Universidade Federal de Pernambuco

las4s e pelados

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Contest (1)

template.cpp
9 lines

```cpp
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
using ll = long long;
using pii = pair<int,int>;
using vi = vector<int>;
```

.bashrc
2 lines

```bash
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \
  -fsanitize=undefined,address'
```

hash.sh
2 lines

```bash
# bash hash.sh file.cpp l1 l2
sed -n $2','$3' p' $1 | sed '/^#w/d' | cpp -dD -P -
    fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

stressTest.sh
20 lines

```bash
P=code  #mude pro filename do codigo
Q=brute  #mude pro filename do brute [correto]
g++ ${P}.cpp -o sol -O2 || exit 1
g++ ${Q}.cpp -o brt -O2 || exit 1
g++ gen.cpp  -o gen -O2 || exit 1
for ((i = 1; ; i++)) do
  echo $i
  ./gen $i > in
  ./sol < in > out
  ./brt < in > out2
  if (! cmp -s out out2) then
    echo "--> entrada:"
    cat in
    echo "--> saida code:"
    cat out
    echo "--> saida brute:"
    cat out2
    break;
  fi
done
```

paperStress.py
26 lines

```python
927 import random
a1a import subprocess
5c9 MAX_N = 100
b5d def gen_case() -> str:
c7e     return f"1\n"

94a random.seed((1 << 9) | 31)

a22 for i in range(100):
d19     print(), print()
a3f     case = gen_case()
266     print(f"Test #{i+1}: ")
ce5     print(case)
d41     # test bruteforce
f60     bf = subprocess.run(['out/b'], input=case, encoding='
    ascii', capture_output=True)
d41     # test solution
37c     sol = subprocess.run(['out/m'], input=case, encoding='
    ascii', capture_output=True)
d55     bf_res = bf.stdout
af9     sol_res = sol.stdout
6b6     print(f"bruteforce {bf_res}, solution {sol_res}")
508     if bf_res == sol_res:
dd4         print("accepted")
f68     else:
ef2         print("WA")
1cb         break
```

troubleshoot.txt
52 lines

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
```

Theoretical (2)

2.1 Mathematics

2.1.1 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \cdots - c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.1.2 Trigonometry

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a\cos x + b \sin x = r\cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2+b^2}, \phi = \operatorname{atan2}(b, a)$.

2.1.3 Geometry

Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1 - \left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\frac{a+b}{a-b} = \frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

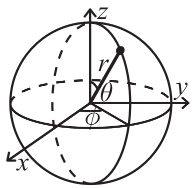
Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

Spherical coordinates



$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \mathrm{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z &= r\cos\theta & \phi &= \mathrm{atan2}(y, x) \end{aligned}$$

Pick's Theorem

The area of a simple polygon whose vertices have integer coordinates is:

$$A = I + \frac{B}{2} - 1$$

where I is the number of interior integer points, and B is the number of integer points in the border of the polygon.

Two Ears Theorem

Every simple polygon with more than 3 vertices has at least two non-overlapping ears (a ear is a vertex whose diagonal induced by its neighbors which lies strictly inside the polygon). Equivalently, every simple polygon can be triangulated.

2.1.4 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\mathrm{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.1.5 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, \ c \neq 1$$

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$\sum_{i=0}^{n} ic^i = \frac{nc^{n+2} - (n+1)c^{n+1} + c}{(c-1)^2}, \ c \neq 1$$

$$g_k(n) = \sum_{i=1}^{n} i^k = \frac{1}{k+1}\left(n^{k+1} + \sum_{j=1}^{k}\binom{k+1}{j+1}(-1)^{j+1}g_{k-j}(n)\right)$$

2.1.6 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, \ (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, \ (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, \ (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, \ (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, \ (-\infty < x < \infty)$$

$$\sum_{i=0}^{\infty} ic^i = \frac{c}{(1-c)^2}, \quad |c| < 1$$

$$(1+x)^n = \sum_{i=0}^{n}\binom{n}{i}x^i$$

$$\frac{1}{1-x} = \sum_{i=0}^{\infty} x^i, \ (-1 < x < 1)$$

$$\frac{1}{(1-x)^n} = \sum_{i=0}^{\infty}\binom{n+i-1}{n-1}x^i, \ (-1 < x < 1)$$

2.1.7 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n, p)$, $n = 1, 2, \ldots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k}p^k(1-p)^{n-k}$$

$$\mu = np, \ \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n, p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\mathrm{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1}, \ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p}, \ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda, \sigma^2 = \lambda$$

2.2 Combinatorial

2.2.1 Binomial Identities

$$\binom{n-1}{k} - \binom{n-1}{k-1} = \frac{n-2k}{k}\binom{n}{k} \qquad \binom{n}{h}\binom{n-h}{k} = \binom{n}{k}\binom{n-k}{h}$$

$$\textstyle\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1} \qquad \sum_{k=0}^{n} k^2\binom{n}{k} = (n+n^2)2^{n-2}$$

$$\textstyle\sum_{j=0}^{k}\binom{m}{j}\binom{n-m}{k-j} = \binom{n}{k} \qquad \sum_{j=0}^{m}\binom{m}{j}^2 = \binom{2m}{m}$$

$$\textstyle\sum_{m=0}^{n}\binom{m}{j}\binom{n-m}{k-j} = \binom{n+1}{k+1} \qquad \sum_{m=0}^{n}\binom{m}{k} = \binom{n+1}{k+1}$$

$$\textstyle\sum_{r=0}^{m}\binom{n+r}{r} = \binom{n+m+1}{m} \qquad \sum_{k=0}^{n}\binom{n-k}{k} = \mathrm{Fib}(n+1)$$

$$\textstyle\sum_{k=0}^{n}\binom{r}{k}\binom{s}{n-k} = \binom{r+s}{n}$$

2.2.2 Permutations

Factorial

n	1 2 3	4	5	6	7	8	9	10
$n!$	1 2 6	24	120	720	5040	40320	362880	3628800

n	11	12	13	14	15	16	17
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14

n	20	25	30	40	50	100	150	171
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX

Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor\frac{n!}{e}\right\rceil$$

Burnside's lemma

Counts the number of distinct colorings of an object under symmetry.

$$\frac{1}{|G|}\sum_{g\in G} k^{\mathrm{cyc}(g)},$$

where G is the symmetry group, k the number of colors, and $\mathrm{cyc}(g)$ the number of cycles induced by g.

Example: number of ways to color a necklace with n beads using k colors (rotations only):

$$g(n) = \frac{1}{n}\sum_{i=0}^{n-1} k^{(\gcd(n,i))}$$

where rotation i shifts the necklace by i positions.

2.2.3 Partitions and subsets

Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}}(-1)^{k+1}p(n-k(3k-1)/2)$$

$$p(n) \sim 0.145/n\cdot\exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	∼2e5	∼2e8

Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_kp^k + ... + n_1p + n_0$ and $m = m_kp^k + ... + m_1p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k}\binom{n_i}{m_i} \pmod p$.

2.2.4 Sum of Binomials (FFT)

Goal: Given freq. array C, compute $Ans[k] = \sum_i C[i]\binom{i}{k}$ for all k. Rewrite: $Ans[k] = \frac{1}{k!}\sum_i(C[i]\cdot i!)\frac{1}{(i-k)!}$.

- Construct P where $P[i] = C[i]\cdot i!$
- Construct Q where $Q[i] = (i!)^{-1}$
- Reverse Q (to handle the $i-k$ subtraction).
- Multiply $R = NTT(P, Q)$.
- Result: $Ans[k] = R[k+|Q|-1]\cdot\frac{1}{k!}$.

2.2.5 General purpose numbers

Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able). $B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k\cdot(n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m+n) \equiv mB(n) + B(n+1) \pmod p$$

Labeled unrooted trees

- on n vertices: n^{n-2}
- on k existing trees of size n_i: $n_1n_2\cdots n_kn^{k-2}$
- with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

2.3 Number Theory

2.3.1 Bézout's identity

For $a \neq$, $b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

2.3.2 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

2.3.3 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 6700 for $n < 1e12$, 200 000 for $n < 1e19$.

2.3.4 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

2.3.5 Theorems

Goldbach's conjecture: Every even integer $n > 2$ can be written as $n = a + b$ with a, b prime.

Legendre's conjecture: There is always at least one prime between n^2 and $(n+1)^2$.

Lagrange's four-square theorem: Every positive integer can be written as

$$n = a^2 + b^2 + c^2 + d^2.$$

Zeckendorf's theorem: Every integer $n \geq 1$ has a unique representation as a sum of non-consecutive Fibonacci numbers:

$$n = F_{i_1} + F_{i_2} + \cdots + F_{i_k}, \quad i_j - i_{j+1} \geq 2.$$

Euclid's formula (primitive Pythagorean triples): The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2),\ \ b = k \cdot (2mn),\ \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

Wilson's theorem: n is prime iff

$$(n-1)! \equiv -1 \pmod{n}.$$

Chicken McNugget theorem: For coprime n, m, the largest integer not representable as $an + bm$ (with $a, b \geq 0$) is

$$nm - n - m.$$

There are $\frac{(n-1)(m-1)}{2}$ non-representable integers, and for each pair $(k,\ nm - n - m - k)$ exactly one is representable.

2.4 Graphs

2.4.1 Flows and Matching

Hall's Theorem

In bipartite graphs, there exists a perfect matching covering the entire side X if and only if for every subset $Y \subseteq X$,

$$|Y| \leq |N(Y)|,$$

where $N(Y)$ denotes the set of neighbors of Y.

Kőnig's Theorem

In a bipartite graph, the size of a Minimum Vertex Cover is equal to the size of a Maximum Matching. A Minimum Vertex Cover is a minimum set of vertices such that every edge of the graph has at least one endpoint in the set.

As a consequence,

$$n - \text{Maximum Matching} = \text{Maximum Independent Set},$$

where a Maximum Independent Set is the largest set of vertices with no edges between them.

Recovering the Minimum Vertex Cover Given a maximum matching in a bipartite graph (X, Y):

- Construct the residual graph by orienting:
  - non-matching edges from X to Y;
  - matching edges from Y to X.
- Perform a BFS or DFS starting from all free (unmatched) vertices in X.
- Let Z_X be the set of reachable vertices in X, and Z_Y the set of reachable vertices in Y.

The Minimum Vertex Cover is given by:

$$(X \setminus Z_X) \cup Z_Y.$$

Node-Disjoint Path Cover

A node-disjoint path cover is a set of paths such that each vertex belongs to exactly one path.

In a directed acyclic graph (DAG),

$$\text{Minimum Node-Disjoint Path Cover} = n - \text{Maximum Matching}.$$

The construction is as follows: for each vertex u, create a copy u'. Add an edge $u \to v'$ if there exists an edge $u \to v$ in the original graph.

Recovering the Paths

- Vertices that do not appear as destinations in the matching are starting points of paths.
- Each matching edge $u \to v'$ corresponds to an edge $u \to v$ in the original DAG.
- Following these edges reconstructs all paths of the path cover.

General Path Cover

A general path cover is a path cover where a vertex may belong to more than one path.

In a DAG, the construction is similar to the node-disjoint case, but an edge $u \to v'$ exists if there is a path from u to v in the original graph.

Recovering the Cover The vertices can be grouped according to the edges used in the matching to form the path cover.

Dilworth's Theorem

An antichain is a set of vertices such that there is no path between any pair of vertices in the set.

In a directed acyclic graph,

$$\text{Minimum General Path Cover} = \text{Maximum Antichain}.$$

Recovering a Maximum Antichain Given a minimum general path cover, selecting one vertex from each path produces a maximum antichain.

2.4.2 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--`, `mat[b][b]++` (and `mat[b][a]--`, `mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

2.4.3 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

2.4.4 Planar Graphs

If G has k connected components, then $n - m + f = k + 1$.

2.5 Optimization tricks

2.5.1 Bit hacks

- `for (int x = m; x; x = (x - 1)&m ) { ... }` loops over all subset masks of `m` (except 0).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
  `if (i & 1 << b) D[i] += D[i^(1 << b)];`
  computes all sums of subsets.

2.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC target("bmi,bmi2,popcnt,lzcnt")` improve bit operations.
- `#pragma GCC optimize("unroll-loops")` self explanatory.

2.6 Various

2.6.1 Master Theorem (Simple)

$T(n) = aT(n/b) + O(n^d)$. Compare a vs b^d:

- $a > b^d \implies O(n^{\log_b a})$ (Work at leaves dominates)
- $a = b^d \implies O(n^d \log n)$ (Work is uniform)
- $a < b^d \implies O(n^d)$ (Work at root dominates)

Data structures (3)

Bit.h

Description: *lower_bound* works the same as on vectors
Time: $\mathcal{O}(\log N)$

5891da, 23 lines

```
8eb struct Bit {
406   vector<ll> bit;
1dd   Bit(int n) : bit(n + 1) {}
265   void update(int i, ll v) {
c38     for (i++; i < sz(bit); i += i & -i) bit[i] += v;
f21   }
74a   ll query(int i) {
b73     ll ret = 0;
71c     for (i++; i > 0; i -= i & -i) ret += bit[i];
edf     return ret;
e40   }
dc8   int lower_bound(ll v){ // min pos st sum[0, pos] >= v
bec     int pos = 0;
a40     for(int j=(1 << 23); j >= 1; j/=2){
3b1       if(pos+j < sz(bit) && bit[pos + j] < v){
b4e         pos += j;
18d         v -= bit[pos];
f6c       }
156     }
d75     return pos;
37b   }
589 };
```

Bit2d.h

Description: Points called on the update function NEED to be on the *pts* vector parameter on build.
Time: $\mathcal{O}\left((\log N)^2\right)$

"Bit.h" 5a98ac, 37 lines

```
9c0 struct Bit2d {
a37   vector<vector<int>> ys;
fe8   vector<Bit> bit;
543   vector<int> cmp_x;
425   Bit2d(){}
521   void put(int x, int y) {
005     for (x++; x < sz(ys); x += x & -x) ys[x].push_back(y);
f3c   }
ce0   int id(const vector<int> &v, int y) {
1e9     return (upper_bound(all(v), y) - v.begin()) - 1;
19a   }
7ff   void build(vector<pii> pts) {
3cb     sort(all(pts));
f99     for(auto p : pts) cmp_x.push_back(p.first);
9a7     cmp_x.erase(unique(all(cmp_x)), cmp_x.end());
f82     ys.resize(cmp_x.size() + 1);
94d     for(auto p : pts) put(id(cmp_x, p.first), p.second);
310     for(auto &v:ys)sort(all(v)), bit.emplace_back(sz(v));
a01   }
767   void update(int x, int y, int val){
f3f     x = id(cmp_x, x);
681     for(x++; x < sz(ys); x+= x&-x)
507       bit[x].update(id(ys[x], y), val);
c88   }
d95   int query(int x, int y){
f3f     x = id(cmp_x, x);
7c9     int ret = 0;
f32     for(x++; x > 0; x-= x&-x)
ea8       ret += bit[x].query(id(ys[x], y));
edf     return ret;
8f7   }
251   int query(int x1, int y1, int x2, int y2){
e4d     int a = query(x2, y2)-query(x2, y1-1);
7d1     return a-query(x1-1, y2)+query(x1-1, y1-1);
c33   }
5a9 };
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$

8ec1c7, 32 lines

```
72c struct Line {
3e2   mutable ll k, m, p;
ca5   bool operator<(const Line& o) const { return k < o.k; }
abf   bool operator<(ll x) const { return p < x; }
7e3 };

781 struct LineContainer : multiset<Line, less<>> {
      // (for doubles, use inf = 1/.0, div(a,b) = a/b)
fd2   static const ll inf = LLONG_MAX;
33a   ll div(ll a, ll b) { // floored division
10f     return a / b - ((a ^ b) < 0 && a % b); }
a1c   bool isect(iterator x, iterator y) {
a95     if (y == end()) return x->p = inf, 0;
9cb     if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
591     else x->p = div(y->m - x->m, x->k - y->k);
870     return x->p >= y->p;
2fa   }
a0c   void add(ll k, ll m) {
116     auto z = insert({k, m, 0}), y = z++, x = y;
7b1     while (isect(y, z)) z = erase(z);
d94     if (x != begin() && isect(--x, y))
c07       isect(x, y = erase(y));
57d     while ((y = x) != begin() && (--x)->p >= y->p)
774       isect(x, erase(y));
086   }
4ad   ll query(ll x) {
229     assert(!empty());
7d1     auto l = *lower_bound(x);
96a     return l.k * x + l.m;
d21   }
577 };
```

Mo.h

Description: For subtree queries, perform an Euler tour and map each node u to the interval $[tin[u], tin[u] + subtree\_size[u] - 1]$. A subtree query becomes a range query over this interval.
For path queries between nodes U and V, Let U be the closest to the root. If V lies in U's subtree, the path corresponds to the interval $[tin[U], tin[V]]$. Otherwise, the path corresponds to the interval $[min(tout[U], tout[V]), max(tin[U], tin[V])]$.
In both cases, nodes on the U–V path appear exactly once in the interval, while all other nodes appear either 0 or 2 times.
Usage: `queries.push(Query(l, r, index of query))`, intervals are `[l, r]`
Time: $\mathcal{O}\left(N\sqrt(Q)\right)$

fb7161, 44 lines

```
626 inline int64_t hilOrd(int x, int y, int pow, int rot) {
51a   if (pow == 0) return 0;
a6e   int hpow = 1 << (pow - 1);
01f   int seg = (x < hpow) ? ((y < hpow) ? 0 : 3) : ((y < hpow) ? 1 : 2);
e08   seg = (seg + rot) & 3;
669   const int rotDelta[4] = { 3, 0, 0, 1 };
d0b   int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
115   int nrot = (rot + rotDelta[seg]) & 3;
fba   int64_t sub = int64_t(1) << (2 * pow - 2);
65b   int64_t ans = seg * sub;
1ae   int64_t add = hilOrd(nx, ny, pow - 1, nrot);
ff7   ans += (seg == 1 || seg == 2) ? add : (sub - add - 1);
ba7   return ans;
ec4 }

