Price Sensitivity (0.1)

Towards 1:1 pricing
Modern/dynamic pricing approaches applied to personal unsecured loans (PUL).

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Preface

- We want to predict a borrower's sensitivity to price and use this information to optimize prices to achieve maximum expected profit (or some other business goal - TBD)

- This deck is a "think aloud" conversation to gain a common understanding of the principles and possible solutions to the pricing sensitivity problem. It also poses questions that need answers to move forward with building a pricing model.

- The deck contains a mostly mathematical tour of the principles because the goal is to discover a computational method(s) that might achieve 1:1 pricing segmentation via a fully automated system.

- We heavily lean upon the work of R. Phillips' (Pricing Credit Products) and referenced materials. A literature review suggests that his work is a definitive baseline. In places, we have formulated the math in a more accessible fashion.

Credit Risk

1. Risk is a variable cost

2. Risk is correlated with price sensitivity: riskier customers tend to be less price sensitive

3. Price influences risk: all else equal, raising prices leads to higher losses - *price-dependent risk*.

Terms

- Exposure at Default (**EAD**) - remaining balance at default

- Loss Given Default (**LGD**) = EAD/Total Loan Amount

- Probability of Default (**PD**) = P(Default) | t<=loan_term

- EAD and LGD are random variables

- (Note: we largely stick to Phillips' terminology to be consistent with his theoretical foundations. Variation from more standard terminology is noted.)

Expected Loss

We care about expected values of random vars

$$PD = \sum_{t=1}^{T} p_d(t)$$

Probability of default at t=1 or t=2 or t=3 ... or t=T

$$EEAD = \frac{\sum_{t=1}^{T} p_d(t) \times EAD_t}{PD}$$

Expected loss is major determinant of expected profitability of a loan and thereby influences price.

$$ELGD = \frac{\sum_{t=1}^{T} p_d(t) \times EAD_t \times LGD_t}{\sum_{t=1}^{T} p_d(t) \times EAD_t}$$

$$EL = \sum_{t=1}^{T} p_d(t) \times EAD_t \times LGD_t$$

$$\therefore EL = PD \times ELGD \times EEAD$$

Simplified Pricing Threshold

Expected profit for $1 1-period loan (to simplify understanding risk/price relationship):

$$E(\pi) = \overset{\text{profit}}{(1-PD)(p-p_c)} - \overset{\text{loss}}{PD \times (1+p_c)}$$
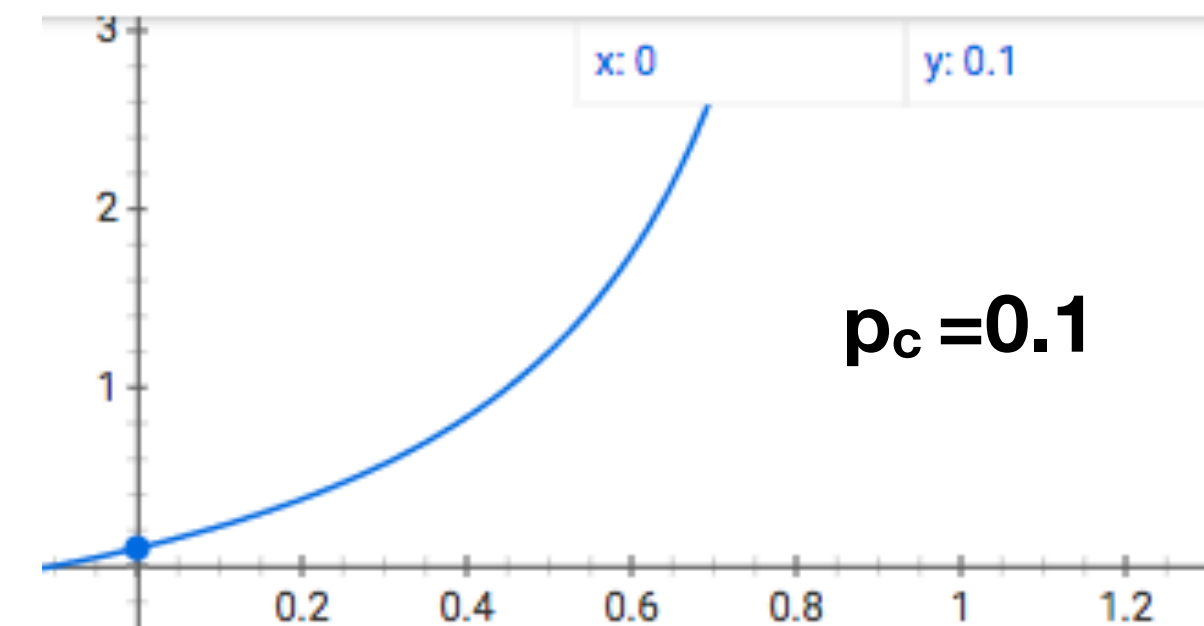
One-period supply capital rate p_c
One-period price p

Risk cut-off equation:

$$p > \frac{PD(1+p_c)}{1-PD} + p_c$$

$p_c = 0.1$

Risk-adjusted premium   cost of capital

A function of PD

Extremely simplified: ignores dynamics of payment/ default in multi-period loan and that PD is not independent of price at which loan offered. But sufficient to show the role of risk.
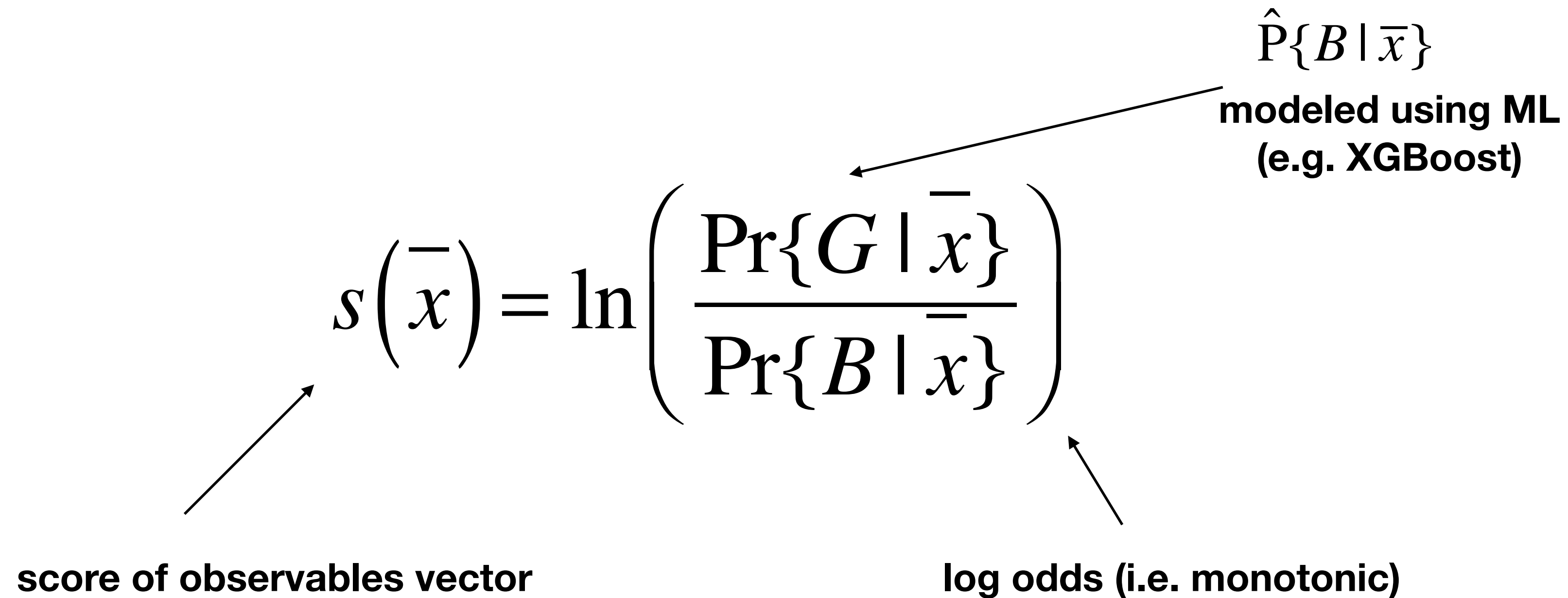
This only suggests a lower bound.
It does not suggest the price:

• Maximize profitability?
• Maximize bookings subject to profit constraint?
• Price sensitivity of customer

Risk Prediction

$$\hat{P}\{B \mid \overline{x}\}$$

modeled using ML (e.g. XGBoost)

$$s\left(\overline{x}\right) = \ln\left(\frac{\Pr\{G \mid \overline{x}\}}{\Pr\{B \mid \overline{x}\}} \right)$$

score of observables vector

log odds (i.e. monotonic)

Presumably decision-tree based for reasons of interpretability (actual adverse actions methodology: see D. Paulsen).
(Look ahead: can we get better sensitivity predictions if we don't care about interpretability?)

Risk Factors

Borrower dependent risk

$$\hat{P}\{B \mid \overline{x}\}$$

- Risk prediction based upon borrower observables

- But also the loan itself

Product dependent risk   Not part of $\overline{x}$

  - Amount

  - Leverage (e.g. LTV, DTI)

  - Period (longer loans riskier due to information asymmetry, random shock etc.)

- And price-dependent risk

Price-dependent Risk

$$E(\pi) = (1 - PD)(p - p_c) - PD \times (1 + p_c)$$

$$\Rightarrow E(\pi) = \big(1 - PD(p)\big)(p - p_c) - PD(p) \times (1 + p_c)$$

$$PD'(p) > 0 \quad \text{i.e. positive slope, or risk increases with price}$$

If $\;PD(p) > \dfrac{p - p_c}{p + 1}\;$ for all values of $\;p \geq p_c$

then there is no price at which a loan is profitable

Price-dependent Risk

- **Fraud** - fraudulent borrowers are insensitive to price

- **Adverse private information** - e.g. unobservable knowledge of a borrower's forthcoming redundancy

- **Competitive alternatives** - less riskier customers have more options and so loans at a certain (higher) price will attract riskier borrowers

- **Behavioral factors** - behavioral economics, mental accounting, time discounting etc. (Dynamic contextual factors unrelated to loan characteristics or borrower credit-worthiness attributes.)

- **Affordability** - e.g. DTI (capacity to pay)

Incremental Profitability

- Optimal price imposes two questions:
  - How will profitability of a loan change with price?
  - How will demand for a loan change with price?
- Profitability for loans non-trivial calculation due to:
  - Time value of money (discounted value of future costs/revenues)
  - Uncertainty: default, prepayment, utilization (for credit line)
- We need a model for incremental profitability for PUL
  - What are the variable costs? (Holds, channels?)
  - What is the profit goal? (For whom?)
    - If we sell/securitize loans, how does this affect optimization?
  - Longer loans are more profitable (via interest)
  - Large-portfolio holders should act more risk-neutral (assuming risk of loans are statistically independent across the portfolio) - see Phillips
  - Also: should pricing model take LTV into account - e.g. cross-selling other products? (Segmentation question?)

Action: construct the incremental profitability model

Price Response Curve

$$d(p) = D\overline{F}(p)$$

D = d(0) - i.e. willing to buy at all
(source: DTU ME)

price-response
(i.e. "demand")

total applications
(i.e. "market size")

take-up rate (i.e. WTP)

- We want to know the price-response function **per product per segment** (vs. market-demand curve per product) - i.e. their Willingness To Pay (WTP)

- Price optimization uses this along with expected incremental profit calculation to price for *maximum expected total profitability* (for a loan portfolio)

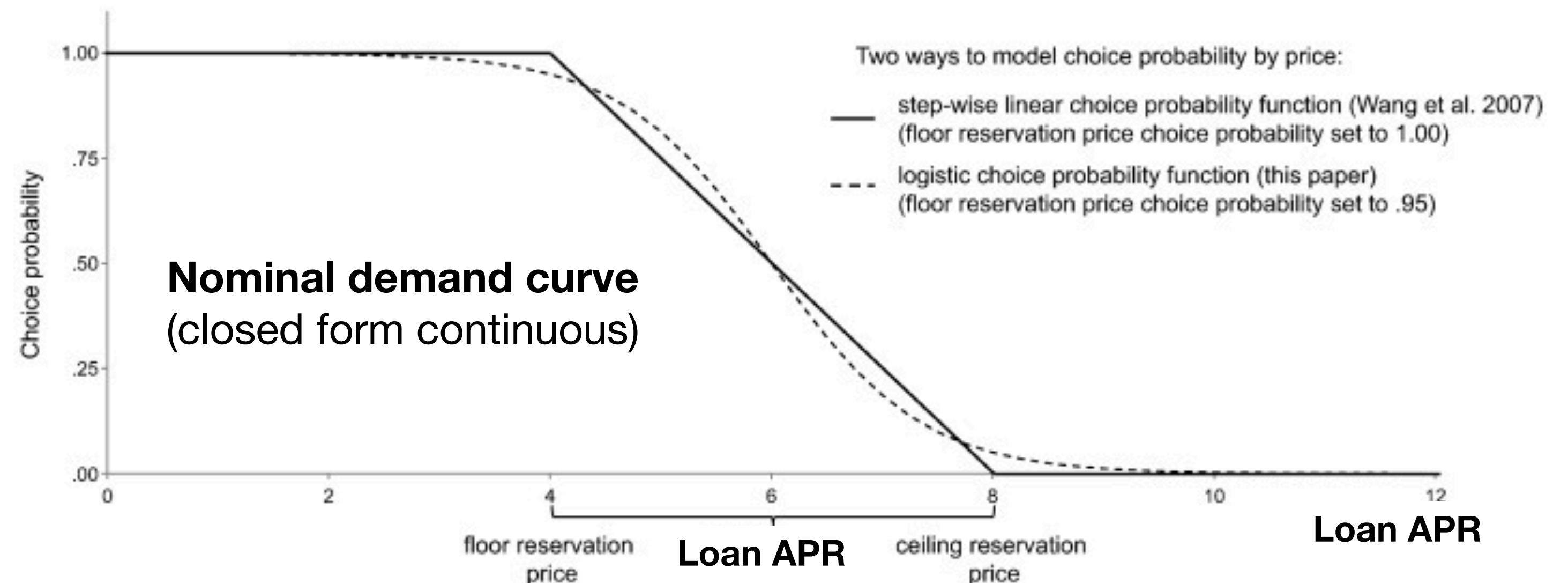$$\overline{F}(p) = \int_{p}^{\infty} f(w)\,dw$$

