SE 160 Homework 4

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8-1) A fuselage skin panel is attached to internal frames and longerons that are spaced 20 inches in the x-direction and 24 inches in the y-direction. The panel is made of aluminum $\left(E = 10 \times 10^6 \text{ lb/in}^2, v = 0.333, \sigma_{\mathrm{y}} = 70 \times 10^3 \mathrm{lb/in}^2\right)$ and is $0.050''$ thick. This fuselage panel can be modeled as a (20" x24 ") rectangular plate that is simply-supported on all four edges. A low-order sinusoidal pressure load is acting over the plate surface given by

$$p_z(x, y) = p_0 \sin(\pi x/a) \sin(\pi y/b) \mathrm{lb/in}^2$$

where $\left(p_0 = 0.10 \mathrm{lb/in}^2\right)$.
a.) Calculate the deflection at the plate center and plate displacement distribution. Plot the deformed shape using MATLAB.

b.) Calculate the stresses $(\sigma_{\mathrm{xx}}, \sigma_{\mathrm{yy}})$ on the upper surface at the plate center. Calculate the stress distribution for $(\sigma_{\mathrm{xx}}, \sigma_{\mathrm{yy}})$ on the upper surface over the plate surface geometry. Plot both stress distributions over plate region using MATLAB.

c.) Apply von Mises failure criteria on the upper surface at the plate center and calculate the margin of safety assuming a safety factor of 1.5 .

d.) What is the maximum allowable applied pressure (p_0) assuming a safety factor of 1.5 , and $MS = 0$? What is the displacement at the plate center?

Problem 8-1) a:

Given:

$a = 20''$
$b = 24''$
$t = 0.05''$
$E = 10 \times 10^6 \frac{lb}{in},$
$\nu = 0.333$
$\sigma_y = 70 \times 10^3 \frac{lb}{in^2}$
$SF = 1.5 P_0 = 0.10 \frac{lb}{in^2}$,
the plate is simply supported on all four sides.

Solution:

$$assume\ w(x,y) = W sin(\frac{\pi x}{a}) sin(\frac{\pi y}{b}) \tag{1}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{P_z}{D} \tag{2}$$

$$\Rightarrow w(x,y) = \frac{a^4 P_0}{D\pi^4} \frac{1}{(1+\frac{a}{b}^2)^2} sin(\frac{\pi x}{a}) sin(\frac{\pi y}{b}) \tag{3}$$

at the plate center $x = \frac{a}{2}$ and $y = \frac{b}{2}$. displacement = 0.488" at the plate center plugging the x and y values into equation (2). The graph of the displacement over the plate is shown in figure 1.

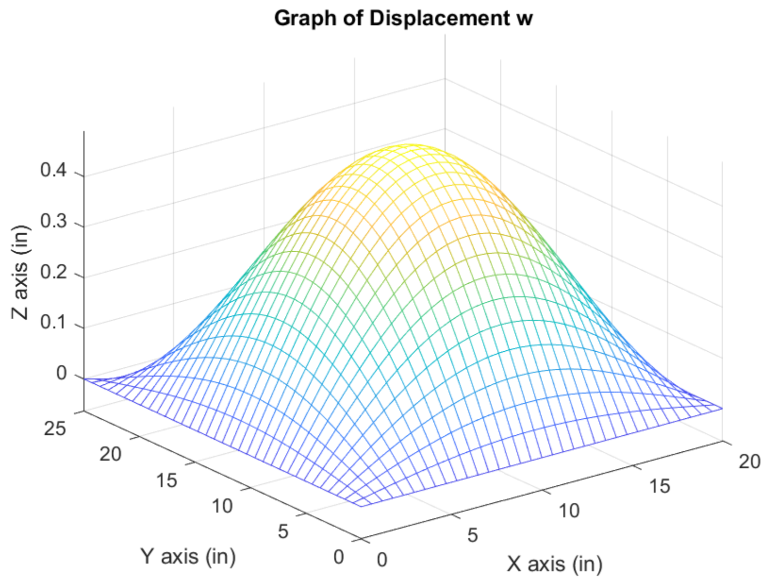

Figure 1:

Problem 8-1) b:
You can now use the equations from the reader to get the stress in the x and y directions

$$M_x = -D(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}) \tag{4}$$

$$M_y = -D(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}) \tag{5}$$

$$\sigma_{xx} = \frac{N_x}{t} + z(12\frac{M_x}{t^3}) \tag{6}$$

$$\sigma_{yy} = \frac{N_y}{t} + z(12\frac{M_Y}{t^3}) \tag{7}$$

Now plug equation (1) into equation (4) and (5). Then plug those new equations into equation (6) and (7)

$$\sigma_{xx} = \frac{12P_0}{\pi^2}(\frac{a}{t})^2\frac{(1+\nu((\frac{a}{b})^2)}{(1+(\frac{a}{b})^2)^2}\frac{z}{t}sin(\frac{\pi x}{a})sin(\frac{\pi y}{b})$$

$$\sigma_{yy} = \frac{12P_0}{\pi^2}(\frac{a}{t})^2\frac{(\nu+((\frac{a}{b})^2)}{(1+(\frac{a}{b})^2)^2}\frac{z}{t}sin(\frac{\pi x}{a})sin(\frac{\pi y}{b})$$

