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las4s e pelados

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Contest (1)

template.cpp
9 lines

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
using ll = long long;
using pii = pair<int,int>;
using vi = vector<int>;
```

.bashrc
2 lines

```
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \
  -fsanitize=undefined,address'
```

hash.sh
2 lines

```
# bash hash.sh file.cpp l1 l2
sed -n $2','$3' p' $1 | sed '/^#w/d' | cpp -dD -P -
    fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

stressTest.sh
20 lines

```
P=code  #mude pro filename do codigo
Q=brute  #mude pro filename do brute [correto]
g++ ${P}.cpp -o sol -O2 || exit 1
g++ ${Q}.cpp -o brt -O2 || exit 1
g++ gen.cpp  -o gen -O2 || exit 1
for ((i = 1; ; i++)) do
  echo $i
  ./gen $i > in
  ./sol < in > out
  ./brt < in > out2
  if (! cmp -s out out2) then
    echo "--> entrada:"
    cat in
    echo "--> saida code:"
    cat out
    echo "--> saida brute:"
    cat out2
    break;
  fi
done
```

paperStress.py
26 lines

```
927 import random
a1a import subprocess
5c9 MAX_N = 100
b5d def gen_case() -> str:
c7e     return f"1\n"

94a random.seed((1 << 9) | 31)

a22 for i in range(100):
d19     print(), print()
a3f     case = gen_case()
266     print(f"Test #{i+1}: ")
ce5     print(case)
d41     # test bruteforce
f60     bf = subprocess.run(['out/b'], input=case, encoding='
    ascii', capture_output=True)
d41     # test solution
37c     sol = subprocess.run(['out/m'], input=case, encoding='
    ascii', capture_output=True)
d55     bf_res = bf.stdout
af9     sol_res = sol.stdout
6b6     print(f"bruteforce {bf_res}, solution {sol_res}")
508     if bf_res == sol_res:
dd4         print("accepted")
f68     else:
ef2         print("WA")
1cb         break
```