670 struct Query {
738   int l, r, idx;
ce8   int64_t ord;
36f   Query(int l, int r, int idx) : l(l), r(r), idx(idx) {
6c4     ord = hilOrd(l, r, 21, 0);
926   }
847   bool operator < (const Query& other) const {
328     return ord < other.ord;
e05   }
315 };

240 vector<Query> queries;
4d5 int ans[ms];
566 void put(int x) {} // F
c29 void remove(int x) {} // F
64b int getAns() {}

1c1 void Mo() {
3d9   int l = 0, r = -1;
bfa   sort(queries.begin(), queries.end());
275   for (Query q : queries) {
482     while (l > q.l) put(--l);
fec     while (r < q.r) put(++r);
5b8     while (l < q.l) remove(l++);
9b5     while (r > q.r) remove(r--);
745     ans[q.idx] = getAns();
5a4   }
2a4 }
```

MoUpdate.h

Description: Block size should be around $(2*N*N)^{\frac{1}{3}}$
Usage: intervals are `[l, r]`, `addQuery(l, r, number of updates happened before this query, index of query)`, `addUpdate(index of updated position, value before update, value after update)`
Time: $\mathcal{O}\left(Q*(2*N*N)^{\frac{1}{3}}*F\right)$

f8eda8, 55 lines

```
496 const int B = 2700;
247 struct MoUpdate {
670   struct Query {
fd6     int l, r, t, idx;
fc8     Query(int l, int r, int t, int idx)
8bf       : l(l), r(r), t(t), idx(idx) {}
f51     bool operator < (const Query& p) const {
f06       if (l / B != p.l / B) return l < p.l;
e80       if (r / B != p.r / B) return r < p.r;
d0c       return t < p.t;
673     }
bc2   };
f2f   struct Upd {
f25     int i, old, now;
f23     Upd(int i, int old, int now): i(i),old(old),now(now){}
c12   };

240   vector<Query> queries;
e2b   vector<Upd> updates;

ac5   void addQuery(int l, int r, int t, int idx) {
fc9     queries.push_back(Query(l, r, t, idx)); }
968   void addUpdate(int i, int old, int now) {
936     updates.push_back(Upd(i, old, now)); }

1aa   void add(int x) {} // F
598   void rem(int x) {} // F
64b   int getAns() {}
0d2   void update(int novo, int idx, int l, int r) {
2b9     if (l <= idx && idx <= r) rem(idx);
4ce     arr[idx] = novo;
ec1     if (l <= idx && idx <= r) add(idx);
100   }

63d   void solve() {
cb1     int l = 0, r = -1, t = 0;
bfa     sort(queries.begin(), queries.end());
275     for (Query q : queries) {
a95       while (l > q.l) add(--l);
875       while (r < q.r) add(++r);
8f6       while (l < q.l) rem(l++);
a38       while (r > q.r) rem(r--);
fda       while (t < q.t) {
df3         auto u = updates[t++];
285         update(u.now, u.i, l, r);
8a4       }
32a       while (t > q.t) {
d69         auto u = updates[--t];
ce2         update(u.old, u.i, l, r);
3bf       }
745       ans[q.idx] = getAns();
f06     }
b09   }
d3e };
```

MinQueue.h

40df8d, 19 lines

```
925 struct MQueue {
fdd     int tin, tout;
375     deque<pair<int, int>> dq;
1ce     MQueue() : tin(0), tout(0) {}
619     void push(int val) {
f0d         while (!dq.empty() && min(dq.back().first, val) == val) dq.pop_back();
9c6         dq.push_back(pair(val, tin++));
769     }
42d     void pop() {
            // assert(!dq.empty());
48c         if (dq.front().second == tout) dq.pop_front();
470         tout++;
b0e     }
f46     int front() {
            // assert(!dq.empty());
651         return dq.front().first;
fa2     }
40d };
```

SegmentTree.h

Description: Zero-indexed max-tree. Bounds are inclusive to the left and inclusive to the right. Can be changed by modifying T, f and unit.
Time: $\mathcal{O}(\log N)$

f609d9, 21 lines

```
5ae struct Tree {
ef4   typedef int T;
cbe   static constexpr T unit = INT_MIN;
e54   T f(T a, T b) { return max(a, b); } // (any associative fn)
6cd   vector<T> s; int n;
3d2   Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {}
6a3   void update(int pos, T val) {
56a     for (s[pos += n] = val; pos /= 2;)
326       s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
0e9   }
b4c   T query(int b, int e) { // query [b, e]
1a3     e++;
0f9     T ra = unit, rb = unit;
fbb     for (b += n, e += n; b < e; b /= 2, e /= 2) {
e83       if (b % 2) ra = f(ra, s[b++]);
064       if (e % 2) rb = f(s[--e], rb);
561     }
cb2     return f(ra, rb);
707   }
f60 };
```

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
Time: $\mathcal{O}(\log N)$

782797, 17 lines

```
c4d #include <bits/extc++.h>
0d7 using namespace __gnu_pbds;

4fc template<class T>
c20 using Tree = tree<T, null_type, less<T>, rb_tree_tag,
3a1     tree_order_statistics_node_update>;

ad0 void example() {
c6f   Tree<int> t, t2; t.insert(8);
559   auto it = t.insert(10).first;
d28   assert(it == t.lower_bound(9));
969   assert(t.order_of_key(10) == 1);
d39   assert(t.order_of_key(11) == 2);
1b7   assert(*t.find_by_order(0) == 8);
a60   t.join(t2); // merge t2 into t
9ad }
```

PersistentSegTree.h

Usage: `SegP(size of the segtree, number of updates)`

`roots = {0}, newRoot = update(roots.back(),...),`
`roots.push(newRoot)`

58842f, 42 lines

```
b17 struct SegP {
709   static constexpr ll neut = 0;
bf2   struct Node {
aa3     ll v; // start with neutral value
74f     int l, r;
9ef     Node(ll v=neut, int l=0, int r=0) : v(v), l(l), r(r){}
945   };
```

```
38f   vector<Node> seg;
068   int n, CNT;
9ea   SegP(int _n, int upd): seg(20*(upd+_n)), n(_n), CNT(1){}
2ce   ll merge(ll a, ll b) { return a + b; }
c97   int update(int root, int pos, int val, int l, int r) {
ec9     int p = CNT++;
77a     seg[p] = seg[root];
893     if (l == r) {
00f       seg[p].v += val;
74e       return p;
3d7     }
ae0     int mid = (l + r) / 2;
8a3     if (pos <= mid){
aa8       seg[p].l = update(seg[p].l, pos, val, l, mid);
583     }else seg[p].r = update(seg[p].r,pos,val,mid+1,r);

85a     seg[p].v=merge(seg[seg[p].l].v, seg[seg[p].r].v);
74e     return p;
a90   }
6a4   int query(int p, int L, int R, int l, int r) {
3c7     if (l > R || r < L) return neut;
c26     if (L <= l && r <= R) return seg[p].v;
ae0     int mid = (l + r) / 2;
864     int left = query(seg[p].l, L, R, l, mid);
195     int right = query(seg[p].r, L, R, mid + 1, r);
90a     return merge(left, right);
e77   }
304   int update(int root, int pos, int val) {
c68     return update(root, pos, val, 0, n - 1);
84e   }
7cc   int query(int root, int L, int R) {
a53     return query(root, L, R, 0, n - 1);
2f9   }
588 };
```

SegBeats.h

Description: In Segment Tree Beats, ‘lazy‘ does NOT mean ”updates still missing here”. The node already reflects all previous updates. Instead, ‘lazy‘ stores what must be propagated to the children before recursing. Always call ‘apply(l,r,p)‘ before descending. This node layout supports range add, range chmin and range chmax operations. Beats conditions:
break: MIN x: mx1 <= x ; MAX x: mi1 >= x
tag: MIN x: x > mx2 ; MAX x: x < mi2
Time: amortized $\mathcal{O}(\log^2 N)$, without range add $\mathcal{O}(\log N)$ fa8527, 47 lines

```
3c9 struct node{
45e   ll mx1, mx2, sum, lazy;
9e5   ll mi1, mi2;
faa   int cMax, cMin, tam;
db3   node(int x=0) : mx1(x),mx2(-inf),mi1(x),mi2(inf),
744             cMax(1),cMin(1),tam(1),sum(x),lazy(0){}
b67   node(node a, node b){
4f5     sum = a.sum+b.sum, tam = a.tam+b.tam;
c60     lazy = 0;
15b     mx1 = max(a.mx1, b.mx1);
9ae     mx2 = max(a.mx2, b.mx2);
f62     if(a.mx1 != b.mx1) mx2 = max(mx2, min(a.mx1, b.mx1));
b60     cMax=(a.mx1==mx1 ? a.cMax:0)+(b.mx1==mx1 ? b.cMax:0);

09f     mi1 = min(a.mi1, b.mi1);
143     mi2 = min(a.mi2, b.mi2);
3bf     if(a.mi1 != b.mi1) mi2=min(mi2, max(a.mi1, b.mi1));
c18     cMin=(a.mi1==mi1 ? a.cMin:0)+(b.mi1==mi1 ? b.cMin:0);
23d   }
38d   void apply_sum(ll x){
2a1     mx1 += x, mx2 += x,mi1 += x, mi2 += x;
99b     sum += tam*x, lazy += x;
b5e   }
cf4   void apply_min(ll x){
e07     if(x >= mx1) return;
c44     sum -= (mx1 - x)*cMax;
be0     if(mi1 == mx1) mi1 = x;
8ef     if(mi2 == mx1) mi2 = x;
ea2     mx1 = x;
908   }
0c8   void apply_max(ll x){
e25     if(x <= mi1) return;
59e     sum -= (mi1 - x)*cMin;
4b1     if(mx1 == mi1) mx1 = x;
d69     if(mx2 == mi1) mx2 = x;
1ff     mi1 = x;
0e4   }
554 };
fdc void apply(int l, int r, int p){
c8e   for(int i=2*p+1; i<=2*p+2; i++){
dbf     seg[i].apply_sum(st[p].lazy);
c90     seg[i].apply_min(st[p].mx1);
61a     seg[i].apply_max(st[p].mi1);
4b8   }
431   seg[p].lazy = 0;
dd0 }
```

RMQ.h

Usage: RMQ rmq(values);
rmq.query(inclusive, inclusive);
Time: $\mathcal{O}(|V| \log |V| + Q)$ bca062, 17 lines

```
76a struct RMQ {
8ac   vector<vector<int>> dp;
dd1   RMQ(const vector<int>& a) : dp(1, a) {
71c     for (int i = 1, pw = 1; pw*2 <= sz(a); pw*=2, i++) {
394       dp.emplace_back(sz(a) - pw*2 + 1);
d17       for (int j = 0; j < sz(dp[i]); j++) {
dcc         dp[i][j] = min(dp[i-1][j], dp[i-1][j+pw]);
75a       }
b68     }
3e9   }
9e3   int query(int l, int r) {
658     assert(l <= r);
884     int k = 31 - __builtin_clz(r - l + 1);
1f9     return min(dp[k][l], dp[k][r - (1 << k) + 1]);
e21   }
bca };
```

UnionFind.h

Description: Disjoint-set data structure with bipartite check 6d2739, 22 lines

```
146 struct Uf{
b54     vector<int> tam, ds, bi, c;
d2c     Uf(int n) : tam(n, 1), ds(n), bi(n, 1), c(n){
244         iota(all(ds), 0);
233     }
001     int find(int i){ return (i==ds[i] ? i : find(ds[i]));}
e5a     int color(int i){
300         return (i==ds[i] ? 0 : (c[i]^color(ds[i])));}
c3b     void merge(int a, int b){
8d0         int ca = color(a), cb = color(b);
605         a = find(a), b = find(b);
a89         if(a == b){
686             if(ca == cb) bi[a] = false;
505             return;
c08         }
226         if(tam[a] < tam[b]) swap(a, b);
1ac         ds[b] = a, tam[a] += tam[b];
27c         bi[a] = (bi[a] && bi[b]);
834         c[b] = (ca ^ cb ^ 1);
a70     }
6d2 };
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().
Usage: int t = uf.time(); ...; uf.rollback(t);
Time: $\mathcal{O}(\log(N))$ d4405e, 23 lines

```
47a struct RollbackUF {
f80   vector<int> e;
919     vector<pii> st;
f6f   RollbackUF(int n) : e(n, -1) {}
84b   int size(int x) { return -e[find(x)]; }
626   int find(int x) { return e[x] < 0 ? x : find(e[x]); }
49f   int time() { return sz(st); }
4db   void rollback(int t) {
314     for (int i = time(); i --> t;)
8d2       e[st[i].first] = st[i].second;
b04     st.resize(t);
30b   }
cf0   bool join(int a, int b) {
605     a = find(a), b = find(b);
5c2     if (a == b) return false;
745     if (e[a] > e[b]) swap(a, b);
bac     st.push_back({a, e[a]});
e6e     st.push_back({b, e[b]});
708     e[a] += e[b]; e[b] = a;
8a6     return true;
6c7   }
d44 };
```

Numerical (4)

4.1 Polynomials and recurrences

Polynomial.h

c9b7b0, 19 lines

```
213 struct Poly {
3a1   vector<double> a;
9a5   double operator()(double x) const {
e3c     double val = 0;
d5c     for (int i = sz(a); i--;) (val *= x) += a[i];
d94     return val;
ae7   }
0ac   void diff() {
7b6     rep(i,1,sz(a)) a[i-1] = i*a[i];
468     a.pop_back();
afc   }
087   void divroot(double x0) {
898     double b = a.back(), c; a.back() = 0;
9cf     for(int i=sz(a)-1; i--;)
406         c = a[i], a[i] = a[i+1]*x0+b, b=c;
468     a.pop_back();
3f8   }
c9b };
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0
Time: $\mathcal{O}(n^2 \log(1/\epsilon))$
"Polynomial.h" b00bfe, 24 lines

```
64a vector<double> polyRoots(Poly p, double xmin, double xmax)
      {
853   if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
539   vector<double> ret;
f55   Poly der = p;
c06   der.diff();
617   auto dr = polyRoots(der, xmin, xmax);
d85   dr.push_back(xmin-1);
12c   dr.push_back(xmax+1);
```

```
423   sort(all(dr));
b98   rep(i,0,sz(dr)-1) {
d85     double l = dr[i], h = dr[i+1];
ad1     bool sign = p(l) > 0;
b41     if (sign ^ (p(h) > 0)) {
03d       rep(it,0,60) { // while (h - l > 1e-8)
761         double m = (l + h) / 2, f = p(m);
0ac         if ((f <= 0) ^ sign) l = m;
193         else h = m;
b69       }
ff5       ret.push_back((l + h) / 2);
fc2     }
d15   }
edf   return ret;
b00 }
```

PolyInverse.h

2745a7, 18 lines

```
747 vector<ll> get_inverse(vector<ll> a) {
e4d   if (a.empty()) return {};
044   int Y = sz(a) - 1, n = 32 - __builtin_clz(Y);
ba5   n = (1 << n);
711   a.resize(n);
e3e   vector<ll> inv = { modpow(a[0], mod - 2) }, f, c;
a2b   inv.reserve(n);
599   for (int tam = 2; tam <= n; tam *= 2) {
d29     while (sz(f) < tam) f.push_back(a[sz(f)]);
fec     c = conv(f, inv);
757     rep(i, 0, tam) c[i] = (c[i] == 0 ? 0 : mod - c[i]);
df6     c[0] += (c[0] + 2 >= mod ? 2 - mod : 2);
f8b     inv = conv(inv, c);
118     inv.resize(tam);
9f4   }
531   return inv;
274 }
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
Time: $\mathcal{O}\left(N^2\right)$

96548b, 21 lines

```
c10 vector<ll> berlekampMassey(vector<ll> s) {
ea1   int n = sz(s), L = 0, m = 0;
2a2   vector<ll> C(n), B(n), T;
2b3   C[0] = B[0] = 1;

d6f   ll b = 1;
36d   rep(i,0,n) { ++m;
b7f     ll d = s[i] % mod;
45a     rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
53a     if (!d) continue;
169     T = C; ll coef = d * modpow(b, mod-2) % mod;
2d1     rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
b6c     if (2 * L > i) continue;
dc3     L = i + 1 - L; B = T; b = d; m = 0;
8c2   }

51b   C.resize(L + 1); C.erase(C.begin());
e98   for (ll& x : C) x = (mod - x) % mod;
a91   return C;
965 }
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \ldots \geq n-1]$ and $tr[0 \ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}\left(n^2 \log k\right)$

547b93, 27 lines

```
437 using Poly = vector<ll>;
2ef ll linearRec(Poly S, Poly tr, ll k) {
327   int n = sz(tr);

0e9   auto combine = [&](Poly a, Poly b) {
b1c     Poly res(n * 2 + 1);
5f7     rep(i,0,n+1) rep(j,0,n+1)
389       res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
bdc     for (int i = 2 * n; i > n; --i) rep(j,0,n)
fc3       res[i-1-j] = (res[i-1-j] + res[i] * tr[j]) % mod;
b76     res.resize(n + 1);
b50     return res;
55c   };

bf8   Poly pol(n + 1), e(pol);
997   pol[0] = e[1] = 1;

e96   for (++k; k; k /= 2) {
491     if (k % 2) pol = combine(pol, e);
0d9     e = combine(e, e);
813   }

cd2   ll res = 0;
e8d   rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
b50   return res;
594 }
```

4.2 Matrices

SolveLinear.h

Description: If inv = 1, finds the inverse of the matrix eq and returns it as a flat vector
Time: $\mathcal{O}\left(\min(n, m)\, nm\right)$

2c134e, 52 lines