The graph of these stress distribution can be seen in figure 2 and 3 below :

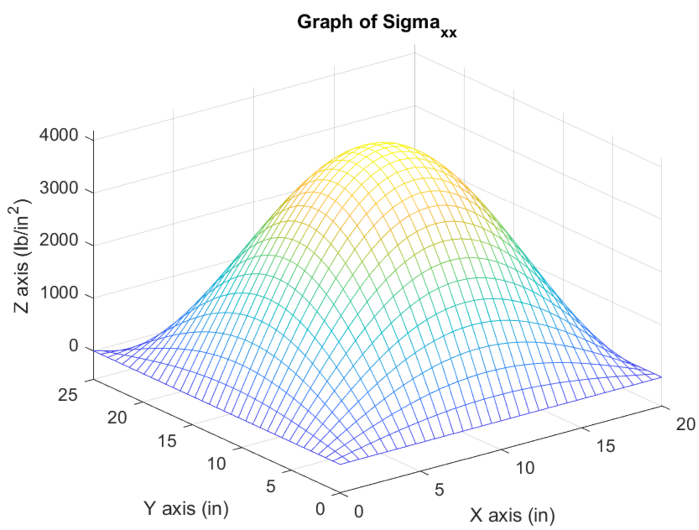

Figure 2:

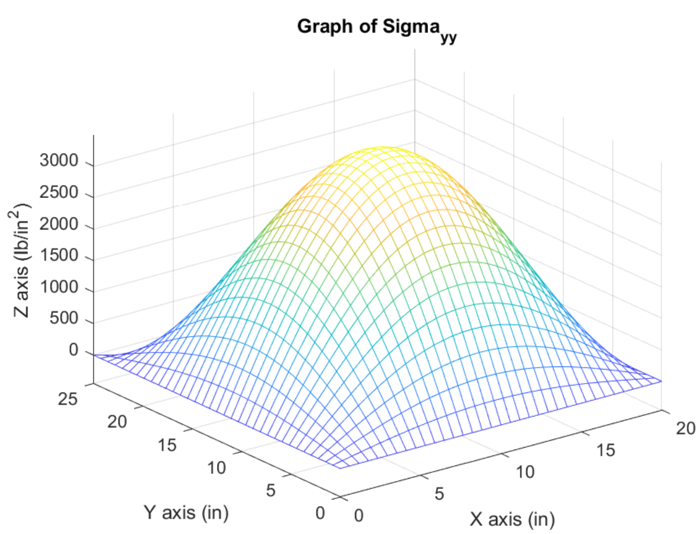

Figure 3:

Problem 8-1) c: The Von Mises failure criteria can be used to calculate the margin of safety for the plate

$$\sigma_{eff} = \sqrt{\frac{(\sigma_{xx} - \sigma_{zz})^2 - (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{xx} - \sigma_{yy})^2}{2}} \tag{8}$$

$$\sigma^* = \frac{\sigma_y}{SF} = 46.6 \times 10^3 \tag{9}$$

$$MS = \frac{\sigma^*}{sigma_{eff}} - 1 = 11.051 \tag{10}$$

Problem 8-1) d:

I solved for P_0 using matlab and syms variables I got $P_0 = 0.1221$

8-2) A fuselage skin panel is attached to internal frames and longerons that are spaced 20 inches in the x-direction and 24 inches in the y-direction. The panel is made of aluminum ($E = 10 \times 10^6 \mathrm{lb/in}^2, v = 0.333, \sigma_{\mathrm{y}} = 70 \times 10^3 \mathrm{lb/in}^2, \rho = 0.100 \mathrm{lb/in}^3$) and is 0.050 " thick. This fuselage panel can be modeled as a (20" ×24 ") rectangular plate that is simply-supported on all four edges. A higher-order pressure loading is applied that has the form:

$$p_z(x, y) = p_0 \sin(3\pi x/a) \sin(3\pi y/b) \mathrm{lb/in}^2$$

where $\left(p_0 = 0.10 \mathrm{lb/in}^2\right)$.

a.) Calculate the plate deflection distribution, assuming that the plate distribution has the form:

$$w(x, y) = W_o \sin(3\pi x/a) \sin(3\pi y/b) \text{ inch}$$

Calculate the deflection at the plate center and plot the displacement distribution using MATLAB.

b.) Calculate the stresses $(\sigma_{\mathrm{xx}}, \sigma_{\mathrm{yy}})$ on the upper surface at the plate center. Calculate the stress distribution for $(\sigma_{\mathrm{xx}}, \sigma_{\mathrm{yy}})$ on the upper surface over the plate surface geometry. Plot both stress distributions over plate region using MATLAB.