troubleshoot.txt
52 lines

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
```

Theoretical (2)

2.1 Mathematics

2.1.1 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \cdots - c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.1.2 Trigonometry

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}$, $\phi = \operatorname{atan2}(b, a)$.

2.1.3 Geometry

Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1 - \left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\dfrac{\alpha+\beta}{2}}{\tan\dfrac{\alpha-\beta}{2}}$

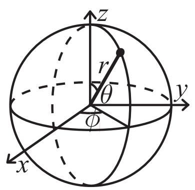
Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

Spherical coordinates



$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \mathrm{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z &= r\cos\theta & \phi &= \mathrm{atan2}(y, x) \end{aligned}$$

Pick's Theorem

The area of a simple polygon whose vertices have integer coordinates is:

$$A = I + \frac{B}{2} - 1$$

where I is the number of interior integer points, and B is the number of integer points in the border of the polygon.

Two Ears Theorem

Every simple polygon with more than 3 vertices has at least two non-overlapping ears (a ear is a vertex whose diagonal induced by its neighbors which lies strictly inside the polygon). Equivalently, every simple polygon can be triangulated.

2.1.4 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\mathrm{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.1.5 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, \; c \neq 1$$

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$\sum_{i=0}^{n} ic^i = \frac{nc^{n+2} - (n+1)c^{n+1} + c}{(c-1)^2}, \; c \neq 1$$

$$g_k(n) = \sum_{i=1}^{n} i^k = \frac{1}{k+1}\left(n^{k+1} + \sum_{j=1}^{k}\binom{k+1}{j+1}(-1)^{j+1}g_{k-j}(n)\right)$$

2.1.6 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, \; (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, \; (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, \; (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, \; (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, \; (-\infty < x < \infty)$$

$$\sum_{i=0}^{\infty} ic^i = \frac{c}{(1-c)^2}, \quad |c| < 1$$

$$(1+x)^n = \sum_{i=0}^{n}\binom{n}{i}x^i$$

$$\frac{1}{1-x} = \sum_{i=0}^{\infty} x^i, \; (-1 < x < 1)$$

$$\frac{1}{(1-x)^n} = \sum_{i=0}^{\infty}\binom{n+i-1}{n-1}x^i, \; (-1 < x < 1)$$

2.1.7 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n, p)$, $n = 1, 2, \ldots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k}p^k(1-p)^{n-k}$$

$$\mu = np, \; \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n, p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\mathrm{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1}, \; k = 1, 2, \ldots$$

$$\mu = \frac{1}{p}, \; \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda, \sigma^2 = \lambda$$

2.2 Combinatorial

2.2.1 Binomial Identities

$$\binom{n-1}{k} - \binom{n-1}{k-1} = \tfrac{n-2k}{k}\binom{n}{k} \qquad \binom{n}{h}\binom{n-h}{k} = \binom{n}{k}\binom{n-k}{h}$$

$$\textstyle\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1} \qquad \sum_{k=0}^{n} k^2\binom{n}{k} = (n+n^2)2^{n-2}$$

$$\textstyle\sum_{j=0}^{k}\binom{m}{j}\binom{n-m}{k-j} = \binom{n}{k} \qquad \sum_{j=0}^{m}\binom{m}{j}^2 = \binom{2m}{m}$$

$$\textstyle\sum_{m=0}^{n}\binom{m}{j}\binom{n-m}{k-j} = \binom{n+1}{k+1} \qquad \sum_{m=0}^{n}\binom{m}{k} = \binom{n+1}{k+1}$$

$$\textstyle\sum_{r=0}^{m}\binom{n+r}{r} = \binom{n+m+1}{m} \qquad \sum_{k=0}^{n}\binom{n-k}{k} = \mathrm{Fib}(n+1)$$

$$\textstyle\sum_{k=0}^{n}\binom{r}{k}\binom{s}{n-k} = \binom{r+s}{n}$$

2.2.2 Permutations

Factorial

n	1 2 3	4	5	6	7	8	9	10
$n!$	1 2 6	24	120	720	5040	40320	362880	3628800

n	11	12	13	14	15	16	17
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14

n	20	25	30	40	50	100	150	171
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX

Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor\frac{n!}{e}\right\rceil$$

Burnside's lemma

Counts the number of distinct colorings of an object under symmetry.

$$\frac{1}{|G|}\sum_{g\in G} k^{\mathrm{cyc}(g)},$$

where G is the symmetry group, k the number of colors, and $\mathrm{cyc}(g)$ the number of cycles induced by g.

Example: number of ways to color a necklace with n beads using k colors (rotations only):

$$g(n) = \frac{1}{n}\sum_{i=0}^{n-1} k^{(\gcd(n,i))}$$

where rotation i shifts the necklace by i positions.

2.2.3 Partitions and subsets

Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k}\binom{n_i}{m_i} \pmod{p}$.

2.2.4 Sum of Binomials (FFT)

Goal: Given freq. array C, compute $Ans[k] = \sum_i C[i]\binom{i}{k}$ for all k. Rewrite: $Ans[k] = \frac{1}{k!}\sum_i (C[i]\cdot i!)\frac{1}{(i-k)!}$.

- Construct P where $P[i] = C[i]\cdot i!$
- Construct Q where $Q[i] = (i!)^{-1}$
- Reverse Q (to handle the $i-k$ subtraction).
- Multiply $R = NTT(P, Q)$.
- Result: $Ans[k] = R[k + |Q| - 1]\cdot\frac{1}{k!}$.

2.2.5 General purpose numbers

Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able). $B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k\cdot(n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

Labeled unrooted trees

- on n vertices: n^{n-2}
- on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
- with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

2.3 Number Theory

2.3.1 Bézout's identity

For $a \neq$, $b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

2.3.2 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

2.3.3 Estimates

$\sum_{d|n} d = O(n \log\log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 6700 for $n < 1e12$, 200 000 for $n < 1e19$.

2.3.4 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \le m \le n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \le m \le n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

2.3.5 Theorems

Goldbach's conjecture: Every even integer $n > 2$ can be written as $n = a + b$ with a, b prime.

Legendre's conjecture: There is always at least one prime between n^2 and $(n+1)^2$.

Lagrange's four-square theorem: Every positive integer can be written as

$$n = a^2 + b^2 + c^2 + d^2.$$

Zeckendorf's theorem: Every integer $n \ge 1$ has a unique representation as a sum of non-consecutive Fibonacci numbers:

$$n = F_{i_1} + F_{i_2} + \cdots + F_{i_k}, \quad i_j - i_{j+1} \ge 2.$$

Euclid's formula (primitive Pythagorean triples): The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2),\ \ b = k \cdot (2mn),\ \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

Wilson's theorem: n is prime iff

$$(n-1)! \equiv -1 \pmod{n}.$$

Chicken McNugget theorem: For coprime n, m, the largest integer not representable as $an + bm$ (with $a, b \ge 0$) is

$$nm - n - m.$$

There are $\frac{(n-1)(m-1)}{2}$ non-representable integers, and for each pair $(k,\ nm - n - m - k)$ exactly one is representable.

2.4 Graphs

2.4.1 Flows and Matching

Hall's Theorem

In bipartite graphs, there exists a perfect matching covering the entire side X if and only if for every subset $Y \subseteq X$,

$$|Y| \le |N(Y)|,$$

where $N(Y)$ denotes the set of neighbors of Y.

Kőnig's Theorem

In a bipartite graph, the size of a Minimum Vertex Cover is equal to the size of a Maximum Matching. A Minimum Vertex Cover is a minimum set of vertices such that every edge of the graph has at least one endpoint in the set.

As a consequence,

$$n - \text{Maximum Matching} = \text{Maximum Independent Set},$$

where a Maximum Independent Set is the largest set of vertices with no edges between them.

Recovering the Minimum Vertex Cover Given a maximum matching in a bipartite graph (X, Y):

- Construct the residual graph by orienting:
  - non-matching edges from X to Y;
  - matching edges from Y to X.
- Perform a BFS or DFS starting from all free (unmatched) vertices in X.
- Let Z_X be the set of reachable vertices in X, and Z_Y the set of reachable vertices in Y.

The Minimum Vertex Cover is given by:

$$(X \setminus Z_X) \cup Z_Y.$$

Node-Disjoint Path Cover

A node-disjoint path cover is a set of paths such that each vertex belongs to exactly one path.

In a directed acyclic graph (DAG),

$$\text{Minimum Node-Disjoint Path Cover} = n - \text{Maximum Matching}.$$

The construction is as follows: for each vertex u, create a copy u'. Add an edge $u \to v'$ if there exists an edge $u \to v$ in the original graph.

Recovering the Paths

- Vertices that do not appear as destinations in the matching are starting points of paths.
- Each matching edge $u \to v'$ corresponds to an edge $u \to v$ in the original DAG.
- Following these edges reconstructs all paths of the path cover.

General Path Cover

A general path cover is a path cover where a vertex may belong to more than one path.

In a DAG, the construction is similar to the node-disjoint case, but an edge $u \to v'$ exists if there is a path from u to v in the original graph.

Recovering the Cover The vertices can be grouped according to the edges used in the matching to form the path cover.

Dilworth's Theorem

An antichain is a set of vertices such that there is no path between any pair of vertices in the set.

In a directed acyclic graph,

$$\text{Minimum General Path Cover} = \text{Maximum Antichain}.$$

Recovering a Maximum Antichain Given a minimum general path cover, selecting one vertex from each path produces a maximum antichain.

2.4.2 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--`, `mat[b][b]++` (and `mat[b][a]--`, `mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

2.4.3 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

2.4.4 Planar Graphs

If G has k connected components, then $n - m + f = k + 1$.

2.5 Optimization tricks

2.5.1 Bit hacks

- `for (int x = m; x; x = (x - 1)&m ) { ... }` loops over all subset masks of `m` (except 0).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
  `if (i & 1 << b) D[i] += D[i^(1 << b)];`
  computes all sums of subsets.

2.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC target("bmi,bmi2,popcnt,lzcnt")` improve bit operations.
- `#pragma GCC optimize("unroll-loops")` self explanatory.

2.6 Various

2.6.1 Master Theorem (Simple)

$T(n) = aT(n/b) + O(n^d)$. Compare a vs b^d:

- $a > b^d \implies O(n^{\log_b a})$ (Work at leaves dominates)
- $a = b^d \implies O(n^d \log n)$ (Work is uniform)
- $a < b^d \implies O(n^d)$ (Work at root dominates)

Data structures (3)

Bit.h

Description: *lower_bound* works the same as on vectors
Time: $\mathcal{O}(\log N)$

5891da, 23 lines

