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las4s e pelados

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- 2 Theoretical
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Contest (1)

template.cpp	9 lines
<pre>#include <bits/stdc++.h> using namespace std; #define rep(i, a, b) for(int i = a; i < (b); ++i) #define all(x) begin(x), end(x) #define sz(x) (int)(x).size() using ll = long long; using pii = pair<int,int>; using vi = vector<int>;</pre>	
.bashrc	2 lines
<pre>alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \ -fsanitize=undefined,address'</pre>	
hash.sh	2 lines
<pre># bash hash.sh file.cpp l1 l2 sed -n \$2', '\$3' p' \$1 sed '/^#w/d' cpp -dD -P - fpreprocessed tr -d '[:space:]' md5sum cut -c-6</pre>	
stressTest.sh	20 lines
<pre>P=code #nude pro filename do codigo Q=brute #nude pro filename do brute [correto] g++ \${P}.cpp -o sol -O2 exit 1 g++ \${Q}.cpp -o brt -O2 exit 1 g++ gen.cpp -o gen -O2 exit 1 for ((i = 1; ; i++)) do echo \$i ./gen \$i > in ./sol < in > out ./brt < in > out2 if (! cmp -s out out2) then echo "--> entrada:" cat in echo "--> saida code:" cat out</pre>	

1	echo "--> saida brute:"
1	cat out2
1	break;
5	fi
5	done
paperStress.py26 lines	
7	import random
	import subprocess
10	MAX_N = 100
10	def gen_case() -> str:
11	return f"1\n"
12	for i in range(100):
12	print(), print()
18	case = gen_case()
18	print(f"Test #{i+1}: ")
23	print(case)
23	# test bruteforce
24	bf = subprocess.run(['out/b'], input=case, encoding='ascii', capture_output=True)
24	# test solution
37c	sol = subprocess.run(['out/m'], input=case, encoding='ascii', capture_output=True)
	bf_res = bf.stdout
	sol_res = sol.stdout
	print(f"bruteforce {bf_res}, solution {sol_res}")
	if bf_res == sol_res:
	print("accepted")
	else:
	print("WA")
	break
troubleshoot.txt52 lines	
Pre-submit:	
Write a few simple test cases if sample is not enough.	
Are time limits close? If so, generate max cases.	
Is the memory usage fine?	
Could anything overflow?	
Make sure to submit the right file.	
Wrong answer:	
Print your solution! Print debug output, as well.	
Are you clearing all data structures between test cases?	
Can your algorithm handle the whole range of input?	
Read the full problem statement again.	
Do you handle all corner cases correctly?	
Have you understood the problem correctly?	
Any uninitialized variables?	
Any overflows?	
Confusing N and M, i and j, etc.?	
Are you sure your algorithm works?	
What special cases have you not thought of?	
Are you sure the STL functions you use work as you think?	
Add some assertions, maybe resubmit.	
Create some testcases to run your algorithm on.	
Go through the algorithm for a simple case.	
Go through this list again.	
Explain your algorithm to a teammate.	
Ask the teammate to look at your code.	
Go for a small walk, e.g. to the toilet.	
Is your output format correct? (including whitespace)	
Rewrite your solution from the start or let a teammate do it.	
Runtime error:	

Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?

Theoretical (2)

2.1 Mathematics

2.1.1 Recurrences

If $a_n = c_1a_{n-1} + \dots + c_k a_{n-k}$, and r_1, \dots, r_k are distinct roots of $x^k - c_1x^{k-1} - \dots - c_k$, there are d_1, \dots, d_k s.t.

$$a_n = d_1r_1^n + \dots + d_kr_k^n.$$

Non-distinct roots r become polynomial factors, e.g.

$$a_n = (d_1n + d_2)r^n.$$

2.1.2 Trigonometry

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$

$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$

$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w .

$$a \cos x + b \sin x = r \cos(x - \phi)$$

$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

2.1.3 Geometry
Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles):
 $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[1 - \left(\frac{a}{b + c} \right)^2 \right]}$$

Law of sines: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\frac{a + b}{a - b} = \frac{\tan \frac{\alpha + \beta}{2}}{\tan \frac{\alpha - \beta}{2}}$

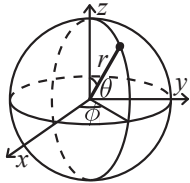
Quadrilaterals

With side lengths a, b, c, d , diagonals e, f , diagonals angle θ , area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° ,
 $ef = ac + bd$, and $A = \sqrt{(p - a)(p - b)(p - c)(p - d)}$.

Spherical coordinates



$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z / \sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

Pick's Theorem

The area of a simple polygon whose vertices have integer coordinates is:

$$A = I + \frac{B}{2} - 1$$

where I is the number of interior integer points, and B is the number of integer points in the border of the polygon.

Two Ears Theorem

Every simple polygon with more than 3 vertices has at least two non-overlapping ears (a ear is a vertex whose diagonal induced by its neighbors which lies strictly inside the polygon). Equivalently, every simple polygon can be triangulated.

2.1.4 Derivatives/Integrals

$$\begin{aligned} \frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1 - x^2}} & \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1 - x^2}} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \frac{d}{dx} \arctan x &= \frac{1}{1 + x^2} \\ \int \tan ax &= -\frac{\ln |\cos ax|}{a} & \int x \sin ax &= \frac{\sin ax - ax \cos ax}{a^2} \\ \int e^{-x^2} &= \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) & \int x e^{ax} dx &= \frac{e^{ax}}{a^2} (ax - 1) \end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.1.5 Sums

$$c^a + c^{a+1} + \dots + c^b = \frac{c^{b+1} - c^a}{c - 1}, \quad c \neq 1$$

$$1^2 + 2^2 + \dots + n^2 = \frac{n(2n + 1)(n + 1)}{6}$$

$$1^3 + 2^3 + \dots + n^3 = \frac{n^2(n + 1)^2}{4}$$

$$1^4 + 2^4 + \dots + n^4 = \frac{n(n + 1)(2n + 1)(3n^2 + 3n - 1)}{30}$$

$$\sum_{i=0}^n ic^i = \frac{nc^{n+2} - (n + 1)c^{n+1} + c}{(c - 1)^2}, \quad c \neq 1$$

$$g_k(n) = \sum_{i=1}^n i^k = \frac{1}{k + 1} \left(n^{k+1} + \sum_{j=1}^k \binom{k + 1}{j + 1} (-1)^{j+1} g_{k-j}(n) \right)$$

2.1.6 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, \quad (-\infty < x < \infty)$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, \quad (-1 < x \leq 1)$$

$$\sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, \quad (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, \quad (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, \quad (-\infty < x < \infty)$$

$$\sum_{i=0}^{\infty} ic^i = \frac{c}{(1 - c)^2}, \quad |c| < 1$$

$$(1 + x)^n = \sum_{i=0}^n \binom{n}{i} x^i$$

$$\frac{1}{1 - x} = \sum_{i=0}^{\infty} x^i, \quad (-1 < x < 1)$$

$$\frac{1}{(1 - x)^n} = \sum_{i=0}^{\infty} \binom{n + i - 1}{n - 1} x^i, \quad (-1 < x < 1)$$

2.1.7 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x . It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y ,

$$V(aX + bY) = a^2 V(X) + b^2 V(Y).$$

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\operatorname{Bin}(n, p)$, $n = 1, 2, \dots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$\mu = np, \quad \sigma^2 = np(1 - p)$$

$\operatorname{Bin}(n, p)$ is approximately $\operatorname{Po}(np)$ for small p .

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\operatorname{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1 - p)^{k-1}, \quad k = 1, 2, \dots$$

$$\mu = \frac{1}{p}, \quad \sigma^2 = \frac{1 - p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\text{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \dots$$

$$\mu = \lambda, \sigma^2 = \lambda$$

2.2 Combinatorial

2.2.1 Binomial Identities

$$\binom{n-1}{k} - \binom{n-1}{k-1} = \frac{n-2k}{k} \binom{n}{k}$$
$$\binom{n}{h} \binom{n-h}{k} = \binom{n}{k} \binom{n-k}{h}$$
$$\sum_{k=0}^n k \binom{n}{k} = n2^{n-1}$$
$$\sum_{k=0}^n k^2 \binom{n}{k} = (n+n^2)2^{n-2}$$
$$\sum_{j=0}^k \binom{m}{j} \binom{n-m}{k-j} = \binom{n}{k}$$
$$\sum_{j=0}^m \binom{m}{j}^2 = \binom{2m}{m}$$
$$\sum_{m=0}^n \binom{m}{j} \binom{n-m}{k-j} = \binom{n+1}{k+1}$$
$$\sum_{m=0}^n \binom{m}{k} = \binom{n+1}{k+1}$$
$$\sum_{r=0}^m \binom{n+r}{r} = \binom{n+m+1}{m}$$
$$\sum_{k=0}^n \binom{n-k}{k} = \text{Fib}(n+1)$$
$$\sum_{k=0}^n \binom{r}{k} \binom{s}{n-k} = \binom{r+s}{n}$$

2.2.2 Permutations

Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

Cycles

Let $g_S(n)$ be the number of n -permutations whose cycle lengths all belong to the set S . Then

$$\sum_{n=0}^\infty g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

Burnside’s lemma

Counts the number of distinct colorings of an object under symmetry.

$$\frac{1}{|G|} \sum_{g \in G} k^{\text{cyc}(g)},$$

where G is the symmetry group, k the number of colors, and $\text{cyc}(g)$ the number of cycles induced by g .

Example: number of ways to color a necklace with n beads using k colors (rotations only):

$$g(n) = \frac{1}{n} \sum_{i=0}^{n-1} k^{\text{gcd}(n,i)}$$

where rotation i shifts the necklace by i positions.

2.2.3 Partitions and subsets

Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \quad p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

Lucas’ Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + \dots + n_1 p + n_0$ and $m = m_k p^k + \dots + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$.

2.2.4 Sum of Binomials (FFT)

Goal: Given freq. array C , compute $\text{Ans}[k] = \sum_i C[i] \binom{i}{k}$ for all k . Rewrite: $\text{Ans}[k] = \frac{1}{k!} \sum_i (C[i] \cdot i!) \frac{1}{(i-k)!}$.

- Construct P where $P[i] = C[i] \cdot i!$
- Construct Q where $Q[i] = (i!)^{-1}$
- Reverse Q (to handle the $i - k$ subtraction).
- Multiply $R = NTT(P, Q)$.
- Result: $\text{Ans}[k] = R[k + |Q| - 1] \cdot \frac{1}{k!}$.

2.2.5 General purpose numbers

Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).
 $B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$

Sums of powers:

$$\sum_{i=1}^n n^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned} \sum_{i=m}^\infty f(i) &= \int_m^\infty f(x) dx - \sum_{k=1}^\infty \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_m^\infty f(x) dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m)) \end{aligned}$$

Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$\begin{aligned} c(n, k) &= c(n-1, k-1) + (n-1)c(n-1, k), \quad c(0, 0) = 1 \\ \sum_{k=0}^n c(n, k) x^k &= x(x+1) \dots (x+n-1) \end{aligned}$$

$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
 $c(n, 2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$

Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k)$$

$$S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

Labeled unrooted trees

- on n vertices: n^{n-2}
- on k existing trees of size n_i : $n_1 n_2 \dots n_k n^{k-2}$
- with degrees d_i : $(n-2)! / ((d_1-1)! \dots (d_n-1)!)$

Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$
$$C_0 = 1, \ C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \ C_{n+1} = \sum C_i C_{n-i}$$
$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n + 1$ leaves (0 or 2 children).
- ordered trees with $n + 1$ vertices.
- ways a convex polygon with $n + 2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

2.3 Number Theory

2.3.1 Bézout’s identity

For $a \neq 0, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{gcd(a,b)}, y - \frac{ka}{gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

2.3.2 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a , except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

2.3.3 Estimates

$$\sum_{d \mid n} d = O(n \log \log n).$$

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 6700 for $n < 1e12$, 200 000 for $n < 1e19$.

2.3.4 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d \mid n} f(d) \Leftrightarrow f(n) = \sum_{d \mid n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$$\sum_{d \mid n} \mu(d) = [n = 1] \text{ (very useful)}$$
$$g(n) = \sum_{n \mid d} f(d) \Leftrightarrow f(n) = \sum_{n \mid d} \mu(d/n) g(d)$$
$$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$$

2.3.5 Theorems

Goldbach’s conjecture: Every even integer $n > 2$ can be written as $n = a + b$ with a, b prime.

Legendre’s conjecture: There is always at least one prime between n^2 and $(n + 1)^2$.

Lagrange’s four-square theorem: Every positive integer can be written as

$$n = a^2 + b^2 + c^2 + d^2.$$

Zeckendorf’s theorem: Every integer $n \geq 1$ has a unique representation as a sum of non-consecutive Fibonacci numbers:

$$n = F_{i_1} + F_{i_2} + \dots + F_{i_k}, \quad i_j - i_{j+1} \geq 2.$$

Euclid’s formula (primitive Pythagorean triples): The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \ b = k \cdot (2mn), \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0, k > 0, m \perp n$, and either m or n even.

Wilson’s theorem: n is prime iff

$$(n - 1)! \equiv -1 \pmod{n}.$$

Chicken McNugget theorem: For coprime n, m , the largest integer not representable as $an + bm$ (with $a, b \geq 0$) is

$$nm - n - m.$$

There are $\frac{(n-1)(m-1)}{2}$ non-representable integers, and for each pair $(k, nm - n - m - k)$ exactly one is representable.

2.4 Graphs

2.4.1 Flows and Matching

Hall’s Theorem

In bipartite graphs, there exists a perfect matching covering the entire side X if and only if for every subset $Y \subseteq X$,

$$|Y| \leq |N(Y)|,$$

where $N(Y)$ denotes the set of neighbors of Y .

König’s Theorem

In a bipartite graph, the size of a Minimum Vertex Cover is equal to the size of a Maximum Matching. A Minimum Vertex Cover is a minimum set of vertices such that every edge of the graph has at least one endpoint in the set.

As a consequence,

$$n - \text{Maximum Matching} = \text{Maximum Independent Set},$$

where a Maximum Independent Set is the largest set of vertices with no edges between them.

Recovering the Minimum Vertex Cover Given a maximum matching in a bipartite graph (X, Y) :

- Construct the residual graph by orienting:
 - non-matching edges from X to Y ;
 - matching edges from Y to X .
- Perform a BFS or DFS starting from all free (unmatched) vertices in X .
- Let Z_X be the set of reachable vertices in X , and Z_Y the set of reachable vertices in Y .

The Minimum Vertex Cover is given by:

$$(X \setminus Z_X) \cup Z_Y.$$

Node-Disjoint Path Cover

A node-disjoint path cover is a set of paths such that each vertex belongs to exactly one path.

In a directed acyclic graph (DAG),

$$\text{Minimum Node-Disjoint Path Cover} = n - \text{Maximum Matching}.$$

The construction is as follows: for each vertex u , create a copy u' . Add an edge $u \rightarrow v'$ if there exists an edge $u \rightarrow v$ in the original graph.

Recovering the Paths

- Vertices that do not appear as destinations in the matching are starting points of paths.
- Each matching edge $u \rightarrow v'$ corresponds to an edge $u \rightarrow v$ in the original DAG.
- Following these edges reconstructs all paths of the path cover.

General Path Cover

A general path cover is a path cover where a vertex may belong to more than one path.

In a DAG, the construction is similar to the node-disjoint case, but an edge $u \rightarrow v'$ exists if there is a path from u to v in the original graph.

Recovering the Cover The vertices can be grouped according to the edges used in the matching to form the path cover.

Dilworth’s Theorem

An antichain is a set of vertices such that there is no path between any pair of vertices in the set.

In a directed acyclic graph,

Minimum General Path Cover = Maximum Antichain.

Recovering a Maximum Antichain Given a minimum general path cover, selecting one vertex from each path produces a maximum antichain.

2.4.2 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \rightarrow b \in G$, do `mat[a][b]--`, `mat[b][b]++` (and `mat[b][a]--`, `mat[a][a]++` if G is undirected). Remove the i th row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

2.4.3 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \dots \geq d_n$ exists iff $d_1 + \dots + d_n$ is even and for every $k = 1 \dots n$,

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k).$$

2.4.4 Planar Graphs

If G has k connected components, then $n - m + f = k + 1$.

2.5 Optimization tricks

2.5.1 Bit hacks

- for (int x = m; x; x = (x - 1)&m) { ... }
loops over all subset masks of m (except 0).
- c = x&-x, r = x+c; ((r^x) >> 2)/c | r is the next number after x with the same number of bits set.
- rep(b,0,K) rep(i,0,(1 << K))
if (i & 1 << b) D[i] += D[i^(1 << b)];
computes all sums of subsets.

2.5.2 Pragmas

- #pragma GCC optimize ("Ofast") will make GCC auto-vectorize loops and optimizes floating points better.
- #pragma GCC target ("avx2") can double performance of vectorized code, but causes crashes on old machines.
- #pragma GCC target ("bmi,bmi2,popcnt,lzcnt") improve bit operations.
- #pragma GCC optimize("unroll-loops") self explanatory.

2.6 Various

2.6.1 Master Theorem (Simple)

$T(n) = aT(n/b) + O(n^d)$. Compare a vs b^d :

- $a > b^d \implies O(n^{\log_b a})$ (Work at leaves dominates)
- $a = b^d \implies O(n^d \log n)$ (Work is uniform)
- $a < b^d \implies O(n^d)$ (Work at root dominates)

Data structures (3)

Bit.h

Description: lower_bound works the same as on vectors

Time: $\mathcal{O}(\log N)$

5891da, 23 lines

8eb struct Bit {
406 vector<ll> bit;
1dd Bit(int n) : bit(n + 1) {}
265 void update(int i, ll v) {
c38 for (i++; i < sz(bit); i += i & -i) bit[i] += v;
f21 }
74a ll query(int i) {
b73 ll ret = 0;
71c for (i++; i > 0; i -= i & -i) ret += bit[i];
edf return ret;
e40 }
dc8 int lower_bound(ll v) { // min pos st sum[0, pos] >= v
bec int pos = 0;
a40 for (int j=(1 << 23); j >= 1; j/=2){
3b1 if (pos+j < sz(bit) && bit[pos + j] < v){
b4e pos += j;
18d v -= bit[pos];
f6c }
156 }
d75 return pos;
37b }
589 };

Bit2d.h

Description: Points called on the update function NEED to be on the pts vector parameter on build.

Time: $\mathcal{O}((\log N)^2)$

5a98ac, 37 lines

9c0 struct Bit2d {
a37 vector<vector<int>> ys;
fe8 vector<Bit> bit;
543 vector<int> cmp_x;
425 Bit2d() {}
521 void put(int x, int y) {
005 for (x++; x < sz(ys); x += x & -x) ys[x].push_back(y);
f3c }

ce0 int id(const vector<int> &v, int y) {
1e9 return (upper_bound(all(v), y) - v.begin()) - 1;
19a }
7ff void build(vector<pii> pts) {
3cb sort(all(pts));
f99 for(auto p : pts) cmp_x.push_back(p.first);
9a7 cmp_x.erase(unique(all(cmp_x)), cmp_x.end());
f82 ys.resize(cmp_x.size() + 1);
94d for(auto p : pts) put(id(cmp_x, p.first), p.second);
310 for(auto &v:ys) sort(all(v)), bit.emplace_back(sz(v));
a01 }
767 void update(int x, int y, int val){
f3f x = id(cmp_x, x);
681 for(x++; x < sz(ys); x+= x&-x)
507 bit[x].update(id(ys[x], y), val);
c88 }
d95 int query(int x, int y){
f3f x = id(cmp_x, x);
7c9 int ret = 0;
f32 for(x++; x > 0; x-= x&-x)
ea8 ret += bit[x].query(id(ys[x], y));
edf return ret;
8f7 }
251 int query(int x1, int y1, int x2, int y2){
e4d int a = query(x2, y2)-query(x2, y1-1);
7d1 return a-query(x1-1, y2)+query(x1-1, y1-1);
c33 }
5a9 };

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming (“convex hull trick”).