probability distribution of WTP

WTP is probability of a customer paying **at most** p - i.e. note that it is the complementary cum. dist. func. or 1 - F(p) (often seen in the literature)

In general, $d(p) = D\,(\,1\text{-}F(p)\,) = D(F(\text{infinity})\text{-}F(p))$.

Choice probability

1.00
.75
.50
.25
.00

0   2   4   6   8   10   12

Nominal demand curve
(closed form continuous)

Two ways to model choice probability by price:

—— step-wise linear choice probability function (Wang et al. 2007)
(floor reservation price choice probability set to 1.00)

‐ ‐ ‐ logistic choice probability function (this paper)
(floor reservation price choice probability set to .95)

floor reservation price   **Loan APR**   ceiling reservation price

Loan APR

Consumer indecisiveness

at these prices, this consumer is *certain to buy* | at these prices, this consumer is *indecisive* | at these prices, this consumer is *certain not to buy*

Adapted from curve in this paper: Journal of Business Research   Additional source: BPO lecture notes.

Some derivations

Slope -ve, hence introduce minus sign

$$\overline{F}(p) = \int_{p}^{\infty} f(w)\, dw \Rightarrow -\overline{F}'(p) = f(p)$$

For a given WTP curve, the corresponding density

$$d(p) = D\overline{F}(p) \Rightarrow \left| d'(p) \right| = -D\overline{F}'(p) = Df(p)$$

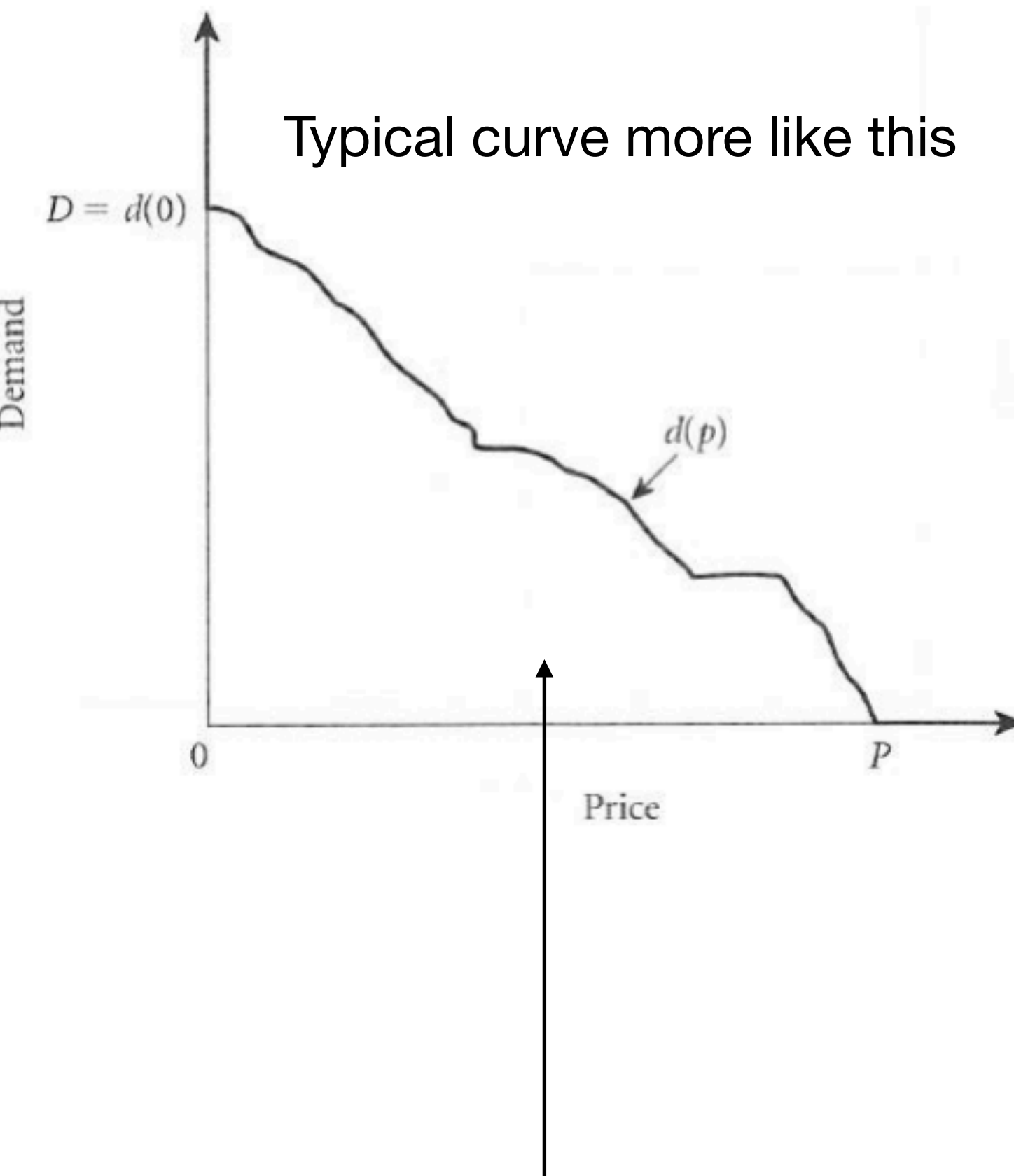In some cases, we are interested in absolute slope of the take-up rate, in which case:

$$\overline{F}'(p) \Rightarrow \left| d'(p) \right| / D = f(p)$$

Note: the derivative can be interpreted as the percentage willing to pay **exactly** p:

$$\overline{F}'(p) \approx \frac{\overline{F}(p+h) - \overline{F}(p)}{\overline{F}(p)}$$

For small h

Measuring Sensitivity

Typical curve more like this

We want to estimate this curve from any actual take-up data.