```
8eb struct Bit {
406   vector<ll> bit;
1dd   Bit(int n) : bit(n + 1) {}
265   void update(int i, ll v) {
c38     for (i++; i < sz(bit); i += i & -i) bit[i] += v;
f21   }
74a   ll query(int i) {
b73     ll ret = 0;
71c     for (i++; i > 0; i -= i & -i) ret += bit[i];
edf     return ret;
e40   }
dc8   int lower_bound(ll v){ // min pos st sum[0, pos] >= v
bec     int pos = 0;
a40     for(int j=(1 << 23); j >= 1; j/=2){
3b1       if(pos+j < sz(bit) && bit[pos + j] < v){
b4e         pos += j;
18d         v -= bit[pos];
f6c       }
156     }
d75     return pos;
37b   }
589 };
```

Bit2d.h

Description: Points called on the update function NEED to be on the *pts* vector parameter on build.
Time: $\mathcal{O}\left((\log N)^2\right)$

"Bit.h" 5a98ac, 37 lines

```
9c0 struct Bit2d {
a37   vector<vector<int>> ys;
fe8   vector<Bit> bit;
543   vector<int> cmp_x;
425   Bit2d(){}
521   void put(int x, int y) {
005     for (x++; x < sz(ys); x += x & -x) ys[x].push_back(y);
f3c   }
ce0   int id(const vector<int> &v, int y) {
1e9     return (upper_bound(all(v), y) - v.begin()) - 1;
19a   }
7ff   void build(vector<pii> pts) {
3cb     sort(all(pts));
f99     for(auto p : pts) cmp_x.push_back(p.first);
9a7     cmp_x.erase(unique(all(cmp_x)), cmp_x.end());
f82     ys.resize(cmp_x.size() + 1);
94d     for(auto p : pts) put(id(cmp_x, p.first), p.second);
310     for(auto &v:ys)sort(all(v)), bit.emplace_back(sz(v));
a01   }
767   void update(int x, int y, int val){
f3f     x = id(cmp_x, x);
681     for(x++; x < sz(ys); x+= x&-x)
507       bit[x].update(id(ys[x], y), val);
c88   }
d95   int query(int x, int y){
f3f     x = id(cmp_x, x);
7c9     int ret = 0;
f32     for(x++; x > 0; x-= x&-x)
ea8       ret += bit[x].query(id(ys[x], y));
edf     return ret;
8f7   }
251   int query(int x1, int y1, int x2, int y2){
e4d     int a = query(x2, y2)-query(x2, y1-1);
7d1     return a-query(x1-1, y2)+query(x1-1, y1-1);
c33   }
5a9 };
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$

8ec1c7, 32 lines

```
72c struct Line {
3e2   mutable ll k, m, p;
ca5   bool operator<(const Line& o) const { return k < o.k; }
abf   bool operator<(ll x) const { return p < x; }
7e3 };

781 struct LineContainer : multiset<Line, less<>> {
      // (for doubles, use inf = 1/.0, div(a,b) = a/b)
fd2   static const ll inf = LLONG_MAX;
33a   ll div(ll a, ll b) { // floored division
10f     return a / b - ((a ^ b) < 0 && a % b); }
a1c   bool isect(iterator x, iterator y) {
a95     if (y == end()) return x->p = inf, 0;
9cb     if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
591     else x->p = div(y->m - x->m, x->k - y->k);
870     return x->p >= y->p;
2fa   }
a0c   void add(ll k, ll m) {
116     auto z = insert({k, m, 0}), y = z++, x = y;
7b1     while (isect(y, z)) z = erase(z);
d94     if (x != begin() && isect(--x, y))
c07       isect(x, y = erase(y));
57d     while ((y = x) != begin() && (--x)->p >= y->p)
774       isect(x, erase(y));
086   }
4ad   ll query(ll x) {
229     assert(!empty());
7d1     auto l = *lower_bound(x);
96a     return l.k * x + l.m;
d21   }
577 };
```

Mo.h

Description: For subtree queries, perform an Euler tour and map each node u to the interval $[tin[u], tin[u] + subtree\_size[u] - 1]$. A subtree query becomes a range query over this interval.
For path queries between nodes U and V, Let U be the closest to the root. If V lies in U's subtree, the path corresponds to the interval $[tin[U], tin[V]]$. Otherwise, the path corresponds to the interval $[min(tout[U], tout[V]), max(tin[U], tin[V])]$.
In both cases, nodes on the U–V path appear exactly once in the interval, while all other nodes appear either 0 or 2 times.
Usage: `queries.push(Query(l, r, index of query))`, `intervals are [l, r]`
Time: $\mathcal{O}\left(N\sqrt{(Q)}\right)$