Time: $\mathcal{O}(\log N)$

8ec1c7, 32 lines

72c struct Line {
3e2 mutable ll k, m, p;
ca5 bool operator<(const Line& o) const { return k < o.k; }
abf bool operator<(ll x) const { return p < x; }
7e3 };

781 struct LineContainer : multiset<Line, less<>> {
// (for doubles, use inf = 1/.0, div(a,b) = a/b)
fd2 static const ll inf = LLONG_MAX;
33a ll div(ll a, ll b) { // floored division
10f return a / b - ((a ^ b) < 0 && a % b); }
a1c bool isect(iterator x, iterator y) {
a95 if (y == end()) return x->p = inf, 0;
9cb if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
591 else x->p = div(y->m - x->m, x->k - y->k);
870 return x->p >= y->p;
2fa }
a0c void add(ll k, ll m) {
116 auto z = insert({k, m, 0}), y = z++, x = y;
7b1 while (isect(y, z)) z = erase(z);
d94 if (x != begin() && isect(--x, y))
c07 isect(x, y = erase(y));
57d while ((y = x) != begin() && (--x)->p >= y->p)
774 isect(x, erase(y));
086 }
4ad ll query(ll x) {
229 assert(!empty());
7d1 auto l = *lower_bound(x);
96a return l.k * x + l.m;
d21 }
577 };

Mo.h
Description: For subtree queries, perform an Euler tour and map each node u to the interval $[tin[u], tin[u] + subtree_size[u] - 1]$. A subtree query becomes a range query over this interval.
For path queries between nodes U and V, Let U be the closest to the root. If V lies in U's subtree, the path corresponds to the interval $[tin[U], tin[V]]$. Otherwise, the path corresponds to the interval $[min(tout[U], tout[V]), max(tin[U], tin[V])]$.
In both cases, nodes on the U-V path appear exactly once in the interval, while all other nodes appear either 0 or 2 times.

Usage: queries.push(Query(l, r, index of query)), intervals are [l, r]

Time: $\mathcal{O}\left(N\sqrt{Q}\right)$

fb7161, 44 lines

```
626 inline int64_t hilOrd(int x, int y, int pow, int rot) {
51a   if (pow == 0) return 0;
a6e   int hpow = 1 << (pow - 1);
01f   int seg = (x < hpow) ? ((y < hpow) ? 0 : 3) : ((y < hpow)
) ? 1 : 2);
e08   seg = (seg + rot) & 3;
669   const int rotDelta[4] = { 3, 0, 0, 1 };
d0b   int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
115   int nrot = (rot + rotDelta[seg]) & 3;
fba   int64_t sub = int64_t(1) << (2 * pow - 2);
65b   int64_t ans = seg * sub;
1ae   int64_t add = hilOrd(nx, ny, pow - 1, nrot);
ff7   ans += (seg == 1 || seg == 2) ? add : (sub - add - 1);
ba7   return ans;
ec4 }
```

```
670 struct Query {
738   int l, r, idx;
ce8   int64_t ord;
36f   Query(int l, int r, int idx) : l(l), r(r), idx(idx) {
6c4     ord = hilOrd(l, r, 21, 0);
926   }
847   bool operator < (const Query& other) const {
328     return ord < other.ord;
e05   }
315   };
```

```
240 vector<Query> queries;
4d5 int ans[ms];
566 void put(int x) {} // F
c29 void remove(int x) {} // F
64b int getAns() {}
```

```
1c1 void Mo() {
3d9   int l = 0, r = -1;
bfa   sort(queries.begin(), queries.end());
275   for (Query q : queries) {
482     while (l > q.l) put(--l);
fec     while (r < q.r) put(++r);
5b8     while (l < q.l) remove(l++);
9b5     while (r > q.r) remove(r--);
745     ans[q.idx] = getAns();
5a4   }
2a4 }
```

MoUpdate.h

Description: Block size should be around $(2 * N * N)^{\frac{1}{3}}$
Usage: intervals are [l, r], addQuery(l, r, number of updates happened before this query, index of query), addUpdate(index of updated position, value before update, value after update)

Time: $\mathcal{O}\left(Q * (2 * N * N)^{\frac{1}{3}} * F\right)$

f8eda8, 55 lines

```
496 const int B = 2700;
```

```
247 struct MoUpdate {
670   struct Query {
fd6     int l, r, t, idx;
fc8     Query(int l, int r, int t, int idx)
8bf       : l(l), r(r), t(t), idx(idx) {}
f51     bool operator < (const Query& p) const {
f06       if (l / B != p.l / B) return l < p.l;
e80       if (r / B != p.r / B) return r < p.r;
d0c       return t < p.t;
673     }
bc2   };
f2f   struct Upd {
f25     int i, old, now;
f23     Upd(int i, int old, int now) : i(i), old(old), now(now) {}
c12   };
```

```
240 vector<Query> queries;
e2b vector<Upd> updates;
```

```
ac5 void addQuery(int l, int r, int t, int idx) {
fc9   queries.push_back(Query(l, r, t, idx)); }
968 void addUpdate(int i, int old, int now) {
936   updates.push_back(Upd(i, old, now)); }
```

```
1aa void add(int x) {} // F
598 void rem(int x) {} // F
64b int getAns() {}
0d2 void update(int novo, int idx, int l, int r) {
2b9   if (l <= idx && idx <= r) rem(idx);
4ce   arr[idx] = novo;
ec1   if (l <= idx && idx <= r) add(idx);
100 }
```

```
63d void solve() {
cb1   int l = 0, r = -1, t = 0;
bfa   sort(queries.begin(), queries.end());
275   for (Query q : queries) {
a95     while (l > q.l) add(--l);
875     while (r < q.r) add(++r);
8f6     while (l < q.l) rem(l++);
a38     while (r > q.r) rem(r--);
fda     while (t < q.t) {
df3       auto u = updates[t++];
285       update(u.now, u.i, l, r);
8a4     }
32a     while (t > q.t) {
d69       auto u = updates[--t];
ce2       update(u.old, u.i, l, r);
3bf     }
745     ans[q.idx] = getAns();
f06   }
b09   }
d3e   };
```

MinQueue.h

40df8d, 19 lines

```
925 struct MQueue {
fdd   int tin, tout;
375   deque<pair<int, int>> dq;
1ce   MQueue() : tin(0), tout(0) {}
619   void push(int val) {
f0d     while (!dq.empty() && min(dq.back().first, val) ==
val) dq.pop_back();
9c6     dq.push_back(pair(val, tin++));
769   }
42d   void pop() {
// assert(!dq.empty());
48c     if (dq.front().second == tout) dq.pop_front();
470     tout++;
```

```
b0e   }
f46   int front() {
// assert(!dq.empty());
651     return dq.front().first;
fa2   }
40d   };
```

SegmentTree.h

Description: Zero-indexed max-tree. Bounds are inclusive to the left and inclusive to the right. Can be changed by modifying T, f and unit.

Time: $\mathcal{O}(\log N)$

f609d9, 21 lines

```
5ae struct Tree {
ef4   typedef int T;
cbe   static constexpr T unit = INT_MIN;
e54   T f(T a, T b) { return max(a, b); } // (any associative
fn)
6cd   vector<T> s; int n;
3d2   Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {}
6a3   void update(int pos, T val) {
56a     for (s[pos += n] = val; pos /= 2;)
326       s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
0e9   }
b4c   T query(int b, int e) { // query [b, e]
1a3     e++;
0f9     T ra = unit, rb = unit;
fbb     for (b += n, e += n; b < e; b /= 2, e /= 2) {
e83       if (b % 2) ra = f(ra, s[b++]);
064       if (e % 2) rb = f(s[--e], rb);
561     }
cb2     return f(ra, rb);
707   }
f60   };
```

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null-type.
Time: $\mathcal{O}(\log N)$

782797, 17 lines

```
c4d #include <bits/extc++.h>
0d7 using namespace __gnu_pbds;

4fc template<class T>
c20 using Tree = tree<T, null_type, less<T>, rb_tree_tag,
3a1   tree_order_statistics_node_update>;
```

```
ad0 void example() {
c6f   Tree<int> t, t2; t.insert(8);
559   auto it = t.insert(10).first;
d28   assert(it == t.lower_bound(9));
969   assert(t.order_of_key(10) == 1);
d39   assert(t.order_of_key(11) == 2);
1b7   assert(*t.find_by_order(0) == 8);
a60   t.join(t2); // merge t2 into t
9ad }
```

PersistentSegTree.h

Usage: SegP(size of the segtree, number of updates)

roots = {0}, newRoot = update(roots.back(), ...), roots.push(newRoot)

58842f, 42 lines

```
b17 struct SegP {
709   static constexpr ll neut = 0;
bf2   struct Node {
aa3     ll v; // start with neutral value
74f     int l, r;
9ef     Node(ll v=neut, int l=0, int r=0) : v(v), l(l), r(r){}
945   };
```

```
38f vector<Node> seg;
068 int n, CNT;
9ea SegP(int _n, int upd): seg(20*(upd+_n)), n(_n), CNT(1){}
2ce ll merge(ll a, ll b) { return a + b; }
c97 int update(int root, int pos, int val, int l, int r) {
ec9     int p = CNT++;
77a     seg[p] = seg[root];
893     if (l == r) {
00f         seg[p].v += val;
74e         return p;
3d7     }
ae0     int mid = (l + r) / 2;
8a3     if (pos <= mid){
aa8         seg[p].l = update(seg[p].l, pos, val, l, mid);
583     }else seg[p].r = update(seg[p].r,pos,val,mid+1,r);

85a     seg[p].v=merge(seg[seg[p].l].v, seg[seg[p].r].v);
74e     return p;
a90 }
6a4 int query(int p, int L, int R, int l, int r) {
3c7     if (l > R || r < L) return neut;
c26     if (L <= l && r <= R) return seg[p].v;
ae0     int mid = (l + r) / 2;
864     int left = query(seg[p].l, L, R, l, mid);
195     int right = query(seg[p].r, L, R, mid + 1, r);
90a     return merge(left, right);
e77 }
304 int update(int root, int pos, int val) {
c68     return update(root, pos, val, 0, n - 1);
84e }
7cc int query(int root, int L, int R) {
a53     return query(root, L, R, 0, n - 1);
2f9 }
588 };
```

SegBeats.h

Description: In Segment Tree Beats, ‘lazy‘ does NOT mean ”updates still missing here”. The node already reflects all previous updates. Instead, ‘lazy‘ stores what must be propagated to the children before recursing. Always call ‘apply(l,r,p)’ before descending. This node layout supports range add, range chmin and range chmax operations. Beats conditions:
break: MIN x: mx1 <= x; MAX x: mi1 >= x
tag: MIN x: x > mx2; MAX x: x < mi2
Time: amortized $\mathcal{O}(\log^2 N)$, without range add $\mathcal{O}(\log N)$

fa8527, 47 lines

```
3c9 struct node{
45e     ll mx1, mx2, sum, lazy;
9e5     ll mi1, mi2;
faa     int cMax, cMin, tam;
db3     node(int x=0) : mx1(x),mx2(-inf),mi1(x),mi2(inf),
744         cMax(1),cMin(1),tam(1),sum(x),lazy(0){}
b67     node(node a, node b){
4f5         sum = a.sum+b.sum, tam = a.tam+b.tam;
c60         lazy = 0;
15b         mx1 = max(a.mx1, b.mx1);
9ae         mx2 = max(a.mx2, b.mx2);
f62         if(a.mx1 != b.mx1) mx2 = max(mx2, min(a.mx1, b.mx1));
b60         cMax=(a.mx1==mx1 ? a.cMax:0)+(b.mx1==mx1 ? b.cMax:0);

09f         mi1 = min(a.mi1, b.mi1);
143         mi2 = min(a.mi2, b.mi2);
3bf         if(a.mi1 != b.mi1) mi2=min(mi2, max(a.mi1, b.mi1));
c18         cMin=(a.mi1==mi1 ? a.cMin:0)+(b.mi1==mi1 ? b.cMin:0);
23d     }
38d void apply_sum(ll x){
2a1         mx1 += x, mx2 += x,mi1 += x, mi2 += x;
99b         sum += tam*x, lazy += x;
b5e     }
cf4 void apply_min(ll x) {
```

```
e07     if(x >= mx1) return;
c44     sum -= (mx1 - x)*cMax;
be0     if(mi1 == mx1) mi1 = x;
8ef     if(mi2 == mx1) mi2 = x;
ea2     mx1 = x;
908 }
0c8 void apply_max(ll x){
e25     if(x <= mi1) return;
59e     sum -= (mi1 - x)*cMin;
4b1     if(mx1 == mi1) mx1 = x;
d69     if(mx2 == mi1) mx2 = x;
1ff     mi1 = x;
0e4 }
554 };
fdc void apply(int l, int r, int p){
c8e     for(int i=2*p+1; i<=2*p+2; i++){
dbf         seg[i].apply_sum(st[p].lazy);
c90         seg[i].apply_min(st[p].mx1);
61a         seg[i].apply_max(st[p].mi1);
4b8     }
431     seg[p].lazy = 0;
dd0 }
```

bca062, 17 lines

RMQ.h

Usage: RMQ rmq(values);
rmq.query(inclusive, inclusive);
Time: $\mathcal{O}(|V|\log|V|+Q)$

```
76a struct RMQ {
8ac     vector<vector<int>> dp;
dd1     RMQ(const vector<int>& a) : dp(1, a) {
71c         for (int i = 1, pw = 1; pw*2 <= sz(a); pw*=2, i++) {
394             dp.emplace_back(sz(a) - pw*2 + 1);
d17             for (int j = 0; j < sz(dp[i]); j++) {
dcc                 dp[i][j] = min(dp[i-1][j], dp[i-1][j+pw]);
75a             }
b68         }
3e9     }
9e3     int query(int l, int r) {
658         assert(l <= r);
884         int k = 31 - __builtin_clz(r - l + 1);
1f9         return min(dp[k][l], dp[k][r - (1 << k) + 1]);
e21     }
bca }
```

UnionFind.h

Description: Disjoint-set data structure with bipartite check

6d2739, 22 lines

```
146 struct Uf{
b54     vector<int> tam, ds, bi, c;
d2c     Uf(int n) : tam(n, 1), ds(n), bi(n, 1), c(n){
244         iota(all(ds), 0);
233     }
001     int find(int i){ return (i==ds[i] ? i : find(ds[i]));}
e5a     int color(int i){
300         return (i==ds[i] ? 0 : (c[i]^color(ds[i])));}
c3b     void merge(int a, int b){
8d0         int ca = color(a), cb = color(b);
605         a = find(a), b = find(b);
a89         if(a == b){
686             if(ca == cb) bi[a] = false;
505             return;
c08         }
226         if(tam[a] < tam[b]) swap(a, b);
1ac         ds[b] = a, tam[a] += tam[b];
27c         bi[a] = (bi[a] && bi[b]);
834         c[b] = (ca ^ cb ^ 1);
a70     }
6d2 };
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().
Usage: int t = uf.time(); ...; uf.rollback(t);
Time: $\mathcal{O}(\log(N))$

d4405e, 23 lines

```
47a struct RollbackUF {
f80     vector<int> e;
919     vector<pii> st;
f6f     RollbackUF(int n) : e(n, -1) {}
84b     int size(int x) { return -e[find(x)]; }
626     int find(int x) { return e[x] < 0 ? x : find(e[x]); }
49f     int time() { return sz(st); }
4db     void rollback(int t) {
314         for (int i = time(); i --> t;)
8d2             e[st[i].first] = st[i].second;
b04         st.resize(t);
30b     }
cf0     bool join(int a, int b) {
605         a = find(a), b = find(b);
5c2         if (a == b) return false;
745         if (e[a] > e[b]) swap(a, b);
bac         st.push_back({a, e[a]});
e6e         st.push_back({b, e[b]});
708         e[a] += e[b]; e[b] = a;
8a6         return true;
6c7     }
d44 };
```

Numerical (4)

4.1 Polynomials and recurrences

Polynomial.h

c9b7b0, 19 lines

```
213 struct Poly {
3a1     vector<double> a;
9a5     double operator()(double x) const {
e3c         double val = 0;
d5c         for (int i = sz(a); i--;) (val *= x) += a[i];
d94         return val;
ae7     }
0ac     void diff() {
7b6         rep(i,1,sz(a)) a[i-1] = i*a[i];
468         a.pop_back();
afc     }
087     void divroot(double x0) {
898         double b = a.back(), c; a.back() = 0;
9cf         for(int i=sz(a)-1; i--;)
406             c = a[i], a[i] = a[i+1]*x0+b, b=c;
468         a.pop_back();
3f8     }
c9b };
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: polyRoots({{2,-3,1}},-1e9,1e9) // solve x²-3x+2 = 0
Time: $\mathcal{O}(n^2 \log(1/\epsilon))$

"Polynomial.h"

b00bfe, 24 lines

```
64a vector<double> polyRoots(Poly p, double xmin, double xmax)
{
853     if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
539     vector<double> ret;
f55     Poly der = p;
c06     der.diff();
617     auto dr = polyRoots(der, xmin, xmax);
d85     dr.push_back(xmin-1);
12c     dr.push_back(xmax+1);
```

```
423 sort(all(dr));
b98 rep(i,0,sz(dr)-1) {
d85     double l = dr[i], h = dr[i+1];
ad1     bool sign = p(l) > 0;
b41     if (sign ^ (p(h) > 0)) {
03d         rep(it,0,60) { // while (h - l > 1e-8)
761             double m = (l + h) / 2, f = p(m);
0ac             if ((f <= 0) ^ sign) l = m;
193             else h = m;
b69         }
ff5         ret.push_back((l + h) / 2);
fc2     }
d15 }
edf return ret;
b00 }
```

PolyInverse.h2745a7, 18 lines

```
747 vector<ll> get_inverse(vector<ll> a) {
e4d     if (a.empty()) return {};
044     int Y = sz(a) - 1, n = 32 - __builtin_clz(Y);
ba5     n = (1 << n);
711     a.resize(n);
e3e     vector<ll> inv = { modpow(a[0], mod - 2) }, f, c;
a2b     inv.reserve(n);
599     for (int tam = 2; tam <= n; tam *= 2) {
d29         while (sz(f) < tam) f.push_back(a[sz(f)]);
fec         c = conv(f, inv);
757         rep(i, 0, tam) c[i] = (c[i] == 0 ? 0 : mod - c[i]);
df6         c[0] += (c[0] + 2 >= mod ? 2 - mod : 2);
f8b         inv = conv(inv, c);
118         inv.resize(tam);
9f4     }
531     return inv;
274 }
```

BerlekampMassey.h

Description: Recovers any n -order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.

Usage: berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}

Time: $\mathcal{O}(N^2)$

96548b, 21 lines

```
c10 vector<ll> berlekampMassey(vector<ll> s) {
ea1     int n = sz(s), L = 0, m = 0;
2a2     vector<ll> C(n), B(n), T;
2b3     C[0] = B[0] = 1;

d6f     ll b = 1;
36d     rep(i,0,n) { ++m;
b7f         ll d = s[i] % mod;
45a         rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
53a         if (!d) continue;
169         T = C; ll coef = d * modpow(b, mod-2) % mod;
2d1         rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
b6c         if (2 * L > i) continue;
dc3         L = i + 1 - L; B = T; b = d; m = 0;
8c2     }
```

```
51b C.resize(L + 1); C.erase(C.begin());
e98 for (ll& x : C) x = (mod - x) % mod;
a91 return C;
965 }
```

LinearRecurrence.h

Description: Generates the k 'th term of an n -order linear recurrence $S[i] = \sum_j S[i - j - 1]tr[j]$, given $S[0 \dots \geq n - 1]$ and $tr[0 \dots n - 1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.

Usage: linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number

Time: $\mathcal{O}(n^2 \log k)$ 547b93, 27 lines

```
437 using Poly = vector<ll>;
2ef ll linearRec(Poly S, Poly tr, ll k) {
327     int n = sz(tr);

0e9     auto combine = [&](Poly a, Poly b) {
b1c         Poly res(n * 2 + 1);
5f7         rep(i,0,n+1) rep(j,0,n+1)
389             res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
bdc         for (int i = 2 * n; i > n; --i) rep(j,0,n)
fc3             res[i-1-j] = (res[i-1-j] + res[i] * tr[j]) % mod;
b76         res.resize(n + 1);
b50         return res;
55c     };

bf8     Poly pol(n + 1), e(pol);
997     pol[0] = e[1] = 1;

e96     for (++k; k; k /= 2) {
491         if (k % 2) pol = combine(pol, e);
0d9         e = combine(e, e);
813     }

cd2     ll res = 0;
e8d     rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
b50     return res;
594 }
```

LagrangeInterpolation.h

Description: Returns $P(x)$, where P is the unique polynomial of degree $\leq |pts| - 1$ such that $\forall i, P(i) = pts[i]$.