Problem 8-2) a:

Equation (2) was used to solve the W_0 and the deflection equation is presented below. The distribution is then plotted in figure 4.

$$\Rightarrow w(x,y) = \frac{a^4 P_0}{D\pi^4 3^4}\frac{1}{(1+\frac{a}{b}^2)^2} sin(\frac{\pi x}{a}) sin(\frac{\pi y}{b}) \tag{11}$$

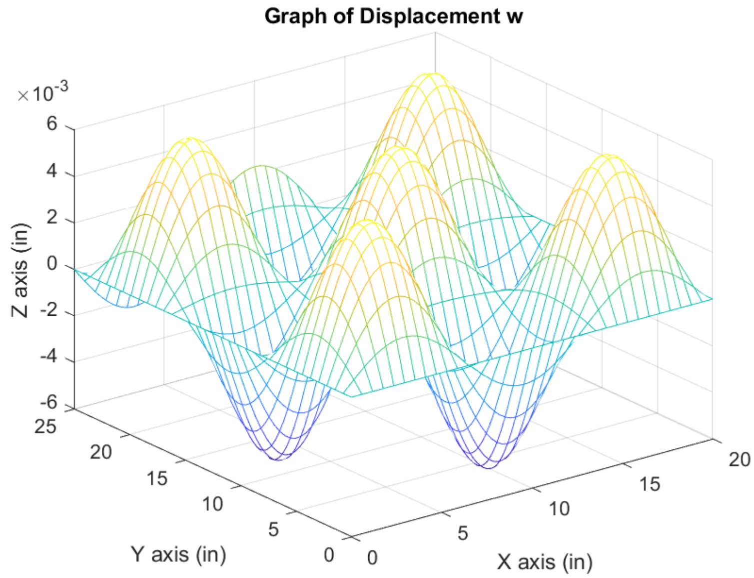

Figure 4:

Problem 8-2) b:

Equations (4), (5), (6) and (7) were used to get the stresses and they are graphed below in figure 5 and 6 :

$$\sigma_{xx} = \frac{12P_0}{\pi^2 3^4}(\frac{a}{t})^2 \frac{(1+\nu((\frac{a}{b})^2)}{(1+(\frac{a}{b})^2)^2}\frac{z}{t} sin(\frac{\pi x}{a}) sin(\frac{\pi y}{b})$$

$$\sigma_{yy} = \frac{12P_0}{\pi^2 3^4}(\frac{a}{t})^2 \frac{(\nu+((\frac{a}{b})^2)}{(1+(\frac{a}{b})^2)^2}\frac{z}{t} sin(\frac{\pi x}{a}) sin(\frac{\pi y}{b})$$

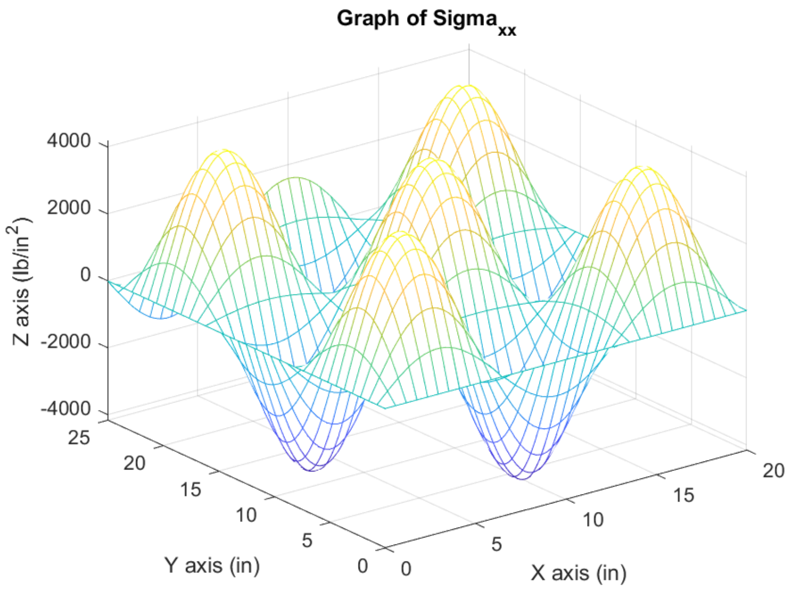

Figure 5:

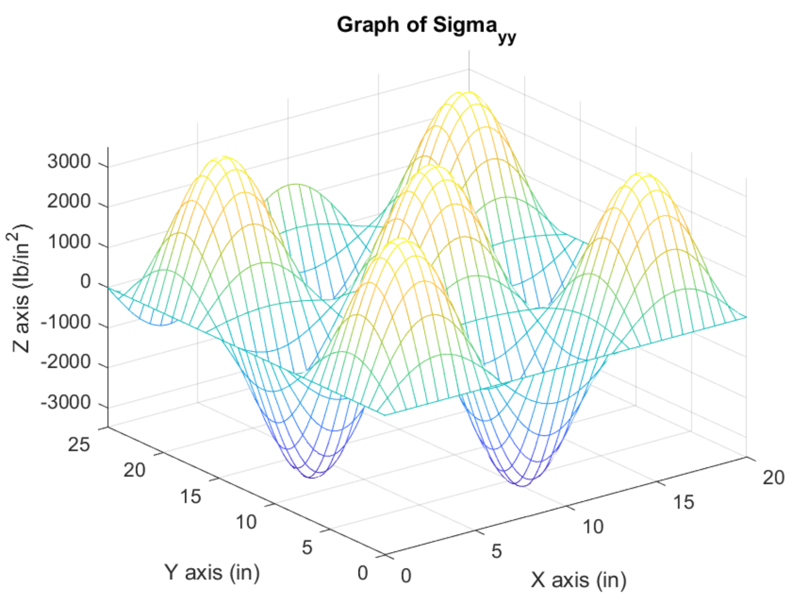

Figure 6:

8-3) Repeat problem (8-1, parts (a) and (b)), where the applied sinusoidal pressure loading is replaced with a uniform pressure load of magnitude $\left(p_0 = 0.05\text{lb/in}^2\right)$. Your solution should be based upon:

a.) One term $(m = n = 1)$
b.) Four terms $(m = n = 1, 3)$.
c.) Nine terms $(m = n = 1, 3, 5)$.
d.) Compare the displacement and stress results for parts (a-c) and comment on the convergence. Comment on why only the odd series terms were used.