fb7161, 44 lines

```
626 inline int64_t hilOrd(int x, int y, int pow, int rot) {
51a   if (pow == 0) return 0;
a6e   int hpow = 1 << (pow - 1);
01f   int seg = (x < hpow) ? ((y < hpow) ? 0 : 3) : ((y < hpow
      ) ? 1 : 2);
e08   seg = (seg + rot) & 3;
669   const int rotDelta[4] = { 3, 0, 0, 1 };
d0b   int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
115   int nrot = (rot + rotDelta[seg]) & 3;
fba   int64_t sub = int64_t(1) << (2 * pow - 2);
65b   int64_t ans = seg * sub;
1ae   int64_t add = hilOrd(nx, ny, pow - 1, nrot);
ff7   ans += (seg == 1 || seg == 2) ? add : (sub - add - 1);
ba7   return ans;
ec4 }

670 struct Query {
738   int l, r, idx;
ce8   int64_t ord;
36f   Query(int l, int r, int idx) : l(l), r(r), idx(idx) {
6c4     ord = hilOrd(l, r, 21, 0);
926   }
847   bool operator < (const Query& other) const {
328     return ord < other.ord;
e05   }
315 };

240 vector<Query> queries;
4d5 int ans[ms];
566 void put(int x) {} // F
c29 void remove(int x) {} // F
64b int getAns() {}

1c1 void Mo() {
3d9   int l = 0, r = -1;
bfa   sort(queries.begin(), queries.end());
275   for (Query q : queries) {
482     while (l > q.l) put(--l);
fec     while (r < q.r) put(++r);
5b8     while (l < q.l) remove(l++);
9b5     while (r > q.r) remove(r--);
745     ans[q.idx] = getAns();
5a4   }
2a4 }
```

MoUpdate.h

Description: Block size should be around $(2 * N * N)^{\frac{1}{3}}$
Usage: `intervals are [l, r], addQuery(l, r, number of updates happened before this query, index of query), addUpdate(index of updated position, value before update, value after update)`
Time: $\mathcal{O}\left(Q * (2 * N * N)^{\frac{1}{3}} * F\right)$

f8eda8, 55 lines

```
496 const int B = 2700;
247 struct MoUpdate {
670   struct Query {
fd6     int l, r, t, idx;
fc8     Query(int l, int r, int t, int idx)
8bf       : l(l), r(r), t(t), idx(idx) {}
f51     bool operator < (const Query& p) const {
f06       if (l / B != p.l / B) return l < p.l;
e80       if (r / B != p.r / B) return r < p.r;
d0c       return t < p.t;
673     }
bc2   };
f2f   struct Upd {
f25     int i, old, now;
f23     Upd(int i, int old, int now): i(i),old(old),now(now){}
c12   };

240   vector<Query> queries;
e2b   vector<Upd> updates;

ac5   void addQuery(int l, int r, int t, int idx) {
fc9     queries.push_back(Query(l, r, t, idx)); }
968   void addUpdate(int i, int old, int now) {
936     updates.push_back(Upd(i, old, now)); }

1aa   void add(int x) {} // F
598   void rem(int x) {} // F
64b   int getAns() {}
0d2   void update(int novo, int idx, int l, int r) {
2b9     if (l <= idx && idx <= r) rem(idx);
4ce     arr[idx] = novo;
ec1     if (l <= idx && idx <= r) add(idx);
100   }

63d   void solve() {
cb1     int l = 0, r = -1, t = 0;
bfa     sort(queries.begin(), queries.end());
275     for (Query q : queries) {
a95       while (l > q.l) add(--l);
875       while (r < q.r) add(++r);
8f6       while (l < q.l) rem(l++);
a38       while (r > q.r) rem(r--);
fda       while (t < q.t) {
df3         auto u = updates[t++];
285         update(u.now, u.i, l, r);
8a4       }
32a       while (t > q.t) {
d69         auto u = updates[--t];
ce2         update(u.old, u.i, l, r);
3bf       }
745       ans[q.idx] = getAns();
f06     }
b09   }
d3e };
```

MinQueue.h

40df8d, 19 lines

```
925 struct MQueue {
fdd     int tin, tout;
375     deque<pair<int, int>> dq;
1ce     MQueue() : tin(0), tout(0) {}
619     void push(int val) {
f0d         while (!dq.empty() && min(dq.back().first, val) ==
        val) dq.pop_back();
9c6         dq.push_back(pair(val, tin++));
769     }
42d     void pop() {
            // assert(!dq.empty());
48c         if (dq.front().second == tout) dq.pop_front();
470         tout++;
b0e     }
f46     int front() {
            // assert(!dq.empty());
651         return dq.front().first;
fa2     }
40d };
```

SegmentTree.h

Description: Zero-indexed max-tree. Bounds are inclusive to the left and inclusive to the right. Can be changed by modifying T, f and unit.
Time: $\mathcal{O}(\log N)$

f609d9, 21 lines

```
5ae struct Tree {
ef4   typedef int T;
cbe   static constexpr T unit = INT_MIN;
e54   T f(T a, T b) { return max(a, b); } // (any associative
    fn)
6cd   vector<T> s; int n;
3d2   Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {}
6a3   void update(int pos, T val) {
56a     for (s[pos += n] = val; pos /= 2;)
326       s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
0e9   }
b4c   T query(int b, int e) { // query [b, e]
1a3     e++;
0f9     T ra = unit, rb = unit;
fbb     for (b += n, e += n; b < e; b /= 2, e /= 2) {
e83       if (b % 2) ra = f(ra, s[b++]);
064       if (e % 2) rb = f(s[--e], rb);
561     }
cb2     return f(ra, rb);
707   }
f60 };
```

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
Time: $\mathcal{O}(\log N)$

782797, 17 lines

```
c4d #include <bits/extc++.h>
0d7 using namespace __gnu_pbds;

4fc template<class T>
c20 using Tree = tree<T, null_type, less<T>, rb_tree_tag,
3a1     tree_order_statistics_node_update>;

ad0 void example() {
c6f   Tree<int> t, t2; t.insert(8);
559   auto it = t.insert(10).first;
d28   assert(it == t.lower_bound(9));
969   assert(t.order_of_key(10) == 1);
d39   assert(t.order_of_key(11) == 2);
1b7   assert(*t.find_by_order(0) == 8);
a60   t.join(t2); // merge t2 into t
9ad }
```