Note: the curve is actually time-dependent, but this is not reflected in the baseline math.

There are many different ways in which product demand might change in response to changing prices but all price-response functions assume:

- non-negative (p≥0)

- continuous (no gaps in market response to prices)

- differentiable (smooth and with well-defined slope at every point)

- downward sloping (raising prices decreases demand)

- *notwithstanding "behavioral economics effects"*

Most common measures are:

• **Slope - d'(p₁) is the derivative of the price-response function at p₁**

• **Hazard rate (slope/demand)** $h(p_1) = \dfrac{|d'(p_1)|}{d(p_1)} = f(p)/\bar{F}(p)$

$r(\xi) = \dfrac{f(\xi)}{1 - F(\xi)}$

"Failure rate" (See Yao) has important underlying properties when calculating optimal pricing.

• **Elasticity: %demand change from 1% change in price:**

$$\varepsilon(p_1) = ph(p_1) = d'(p_1)p_1/d(p_1) = pf(p)/\bar{F}(p)$$

point elasticity

Sensitivity and risk

- As said, higher prices = higher default rates AND higher risk customers are less price sensitive (e.g. price elasticity higher for higher FICO)

- But the two phenomenon are the same: if price sensitivity decreases with risk in a population, then the populate will demonstrate price-dependent risk.

$$D = D_g + D_b$$

$$d_b(p) = D_b \bar{F}_b(p)$$

$$d_g(p) = D_g \bar{F}_g(p)$$

#goods   **#bads**

#applicants   **take-up rate**

$$DR = D_b \, / \, D_g + D_b$$

However, the rate is price dependent, so: $$DR(p) = \frac{D_b \bar{F}_b(p)}{D_b \bar{F}_b(p) + D_g \bar{F}_g(p)}$$

$$h_g(p) = -\bar{F}_g'(p) \, / \, \bar{F}_g(p) = f_g(p) \, / \, \bar{F}_g(p)$$
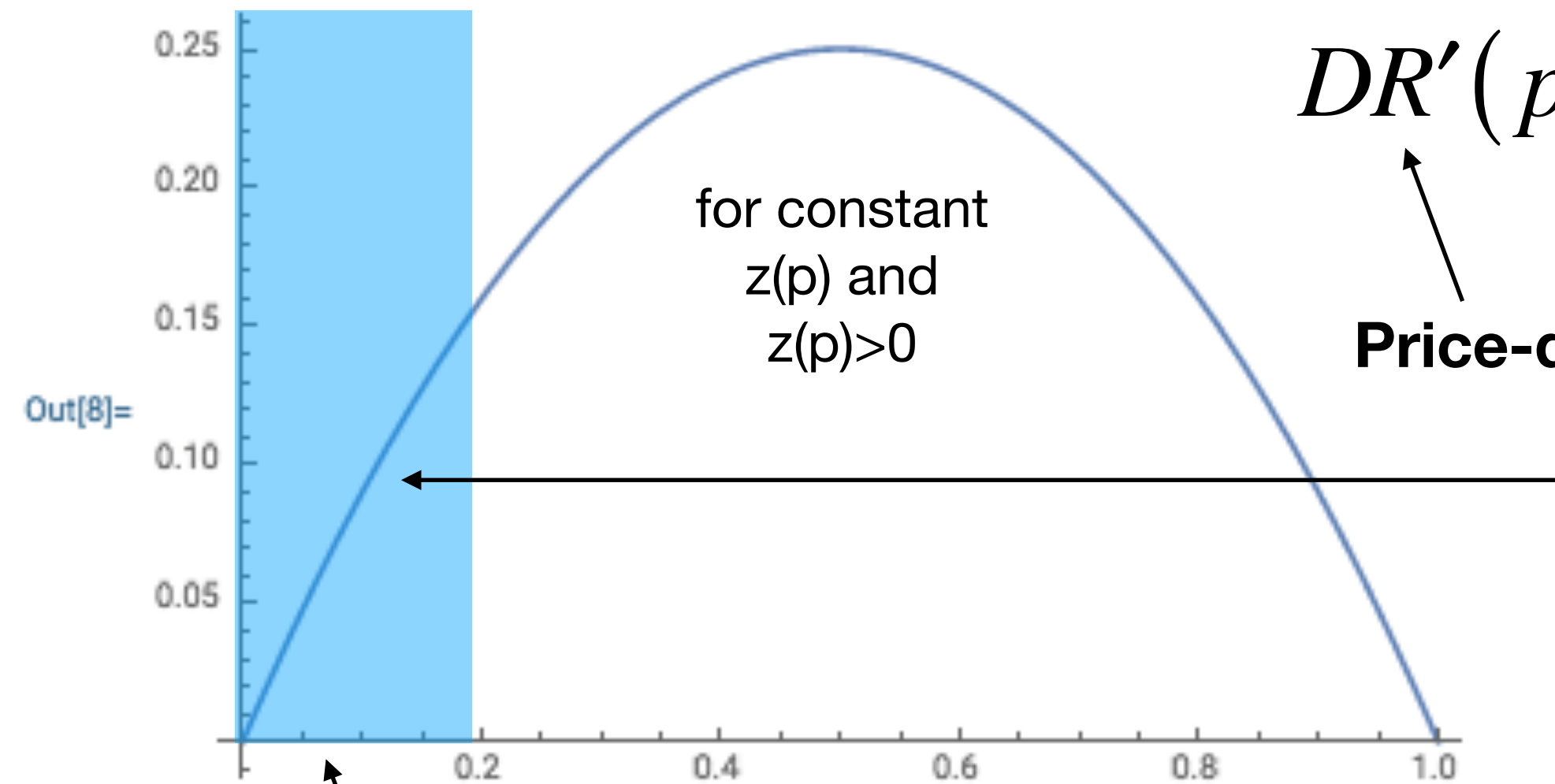$$h_b(p) = -\bar{F}_b'(p) \, / \, \bar{F}_b(p) = f_b(p) \, / \, \bar{F}_b(p)$$

Differentiating wrt price, we get: $$DR'(p) = (h_g(p) - h_b(p)) DR(p)(1 - DR(p))$$

via substitution, chain-rule and product rule

failure rate difference between goods and bads

Sensitivity and risk

`In[8]:= Plot[(1 - x) x, {x, 0, 1}]`

Out[8]=

for constant
z(p) and
z(p)>0

commercial lending region

$$z(p) = h_g(p) - h_b(p)$$ **Difference in hazard rate**

$$DR'(p) = z(p)DR(p)(1 - DR(p))$$

Price-dependent risk rate (or *PDR rate*) - i.e. rate at which risk changes with price

In this part of the curve, price-dependent risk more salient for higher-risk populations.