```
320 struct Gauss {
d6d   const double eps = 1e-9;
93d   vector<vector<double>> eq;
754   void addEquation(const vector<double>& e) {
503     eq.push_back(e); }
04f   pair<int, vector<double>> solve(int inv=0) {
214     int n = sz(eq), m = sz(eq[0]) - 1 + inv;
f9c     if(inv){
d33       rep(i, 0, n) eq[i].resize(2*n), eq[i][n+i] = 1;
2e2     }
3cb     vector<int> where(m, -1);
a73     for (int col = 0, row = 0; col < m && row < n; col++)
    {
f05       int sel = row;
53c       rep(i, row, n) {
664         if (abs(eq[i][col]) > abs(eq[sel][col])) sel = i;
e04       }
68b       if (abs(eq[sel][col]) < eps) continue;
3ad       rep(i, col, sz(eq[0])) swap(eq[sel][i], eq[row][i]);
2c3       where[col] = row;
dff       rep(i, 0, n) if (i != row) {
184         double c = eq[i][col] / eq[row][col];
7f1         rep(j, col, sz(eq[0])) eq[i][j] -= eq[row][j] * c;
17d       }
4ef       ++row;
9b8     }
f9c     if(inv){
208       vector<double> res;
fea       rep(i, 0, n) {
420         if (where[i] == -1) return {0, {}}; // Singular
3af         rep(j, n, 2*n)
f89           res.push_back(eq[where[i]][j] / eq[where[i]][i])
    ;
d81       }
3b1       return {1, res};
700     }

233     vector<double> ans(m, 0);
670     rep(i, 0, m) {
c19       if (where[i] != -1)
02c         ans[i] = eq[where[i]][m] / eq[where[i]][i];
5bb     }
fea     rep(i, 0, n) {
68c       double sum = 0;
5f8       rep(j, 0, m) {
f48         sum = sum + ans[j] * eq[i][j];
fa6       }
3c8       if(abs(sum - eq[i][m]) > eps)return {0, {}};
bf2     }
260     rep(i, 0, m) if (where[i] == -1) return {2, ans};
d4a     return {1, ans};
a95   }
2c1 };
```

SolveLinearBinary.h

Time: $\mathcal{O}\left(\frac{\min(n,m)\, nm}{64}\right)$

28c946, 32 lines

```
e81 pair<int, bitset<M>> gauss(vector<bitset<M>> eq) {
579     int n = eq.size(), m = M - 1;
3cb     vector<int> where(m, -1);
a73     for(int col = 0, row = 0; col < m && row < n; col++){
dbb         rep(i,row,n)
926             if (eq[i][col]) {
c35                 swap(eq[i], eq[row]);
c2b                 break;
177             }
f4f         if (!eq[row][col]) continue;
2c3         where[col] = row;

fea         rep(i, 0, n) {
b60             if (i != row && eq[i][col]) eq[i] ^= eq[row];
981         }
4ef         ++row;
c74     }
7eb     bitset<M> ans;
670     rep(i,0,m) {
713         if (where[i] != -1) ans[i] = eq[where[i]][m];
691     }
fea     rep(i,0,n) {
e5c         int sum = (ans & eq[i]).count();
53f         sum %= 2;
36a         if (sum != eq[i][m]) return pair(0, bitset<M>());
29e     }
670     rep(i,0,m) {
be2         if (where[i] == -1) return pair(INF, ans);
958     }
280     return pair(1, ans);
28c }
```

XorGauss.h

5a1957, 30 lines

```
b94 struct XorGauss {
060     int N;
471     vector<ll> basis, who, mask;
47b     XorGauss(int N) : N(N), basis(N), who(N), mask(N) {}
        // if(ans & (1ll << j)) who[j] was used to form x
221     bool belong(ll x){
04b         ll ans = 0;
042         for(int i=N-1; i>=0; i--){
e13             if((x ^ basis[i]) < x){
4ec                 ans ^= mask[i];
6b0                 x ^= basis[i];
```

```
254             }
2ad         }
069         return (x == 0);
c26     }
397     void add(ll v, int idx) {
a4d         ll msk = 0;
042         for (int i = N - 1; i >= 0; i--) {
80f             if (!(v & (1ll << i))) continue;
bf3             if (basis[i] == 0) {
1c7                 basis[i] = v, who[i] = idx;
940                 mask[i] = (msk | (1ll << i));
505                 return;
bc8             }
00e             msk ^= mask[i];
647             v ^= basis[i];
25b         }
fcc     }
5a1 };
```

4.3 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x]\exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$)

773fcd, 44 lines

```
bcc typedef complex<double> C;

7c0 void fft(vector<C>& a) {
a5b   int n = a.size(), L = 31 - __builtin_clz(n);
f82   static vector<complex<long double>> R(2, 1); // 10%
    faster if double
991   static vector<C> rt(2, 1);
ad8   for (static int k = 2; k < n; k *= 2) {
9d9     R.resize(n);
335     rt.resize(n);
411     auto x = polar(1.0L, acos(-1.0L) / k);
cdb     rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
a8a   }
e66   vector<ll> rev(n);
dcb   rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
47b   rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);

d3f   for (int k = 1; k < n; k *= 2) {
cda     for (int i = 0; i < n; i += 2 * k) {
0c2       for (int j = 0; j < k; j++) {
30c         auto x = (double*)&rt[j + k];
ebe         auto y = (double*)&a[i + j + k];
15c         C z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]);
20a         a[i + j + k] = a[i + j] - z;
1b0         a[i + j] += z;
b5b       }
1fe     }
fa0   }
b33 }

ccc vector<ll> conv(const vector<ll>& a, const vector<ll>& b){
f88   if (a.empty() || b.empty()) return {};
920   vector<ll> res(sz(a) + sz(b) - 1);
441   int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
060   vector<C> in(n), out(n);
b1a   copy(all(a), in.begin());
fef   rep(i,0,sz(b)) in[i].imag(b[i]);
21a   fft(in);
6fb   for (C& x : in) x *= x;
4d7   rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
3d7   fft(out);
aa3   rep(i,0,sz(res)) res[i]=round(imag(out[i]) / (4 * n));
b50   return res;
7f4 }
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.
Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h" b82773, 23 lines

```
192 typedef vector<ll> vl;
3fe template<int M> vl convMod(const vl &a, const vl &b) {
f88  if (a.empty() || b.empty()) return {};
19d  vl res(sz(a) + sz(b) - 1);
a6f  int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M));
3dd  vector<C> L(n), R(n), outs(n), outl(n);
a1d  rep(i,0,sz(a)) L[i] =C((int)a[i] / cut, (int)a[i] % cut);
97d  rep(i,0,sz(b)) R[i] =C((int)b[i] / cut, (int)b[i] % cut);
5d5  fft(L), fft(R);
fea  rep(i,0,n) {
39d    int j = -i & (n - 1);
65e    outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
91a    outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
cb3  }
d08  fft(outl), fft(outs);
35e  rep(i,0,sz(res)) {
351    ll av = ll(real(outl[i])+.5), cv =ll(imag(outs[i])+.5);
988    ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
6a3    res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
58f  }
b50  return res;
c1f }
```

NumberTheoreticTransform.h

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x]g^{xk}$ for all k, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).
Time: $\mathcal{O}(N \log N)$

84c11e, 34 lines

```
376 const int mod = 998244353, root = 62;
192 typedef vector<ll> vl;
8ec void ntt(vl &a) {
6ae  int n = sz(a), L = 31 - __builtin_clz(n);
7c9  static vl rt(2, 1);
8ee  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
335    rt.resize(n);
d43    ll z[] = {1, modpow(root, mod >> s)};
8e7    rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
f39  }
808  vector<int> rev(n);
dcb  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
47b  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
657  for (int k = 1; k < n; k *= 2)
2cb    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
86e      ll z = rt[j+k] * a[i+j+k] % mod, &ai = a[i+j];
598      a[i + j + k] = ai - z + (z > ai ? mod : 0);
589      ai += (ai + z >= mod ? z - mod : z);
9a8    }
de9 }
08f vl conv(const vl &a, const vl &b) {
f88  if (a.empty() || b.empty()) return {};
f51  int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s),
570      n = 1 << B;
9ef  int inv = modpow(n, mod - 2);
e4c  vl L(a), R(b), out(n);
6b4  L.resize(n), R.resize(n);
d9e  ntt(L), ntt(R);
dfc  rep(i,0,n)
0db    out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
ec9  ntt(out);
c20  return {out.begin(), out.begin() + s};
387 }
```

FWHT.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

124c14, 20 lines

```
5ad void FST(vector<ll>& a, bool inv) {
a9d   for (int n = sz(a), step = 1; step < n; step *= 2) {
5bd     for (int i = 0; i < n; i += 2 * step) {
4ee       for (int j = i; j < i + step; j++) {
2fe         ll& u = a[j], &v = a[j + step];
c6f         tie(u, v) =
2d3         inv ? pair(v - u, u) : pair(v, u + v); // AND
aba         inv ? pair(v, u - v) : pair(u + v, u); // OR
a5a         pair(u + v, u - v);   // XOR
0b4       }
fb4     }
cd3   }
c9b   if(inv) for(ll& x : a) x /= sz(a); // XOR only
075 }
eb2 vector<ll> conv(vector<ll> a, vector<ll> b) {
595   FST(a, 0); FST(b, 0);
2dd   for (int i = 0; i < sz(a); i++) a[i]*=b[i];
062   FST(a, 1); return a;
7bf }
```

Number theory (5)

5.1 Modular arithmetic

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

c375f5, 5 lines

```
88a const ll mod = 1000000007, LIM = 200000;
0f2 inv[1] = 1;
379 for(int i=2; i<LIM; i++)
86c     inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.
Time: $\mathcal{O}(1)$ for `modmul`, $\mathcal{O}(\log b)$ for `modpow`

bbbd8f, 12 lines

```
f4c typedef unsigned long long ull;
f85 ull modmul(ull a, ull b, ull M) {
2dd   ll ret = a * b - M * ull(1.L / M * a * b);
964   return ret + M * (ret < 0) - M * (ret >= (ll)M);
e93 }
4f6 ull modpow(ull b, ull e, ull mod) {
c1a   ull ans = 1;
a18   for (; e; b = modmul(b, b, mod), e /= 2)
9e8     if (e & 1) ans = modmul(ans, b, mod);
ba7   return ans;
100 }
```

ModPow.h

b83e45, 9 lines

```
e2e  const ll mod = 1000000007; // faster if const

9d8  ll modpow(ll b, ll e) {
d54    ll ans = 1;
36e    for (; e; b = b * b % mod, e /= 2)
b46      if (e & 1) ans = ans * b % mod;
ba7    return ans;
d1e  }
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).
Time: $\mathcal{O}\left(\log^2 p\right)$ worst case, $\mathcal{O}\left(\log p\right)$ for most p

"ModPow.h" 19a793, 25 lines

```
a77  ll sqrt(ll a, ll p) {
5de    a %= p; if (a < 0) a += p;
b47    if (a == 0) return 0;
5c6    assert(modpow(a, (p-1)/2, p) == 1); // else no solution
a75    if (p % 4 == 3) return modpow(a, (p+1)/4, p);
       // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
b94    ll s = p - 1, n = 2;
ee5    int r = 0, m;
084    while (s % 2 == 0)
082      ++r, s /= 2;
eaa    while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
0c3    ll x = modpow(a, (s + 1) / 2, p);
b74    ll b = modpow(a, s, p), g = modpow(n, s, p);
1af    for (;; r = m) {
4fd      ll t = b;
713      for (m = 0; m < r && t != 1; ++m)
c58        t = t * t % p;
ae0      if (m == 0) return x;
20e      ll gs = modpow(g, 1LL << (r - m - 1), p);
fba      g = gs * gs % p;
4fb      x = x * gs % p;
c5c      b = b * g % p;
e3a    }
19a  }
```

DiscreteLog.h

Description: Returns the smallest x such that $a^x \bmod m = b \bmod m$. If no such x exists, returns -1.
Time: O(sqrt(m) * log(sqrt(m))

2f126b, 32 lines

```
758  int solve(int a, int b, int m) {
a6e      a %= m, b %= m;
ec4      if (a == 0) return (b ? -1 : 1);
         // caso gcd(a, m) > 1
6af      int k = 1, add = 0, g;
553      while ((g = gcd(a, m)) > 1) {
d90          if (b == k) return add;
642          if (b % g) return -1;
92a          b /= g, m /= g, ++add;
803          k = (k * 1ll * a / g) % m;
8a0      }

16c      int sq = sqrt(m) + 1;
b51      int big = 1;
4e1      for (int i = 0; i < sq; i++) big = (1ll * big * a) % m
     ;

053      vector<pii> vals;
3c2      for (int q = 0, cur = b; q <= sq; q++) {
b53          vals.push_back({ cur, q });
b50          cur = (1ll * cur * a) % m;
837      }
62b      sort(all(vals));
90c      for (int p = 1, cur = k; p <= sq; p++) {
5d3          cur = (1ll * cur * big) % m;
958          auto it = lower_bound(all(vals), pair(cur, INF));
721          if (it != vals.begin() && (--it)->first == cur) {
a30              return sq * p - it->second + add;
6fe          }
f22      }
daa      return -1;
2f1  }
```

DiscreteRoot.h

Description: Returns x such that $x^k \bmod m = a \bmod m$. If no such x exists, returns -1.
Time: O(sqrt(m) * log(sqrt(m))

"PrimitiveRoot.h", "DiscreteLog.h" 1d582e, 11 lines

```
     // Discrete Root

27c  ll discreteRoot(ll k, ll a, ll m) {
738      ll g = primitiveRoot(m);
58b      ll y = discreteLog(fexp(g, k, m), a, m);
f31      if (y == -1) return y;
a58      return fexp(g, y, m);
1d5  }
```

5.2 Primality

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \bmod c$.

"ModMulLL.h" 66fe73, 13 lines

```
da4  bool isPrime(ull n) {
c16    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
062    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 17952650
     22};
ae0    ull s = __builtin_ctzll(n-1), d = n >> s;
e80    for (ull a : A) {   // ^ count trailing zeroes
6b4      ull p = modpow(a%n, d, n), i = s;
274      while (p != 1 && p != n - 1 && a % n && i--)
c77        p = modmul(p, p, n);
e28      if (p != n-1 && i != s) return 0;
edf    }
6a5    return 1;
66f  }
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" da0c7c, 19 lines

```
7eb  ull pollard(ull n) {
222    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
5f5    auto f = [&](ull x) { return modmul(x, x, n) + i; };
f51    while (t++ % 40 || gcd(prd, n) == 1) {
be9      if (x == y) x = ++i, y = f(x);
70f      if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
b78      x = f(x), y = f(f(y));
bf8    }
002    return gcd(prd, n);
d1b  }
591  vector<ull> factor(ull n) {
1b9    if (n == 1) return {};
6b5    if (isPrime(n)) return {n};
bc6    ull x = pollard(n);
52a    auto l = factor(x), r = factor(n / x);
7af    l.insert(l.end(), all(r));
792    return l;
d54  }
```

PrimitiveRoot.h

18a01e, 15 lines

```
     //is n primitive root of p ?
ad0  bool test(ll x, ll p) {
a56      ll m = p - 1;
845      for (ll i = 2; i * i <= m; ++i) if (!(m % i)) {
e64          if (modpow(x, i, p) == 1) return false;
599          if (modpow(x, m / i, p) == 1) return false;
53a      }
8a6      return true;
c4e  }
     //find the smallest primitive root for p
220  ll search(ll p) {
1bf      for (ll i = 2; i < p; i++) if (test(i, p)) return i;
daa      return -1;
a3c  }
```

5.3 Divisibility

Euclid.h

Description: Find x, y such that $Ax + By = \gcd(A, B)$. If $\gcd(A, B) = 1$, then $x = A^{-1} \pmod B$ and $y = B^{-1} \pmod A$.
Time: $\mathcal{O}\left(\log\right)$

33ba8f, 6 lines

```
c22  ll euclid(ll a, ll b, ll &x, ll &y) {
1ee    if (!b) return x = 1, y = 0, a;
e3d    ll d = euclid(b, a % b, y, x);
0a4    return y -= a/b * x, d;
33b  }
```

CRT.h

ba1a4a, 25 lines

```
bc9  ll modinverse(ll a, ll b, ll s0 = 1, ll s1 = 0) {
a76    return !b ? s0 : modinverse(b, a % b, s1, s0 - s1 * (a /
     b)); }

d8b  ll mul(ll a, ll b, ll m) {
a6f    return (((__int128_t)a*b)%m + m)%m;
0bc  }

28d  struct Equation {
4c5    ll mod, ans;
08f    bool valid;
145    Equation(ll a, ll m) { mod = m, ans = a, valid = true; }
0fc    Equation() { valid = false; }
4d3    Equation(Equation a, Equation b) {
515      valid = false;
1a0      if (!a.valid || !b.valid) return;
85c      ll g = gcd(a.mod, b.mod);
44d      if ((a.ans - b.ans) % g != 0) return;
af0      valid = true;
b98      mod = a.mod * (b.mod / g);
81a      ll x = mul(a.mod, modinverse(a.mod, b.mod), mod);
38a      ans = a.ans + mul(x, (b.ans - a.ans) / g, mod);
c4c      ans = (ans % mod + mod) % mod;
6f5    }
f48  };
```

DivisionTrick.h

02aebb, 15 lines

```
7f1  void floor_ranges(int n) {
79c      for (int l = 1, r; l <= n; l = r + 1) {
746          r = n / (n / l);
             // floor(n/y) has the same value for y in [l..r]
5bf      }
eee  }
678  void ceil_ranges(int n) {
79c      for (int l = 1, r; l <= n; l = r + 1) {
```

```
d47         int x = (n + l - 1) / l;
374         if (x == 1) r = n;
21b         else r = (n - 1) / (x - 1);
            // ceil(n/y) has the same value for y in [l..r]
06c       }
57c   }
```

Phi.h

Description: *Euler's* ϕ function is defined as $\phi(n) := \#$ of positive integers $\le n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} ... (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \le k \le n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$.
Euler's thm (generalized): a, m arbitrary, $n \ge \log_2 m \Rightarrow a^n \equiv a^{\phi(m)+(n \bmod \phi(m))} \pmod{m}$.

e58bf0, 6 lines