PersistentSegTree.h

Usage: `SegP(size of the segtree, number of updates)`

`roots = {0}, newRoot = update(roots.back(),...),`
`roots.push(newRoot)`

58842f, 42 lines

```
b17 struct SegP {
709   static constexpr ll neut = 0;
bf2   struct Node {
aa3     ll v; // start with neutral value
74f     int l, r;
9ef     Node(ll v=neut, int l=0, int r=0) : v(v), l(l), r(r){}
945   };
```

```
38f    vector<Node> seg;
068    int n, CNT;
9ea    SegP(int _n, int upd): seg(20*(upd+_n)), n(_n), CNT(1){}
2ce    ll merge(ll a, ll b) { return a + b; }
c97    int update(int root, int pos, int val, int l, int r) {
ec9      int p = CNT++;
77a      seg[p] = seg[root];
893      if (l == r) {
00f        seg[p].v += val;
74e        return p;
3d7      }
ae0      int mid = (l + r) / 2;
8a3      if (pos <= mid){
aa8        seg[p].l = update(seg[p].l, pos, val, l, mid);
583      }else seg[p].r = update(seg[p].r,pos,val,mid+1,r);

85a      seg[p].v=merge(seg[seg[p].l].v, seg[seg[p].r].v);
74e      return p;
a90    }
6a4    int query(int p, int L, int R, int l, int r) {
3c7      if (l > R || r < L) return neut;
c26      if (L <= l && r <= R) return seg[p].v;
ae0      int mid = (l + r) / 2;
864      int left = query(seg[p].l, L, R, l, mid);
195      int right = query(seg[p].r, L, R, mid + 1, r);
90a      return merge(left, right);
e77    }
304    int update(int root, int pos, int val) {
c68      return update(root, pos, val, 0, n - 1);
84e    }
7cc    int query(int root, int L, int R) {
a53      return query(root, L, R, 0, n - 1);
2f9    }
588  };
```

SegBeats.h

Description: In Segment Tree Beats, 'lazy' does NOT mean "updates still missing here". The node already reflects all previous updates. Instead, 'lazy' stores what must be propagated to the children before recursing. Always call 'apply(l,r,p)' before descending. This node layout supports range add, range chmin and range chmax operations. Beats conditions:
break: MIN x: mx1 <= x ; MAX x: mi1 >= x
tag: MIN x: x > mx2 ; MAX x: x < mi2
Time: amortized $\mathcal{O}(\log^2 N)$, without range add $\mathcal{O}(\log N)$ fa8527, 47 lines

```
3c9  struct node{
45e    ll mx1, mx2, sum, lazy;
9e5    ll mi1, mi2;
faa    int cMax, cMin, tam;
db3    node(int x=0) : mx1(x),mx2(-inf),mi1(x),mi2(inf),
744              cMax(1),cMin(1),tam(1),sum(x),lazy(0){}
b67    node(node a, node b){
4f5      sum = a.sum+b.sum, tam = a.tam+b.tam;
c60      lazy = 0;
15b      mx1 = max(a.mx1, b.mx1);
9ae      mx2 = max(a.mx2, b.mx2);
f62      if(a.mx1 != b.mx1) mx2 = max(mx2, min(a.mx1, b.mx1));
b60      cMax=(a.mx1==mx1 ? a.cMax:0)+(b.mx1==mx1 ? b.cMax:0);

09f      mi1 = min(a.mi1, b.mi1);
143      mi2 = min(a.mi2, b.mi2);
3bf      if(a.mi1 != b.mi1) mi2=min(mi2, max(a.mi1, b.mi1));
c18      cMin=(a.mi1==mi1 ? a.cMin:0)+(b.mi1==mi1 ? b.cMin:0);
23d    }
38d    void apply_sum(ll x){
2a1      mx1 += x, mx2 += x,mi1 += x, mi2 += x;
99b      sum += tam*x, lazy += x;
b5e    }
cf4    void apply_min(ll x){
e07      if(x >= mx1) return;
c44      sum -= (mx1 - x)*cMax;
be0      if(mi1 == mx1) mi1 = x;
8ef      if(mi2 == mx1) mi2 = x;
ea2      mx1 = x;
908    }
0c8    void apply_max(ll x){
e25      if(x <= mi1) return;
59e      sum -= (mi1 - x)*cMin;
4b1      if(mx1 == mi1) mx1 = x;
d69      if(mx2 == mi1) mx2 = x;
1ff      mi1 = x;
0e4    }
554  };
fdc  void apply(int l, int r, int p){
c8e    for(int i=2*p+1; i<=2*p+2; i++){
dbf      seg[i].apply_sum(st[p].lazy);
c90      seg[i].apply_min(st[p].mx1);
61a      seg[i].apply_max(st[p].mi1);
4b8    }
431    seg[p].lazy = 0;
dd0  }
```

RMQ.h

Usage: RMQ rmq(values);
rmq.query(inclusive, inclusive);
Time: $\mathcal{O}(|V|\log|V|+Q)$ bca062, 17 lines

```
76a  struct RMQ {
8ac    vector<vector<int>> dp;
dd1    RMQ(const vector<int>& a) : dp(1, a) {
71c      for (int i = 1, pw = 1; pw*2 <= sz(a); pw*=2, i++) {
394        dp.emplace_back(sz(a) - pw*2 + 1);
d17        for (int j = 0; j < sz(dp[i]); j++) {
dcc          dp[i][j] = min(dp[i-1][j], dp[i-1][j+pw]);
75a        }
b68      }
3e9    }
9e3    int query(int l, int r) {
658      assert(l <= r);
884      int k = 31 - __builtin_clz(r - l + 1);
1f9      return min(dp[k][l], dp[k][r - (1 << k) + 1]);
e21    }
bca  };
```

