A Non-Negative Bilinear DFEM Scheme for S_N Transport

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M&C 2015
April 20, 2015

Outline

1. Goal/Motivation
2. Basics
3. BCSZ Method
4. Initial Results

Goal

Long term goal: Accurate methods for multi-dimensional radiative transfer Near term goal: Non-negative bilinear DFEM for neutron transport -Why bilinear? Bilinear DFEM maintains thick diffusion limit, radiative transfer requires optically thick cells History: Extension of non-negative linear $(1, x, y)$ scheme developed on rectangles

Formalities

Solving

$$\vec{\Omega}_d \cdot \nabla \psi_d(x,y) + \sigma_t(x,y)\psi_d(x,y) = S_d(x,y) \tag{1}$$

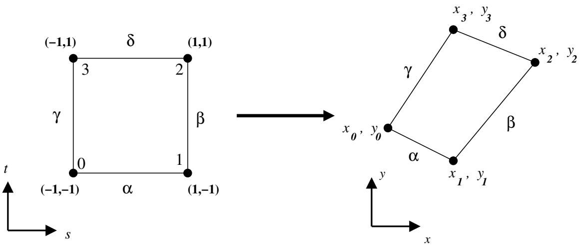
for unstructured quadrilaterals using bilinear (Q^1) DFEM.

- Using the standard interpolatory basis functions

$$\begin{aligned}
B_0(s,t) &= \frac{1-s}{2}\frac{1-t}{2} \\
B_1(s,t) &= \frac{s+1}{2}\frac{1-t}{2} \\
B_2(s,t) &= \frac{s+1}{2}\frac{t+1}{2} \\
B_3(s,t) &= \frac{1-s}{2}\frac{t+1}{2}
\end{aligned}$$

Mapping to Reference Coordinates

All work carried out on a reference element, $s \in [-1, 1]$, $t \in [-1, 1]$



$$x = x_0 B_0(s,t) + x_1 B_1(s,t) + x_2 B_2(s,t) + x_3 B_3(s,t) \quad (2)$$
$$y = y_0 B_0(s,t) + y_1 B_1(s,t) + y_2 B_2(s,t) + y_3 B_3(s,t) \quad (3)$$

Methods to Compare

1. UBLD: Unlumped Bilinear DFEM- Galerkin DFEM, no explanation necessary
2. FLBLD: Fully Lumped Bilinear DFEM- UBLD with mass matrix lumping, surface matrix lumping, and other manipulations. Equivalent to sub-cell corner balance.
3. BCSZ: Bilinear consistent set-to-zero: Non-linear, Petrov-Galerkin DFEM, satisfies all bilinear moments of Eq. (1)

BCSZ Definition

Solution representation, $\widetilde{\psi}_{BCSZ}(s,t)$:

$$\widetilde{\psi}_{BCSZ}(s,t) = \begin{cases} \widehat{\psi}_{BCSZ}(s,t) & \widehat{\psi}_{BCSZ}(s,t) > 0 \\ 0 & \text{otherwise} \end{cases} . \tag{4}$$

Bilinear function, $\widehat{\psi}_{BCSZ}$, to search for,

$$\widehat{\psi}_{BCSZ}(s,t) = \sum_{i=0}^{3} \psi_{i,BCSZ} B_i(s,t) , \tag{5}$$

Moment Equation Edge Integration

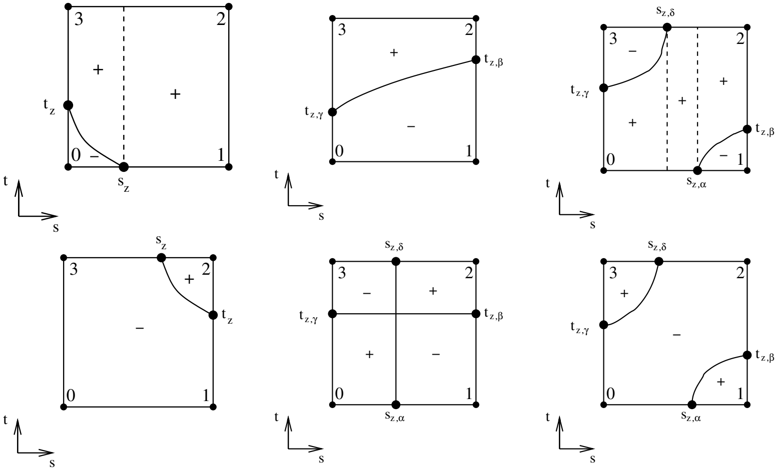
DFEM moment equations will need to integrate quantities like this on cell edges:

$$(\vec{\Omega}_d \cdot \vec{n}_\alpha) \int_\alpha B_i(s,-1)\widetilde{\psi}_{BCSZ}\ ds$$

1. Check vertices for negativity
2. By definition of $\widetilde{\psi}_{BCSZ}$, integrate $\widehat{\psi}_{BCSZ}$ only over portion of the interval where $\widehat{\psi}_{BCSZ} \geq 0$
   - 4 possible cases

Cell Interior Integration

Must integrate terms like, $B_i \widetilde{\psi}_{BCSZ} |J|$, over areas like these:



Enabling Idea

Along every integration area curve:

$$\widehat{\psi}_{BCSZ}(s,t) = 0$$

Transform interpolatory $\widehat{\psi}_{BCSZ}$ to moment based $f(s,t)$:

$$f(s,t) = f_c + sf_s + tf_t + stf_{st} \tag{6}$$

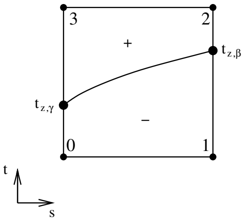
$$\begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} f_c \\ f_s \\ f_t \\ f_{st} \end{bmatrix} = \vec{\psi}_{BCSZ} \tag{7}$$

Enabling Idea- 2

Along each integration area defined by a curve, $f(s,t)=0$, and

$$\hat{l}_t = -\frac{f_c + f_s s}{f_t + f_{st} s}\,, \tag{8}$$

enabling the use of variable limits of integration. Consider integration over the region where $\widehat{\psi}_{BCSZ} > 0$, R_+, of a generic function $M(s,t)$:



$$\int\int_{R_+} M(s,t) = \int_{-1}^{1} ds \int_{\hat{l}_t}^{1} dt\; M(s,t)$$