"As found in practice - i.e. effect of price on risk much stronger in sub-prime than prime. Very little impact in super prime."

What does it imply for pricing? Net interest income for a loan of value 1 (for simplicity) and loss-given default: $0 < l \leq 1$

$$\Pi(p) = (p - c)d_g(p) - ld_b(p)$$

There is a positive number k such that z(p)>k for all p >=c in which case the lender cannot achieve a profit at any price.

Whether or not it is profitable to offer a loan depends not only on bads:goods but on their difference in price sensitivity as reflected in the differential hazard rate z(p)

Pricing/Underwriting
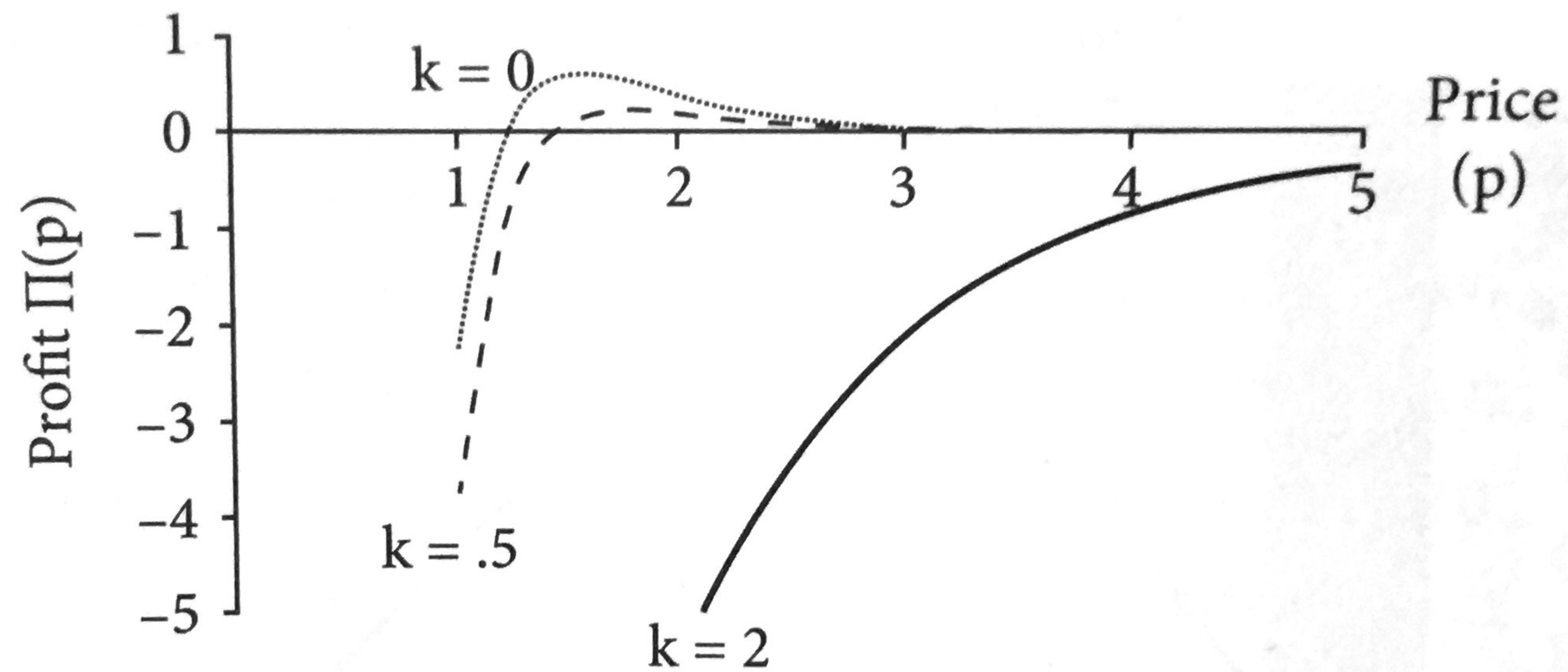
$$k = h_b(p) - h_g(p)$$

$$\Pi(p) = (p - c)d_g(p) - ld_b(p)$$

There is a positive number k such that z(p)>k for all p >=c in which case the lender cannot achieve a profit at any price.

Whether or not it is profitable to offer a loan depends not only on bads:goods but on their difference in price sensitivity as reflected in the differential hazard rate z(p)

Estimating d(p)

- We assume a data-driven approach based upon random testing (of prices), but we need to evaluate the current random-testing method/data

  - Are we storing all endogenous variables? (if any)

  - Do we have sufficient test data for each segment?

  - (Note: we can model using available data and also estimate with piecewise numerical estimator - see later.)

- Given the historical data, we now estimate a price-response model using ML (GLMs or ANN?)

- Do we have endogeneity?

  - What might cause it? e.g. agent intervention in the sale of the loan

Model

- Considerations
  - Feature selection (for price sensitivity)
  - Feature engineering overhead
  - Monotonicity - The assumption of pricing optimization is that prices change monotonically
- Can we short-cut feature engineering stage with use of a novel DL method, like Deep Lattice Networks?
  - Pros: shortcut feature engineering, Tensorflow models, organizational learning
  - Cons: lack of proven field experience (with DLN). Expertise better used elsewhere (portfolio optimization?)
- Updating: frequency and method (to track temporal changes)
  - To what extent can this be automated?
- Should we include other product co-joint analysis? ("Price to entice" as gateway to higher LTV.)