UnionFind.h

Description: Disjoint-set data structure with bipartite check 6d2739, 22 lines

```
146  struct Uf{
b54      vector<int> tam, ds, bi, c;
d2c      Uf(int n) : tam(n, 1), ds(n), bi(n, 1), c(n){
244          iota(all(ds), 0);
233      }
001      int find(int i){ return (i==ds[i] ? i : find(ds[i]));}
e5a      int color(int i){
300          return (i==ds[i] ? 0 : (c[i]^color(ds[i])));}
c3b      void merge(int a, int b){
8d0          int ca = color(a), cb = color(b);
605          a = find(a), b = find(b);
a89          if(a == b){
686              if(ca == cb) bi[a] = false;
505              return;
c08          }
226          if(tam[a] < tam[b]) swap(a, b);
1ac          ds[b] = a, tam[a] += tam[b];
27c          bi[a] = (bi[a] && bi[b]);
834          c[b] = (ca ^ cb ^ 1);
a70      }
6d2  };
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().
Usage: int t = uf.time(); ...; uf.rollback(t);
Time: $\mathcal{O}(\log(N))$ d4405e, 23 lines

```
47a  struct RollbackUF {
f80    vector<int> e;
919       vector<pii> st;
f6f    RollbackUF(int n) : e(n, -1) {}
84b    int size(int x) { return -e[find(x)]; }
626    int find(int x) { return e[x] < 0 ? x : find(e[x]); }
49f    int time() { return sz(st); }
4db    void rollback(int t) {
314      for (int i = time(); i --> t;)
8d2        e[st[i].first] = st[i].second;
b04      st.resize(t);
30b    }
cf0    bool join(int a, int b) {
605      a = find(a), b = find(b);
5c2      if (a == b) return false;
745      if (e[a] > e[b]) swap(a, b);
bac      st.push_back({a, e[a]});
e6e      st.push_back({b, e[b]});
708      e[a] += e[b]; e[b] = a;
8a6      return true;
6c7    }
d44  };
```

Numerical (4)

4.1 Polynomials and recurrences

Polynomial.h

c9b7b0, 19 lines

```
213  struct Poly {
3a1    vector<double> a;
9a5    double operator()(double x) const {
e3c      double val = 0;
d5c      for (int i = sz(a); i--;) (val *= x) += a[i];
d94      return val;
ae7    }
0ac    void diff() {
7b6      rep(i,1,sz(a)) a[i-1] = i*a[i];
468      a.pop_back();
afc    }
087    void divroot(double x0) {
898      double b = a.back(), c; a.back() = 0;
9cf      for(int i=sz(a)-1; i--;)
406          c = a[i], a[i] = a[i+1]*x0+b, b=c;
468      a.pop_back();
3f8    }
c9b  };
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0
Time: $\mathcal{O}(n^2\log(1/\epsilon))$
"Polynomial.h" b00bfe, 24 lines

```
64a  vector<double> polyRoots(Poly p, double xmin, double xmax)
       {
853    if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
539    vector<double> ret;
f55    Poly der = p;
c06    der.diff();
617    auto dr = polyRoots(der, xmin, xmax);
d85    dr.push_back(xmin-1);
12c    dr.push_back(xmax+1);
```

```
423   sort(all(dr));
b98   rep(i,0,sz(dr)-1) {
d85     double l = dr[i], h = dr[i+1];
ad1     bool sign = p(l) > 0;
b41     if (sign ^ (p(h) > 0)) {
03d       rep(it,0,60) { // while (h - l > 1e-8)
761         double m = (l + h) / 2, f = p(m);
0ac         if ((f <= 0) ^ sign) l = m;
193         else h = m;
b69       }
ff5       ret.push_back((l + h) / 2);
fc2     }
d15   }
edf   return ret;
b00 }
```

PolyInverse.h

2745a7, 18 lines

```
747 vector<ll> get_inverse(vector<ll> a) {
e4d   if (a.empty()) return {};
044   int Y = sz(a) - 1, n = 32 - __builtin_clz(Y);
ba5   n = (1 << n);
711   a.resize(n);
e3e   vector<ll> inv = { modpow(a[0], mod - 2) }, f, c;
a2b   inv.reserve(n);
599   for (int tam = 2; tam <= n; tam *= 2) {
d29     while (sz(f) < tam) f.push_back(a[sz(f)]);
fec     c = conv(f, inv);
757     rep(i, 0, tam) c[i] = (c[i] == 0 ? 0 : mod - c[i]);
df6     c[0] += (c[0] + 2 >= mod ? 2 - mod : 2);
f8b     inv = conv(inv, c);
118     inv.resize(tam);
9f4   }
531   return inv;
274 }
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
Time: $\mathcal{O}(N^2)$

96548b, 21 lines

```
c10 vector<ll> berlekampMassey(vector<ll> s) {
ea1   int n = sz(s), L = 0, m = 0;
2a2   vector<ll> C(n), B(n), T;
2b3   C[0] = B[0] = 1;

d6f   ll b = 1;
36d   rep(i,0,n) { ++m;
b7f     ll d = s[i] % mod;
45a     rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
53a     if (!d) continue;
169     T = C; ll coef = d * modpow(b, mod-2) % mod;
2d1     rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
b6c     if (2 * L > i) continue;
dc3     L = i + 1 - L; B = T; b = d; m = 0;
8c2   }

51b   C.resize(L + 1); C.erase(C.begin());
e98   for (ll& x : C) x = (mod - x) % mod;
a91   return C;
965 }
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \ldots \geq n-1]$ and $tr[0 \ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}(n^2 \log k)$

547b93, 27 lines

```
437 using Poly = vector<ll>;
2ef ll linearRec(Poly S, Poly tr, ll k) {
327  int n = sz(tr);

0e9  auto combine = [&](Poly a, Poly b) {
b1c    Poly res(n * 2 + 1);
5f7    rep(i,0,n+1) rep(j,0,n+1)
389      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
bdc    for (int i = 2 * n; i > n; --i) rep(j,0,n)
fc3      res[i-1-j] = (res[i-1-j] + res[i] * tr[j]) % mod;
b76    res.resize(n + 1);
b50    return res;
55c  };

bf8  Poly pol(n + 1), e(pol);
997  pol[0] = e[1] = 1;

e96  for (++k; k; k /= 2) {
491    if (k % 2) pol = combine(pol, e);
0d9    e = combine(e, e);
813  }

cd2  ll res = 0;
e8d  rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
b50  return res;
594 }
```