```
d08 void calculatePhi() {
265   for(int i=0; i<LIM; i++) phi[i] = i&1 ? i : i/2;
c83   for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
dc2     for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
e58 }
```

Combinatorial (6)

PartitionSolver.h

e50fb7, 61 lines

```
d38 template<const int N>
182 struct PartitionSolver {
4ce   vector<vector<int>> part, to, from;
621   PartitionSolver() {
a9d     vector<int> a;
1ed     part.push_back(a);
77f     gen(1, N, a);
796     sort(all(part));
ed4     to.assign(sz(part), vector<int>(N + 1, -1));
9a5     from = to;
ddd     for (int i = 0; i < sz(part); i++) {
a93       int sum = 0;
87f       auto arr = part[i];
bca       for (auto x : arr) sum += x;
4fa       to[i][0] = i;
615       from[i][0] = i;
afc       for (int j = 1; j + sum <= N; j++) {
123         arr = part[i];
9d6         arr.push_back(j);
ceb         sort(all(arr));
d02         to[i][j] = getIndex(arr);
942         from[to[i][j]][j] = i;
20d       }
bef     }
283   }

810   int size() const { return sz(part); }
9ee   int getIndex(const vector<int>& arr) const {
168     return lower_bound(all(part), arr) - part.begin(); }
b49   int add(int id, int num) const { return to[id][num]; }
944   int rem(int id, int num) const { return from[id][num]; }
168   vector<int> getPartition(int id) const {
37b     return part[id]; }

1ba   void gen(int i, int sum, vector<int>& a) {
a05     if (i > sum) { return; }
726     a.push_back(i);
1ed     part.push_back(a);
278     gen(i, sum - i, a);
468     a.pop_back();
48f     gen(i + 1, sum, a);
537   }
f4f };

    // Number of partitions for all integers <= n
75c vector<ll> partitionNumber(int n) {
d9c   vector<ll> ans(n + 1, 0);
82f   ans[0] = 1;
78a   for (int i = 1; i <= n; i++) {
87f     for (int j = 1; j * (3 * j + 1) / 2 <= i; j++) {
b6b       ll here = ans[i - j * (3 * j + 1) / 2];
c91       ans[i] = (ans[i] + (j & 1 ? here : -here));
365     }
7c6     for (int j = 1; j * (3 * j - 1) / 2 <= i; j++) {
a1a       ll here = ans[i - j * (3 * j - 1) / 2];
c91       ans[i] = (ans[i] + (j & 1 ? here : -here));
162     }
4a3   }
ba7   return ans;
08b }
```

Graph (7)

7.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max |w_i| < \sim 2^{63}$.
Time: $\mathcal{O}(VE)$

529834, 24 lines

```
f5e const ll inf = LLONG_MAX;
83a struct Ed { int a, b, w, s() { return a < b ? a : -a; }};
9ac struct Node { ll dist = inf; int prev = -1; };

6fc void bell(vector<Node>& nodes, vector<Ed>& eds, int s) {
97b  nodes[s].dist = 0;
eb9  sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s(); });

74e  int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled
     vertices
c5a  rep(i,0,lim) for (Ed ed : eds) {
905    Node cur = nodes[ed.a], &dest = nodes[ed.b];
d7d    if (abs(cur.dist) == inf) continue;
6ab    ll d = cur.dist + ed.w;
6ec    if (d < dest.dist) {
956      dest.prev = ed.a;
4c2      dest.dist = (i < lim-1 ? d : -inf);
452    }
75a  }
ced  rep(i,0,lim) for (Ed e : eds) {
3ab    if (nodes[e.a].dist == -inf)
5ff      nodes[e.b].dist = -inf;
1d7  }
166 }
```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m, where $m[i][j] = \texttt{inf}$ if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, inf if no path, or -inf if the path goes through a negative-weight cycle.
Time: $\mathcal{O}(N^3)$

531245, 13 lines

```
964 const ll inf = 1LL << 62;
914 void floydWarshall(vector<vector<ll>>& m) {
e9d  int n = sz(m);
831  rep(i,0,n) m[i][i] = min(m[i][i], 0LL);
99d  rep(k,0,n) rep(i,0,n) rep(j,0,n)
19b    if (m[i][k] != inf && m[k][j] != inf) {
6e8      auto newDist = max(m[i][k] + m[k][j], -inf);
e89      m[i][j] = min(m[i][j], newDist);
f38    }
a69  rep(k,0,n) if (m[k][k] < 0) rep(i,0,n) rep(j,0,n)
ffd    if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
f12 }
```

7.2 Network flow and Matching

Dinic.h

Time: - $\mathcal{O}(\min(m \cdot \text{max\_flow},\ n^2 m))$.
- For graphs with unit capacities: $\mathcal{O}\left(\min(m\sqrt{m},\ mn^{2/3})\right)$.
- If every vertex has in-degree 1 or out-degree 1: $\mathcal{O}(m\sqrt{n})$.
- With capacity scaling: $\mathcal{O}(nm \log(\text{MAXCAP}))$ with high constant factor.

892d6e, 56 lines

```
14d struct Dinic {
61f   const bool scaling = false;
206   int lim;
670   struct edge {
c63     int to, rev;
a14     ll cap, flow;
7f9     bool res;
6dd     edge(int to_, ll cap_, int rev_, bool res_)
a94       : to(to_), cap(cap_), rev(rev_), flow(0), res(res_){}
477   };

002   vector<vector<edge>> g;
216   vector<int> lev, beg;
a71   ll F;
63f   Dinic(int n) : g(n), lev(n), beg(n), F(0) {}

0c5   void add(int a, int b, ll c, ll other = 0) {
de2     g[a].emplace_back(b, c, sz(g[b]), false);
fa5     g[b].emplace_back(a, other, sz(g[a])-1, true);
14f   }
123   bool bfs(int s, int t) {
e59     fill(all(lev), -1);
4e7     fill(all(beg), 0);
0a4         lev[s] = 0;
8b2     queue<int> q; q.push(s);
647     while (sz(q)) {
be1       int u = q.front(); q.pop();
bd9       for (auto& i : g[u]) {
dbc         if (lev[i.to] != -1 or (i.flow == i.cap)) continue;
b4f         if (scaling and i.cap - i.flow < lim) continue;
185         lev[i.to] = lev[u] + 1;
8ca         q.push(i.to);
f97       }
b1b     }
0de     return lev[t] != -1;
310   }
1dc   ll dfs(int v, int s, ll f = INF) {
50b     if (!f or v == s) return f;
84d     for (int& i = beg[v]; i < sz(g[v]); i++) {
027       auto& e = g[v][i];
206       if (lev[e.to] != lev[v] + 1) continue;
a30       ll foi = dfs(e.to, s, min(f, e.cap - e.flow));
749       if (!foi) continue;
3c5       e.flow += foi, g[e.to][e.rev].flow -= foi;
45c       return foi;
e08     }
bb3     return 0;
b98   }
2b4   ll maxFlow(int s, int t) {
a86     for (lim = scaling ? (1<<30) : 1; lim; lim /= 2)
69c       while (bfs(s, t)) while (ll ff = dfs(s, t)) F += ff;
4ff     return F;
```

```
6c8   }
0fe       bool inCut(int u){ return lev[u] != -1; }
892 };
```

LowerBoundFlow.h

Description: Calculates maximum flow with lower/upper bounds on edges. Returns -1 if no feasible flow exists. `add(a, b, l, r)` adds edge $a \to b$ where flow f must satisfy $l \le f \le r$. `add(a, b, c)` adds edge $a \to b$ with capacity c (implies $0 \le f \le c$). Same complexity as Dinic.

"Dinic.h" 756539, 36 lines

```
0ca struct lb_max_flow : Dinic {
96f   vector<ll> d;
be9   lb_max_flow(int n) : Dinic(n + 2), d(n, 0) {}
b12   void add(int a, int b, int l, int r) {
c97     d[a] -= l;
f1b     d[b] += l;
cb6     Dinic::add(a, b, r - l);
989   }
087   void add(int a, int b, int c) {
610     Dinic::add(a, b, c);
330   }
7a1   bool has_circulation() {
ac0     int n = sz(d);
854     ll cost = 0;
fea     rep(i, 0, n){
c69       if (d[i] > 0) {
f56         cost += d[i];
4f6         Dinic::add(n, i, d[i]);
551       } else if (d[i] < 0) {
bd2         Dinic::add(i, n+1, -d[i]);
bd9       }
a13     }

9f2     return (Dinic::maxFlow(n, n+1) == cost);
cc6   }
7bd   bool has_flow(int src, int snk) {
eda     Dinic::add(snk, src, INF);
e40     return has_circulation();
4aa   }
4eb   ll max_flow(int src, int snk) {
ee8     if (!has_flow(src, snk)) return -1;
99c     Dinic::F = 0;
703     return Dinic::maxFlow(src, snk);
0bb   }
756 };
```

MinCost.h

Description: Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only. If graph is a DAG pi can be calculated with DP instead of Bellman ford.

Time: $\mathcal{O}(FE\log(V))$ where F is max flow. $\mathcal{O}(VE)$ for setpi.

6f4fae, 95 lines

```
c4d #include <bits/extc++.h>

9f4 const ll INF = numeric_limits<ll>::max() / 4;

6f3 struct MCMF {
670   struct edge {
ede     int from, to, rev;
e20     ll cap, cost, flow;
092   };
060   int N;
091   vector<vector<edge>> ed;
a83   vector<int> seen, vis;
0ec   vector<ll> dist, pi;
c45   vector<edge*> par;

2cc   MCMF(int N) : N(N), ed(N), seen(N), vis(N),
dc7     dist(N), pi(N), par(N) {}

6f3   void addEdge(int from, int to, ll cap, ll cost) {
ad8     if (from == to || cap == 0) return;
1af     ed[from].push_back(edge{from,to,sz(ed[to]),cap,cost,0});
700     ed[to].push_back(edge{to,from,sz(ed[from])-1,0,-cost,0});
dad   }

975   void path(int s) {
7d4     fill(all(seen), 0);
04e     fill(all(dist), INF);
a93     dist[s] = 0;
841     ll di;
937     __gnu_pbds::priority_queue<pair<ll, int>> q;
9fb     vector<decltype(q)::point_iterator> its(N);
23b     q.push({ 0, s });

14d     while (!q.empty()) {
eda       s = q.top().second; q.pop();
2af       seen[s] = 1; di = dist[s] + pi[s];
6bd       for (edge& e : ed[s]) {
d20         if (!seen[e.to]) {
f1f           ll val = di - pi[e.to] + e.cost;
f3c           if(e.cap - e.flow > 0 && val < dist[e.to]){
0c7             dist[e.to] = val;
fb6             par[e.to] = &e;
22d             if (its[e.to] == q.end()) {
aac               its[e.to] = q.push({-dist[e.to], e.to});
388             }
6f8             else q.modify(its[e.to], {-dist[e.to], e.to});
80b           }
fce         }
013       }
e16     }
faa     for (int i = 0; i < N; i++) {
0ef       pi[i] = min(pi[i] + dist[i], INF);
ded     }
17b   }

310   pair<ll, ll> maxflow(int s, int t) {
923     setpi(s, t);
3d3     ll totflow = 0, totcost = 0;
8dd     while (path(s), seen[t]) {
535       ll fl = INF;
733       for (edge* x = par[t]; x; x = par[x->from]) {
8ed         fl = min(fl, x->cap - x->flow);
ddf       }
f9f       totflow += fl;
733       for (edge* x = par[t]; x; x = par[x->from]) {
10b         x->flow += fl;
e58         ed[x->to][x->rev].flow -= fl;
3bf       }
219     }
faa     for (int i = 0; i < N; i++) {
a18       for (edge& e : ed[i]) {
7a0         totcost += e.cost * e.flow;
774       }
a06     }
17e     return { totflow, totcost / 2 };
411   }

      // If some costs can be negative, call this before
      //     maxflow:
eda   void setpi(int s, int t) {
3ef     fill(all(pi), INF);
156     pi[s] = 0;
45c     int it = N, ch = 1;
aa3     ll v;
5e8     while (ch-- && it--) {
faa       for (int i = 0; i < N; i++) {
c9b         if (pi[i] != INF)
fb0           for (edge& e : ed[i]) if (e.cap)
257             if((v= pi[i] + e.cost)< pi[e.to])
a43               pi[e.to] = v, ch = 1;
d0b       }
250     }
38b     assert(it >= 0); // negative cost cycle
545   }
f1d };
```

PushRelabel.h

Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

Time: $\mathcal{O}\left(V^2\sqrt{E}\right)$

a7bbd5, 55 lines

```
49f struct PushRelabel {
e9b   struct Edge {
548     int dest, back;
e00     ll f, c;
571   };
ed3   vector<vector<Edge>> g;
51c   vector<ll> ec;
658   vector<Edge*> cur;
b08   vector<vector<int>> hs;
4d4   vector<int> H;
4e1   PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n){}

b1c   void addEdge(int s, int t, ll cap, ll rcap=0) {
50b     if (s == t) return;
cc8     g[s].push_back({t, sz(g[t]), 0, cap});
2aa     g[t].push_back({s, sz(g[s])-1, 0, rcap});
817   }

359   void addFlow(Edge& e, ll f) {
759     Edge &back = g[e.dest][e.back];
f7e     if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
d2e     e.f += f; e.c -= f; ec[e.dest] += f;
c47     back.f -= f; back.c += f; ec[back.dest] -= f;
340   }
0e0   ll calc(int s, int t) {
f00     int v = sz(g); H[s] = v; ec[t] = 1;
fbb     vector<int> co(2*v); co[0] = v-1;
e20     for(int i=0; i<v; i++) cur[i] = g[i].data();
8c2     for (Edge& e : g[s]) addFlow(e, e.c);

604     for (int hi = 0;;) {
ae9       while (hs[hi].empty()) if (!hi--) return -ec[s];
c6f       int u = hs[hi].back(); hs[hi].pop_back();
a3e       while (ec[u] > 0)  // discharge u
457         if (cur[u] == g[u].data() + sz(g[u])) {
e94           H[u] = 1e9;
5fa           for (Edge& e : g[u]){
256             if (e.c && H[u] > H[e.dest]+1)
740               H[u] = H[e.dest]+1, cur[u] = &e;
88f           }
f04           if (++co[H[u]], !--co[hi] && hi < v){
10d             for(int i=0; i<v; i++){
4be               if (hi < H[i] && H[i] < v)
021                 --co[H[i]], H[i] = v + 1;
a21             }
cc1           }
3a2           hi = H[u];
b6b         } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1){
779           addFlow(*cur[u], min(ec[u], cur[u]->c));
e91         }else ++cur[u];
```

```
4d7     }
b65   }
385   bool inCut(int a) { return H[a] >= sz(g); }
a7b };
```

Blossom.h

Description: Max matching on general Graph. $mate[i]$ = match of i
Time: $\mathcal{O}(N^3)$

21cc7b, 56 lines

```
40f vector<int> Blossom(vector<vector<int>>& g) {
10a     int n = sz(g), timer = -1;
f55     vector<int> mate(n, -1),label(n),par(n),orig(n),aux(n,
      -1),q;

060     auto lca = [&](int x, int y) {
7b8         for (timer++; ; swap(x, y)) {
583             if (x == -1) continue;
4be             if (aux[x] == timer) return x;
90d             aux[x] = timer;
fb4             x=(mate[x] == -1 ? -1 : orig[par[mate[x]]]);
f6a         }
aba     };
be4     auto blossom = [&](int v, int w, int a) {
509         while (orig[v] != a) {
721             par[v] = w; w = mate[v];
1e2             if(label[w] == 1) label[w] = 0,q.push_back(w);
8c7             orig[v] = orig[w] = a;
3d0             v = par[w];
eae         }
068     };
a0f     auto aug = [&](int v) {
8c8         while (v != -1) {
86a             int pv = par[v], nv = mate[pv];
941             mate[v] = pv; mate[pv] = v; v = nv;
ba8         }
54c     };
9f9     auto bfs = [&](int root) {
be5         fill(all(label), -1);
652         iota(all(orig), 0);
4b6         q.clear();
594         label[root] = 0; q.push_back(root);
a43         rep(i, 0, sz(q)) {
4c1             int v = q[i];
5aa             for (auto x : g[v]) {
464                 if (label[x] == -1) {
73a                     label[x] = 1; par[x] = v;
1bd                     if (mate[x] == -1) return aug(x), 1;
8d9                     label[mate[x]] = 0;
de3                     q.push_back(mate[x]);
641                 }
018                 else if(!label[x] && orig[v] != orig[x]){
37f                     int a = lca(orig[v], orig[x]);
f12                     blossom(x, v, a);
183                     blossom(v, x, a);
405                 }
ab5             }
9e2         }
bb3         return 0;
139     };
        // Time halves if you start with (any) maximal
            matching.
fea     rep(i, 0, n) {
698         if (mate[i] == -1) bfs(i);
7b5     }
568     return mate;
21c }
```

HopcroftKarp.h

Description: *ans* is the size of the max matching.
The match of x is $l[x]$
Usage: `HopcroftKarp(|X|, |Y|, edges(x, y))`
Time: $\mathcal{O}\left(\sqrt{V}E\right)$

c4f2f2, 46 lines

```
725 struct HopcroftKarp {
e40     vector<int> g, l, r;
959     int ans;
b82     HopcroftKarp(int n, int m, vector<pii> e)
aa0         : g(sz(e)), l(n, -1), r(m, -1), ans(0) {
bb0         shuffle(all(e), rng);
322         vector<int> deg(n + 1);
235         for (auto& [x, y] : e) deg[x]++;
b4a         rep(i, 1, n+1) deg[i] += deg[i - 1];
85a         for (auto& [x, y] : e) g[--deg[x]] = y;