Problem 8-3) a-d:

$$P_z(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} P_{mn} sin(\frac{m\pi x}{a}) sin(\frac{n\pi y}{b}) \tag{12}$$

$$P_{mn} = \int_0^b \int_0^a P_z(x,y) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) dx\, dy \tag{13}$$

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn} sin(\frac{m\pi x}{a}) sin(\frac{n\pi y}{b}) \tag{14}$$

Using equation (12) (13) (14) and (2) I determined the displacement distribution equation:

$$\Rightarrow w(x,y) = \frac{a^4}{\pi^4 D} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} w_{mn} \frac{P_{mn}}{(m^2 + n^2 \left(\frac{a}{b}\right)^2)^2} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

I then used matlab to find the displacement and the plate center. The top left term in the matrix represent the m = 1, n = 1, and the bottom right term represents m = 5, n = 5. To get the total displacement we just sum all of this. These summations are represented in the table below the matrix

$$\begin{bmatrix} 0.3958 & -0.0072 & 6.74 \times 10^{-4} \\ -0.004 & 5.43 \times 10^{-4} & -1.09 \times 10^{-4} \\ 3.44 \times 10^{-4} & -7.75 \times 10^{-5} & 2.5332 \times 10^{-5} \end{bmatrix}$$

Terms	Total Displacement(in)	Error
1	0.3958	2.59%
4	0.3851	0.181%
9	0.3860	0.05%
∞	0.3858	0%

The stresses were then again found using equations (4), (5), (6) and (7) again. The plots for all the m and n values for part a-c are in figures 7 through 15.

The reason that we only use the odd terms is because $(1 - cos(n\pi)) = 0$ for even integers in the equation 13 $P_m n$ when you integrate.

$$\sigma_{xx} = \frac{192a^2 P_0 z}{\pi^4 t^3} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} w_{mn} \frac{(m^2 + \nu n^2 \left(\frac{a}{b}\right)^2)^2}{(m^2 + n^2 \left(\frac{a}{b}\right)^2)^2} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$\sigma_{yy} = \frac{192a^2 P_0 z}{\pi^4 t^3} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} w_{mn} \frac{(\nu m^2 + n^2 \left(\frac{a}{b}\right)^2)^2}{(m^2 + n^2 \left(\frac{a}{b}\right)^2)^2} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

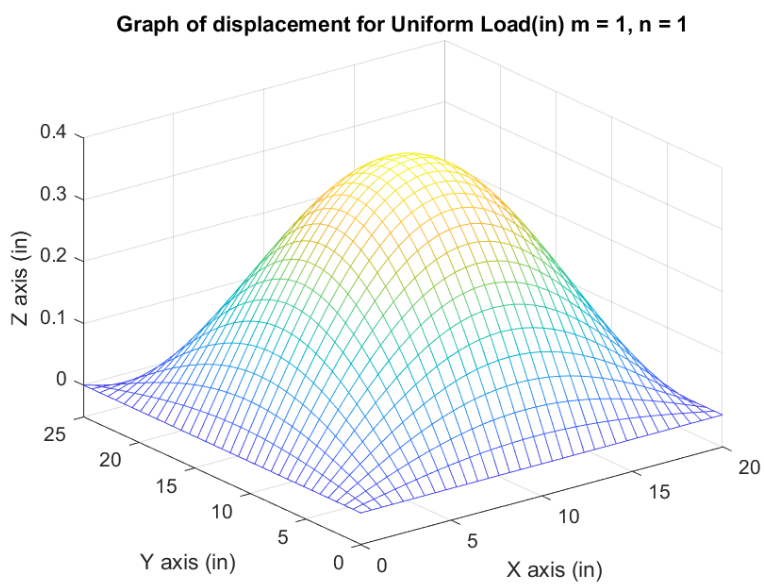

Figure 7:

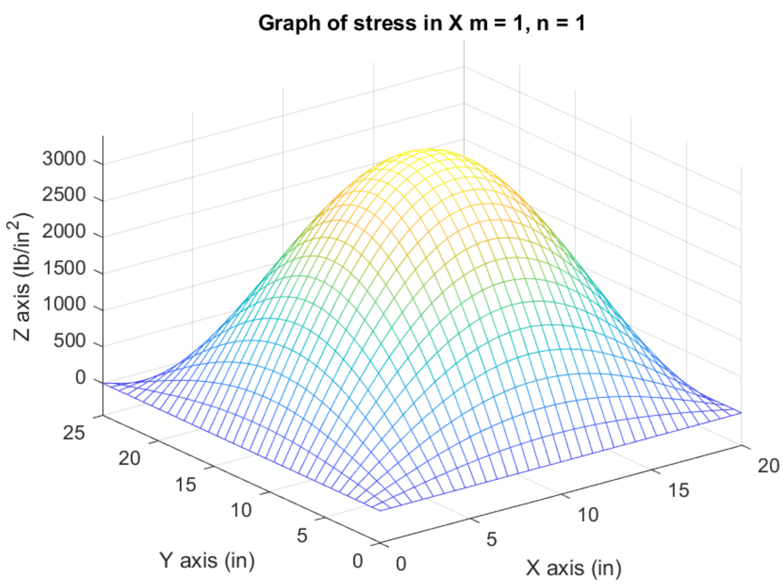

Figure 8:

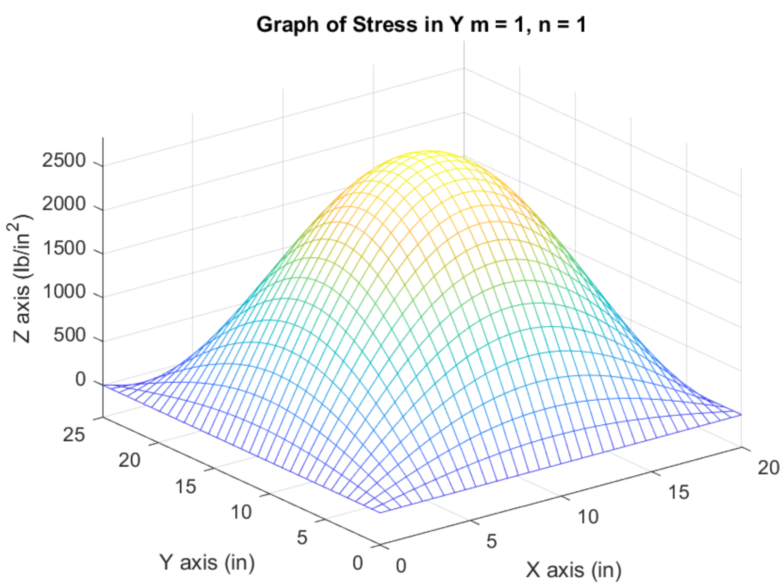

Figure 9:

Problem 8-3) b:

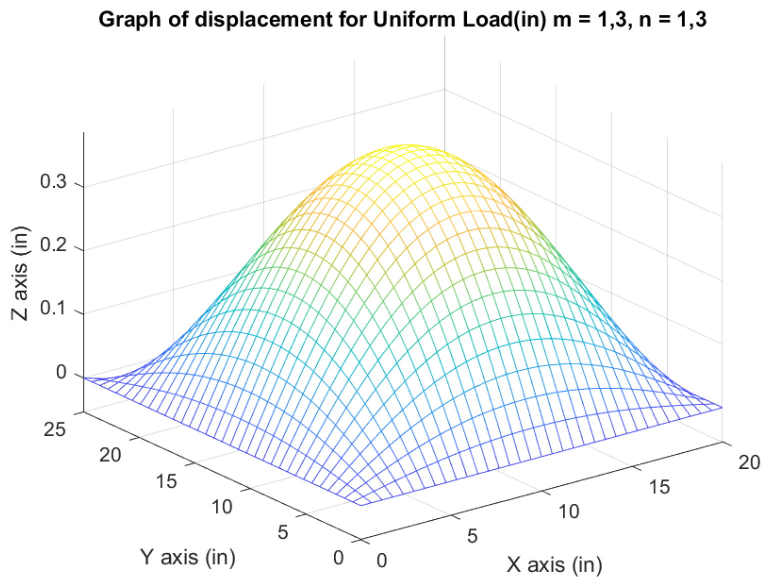

Figure 10:

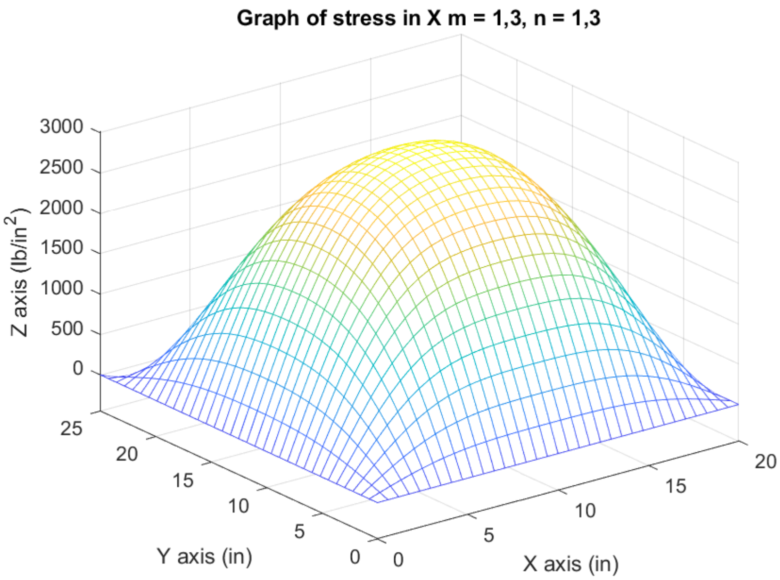

Figure 11:

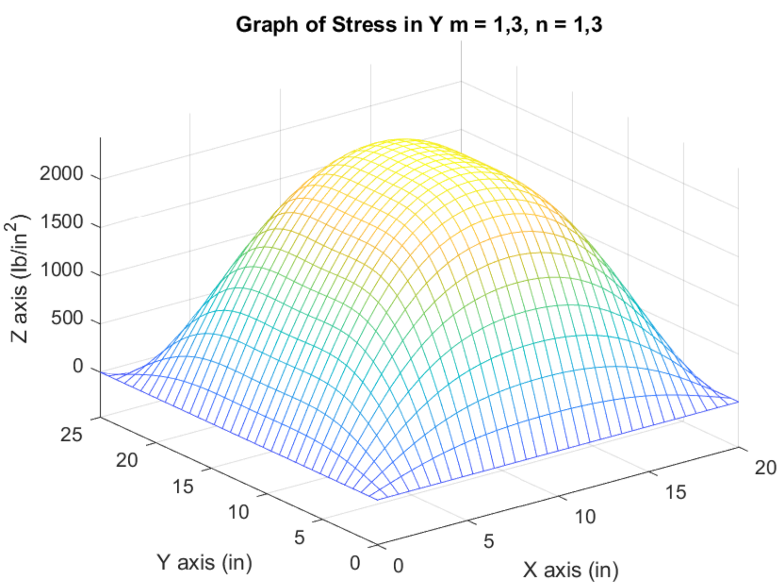

Figure 12:

Problem 8-3) c:

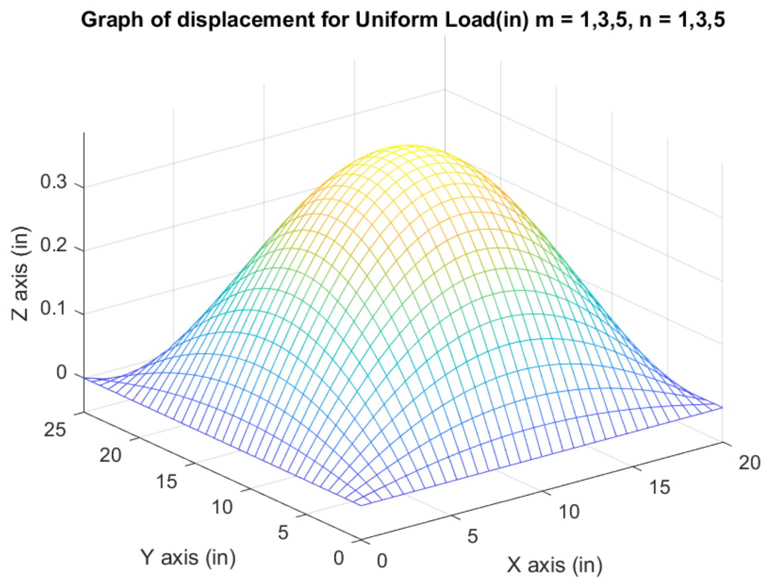

Figure 13:

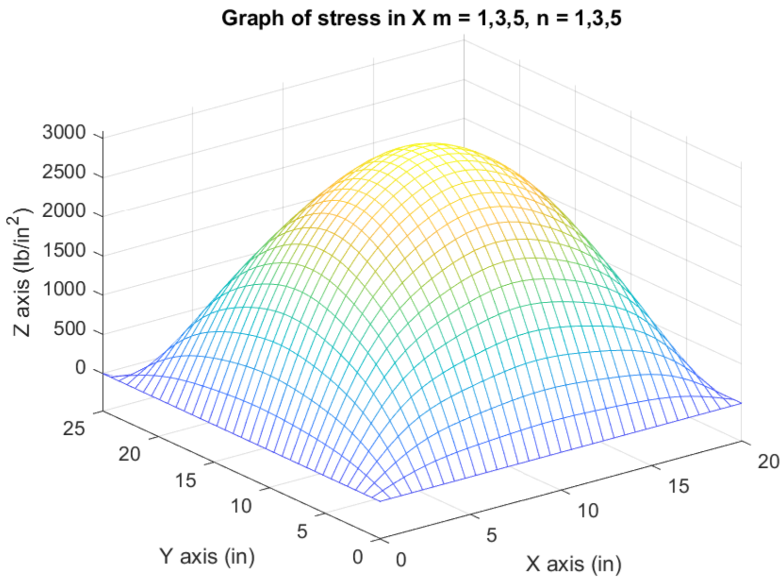

Figure 14:

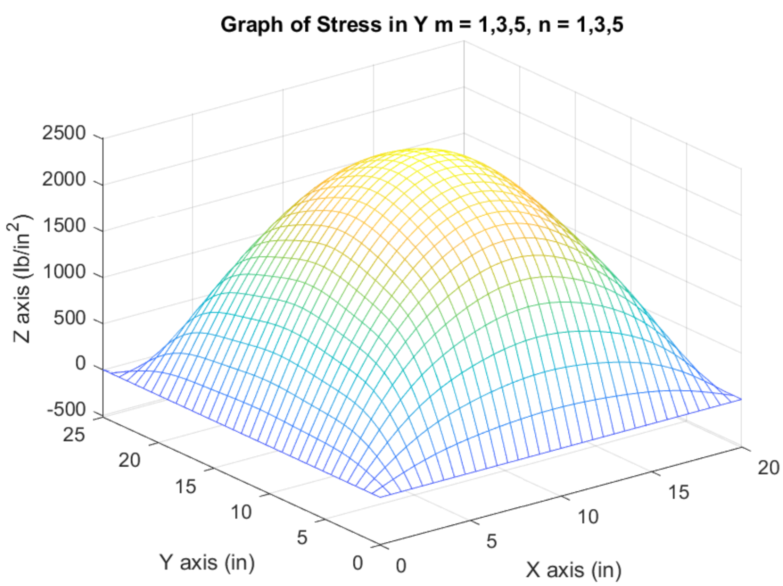

Figure 15:

8-4) A simply-supported aluminum plate with ($a=12$ inch, $b=12$ inch, and $t=0.050$ inch), is subjected to loads ($P_1=50$lbs) and ($P_2=150$lbs) at points ($x=a/2, y=b/4$) and ($x=a/2, y=3b/4$), respectively. The aluminum plate properties are $\left(E=10\times10^6 \mathrm{lb}^2/\mathrm{in}^2, v=0.333, \sigma_{\mathrm{y}}=70\times10^3\ \mathrm{lb}/\ \mathrm{in}\ ^2\right)$. Using the first nine terms of the series ($n, m=1,2,3$),

a.) Calculate the center deflection and plot the displacement distribution using MATLAB

b.) Calculate the stresses ($\sigma_{\mathrm{xx}}, \sigma_{\mathrm{yy}}$) on the upper surface at plate center.

c.) Calculate the ($\sigma_{\mathrm{xx}}, \sigma_{\mathrm{yy}}$) distributions on the upper surface and plot using MATLAB.

d.) Calculate the margin of safety (MS) on the upper surface at plate center using the two stresses from part (c) and along with a von Mises failure criteria and (SF = 1.5).

e.) Calculate the margin of safety (MS) distribution on the upper surface and plot using MATLAB.

Problem 8-4) a:

Point loads are simulated on a plate by having the bounds on the itegral for the load go to zero and converge at a point. When we assume our area is infinitely small we get the equations below. I then graphed the displacement below.

$$P_{mn}=\frac{4P_1}{ab}sin(\frac{m\pi x_1}{a})sin(\frac{n\pi y_1}{b})+\frac{4P_2}{ab}sin(\frac{m/pix_2}{a})sin(\frac{n\pi y_2}{b})$$

$$w(x,y)=\frac{4a^2}{\pi^4 D}\sum_{m=1,2,3}^{\infty}\sum_{n=1,2,3}^{\infty}\frac{(P_1sin(\frac{m\pi x_1}{a})sin(\frac{n\pi y_1}{b})+P_2sin(\frac{m/pix_2}{a})sin(\frac{n\pi y_2}{b}))(sin(\frac{m\pi x}{a})sin(\frac{n\pi y}{b}))}{(m^2+n^2(\frac{a}{b})^2)^2}$$

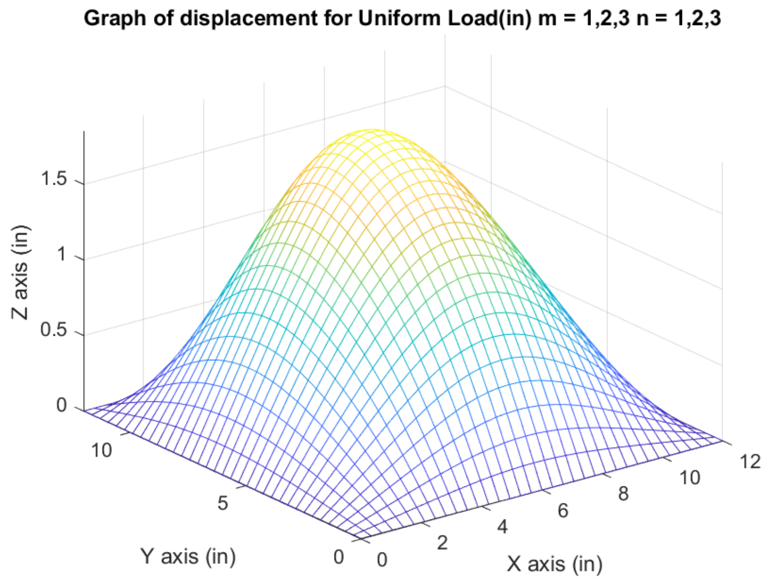

Figure 16:

Problem 8-4) b and c:

Equations (4) (5) (6) and (7) were used to calculate the stresses again. I calculated the equations on matlab and plotted the stresses in the figures below. The stress at the center of the plate was calculated to be:$\sigma_{xxcenter} = 48,087$ PSI and $\sigma_{yycenter} = 33407 PSI$