5cbe52, 16 lines

```
1d7 ll lagrangeiro(const vector<ll> &pts, ll x)
f95 {
c10     int n = sz(pts);
e52     vector<ll> pref(n, 1), suf(n, 1);
a7c     for (int i=1; i<n; i++) pref[i] = mul(pref[i-1], x-(i-1))
;
71b     for (int i=n-2; i>=0; i--) suf[i] = mul(suf[i+1], x-(i+1))
);
a3e     ll y = 0;
603     for (int i=0; i<n; i++) {
591         ll oi = pts[i];
08c         oi = mul(oi, mul(pref[i], mul(suf[i], mul(ifat[i], ifat
[n-i-1]))));
b67         if ((n-i-1) % 2 == 1) oi *= -1;
ded         y = (y + oi) % mod;
b2d     }
6d1     return (y + mod) % mod;
f9e }
```

4.2 Optimization

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a, b]$ assuming f is unimodal on the interval, i.e. has only one local minimum and no local maximum. The maximum error in the result is eps . Works equally well for maximization with a small change in the code. See Ternary-Search.h in the Various chapter for a discrete version.

Usage: double func(double x) { return 4+x+.3*x*x; }

double xmin = gss(-1000,1000,func);

Time: $\mathcal{O}(\log((b - a)/\epsilon))$

31d45b, 15 lines

```
eb1 double gss(double a, double b, double (*f)(double)) {
97e     double r = (sqrt(5)-1)/2, eps = 1e-7;
b87     double x1 = b - r*(b-a), x2 = a + r*(b-a);
47d     double f1 = f(x1), f2 = f(x2);
708     while (b-a > eps)
```

```
f4d     if (f1 < f2) { //change to > to find maximum
da5         b = x2; x2 = x1; f2 = f1;
dfb         x1 = b - r*(b-a); f1 = f(x1);
451     } else {
d6e         a = x1; x1 = x2; f1 = f2;
815         x2 = a + r*(b-a); f2 = f(x2);
2fe     }
3f5     return a;
31d }
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b, x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.

Usage: vvd A = {{1,-1}, {-1,1}, {-1,-2}};

vd b = {1,1,-4}, c = {-1,-1}, x;

T val = LPSolver(A, b, c).solve(x);

Time: $\mathcal{O}(NM * \#pivots)$, where a pivot may be e.g. an edge relaxation.

$\mathcal{O}(2^n)$ in the general case.

aa8530, 69 lines

```
943 typedef double T; // long double, Rational, double + modkP
>...
487 typedef vector<T> vd;
840 typedef vector<vd> vvd;

8cb     const T eps = 1e-8, inf = 1/.0;
85f     #define MP make_pair
90c     #define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])
) s=j

34b     struct LPSolver {
b5c         int m, n;
14e         vi N, B;
d5f         vvd D;

9b8         LPSolver(const vvd& A, const vd& b, const vd& c) :
f40             m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
27d             rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
f03             rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[
i];}

59d             rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
24a             N[n] = -1; D[m+1][n] = 1;
6ff         }

333     void pivot(int r, int s) {
3cd         T *a = D[r].data(), inv = 1 / a[s];
12b         rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
449             T *b = D[i].data(), inv2 = b[s] * inv;
e07             rep(j,0,n+2) b[j] -= a[j] * inv2;
e78             b[s] = a[s] * inv2;
ca4         }
485         rep(j,0,n+2) if (j != s) D[r][j] *= inv;
3b7         rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
dbd         D[r][s] = inv;
c97         swap(B[r], N[s]);
9cd     }

24e     bool simplex(int phase) {
8b8         int x = m + phase - 1;
1de         for (;;) {
7a6             int s = -1;
aea             rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
4dc             if (D[x][s] >= -eps) return true;
56d             int r = -1;
670             rep(i,0,m) {
776                 if (D[i][s] <= eps) continue;
```

```
c95         if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
133             < MP(D[r][n+1] / D[r][s], B[r])) r = i
;
468     }
fbf     if (r == -1) return false;
dba     pivot(r, s);
7d8     }
f15     }

859 T solve(vd &x) {
898     int r = 0;
435     rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
32d     if (D[r][n+1] < -eps) {
f65         pivot(r, n);
fda         if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
939         rep(i,0,m) if (B[i] == -1) {
37f             int s = 0;
c66             rep(j,1,n+1) ltj(D[i]);
b98             pivot(i, s);
683         }
b65     }
203     bool ok = simplex(1); x = vd(n);
972     rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
d3a     return ok ? D[m][n+1] : inf;
396 }
c57 };
```

4.3 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.

Time: $\mathcal{O}(N^3)$

bd5cec, 16 lines

```
e36 double det(vector<vector<double>>& a) {
70e     int n = sz(a); double res = 1;
fea     rep(i,0,n) {
281         int b = i;
b0b         rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
311         if (i != b) swap(a[i], a[b]), res *= -1;
9b1         res *= a[i][i];
d5c         if (res == 0) return 0;
3e3         rep(j,i+1,n) {
f15             double v = a[j][i] / a[i][i];
353             if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
4ec         }
ee1     }
b50     return res;
bd5 }
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.

Time: $\mathcal{O}(N^3)$

3313dc, 19 lines

```
031 const ll mod = 12345;
38e ll det(vector<vector<ll>>& a) {
da9     int n = sz(a); ll ans = 1;
fea     rep(i,0,n) {
3e3         rep(j,i+1,n) {
f36             while (a[j][i] != 0) { // gcd step
479                 ll t = a[i][i] / a[j][i];
b87                 if (t) rep(k,i,n)
e5b                     a[i][k] = (a[i][k] - a[j][k] * t) % mod;
332                 swap(a[i], a[j]);
17c                 ans *= -1;
e81             }
30d         }
a97         ans = ans * a[i][i] % mod;
f4e         if (!ans) return 0;
f39     }
```

```
d38     return (ans + mod) % mod;
5e8 }
```

SolveLinear.h

Description: If inv = 1, finds the inverse of the matrix eq and returns it as a flat vector

Time: $\mathcal{O}(\min(n,m)nm)$

2c134e, 52 lines

```
320 struct Gauss {
d6d     const double eps = 1e-9;
93d     vector<vector<double>> eq;
754     void addEquation(const vector<double>& e) {
503         eq.push_back(e); }
04f     pair<int, vector<double>> solve(int inv=0) {
214         int n = sz(eq), m = sz(eq[0]) - 1 + inv;
f9c         if(inv){
d33             rep(i, 0, n) eq[i].resize(2*n), eq[i][n+i] = 1;
2e2         }
3cb         vector<int> where(m, -1);
a73         for (int col = 0, row = 0; col < m && row < n; col++)
{
f05             int sel = row;
53c             rep(i, row, n) {
664                 if (abs(eq[i][col]) > abs(eq[sel][col])) sel = i;
e04             }
68b             if (abs(eq[sel][col]) < eps) continue;
3ad             rep(i, col, sz(eq[0])) swap(eq[sel][i], eq[row][i]);
2c3             where[col] = row;
dff             rep(i, 0, n) if (i != row) {
184                 double c = eq[i][col] / eq[row][col];
7f1                 rep(j, col, sz(eq[0])) eq[i][j] -= eq[row][j] * c;
17d             }
4ef             ++row;
9b8         }
f9c         if(inv){
208             vector<double> res;
fea             rep(i, 0, n) {
420                 if (where[i] == -1) return {0, {}}; // Singular
3af                 rep(j, n, 2*n)
f89                     res.push_back(eq[where[i]][j] / eq[where[i]][i])
;
d81             }
3b1             return {1, res};
700         }

233     vector<double> ans(m, 0);
670     rep(i, 0, m) {
c19         if (where[i] != -1)
02c             ans[i] = eq[where[i]][m] / eq[where[i]][i];
5bb     }
fea     rep(i, 0, n) {
68c         double sum = 0;
5f8         rep(j, 0, m) {
f48             sum = sum + ans[j] * eq[i][j];
fa6         }
3c8         if(abs(sum - eq[i][m]) > eps)return {0, {}};
bf2     }
260     rep(i, 0, m) if (where[i] == -1) return {2, ans};
d4a     return {1, ans};
a95 }
2c1 };
```

SolveLinearBinary.h

Time: $\mathcal{O}\left(\frac{\min(n,m)nm}{64}\right)$

28c946, 32 lines

```
e81 pair<int, bitset<M>> gauss(vector<bitset<M>> eq) {
579     int n = eq.size(), m = M - 1;
3cb     vector<int> where(m, -1);
a73     for(int col = 0, row = 0; col < m && row < n; col++){
```

```
dbb         rep(i,row,n)
926             if (eq[i][col]) {
c35                 swap(eq[i], eq[row]);
c2b                 break;
177             }
f4f             if (!eq[row][col]) continue;
2c3             where[col] = row;

fea         rep(i, 0, n) {
b60             if (i != row && eq[i][col]) eq[i] ^= eq[row];
981         }
4ef         ++row;
c74     }
7eb     bitset<M> ans;
670     rep(i,0,m) {
713         if (where[i] != -1) ans[i] = eq[where[i]][m];
691     }
fea     rep(i,0,n) {
e5c         int sum = (ans & eq[i]).count();
53f         sum %= 2;
36a         if (sum != eq[i][m]) return pair(0, bitset<M>());
29e     }
670     rep(i,0,m) {
be2         if (where[i] == -1) return pair(INF, ans);
958     }
280     return pair(1, ans);
28c }
```

XorGauss.h

5a1957, 30 lines

```
b94 struct XorGauss {
060     int N;
471     vector<ll> basis, who, mask;
47b     XorGauss(int N) : N(N), basis(N), who(N), mask(N) {}
// if(ans & (1ll << j)) who[j] was used to form x
221     bool belong(ll x){
04b         ll ans = 0;
042         for(int i=N-1; i>=0; i--){
e13             if((x ^ basis[i]) < x){
4ec                 ans ^= mask[i];
6b0                 x ^= basis[i];
254             }
2ad         }
069         return (x == 0);
c26     }
397     void add(ll v, int idx) {
a4d         ll msk = 0;
042         for (int i = N - 1; i >= 0; i--) {
80f             if (!(v & (1ll << i))) continue;
bf3             if (basis[i] == 0) {
1c7                 basis[i] = v, who[i] = idx;
940                 mask[i] = (msk | (1ll << i));
505                 return;
bc8             }
00e             msk ^= mask[i];
647             v ^= basis[i];
25b         }
fcc     }
5a1 };
```

4.4 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all k . N must be a power of 2. Useful for convolution: $\text{conv}(a, b) = c$, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16} ; higher for random inputs). Otherwise, use NTT/FFTMod.

Time: $\mathcal{O}(N \log N)$ with $N = A + B $ ($\sim 1s$ for $N = 2^{22}$)		773fcd, 44 lines
bcc typedef complex<double> C;		
7c0	void fft(vector<C>& a) {	
a5b	int n = a.size(), L = 31 - __builtin_clz(n);	
f82	static vector<complex<long double>> R(2, 1); <i>// 10%</i>	
	<i>faster if double</i>	
991	static vector<C> rt(2, 1);	
ad8	for (static int k = 2; k < n; k *= 2) {	
9d9	R.resize(n);	
335	rt.resize(n);	
411	auto x = polar(1.0L, acos(-1.0L) / k);	
cdb	rep (i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];	
a8a	}	
e66	vector<ll> rev(n);	
dcb	rep (i,0,n) rev[i] = (rev[i / 2] (i & 1) << L) / 2;	
47b	rep (i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);	
d3f	for (int k = 1; k < n; k *= 2) {	
cda	for (int i = 0; i < n; i += 2 * k) {	
0c2	for (int j = 0; j < k; j++) {	
30c	auto x = (double*)&rt[j + k];	
ebe	auto y = (double*)&a[i + j + k];	
15c	C z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]);	
20a	a[i + j + k] = a[i + j] - z;	
1b0	a[i + j] += z;	
b5b	}	
1fe	}	
fa0	}	
b33	}	
ccc	vector<ll> conv(const vector<ll>& a, const vector<ll>& b) {	
f88	if (a.empty() b.empty()) return {};	
920	vector<ll> res(sz(a) + sz(b) - 1);	
441	int L = 32 - __builtin_clz(sz(res)), n = 1 << L;	
060	vector<C> in(n), out(n);	
b1a	copy(all(a), in.begin());	
fef	rep (i,0,sz(b)) in[i].imag(b[i]);	
21a	fft(in);	
6fb	for (C& x : in) x *= x;	
4d7	rep (i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);	
3d7	fft(out);	
aa3	rep (i,0,sz(res)) res[i]=round(imag(out[i]) / (4 * n));	
b50	return res;	
7f4	}	

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.

Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h"		b82773, 23 lines
192	typedef vector<ll> vl;	
3fe	template < int M> vl convMod(const vl &a, const vl &b) {	
f88	if (a.empty() b.empty()) return {};	
19d	vl res(sz(a) + sz(b) - 1);	
a6f	int B=32-__builtin_clz(sz(res)), n=1<<B,cut= int (sqrt(M));	
3dd	vector<C> L(n), R(n), outs(n), outl(n);	
a1d	rep (i,0,sz(a)) L[i] =C((int)a[i] / cut, (int)a[i] % cut);	
97d	rep (i,0,sz(b)) R[i] =C((int)b[i] / cut, (int)b[i] % cut);	
5d5	fft(L), fft(R);	
fea	rep (i,0,n) {	
39d	int j = -i & (n - 1);	
65e	outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);	
91a	outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;	
cb3	}	
d08	fft(outl), fft(outs);	
35e	rep (i,0,sz(res)) {	

351	ll av = ll (real(outl[i])+.5), cv = ll (imag(outs[i])+.5);
988	ll bv = ll (imag(outl[i])+.5) + ll (real(outs[i])+.5);
6a3	res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
58f	}
b50	return res;
c1f	}

NumberTheoreticTransform.h

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x]g^{xk}$ for all k , where $g = \text{root}^{(\text{mod}-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a . For arbitrary modulo, see FFTMod. conv(a, b) = c, where $c[x] = \sum a[i]b[x - i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in $[0, \text{mod})$.

Time: $\mathcal{O}(N \log N)$		84c11e, 34 lines
376	const int mod = 998244353, root = 62;	
192	typedef vector<ll> vl;	
8ec	void ntt(vl &a) {	
6ae	int n = sz(a), L = 31 - __builtin_clz(n);	
7c9	static vl rt(2, 1);	
8ee	for (static int k = 2, s = 2; k < n; k *= 2, s++) {	
335	rt.resize(n);	
d43	ll z[] = {1, modpow(root, mod >> s)};	
8e7	rep (i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;	
f39	}	
808	vector< int > rev(n);	
dcb	rep (i,0,n) rev[i] = (rev[i / 2] (i & 1) << L) / 2;	
47b	rep (i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);	
657	for (int k = 1; k < n; k *= 2)	
2cb	for (int i = 0; i < n; i += 2 * k) rep (j,0,k) {	
86e	ll z = rt[j+k] * a[i+j+k] % mod, &ai = a[i+j];	
598	a[i + j + k] = ai - z + (z > ai ? mod : 0);	
589	ai += (ai + z >= mod ? z - mod : z);	
9a8	}	
de9	}	
08f	vl conv(const vl &a, const vl &b) {	
f88	if (a.empty() b.empty()) return {};	
f51	int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s),	
570	n = 1 << B;	
9ef	int inv = modpow(n, mod - 2);	
e4c	vl L(a), R(b), out(n);	
6b4	L.resize(n), R.resize(n);	
d9e	ntt(L), ntt(R);	
dfc	rep (i,0,n)	
0db	out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;	
ec9	ntt(out);	
c20	return {out.begin(), out.begin() + s};	
387	}	

FWHT.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$, where \oplus is one of AND, OR, XOR. The size of a must be a power of two.

Time: $\mathcal{O}(N \log N)$		124c14, 20 lines
5ad	void FST(vector<ll>& a, bool inv) {	
a9d	for (int n = sz(a), step = 1; step < n; step *= 2) {	
5bd	for (int i = 0; i < n; i += 2 * step) {	
4ee	for (int j = i; j < i + step; j++) {	
2fe	ll & u = a[j], &v = a[j + step];	
c6f	tie(u, v) =	
2d3	inv ? pair(v - u, u) : pair(v, u + v); <i>// AND</i>	
aba	inv ? pair(v, u - v) : pair(u + v, u); <i>// OR</i>	
a5a	pair(u + v, u - v); <i>// XOR</i>	
0b4	}	
fb4	}	

cd3	}
c9b	if (inv) for (ll & x : a) x /= sz(a); <i>// XOR only</i>
075	}
eb2	vector<ll> conv(vector<ll> a, vector<ll> b) {
595	FST(a, 0); FST(b, 0);
2dd	for (int i = 0; i < sz(a); i++) a[i]*=b[i];
062	FST(a, 1); return a;
7bf	}

Number theory (5)

5.1 Modular arithmetic

ModInverse.h

Description: Pre-computation of modular inverses. Assumes $\text{LIM} \leq \text{mod}$ and that mod is a prime.

		c375f5, 5 lines
88a	const ll mod = 1000000007, LIM = 200000;	
0f2	inv[1] = 1;	
379	for (int i=2; i<LIM; i++)	
86c	inv[i] = mod - (mod / i) * inv[mod % i] % mod;	

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.
Time: $\mathcal{O}(1)$ for modmul, $\mathcal{O}(\log b)$ for modpow

		bbbd8f, 12 lines
f4c	typedef unsigned long long ull;	
f85	ull modmul(ull a, ull b, ull M) {	
2dd	ll ret = a * b - M * ull(1.L / M * a * b);	
964	return ret + M * (ret < 0) - M * (ret >= (ll)M);	
e93	}	
4f6	ull modpow(ull b, ull e, ull mod) {	
c1a	ull ans = 1;	
a18	for (; e; b = modmul(b, b, mod), e /= 2)	
9e8	if (e & 1) ans = modmul(ans, b, mod);	
ba7	return ans;	
100	}	

ModPow.h

		b83e45, 9 lines
e2e	const ll mod = 1000000007; <i>// faster if const</i>	
9d8	ll modpow(ll b, ll e) {	
d54	ll ans = 1;	
36e	for (; e; b = b * b % mod, e /= 2)	
b46	if (e & 1) ans = ans * b % mod;	
ba7	return ans;	
d1e	}	

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).
Time: $\mathcal{O}(\log^2 p)$ worst case, $\mathcal{O}(\log p)$ for most p

"ModPow.h"		19a793, 25 lines
a77	ll sqrt(ll a, ll p) {	
5de	a %= p; if (a < 0) a += p;	
b47	if (a == 0) return 0;	
5c6	assert(modpow(a, (p-1)/2, p) == 1); <i>// else no solution</i>	
a75	if (p % 4 == 3) return modpow(a, (p+1)/4, p);	
	<i>// a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5</i>	
b94	ll s = p - 1, n = 2;	
ee5	int r = 0, m;	
084	while (s % 2 == 0)	
082	++r, s /= 2;	
ea4	while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;	
0c3	ll x = modpow(a, (s + 1) / 2, p);	
b74	ll b = modpow(a, s, p), g = modpow(n, s, p);	
1af	for (; r = m) {	

```
4fd  ll t = b;
713  for (m = 0; m < r && t != 1; ++m)
c58      t = t * t % p;
ae0  if (m == 0) return x;
20e  ll gs = modpow(g, 1LL << (r - m - 1), p);
fba  g = gs * gs % p;
4fb  x = x * gs % p;
c5c  b = b * g % p;
e3a  }
19a }
```

DiscreteLog.h
Description: Returns the smallest x such that $a^x \bmod m = b \bmod m$. If no such x exists, returns -1 .
Time: $O(\sqrt{m}) \cdot \log(\sqrt{m})$

```
758 int solve(int a, int b, int m) {
a6e     a %= m, b %= m;
ec4     if (a == 0) return (b ? -1 : 1);
        // caso gcd(a, m) > 1
6af     int k = 1, add = 0, g;
553     while ((g = gcd(a, m)) > 1) {
d90         if (b == k) return add;
642         if (b % g) return -1;
92a         b /= g, m /= g, ++add;
803         k = (k * 111 * a / g) % m;
8a0     }

16c     int sq = sqrt(m) + 1;
b51     int big = 1;
4e1     for (int i = 0; i < sq; i++) big = (111 * big * a) % m
;

053     vector<pii> vals;
3c2     for (int q = 0, cur = b; q <= sq; q++) {
b53         vals.push_back({ cur, q });
b50         cur = (111 * cur * a) % m;
837     }
62b     sort(all(vals));
90c     for (int p = 1, cur = k; p <= sq; p++) {
5d3         cur = (111 * cur * big) % m;
958         auto it = lower_bound(all(vals), pair(cur, INF));
721         if (it != vals.begin() && (--it)->first == cur) {
a30             return sq * p - it->second + add;
6fe         }
f22     }
daa     return -1;
2f1 }
```