Pricing Segmentation

N Customer dimensions
(e.g. scores, employment)
observable/explainable

Price	-	Price	-
-	Price	Price	-
-	Price	Price	-
-	-	-	Price

M Product dimensions
(e.g. amount, term)

NxM buckets
(upper bound)
logical limit 1:1 pricing
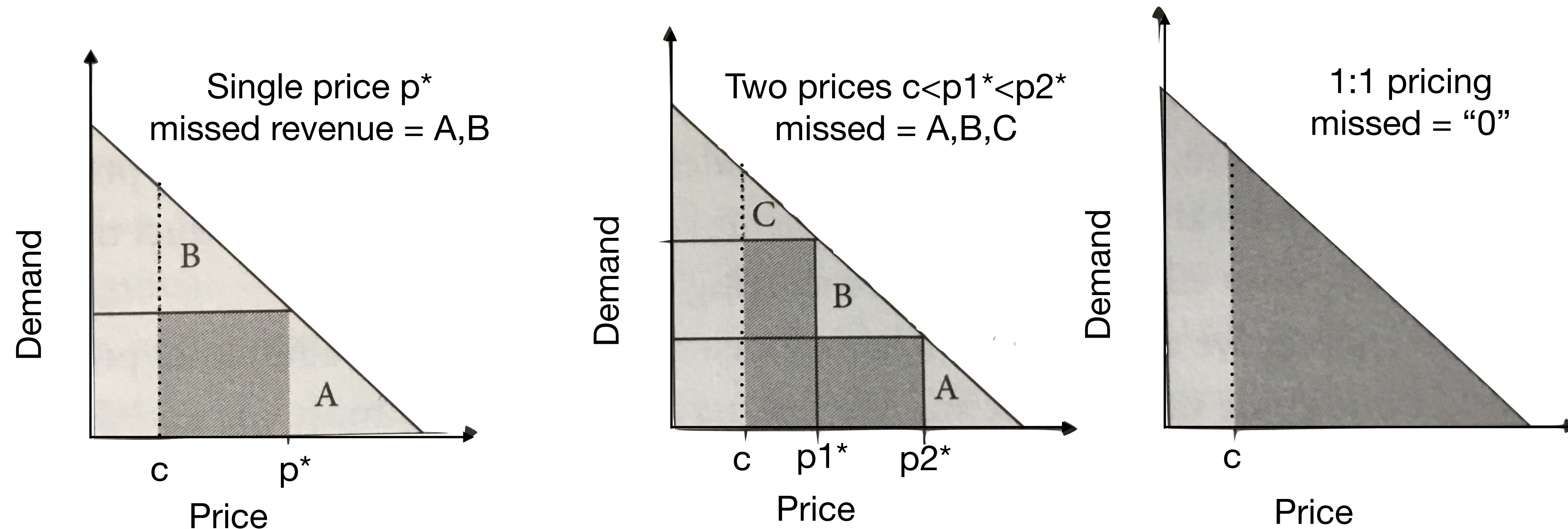
or.. **NxM dimensional surface**
whose geometry determines revenue/profit
and should be correlated to:
• incremental profitability
• price sensitivity

- Smaller (more targeted) buckets => profitability, but also complexity. What is the trade-off? (We need to do work to analyze the frontier of ROI trade-off curves.)
- All else being equal, charge lower prices to segments with:
    - higher inc. profitability
    - higher price sensitivity
    - other business incentive e.g. promote certain channels? (Again, what are the business constraints.)

- Customer dimensions:
    - Risk, [age], loyalty, [geography] etc.
- Product dimensions:
    - Loan amount, term
- Term - should we consider a higher resolution of term (than just 3/5 year) to obtain better results? (And give a more satisfactory UX.)?

Are there other ways to differentiate product besides price? e.g. "no-charge term rescheduling or repayment rewards" i.e. customer segmentation achieved via product segmentation correlated to price sensitivity

WTP: Pricing

Single price p*
missed revenue = A,B

Demand

Price

B

A

c   p*

Two prices c<p1*<p2*
missed = A,B,C

Demand

Price

C

B

A

c   p1*   p2*

1:1 pricing
missed = "0"

Demand

Price

c

- We now see the importance of predicting WTP = revenue/profit
- However, WTP extends beyond risk-based customer variables e.g. to include things like financial sophistication, brand affinity, urgency, competitive environment etc.
  - Note: are some of these observable, say via random experiments in design (or other means). Do we capture all the necessary data? What other data sources can we use?
  - i.e. pricing (sensitivity) is not a "backend" function only
  - Can we gather more user data? (e.g. for DM clients can we afford "second stage?")

Dynamic segmentation
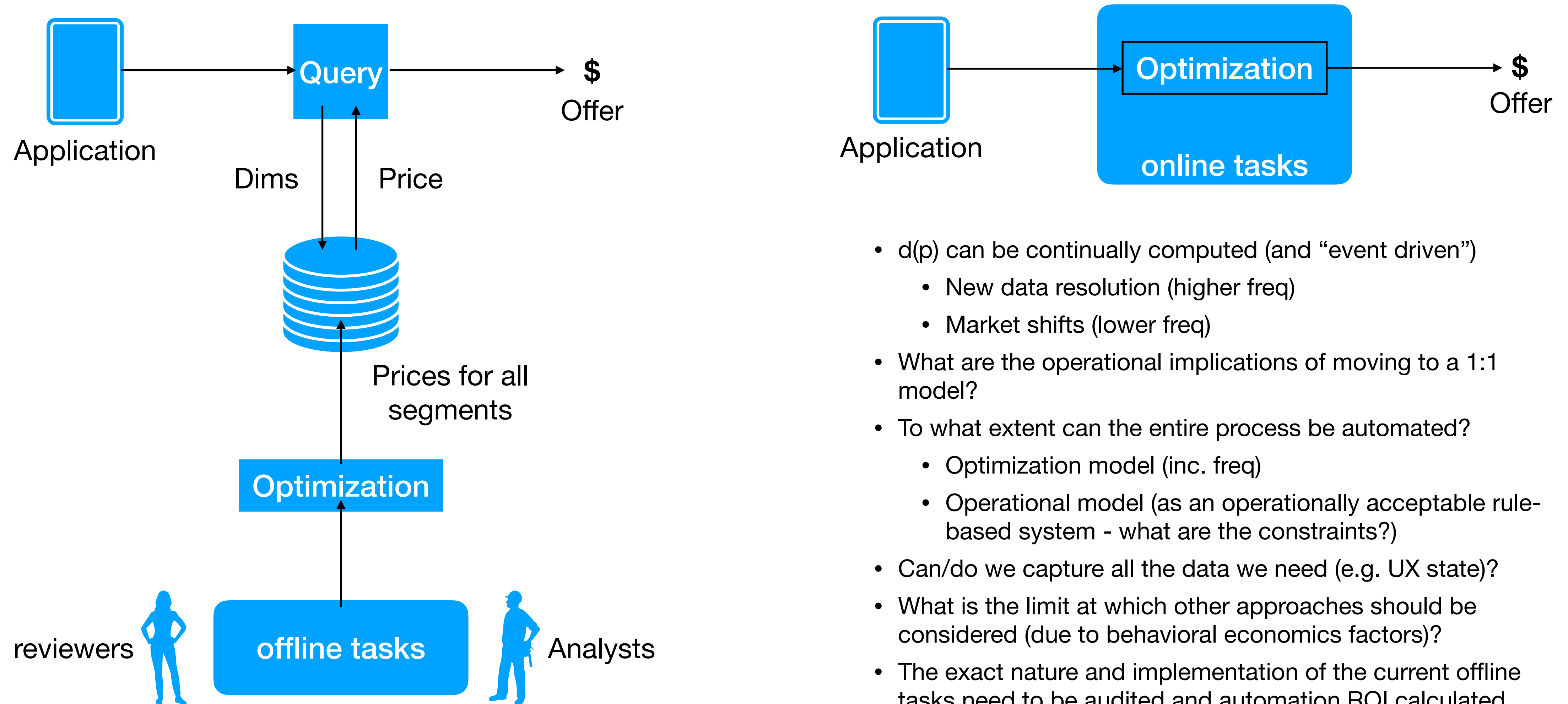
- d(p) can be continually computed (and "event driven")
  - New data resolution (higher freq)
  - Market shifts (lower freq)
- What are the operational implications of moving to a 1:1 model?
- To what extent can the entire process be automated?
  - Optimization model (inc. freq)
  - Operational model (as an operationally acceptable rule-based system - what are the constraints?)
- Can/do we capture all the data we need (e.g. UX state)?
- What is the limit at which other approaches should be considered (due to behavioral economics factors)?
- The exact nature and implementation of the current offline tasks need to be audited and automation ROI calculated