5ae         vector<int> q(n);
667         while (true) {
661             vector<int> a(n, -1), p(n, -1);
6bb             int t = 0;
fea             rep(i, 0, n){
4b1                 if (l[i] == -1) {
b53                     q[t++] = a[i] = p[i] = i;
4b6                 }
62e             }
a15             bool match = false;
edb             rep(i, 0, t){
912                 int x = q[i];
08c                 if (~l[a[x]]) continue;
0ba                 rep(j, deg[x], deg[x+1]) {
360                     int y = g[j];
89a                     if (r[y] == -1) {
d3b                         while (~y) {
ee7                             r[y] = x;
dbb                             swap(l[x], y);
2a5                             x = p[x];
ebf                         }
6aa                         match = true, ans++;
c2b                         break;
b54                     }
f06                     if (p[r[y]] == -1) {
a74                         q[t++] = y = r[y];
d11                         p[y] = x, a[y] = a[x];
9ef                     }
e8a                 }
0ab             }
984             if (!match) break;
bc5         }
6ec     }
c4f };
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \le M$.
Time: $\mathcal{O}\left(N^2M\right)$

4a75d2, 41 lines

```
d57 pair<ll, vector<int>> hunga(const vector<vector<ll>>& a) {
c04     if (a.empty()) return { 0, {} };
1a9     int n = sz(a) + 1, m = sz(a[0]) + 1;
fc8     vector<ll> u(n), v(m), p(m);
5bd     vector<int> ans(n - 1);
6f5     for (int i = 1; i < n; i++) {
8c9         p[0] = i;
625         int j0 = 0;
91d         vector<ll> dist(m, LLONG_MAX), pre(m, -1);
910         vector<bool> done(m + 1);
016         do {
781             done[j0] = true;
507             ll i0 = p[j0], j1 = -1, delta = LLONG_MAX;
b84             for (int j = 1; j < m; j++) {
10a                 if (!done[j]) {
ed6                     ll cur = a[i0-1][j-1] - u[i0] - v[j];
607                     if (cur < dist[j])
29f                         dist[j] = cur, pre[j] = j0;
172                     if(dist[j] < delta)
4ab                         delta = dist[j], j1 = j;
103                 }
bb2             }
891             for (int j = 0; j < m; j++) {
7a9                 if (done[j])
3bc                     u[p[j]] += delta, v[j] -= delta;
202                 else dist[j] -= delta;
11a             }
e73             assert(j1 != -1);
6d4             j0 = j1;
ac1         } while (p[j0]);
4b9         while (j0) {
196             int j1 = pre[j0];
0c1             p[j0] = p[j1], j0 = j1;
f55         }
193     }
b84     for (int j = 1; j < m; j++) {
eb3         if (p[j]) ans[p[j] - 1] = j - 1;
c9a     }
def     return { -v[0], ans }; // min cost
4a7 }
```

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}\left(V^3\right)$

8b0e19, 22 lines

```
192 pair<int, vi> globalMinCut(vector<vi> mat) {
afa   pair<int, vi> best = {INT_MAX, {}};
755   int n = sz(mat);
91d   vector<vi> co(n);
d0f   rep(i,0,n) co[i] = {i};
488   rep(ph,1,n) {
2e9     vi w = mat[0];
e44     size_t s = 0, t = 0;
694     rep(it,0,n-ph) { // O(V^2) -> O(E log V) with prio.
    queue
d6e       w[t] = INT_MIN;
a5f       s = t, t = max_element(all(w)) - w.begin();
d39       rep(i,0,n) w[i] += mat[t][i];
ec9     }
3df     best = min(best, {w[t] - mat[t][t], co[t]});
096     co[s].insert(co[s].end(), all(co[t]));
959     rep(i,0,n) mat[s][i] += mat[t][i];
984     rep(i,0,n) mat[i][s] = mat[s][i];
5dd     mat[0][t] = INT_MIN;
ca0   }
f26   return best;
8b0 }
```

7.3 DFS algorithms

Bridges.h

1fa56b, 24 lines

```
cd9 vector<int> g[ms];
9e4 int low[ms], tin[ms], vis[ms], t;

403 void dfs(int u = 0, int p = -1) {
b9c     vis[u] = true;
b4a     low[u] = tin[u] = t++;
```

```
    for (auto v : g[u]) {
        if (v == p) continue;
        if (vis[v]) {
            low[u] = min(low[u], tin[v]);
        }
        else {
            dfs(v, u);
            low[u] = min(low[u], low[v]);
            // if (low[v] >= tin[u] && p != -1), U is an articulation point
            if (low[v] > tin[u]) {
                // edge from U to V is a bridge
            }
            // children++
        }
    }
    // if(children > 1 && p == -1) root is an articulation point
}
```

BridgeOnline.h

Description: Maintains bridges and 2-edge-connected components (2-ECC) incrementally. `ds[0]` tracks Connected Components (CC). `ds[1]` tracks 2-ECCs. Nodes u, v are in the same 2-ECC iff `dsfind(u, 1) == dsfind(v, 1)`. `g` stores the spanning forest edges (edges that were bridges when added). An edge $(u, v) \in g$ is a current bridge iff `dsfind(u, 1) != dsfind(v, 1)`. `bridges` tracks the total count of active bridges. Use `init()` before starting.
Time: Amortized $\mathcal{O}(\log N)$

ef24c8, 75 lines

```
int bridges;
int ds[2][ms], sz[2][ms];
int h[ms], pai[ms], old[ms];
vector<int> g[ms];

void init() {
    bridges = 0;
    rep(i, 0, ms) {
        g[i].clear(), h[i] = 0;
        ds[0][i] = ds[1][i] = i;
        sz[0][i] = sz[1][i] = 1;
    }
}

int dsfind(int j, int i) {
    if(j == ds[i][j]) return ds[i][j];
    return ds[i][j] = dsfind(ds[i][j], i);
}

void dfs(int u, int p, int l) {
    h[u] = l;
    pai[u] = p;
    old[u] = dsfind(u, 1);
    for (int v : g[u]) {
        if (v == p) continue;
        dfs(v, u, l + 1);
    }
}

void updateNodes(int u, int p) {
    if (old[u] == old[p]) {
        ds[1][u] = ds[1][p];
    }
    else ds[1][u] = u;
    for (int v : g[u]) {
        if (v == p) continue;
        updateNodes(v, u);
    }
}

void mergeTrees(int a, int b) {
    bridges++;
    int iniA = a, iniB = b;
    a = dsfind(a, 0), b = dsfind(b, 0);
    if (sz[0][a] < sz[0][b]) swap(a, b), swap(iniA, iniB);
    dfs(iniB, iniA, h[iniA] + 1);
    old[iniA] = -1;
    updateNodes(iniB, iniA);
    ds[0][b] = a;
    sz[0][a] += sz[0][b];
}

void removeBridges(int a, int b) {
    a = dsfind(a, 1), b = dsfind(b, 1);
    while (a != b) {
        bridges--;
        if (h[a] < h[b]) swap(a, b);
        // ponte entre (a, pai[a]) deixou de existir
        ds[1][a] = dsfind(pai[a], 1);
        a = ds[1][a];
    }
}

void addEdge(int a, int b) {
    if (dsfind(a, 0) == dsfind(b, 0)) {
        removeBridges(a, b);
    }
    else {
        // nova ponte entre (a, b)
        g[a].push_back(b);
        g[b].push_back(a);
        mergeTrees(a, b);
    }
}
```

BlockCutTree.h

Description: Constructs the Block-Cut Tree, which is a bipartite graph with blocks (maximal 2-vertex-connected components) on one side and articulation points on the other. Works for disconnected graphs. Tree size is $\leq 2N$. Be careful with self loops and multi edges. `art[i]`: number of new components created by removing i (AP if ≥ 1). `blocks[i]`, `edgblocks[i]`: vertices/edges of block i. `tree[i]`: the tree node index corresponding to block i. `pos[i]`: the tree node index corresponding to vertex i.
Time: $\mathcal{O}(N + M)$

e55ab0, 66 lines

```
struct block_cut_tree {
 vector<vector<int>> g, blocks, tree;
 vector<vector<pair<int, int>>> edgblocks;
 stack<int> s;
 stack<pair<int, int>> s2;
 vector<int> id, art, pos;

 block_cut_tree(vector<vector<int>> g_) : g(g_) {
   int n = sz(g);
   id.resize(n, -1), art.resize(n), pos.resize(n);
   build();
 }

 int dfs(int i, int& t, int p = -1) {
   int lo = id[i] = t++;
   s.push(i);

   if (p != -1) s2.emplace(i, p);
   for (int j : g[i])
           if (j != p and id[j] != -1) s2.emplace(i, j);

   for (int j : g[i]) if (j != p) {
     if (id[j] == -1) {
       int val = dfs(j, t, i);
       lo = min(lo, val);

       if (val >= id[i]) {
         art[i]++;
         blocks.emplace_back(1, i);
         while (blocks.back().back() != j)
           blocks.back().push_back(s.top()), s.pop();

         edgblocks.emplace_back(1, s2.top()), s2.pop();
         while (edgblocks.back().back() != pii(j, i))
           edgblocks.back().push_back(s2.top()), s2.pop();
       }
     }
     else lo = min(lo, id[j]);
   }
   if (p == -1) {
           if(art[i]) art[i]--;
           else{
               blocks.emplace_back(1, i);
               edgblocks.emplace_back();
           }
      }
   return lo;
 }

 void build() {
   int t = 0;
   rep(i, 0, sz(g)) if(id[i] == -1) dfs(i, t, -1);
   tree.resize(sz(blocks));
   rep(i, 0, sz(g)) if (art[i])
     pos[i] = sz(tree), tree.emplace_back();

   rep(i, 0, sz(blocks)) for (int j : blocks[i]) {
     if (!art[j]) pos[j] = i;
     else {
               tree[i].push_back(pos[j]);
               tree[pos[j]].push_back(i);
           }
   }
 }
};
```

DominatorTree.h

Description: Builds the Dominator Tree of a directed graph rooted at `root`. Node u dominates v if every path from `root` to v passes through u. The immediate dominator of v is the unique dominator closest to v (excluding v). Returns a vector `par` where `par[u]` is the parent of u in the tree. Roots and unreachable nodes satisfy `par[u] = u`.
Time: $\mathcal{O}(M \log N)$

8c4613, 55 lines

```
struct dominator_tree {
 int n, t;
 vector<vector<int>> g, rg, bucket;
 vector<int> arr, par, rev, sdom, dom, ds, lbl;

 dominator_tree(int n) : n(n),t(0),g(n),rg(n),bucket(n),
   arr(n,-1),par(n),rev(n),sdom(n),dom(n),ds(n),lbl(n) {}

 void add_edge(int u, int v) { g[u].push_back(v); }

 void dfs(int u) {
   arr[u] = t;
   rev[t] = u;
   lbl[t] = sdom[t] = ds[t] = t;
    t++;
   for (int w : g[u]) {
     if (arr[w] == -1) {
       dfs(w);
       par[arr[w]] = arr[u];
```

```
869       }
f8e       rg[arr[w]].push_back(arr[u]);
93a     }
b04   }
792   int find(int u, int x=0) {
9fe     if (u == ds[u]) return x ? -1 : u;
41f     int v = find(ds[u], x+1);
388     if (v < 0) return u;
b30     if(sdom[lbl[ds[u]]] < sdom[lbl[u]]) lbl[u]= lbl[ds[u]];
300     ds[u] = v;
784     return x ? v : lbl[u];
a59   }

46f   vector<int> run(int root) {
14e     dfs(root);
b81     iota(all(dom), 0);
da8     for (int i=t-1; i>=0; i--) {
76c       for(int w : rg[i]) sdom[i] = min(sdom[i],sdom[find(w)]);
c94       if (i) bucket[sdom[i]].push_back(i);
3b2       for (int w : bucket[i]) {
46a         int v = find(w);
ae4         if (sdom[v] == sdom[w]) dom[w] = sdom[w];
41c         else dom[w] = v;
1e6       }
fd8       if (i > 1) ds[i] = par[i];
b9e     }
e8f     rep(i, 1, t) {
7d7       if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
32d     }
af8     vector<int> par(n);
2c2     iota(all(par), 0);
533     rep(i, 0, t) par[rev[i]] = rev[dom[i]];
148     return par;
900   }
8c4 };
```

EulerPath.h

Description: Receives as input graph(node, edge index), number of edges and source. Returns list of node, index of edge he came from, if path/circuit does not exists returns empty list.

a3ed13, 27 lines

```
b4a  vector<pii> eulerPath(const vector<vector<pii>>& g, int nedges, int src) {
625      int n = sz(g);
b47      vector<int> deg(n, 0), its(n, 0), used(nedges + 1, 0);
a42      vector<pii> s = { {src, -1} };
         //deg[src]++; //to allow paths, not only circuits
a5f      vector<pii> ret;
980      while (!s.empty()) {
d0b          int u = s.back().first, &it = its[u];
c45          if (it == sz(g[u])) {
5e3              ret.push_back(s.back());
342              s.pop_back();
5e2              continue;
8e8          }
f7f          auto& [nxt, id] = g[u][it++];
b25          if (!used[id]) {
e48              deg[u]--, deg[nxt]++;
029              used[id] = 1;
e1c              s.push_back({ nxt, id });
777          }
388      }
8d8      for (int x : deg) {
518          if (x < 0 || sz(ret) != (nedges + 1)) return {};
26e      }
969      reverse(ret.begin(), ret.end());
edf      return ret;
a3e  }
```

SCC.h

Description: Kosaraju algorithm for calculating strongly connected components. Components are ordered in topological order.

008ff2, 36 lines

```
bf0  struct SCC {
dab      int n, ncomp;
0e3      vector<vector<int>> g, inv;
829      vector<int> comp, vis, stk;
8b6      SCC(){}
471      SCC(int n)
464          : n(n),ncomp(0),g(n),inv(n),comp(n, -1),vis(n){}

315      void dfs(int u) {
150          vis[u] = 1;
a35          for (int v : g[u]) if (!vis[v]) dfs(v);
967          stk.push_back(u);
37b      }
f20      void dfs_inv(int u) {
62c          comp[u] = ncomp;
3a5          for (int v : inv[u]) {
df4              if (comp[v] == -1) dfs_inv(v);
0a0          }
984      }
63d      void solve() {
603          for (int i = 0; i < n; i++) {
b65              if (!vis[i]) dfs(i);
358          }
340          reverse(all(stk));
49b          for (int u : stk) {
9ef              if (comp[u] != -1) continue;
672              dfs_inv(u);
a8f              ncomp++;
ecb          }
ef8      }
010      void add_edge(int a, int b) {
025          g[a].push_back(b);
a6a          inv[b].push_back(a);
1ec      }
008  };
```

TwoSat.h

Usage: not A = ~A

"SCC.h" c8b989, 37 lines

```
d9d  struct TwoSat{
1a8   int n;
3c9   SCC scc;
7c7   vector<int> value;
425   vector<pii> e;
e2c   TwoSat(int n) : n(n){}
6c0   bool solve(){
b36     value.resize(n);
8cc     scc = SCC(2*n);
1f3     for(auto &x : e) scc.add_edge(x.first, x.second);
7f9     scc.solve();
3df     for(int i=0; i<2*n; i++)
f83       if(scc.comp[i] == scc.comp[i^1]) return false;
830     for(int i=0; i<n; i++)
733       value[i] = scc.comp[id(i)] > scc.comp[id(~i)];
8a6     return true;
949   }
a0a   void atMostOne(vector<int> &li){
615     if(sz(li) <= 1) return;
da9     int cur = ~li[0];
b25     for(int i = 2; i < sz(li); i++) {
abb       int next = n++;
e0a       addOr(cur, ~li[i]);
f26       addOr(cur, next);
7ba       addOr(~li[i], next);
072       cur = ~next;
e3d     }
921     addOr(cur, ~li[1]);
bbb   }
41b   int id(int v) { return v < 0 ? (~v) * 2 ^ 1 : v * 2; }
276   void add(int a, int b) { e.push_back({id(a), id(b)}); }
bc7   void addOr(int a, int b) { add(~a, b); add(~b, a); }
671   void addImp(int a, int b) { addOr(~a, b); }
d9d   void addEqual(int a, int b){ addOr(a, ~b); addOr(~a, b); }
ec3   void isFalse(int a) { addImp(a, ~a); }
c8b  };
```

7.4 Coloring

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)

Time: $\mathcal{O}(NM)$

e210e2, 32 lines

```
f41  vi edgeColoring(int N, vector<pii> eds) {
727    vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
10d    for (pii e : eds) ++cc[e.first], ++cc[e.second];
e2f    int u, v, ncols = *max_element(all(cc)) + 1;
fda    vector<vi> adj(N, vi(ncols, -1));
6ec    for (pii e : eds) {
119      tie(u, v) = e;
e51      fan[0] = v;
0f4      loc.assign(ncols, 0);
696      int at = u, end = u, d, c = free[u], ind = 0, i = 0;
3b2      while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
3e1        loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
01e      cc[loc[d]] = c;
997      for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
4ff        swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
79f      while (adj[fan[i]][d] != -1) {
a9f        int left = fan[i], right = fan[++i], e = cc[i];
99b        adj[u][e] = left;
ccb        adj[left][e] = u;
f7e        adj[right][e] = -1;
d99        free[right] = e;
316      }
dfd      adj[u][d] = fan[i];
c45      adj[fan[i]][d] = u;
0e1      for (int y : {fan[0], u, end})
3fa        for (int& z = free[y] = 0; adj[y][z] != -1; z++);
fdc    }
29d    rep(i,0,sz(eds))
961      for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
edf    return ret;
e21  }
```

7.5 Heuristics

MaxClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

2eeaf4, 53 lines

```
db9  using vb = vector<bitset<200>>;
c7d  struct Maxclique {
24e    double limit=0.025, pk=0;
c04    struct Vertex { int i, d=0; };
547    using vv = vector<Vertex>;
d44    vb e;
```

```
df7   vv V;
e5c   vector<vector<int>> C;
497   vector<int> qmax, q, S, old;
fe3   void init(vv& r) {
fd3     for (auto& v : r) v.d = 0;
583     for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
0f1     sort(all(r), [](auto a, auto b) { return a.d > b.d; });
c43     int mxD = r[0].d;
3f8     for(int i=0; i<sz(r); i++) r[i].d = min(i, mxD) + 1;
526   }
bc8   void expand(vv& R, int lev = 1) {
ac1     S[lev] += S[lev - 1] - old[lev];
92c     old[lev] = S[lev - 1];
d18     while (sz(R)) {
3fd       if (sz(q) + R.back().d <= sz(qmax)) return;
d62       q.push_back(R.back().i);
f28       vv T;
7fb       for(auto v : R)
740         if (e[R.back().i][v.i]) T.push_back({v.i});
d21       if (sz(T)) {
eea         if (S[lev]++ / ++pk < limit) init(T);
457         int j = 0, mxk = 1, mnk = max(sz(qmax)-sz(q)+1, 1);
9bc         C[1].clear(), C[2].clear();
969         for (auto v : T) {
bfe           int k = 1;
8f5           auto f = [&](int i) { return e[v.i][i]; };
5c6           while (any_of(all(C[k]), f)) k++;
782           if (k > mxk) mxk = k, C[mxk + 1].clear();
18a           if (k < mnk) T[j++].i = v.i;
0e6           C[k].push_back(v.i);
322         }
238         if (j > 0) T[j - 1].d = 0;
d2f         for(int k=mnk; k<mxk + 1; k++){
5bf                 for (int i : C[k])
361               T[j].i = i, T[j++].d = k;
9dc             }
22d         expand(T, lev + 1);
61f       } else if (sz(q) > sz(qmax)) qmax = q;
c81       q.pop_back(), R.pop_back();
3e0     }
81d   }
b2d   vector<int> maxClique(){ init(V),expand(V); return qmax;}
b40   Maxclique(vb conn) : e(conn),C(sz(e)+1),S(sz(C)),old(S){
01d     for(int i=0; i<sz(e); i++) V.push_back({i});
b60   }
534 };
```

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

b0d5b1, 13 lines

```
753 typedef bitset<128> B;
044 template<class F>
6a9 void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R
    ={}) {
9bb   if (!P.any()) { if (!X.any()) f(R); return; }
a8e   auto q = (P | X)._Find_first();
cd1   auto cands = P & ~eds[q];
3d7   rep(i,0,sz(eds)) if (cands[i]) {
a75     R[i] = 1;
e78     cliques(eds, f, P & eds[i], X & eds[i], R);
bb6     R[i] = P[i] = 0; X[i] = 1;
181   }
c9d }
```

7.6 Trees

Centroid.h

Description: Call decomp(0) to solve, marked array should be initially set to zero.