DiscreteRoot.h
Description: Returns x such that $x^k \bmod m = a \bmod m$. If no such x exists, returns -1 .
Time: $O(\sqrt{m}) \cdot \log(\sqrt{m})$

```
"PrimitiveRoot.h", "DiscreteLog.h"
// Discrete Root

27c ll discreteRoot(ll k, ll a, ll m) {
738     ll g = primitiveRoot(m);
58b     ll y = discreteLog(fexp(g, k, m), a, m);
f31     if (y == -1) return y;
a58     return fexp(g, y, m);
1d5 }
```

5.2 Primality

MillerRabin.h
Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.

```
Time: 7 times the complexity of a^b mod c.
"ModMullLL.h"
66fe73, 13 lines

da4 bool isPrime(ull n) {
c16     if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
062     ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 17952650
22};
ae0     ull s = __builtin_ctzll(n-1), d = n >> s;
e80     for (ull a : A) { // ^ count trailing zeroes
6b4         ull p = modpow(a%n, d, n), i = s;
274         while (p != 1 && p != n - 1 && a % n && i--)
c77             p = modmul(p, p, n);
e28         if (p != n-1 && i != s) return 0;
edf     }
6a5     return 1;
66f }
```

Factor.h
Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $O(n^{1/4})$, less for numbers with small factors.

```
"ModMullLL.h", "MillerRabin.h"
da0c7c, 19 lines

7eb ull pollard(ull n) {
222     ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
5f5     auto f = [&](ull x) { return modmul(x, x, n) + i; };
f51     while (t++ % 40 || gcd(prd, n) == 1) {
be9         if (x == y) x = ++i, y = f(x);
70f         if ((q = modmul(prd, max(x,y) - min(x,y), n)) prd = q;
b78             x = f(x), y = f(f(y));
bf8     }
002     return gcd(prd, n);
dlb }

591 vector<ull> factor(ull n) {
1b9     if (n == 1) return {};
6b5     if (isPrime(n)) return {n};
bc6     ull x = pollard(n);
52a     auto l = factor(x), r = factor(n / x);
7af     l.insert(l.end(), all(r));
792     return l;
d54 }
```

```
PrimitiveRoot.h
18a01e, 15 lines

//is n primitive root of p ?
ad0 bool test(ll x, ll p) {
a56     ll m = p - 1;
845     for (ll i = 2; i * i <= m; ++i) if (!(m % i)) {
e64         if (modpow(x, i, p) == 1) return false;
599         if (modpow(x, m / i, p) == 1) return false;
53a     }
8a6     return true;
c4e }

//find the smallest primitive root for p
220 ll search(ll p) {
1bf     for (ll i = 2; i < p; i++) if (test(i, p)) return i;
daa     return -1;
a3c }
```

5.3 Divisibility

Euclid.h
Description: Find x, y such that $Ax + By = \gcd(A, B)$. If $\gcd(A, B) = 1$, then $x = A^{-1} \pmod B$ and $y = B^{-1} \pmod A$.
Time: $O(\log)$

```
c22 ll euclid(ll a, ll b, ll &x, ll &y) {
1ee     if (!b) return x = 1, y = 0, a;
e3d     ll d = euclid(b, a % b, y, x);
0a4     return y -= a/b * x, d;
```

```
33b }
```

CRT.h

```
ba1a4a, 25 lines

bc9 ll modinverse(ll a, ll b, ll s0 = 1, ll s1 = 0) {
a76     return !b ? s0 : modinverse(b, a % b, s1, s0 - s1 * (a /
b)); }

d8b ll mul(ll a, ll b, ll m) {
a6f     return (((__int128_t)a*b)%m + m)%m;
0bc }

28d struct Equation {
4c5     ll mod, ans;
08f     bool valid;
145     Equation(ll a, ll m) { mod = m, ans = a, valid = true; }
0fc     Equation() { valid = false; }
4d3     Equation(Equation a, Equation b) {
515         valid = false;
1a0         if (!a.valid || !b.valid) return;
85c         ll g = gcd(a.mod, b.mod);
44d         if ((a.ans - b.ans) % g != 0) return;
af0         valid = true;
b98         mod = a.mod * (b.mod / g);
81a         ll x = mul(a.mod, modinverse(a.mod, b.mod), mod);
38a         ans = a.ans + mul(x, (b.ans - a.ans) / g, mod);
c4c         ans = (ans % mod + mod) % mod;
6f5     }
f48 };
```

DivisionTrick.h

```
02aebb, 15 lines

7f1 void floor_ranges(int n) {
79c     for (int l = 1, r; l <= n; l = r + 1) {
746         r = n / (n / l);
        // floor(n/y) has the same value for y in [l..r]
5bf     }
eee }

678 void ceil_ranges(int n) {
79c     for (int l = 1, r; l <= n; l = r + 1) {
d47         int x = (n + l - 1) / l;
374         if (x == 1) r = n;
21b         else r = (n - 1) / (x - 1);
        // ceil(n/y) has the same value for y in [l..r]
06c     }
57c }
```

Phi.h
Description: Euler's ϕ function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n . $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p - 1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} \dots (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n} (1 - 1/p)$. $\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k, n) = 1} k = n\phi(n)/2, n > 1$.
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.
Euler's thm (generalized): a, m arbitrary, $n \geq \log_2 m \Rightarrow a^n \equiv a^{\phi(m) + (n \bmod \phi(m))} \pmod m$.

```
e58bf0, 6 lines

d08 void calculatePhi() {
265     for(int i=0; i<LIM; i++) phi[i] = i&1 ? i : i/2;
c83     for (int i = 3; i < LIM; i += 2) if (phi[i] == i)
dc2         for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
e58 }
```

Combinatorial (6)

PartitionSolver.h

e50fb7, 61 lines

```

d38  template<const int N>
182  struct PartitionSolver {
4ce      vector<vector<int>> part, to, from;
621  PartitionSolver() {
a9d      vector<int> a;
1ed      part.push_back(a);
77f      gen(1, N, a);
796      sort(all(part));
ed4      to.assign(sz(part), vector<int>(N + 1, -1));
9a5      from = to;
ddd      for (int i = 0; i < sz(part); i++) {
a93          int sum = 0;
87f          auto arr = part[i];
bca          for (auto x : arr) sum += x;
4fa          to[i][0] = i;
615          from[i][0] = i;
afc          for (int j = 1; j + sum <= N; j++) {
123              arr = part[i];
9d6              arr.push_back(j);
ceb              sort(all(arr));
d02              to[i][j] = getIndex(arr);
942              from[to[i][j]][j] = i;
20d          }
bef      }
283  }

810  int size() const { return sz(part); }
9ee  int getIndex(const vector<int>& arr) const {
168      return lower_bound(all(part), arr) - part.begin(); }
b49  int add(int id, int num) const { return to[id][num]; }
944  int rem(int id, int num) const { return from[id][num]; }
168  vector<int> getPartition(int id) const {
37b      return part[id]; }

1ba  void gen(int i, int sum, vector<int>& a) {
a05      if (i > sum) { return; }
726      a.push_back(i);
1ed      part.push_back(a);
278      gen(i, sum - i, a);
468      a.pop_back();
48f      gen(i + 1, sum, a);
537  }
f4f  };

// Number of partitions for all integers <= n
75c  vector<ll> partitionNumber(int n) {
d9c      vector<ll> ans(n + 1, 0);
82f      ans[0] = 1;
78a      for (int i = 1; i <= n; i++) {
87f          for (int j = 1; j * (3 * j + 1) / 2 <= i; j++) {
b6b              ll here = ans[i - j * (3 * j + 1) / 2];
c91              ans[i] = (ans[i] + (j * 1 ? here : -here));
365          }
7c6          for (int j = 1; j * (3 * j - 1) / 2 <= i; j++) {
a1a              ll here = ans[i - j * (3 * j - 1) / 2];
c91              ans[i] = (ans[i] + (j * 1 ? here : -here));
162          }
4a3      }
ba7      return ans;
08b  }

```

Graph (7)

7.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get $\text{dist} = \text{inf}$; nodes reachable through negative-weight cycles get $\text{dist} = -\text{inf}$. Assumes $V^2 \max |w_i| < \sim 2^{63}$.
Time: $\mathcal{O}(VE)$

529834, 24 lines

```

f5e  const ll inf = LLONG_MAX;
83a  struct Ed { int a, b, w, s() { return a < b ? a : -a; }};
9ac  struct Node { ll dist = inf; int prev = -1; };

6fc  void bell(vector<Node>& nodes, vector<Ed>& eds, int s) {
97b      nodes[s].dist = 0;
eb9      sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s(); });

74e  int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled
vertices
c5a      rep(i, 0, lim) for (Ed ed : eds) {
905          Node cur = nodes[ed.a], &dest = nodes[ed.b];
d7d          if (abs(cur.dist) == inf) continue;
6ab          ll d = cur.dist + ed.w;
6ec          if (d < dest.dist) {
956              dest.prev = ed.a;
4c2              dest.dist = (i < lim-1 ? d : -inf);
452          }
75a      }
ced      rep(i, 0, lim) for (Ed e : eds) {
3ab          if (nodes[e.a].dist == -inf)
5ff              nodes[e.b].dist = -inf;
1d7      }
166  }

```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m , where $m[i][j] = \text{inf}$ if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j , inf if no path, or $-\text{inf}$ if the path goes through a negative-weight cycle.

Time: $\mathcal{O}(N^3)$

531245, 13 lines

```

964  const ll inf = 1LL << 62;
914  void floydWarshall(vector<vector<ll>>& m) {
e9d      int n = sz(m);
831      rep(i, 0, n) m[i][i] = min(m[i][i], 0LL);
99d      rep(k, 0, n) rep(i, 0, n) rep(j, 0, n)
19b          if (m[i][k] != inf && m[k][j] != inf) {
6e8              auto newDist = max(m[i][k] + m[k][j], -inf);
e89              m[i][j] = min(m[i][j], newDist);
f38          }
a69      rep(k, 0, n) if (m[k][k] < 0) rep(i, 0, n) rep(j, 0, n)
ffd          if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
f12  }

```

7.2 Network flow and Matching

Dinic.h

Time: $\mathcal{O}(\min(m \cdot \text{max_flow}, n^2 m))$.- For graphs with unit capacities: $\mathcal{O}(\min(m\sqrt{m}, mn^{2/3}))$.- If every vertex has in-degree 1 or out-degree 1: $\mathcal{O}(m\sqrt{n})$.- With capacity scaling: $\mathcal{O}(nm \log(\text{MAXCAP}))$ with high constant factor.

892d6e, 36 lines

```

14d  struct Dinic {
61f      const bool scaling = false;
206      int lim;
670      struct edge {
c63          int to, rev;

```

```

a14      ll cap, flow;
7f9      bool res;
6dd      edge(int to_, ll cap_, int rev_, bool res_)
a94          : to(to_), cap(cap_), rev(rev_), flow(0), res(res_){}
477  };

```

```

002      vector<vector<edge>> g;
216      vector<int> lev, beg;
a71      ll F;
63f      Dinic(int n) : g(n), lev(n), beg(n), F(0) {}

```

```

0c5  void add(int a, int b, ll c, ll other = 0) {
de2      g[a].emplace_back(b, c, sz(g[b]), false);
fa5      g[b].emplace_back(a, other, sz(g[a])-1, true);
14f  }
123  bool bfs(int s, int t) {
e59      fill(all(lev), -1);
4e7      fill(all(beg), 0);
0a4      lev[s] = 0;
8b2      queue<int> q; q.push(s);
647      while (sz(q)) {
be1          int u = q.front(); q.pop();
bd9          for (auto& i : g[u]) {
dbc              if (lev[i.to] != -1 or (i.flow == i.cap)) continue;
b4f              if (scaling and i.cap - i.flow < lim) continue;
185              lev[i.to] = lev[u] + 1;
8ca              q.push(i.to);
f97          }
b1b      }
0de      return lev[t] != -1;
310  }
1dc  ll dfs(int v, int s, ll f = INF) {
50b      if (!f or v == s) return f;
84d      for (int& i = beg[v]; i < sz(g[v]); i++) {
027          auto& e = g[v][i];
206          if (lev[e.to] != lev[v] + 1) continue;
a30          ll foi = dfs(e.to, s, min(f, e.cap - e.flow));
749          if (!foi) continue;
3c5          e.flow += foi, g[e.to][e.rev].flow -= foi;
45c          return foi;
e08      }
bb3      return 0;
b98  }
2b4  ll maxFlow(int s, int t) {
a86      for (lim = scaling ? (1<<30) : 1; lim; lim /= 2)
69c          while (bfs(s, t)) while (ll ff = dfs(s, t)) F += ff;
4ff          return F;
6c8  }
0fe      bool inCut(int u) { return lev[u] != -1; }
892  };

```

LowerBoundFlow.h

Description: Calculates maximum flow with lower/upper bounds on edges. Returns -1 if no feasible flow exists. $\text{add}(a, b, l, r)$ adds edge $a \rightarrow b$ where flow f must satisfy $l \leq f \leq r$. $\text{add}(a, b, c)$ adds edge $a \rightarrow b$ with capacity c (implies $0 \leq f \leq c$). Same complexity as Dinic.

"Dinic.h"

756539, 36 lines

```

0ca  struct lb_max_flow : Dinic {
96f      vector<ll> d;
be9      lb_max_flow(int n) : Dinic(n + 2), d(n, 0) {}
b12      void add(int a, int b, int l, int r) {
c97          d[a] -= l;
f1b          d[b] += l;
cb6          Dinic::add(a, b, r - l);
989      }
087      void add(int a, int b, int c) {
610          Dinic::add(a, b, c);
330      }
7a1      bool has_circulation() {

```

```

ac0  int n = sz(d);
854  ll cost = 0;
fea  rep(i, 0, n){
c69   if (d[i] > 0) {
f56     cost += d[i];
4f6     Dinic::add(n, i, d[i]);
551   } else if (d[i] < 0) {
bd2     Dinic::add(i, n+1, -d[i]);
bd9   }
a13  }

9f2  return (Dinic::maxFlow(n, n+1) == cost);
cc6  }
7bd  bool has_flow(int src, int snk) {
eda  Dinic::add(snk, src, INF);
e40  return has_circulation();
4aa  }
4eb  ll max_flow(int src, int snk) {
ee8  if (!has_flow(src, snk)) return -1;
99c  Dinic::F = 0;
703  return Dinic::maxFlow(src, snk);
0bb  }
756  };

```

MinCost.h

Description: Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only. If graph is a DAG pi can be calculated with DP instead of Bellman ford.

Time: $\mathcal{O}(FE \log(V))$ where F is max flow. $\mathcal{O}(VE)$ for setpi.

```

c4d  #include <bits/extc++.h>

9f4  const ll INF = numeric_limits<ll>::max() / 4;

6f3  struct MCMF {
670    struct edge {
ede      int from, to, rev;
e20      ll cap, cost, flow;
092    };
060    int N;
091    vector<vector<edge>> ed;
a83    vector<int> seen, vis;
0ec    vector<ll> dist, pi;
c45    vector<edge*> par;

2cc    MCMF(int N) : N(N), ed(N), seen(N), vis(N),
dc7      dist(N), pi(N), par(N) {}

6f3    void addEdge(int from, int to, ll cap, ll cost) {
ad8      if (from == to || cap == 0) return;
1af      ed[from].push_back(edge{from, to, sz(ed[to]), cap, cost, 0});
700      ed[to].push_back(edge{to, from, sz(ed[from]) - 1, 0, -cost, 0});
dad    }

975    void path(int s) {
7d4      fill(all(seen), 0);
04e      fill(all(dist), INF);
a93      dist[s] = 0;
841      ll di;
937      __gnu_pbds::priority_queue<pair<ll, int>> q;
9fb      vector<decltype(q)::point_iterator> its(N);
23b      q.push({0, s});

14d      while (!q.empty()) {
eda        s = q.top().second; q.pop();
2af        seen[s] = 1; di = dist[s] + pi[s];

```

```

6bd    for (edge& e : ed[s]) {
d20      if (!seen[e.to]) {
f1f        ll val = di - pi[e.to] + e.cost;
f3c        if (e.cap - e.flow > 0 && val < dist[e.to]){
0c7          dist[e.to] = val;
fb6          par[e.to] = &e;
22d          if (its[e.to] == q.end()) {
aac            its[e.to] = q.push({-dist[e.to], e.to});
388          }
6f8          else q.modify(its[e.to], {-dist[e.to], e.to});
80b        }
fce      }
013    }
e16  }
faa  for (int i = 0; i < N; i++) {
0ef    pi[i] = min(pi[i] + dist[i], INF);
ded  }
17b  }

310  pair<ll, ll> maxflow(int s, int t) {
923    setpi(s, t);
3d3    ll totflow = 0, totcost = 0;
8dd    while (path(s), seen[t]) {
535      ll fl = INF;
733      for (edge* x = par[t]; x; x = par[x->from]) {
8ed        fl = min(fl, x->cap - x->flow);
ddf      }
f9f      totflow += fl;
733      for (edge* x = par[t]; x; x = par[x->from]) {
10b        x->flow += fl;
e58        ed[x->to][x->rev].flow -= fl;
3bf      }
219    }
faa    for (int i = 0; i < N; i++) {
a18      for (edge& e : ed[i]) {
7a0        totcost += e.cost * e.flow;
774      }
a06    }
17e    return { totflow, totcost / 2 };
411  }

// If some costs can be negative, call this before
// maxflow:
eda  void setpi(int s, int t) {
3ef    fill(all(pi), INF);
156    pi[s] = 0;
45c    int it = N, ch = 1;
aa3    ll v;
5e8    while (ch-- && it--) {
faa      for (int i = 0; i < N; i++) {
c9b        if (pi[i] != INF)
fb0          for (edge& e : ed[i]) if (e.cap)
257            if ((v = pi[i] + e.cost) < pi[e.to])
a43              pi[e.to] = v, ch = 1;
d0b      }
250    }
38b    assert(it >= 0); // negative cost cycle
545  }
fid  };

```

PushRelabel.h

Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

Time: $\mathcal{O}(V^2\sqrt{E})$

```

49f  struct PushRelabel {
e9b    struct Edge {
548      int dest, back;

```

```

e00    ll f, c;
571  };
ed3  vector<vector<Edge>> g;
51c  vector<ll> ec;
658  vector<Edge*> cur;
b08  vector<vector<int>> hs;
4d4  vector<int> H;
4e1  PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n) {}

b1c  void addEdge(int s, int t, ll cap, ll rcap=0) {
50b    if (s == t) return;
cc8    g[s].push_back({t, sz(g[t]), 0, cap});
2aa    g[t].push_back({s, sz(g[s]) - 1, 0, rcap});
817  }

359  void addFlow(Edge& e, ll f) {
759    Edge &back = g[e.dest][e.back];
f7e    if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
d2e    e.f += f; e.c -= f; ec[e.dest] += f;
c47    back.f -= f; back.c += f; ec[back.dest] -= f;
340  }
0e0  ll calc(int s, int t) {
f00    int v = sz(g); H[s] = v; ec[t] = 1;
fbb    vector<int> co(2*v); co[0] = v - 1;
e20    for (int i = 0; i < v; i++) cur[i] = g[i].data();
8c2    for (Edge& e : g[s]) addFlow(e, e.c);

604    for (int hi = 0;;) {
ae9      while (hs[hi].empty()) if (!hi--) return -ec[s];
int u = hs[hi].back(); hs[hi].pop_back();
a3e      while (ec[u] > 0) // discharge u
457        if (cur[u] == g[u].data() + sz(g[u])) {
e94          H[u] = 1e9;
5fa          for (Edge& e : g[u]){
256            if (e.c && H[u] > H[e.dest] + 1)
740              H[u] = H[e.dest] + 1, cur[u] = &e;
88f          }
f04          if (++co[H[u]], !--co[hi] && hi < v){
10d            for (int i = 0; i < v; i++){
4be              if (hi < H[i] && H[i] < v)
021                --co[H[i]], H[i] = v + 1;
a21            }
cc1          }
3a2          hi = H[u];
b6b          } else if (cur[u] -> c && H[u] == H[cur[u] -> dest] + 1){
779            addFlow(*cur[u], min(ec[u], cur[u] -> c));
e91          } else ++cur[u];
4d7        }
b65      }
385      bool inCut(int a) { return H[a] >= sz(g); }
a7b    };

```

Blossom.h

Description: Max matching on general Graph. $mate[i]$ = match of i

Time: $\mathcal{O}(N^3)$

```

40f  vector<int> Blossom(vector<vector<int>>& g) {
10a    int n = sz(g), timer = -1;
f55    vector<int> mate(n, -1), label(n), par(n), orig(n), aux(n,
-1), q;

060    auto lca = [&](int x, int y) {
7b8      for (timer++; ; swap(x, y)) {
583        if (x == -1) continue;
4be        if (aux[x] == timer) return x;
90d        aux[x] = timer;
fb4        x = (mate[x] == -1 ? -1 : orig[par[mate[x]]]);
f6a      }
aba    };

```

```

be4     auto blossom = [&](int v, int w, int a) {
509         while (orig[v] != a) {
721             par[v] = w; w = mate[v];
1e2             if(label[w] == 1) label[w] = 0, q.push_back(w);
8c7             orig[v] = orig[w] = a;
3d0             v = par[w];
eae         }
068     };
a0f     auto aug = [&](int v) {
8c8         while (v != -1) {
86a             int pv = par[v], nv = mate[pv];
941             mate[v] = pv; mate[pv] = v; v = nv;
ba8         }
54c     };
9f9     auto bfs = [&](int root) {
be5         fill(all(label), -1);
652         iota(all(orig), 0);
4b6         q.clear();
594         label[root] = 0; q.push_back(root);
a43         rep(i, 0, sz(q)) {
4c1             int v = q[i];
5aa             for (auto x : g[v]) {
464                 if (label[x] == -1) {
73a                     label[x] = 1; par[x] = v;
1bd                     if (mate[x] == -1) return aug(x), 1;
8d9                     label[mate[x]] = 0;
de3                     q.push_back(mate[x]);
641                 }
018                 else if(!label[x] && orig[v] != orig[x]){
37f                     int a = lca(orig[v], orig[x]);
f12                     blossom(x, v, a);
183                     blossom(v, x, a);
405                 }
ab5             }
9e2         }
bb3         return 0;
139     };
// Time halves if you start with (any) maximal
// matching.
fea     rep(i, 0, n) {
698         if (mate[i] == -1) bfs(i);
7b5     }
568     return mate;
21c }

```

HopcroftKarp.h

Description: *ans* is the size of the max matching.