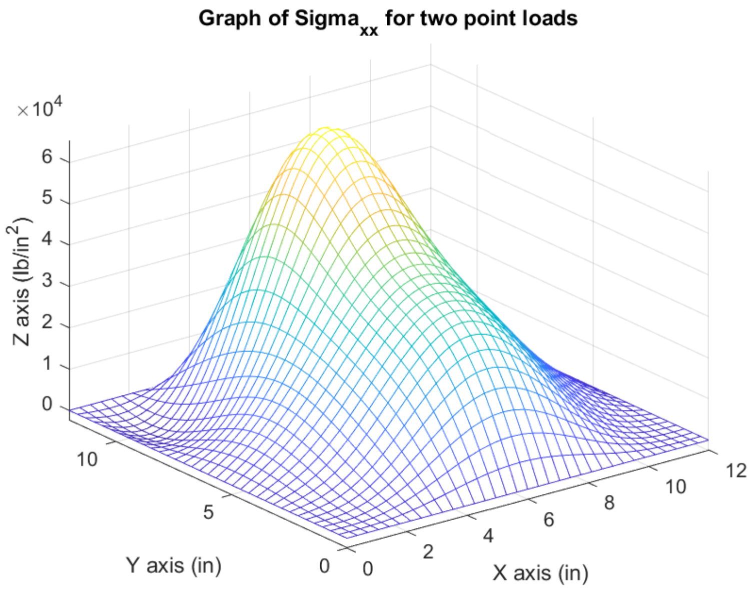

Figure 17:

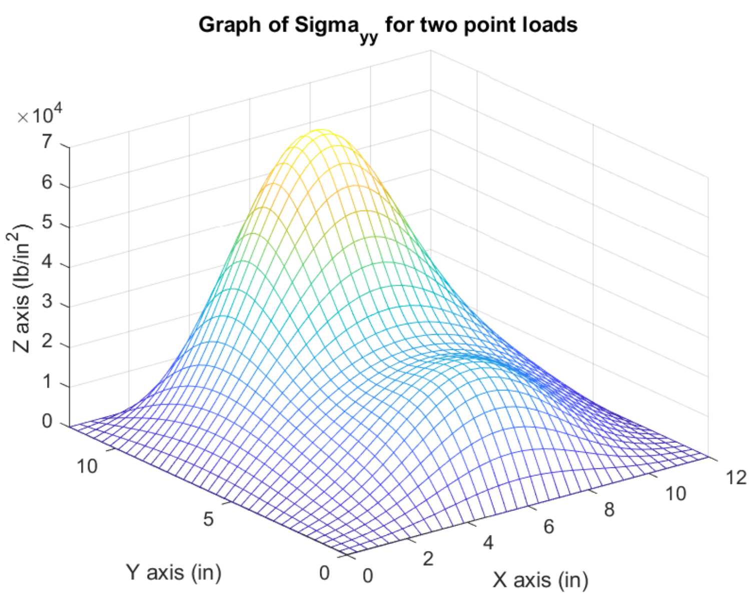

Figure 18:

Problem 8-4) d and e:

The margin of safety was calculated using equations (8) (9) and (10). At the center of the plate the p=margin of safety was calculated to be 0.0933. A graph of the margin of safety over the plate is in the figure below. I did not graph it over the entire plate because the margin of safety because extremely large at the boundaries.

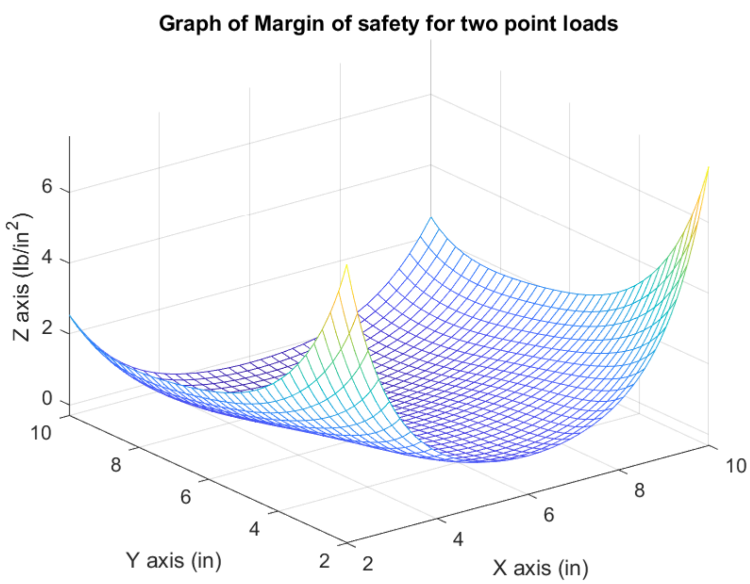

Figure 19:

3. (15 points) You would like to solve the problem

$$\dot{y} = g(t, y), \quad y(0) = 4$$

up to time $T = 6$. Here, $g(t, y)$ is the solution of

$$z = e^{-(1+\sin(z))} - \sin^2(t + y)\left(1 + z^2\right)^{1/3}.$$

For example, if $y = \pi/2$ and $t = 1$, then $g(t, y)$ is the solution of fixed-point equation $z = e^{-(1+\sin(z))} - \sin^2(1 + \pi/2)\left(1 + z^2\right)^{1/3}$. Use your fourth-order Runge-Kutta code. The function evaluations inside the Runge-Kutta code will use some sort of fixedpoint or root-finding code. This function evaluation must be (or be inside) a separate function called by the Runge-Kutta code. Plot your solution approximations for several n values, say $n \in \{8, 16, 32, 64, 128, 256, 512\}$, for example.