Time: $\mathcal{O}(N \log N)$

b73755, 27 lines

```
6b6 int tam[ms], marked[ms];

2a1 int calc_tam(int u, int p) {
5d1   tam[u] = 1;
4d5   for (int v : g[u]) {
5ea     if (v != p && !marked[v]) tam[u] += calc_tam(v, u);
d09   }
f95   return tam[u];
d5d }

5fb int get_centroid(int u, int p, int tot) {
4d5   for (int v : g[u]) {
38c     if (v != p && !marked[v] && (tam[v] > (tot / 2)))
32c       return get_centroid(v, u, tot);
b6c   }
03f   return u;
0c7 }
    // Cent is a child of P in the centroid tree
179 void decomp(int u, int p = -1) {
308   calc_tam(u, -1);
bd4   int cent = get_centroid(u, -1, tam[u]);
83d   marked[cent] = 1;
9f1   for (int v : g[cent]) {
c6e     if (!marked[v]) decomp(v, cent);
194   }
dc1 }
```

HLD.h

Description: If values are stored on edges, set EDGE = true and store each edge's value at the endpoint farther from the root (the deeper node). rp[i] is the representative (head) of the heavy path containing node i: it is the node in that chain that is closest to the root.

a129d6, 51 lines

```
5f2 template<bool EDGE> struct HLD {
577     int n, t;
789     vector<vector<int>> g;
003     vector<int> pai, rp, tam, pos, val, arr;
f1e     Seg seg;
bcf     HLD(int n, vector<vector<int>>& g, vector<int>& val)
ac9         : n(n), t(0), g(g), pai(n), rp(n), tam(n, 1),
616           pos(n), val(val), arr(n) {
f80         calc_tam(0, -1);
c91         dfs(0, -1);
d14         seg.build(arr);
a43     }

2a1     int calc_tam(int u, int p) {
49e         pai[u] = p;
704         for (int& v : g[u]) {
730             if (v == p) continue;
2e4             tam[u] += calc_tam(v, u);
2d5             if(tam[v] > tam[g[u][0]] || g[u][0] == p)
a7f                 swap(g[u][0], v);
0a3         }
f95         return tam[u];
c19     }

fb6     void dfs(int u, int p) {
4c8         pos[u] = t++;
d7b         arr[pos[u]] = val[u];
4d5         for (int v : g[u]) {
730             if (v == p) continue;
84d             rp[v] = (v == g[u][0] ? rp[u] : v);
95e             dfs(v, u);
42d         }
de1     }

4ea     int query(int a, int b) { // query on the path from a
        to b
1a4         int ans = 0; // neutral value
34d         while (rp[a] != rp[b]) {
aa1             if (pos[a] < pos[b]) swap(a, b);
9a5             ans = max(ans, seg.query(pos[rp[a]], pos[a]));
677             a = pai[rp[a]];
ebd         }
9bc         if(pos[a] > pos[b]) swap(a, b);
0f8         ans = max(ans, seg.query(pos[a] + EDGE, pos[b]));
ba7         return ans;
e8a     }

534     void update(int a, int x) {
e5e         seg.update(pos[a], x);
5db     }
a12 };
```

LCA.h

Description: LCA algorithm using binary lifting, $is\_ancestor(a, b)$ returns true if a is an ancestral of b and false otherwise.

Time: $\mathcal{O}(N \log N)$

db7791, 26 lines

```
67e int tin[MAXN], tout[MAXN], timer=0;
768 int up[MAXN][BITS];
fb6 void dfs(int u, int p){
545     tin[u] = timer++, up[u][0] = p;
532     for (int i=1; i<BITS; i++) {
88a         up[u][i] = up[up[u][i-1]][i-1];
4a0     }
712     for (int v: g[u]) if (v != p) dfs(v, u);
4f8     tout[u] = timer;
4a1 }

f31 bool is_ancestor(int u, int v){
d34     return (tin[u] <= tin[v] && tout[u] >= tout[v]);
f9f }

310 int lca(int u, int v){
bd5     if (is_ancestor(u, v)) return u;
6fc     if (is_ancestor(v, u)) return v;
3c3     for (int i=BITS-1; i>=0; i--) {
3a3         if (up[u][i] && !is_ancestor(up[u][i], v)) {
c3f             u = up[u][i];
49e         }
dc4     }
c15     return up[u][0];
001 }
```

VirtualTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S|-1$) pairwise LCA's and compressing edges. virt[u] is the adjacency list of the virtual tree: it stores pairs (v, dist), where v is a neighbor of u in the virtual tree and dist is the distance between u and v in the original tree.

Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h" 11157a, 24 lines

```
0b1 vector<pair<int, int>> virt[ms];

d0c void build_virt(vector<int>& v) {
078   auto cmp = [&](int i, int j){ return tin[i] < tin[j]; };
b84   sort(all(v), cmp);
1ee   for (int i = 0, n = sz(v);i + 1 < n; i++)
4cf     v.push_back(lca(v[i], v[i + 1]));
b84   sort(all(v), cmp);
```

```
64f    v.erase(unique(all(v)), v.end());
7b4    stack<int> st;
3a7    for (auto u : v) {
c53      if (st.empty()) {
4a6        st.push(u);
e82      }
4e6      else {
7eb        while(sz(st) && !is_ancestor(st.top(), u)) st.pop();
88b        int p = st.top();
bfa        virt[p].emplace_back(u, abs(lvl[u] - lvl[p]));
0a5        virt[u].emplace_back(p, abs(lvl[u] - lvl[p]));
4a6        st.push(u);
92c      }
f46    }
c83  }
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.

Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" 39e620, 61 lines

```
030  struct Edge { int a, b; ll w; };
bf2  struct Node {
25f    Edge key;
c17    Node *l, *r;
981    ll delta;
a9c    void prop() {
6f9      key.w += delta;
d2d      if (l) l->delta += delta;
d86      if (r) r->delta += delta;
978      delta = 0;
0d3    }
866    Edge top() { prop(); return key; }
ab4  };
3eb  Node *merge(Node *a, Node *b) {
b9f    if (!a || !b) return a ?: b;
626    a->prop(), b->prop();
dc2    if (a->key.w > b->key.w) swap(a, b);
485    swap(a->l, (a->r = merge(b, a->r)));
3f5    return a;
c51  }
7bb  void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

002  pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
8df    RollbackUF uf(n);
3f8    vector<Node*> heap(n);
563    for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
cd2    ll res = 0;
517    vi seen(n, -1), path(n), par(n);
559    seen[r] = r;
dd6    vector<Edge> Q(n), in(n, {-1,-1}), comp;
111    deque<tuple<int, int, vector<Edge>>> cycs;
328    rep(s,0,n) {
3cb      int u = s, qi = 0, w;
a0a      while (seen[u] < 0) {
572        if (!heap[u]) return {-1,{}};
ebe        Edge e = heap[u]->top();
5ed        heap[u]->delta -= e.w, pop(heap[u]);
952        Q[qi] = e, path[qi++] = u, seen[u] = s;
d56        res += e.w, u = uf.find(e.a);
9e2        if (seen[u] == s) {
28d          Node* cyc = 0;
cab          int end = qi, time = uf.time();
f38          do cyc = merge(cyc, heap[w = path[--qi]]);
4f9          while (uf.join(u, w));
562          u = uf.find(u), heap[u] = cyc, seen[u] = -1;
c06          cycs.push_front({u, time, {&Q[qi], &Q[end]}});
00a        }
c8f      }
068      rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
fa3    }

e41    for (auto& [u,t,comp] : cycs) { // restore sol (optional)
36c      uf.rollback(t);
1d0      Edge inEdge = in[u];
251      for (auto& e : comp) in[uf.find(e.b)] = e;
56d      in[uf.find(inEdge.b)] = inEdge;
4f9    }
427    rep(i,0,n) par[i] = in[i].a;
efb    return {res, par};
efa  }
```

Geometry (8)

8.1 Geometric primitives

Point.h

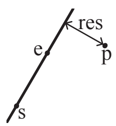
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

47ec0a, 29 lines

```
48b  template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
4fc  template<class T>
f26  struct Point {
ea4    typedef Point P;
645    T x, y;
ea6    explicit Point(T x=0, T y=0) : x(x), y(y) {}
0d0    bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
ec7    bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
279    P operator+(P p) const { return P(x+p.x, y+p.y); }
40d    P operator-(P p) const { return P(x-p.x, y-p.y); }
e03    P operator*(T d) const { return P(x*d, y*d); }
0b9    P operator/(T d) const { return P(x/d, y/d); }
57b    T dot(P p) const { return x*p.x + y*p.y; }
460    T cross(P p) const { return x*p.y - y*p.x; }
b3f    T cross(P a, P b) const { return (a-*this).cross(b-*this); }
f68    T dist2() const { return x*x + y*y; }
18b    double dist() const { return sqrt((double)dist2()); }
       // angle to x-axis in interval [-pi, pi]
907    double angle() const { return atan2(y, x); }
d06    P unit() const { return *this/dist(); } // makes dist()=1
200    P perp() const { return P(-y, x); } // rotates +90 degrees
852    P normal() const { return perp().unit(); }
       // returns point rotated 'a' radians ccw around the origin
f23    P rotate(double a) const {
482      return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
902    friend ostream& operator<<(ostream& os, P p) {
9a9      return os << "(" << p.x << "," << p.y << ")"; }
d2d  };
```

lineDistance.h

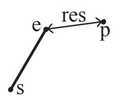
Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.



"Point.h" f6bf6b, 5 lines

```
7dc  template<class P>
2ff  double lineDist(const P& a, const P& b, const P& p) {
e07    return (double)(b-a).cross(p-a)/(b-a).dist();
008  }
```

SegmentDistance.h



Description:
Returns the shortest distance between point p and the line segment from point s to e.

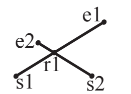
Usage: `Point<double> a, b(2,2), p(1,1);`
`bool onSegment = segDist(a,b,p) < 1e-10;`

"Point.h" 5c88f4, 7 lines

```
626  typedef Point<double> P;
929  double segDist(P& s, P& e, P& p) {
a44    if (s==e) return (p-s).dist();
f81    auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
2c1    return ((p-s)*d-(e-s)*t).dist()/d;
ae7  }
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



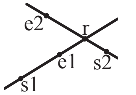
Usage: `vector<P> inter = segInter(s1,e1,s2,e2);`
`if (sz(inter)==1)`
`cout << "segments intersect at " << inter[0] << endl;`

"Point.h", "OnSegment.h" 9d57f2, 14 lines

```
dae  template<class P> vector<P> segInter(P a, P b, P c, P d) {
0b6    auto oa = c.cross(d, a), ob = c.cross(d, b),
318         oc = a.cross(b, c), od = a.cross(b, d);
       // Checks if intersection is single non-endpoint point.
914    if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
e5b      return {(a * ob - b * oa) / (ob - oa)};
4c1    set<P> s;
ccb    if (onSegment(c, d, a)) s.insert(a);
0ad    if (onSegment(c, d, b)) s.insert(b);
3d8    if (onSegment(a, b, c)) s.insert(c);
2fa    if (onSegment(a, b, d)) s.insert(d);
a35    return {all(s)};
9d5  }
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage: `auto res = lineInter(s1,e1,s2,e2);`
`if (res.first == 1)`
`cout << "intersection point at " << res.second << endl;`

"Point.h" a01f81, 9 lines

```
7dc  template<class P>
0bf  pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
14f    auto d = (e1 - s1).cross(e2 - s2);
8cc    if (d == 0) // if parallel
d99      return {-(s1.cross(e1, s2) == 0), P(0, 0)};
f6b    auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
```

```
9b8   return {1, (s1 * p + e1 * q) / d};
472 }
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.
Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h" 3af81c, 10 lines

```
7dc template<class P>
70b int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

7dc template<class P>
b5e int sideOf(const P& s, const P& e, const P& p, double eps)
    {
79e   auto a = (e-s).cross(p-s);
653   double l = (e-s).dist()*eps;
c32   return (a > l) - (a < -l);
33f }
```

OnSegment.h

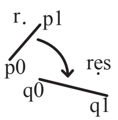
Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

"Point.h" c597e8, 4 lines

```
514 template<class P> bool onSegment(P s, P e, P p) {
5fb   return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
c59 }
```

linearTransformation.h

Description:



Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.

"Point.h" 03a306, 7 lines

```
626 typedef Point<double> P;
664 P linearTransformation(const P& p0, const P& p1,
f06     const P& q0, const P& q1, const P& r) {
99f   P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
0aa   return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist
    2();
45e }
```

LineProjectionReflection.h

Description: Projects point p onto line ab. Set refl=true to get reflection of point p across line ab instead. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h" b5562d, 6 lines

```
7dc template<class P>
981 P lineProj(P a, P b, P p, bool refl=false) {
de3   P v = b - a;
3fc   return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
4b7 }
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.
Usage: `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; } // sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i`

0f0602, 36 lines

```
755 struct Angle {
e91   int x, y;
8bd   int t;
5ac   Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
de8   Angle operator-(Angle b) const { return {x-b.x, y-b.y, t
    }; }
3cd   int half() const {
840     assert(x || y);
aa4     return y < 0 || (y == 0 && x < 0);
c93   }
dfc   Angle t90() const { return {-y, x, t + (half() && x >= 0)
    }; }
726   Angle t180() const { return {-x, -y, t + half()}; }
925   Angle t360() const { return {x, y, t + 1}; }
e25 };
a92 bool operator<(Angle a, Angle b) {
    // add a.dist2() and b.dist2() to also compare distances
ea7   return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
05f          make_tuple(b.t, b.half(), a.x * (ll)b.y);
ce5 }

    // Given two points, this calculates the smallest angle
        between
    // them, i.e., the angle that covers the defined line
        segment.
908 pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
ee4   if (b < a) swap(a, b);
423   return (b < a.t180() ?
c35           make_pair(a, b) : make_pair(b, a.t360()));
5ea }
784 Angle operator+(Angle a, Angle b) { // point a + vector b
eb1   Angle r(a.x + b.x, a.y + b.y, a.t);
8ca   if (a.t180() < r) r.t--;
d9f   return r.t180() < a ? r.t360() : r;
3d8 }
106 Angle angleDiff(Angle a, Angle b) { // angle b - angle a
125   int tu = b.t - a.t; a.t = b.t;
e63   return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a
    )};
ba3 }
```

HalfPlane.h

Description: Computes the intersection of a set of half-planes. Half-planes are sorted by angle and processed with a deque, removing redundant or conflicting constraints. Parallel half-planes are handled explicitly. Returns the convex polygon of the intersection, or empty if infeasible.
Time: $\mathcal{O}(n \log n)$

"Point.h" cf24a8, 72 lines

```
984 using ld = long double;
207 using P = Point<ld>;

533 struct Hp { // Half plane struct
      // 'p' is a passing point of the line and 'pq' is the
          direction vector of the line.
812   P p, pq;
d29   ld angle;

b93   Hp() {}
65d   Hp(const P& a, const P& b) : p(a), pq(b - a) {
0e3     angle = atan2l(pq.y, pq.x);
2ff   }
8ce   bool out(const P& r) { return pq.cross(r - p) < -eps; }
d36   bool operator < (const Hp& e) const {
1dd     return angle < e.angle;
44e   }
ea9   friend P inter(const Hp& s, const Hp& t) {
020     ld alpha = (t.p - s.p).cross(t.pq) / s.pq.cross(t.pq);
93b     return s.p + (s.pq * alpha);
825   }
b46 };

fa5 vector<P> hp_intersect(vector<Hp>& H) {
12f   P box[4] = { P(inf, inf), P(-inf, inf),
9c8                P(-inf, -inf), P(inf, -inf) };