The match of x is $l[x]$

Usage: HopcroftKarp($|X|$, $|Y|$, edges(x , y))

Time: $\mathcal{O}(\sqrt{VE})$

c4f2f2, 46 lines

```

725 struct HopcroftKarp {
e40     vector<int> g, l, r;
959     int ans;
b82     HopcroftKarp(int n, int m, vector<pii> e)
aa0         : g(sz(e)), l(n, -1), r(m, -1), ans(0) {
bb0         shuffle(all(e), rng);
322         vector<int> deg(n + 1);
235         for (auto& [x, y] : e) deg[x]++;
b4a         rep(i, 1, n+1) deg[i] += deg[i - 1];
85a         for (auto& [x, y] : e) g[--deg[x]] = y;

5ae         vector<int> q(n);
667         while (true) {
661             vector<int> a(n, -1), p(n, -1);
6bb             int t = 0;
fea             rep(i, 0, n){
4b1                 if (l[i] == -1) {
b53                     q[t++] = a[i] = p[i] = i;

```

```

4b6     }
62e     }
a15     bool match = false;
edb     rep(i, 0, t){
912         int x = q[i];
08c         if (~l[a[x]]) continue;
0ba         rep(j, deg[x], deg[x+1]) {
360             int y = g[j];
89a             if (r[y] == -1) {
d3b                 while (~y) {
ee7                     r[y] = x;
dbb                     swap(l[x], y);
2a5                     x = p[x];
ebf                 }
6aa                 match = true, ans++;
c2b                 break;
b54             }
f06             if (p[r[y]] == -1) {
a74                 q[t++] = y = r[y];
d11                 p[y] = x, a[y] = a[x];
9ef             }
e8a         }
0ab     }
984     if (!match) break;
bc5 }
6ec }
c4f };

```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \leq M$.

Time: $\mathcal{O}(N^2M)$

4a75d2, 41 lines

```

d57 pair<ll, vector<int>> hunga(const vector<vector<ll>>& a) {
c04     if (a.empty()) return { 0, {} };
1a9     int n = sz(a) + 1, m = sz(a[0]) + 1;
fc8     vector<ll> u(n), v(m), p(m);
5bd     vector<int> ans(n - 1);
6f5     for (int i = 1; i < n; i++) {
8c9         p[0] = i;
625         int j0 = 0;
91d         vector<ll> dist(m, LLONG_MAX), pre(m, -1);
910         vector<bool> done(m + 1);
016         do {
781             done[j0] = true;
507             ll i0 = p[j0], j1 = -1, delta = LLONG_MAX;
b84             for (int j = 1; j < m; j++) {
10a                 if (!done[j]) {
ed6                     ll cur = a[i0-1][j-1] - u[i0] - v[j];
607                     if (cur < dist[j])
29f                         dist[j] = cur, pre[j] = j0;
172                     if (dist[j] < delta)
4ab                         delta = dist[j], j1 = j;
103                 }
bb2             }
891             for (int j = 0; j < m; j++) {
7a9                 if (done[j])
3bc                     u[p[j]] += delta, v[j] -= delta;
202                 else dist[j] -= delta;
11a             }
e73             assert(j1 != -1);
6d4             j0 = j1;
ac1         } while (p[j0]);
4b9         while (j0) {
196             int j1 = pre[j0];
0c1             p[j0] = p[j1], j0 = j1;

```

```

f55     }
193     }
b84     for (int j = 1; j < m; j++) {
eb3         if (p[j]) ans[p[j] - 1] = j - 1;
c9a     }
def     return { -v[0], ans }; // min cost
4a7 }

```

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.

Time: $\mathcal{O}(V^3)$

8b0e19, 22 lines

```

192 pair<int, vi> globalMinCut(vector<vi> mat) {
afa     pair<int, vi> best = {INT_MAX, {}};
755     int n = sz(mat);
91d     vector<vi> co(n);
d0f     rep(i, 0, n) co[i] = {i};
488     rep(ph, 1, n) {
2e9         vi w = mat[0];
e44         size_t s = 0, t = 0;
694         rep(it, 0, n-ph) { //  $\mathcal{O}(V^2) \rightarrow \mathcal{O}(E \log V)$  with prio.
queue
d6e             w[t] = INT_MIN;
a5f             s = t, t = max_element(all(w)) - w.begin();
d39             rep(i, 0, n) w[i] += mat[t][i];
ec9         }
3df         best = min(best, {w[t] - mat[t][t], co[t]});
096         co[s].insert(co[s].end(), all(co[t]));
959         rep(i, 0, n) mat[s][i] += mat[t][i];
984         rep(i, 0, n) mat[i][s] = mat[s][i];
5dd         mat[0][t] = INT_MIN;
ca0     }
f26     return best;
8b0 }

```

7.3 DFS algorithms

Bridges.h

1fa56b, 24 lines

```

cd9 vector<int> g[ms];
9e4 int low[ms], tin[ms], vis[ms], t;

403 void dfs(int u = 0, int p = -1) {
b9c     vis[u] = true;
b4a     low[u] = tin[u] = t++;
7b9     for (auto v : g[u]) {
730         if (v == p) continue;
c84         if (vis[v]) {
34f             low[u] = min(low[u], tin[v]);
728         }
4e6         else {
95e             dfs(v, u);
ab6             low[u] = min(low[u], low[v]);
// if (low[v] >= tin[u] && p != -1), U is an
// articulation point
975             if (low[v] > tin[u]) {
// edge from U to V is a bridge
4b8             }
// children++
862         }
677         // if (children > 1 && p == -1) root is an articulation
// point
30c     }

```

BridgeOnline.h

Description: Maintains bridges and 2-edge-connected components (2-ECC) incrementally. `ds[0]` tracks Connected Components (CC). `ds[1]` tracks 2-ECCs. Nodes u, v are in the same 2-ECC iff `dsfind(u, 1) == dsfind(v, 1)`. `g` stores the spanning forest edges (edges that were bridges when added). An edge $(u, v) \in g$ is a current bridge iff `dsfind(u, 1) != dsfind(v, 1)`. `bridges` tracks the total count of active bridges. Use `init()` before starting. **Time:** Amortized $\mathcal{O}(\log N)$

ef24c8, 75 lines

```
4dd int bridges;
801 int ds[2][ms], sz[2][ms];
87b int h[ms], pai[ms], old[ms];
cd9 vector<int> g[ms];

ca2 void init() {
786     bridges = 0;
f0d     rep(i, 0, ms) {
a4e         g[i].clear(), h[i] = 0;
606         ds[0][i] = ds[1][i] = i;
8f3         sz[0][i] = sz[1][i] = 1;
4a6     }
c1e }

243 int dsfind(int j, int i) {
7fa     if(j == ds[i][j]) return ds[i][j];
db7     return ds[i][j] = dsfind(ds[i][j], i);
4a4 }

b55 void dfs(int u, int p, int l) {
40d     h[u] = l;
49e     pai[u] = p;
a32     old[u] = dsfind(u, 1);
4d5     for (int v : g[u]) {
730         if (v == p) continue;
0c5         dfs(v, u, l + 1);
11d     }
f2e }

94c void updateNodes(int u, int p) {
840     if (old[u] == old[p]) {
dc4         ds[1][u] = ds[1][p];
574     }
e79     else ds[1][u] = u;
4d5     for (int v : g[u]) {
730         if (v == p) continue;
01c         updateNodes(v, u);
42a     }
329 }

814 void mergeTrees(int a, int b) {
cbf     bridges++;
5cb     int iniA = a, iniB = b;
19d     a = dsfind(a, 0), b = dsfind(b, 0);
834     if (sz[0][a] < sz[0][b]) swap(a, b), swap(iniA, iniB);
e14     dfs(iniB, iniA, h[iniA] + 1);
376     old[iniA] = -1;
ee0     updateNodes(iniB, iniA);
86b     ds[0][b] = a;
013     sz[0][a] += sz[0][b];
c9a }

416 void removeBridges(int a, int b) {
532     a = dsfind(a, 1), b = dsfind(b, 1);
984     while (a != b) {
e7a         bridges--;
54b         if (h[a] < h[b]) swap(a, b);
// ponte entre (a, pai[a]) deixou de existir
9f6         ds[1][a] = dsfind(pai[a], 1);
e40         a = ds[1][a];
cda     }
```

```
a78 }

02b void addEdge(int a, int b) {
7b9     if (dsfind(a, 0) == dsfind(b, 0)) {
69d         removeBridges(a, b);
221     }
4e6     else {
// nova ponte entre (a, b)
025         g[a].push_back(b);
3e9         g[b].push_back(a);
f8e         mergeTrees(a, b);
447     }
e57 }
```

BlockCutTree.h

Description: Constructs the Block-Cut Tree, which is a bipartite graph with blocks (maximal 2-vertex-connected components) on one side and articulation points on the other. Works for disconnected graphs. Tree size is $\leq 2N$. Be careful with self loops and multi edges. `art[i]`: number of new components created by removing i (AP if ≥ 1). `blocks[i]`, `edgblocks[i]`: vertices/edges of block i . `tree[i]`: the tree node index corresponding to block i . `pos[i]`: the tree node index corresponding to vertex i .

Time: $\mathcal{O}(N + M)$

e55ab0, 66 lines

```
d10 struct block_cut_tree {
d8e     vector<vector<int>> g, blocks, tree;
43b     vector<vector<pair<int, int>>> edgblocks;
4ce     stack<int> s;
6c0     stack<pair<int, int>> s2;
2bb     vector<int> id, art, pos;

763     block_cut_tree(vector<vector<int>> g_) : g(g_) {
625         int n = sz(g);
37a         id.resize(n, -1), art.resize(n), pos.resize(n);
6f2         build();
246     }

df6 int dfs(int i, int& t, int p = -1) {
cf0     int lo = id[i] = t++;
18e     s.push(i);

827     if (p != -1) s2.emplace(i, p);
43f     for (int j : g[i])
6bf         if (j != p and id[j] != -1) s2.emplace(i, j);

cac     for (int j : g[i]) if (j != p) {
9a3         if (id[j] == -1) {
121             int val = dfs(j, t, i);
0c3             lo = min(lo, val);

588             if (val >= id[i]) {
66a                 art[i]++;
483                 blocks.emplace_back(1, i);
110                 while (blocks.back().back() != j)
138                     blocks.back().push_back(s.top()), s.pop();

128                 edgblocks.emplace_back(1, s2.top()), s2.pop();
904                 while (edgblocks.back().back() != pii(j, i))
bce                     edgblocks.back().push_back(s2.top()), s2.pop();
041             }
38c         }
328         else lo = min(lo, id[j]);
5b6     }
924     if (p == -1) {
2db         if (art[i]) art[i]--;
4e6         else{
483             blocks.emplace_back(1, i);
433             edgblocks.emplace_back(i);
333         }
```

```
384     }
253     return lo;
6d7 }

0a8 void build() {
6bb     int t = 0;
c80     rep(i, 0, sz(g)) if(id[i] == -1) dfs(i, t, -1);
de0     tree.resize(sz(blocks));
008     rep(i, 0, sz(g)) if (art[i])
b9a         pos[i] = sz(tree), tree.emplace_back();

05c     rep(i, 0, sz(blocks)) for (int j : blocks[i]) {
403         if (!art[j]) pos[j] = i;
4e6         else {
49d             tree[i].push_back(pos[j]);
9a7             tree[pos[j]].push_back(i);
01e         }
27c     }
5a7 }
e55 };
```

DominatorTree.h

Description: Builds the Dominator Tree of a directed graph rooted at `root`. Node u dominates v if every path from `root` to v passes through u . The immediate dominator of v is the unique dominator closest to v (excluding v). Returns a vector `par` where `par[u]` is the parent of u in the tree. Roots and unreachable nodes satisfy `par[u] = u`.

Time: $\mathcal{O}(M \log N)$

8c4613, 55 lines

```
3db struct dominator_tree {
577     int n, t;
324     vector<vector<int>> g, rg, bucket;
7f3     vector<int> arr, par, rev, sdom, dom, ds, lbl;

226     dominator_tree(int n) : n(n), t(0), g(n), rg(n), bucket(n),
7a1         arr(n, -1), par(n), rev(n), sdom(n), dom(n), ds(n), lbl(n) {}

c2b void add_edge(int u, int v) { g[u].push_back(v); }

315 void dfs(int u) {
12e     arr[u] = t;
64f     rev[t] = u;
bad     lbl[t] = sdom[t] = ds[t] = t;
c82     t++;
6f1     for (int w : g[u]) {
0c2         if (arr[w] == -1) {
8c6             dfs(w);
81a             par[arr[w]] = arr[u];
869         }
f8e         rg[arr[w]].push_back(arr[u]);
93a     }
b04 }
792 int find(int u, int x=0) {
9fe     if (u == ds[u]) return x ? -1 : u;
41f     int v = find(ds[u], x+1);
388     if (v < 0) return u;
b30     if (sdom[lbl[ds[u]]] < sdom[lbl[u]]) lbl[u] = lbl[ds[u]];
300     ds[u] = v;
784     return x ? v : lbl[u];
a59 }

46f vector<int> run(int root) {
14e     dfs(root);
b81     iota(all(dom), 0);
da8     for (int i=t-1; i>=0; i--) {
76c         for(int w : rg[i]) sdom[i] = min(sdom[i], sdom[find(w)]);

c94         if (i) bucket[sdom[i]].push_back(i);
3b2         for (int w : bucket[i]) {
```

```

46a         int v = find(w);
ae4         if (sdom[v] == sdom[w]) dom[w] = sdom[w];
41c         else dom[w] = v;
1e6     }
fd8     if (i > 1) ds[i] = par[i];
b9e     }
e8f     rep(i, 1, t) {
7d7         if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
32d     }
af8     vector<int> par(n);
2c2     iota(all(par), 0);
533     rep(i, 0, t) par[rev[i]] = rev[dom[i]];
148     return par;
900 }
8c4 };

```

EulerPath.h

Description: Receives as input graph(node, edge index), number of edges and source. Returns list of node, index of edge he came from, if path/circuit does not exists returns empty list.

a3ed13, 27 lines

```

b4a vector<pii> eulerPath(const vector<vector<pii>>& g, int
nedges, int src) {
    int n = sz(g);
    vector<int> deg(n, 0), its(n, 0), used(nedges + 1, 0);
    vector<pii> s = { {src, -1} };
    //deg[src]++; //to allow paths, not only circuits
    vector<pii> ret;
    while (!s.empty()) {
        int u = s.back().first, &it = its[u];
        if (it == sz(g[u])) {
            ret.push_back(s.back());
            s.pop_back();
            continue;
        }
        auto& [nxt, id] = g[u][it++];
        b25 if (!used[id]) {
            deg[u]--; deg[nxt]++;
            e48 used[id] = 1;
            e1c s.push_back({ nxt, id });
            777 }
        388 }
        8d8 for (int x : deg) {
            518 if (x < 0 || sz(ret) != (nedges + 1)) return {};
        }
        26e reverse(ret.begin(), ret.end());
        edf return ret;
        a3e }

```

SCC.h

Description: Kosaraju algorithm for calculating strongly connected components. Components are ordered in topological order.

008ff2, 36 lines

```

bf0 struct SCC {
dab     int n, ncomp;
0e3     vector<vector<int>> g, inv;
829     vector<int> comp, vis, stk;
8b6     SCC(){}
471     SCC(int n)
464         : n(n), ncomp(0), g(n), inv(n), comp(n, -1), vis(n) {}

315     void dfs(int u) {
150         vis[u] = 1;
a35         for (int v : g[u]) if (!vis[v]) dfs(v);
967         stk.push_back(u);
37b     }
f20     void dfs_inv(int u) {
62c         comp[u] = ncomp;
3a5         for (int v : inv[u]) {

```

```

df4         if (comp[v] == -1) dfs_inv(v);
0a0     }
984     }
63d     void solve() {
603         for (int i = 0; i < n; i++) {
b65             if (!vis[i]) dfs(i);
358         }
340         reverse(all(stk));
49b         for (int u : stk) {
9ef             if (comp[u] != -1) continue;
672             dfs_inv(u);
a8f             ncomp++;
ecb         }
ef8     }
010     void add_edge(int a, int b) {
025         g[a].push_back(b);
a6a         inv[b].push_back(a);
1ec     }
008 };

```

TwoSat.h

Usage: not A ~A

"scc.h"

c8b989, 37 lines

```

d9d struct TwoSat{
1a8     int n;
3c9     SCC scc;
7c7     vector<int> value;
425     vector<pii> e;
e2c     TwoSat(int n) : n(n){}
6c0     bool solve(){
b36         value.resize(n);
8cc         scc = SCC(2*n);
1f3         for(auto &x : e) scc.add_edge(x.first, x.second);
7f9         scc.solve();
3df         for(int i=0; i<2*n; i++)
f83             if(scc.comp[i] == scc.comp[i^1]) return false;
830         for(int i=0; i<n; i++)
733             value[i] = scc.comp[id(i)] > scc.comp[id(~i)];
8a6         return true;
949     }
a0a     void atMostOne(vector<int> &li){
615         if(sz(li) <= 1) return;
da9         int cur = ~li[0];
b25         for(int i = 2; i < sz(li); i++) {
abb             int next = n++;
e0a             addOr(cur, ~li[i]);
f26             addOr(cur, next);
7ba             addOr(~li[i], next);
072             cur = ~next;
e3d         }
921         addOr(cur, ~li[1]);
bbb     }
41b     int id(int v) { return v < 0 ? (~v) * 2 ^ 1 : v * 2; }
276     void add(int a, int b) { e.push_back({id(a), id(b)}); }
bc7     void addOr(int a, int b) { add(~a, b); add(~b, a); }
671     void addImp(int a, int b) { addOr(~a, b); }
d9d     void addEqual(int a, int b){ addOr(a, ~b); addOr(~a, b); }
ec3     void isFalse(int a) { addImp(a, ~a); }
c8b };

```

7.4 Coloring

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D , computes a $(D+1)$ -coloring of the edges such that no neighboring edges share a color. (D -coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)

Time: $\mathcal{O}(NM)$

e210e2, 32 lines

```

f41 vi edgeColoring(int N, vector<pii> eds) {
727     vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
10d     for (pii e : eds) ++cc[e.first], ++cc[e.second];
e2f     int u, v, ncols = *max_element(all(cc)) + 1;
fda     vector<vi> adj(N, vi(ncols, -1));
6ec     for (pii e : eds) {
119         tie(u, v) = e;
e51         fan[0] = v;
0f4         loc.assign(ncols, 0);
696         int at = u, end = u, d, c = free[u], ind = 0, i = 0;
3b2         while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
3e1             loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
01e         cc[loc[d]] = c;
997         for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd
])
4ff             swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
79f         while (adj[fan[i]][d] != -1) {
a9f             int left = fan[i], right = fan[++i], e = cc[i];
99b             adj[u][e] = left;
ccb             adj[left][e] = u;
f7e             adj[right][e] = -1;
d99             free[right] = e;
316         }
dfd         adj[u][d] = fan[i];
c45         adj[fan[i]][d] = u;
0e1         for (int y : {fan[0], u, end})
3fa             for (int& z = free[y] = 0; adj[y][z] != -1; z++);
fdc     }
29d     rep(i, 0, sz(eds))
961         for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i
];
edf     return ret;
e21 }