Compute power is cheap. If we can find an end-to-end computational solution, even highly complex (AI), it potentially pays! What are the strategic (tech) imperatives in a commodity PUL market?

Optimizing Prices (Simple loan)

Consider a single two-period loan (just makes math simpler initially) [And assume LGD independent of price, **which it is not.**]

$$E\left(\pi\left(p\right)\right) = \overline{F}\left(p\right)[(1-PD)\times\left(p-c\right)-PD\times l] \qquad l = LGD + c$$

price response

[simplistic incremental profit]

loss given default

cost of funds

$$= \overline{F}\left(p\right)[(1-PD)\times\left(p-c-\frac{l}{o}\right)] \qquad o = (1-PD)/PD \quad \text{odds that loan will \textbf{not} default}$$

$$\Longrightarrow o^* = \frac{LGD+c}{p-c} \qquad \text{underwriting criterion for a price-taking lender - i.e. take any loans with odds > \textbf{o*}}$$

But we are interested in price-setting lender. In which case we want to charge a price **p*** that would maximize $E\left(\pi\left(p\right)\right)$

Here we omit the analytical solution for $E'(\pi(p))$ as there is no closed-form solution for any non-linear price response function. Rather, we can see that if we can compute $\overline{F}(p)$ then we can calculate the maximum using numerical methods.

Price-dependent risk

$$h\left(p^{*}\right) = \cfrac{1}{p^{*} - c - \cfrac{l}{o}}$$

Analytical solution for $E'\left(\pi\left(p\right)\right) = 0$

But we have so far assumed that PD and loss are independent of price of loan, which is incorrect.
If we now assume that we have a population **D_g** of good and **D_b** bad customers.

$$E\left(\pi\left(p\right)\right) = \bar{F}_g\left(p\right) D_g\left(p - c\right) - \bar{F}_b\left(p\right) D_b l$$

$$E'\left(\pi\left(p\right)\right) \xrightarrow{E'=0} h_g\left(p^{*}\right) = \cfrac{1}{p^{*} - c - \left(\cfrac{f_b\left(p^{*}\right)}{f_g\left(p^{*}\right)}\right)\left(\cfrac{l}{o_0}\right)}$$

This holds for price-dependent risk (different dist. for bads/goods)

⟵ population odds

$$E'\left(\pi\left(p\right)\right) \xrightarrow{E'=0} h_g\left(\hat{p}\right) = \cfrac{1}{\hat{p} - c - \left(\cfrac{l}{o\left(\hat{p}\right)}\right)}$$

If we assume the odds will not change (i.e. ignore price-dependent risk)

BUT: we do expect goods have higher failure rate => $f_b\left(p\right) / f_g\left(p\right) > \bar{F}_b\left(p\right) / \bar{F}_g\left(p\right)$

And therefore (plugging into above equations): $p^{*} < \hat{p}$
Which means the (amateur) lender who does not consider influence of price upon risk will set a price that is higher than the expert lender, which means higher losses and lower profit. Note: this affects competitors profitability - see Nomis white paper.

Adaptive Optimization

What if we don't know h(p)?
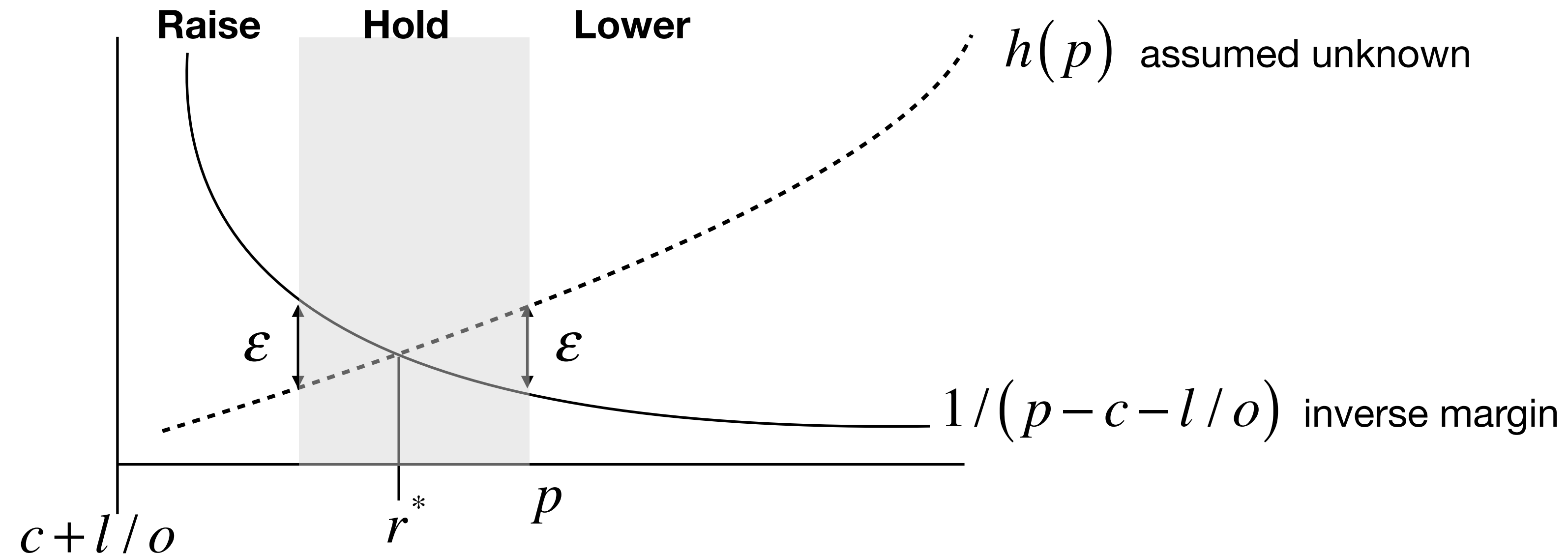
1. Iterate k pricing adjustments $\alpha_1 > \alpha_2 > \alpha_3 ... > 0$