1cd   for(int i = 0; i<4; i++) {
1a8     Hp aux(box[i], box[(i+1) % 4]);
d82     H.push_back(aux);
560   }
f1a   sort(all(H));
6c5   deque<Hp> dq;
486   int len = 0;
908   for(int i = 0; i < sz(H); i++) {
3fb     while(len>1 && H[i].out(inter(dq[len-1], dq[len-2]))){
c70       dq.pop_back();
654       --len;
a31     }
757     while (len > 1 && H[i].out(inter(dq[0], dq[1]))){
c68       dq.pop_front();
654       --len;
1eb     }
a5a     if(len && fabsl(H[i].pq.cross(dq[len-1].pq)) < eps){
25f       if (H[i].pq.dot(dq[len-1].pq) < 0.0)
282         return vector<P>();
e7b       if (H[i].out(dq[len-1].p)) {
c70         dq.pop_back();
654         --len;
2dc       }
64e       else continue;
9a0     }
fc2     dq.push_back(H[i]);
250     ++len;
8ed   }

337   while(len> 2 && dq[0].out(inter(dq[len-1], dq[len-2]))){
c70     dq.pop_back();
654     --len;
faa   }
81e   while (len > 2 && dq[len-1].out(inter(dq[0], dq[1]))){
c68     dq.pop_front();
654     --len;
694   }
1a3   if (len < 3) return vector<P>();
7e7   vector<P> ret(len);
cc7   for(int i = 0; i+1 < len; i++) {
01e     ret[i] = inter(dq[i], dq[i+1]);
00f   }
4fd   ret.back() = inter(dq[len-1], dq[0]);
edf   return ret;
deb }
```

8.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" ba7267, 12 lines

```
626 typedef Point<double> P;
27f bool circleInter(P a,P b,double r1,double r2,pair<P, P>*
    out) {
b48   if (a == b) { assert(r1 != r2); return false; }
f30   P vec = b - a;
6c8   double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2;
c28   double p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*
    d2;
5b0   if (sum*sum < d2 || dif*dif > d2) return false;
84d   P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) /
    d2);
21e   *out = {mid + per, mid - per};
```

```
8a6   return true;
170 }
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" b0153d, 14 lines

```
7dc template<class P>
3a5 vector<pair<P, P>> tangents(P c1, double r1, P c2, double
      r2) {
c0b   P d = c2 - c1;
432   double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
018   if (d2 == 0 || h2 < 0)  return {};
c14   vector<pair<P, P>> out;
092   for (double sign : {-1, 1}) {
2ad     P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
2e3     out.push_back({c1 + v * r1, c2 + v * r2});
e25   }
b21   if (h2 == 0) out.pop_back();
fe8   return out;
483 }
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" e0cfba, 10 lines

```
7dc template<class P>
195 vector<P> circleLine(P c, double r, P a, P b) {
33b   P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
55a   double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
3e4   if (h2 < 0) return {};
071   if (h2 == 0) return {p};
7cd   P h = ab.unit() * sqrt(h2);
d65   return {p - h, p + h};
59a }
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

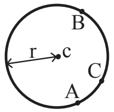
"../../content/geometry/Point.h" 19add1, 20 lines

```
626 typedef Point<double> P;
361 #define arg(p, q) atan2(p.cross(q), p.dot(q))
bb9 double circlePoly(P c, double r, vector<P> ps) {
6d1   auto tri = [&](P p, P q) {
c9c     auto r2 = r * r / 2;
291     P d = q - p;
127     auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist
      2();
eea     auto det = a * a - b;
691     if (det <= 0) return arg(p, q) * r2;
f43     auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det
      ));
aba     if (t < 0 || 1 <= s) return arg(p, q) * r2;
57f     P u = p + d * s, v = q + d * (t-1);
8c0     return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
a52   };
bef   auto sum = 0.0;
8f4   rep(i,0,sz(ps))
3b7     sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
e66   return sum;
```

```
f08 }
```

circumcircle.h

Description:



The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.

"Point.h" 1caa3a, 10 lines

```
626 typedef Point<double> P;
510 double ccRadius(const P& A, const P& B, const P& C) {
14b   return (B-A).dist()*(C-B).dist()*(A-C).dist()/
f73       abs((B-A).cross(C-A))/2;
607 }
c0d P ccCenter(const P& A, const P& B, const P& C) {
28a   P b = C-A, c = B-A;
680   return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
793 }
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h" 09dd0a, 18 lines

```
a28 pair<P, double> mec(vector<P> ps) {
4da   shuffle(all(ps), mt19937(time(0)));
f6a   P o = ps[0];
328   double r = 0, EPS = 1 + 1e-8;
2be   rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
5cc     o = ps[i], r = 0;
4da     rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
a30       o = (ps[i] + ps[j]) / 2;
6f7       r = (o - ps[i]).dist();
102       rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
fa9         o = ccCenter(ps[i], ps[j], ps[k]);
6f7         r = (o - ps[i]).dist();
648       }
7b0     }
dcf   }
645   return {o, r};
09d }
```

8.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`
Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 2bf504, 12 lines

```
7dc template<class P>
0cc bool inPolygon(vector<P> &p, P a, bool strict = true) {
8b7   int cnt = 0, n = sz(p);
fea   rep(i,0,n) {
444     P q = p[(i + 1) % n];
cbd     if (onSegment(p[i], q, a)) return !strict;
        //or: if (segDist(p[i], q, a) <= eps) return !strict;
007     cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) >
      0;
1b9   }
70a   return cnt;
c72 }
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" f12300, 7 lines

```
4fc template<class T>
a51 T polygonArea2(vector<Point<T>>& v) {
2f8   T a = v.back().cross(v[0]);
06e   rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
3f5   return a;
693 }
```

PolygonCenter.h

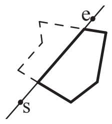
Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

"Point.h" 9706dc, 10 lines

```
626 typedef Point<double> P;
6d9 P polygonCenter(const vector<P>& v) {
f9f   P res(0, 0); double A = 0;
70b   for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
346     res = res + (v[i] + v[j]) * v[j].cross(v[i]);
3ea     A += v[j].cross(v[i]);
307   }
33c   return res / A / 3;
0d0 }
```

PolygonCut.h

Description:



Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h" d07181, 14 lines

```
626 typedef Point<double> P;
37d vector<P> polygonCut(const vector<P>& poly, P s, P e) {
fe2   vector<P> res;
d48   rep(i,0,sz(poly)) {
21c     P cur = poly[i], prev = i ? poly[i-1] : poly.back();
c5f     auto a = s.cross(e, cur), b = s.cross(e, prev);
2dc     if ((a < 0) != (b < 0))
380       res.push_back(cur + (prev - cur) * (a / (a - b)));
c5c     if (a < 0)
a5f       res.push_back(cur);
757   }
b50   return res;
42c }
```

PolygonUnion.h

Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)
Time: $\mathcal{O}(N^2)$, where N is the total number of points

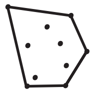
"Point.h", "sideOf.h" 3931c6, 34 lines

```
626 typedef Point<double> P;
142 double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y
      ; }
61d double polyUnion(vector<vector<P>>& poly) {
499   double ret = 0;
9af   rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
9c8     P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
05c     vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
cbd     rep(j,0,sz(poly)) if (i != j) {
cc1       rep(u,0,sz(poly[j])) {
418         P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])
      ];
688         int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
68b         if (sc != sd) {
295           double sa = C.cross(D, A), sb = C.cross(D, B);
```

```
        if (min(sc, sd) < 0)
          segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
      } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))>0){
        segs.emplace_back(rat(C - A, B - A), 1);
        segs.emplace_back(rat(D - A, B - A), -1);
      }
    }
  }
  sort(all(segs));
  for (auto& s : segs) s.first = min(max(s.first, 0.0), 1.0);
  double sum = 0;
  int cnt = segs[0].second;
  rep(j,1,sz(segs)) {
    if (!cnt) sum += segs[j].first - segs[j - 1].first;
    cnt += segs[j].second;
  }
  ret += A.cross(B) * sum;
 }
 return ret / 2;
}
```

ConvexHull.h

Description:
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull. If you want to keep the collinear points in the convex hull, change the comparison to $h[t-2].cross(h[t-1], p) < 0$ and the size of the vector h to $2 * sz(pts) + 1$.



Time: $\mathcal{O}(n \log n)$

"Point.h" 310954, 14 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
 if (sz(pts) <= 1) return pts;
 sort(all(pts));
 vector<P> h(sz(pts)+1);
 int s = 0, t = 0;
 for (int it = 2; it--; s = --t, reverse(all(pts)))
   for (P p : pts) {
     while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
     h[t++] = p;
   }
 return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

Time: $\mathcal{O}(n)$

"Point.h" c571b8, 13 lines

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
 int n = sz(S), j = n < 2 ? 0 : 1;
 pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
 rep(i,0,j)
   for (;; j = (j + 1) % n) {
     res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
     if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
       break;
   }
 return res.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446b, 15 lines

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict = true) {
 int a = 1, b = sz(l) - 1, r = !strict;
 if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
 if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
 if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
   return false;
 while (abs(a - b) > 1) {
   int c = (a + b) / 2;
   (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
 }
 return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.

Time: $\mathcal{O}(\log n)$

"Point.h" 7cf45b, 40 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
 int n = sz(poly), lo = 0, hi = n;
 if (extr(0)) return 0;
 while (lo + 1 < hi) {
   int m = (lo + hi) / 2;
   if (extr(m)) return m;
   int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
   (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
 }
 return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
 int endA = extrVertex(poly, (a - b).perp());
 int endB = extrVertex(poly, (b - a).perp());
 if (cmpL(endA) < 0 || cmpL(endB) > 0)
   return {-1, -1};
 array<int, 2> res;
 rep(i,0,2) {
   int lo = endB, hi = endA, n = sz(poly);
   while ((lo + 1) % n != hi) {
     int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
     (cmpL(m) == cmpL(endB) ? lo : hi) = m;
   }
   res[i] = (lo + !cmpL(hi)) % n;
   swap(endA, endB);
 }
 if (res[0] == res[1]) return {res[0], -1};
 if (!cmpL(res[0]) && !cmpL(res[1]))
   switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
     case 0: return {res[0], res[0]};
     case 2: return {res[1], res[1]};
   }
 return res;
}
```

Minkowski.h

Description: Computes the Minkowski sum of two convex polygons. Polygons must be convex and given in CCW order. Returns the vertices of the Minkowski sum polygon in CCW order.

Time: $\mathcal{O}(n+m)$

"Point.h" 664d67, 24 lines

```
using P = Point<ll>;

vector<P> minkowski(vector<P> p, vector<P> q) {
  auto fix = [](vector<P>& A) {
    int pos = 0;
    for (int i = 1; i < sz(A); i++) {
      if(A[i].y < A[pos].y || (A[i].y == A[pos].y && A[i].x < A[pos].x))
        pos = i;
    }
    rotate(A.begin(), A.begin() + pos, A.end());
    A.push_back(A[0]), A.push_back(A[1]);
  };
  fix(p), fix(q);
  vector<P> result;
  int i = 0, j = 0;
  while (i < sz(p) - 2 || j < sz(q) - 2) {
    result.push_back(p[i] + q[j]);
    auto cross = (p[i + 1] - p[i]).cross(q[j + 1] - q[j]);
    if (cross >= 0 && i < sz(p) - 2) i++;
    if (cross <= 0 && j < sz(q) - 2) j++;
  }
  return result;
}
```

Extreme.h

Description: Finds an extreme vertex of a convex polygon according to a unimodal comparator. The comparator defines a total order along the polygon (given in CCW order).

Time: $\mathcal{O}(\log n)$

"Point.h" 70b181, 26 lines

```
using P = Point<ll>;
int extreme(vector<P> &pol, const function<bool(P, P)>& cmp) {
  int n = pol.size();
  auto extr = [&](int i, bool& cur_dir) {
    cur_dir = cmp(pol[(i+1)%n], pol[i]);
    return !cur_dir and !cmp(pol[(i+n-1)%n], pol[i]);
  };
  bool last_dir, cur_dir;
  if (extr(0, last_dir)) return 0;
  int l = 0, r = n;
  while (l+1 < r) {
    int m = (l+r)/2;
    if (extr(m, cur_dir)) return m;
    bool rel_dir = cmp(pol[m], pol[l]);
    if ((!last_dir and cur_dir) or
        (last_dir == cur_dir and rel_dir == cur_dir)) {
      l = m;
      last_dir = cur_dir;
    } else r = m;
```

```
606     }
792     return l;
985   }
cad   int max_dot(vector<P> &pol, P v) {
a98     return extreme([&](P p, P q) { return p.dot(v) > q.dot(v
      ); });
27e   }
```

Tangents.h

Description: Finds the left and right tangent points from an external point p to a convex polygon given in CCW order. A tangent point is a vertex where the segment p->v touches the polygon without intersecting its interior, defining the limits of visibility from p. Returns the indices of the left and right tangent vertices.
Time: $\mathcal{O}(\log n)$

"Point.h", "Extreme.h" dcf85f, 11 lines

```
780   using P = Point<ll>;

08d   bool ccw(P p, P q, P r) {
274     return (q-p).cross(r-q) > 0;
0f3   }
826   pair<int, int> tangents(vector<P> &pol, P p) {
ae2       auto L = [&](P q, P r) { return ccw(p, r, q); };
98c       auto R = [&](P q, P r) { return ccw(p, q, r); };
861       return {extreme(pol, L), extreme(pol, R)};
3dc   }
```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" ac41a6, 18 lines

```
2c0   typedef Point<ll> P;
24b   pair<P, P> closest(vector<P> v) {
7f9    assert(sz(v) > 1);
7f7    set<P> S;
879    sort(all(v), [](P a, P b) { return a.y < b.y; });
571    pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
eec    int j = 0;
813    for (P p : v) {
3fb      P d{1 + (ll)sqrt(ret.first), 0};
8be      while (v[j].y <= p.y - d.x) S.erase(v[j++]);
a5a      auto lo = S.lower_bound(p - d), hi = S.upper_bound(p +
      d);
c77      for (; lo != hi; ++lo)
113        ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
8aa      S.insert(p);
5b0    }
70d    return ret.second;
bf2   }
```

ManhattanMST.h

Description: Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights w(p, q) = —p.x - q.x— + —p.y - q.y—. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
Time: $\mathcal{O}(N \log N)$

"Point.h" df6f59, 24 lines

```
bbe   typedef Point<int> P;
ea9   vector<array<int, 3>> manhattanMST(vector<P> ps) {
850    vi id(sz(ps));
27c    iota(all(id), 0);
8c1    vector<array<int, 3>> edges;
8de    rep(k,0,4) {
1dd      sort(all(id), [&](int i, int j) {
02b           return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
702      map<int, int> sweep;
1e2      for (int i : id) {
84d        for (auto it = sweep.lower_bound(-ps[i].y);
904                    it != sweep.end(); sweep.erase(it++)) {
61d          int j = it->second;
6f3          P d = ps[i] - ps[j];
d18          if (d.y > d.x) break;
537          edges.push_back({d.y + d.x, i, j});
271        }
923        sweep[-ps[i].y] = i;
e69      }
4eb      for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p
      .y);
a11    }
da2    return edges;
a11   }
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" bac5b0, 64 lines

```
9a6   typedef long long T;
293   typedef Point<T> P;
305   const T INF = numeric_limits<T>::max();

173   bool on_x(const P& a, const P& b) { return a.x < b.x; }
0bd   bool on_y(const P& a, const P& b) { return a.y < b.y; }

bf2   struct Node {
975    P pt; // if this is a leaf, the single point in it
877    T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
a23    Node *first = 0, *second = 0;

86a    T distance(const P& p) { // min squared distance to a
      point
28b      T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
88e      T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
d98      return (P(x,y) - p).dist2();
ca4    }

d97    Node(vector<P>&& vp) : pt(vp[0]) {
741      for (P p : vp) {
ad3        x0 = min(x0, p.x); x1 = max(x1, p.x);
e5d        y0 = min(y0, p.y); y1 = max(y1, p.y);
310      }
994      if (vp.size() > 1) {
           // split on x if width >= height (not ideal...)
9b7        sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
           // divide by taking half the array for each child (
             not
           // best performance with many duplicates in the
             middle)
0f9        int half = sz(vp)/2;
48e        first = new Node({vp.begin(), vp.begin() + half});
902        second = new Node({vp.begin() + half, vp.end()});
66e      }
204    }
a77   };

dad   struct KDTree {
95f    Node* root;
c30    KDTree(const vector<P>& vp) : root(new Node({all(vp)}))
      {}

113    pair<T, P> search(Node *node, const P& p) {
ec4      if (!node->first) {
           // uncomment if we should not find the point itself:
           // if (p == node->pt) return {INF, P()};
47e        return make_pair((p - node->pt).dist2(), node->pt);
119      }
ea4      Node *f = node->first, *s = node->second;
d40      T bfirst = f->distance(p), bsec = s->distance(p);
a16      if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

         // search closest side first, other side if needed
86c      auto best = search(f, p);
314      if (bsec < best.first)
509        best = min(best, search(s, p));
f26      return best;
74c    }

       // find nearest point to a point, and its squared
         distance
       // (requires an arbitrary operator< for Point)
9b6    pair<T, P> nearest(const P& p) {
195      return search(root, p);
94c    }
6f5   };
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" eefdf5, 89 lines

```
2c0   typedef Point<ll> P;
806   typedef struct Quad* Q;
449   typedef __int128_t lll; // (can be ll if coords are < 2e4)
59b   P arb(LLONG_MAX,LLONG_MAX); // not equal to any other
      point