```

7.5 Heuristics

MaxClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

2ecaf4, 53 lines

```

db9 using vb = vector<bitset<200>>;
c7d struct Maxclique {
24e     double limit=0.025, pk=0;
c04     struct Vertex { int i, d=0; };
547     using vv = vector<Vertex>;
d44     vb e;
df7     vv V;
e5c     vector<vector<int>> C;
497     vector<int> qmax, q, S, old;
fe3     void init(vv& r) {
fd3         for (auto& v : r) v.d = 0;
583         for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
0f1         sort(all(r), [](auto a, auto b) { return a.d > b.d; });
c43         int mxD = r[0].d;
3f8         for(int i=0; i<sz(r); i++) r[i].d = min(i, mxD) + 1;
526     }
bc8     void expand(vv& R, int lev = 1) {
ac1         S[lev] += S[lev - 1] - old[lev];
92c         old[lev] = S[lev - 1];
d18         while (sz(R)) {
3fd             if (sz(q) + R.back().d <= sz(qmax)) return;
d62             q.push_back(R.back().i);
f28             vv T;
7fb             for(auto v : R)
740                 if (e[R.back().i][v.i]) T.push_back({v.i});

```

```
d21     if (sz(T)) {
eea         if (S[lev]++ / ++pk < limit) init(T);
457         int j = 0, mxk = 1, mnk = max(sz(qmax)-sz(q)+1, 1);
9bc         C[1].clear(), C[2].clear();
969         for (auto v : T) {
bfe             int k = 1;
8f5             auto f = [&](int i) { return e[v.i][i]; };
5c6             while (any_of(all(C[k]), f)) k++;
782             if (k > mxk) mxk = k, C[mxk + 1].clear();
18a             if (k < mnk) T[j++].i = v.i;
0e6             C[k].push_back(v.i);
322         }
238         if (j > 0) T[j - 1].d = 0;
d2f         for(int k=mnk; k<mxk + 1; k++){
5bf             for (int i : C[k])
361                 T[j].i = i, T[j++].d = k;
9dc             }
22d             expand(T, lev + 1);
61f         } else if (sz(q) > sz(qmax)) qmax = q;
c81         q.pop_back(), R.pop_back();
3e0     }
81d }
b2d vector<int> maxClique(){ init(V),expand(V); return qmax;}
b40 Maxclique(vb conn) : e(conn),C(sz(e)+1),S(sz(C)),old(S){
01d     for(int i=0; i<sz(e); i++) V.push_back({i});
b60 }
534 };
```

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

```
753 typedef bitset<128> B;
044 template<class F>
6a9 void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R
={}) {
9bb     if (!P.any()) { if (!X.any()) f(R); return; }
a8e     auto q = (P | X)._Find_first();
cd1     auto cands = P & ~eds[q];
3d7     rep(i,0,sz(eds)) if (cands[i]) {
a75         R[i] = 1;
e78         cliques(eds, f, P & eds[i], X & eds[i], R);
bb6         R[i] = P[i] = 0; X[i] = 1;
181     }
c9d }
```

7.6 Trees

Centroid.h

Description: Call decomp(0) to solve, marked array should be initially set to zero.

Time: $\mathcal{O}(N \log N)$

```
6b6 int tam[ms], marked[ms];

2a1 int calc_tam(int u, int p) {
5d1     tam[u] = 1;
4d5     for (int v : g[u]) {
5ea         if (v != p && !marked[v]) tam[u] += calc_tam(v, u);
d09     }
f95     return tam[u];
d5d }

5fb int get_centroid(int u, int p, int tot) {
4d5     for (int v : g[u]) {
38c         if (v != p && !marked[v] && (tam[v] > (tot / 2)))
32c             return get_centroid(v, u, tot);
```

```
b6c     }
03f     return u;
0c7 }

// Cent is a child of P in the centroid tree
179 void decomp(int u, int p = -1) {
308     calc_tam(u, -1);
bd4     int cent = get_centroid(u, -1, tam[u]);
83d     marked[cent] = 1;
9f1     for (int v : g[cent]) {
c6e         if (!marked[v]) decomp(v, cent);
194     }
dc1 }
```

HLD.h

Description: If values are stored on edges, set EDGE = true and store each edge's value at the endpoint farther from the root (the deeper node). rp[i] is the representative (head) of the heavy path containing node i: it is the node in that chain that is closest to the root.

```
a129d6, 51 lines
5f2 template<bool EDGE> struct HLD {
577     int n, t;
789     vector<vector<int>> g;
003     vector<int> pai, rp, tam, pos, val, arr;
f1e     Seg seg;
bcf     HLD(int n, vector<vector<int>>& g, vector<int>& val)
ac9         : n(n), t(0), g(g), pai(n), rp(n), tam(n, 1),
616             pos(n), val(val), arr(n) {
f80             calc_tam(0, -1);
c91             dfs(0, -1);
d14             seg.build(arr);
a43     }

2a1     int calc_tam(int u, int p) {
49e         pai[u] = p;
704         for (int& v : g[u]) {
730             if (v == p) continue;
2e4             tam[u] += calc_tam(v, u);
2d5             if(tam[v] > tam[g[u][0]] || g[u][0] == p)
a7f                 swap(g[u][0], v);
0a3         }
f95         return tam[u];
c19     }

fb6     void dfs(int u, int p) {
4c8         pos[u] = t++;
d7b         arr[pos[u]] = val[u];
4d5         for (int v : g[u]) {
730             if (v == p) continue;
84d             rp[v] = (v == g[u][0] ? rp[u] : v);
95e             dfs(v, u);
42d         }
de1     }

4ea     int query(int a, int b) { // query on the path from a
to b
1a4         int ans = 0; // neutral value
34d         while (rp[a] != rp[b]) {
aa1             if (pos[a] < pos[b]) swap(a, b);
9a5             ans = max(ans, seg.query(pos[rp[a]], pos[a]));
677             a = pai[rp[a]];
ebd         }
9bc         if(pos[a] > pos[b]) swap(a, b);
0f8         ans = max(ans, seg.query(pos[a] + EDGE, pos[b]));
ba7         return ans;
e8a     }

534     void update(int a, int x) {
e5e         seg.update(pos[a], x);
5db     }
```

```
a12 };
```

LCA.h

Description: LCA algorithm using binary lifting, *is_ancestor(a,b)* returns true if *a* is an ancestral of *b* and false otherwise.

Time: $\mathcal{O}(N \log N)$

```
db7791, 26 lines
67e int tin[MAXN], tout[MAXN], timer=0;
768 int up[MAXN][BITS];
fb6 void dfs(int u, int p){
545     tin[u] = timer++; up[u][0] = p;
532     for (int i=1; i<BITS; i++) {
88a         up[u][i] = up[up[u][i-1]][i-1];
4a0     }
712     for (int v: g[u]) if (v != p) dfs(v, u);
4f8     tout[u] = timer;
4a1 }

f31 bool is_ancestor(int u, int v){
d34     return (tin[u] <= tin[v] && tout[u] >= tout[v]);
f9f }

310 int lca(int u, int v){
bd5     if (is_ancestor(u, v)) return u;
6fc     if (is_ancestor(v, u)) return v;
3c3     for (int i=BITS-1; i>=0; i--) {
3a3         if (up[u][i] && !is_ancestor(up[u][i], v)) {
c3f             u = up[u][i];
49e         }
dc4     }
c15     return up[u][0];
001 }
```

VirtualTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S| - 1$) pairwise LCA's and compressing edges. virt[u] is the adjacency list of the virtual tree: it stores pairs (v, dist), where v is a neighbor of u in the virtual tree and dist is the distance between u and v in the original tree.

Time: $\mathcal{O}(|S| \log |S|)$

```
"LCA.h"
11157a, 24 lines
0b1 vector<pair<int, int>> virt[ms];

d0c void build_virt(vector<int>& v) {
078     auto cmp = [&](int i, int j){ return tin[i] < tin[j]; };
b84     sort(all(v), cmp);
1ee     for (int i = 0, n = sz(v); i + 1 < n; i++)
4cf         v.push_back(lca(v[i], v[i + 1]));
b84     sort(all(v), cmp);
64f     v.erase(unique(all(v)), v.end());
7b4     stack<int> st;
3a7     for (auto u : v) {
c53         if (st.empty()) {
4a6             st.push(u);
e82         }
4e6         else {
7eb             while(sz(st) && !is_ancestor(st.top(), u)) st.pop();
88b             int p = st.top();
bfa             virt[p].emplace_back(u, abs(lvl[u] - lvl[p]));
0a5             virt[u].emplace_back(p, abs(lvl[u] - lvl[p]));
4a6             st.push(u);
92c         }
f46     }
c83 }
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.

Time: $\mathcal{O}(E \log V)$

../data-structures/UnionFindRollback.h" 39e620, 61 lines

```
030 struct Edge { int a, b; ll w; };
bf2 struct Node {
25f   Edge key;
c17   Node *l, *r;
981   ll delta;
a9c   void prop() {
6f9     key.w += delta;
d2d     if (l) l->delta += delta;
d86     if (r) r->delta += delta;
978     delta = 0;
0d3   }
866   Edge top() { prop(); return key; }
ab4 };
3eb Node *merge(Node *a, Node *b) {
b9f   if (!a || !b) return a ? b;
626   a->prop(), b->prop();
dc2   if (a->key.w > b->key.w) swap(a, b);
485   swap(a->l, (a->r = merge(b, a->r)));
3f5   return a;
c51 }
7bb void pop(Node& a) { a->prop(); a = merge(a->l, a->r); }
```

```
002 pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
8df   RollbackUF uf(n);
3f8   vector<Node*> heap(n);
563   for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node(e
));
cd2   ll res = 0;
517   vi seen(n, -1), path(n), par(n);
559   seen[r] = r;
dd6   vector<Edge> Q(n), in(n, {-1,-1}), comp;
111   deque<tuple<int, int, vector<Edge>>> cys;
328   rep(s,0,n) {
3cb     int u = s, qi = 0, w;
a0a     while (seen[u] < 0) {
572       if (!heap[u]) return {-1, {}};
ebe       Edge e = heap[u]->top();
5ed       heap[u]->delta -= e.w, pop(heap[u]);
952       Q[qi] = e, path[qi++] = u, seen[u] = s;
d56       res += e.w, u = uf.find(e.a);
9e2       if (seen[u] == s) {
28d         Node* cyc = 0;
cab         int end = qi, time = uf.time();
f38         do cyc = merge(cyc, heap[w = path[--qi]]);
4f9         while (uf.join(u, w));
562         u = uf.find(u), heap[u] = cyc, seen[u] = -1;
c06         cys.push_front({u, time, {Q[qi], &Q[end]}});
00a       }
c8f     }
068     rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
fa3   }
```

```
e41 for (auto& [u,t,comp] : cys) { // restore sol (optional)
36c   uf.rollback(t);
1d0   Edge inEdge = in[u];
251   for (auto& e : comp) in[uf.find(e.b)] = e;
56d   in[uf.find(inEdge.b)] = inEdge;
4f9 }
427 rep(i,0,n) par[i] = in[i].a;
efb return {res, par};
efa }
```

Geometry (8)

8.1 Geometric primitives

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

47ec0a, 29 lines

```
48b template <class T> int sgn(T x) { return (x > 0) - (x < 0)
; }
4fc template<class T>
f26 struct Point {
ea4   typedef Point P;
645   T x, y;
ea6   explicit Point(T x=0, T y=0) : x(x), y(y) {}
0d0   bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y
); }
ec7   bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y
); }
279   P operator+(P p) const { return P(x+p.x, y+p.y); }
40d   P operator-(P p) const { return P(x-p.x, y-p.y); }
e03   P operator*(T d) const { return P(x*d, y*d); }
0b9   P operator/(T d) const { return P(x/d, y/d); }
57b   T dot(P p) const { return x*p.x + y*p.y; }
460   T cross(P p) const { return x*p.y - y*p.x; }
b3f   T cross(P a, P b) const { return (a-*this).cross(b-*this
); }
f68   T dist2() const { return x*x + y*y; }
18b   double dist() const { return sqrt((double)dist2()); }
// angle to x-axis in interval [-pi, pi]
907   double angle() const { return atan2(y, x); }
d06   P unit() const { return *this/dist(); } // makes dist()==1
200   P perp() const { return P(-y, x); } // rotates +90
degrees
852   P normal() const { return perp().unit(); }
// returns point rotated 'a' radians ccw around the
origin
f23   P rotate(double a) const {
482     return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
902   friend ostream& operator<<(ostream& os, P p) {
9a9     return os << "(" << p.x << ", " << p.y << ")"; }
d2d   };
```

lineDistance.h

Description:

Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

f6bf6b, 5 lines

```
7dc template<class P>
2ff double lineDist(const P& a, const P& b, const P& p) {
e07   return (double) (b-a).cross(p-a)/(b-a).dist();
008   }
```

SegmentDistance.h

Description:

Returns the shortest distance between point p and the line segment from point s to e.

Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;

"Point.h" 5c88f4, 7 lines

```
626 typedef Point<double> P;
929 double segDist(P& s, P& e, P& p) {
a44   if (s==e) return (p-s).dist();
```

```
f81   auto d = (e-s).dist2(), t = min(d,max(.0, (p-s).dot(e-s)))
;
2c1   return ((p-s)*d-(e-s)*t).dist()/d;
ae7   }
```

SegmentIntersection.h

Description:

If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

Usage: vector<P> inter = segInter(s1,e1,s2,e2);

if (sz(inter)==1)

cout << "segments intersect at " << inter[0] << endl;

"Point.h", "OnSegment.h" 9d57f2, 14 lines

```
dae template<class P> vector<P> segInter(P a, P b, P c, P d) {
0b6   auto oa = c.cross(d, a), ob = c.cross(d, b),
318     oc = a.cross(b, c), od = a.cross(b, d);
// Checks if intersection is single non-endpoint point.
914   if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
e5b     return {(a * ob - b * oa) / (ob - oa)};
4c1   set<P> s;
ccb   if (onSegment(c, d, a)) s.insert(a);
0ad   if (onSegment(c, d, b)) s.insert(b);
3d8   if (onSegment(a, b, c)) s.insert(c);
2fa   if (onSegment(a, b, d)) s.insert(d);
a35   return {all(s)};
9d5   }
```

lineIntersection.h

Description:

If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.

Usage: auto res = lineInter(s1,e1,s2,e2);

if (res.first == 1)

cout << "intersection point at " << res.second << endl;

"Point.h" a01f81, 9 lines

```
7dc template<class P>
0bf pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
14f   auto d = (e1 - s1).cross(e2 - s2);
8cc   if (d == 0) // if parallel
d99     return {- (s1.cross(e1, s2) == 0), P(0, 0)};
f6b   auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
9b8   return {1, (s1 * p + e1 * q) / d};
472   }
```

sideOf.h

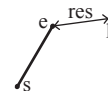
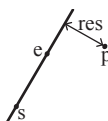
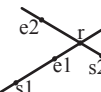
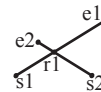
Description: Returns where p is as seen from s towards e. $1/0/-1 \Leftrightarrow$ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: bool left = sideOf(p1,p2,q)==1;

"Point.h" 3af81c, 10 lines

```
7dc template<class P>
70b int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }
```

7dc template<class P>



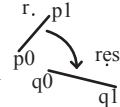
```
b5e int sideOf(const P& s, const P& e, const P& p, double eps)
{
79e auto a = (e-s).cross(p-s);
653 double l = (e-s).dist()*eps;
c32 return (a > l) - (a < -l);
33f }
```

OnSegment.h
Description: Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

```
"Point.h" c597e8, 4 lines
514 template<class P> bool onSegment(P s, P e, P p) {
5fb return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
c59 }
```

linearTransformation.h

Description:
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



```
"Point.h" 03a306, 7 lines
626 typedef Point<double> P;
664 P linearTransformation(const P& p0, const P& p1,
f06 const P& q0, const P& q1, const P& r) {
99f P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
0aa return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist
2();
45e }
```

LineProjectionReflection.h

Description: Projects point p onto line ab. Set refl=true to get reflection of point p across line ab instead. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

```
"Point.h" b5562d, 6 lines
7dc template<class P>
981 P lineProj(P a, P b, P p, bool refl=false) {
de3 P v = b - a;
3fc return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
4b7 }
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage: vector<Angle> v = {w[0], w[0].t360() ...}; // sorted
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }
// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i

```
0f0602, 36 lines
755 struct Angle {
e91 int x, y;
8bd int t;
5ac Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
de8 Angle operator-(Angle b) const { return {x-b.x, y-b.y, t
}; }
3cd int half() const {
840 assert(x || y);
aa4 return y < 0 || (y == 0 && x < 0);
c93 }
dfc Angle t90() const { return {-y, x, t + (half() && x >= 0)
}; }
726 Angle t180() const { return {-x, -y, t + half()}; }
925 Angle t360() const { return {x, y, t + 1}; }
e25 };
a92 bool operator<(Angle a, Angle b) {
// add a.dist2() and b.dist2() to also compare distances
```

```
ea7 return make_tuple(a.t, a.half(), a.y * (11)b.x) <
05f make_tuple(b.t, b.half(), a.x * (11)b.y);
ce5 }

// Given two points, this calculates the smallest angle
// between
// them, i.e., the angle that covers the defined line
// segment.
908 pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
ee4 if (b < a) swap(a, b);
423 return (b < a.t180() ?
c35 make_pair(a, b) : make_pair(b, a.t360()));
5ea }
784 Angle operator+(Angle a, Angle b) { // point a + vector b
eb1 Angle r(a.x + b.x, a.y + b.y, a.t);
8ca if (a.t180() < r) r.t--;
d9f return r.t180() < a ? r.t360() : r;
3d8 }
106 Angle angleDiff(Angle a, Angle b) { // angle b - angle a
125 int tu = b.t - a.t; a.t = b.t;
e63 return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a
)};
ba3 }
```

HalfPlane.h

Description: Computes the intersection of a set of half-planes. Half-planes are sorted by angle and processed with a deque, removing redundant or conflicting constraints. Parallel half-planes are handled explicitly. Returns the convex polygon of the intersection, or empty if infeasible.

Time: $O(n \log n)$

```
"Point.h" cf24a8, 72 lines
984 using ld = long double;
207 using P = Point<ld>;

533 struct Hp { // Half plane struct
// 'p' is a passing point of the line and 'pq' is the
// direction vector of the line.
812 P p, pq;
d29 ld angle;

b93 Hp() {}
65d Hp(const P& a, const P& b) : p(a), pq(b - a) {
0e3 angle = atan21(pq.y, pq.x);
2ff }
8ce bool out(const P& r) { return pq.cross(r - p) < -eps; }
d36 bool operator < (const Hp& e) const {
1dd return angle < e.angle;
44e }
ea9 friend P inter(const Hp& s, const Hp& t) {
020 ld alpha = (t.p - s.p).cross(t.pq) / s.pq.cross(t.pq);
93b return s.p + (s.pq * alpha);
825 }
b46 };

fa5 vector<P> hp_intersect(vector<Hp>& H) {
12f P box[4] = { P(Inf, Inf), P(-Inf, Inf),
9c8 P(-Inf, -Inf), P(Inf, -Inf) };
```

```
1cd for(int i = 0; i<4; i++) {
1a8 Hp aux(box[i], box[(i+1) % 4]);
d82 H.push_back(aux);
560 }
f1a sort(all(H));
6c5 deque<Hp> dq;
486 int len = 0;
908 for(int i = 0; i < sz(H); i++) {
3fb while(len>1 && H[i].out(inter(dq[len-1], dq[len-2]))){
c70 dq.pop_back();
654 --len;
```

```
a31 }
757 while (len > 1 && H[i].out(inter(dq[0], dq[1]))){
c68 dq.pop_front();
654 --len;
1eb }
a5a if(len && fabs1(H[i].pq.cross(dq[len-1].pq)) < eps){
25f if (H[i].pq.dot(dq[len-1].pq) < 0.0)
282 return vector<P>();
e7b if (H[i].out(dq[len-1].p)) {
c70 dq.pop_back();
654 --len;
2dc }
64e else continue;
9a0 }
fc2 dq.push_back(H[i]);
250 ++len;
8ed }

337 while(len> 2 && dq[0].out(inter(dq[len-1], dq[len-2]))){
c70 dq.pop_back();
654 --len;
faa }
81e while (len > 2 && dq[len-1].out(inter(dq[0], dq[1]))){
c68 dq.pop_front();
654 --len;
694 }
1a3 if (len < 3) return vector<P>();
7e7 vector<P> ret(len);
cc7 for(int i = 0; i+1 < len; i++) {
01e ret[i] = inter(dq[i], dq[i+1]);
00f }
4fd ret.back() = inter(dq[len-1], dq[0]);
edf return ret;
deb }
```

8.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

```
"Point.h" ba7267, 12 lines
626 typedef Point<double> P;
27f bool circleInter(P a,P b,double r1,double r2,pair<P, P>*
out) {
b48 if (a == b) { assert(r1 != r2); return false; }
f30 P vec = b - a;
6c8 double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2;
c28 double p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*
d2;
5b0 if (sum*sum < d2 || dif*dif > d2) return false;
84d P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) /
d2);
21e *out = {mid + per, mid - per};
8a6 return true;
170 }
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

```
"Point.h" b0153d, 14 lines
7dc template<class P>
3a5 vector<pair<P, P>> tangents(P c1, double r1, P c2, double
r2) {
c0b P d = c2 - c1;
```

```
432 double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
018 if (d2 == 0 || h2 < 0) return {};
c14 vector<pair<P, P>> out;
092 for (double sign : {-1, 1}) {
2ad     P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
2e3     out.push_back({c1 + v * r1, c2 + v * r2});
e25 }
b21 if (h2 == 0) out.pop_back();
fe8 return out;
483 }
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

```
Point.h" e0cfba, 10 lines
7dc template<class P>
195 vector<P> circleLine(P c, double r, P a, P b) {
33b P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
55a double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
3e4 if (h2 < 0) return {};
071 if (h2 == 0) return {p};
7cd P h = ab.unit() * sqrt(h2);
d65 return {p - h, p + h};
59a }
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.