4.2 Matrices

SolveLinear.h

Description: If inv = 1, finds the inverse of the matrix eq and returns it as a flat vector
Time: $\mathcal{O}(\min(n,m)\, nm)$

2c134e, 52 lines

```
320 struct Gauss {
d6d   const double eps = 1e-9;
93d   vector<vector<double>> eq;
754   void addEquation(const vector<double>& e) {
503     eq.push_back(e); }
04f   pair<int, vector<double>> solve(int inv=0) {
214     int n = sz(eq), m = sz(eq[0]) - 1 + inv;
f9c     if(inv){
d33       rep(i, 0, n) eq[i].resize(2*n), eq[i][n+i] = 1;
2e2     }
3cb     vector<int> where(m, -1);
a73     for (int col = 0, row = 0; col < m && row < n; col++)
      {
f05       int sel = row;
53c       rep(i, row, n) {
664         if (abs(eq[i][col]) > abs(eq[sel][col])) sel = i;
e04       }
68b       if (abs(eq[sel][col]) < eps) continue;
3ad       rep(i, col, sz(eq[0])) swap(eq[sel][i], eq[row][i]);
2c3       where[col] = row;
dff       rep(i, 0, n) if (i != row) {
184         double c = eq[i][col] / eq[row][col];
7f1         rep(j, col, sz(eq[0])) eq[i][j] -= eq[row][j] * c;
17d       }
4ef       ++row;
9b8     }
f9c     if(inv){
208       vector<double> res;
fea       rep(i, 0, n) {
420         if (where[i] == -1) return {0, {}}; // Singular
3af         rep(j, n, 2*n)
f89           res.push_back(eq[where[i]][j] / eq[where[i]][i])
      ;
d81       }
3b1       return {1, res};
700     }

233     vector<double> ans(m, 0);
670     rep(i, 0, m) {
c19       if (where[i] != -1)
02c         ans[i] = eq[where[i]][m] / eq[where[i]][i];
5bb     }
fea     rep(i, 0, n) {
68c       double sum = 0;
5f8       rep(j, 0, m) {
f48         sum = sum + ans[j] * eq[i][j];
fa6       }
3c8       if(abs(sum - eq[i][m]) > eps)return {0, {}};
bf2     }
260     rep(i, 0, m) if (where[i] == -1) return {2, ans};
d4a     return {1, ans};
a95   }
2c1 };
```

SolveLinearBinary.h

Time: $\mathcal{O}\left(\frac{\min(n,m)\, nm}{64}\right)$

28c946, 32 lines

```
e81 pair<int, bitset<M>> gauss(vector<bitset<M>> eq) {
579     int n = eq.size(), m = M - 1;
3cb     vector<int> where(m, -1);
a73     for(int col = 0, row = 0; col < m && row < n; col++){
dbb         rep(i,row,n)
926             if (eq[i][col]) {
c35                 swap(eq[i], eq[row]);
c2b                 break;
177             }
f4f         if (!eq[row][col]) continue;
2c3         where[col] = row;

fea         rep(i, 0, n) {
b60             if (i != row && eq[i][col]) eq[i] ^= eq[row];
981         }
4ef         ++row;
c74     }
7eb     bitset<M> ans;
670     rep(i,0,m) {
713         if (where[i] != -1) ans[i] = eq[where[i]][m];
691     }
fea     rep(i,0,n) {
e5c         int sum = (ans & eq[i]).count();
53f         sum %= 2;
36a         if (sum != eq[i][m]) return pair(0, bitset<M>());
29e     }
670     rep(i,0,m) {
be2         if (where[i] == -1) return pair(INF, ans);
958     }
280     return pair(1, ans);
28c }
```

XorGauss.h

5a1957, 30 lines

```
b94 struct XorGauss {
060     int N;
471     vector<ll> basis, who, mask;
47b     XorGauss(int N) : N(N), basis(N), who(N), mask(N) {}
        // if(ans & (1ll << j)) who[j] was used to form x
221     bool belong(ll x){
04b         ll ans = 0;
042         for(int i=N-1; i>=0; i--){
e13             if((x ^ basis[i]) < x){
4ec                 ans ^= mask[i];
6b0                 x ^= basis[i];
```

```
254             }
2ad         }
069         return (x == 0);
c26     }
397     void add(ll v, int idx) {
a4d         ll msk = 0;
042         for (int i = N - 1; i >= 0; i--) {
80f             if (!(v & (1ll << i))) continue;
bf3             if (basis[i] == 0) {
1c7                 basis[i] = v, who[i] = idx;
940                 mask[i] = (msk | (1ll << i));
505                 return;
bc8             }
00e             msk ^= mask[i];
647             v ^= basis[i];
25b         }
fcc     }
5a1 };
```

4.3 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x]\exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim$1s for $N = 2^{22}$)