070   struct Quad {
461    Q rot, o; P p = arb; bool mark;
b38    P& F() { return r()->p; }
23a    Q& r() { return rot->rot; }
f4f    Q prev() { return rot->o->rot; }
57e    Q next() { return r()->prev(); }
180   } *H;

d15   bool circ(P p, P a, P b, P c) { // is p in the
      circumcircle?
4b4    lll p2 = p.dist2(), A = a.dist2()-p2,
ffa        B = b.dist2()-p2, C = c.dist2()-p2;
59a    return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B >
       0;
6af   }
00a   Q makeEdge(P orig, P dest) {
bdf    Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
516    H = r->o; r->r()->r() = r;
2c3    rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->
      r();
ed2    r->p = orig; r->F() = dest;
4c1    return r;
b3b   }
d8d   void splice(Q a, Q b) {
686    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
86c   }
e92   Q connect(Q a, Q b) {
fc2    Q q = makeEdge(a->F(), b->p);
6e6    splice(q, a->next());
642    splice(q->r(), b);
bef    return q;
4a4   }

196   pair<Q,Q> rec(const vector<P>& s) {
e63    if (sz(s) <= 3) {
```

```
  Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
  if (sz(s) == 2) return { a, a->r() };
  splice(a->r(), b);
  auto side = s[0].cross(s[1], s[2]);
  Q c = side ? connect(b, a) : 0;
  return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
 }

 #define H(e) e->F(), e->p
 #define valid(e) (e->F().cross(H(base)) > 0)
 Q A, B, ra, rb;
 int half = sz(s) / 2;
 tie(ra, A) = rec({all(s) - half});
 tie(B, rb) = rec({sz(s) - half + all(s)});
 while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
        (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
 Q base = connect(B->r(), A);
 if (A->p == ra->p) ra = base->r();
 if (B->p == rb->p) rb = base;

 #define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
   while (circ(e->dir->F(), H(base), e->F())) { \
     Q t = e->dir; \
     splice(e, e->prev()); \
     splice(e->r(), e->r()->prev()); \
     e->o = H; H = e; e = t; \
   }
 for (;;) {
   DEL(LC, base->r(), o);  DEL(RC, base, prev());
   if (!valid(LC) && !valid(RC)) break;
   if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
     base = connect(RC, base->r());
   else
     base = connect(base->r(), LC->r());
 }
 return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
 sort(all(pts));  assert(unique(all(pts)) == pts.end());
 if (sz(pts) < 2) return {};
 Q e = rec(pts).first;
 vector<Q> q = {e};
 int qi = 0;
 while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
 #define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
 q.push_back(c->r()); c = c->next(); } while (c != e); }
 ADD; pts.clear();
 while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
 return pts;
}
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 7 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
 double v = 0;
 for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
 return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 33 lines

```
template<class T> struct Point3D {
 typedef Point3D P;
 typedef const P& R;
 T x, y, z;
 explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
 bool operator<(R p) const {
   return tie(x, y, z) < tie(p.x, p.y, p.z); }
 bool operator==(R p) const {
   return tie(x, y, z) == tie(p.x, p.y, p.z); }
 P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
 P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
 P operator*(T d) const { return P(x*d, y*d, z*d); }
 P operator/(T d) const { return P(x/d, y/d, z/d); }
 T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
 P cross(R p) const {
   return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
 }
 T dist2() const { return x*x + y*y + z*z; }
 double dist() const { return sqrt((double)dist2()); }
 //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
 double phi() const { return atan2(y, x); }
 //Zenith angle (latitude) to the z-axis in interval [0, pi]
 double theta() const { return atan2(sqrt(x*x+y*y),z); }
 P unit() const { return *this/(T)dist(); } //makes dist()=1
 //returns unit vector normal to *this and p
 P normal(P p) const { return cross(p).unit(); }
 //returns point rotated 'angle' radians ccw around axis
 P rotate(double angle, P axis) const {
   double s = sin(angle), c = cos(angle); P u = axis.unit();
   return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
 }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.

Time: $\mathcal{O}(n^2)$

"Point3D.h" 5b45fc, 50 lines

```
typedef Point3D<double> P3;

struct PR {
 void ins(int x) { (a == -1 ? a : b) = x; }
 void rem(int x) { (a == x ? a : b) = -1; }
 int cnt() { return (a != -1) + (b != -1); }
 int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
 assert(sz(A) >= 4);
 vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
 vector<F> FS;
 auto mf = [&](int i, int j, int k, int l) {
   P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
   if (q.dot(A[l]) > q.dot(A[i]))
     q = q * -1;
   F f{q, i, j, k};
   E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
   FS.push_back(f);
 };
 rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
   mf(i, j, k, 6 - i - j - k);

 rep(i,4,sz(A)) {
   rep(j,0,sz(FS)) {
     F f = FS[j];
     if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
       E(a,b).rem(f.c);
       E(a,c).rem(f.b);
       E(b,c).rem(f.a);
       swap(FS[j--], FS.back());
       FS.pop_back();
     }
   }
   int nw = sz(FS);
   rep(j,0,nw) {
     F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
     C(a, b, c); C(a, c, b); C(b, c, a);
   }
 }
 for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
   A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
 return FS;
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f07, 9 lines

```
double sphericalDistance(double f1, double t1,
   double f2, double t2, double radius) {
 double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
 double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
 double dz = cos(t2) - cos(t1);
 double d = sqrt(dx*dx + dy*dy + dz*dz);
 return radius*2*asin(d/2);
}
```

Strings (9)

AhoCorasick.h

95b3e7, 46 lines

```
int trie[ms][sigma], fail[ms], terminal[ms], superfail[ms];
bool present[ms];
int z = 1;

int val(char c) { return c - 'a'; }

void add(string& p) {
    int cur = 0;
    for (int i = 0; i < (int)p.size(); i++) {
        int& nxt = trie[cur][val(p[i])];
        if (nxt == 0) nxt = z++;
        cur = nxt;
    }
    present[cur] = true;
```

```
b07         terminal[cur]++;
6aa     }

0a8     void build() {
26a         queue<int> q;
f47         for (q.push(0); !q.empty(); q.pop()) {
fb5             int on = q.front();
0b2             for (int i = 0; i < sigma; i++) {
df1                 int& to = trie[on][i];
279                 int f = (on == 0 ? 0 : trie[fail[on]][i]);
de7                 int sf = (present[f] ? f : superfail[f]);
24d                 if (!to) {
c4e                     to = f;
6fd                 }
4e6                 else {
3ef                     fail[to] = f;
b86                     superfail[to] = sf;
                        // merge infos (ex: terminal[to] +=
                            terminal[f])
91b                     q.push(to);
692                 }
bff             }
e61         }
b89     }

54e     void search(string& s) {
b3d         int cur = 0;
b4f         for (char c : s) {
3ba             cur = trie[cur][val(c)];
                // process infos on current node (ex: ocurrences
                    += terminal[cur])
5ac         }
d1b     }
```

Hash.h

Description: C can also be random, operator is $[l, r]$

79e7f5, 28 lines

```
541 using ull = uint64_t;
54d struct H {
858     ull x; H(ull x = 0) : x(x) {}
c9b     H operator+(H o) { return x + o.x + (x + o.x < x); }
5cd     H operator-(H o) { return *this + ~o.x; }
167     H operator*(H o) {
2f3         auto m = (__uint128_t)x * o.x;
540         return H((ull)m) + (ull)(m >> 64);
681     }
bf2     ull get() const { return x + !~x; }
03c     bool operator==(H o) const{ return get() == o.get();}
0ab     bool operator<(H o) const{ return get() < o.get();}
bf6 };
862 static const H C = (ll)1e11 + 3;
61c struct Hash {
2f2     vector<H> h, pw;
1df     Hash(string& str) : h(str.size()), pw(str.size()) {
9bc         pw[0] = 1, h[0] = str[0];
1c5         for (int i = 1; i < str.size(); i++) {
90a             h[i] = h[i - 1] * C + str[i];
b3c             pw[i] = pw[i - 1] * C;
57e         }
f1b     }
75e     H operator()(int l, int r) {
91f         return h[r] - (l ? h[l - 1] * pw[r - l + 1] : 0);
9cf     }
c36 };
```

KMP.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123).

c7cf15, 10 lines

```
a56 vector<int> pi(const string& s) {
627     vector<int> p(sz(s));
edb     for(int i = 1; i < sz(s); i++) {
052         int g = p[i-1];
6c0         while (g && s[i] != s[g]) g = p[g-1];
7cf         p[i] = g + (s[i] == s[g]);
a2e     }
74e     return p;
c7c }
```

KmpAutomaton.h

Description: $go[i][j]$ = length of the longest prefix of s that is a suffix of $s[0..i]$ followed by the letter j (i.e., the next matched prefix length if, at state i, we read letter j).

8833cb, 17 lines

```
ab6 int go[ms][sigma];
ca3 int val(char c) { return c - 'a'; }
8cf void automaton(string& s) {
3cc     for (int i = 0; i < sigma; i++)
48d         go[0][i] = (i == val(s[0]));

8cc     for (int i = 1, bdr = 0; i <= (int)s.size(); i++) {
782         for (int j = 0; j < sigma; j++) {
6ef             go[i][j] = go[bdr][j];
87c         }
f8d         if (i < (int)s.size()) {
02f             go[i][val(s[i])] = i + 1;
364             bdr = go[bdr][val(s[i])];
63b         }
d7e     }
0c5 }
```

Manacher.h

Description: $p[0][i+1]$ is the length of matches of even length palindrome, starting from $[i, i+1]$.
$p[1][i]$ is the length of matches of odd length palindrome, starting from $[i, i]$.
(abaxx -> p[0] = 00001)
(abaxx -> p[1] = 01000)

7dfe41, 17 lines

```
aa9 array<vector<int>, 2> manacher(const string& s) {
f89     int n = sz(s);
ca1     array<vector<int>,2> p={vector<int>(n+1),vector<int>(n
        )};
6b7     for (int z = 0; z < 2; z++) {
22c         for (int i = 0, l = 0, r = 0; i < n; i++) {
24e             int t = r - i + !z;
e70             if (i < r) p[z][i] = min(t, p[z][l + t]);
fff             int L = i - p[z][i], R = i + p[z][i] - !z;
40c             while(L >= 1 && R+1 < n && s[L-1] == s[R+1]){
895                 p[z][i]++, L--, R++;
48e             }
f28             if (R > r) l = L, r = R;
e05         }
7a3     }
74e     return p;
7df }
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(s.begin(), s.begin()+minRotation(s), s.end());`
Time: $\mathcal{O}(N)$

19c4ce, 14 lines

```
5fa int minRotation(string s) {
a3e     int a = 0, N = s.size(); s += s;
239     for (int b = 0; b < N; b++) {
e0d         for (int k = 0; k < N; k++) {
32f             if (a+k == b || s[a+k] < s[b+k]) {
313                 b += max(0, k-1);
c2b                 break;
873             }
068             if (s[a+k] > s[b+k]) { a = b; break; }
9b5         }
193     }
3f5     return a;
19c }
```

SuffixArray.h

Description: $lcp[i]$ is the length of the longest common prefix between the suffixes $s[sa[i]..n-1]$ and $s[sa[i-1]..n-1]$.
If we concatenate multiple strings using separator characters, the separator that appears furthest to the right must be the smallest character in the alphabet.

048424, 31 lines

```
3f4 struct SuffixArray {
716  vector<int> sa, lcp;
d91  SuffixArray(string s, int lim=256) {
59b    s.push_back('$');
323    int n = sz(s), k = 0, a, b;
9f1    vector<int> x(all(s)), y(n), ws(max(n, lim));
af4    sa = lcp = y, iota(all(sa), 0);
25d    for(int j = 0, p = 0; p < n; j= max(1, j*2), lim = p) {
3cd      p = j, iota(all(y), n - j);
603      for(int i=0; i<n; i++){
071        if (sa[i] >= j) y[p++] = sa[i] - j;
cb4      }
911      fill(all(ws), 0);
483      for(int i=0; i<n; i++) ws[x[i]]++;
5d9      for(int i=1; i<lim; i++) ws[i] += ws[i - 1];
a9e      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
c7d      swap(x, y), p = 1, x[sa[0]] = 0;
6f5      for(int i=1; i<n; i++){
93f        a = sa[i - 1], b = sa[i];
ddb        x[b] = p-1;
a32        if(y[a] != y[b] || y[a+j] != y[b+j]) x[b] = p++;
1ba      }
c36    }
65b    for (int i = 0, j; i < n - 1; lcp[x[i++]] = k)
904      for (k && k--, j = sa[x[i] - 1];
262          s[i + k] == s[j + k]; k++);
68a    sa = vector<int>(sa.begin() + 1, sa.end());
5d4    lcp = vector<int>(lcp.begin() + 1, lcp.end());
4db  }
048 };
```

Zfunc.h

Description: z[i] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)

495392, 13 lines

```
572 vector<int> ZFunc(const string& s) {
d6b     int n = sz(s), a = 0, b = 0;
2b1     vector<int> z(n, 0);
29a     if (!z.empty()) z[0] = 0;
6f5     for (int i = 1; i < n; i++) {
fe0         int end = i;
98f         if (i < b) end = min(i + z[i - a], b);
65f         while (end < n && s[end] == s[end - i]) ++end;
816         z[i] = end - i; if (end > b) a = i, b = end;
253     }
070     return z;
495 }
```

Various (10)

10.1 Misc. algorithms

Dates.h

Description: dateToInt converts Gregorian date to integer (Julian day number). intToDate converts integer (Julian day number) to Gregorian date: month/day/year. intToDay converts Julian day number to day of the week

688e56, 23 lines

```
37c string day[] = { "Mon", "Tue", "Wed", "Thu", "Fri", "Sat",
      "Sun" };
fb9 int dateToInt(int m, int d, int y) {
e70     return
773         1461 * (y + 4800 + (m - 14) / 12) / 4 +
649         367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
fa0         3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
3aa         d - 32075;
a73 }
3fe void intToDate(int jd, int& m, int& d, int& y) {
ee1     int x, n, i, j;
33a     x = jd + 68569;
403     n = 4 * x / 146097;
33e     x -= (146097 * n + 3) / 4;
6fc     i = (4000 * (x + 1)) / 1461001;
b1d     x -= 1461 * i / 4 - 31;
fc9     j = 80 * x / 2447;
c8d     d = x - 2447 * j / 80;
179     x = j / 11;
335     m = j + 2 - 12 * x;
23d     y = 100 * (n - 49) + i + x;
cbb }
04e string intToDay(int jd) { return day[jd % 7]; }
```

MultisetHash.h

5648da, 8 lines

```
cdc ull hashify(ull sum) {
7b8     sum += FIXED_RANDOM;
6ec     sum += 0x9e3779b97f4a7c15;
dc6     sum = (sum ^ (sum >> 30)) * 0xbf58476d1ce4e5b9;
005     sum = (sum ^ (sum >> 27)) * 0x94d049bb133111eb;
358     return sum ^ (sum >> 31);
564 }
```

Rand.h

2de3f8, 8 lines

```
c8a mt19937 rng(chrono::steady_clock::now().time_since_epoch()
      .count());
    // _64

463 int uniform(int l, int r) { // [l, r]
a7f     uniform_int_distribution<int> uid(l, r);
f54     return uid(rng);
d9e }
```

10.2 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: $dp[i][j] = \min_{i<k<j}(dp[i][k] + dp[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \leq f(a,d)$ and $f(a,c) + f(b,d) \leq f(a,d) + f(b,c)$ for all $a \leq b \leq c \leq d$. Another sufficient criteria is: $opt[i][j-1] \leq opt[i][j] \leq opt[i+1][j]$

Time: $\mathcal{O}\left(N^2\right)$

fea016, 22 lines

```
7cc ll knuth(){
6a7     memset(opt, -1, sizeof opt);
45b     for(int i=n-1; i>=0; i--){
8e7         dp[i][i] = 0; // base case
b28         opt[i][i] = i;
94f         for(int j=i+1; j<n; j++){
2e2             int optL = (!j ? 0 : opt[i][j-1]);
dc4             int optR = (~opt[i+1][j] ? opt[i+1][j] : n-1);
554             ll cst = cost(i, j);
f12             dp[i][j] = INF;
3bb             optL = max(i, optL), optR = min(j-1, optR);
349             for(int k=optL; k<=optR; k++){
f8b                 ll now = dp[i][k] + dp[k+1][j] + cst;
e83                 if(now <= dp[i][j]){
960                     dp[i][j] = now;
14d                     opt[i][j] = k;
5fc                 }
114             }
4ce         }
96c     }
fea }
```

DivideAndConquerDP.h

Description: Divide and Conquer DP maintaining cost, can be used when $opt[i][j] \leq opt[i][j+1]$. In this code everything is 1-based. Memory can be optmized by keeping only the last row

Time: $\mathcal{O}(MN \log N)$

c7cb38, 42 lines

```
129 void add(int idx) {}
404 void rem(int idx) {}

749 void deC(int i, int l, int r, int optL, int optR) {
de6     if (l > r) return;
995     int j = (l + r) / 2;
d9a     for (int k = r; k > j; k--) rem(k);
c45     int opt = optL;
364     for (int k = optL; k <= min(optR, j); k++) {
            // cost = cost[k, j]
597         int val = dp[i - 1][k - 1] + cost;
532         if (val < dp[i][j]) {
482             dp[i][j] = val;
613             opt = k;
178         }
183         rem(k);
93f     }
5d9     for (int k = min(optR, j); k >= optL; k--) add(k);
446     rem(j);
ace     deC(i, l, j - 1, optL, opt);

ebd     for (int k = j; k <= r; k++) add(k);
648     for (int k = optL; k < opt; k++) rem(k);
0b6     deC(i, j + 1, r, opt, optR);

9bb     for (int k = optL; k < opt; k++) add(k);
460 }

d57 int solve(int N, int M) { // 1-based
d9f     for (int i = 0; i <= M; i++) {
138         for (int j = 0; j <= N; j++){
3db             dp[i][j] = inf; // base case
a26         }
e0f     }
c21     cost = 0; // neutral value
c62     for (int i = 1; i <= N; i++) add(i);
143     for (int i = 1; i <= M; i++) {
156         deC(i, 1, N, 1, N);
c97     }
01a     return dp[M][N];
3ab }
```