```

Time:  $\mathcal{O}(n)$ 
"../../content/geometry/Point.h"
19add1, 20 lines

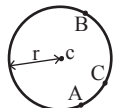
626 typedef Point<double> P;
361 #define arg(p, q) atan2(p.cross(q), p.dot(q))
bb9 double circlePoly(P c, double r, vector<P> ps) {
6d1     auto tri = [&](P p, P q) {
c9c         auto r2 = r * r / 2;
291         P d = q - p;
127         auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist
2();
eea     auto det = a * a - b;
691     if (det <= 0) return arg(p, q) * r2;
f43     auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det
));
aba     if (t < 0 || 1 <= s) return arg(p, q) * r2;
57f     P u = p + d * s, v = q + d * (t-1);
8c0     return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
a52 };
bef     auto sum = 0.0;
8f4     rep(i,0,sz(ps))
3b7         sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
e66     return sum;
f08 }

```

circumcircle.h

Description:

The circumcircle of a triangle is the circle intersecting all three vertices. `ccRadius` returns the radius of the circle going through points A, B and C and `ccCenter` returns the center of the same circle.



```

Point.h" 1caa3a, 10 lines
626 typedef Point<double> P;
510 double ccRadius(const P& A, const P& B, const P& C) {
14b     return (B-A).dist()*(C-B).dist()*(A-C).dist()/
f73         abs((B-A).cross(C-A))/2;
607 }
c0d P ccCenter(const P& A, const P& B, const P& C) {
28a     P p = C-A, c = B-A;

```

```
680     return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
793 }
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.

Time: expected $\mathcal{O}(n)$

```

circumcircle.h"                                09dd0a, 18 lines
a28 pair<P, double> mec(vector<P> ps) {
4da  shuffle(all(ps), mt19937(time(0)));
f6a  P o = ps[0];
328  double r = 0, EPS = 1 + 1e-8;
2be  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
5cc    o = ps[i], r = 0;
4da    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
a30      o = (ps[i] + ps[j]) / 2;
6f7      r = (o - ps[i]).dist();
102      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
fa9        o = ccCenter(ps[i], ps[j], ps[k]);
6f7        r = (o - ps[i]).dist();
648      }
7b0    }
dcf  }
645  return {o, r};
09d }

```

8.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If `strict` is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

```
Usage: vector<P> v = {P{4,4}, P{1,2}, P{2,1}};  
bool in = inPolygon(v, P{3, 3}, false);
```

Time: $\mathcal{O}(n)$

```
Point.h", "OnSegment.h", "SegmentDistance.h" 2bf504, 12 lines
7dc  template<class P>
0cc  bool inPolygon(vector<P> &p, P a, bool strict = true) {
8b7  int cnt = 0, n = sz(p);
fea  rep(i,0,n) {
444      P q = p[(i + 1) % n];
cbd      if (onSegment(p[i], q, a)) return !strict;
//or: if (segDist(p[i], q, a) <= eps) return !strict;
007  cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) >
0;
1b9  }
70a  return cnt;
c72  }
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

```

"Point.h" f12300, 7 lines
4fc template<class T>
a51 T polygonArea2(vector<Point<T>>& v) {
2f8     T a = v.back().cross(v[0]);
06e     rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
3f5     return a;
693 }

```

PolygonCenter.h

Description: Returns the center of mass for a polygon.

Time: $\mathcal{O}(n)$

```
"Point.h" 9706dc, 10 lines
626 typedef Point<double> P;
6d9 P polygonCenter(const vector<P>& v) {
f9f     P res(0, 0); double A = 0;
70b     for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
```

```

346     res = res + (v[i] + v[j]) * v[j].cross(v[i]);
3ea     A += v[j].cross(v[i]);
307 }
33c return res / A / 3;
0d0 }

```

PolygonCut.h

Description:

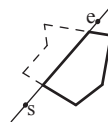
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.

```
Usage: vector<P> p = ...;  
p = polygonCut(p, P(0,0), P(1,0));
```

```

"Point.h" d07181, 14 lines
626 typedef Point<double> P;
37d vector<P> polygonCut(const vector<P>& poly, P s, P e) {
fe2     vector<P> res;
d48     rep(i,0,sz(poly)) {
21c         P cur = poly[i], prev = i ? poly[i-1] : poly.back();
c5f         auto a = s.cross(e, cur), b = s.cross(e, prev);
2dc         if ((a < 0) != (b < 0))
380             res.push_back(cur + (prev - cur) * (a / (a - b)));
c5c         if (a < 0)
a5f             res.push_back(cur);
757     }
b50     return res;
42c }

```



PolygonUnion.h

Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)

Time: $\mathcal{O}(N^2)$, where N is the total number of points

```

Point.h", "sideOf.h"
3931c6, 34 lines
626 typedef Point<double> P;
142 double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y
; }
61d double polyUnion(vector<vector<P>>& poly) {
499 double ret = 0;
9af rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
9c8 P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
05c vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
cbd rep(j,0,sz(poly)) if (i != j) {
cc1 rep(u,0,sz(poly[j])) {
418 P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])
];
688 int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
68b if (sc != sd) {
295 double sa = C.cross(D, A), sb = C.cross(D, B);
e90 if (min(sc, sd) < 0)
dac segs.emplace_back(sa / (sa - sb), sgn(sc - sd))
;
cf7 } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))
>0){
5b4 segs.emplace_back(rat(C - A, B - A), 1);
e96 segs.emplace_back(rat(D - A, B - A), -1);
313 }
0d1 }
fdc }
861 sort(all(segs));
153 for (auto& s : segs) s.first = min(max(s.first, 0.0), 1
.0);
68c double sum = 0;
723 int cnt = segs[0].second;
067 rep(j,1,sz(segs)) {
081 if (!cnt) sum += segs[j].first - segs[j - 1].first;
6e9 cnt += segs[j].second;
f58 }

```

```
320     ret += A.cross(B) * sum;
191   }
ad6   return ret / 2;
6e8 }
```

ConvexHull.h

Description: Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull. If you want to keep the collinear points in the convex hull, change the comparison to $h[t-2].cross(h[t-1],p) < 0$ and the size of the vector h to $2 * sz(pts) + 1$.
Time: $\mathcal{O}(n \log n)$

"Point.h"	310954, 14 lines
<pre>2c0 typedef Point<ll> P; f16 vector<P> convexHull(vector<P> pts) { f78 if (sz(pts) <= 1) return pts; 3cb sort(all(pts)); abf vector<P> h(sz(pts)+1); 573 int s = 0, t = 0; 628 for (int it = 2; it--; s = --t, reverse(all(pts))) 4eb for (P p : pts) { 3da while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t --; f39 h[t++] = p; bf0 } 036 return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])}; ec8 }</pre>	

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).
Time: $\mathcal{O}(n)$

"Point.h"	c571b8, 13 lines
<pre>2c0 typedef Point<ll> P; d31 array<P, 2> hullDiameter(vector<P> S) { e79 int n = sz(S), j = n < 2 ? 0 : 1; 354 pair<ll, array<P, 2>> res({0, {S[0], S[0]}}); e4d rep(i,0,j) 42e for (; j = (j + 1) % n) { ca1 res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}}) ; be8 if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0) c2b break; 56c } 3f2 return res.second; 5f7 }</pre>	

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h"	71446b, 15 lines
<pre>2c0 typedef Point<ll> P; 2d4 bool inHull(const vector<P>& l, P p, bool strict = true) { d44 int a = 1, b = sz(l) - 1, r = !strict; 5cc if (sz(l) < 3) return r && onSegment(l[0], l.back(), p); 6bc if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b); 456 if (sideOf(l[0], l[a], p) >= r sideOf(l[0], l[b], p) <= -r) d1f return false; 48a while (abs(a - b) > 1) { 4f7 int c = (a + b) / 2;</pre>	

```
ac8     (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
b26   }
06f   return sgn(l[a].cross(l[b], p)) < r;
c74 }
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: $\bullet(-1, -1)$ if no collision, $\bullet(i, -1)$ if touching the corner i , $\bullet(i, i)$ if along side $(i, i + 1)$, $\bullet(i, j)$ if crossing sides $(i, i + 1)$ and $(j, j + 1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i, i + 1)$. The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log n)$

"Point.h"	7cf45b, 40 lines
<pre>530 #define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j) %n])) f84 #define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0 e7e template <class P> int extrVertex(vector<P>& poly, P dir) { 747 int n = sz(poly), lo = 0, hi = n; fdf if (extr(0)) return 0; 3d1 while (lo + 1 < hi) { 591 int m = (lo + hi) / 2; 855 if (extr(m)) return m; c0c int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m); f48 (ls < ms (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m; 68a } 253 return lo; 7f0 }</pre>	

<pre>8e0 #define cmpL(i) sgn(a.cross(poly[i], b)) 7dc template <class P> ec4 array<int, 2> lineHull(P a, P b, vector<P>& poly) { 409 int endA = extrVertex(poly, (a - b).perp()); 761 int endB = extrVertex(poly, (b - a).perp()); 1a8 if (cmpL(endA) < 0 cmpL(endB) > 0) 423 return {-1, -1}; 649 array<int, 2> res; f4b rep(i,0,2) { 234 int lo = endB, hi = endA, n = sz(poly); c2d while ((lo + 1) % n != hi) { 57e int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n; 7f6 (cmpL(m) == cmpL(endB) ? lo : hi) = m; 525 } 7dd res[i] = (lo + !cmpL(hi)) % n; 356 swap(endA, endB); c05 } e00 if (res[0] == res[1]) return {res[0], -1}; 3d1 if (!cmpL(res[0]) && !cmpL(res[1])) 959 switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) { 3f3 case 0: return {res[0], res[0]}; 223 case 2: return {res[1], res[1]}; 8fa } b50 return res; 36f }</pre>	
--	--

Minkowski.h

Description: Computes the Minkowski sum of two convex polygons. Polygons must be convex and given in CCW order. Returns the vertices of the Minkowski sum polygon in CCW order.
Time: $\mathcal{O}(n + m)$

"Point.h"	664d67, 24 lines
<pre>780 using P = Point<ll>;</pre>	

<pre>89f vector<P> minkowski(vector<P> p, vector<P> q) { a8e auto fix = [](vector<P>& A) { bec int pos = 0; 2bb for (int i = 1; i < sz(A); i++) { 609 if(A[i].y < A[pos].y (A[i].y == A[pos].y && A[i]. x < A[pos].x)) e4c pos = i; f76 } 703 rotate(A.begin(), A.begin() + pos, A.end()); 9e5 A.push_back(A[0]), A.push_back(A[1]); 236 }; 889 fix(p), fix(q); db6 vector<P> result; 692 int i = 0, j = 0; 98a while (i < sz(p) - 2 j < sz(q) - 2) { 942 result.push_back(p[i] + q[j]); 3bd auto cross = (p[i + 1] - p[i]).cross(q[j + 1] - q[j]); c3c if (cross >= 0 && i < sz(p) - 2) i++; f33 if (cross <= 0 && j < sz(q) - 2) j++; 801 } dc8 return result; 2f9 }</pre>	
---	--

Extreme.h

Description: Finds an extreme vertex of a convex polygon according to a unimodal comparator. The comparator defines a total order along the polygon (given in CCW order).
Time: $\mathcal{O}(\log n)$

"Point.h"	70b181, 26 lines
<pre>780 using P = Point<ll>; c88 int extreme(vector<P> &pol, const function<bool(P, P)>& cmp) { b1c int n = pol.size(); 4a2 auto extr = [&](int i, bool& cur_dir) { 22a cur_dir = cmp(pol[(i+1)%n], pol[i]); 61a return !cur_dir and !cmp(pol[(i+n-1)%n], pol[i]); 364 }; 63d bool last_dir, cur_dir; a0d if (extr(0, last_dir)) return 0; 993 int l = 0, r = n; ead while (l+1 < r) { ee4 int m = (l+r)/2; f29 if (extr(m, cur_dir)) return m; 44a bool rel_dir = cmp(pol[m], pol[l]); b18 if ((!last_dir and cur_dir) or 261 (last_dir == cur_dir and rel_dir == cur_dir)) { 8a6 l = m; 1f1 last_dir = cur_dir; 94a } else r = m; 606 } 792 return l; 985 } cad int max_dot(vector<P> &pol, P v) { a98 return extreme([&](P p, P q) { return p.dot(v) > q.dot(v) }); }); 27e }</pre>	

Tangents.h

Description: Finds the left and right tangent points from an external point p to a convex polygon given in CCW order. A tangent point is a vertex where the segment p->v touches the polygon without intersecting its interior, defining the limits of visibility from p. Returns the indices of the left and right tangent vertices.
Time: $\mathcal{O}(\log n)$

"Point.h", "Extreme.h"	dcf85f, 11 lines
<pre>780 using P = Point<ll>; 08d bool ccw(P p, P q, P r) {</pre>	

```

274     return (q-p).cross(r-q) > 0;
0f3 }
826 pair<int, int> tangents(vector<P> &pol, P p) {
ae2     auto L = [&](P q, P r) { return ccw(p, r, q); };
98c     auto R = [&](P q, P r) { return ccw(p, q, r); };
861     return {extreme(pol, L), extreme(pol, R)};
3dc }

```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.

Time: $O(n \log n)$

```

"Point.h" ac41a6, 18 lines
2c0 typedef Point<ll> P;
24b pair<P, P> closest(vector<P> v) {
7f9     assert(sz(v) > 1);
7f7     set<P> S;
879     sort(all(v), [](P a, P b) { return a.y < b.y; });
571     pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
eec     int j = 0;
813     for (P p : v) {
3fb         P d{1 + (ll)sqrt(ret.first), 0};
8be         while (v[j].y <= p.y - d.x) S.erase(v[j++]);
a5a         auto lo = S.lower_bound(p - d), hi = S.upper_bound(p +
d);
c77         for (; lo != hi; ++lo)
113             ret = min(ret, {(p - lo).dist2(), {lo, p}});
8aa         S.insert(p);
5b0     }
70d     return ret.second;
bf2 }

```

ManhattanMST.h

Description: Given N points, returns up to $4N$ edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights $w(p, q) = -p.x - q.x - p.y - q.y$. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.

Time: $O(N \log N)$

```

"Point.h" df6f59, 24 lines
bbe typedef Point<int> P;
ea9 vector<array<int, 3>> manhattanMST(vector<P> ps) {
850     vi id(sz(ps));
27c     iota(all(id), 0);
8c1     vector<array<int, 3>> edges;
8de     rep(k, 0, 4) {
1dd         sort(all(id), [&](int i, int j) {
02b             return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y; });
702         map<int, int> sweep;
1e2         for (int i : id) {
84d             for (auto it = sweep.lower_bound(-ps[i].y);
904                 it != sweep.end(); sweep.erase(it++)) {
61d                 int j = it->second;
6f3                 P d = ps[i] - ps[j];
d18                 if (d.y > d.x) break;
537                 edges.push_back({d.y + d.x, i, j});
271             }
923             sweep[-ps[i].y] = i;
e69         }
4eb         for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p
.y);
a11     }
da2     return edges;
a11 }

```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" bac5b0, 64 lines

```

9a6 typedef long long T;
293 typedef Point<T> P;
305 const T INF = numeric_limits<T>::max();

173 bool on_x(const P& a, const P& b) { return a.x < b.x; }
0bd bool on_y(const P& a, const P& b) { return a.y < b.y; }

bf2 struct Node {
975     P pt; // if this is a leaf, the single point in it
877     T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
a23     Node *first = 0, *second = 0;

86a     T distance(const P& p) { // min squared distance to a
point
28b         T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
88e         T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
d98         return (P(x,y) - p).dist2();
ca4     }

d97     Node(vector<P>&& vp) : pt(vp[0]) {
741         for (P p : vp) {
ad3             x0 = min(x0, p.x); x1 = max(x1, p.x);
e5d             y0 = min(y0, p.y); y1 = max(y1, p.y);
310         }
994         if (vp.size() > 1) {
// split on x if width >= height (not ideal...)
9b7         sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
// divide by taking half the array for each child (
not
// best performance with many duplicates in the
middle)
0f9         int half = sz(vp)/2;
48e         first = new Node({vp.begin(), vp.begin() + half});
902         second = new Node({vp.begin() + half, vp.end()});
66e     }
204     }
a77 };

dad struct KDTree {
95f     Node* root;
c30     KDTree(const vector<P>& vp) : root(new Node({all(vp)}))
{}

113     pair<T, P> search(Node *node, const P& p) {
ec4         if (!node->first) {
// uncomment if we should not find the point itself:
// if (p == node->pt) return {INF, P()};
47e         return make_pair((p - node->pt).dist2(), node->pt);
119     }

ea4     Node *f = node->first, *s = node->second;
d40     T bfirst = f->distance(p), bsec = s->distance(p);
a16     if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

// search closest side first, other side if needed
86c     auto best = search(f, p);
314     if (bsec < best.first)
509         best = min(best, search(s, p));
f26     return best;
74c }

// find nearest point to a point, and its squared
distance
// (requires an arbitrary operator< for Point)
9b6     pair<T, P> nearest(const P& p) {
195         return search(root, p);
94c     }
6f5 };

```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.

