sources subjectives*. Ann. Inst. Henri Poincaré **7**^{1}, 1–68. Transl. in **kyburgetal1964_r1980**, pp. 53–118, by Henry E. Kyburg, Jr.

Heath, D., Sudderth, W. (1976): *De Finetti's theorem on exchangeable variables*. American Statistician **30**^{4}, 188–189.

ISO (2006): *ISO 3534-1:2006: Statistics – Vocabulary and symbols – Part 1: General statistical terms and terms used in probability*. International Organization for Standardization.

— (2009): *ISO 80000:2009: Quantities and units*. International Organization for Standardization. First publ. 1993.