2. Use 2-point price estimation (testing) to estimate $\hat{h}(p_k) = 2(d_1 D_2 - d_2 D_1)/[\delta((d_1 D_2 + d_2 D_1))]$

3. Calculate price gap $g(p_k) = \hat{h}(p_k) - 1/(p_k - c - l/o)$

4. if $-\varepsilon \leq g(p_k) \leq \varepsilon$ set $p^* = p_k$ and stop

5. if $g(p_k) < -\varepsilon$ set $p_{k+1} = p_k + \alpha_k$ and go to step 2

6. if $g(p_k) > \varepsilon$ set $p_{k+1} = p_k - \alpha_k$ and go to step 2

Optimization Over Segments

$$p^* = \arg_p \max \left(D\bar{F}(p)\pi(p) \right)$$

$$\vec{p}^* = \arg_p \max \sum_{i=1}^{N} D_i \bar{F}_i(p_i)\pi_i(p_i) \qquad \text{(assume independently solvable)}$$

$$\vec{p} = \arg_p \max \sum_{i=1}^{N} D_i \bar{F}_i(p_i) R_i(p_i)$$

$$\vec{p} = \arg_p \max \left[\sum_{i=1}^{N} D_i \bar{F}_i(p_i)\pi_i(p_i) + \sum_{i=1}^{N} D_i \bar{F}_i(p_i) R_i(p_i) \right]$$
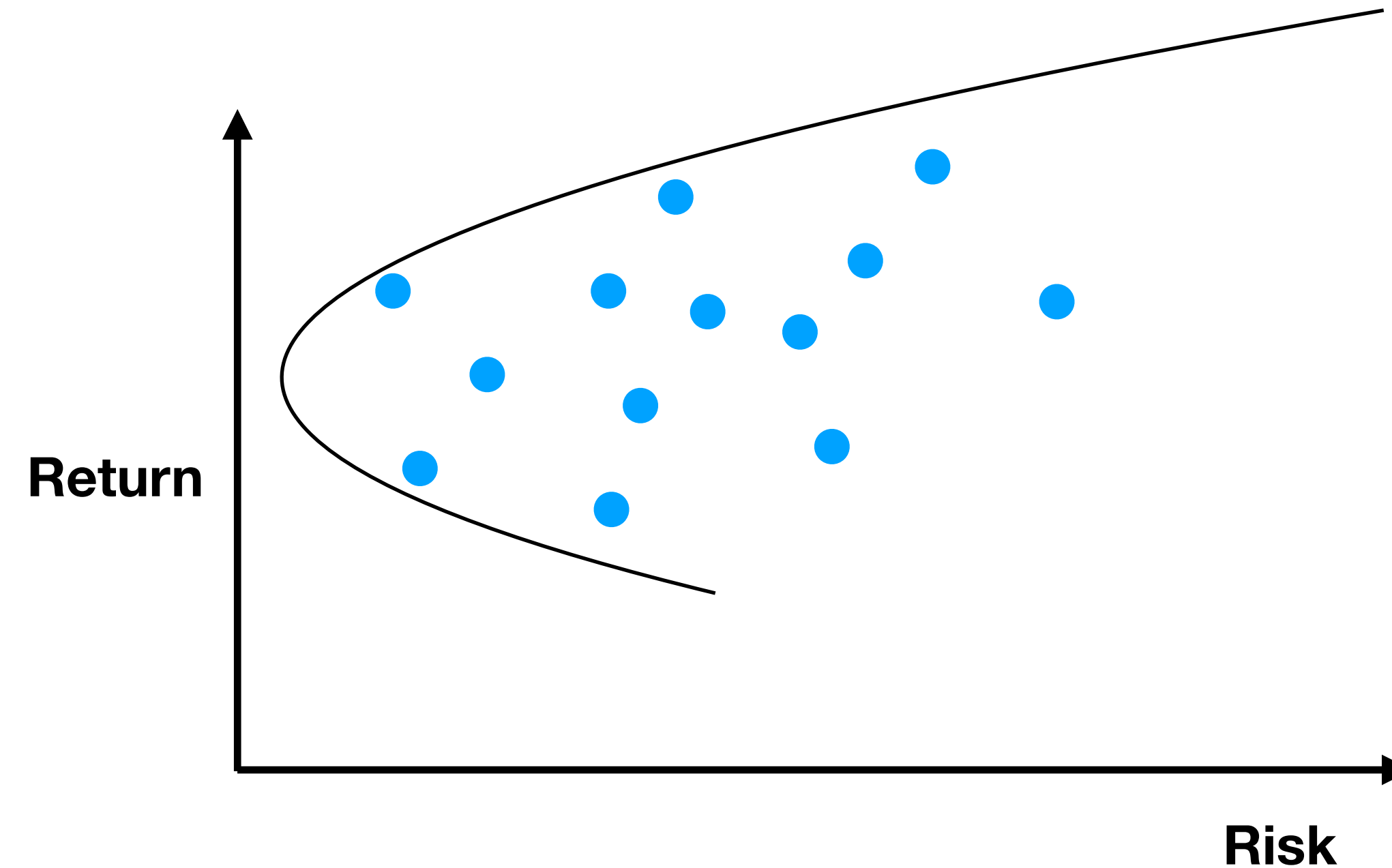
Or with constraints:
e.g Bounds, structural constraints, monotonicity of pricing, segment bounds, rate endings

$$\vec{p} = \arg_p \max \left[\sum_{c=1}^{M} \left[\Phi_C (\sum_{i=1}^{N} D_i \bar{F}_i(p_i)\theta_i(p_i)) \right] \right]$$

Subject to feasibility whilst trying to avoid post-hoc pricing adjustments ("overlays") due to "shortcomings" in the model.
And subject to idealized segmentation (that avoids mixed [separable] failure curves with implied lack of global maximum).
And subject to location of the portfolio on the efficient frontier (next).
Where/what are the automation opportunities for constraint review and management?

Efficient Frontier

Can I have the highest revenue with greatest profit and lowest risk please? No.

- We can estimate the frontier and where our portfolio lies within it
- But we may have chosen unrealistic (or too many) constraints
- We might have chosen constraints outside of the model (i.e. tuned prices manually)
- Optimal portfolio selection is an area of R&D opportunity (reinforcement learning, genetic algorithms, etc.)
- Optimal segmentation is also an area for R&D opportunity