Time: $O(n \log n)$

```

"Point.h" eefdf5, 89 lines
2c0 typedef Point<ll> P;
806 typedef struct Quad* Q;
449 typedef __int128_t lll; // (can be ll if coords are < 2e4)
59b P arb(LLONG_MAX, LLONG_MAX); // not equal to any other
point

070 struct Quad {
461     Q rot, o; P p = arb; bool mark;
b38     P& F() { return r()->p; }
23a     Q& r() { return rot->rot; }
f4f     Q prev() { return rot->o->rot; }
57e     Q next() { return r()->prev(); }
180 } *H;

d15 bool circ(P p, P a, P b, P c) { // is p in the
circumcircle?
4b4     lll p2 = p.dist2(), A = a.dist2()-p2,
ffa     B = b.dist2()-p2, C = c.dist2()-p2;
59a     return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B >
0;
6af }
00a Q makeEdge(P orig, P dest) {
bdf     Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
516     H = r->o; r->r()->r() = r;
2c3     rep(i, 0, 4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->
r();
ed2     r->p = orig; r->F() = dest;
4c1     return r;
b3b }
d8d void splice(Q a, Q b) {
686     swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
86c }
e92 Q connect(Q a, Q b) {
fc2     Q q = makeEdge(a->F(), b->p);
6e6     splice(q, a->next());
642     splice(q->r(), b);
bef     return q;
4a4 }

196 pair<Q, Q> rec(const vector<P>& s) {
e63     if (sz(s) <= 3) {
4a0         Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back()
);
2ba         if (sz(s) == 2) return { a, a->r() };
19e         splice(a->r(), b);
5f8         auto side = s[0].cross(s[1], s[2]);
b9f         Q c = side ? connect(b, a) : 0;
3d8         return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
c9e     }

5ef #define H(e) e->F(), e->p
c98 #define valid(e) (e->F().cross(H(base)) > 0)
a3e     Q A, B, ra, rb;
f5e     int half = sz(s) / 2;
391     tie(ra, A) = rec({all(s) - half});
d9b     tie(B, rb) = rec({sz(s) - half + all(s)});
f80     while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
b08         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
76d     Q base = connect(B->r(), A);
87f     if (A->p == ra->p) ra = base->r();
b58     if (B->p == rb->p) rb = base;

```

```
3e6 #define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
f02 while (circ(e->dir->F(), H(base), e->F())) { \
936 Q t = e->dir; \
6d3 splice(e, e->prev()); \
16e splice(e->r(), e->r()->prev()); \
d47 e->o = H; H = e; e = t; \
a2e }
1de for (;;) {
eaa DEL(LC, base->r(), o); DEL(RC, base, prev());
6fa if (!valid(LC) && !valid(RC)) break;
e09 if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
b74 base = connect(RC, base->r());
295 else
271 base = connect(base->r(), LC->r());
fcf }
345 return { ra, rb };
7cf }
```

```
dal vector<P> triangulate(vector<P> pts) {
af6 sort(all(pts)); assert(unique(all(pts)) == pts.end());
e00 if (sz(pts) < 2) return {};
235 Q e = rec(pts).first;
50c vector<Q> q = {e};
6c1 int qi = 0;
7a5 while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
806 #define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->
p); \
43e q.push_back(c->r()); c = c->next(); } while (c != e); }
9d6 ADD; pts.clear();
b58 while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
a42 return pts;
a02 }
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 7 lines

```
f9c template<class V, class L>
cb3 double signedPolyVolume(const V& p, const L& trilst) {
9e8 double v = 0;
b72 for (auto i : trilst) v += p[i.a].cross(p[i.b]).dot(p[i.
c]);
fb8 return v / 6;
fca }
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 33 lines

```
f10 template<class T> struct Point3D {
f07 typedef Point3D P;
d0e typedef const P& R;
329 T x, y, z;
cf2 explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z)
{}
803 bool operator<(R p) const {
8ee return tie(x, y, z) < tie(p.x, p.y, p.z); }
236 bool operator==(R p) const {
bd6 return tie(x, y, z) == tie(p.x, p.y, p.z); }
9ae P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
54a P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
743 P operator*(T d) const { return P(x*d, y*d, z*d); }
17b P operator/(T d) const { return P(x/d, y/d, z/d); }
e49 T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
270 P cross(R p) const {
923 return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
a77 }
```

```
b70 T dist2() const { return x*x + y*y + z*z; }
18b double dist() const { return sqrt((double)dist2()); }
//Azimuthal angle (longitude) to x-axis in interval [-pi,
pi]
3d6 double phi() const { return atan2(y, x); }
//Zenith angle (latitude) to the z-axis in interval [0,
pi]
0fa double theta() const { return atan2(sqrt(x*x+y*y),z); }
55e P unit() const { return *this/(T)dist(); } //makes dist()
=1
//returns unit vector normal to *this and p
P normal(P p) const { return cross(p).unit(); }
//returns point rotated 'angle' radians ccw around axis
c67 P rotate(double angle, P axis) const {
7cd double s = sin(angle), c = cos(angle); P u = axis.unit
();
6b7 return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
73a }
805 };
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.

Time: $\mathcal{O}(n^2)$

"Point3D.h"5b45fc, 50 lines

```
b8e typedef Point3D<double> P3;

9ce struct PR {
1fc void ins(int x) { (a == -1 ? a : b) = x; }
82f void rem(int x) { (a == x ? a : b) = -1; }
2ad int cnt() { return (a != -1) + (b != -1); }
ba2 int a, b;
cf7 };
```

```
5e4 struct F { P3 q; int a, b, c; };
```

```
b6d vector<F> hull3d(const vector<P3>& A) {
cd9 assert(sz(A) >= 4);
ec1 vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
394 #define E(x,y) E[f.x][f.y]
afe vector<F> FS;
9e0 auto mf = [&](int i, int j, int k, int l) {
2ce P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
fa1 if (q.dot(A[l]) > q.dot(A[i]))
eaa q = q * -1;
f22 F f{q, i, j, k};
ee5 E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
471 FS.push_back(f);
d73 };
30c rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
047 mf(i, j, k, 6 - i - j - k);
```

```
3ef rep(i,4,sz(A)) {
3b5 rep(j,0,sz(FS)) {
068 F f = FS[j];
04f if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
412 E(a,b).rem(f.c);
b61 E(a,c).rem(f.b);
e5c E(b,c).rem(f.a);
8d5 swap(FS[j--], FS.back());
eef FS.pop_back();
5cd }
220 }
97f int nw = sz(FS);
c63 rep(j,0,nw) {
068 F f = FS[j];
561 #define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i,
f.c);
```

```
3da C(a, b, c); C(a, c, b); C(b, c, a);
248 }
472 }
864 for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
770 A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
311 return FS;
be2 };
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f07, 9 lines

```
c5f double sphericalDistance(double f1, double t1,
3e8 double f2, double t2, double radius) {
284 double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
277 double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
c7e double dz = cos(t2) - cos(t1);
c09 double d = sqrt(dx*dx + dy*dy + dz*dz);
154 return radius*2*asin(d/2);
4fa }
```

Strings (9)

AhoCorasick.h

95b3e7, 46 lines

```
c2e int trie[ms][sigma], fail[ms], terminal[ms], superfail[ms
];
1e1 bool present[ms];
965 int z = 1;
```

```
ca3 int val(char c) { return c - 'a'; }
```

```
f97 void add(string& p) {
b3d int cur = 0;
b4b for (int i = 0; i < (int)p.size(); i++) {
9e4 int& nxt = trie[cur][val(p[i])];
b6e if (nxt == 0) nxt = z++;
1bc cur = nxt;
a92 }
c0e present[cur] = true;
b07 terminal[cur]++;
6aa }
```

```
0a8 void build() {
26a queue<int> q;
f47 for (q.push(0); !q.empty(); q.pop()) {
fb5 int on = q.front();
0b2 for (int i = 0; i < sigma; i++) {
df1 int& to = trie[on][i];
279 int f = (on == 0 ? 0 : trie[fail[on]][i]);
de7 int sf = (present[f] ? f : superfail[f]);
24d if (!to) {
c4e to = f;
6fd }
4e6 else {
3ef fail[to] = f;
b86 superfail[to] = sf;
// merge infos (ex: terminal[to] +=
terminal[f])
q.push(to);
91b }
692 }
bff }
```

```
e61      }
b89    }

54e void search(string& s) {
b3d   int cur = 0;
b4f   for (char c : s) {
3ba       cur = trie[cur][val(c)];
           // process infos on current node (ex: ocurrences
           += terminal[cur])

5ac     }
d1b  }
```

Hash.h

Description: C can also be random, operator is $[l, r]$

79e7f5, 28 lines

```
541 using ull = uint64_t;
54d struct H {
858     ull x; H(ull x = 0) : x(x) {}
c9b     H operator+(H o) { return x + o.x + (x + o.x < x); }
5cd     H operator-(H o) { return *this + ~o.x; }
167     H operator*(H o) {
2f3         auto m = (__uint128_t)x * o.x;
540         return H((ull)m) + (ull)(m >> 64);
681     }
bf2     ull get() const { return x + !~x; }
03c     bool operator==(H o) const{ return get() == o.get();}
0ab     bool operator<(H o) const{ return get() < o.get();}
bf6 };
862 static const H C = (1l)1e11 + 3;
61c struct Hash {
2f2     vector<H> h, pw;
1df     Hash(string& str) : h(str.size()), pw(str.size()) {
9bc         pw[0] = 1, h[0] = str[0];
1c5         for (int i = 1; i < str.size(); i++) {
90a             h[i] = h[i - 1] * C + str[i];
b3c             pw[i] = pw[i - 1] * C;
57e         }
flb     }
75e     H operator() (int l, int r) {
91f         return h[r] - (l ? h[l - 1] * pw[r - l + 1] : 0);
9cf     }
c36 };
```

KMP.h

Description: $pi[x]$ computes the length of the longest prefix of s that ends at x , other than $s[0...x]$ itself (abacaba -> 0010123).

c7cf15, 10 lines

```
a56 vector<int> pi(const string& s) {
627     vector<int> p(sz(s));
edb     for(int i = 1; i < sz(s); i++) {
052         int g = p[i-1];
6c0         while (g && s[i] != s[g]) g = p[g-1];
7cf         p[i] = g + (s[i] == s[g]);
a2e     }
74e     return p;
c7c }
```

KmpAutomaton.h

Description: $go[i][j]$ = length of the longest prefix of s that is a suffix of $s[0..i]$ followed by the letter j (i.e., the next matched prefix length if, at state i , we read letter j).

8833cb, 17 lines

```
ab6 int go[ms][sigma];
ca3 int val(char c) { return c - 'a'; }
8cf void automaton(string& s) {
3cc     for (int i = 0; i < sigma; i++)
48d         go[0][i] = (i == val(s[0]));

8cc     for (int i = 1, bdr = 0; i <= (int)s.size(); i++) {
```

```
782     for (int j = 0; j < sigma; j++) {
6ef         go[i][j] = go[bdr][j];
87c     }
f8d     if (i < (int)s.size()) {
02f         go[i][val(s[i])] = i + 1;
364         bdr = go[bdr][val(s[i])];
63b     }
d7e     }
0c5 }
```

Manacher.h

Description: $p[0][i + 1]$ is the length of matches of even length palindrome, starting from $[i, i + 1]$. $p[1][i]$ is the length of matches of odd length palindrome, starting from $[i, i]$. (abaxx -> $p[0] = 00001$) (abaxx -> $p[1] = 01000$)

e7ad79, 14 lines

```
fc1 array<vi, 2> manacher(const string& s) {
f89     int n = sz(s);
f77     array<vi, 2> p = {vi(n+1), vi(n)};
c9a     rep(z, 0, 2) for (int i=0, l=0, r=0; i<n; i++) {
24e         int t = r-i+!z;
e70         if (i<r) p[z][i] = min(t, p[z][l+t]);
fff         int L = i-p[z][i], R = i+p[z][i]-!z;
649         while (L>=1 && R+1<n && s[L-1] == s[R+1])
895             p[z][i]++, L--, R++;
f28         if (R>r) l=L, r=R;
a84     }
74e     return p;
e7a }
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.

Usage: rotate(s.begin(), s.begin()+minRotation(s), s.end());

Time: $\mathcal{O}(N)$

d07a42, 10 lines

```
5fa int minRotation(string s) {
c6c     int a=0, N=sz(s); s += s;
840     rep(b, 0, N) rep(k, 0, N) {
32f         if (a+k == b || s[a+k] < s[b+k]) {
873             b += max(0, k-1); break;}
068         if (s[a+k] > s[b+k]) { a = b; break; }
937     }
3f5     return a;
d07 }
```

SuffixArray.h

Description: $lcp[i]$ is the length of the longest common prefix between the suffixes $s[sa[i]..n - 1]$ and $s[sa[i - 1]..n - 1]$. If we concatenate multiple strings using separator characters, the separator that appears furthest to the right must be the smallest character in the alphabet.

048424, 31 lines

```
3f4 struct SuffixArray {
716     vector<int> sa, lcp;
d91     SuffixArray(string s, int lim=256) {
59b         s.push_back('$');
323         int n = sz(s), k = 0, a, b;
9f1         vector<int> x(all(s)), y(n), ws(max(n, lim));
af4         sa = lcp = y, iota(all(sa), 0);
25d         for(int j = 0, p = 0; p < n; j= max(1, j*2), lim = p) {
3cd             p = j, iota(all(y), n - j);
603             for(int i=0; i<n; i++){
071                 if (sa[i] >= j) y[p++] = sa[i] - j;
cb4             }
911             fill(all(ws), 0);
483             for(int i=0; i<n; i++) ws[x[i]]++;
5d9             for(int i=1; i<lim; i++) ws[i] += ws[i - 1];
a9e             for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
```

```
c7d     swap(x, y), p = 1, x[sa[0]] = 0;
6f5     for(int i=1; i<n; i++){
93f         a = sa[i - 1], b = sa[i];
ddb         x[b] = p-1;
a32         if (y[a] != y[b] || y[a+j] != y[b+j]) x[b] = p++;
1ba     }
c36 }
65b     for (int i = 0, j; i < n - 1; lcp[x[i++]] = k)
904         for (k && k--, j = sa[x[i] - 1];
262             s[i + k] == s[j + k]; k++);
68a     sa = vector<int>(sa.begin() + 1, sa.end());
5d4     lcp = vector<int>(lcp.begin() + 1, lcp.end());
4db }
048 };
```

Zfunc.h

Description: $z[i]$ computes the length of the longest common prefix of $s[i:]$ and s , except $z[0] = 0$. (abacaba -> 0010301)

495392, 13 lines

```
572 vector<int> ZFunc(const string& s) {
d6b     int n = sz(s), a = 0, b = 0;
2b1     vector<int> z(n, 0);
29a     if (!z.empty()) z[0] = 0;
6f5     for (int i = 1; i < n; i++) {
fe0         int end = i;
98f         if (i < b) end = min(i + z[i - a], b);
65f         while (end < n && s[end] == s[end - i]) ++end;
816         z[i] = end - i; if (end > b) a = i, b = end;
253     }
070     return z;
495 }
```

Various (10)

10.1 Misc. algorithms

Dates.h

Description: dateToInt converts Gregorian date to integer (Julian day number). intToDate converts integer (Julian day number) to Gregorian date: month/day/year. intToDay converts Julian day number to day of the week.

688e36, 23 lines

```
37c string day[] = { "Mon", "Tue", "Wed", "Thu", "Fri", "Sat",
                    "Sun" };
fb9 int dateToInt(int m, int d, int y) {
e70     return
773         1461 * (y + 4800 + (m - 14) / 12) / 4 +
649         367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
fa0         3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
3aa         d - 32075;
a73 }
3fe void intToDate(int jd, int& m, int& d, int& y) {
ee1     int x, n, i, j;
33a     x = jd + 68569;
403     n = 4 * x / 146097;
33e     x -= (146097 * n + 3) / 4;
6fc     i = (4000 * (x + 1)) / 1461001;
b1d     x -= 1461 * i / 4 - 31;
fc9     j = 80 * x / 2447;
c8d     d = x - 2447 * j / 80;
179     x = j / 11;
335     m = j + 2 - 12 * x;
23d     y = 100 * (n - 49) + i + x;
cbb }
04e string intToDay(int jd) { return day[jd % 7]; }
```

MultisetHash.h

cdc ull hashify(ull sum) {

```
7b8      sum += FIXED_RANDOM;
6ec      sum += 0x9e3779b97f4a7c15;
dc6      sum = (sum ^ (sum >> 30)) * 0xbf58476d1ce4e5b9;
005      sum = (sum ^ (sum >> 27)) * 0x94d049bb133111eb;
358      return sum ^ (sum >> 31);
564  }
```

Rand.h

2de3f8, 8 lines

```
c8a  mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
    // _64

463 int uniform(int l, int r) { // [l, r]
a7f      uniform_int_distribution<int> uid(l, r);
f54      return uid(rng);
d9e  }
```

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive). Time: O(log N)

edce47, 24 lines

```
d91  set<pii>::iterator addInterval(set<pii>& is, int L, int R)
{
bb3      if (L == R) return is.end();
d4c      auto it = is.lower_bound({L, R}), before = it;
dc6      while (it != is.end() && it->first <= R) {
164          R = max(R, it->second);
1a5          before = it = is.erase(it);
fe9      }
1af      if (it != is.begin() && (--it)->second >= L) {
3ca          L = min(L, it->first);
164          R = max(R, it->second);
861          is.erase(it);
0de      }
aa0      return is.insert(before, {L,R});
d57  }

675 void removeInterval(set<pii>& is, int L, int R) {
17b      if (L == R) return;
bef      auto it = addInterval(is, L, R);
e14      auto r2 = it->second;
655      if (it->first == L) is.erase(it);
016      else (int&)it->second = L;
ee9      if (R != r2) is.emplace(R, r2);
059  }
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add || R.empty(). Returns empty set on failure (or if G is empty). Time: O(N log N)

9e9d8d, 20 lines

```
4fc  template<class T>
dbe  vi cover(pair<T, T> G, vector<pair<T, T>> I) {
3d5      vi S(sz(I)), R;
d00      iota(all(S), 0);
591      sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
d10      T cur = G.first;
05e      int at = 0;
336      while (cur < G.second) { // (A)
438          pair<T, int> mx = make_pair(cur, -1);
f07          while (at < sz(I) && I[S[at]].first <= cur) {
032              mx = max(mx, make_pair(I[S[at]].second, S[at]));
69a              at++;
c42          }
```

```
c54      if (mx.second == -1) return {};
953      cur = mx.first;
fbf      R.push_back(mx.second);
dd1      }
bla      return R;
b8d      }
```

TernarySearch.h

Description: Find the smallest i in [a,b] that maximizes f(i), assuming that f(a) < ... < f(i) ≥ ... ≥ f(b). To reverse which of the sides allows non-strict inequalities, change the < marked with (A) to <=, and reverse the loop at (B). To minimize f, change it to >, also at (B). Usage: int ind = ternSearch(0,n-1); Time: O(log(b-a))

a995fb, 11 lines

```
53a  int ternSearch(int a, int b) {
25b      assert(a <= b);
329      while (b - a >= 5) {
924          int mid = (a + b) / 2;
c9e          if (f(mid) < f(mid+1)) a = mid; // (A)
ceb          else b = mid+1;
ce7      }
95e      rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
3f5      return a;
a99  }
```

10.2 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: dp[i][j] = min_{i<k<j} (dp[i][k] + dp[k][j]) + f(i, j), where the (minimal) optimal k increases with both i and j. This is known as Knuth DP. Sufficient criteria for this are if f(b, c) ≤ f(a, d) and f(a, c) + f(b, d) ≤ f(a, d) + f(b, c) for all a ≤ b ≤ c ≤ d. Another sufficient criteria is: opt[i][j-1] ≤ opt[i][j] ≤ opt[i+1][j] Time: O(N^2)

fea016, 22 lines

```
7cc  ll knuth(){
6a7      memset(opt, -1, sizeof opt);
45b      for(int i=n-1; i>=0; i--){
8e7          dp[i][i] = 0; // base case
b28          opt[i][i] = i;
94f          for(int j=i+1; j<n; j++){
2e2              int optL = (!j ? 0 : opt[i][j-1]);
dc4              int optR = (~opt[i+1][j] ? opt[i+1][j] : n-1);
554              ll cst = cost(i, j);
f12              dp[i][j] = INF;
3bb              optL = max(i, optL), optR = min(j-1, optR);
349              for(int k=optL; k<=optR; k++){
f8b                  ll now = dp[i][k] + dp[k+1][j] + cst;
e83                  if(now <= dp[i][j]){
960                      dp[i][j] = now;
14d                      opt[i][j] = k;
5fc                  }
114              }
4ce          }
96c      }
fea  }
```

DivideAndConquerDP.h

Description: Divide and Conquer DP maintaining cost, can be used when opt[i][j] ≤ opt[i][j+1]. In this code everything is 1-based. Memory can be optimized by keeping only the last row Time: O(MN log N)

c7cb38, 42 lines

```
129 void add(int idx) {}
404 void rem(int idx) {}

749 void deC(int i, int l, int r, int optL, int optR) {
de6     if (l > r) return;
995     int j = (l + r) / 2;
```

```
d9a      for (int k = r; k > j; k--) rem(k);
c45      int opt = optL;
364      for (int k = optL; k <= min(optR, j); k++) {
    // cost = cost[k, j]
597          int val = dp[i-1][k-1] + cost;
532          if (val < dp[i][j]) {
482              dp[i][j] = val;
613              opt = k;
178          }
183          rem(k);
93f      }
5d9      for (int k = min(optR, j); k >= optL; k--) add(k);
446      rem(j);
ace      deC(i, l, j-1, optL, opt);

ebd      for (int k = j; k <= r; k++) add(k);
648      for (int k = optL; k < opt; k++) rem(k);
0b6      deC(i, j+1, r, opt, optR);

9bb      for (int k = optL; k < opt; k++) add(k);
460  }

d57 int solve(int N, int M) { // 1-based
d9f     for (int i = 0; i <= M; i++) {
138         for (int j = 0; j <= N; j++){
3db             dp[i][j] = inf; // base case
a26         }
e0f     }
c21     cost = 0; // neutral value
c62     for (int i = 1; i <= N; i++) add(i);
143     for (int i = 1; i <= M; i++) {
156         deC(i, 1, N, 1, N);
c97     }
01a     return dp[M][N];
3ab }
```