773fcd, 44 lines

```
bcc typedef complex<double> C;

7c0 void fft(vector<C>& a) {
a5b   int n = a.size(), L = 31 - __builtin_clz(n);
f82   static vector<complex<long double>> R(2, 1); // 10% faster if double
991   static vector<C> rt(2, 1);
ad8   for (static int k = 2; k < n; k *= 2) {
9d9     R.resize(n);
335     rt.resize(n);
411     auto x = polar(1.0L, acos(-1.0L) / k);
cdb     rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
a8a   }
e66   vector<ll> rev(n);
dcb   rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
47b   rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);

d3f   for (int k = 1; k < n; k *= 2) {
cda     for (int i = 0; i < n; i += 2 * k) {
0c2       for (int j = 0; j < k; j++) {
30c         auto x = (double*)&rt[j + k];
ebe         auto y = (double*)&a[i + j + k];
15c         C z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]);
20a         a[i + j + k] = a[i + j] - z;
1b0         a[i + j] += z;
b5b       }
1fe     }
fa0   }
b33 }

ccc vector<ll> conv(const vector<ll>& a, const vector<ll>& b){
f88   if (a.empty() || b.empty()) return {};
920   vector<ll> res(sz(a) + sz(b) - 1);
441   int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
060   vector<C> in(n), out(n);
b1a   copy(all(a), in.begin());
fef   rep(i,0,sz(b)) in[i].imag(b[i]);
21a   fft(in);
6fb   for (C& x : in) x *= x;
4d7   rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
3d7   fft(out);
aa3   rep(i,0,sz(res)) res[i]=round(imag(out[i]) / (4 * n));
b50   return res;
7f4 }
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \mathrm{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \mathrm{mod})$.
Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h" b82773, 23 lines

```
192 typedef vector<ll> vl;
3fe template<int M> vl convMod(const vl &a, const vl &b) {
f88  if (a.empty() || b.empty()) return {};
19d  vl res(sz(a) + sz(b) - 1);
a6f  int B=32-__builtin_clz(sz(res)), n=1<<B,cut=int(sqrt(M));
3dd  vector<C> L(n), R(n), outs(n), outl(n);
a1d  rep(i,0,sz(a)) L[i] =C((int)a[i] / cut, (int)a[i] % cut);
97d  rep(i,0,sz(b)) R[i] =C((int)b[i] / cut, (int)b[i] % cut);
5d5  fft(L), fft(R);
fea  rep(i,0,n) {
39d    int j = -i & (n - 1);
65e    outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
91a    outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
cb3  }
d08  fft(outl), fft(outs);
35e  rep(i,0,sz(res)) {
351    ll av = ll(real(outl[i])+.5), cv =ll(imag(outs[i])+.5);
988    ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
6a3    res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
58f  }
b50  return res;
c1f }
```

NumberTheoreticTransform.h

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x]g^{xk}$ for all k, where $g = \mathrm{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).
Time: $\mathcal{O}(N \log N)$

84c11e, 34 lines

```
376 const int mod = 998244353, root = 62;
192 typedef vector<ll> vl;
8ec void ntt(vl &a) {
6ae  int n = sz(a), L = 31 - __builtin_clz(n);
7c9  static vl rt(2, 1);
8ee  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
335    rt.resize(n);
d43    ll z[] = {1, modpow(root, mod >> s)};
8e7    rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
f39  }
808  vector<int> rev(n);
dcb  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
47b  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
657  for (int k = 1; k < n; k *= 2)
2cb    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
86e      ll z = rt[j+k] * a[i+j+k] % mod, &ai = a[i+j];
598      a[i + j + k] = ai - z + (z > ai ? mod : 0);
589      ai += (ai + z >= mod ? z - mod : z);
9a8    }
de9 }
08f vl conv(const vl &a, const vl &b) {
f88  if (a.empty() || b.empty()) return {};
f51  int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s),
570      n = 1 << B;
9ef  int inv = modpow(n, mod - 2);
e4c  vl L(a), R(b), out(n);
6b4  L.resize(n), R.resize(n);
d9e  ntt(L), ntt(R);
dfc  rep(i,0,n)
0db    out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
ec9  ntt(out);
c20  return {out.begin(), out.begin() + s};
387 }
```

FWHT.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

124c14, 20 lines

```
5ad void FST(vector<ll>& a, bool inv) {
a9d   for (int n = sz(a), step = 1; step < n; step *= 2) {
5bd     for (int i = 0; i < n; i += 2 * step) {
4ee       for (int j = i; j < i + step; j++) {
2fe         ll& u = a[j], &v = a[j + step];
c6f         tie(u, v) =
2d3         inv ? pair(v - u, u) : pair(v, u + v); // AND
aba         inv ? pair(v, u - v) : pair(u + v, u); // OR
a5a         pair(u + v, u - v);  // XOR
0b4       }
fb4     }
cd3   }
c9b   if(inv) for(ll& x : a) x /= sz(a); // XOR only
075 }
eb2 vector<ll> conv(vector<ll> a, vector<ll> b) {
595   FST(a, 0); FST(b, 0);
2dd   for (int i = 0; i < sz(a); i++) a[i]*=b[i];
062   FST(a, 1); return a;
7bf }
```

Number theory (5)

5.1 Modular arithmetic

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

c375f5, 5 lines

```
88a const ll mod = 1000000007, LIM = 200000;
0f2 inv[1] = 1;
379 for(int i=2; i<LIM; i++)
86c     inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.
Time: $\mathcal{O}(1)$ for `modmul`, $\mathcal{O}(\log b)$ for `modpow`

bbbd8f, 12 lines

```
f4c typedef unsigned long long ull;
f85 ull modmul(ull a, ull b, ull M) {
2dd  ll ret = a * b - M * ull(1.L / M * a * b);
964  return ret + M * (ret < 0) - M * (ret >= (ll)M);
e93 }
4f6 ull modpow(ull b, ull e, ull mod) {
c1a  ull ans = 1;
a18  for (; e; b = modmul(b, b, mod), e /= 2)
9e8    if (e & 1) ans = modmul(ans, b, mod);
ba7  return ans;
100 }
```

ModPow.h

b83e45, 9 lines

```
e2e const ll mod = 1000000007; // faster if const

9d8 ll modpow(ll b, ll e) {
d54   ll ans = 1;
36e   for (; e; b = b * b % mod, e /= 2)
b46     if (e & 1) ans = ans * b % mod;
ba7   return ans;
d1e }
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).
Time: $\mathcal{O}(\log^2 p)$ worst case, $\mathcal{O}(\log p)$ for most p

"ModPow.h" 19a793, 25 lines

```
a77 ll sqrt(ll a, ll p) {
5de   a %= p; if (a < 0) a += p;
b47   if (a == 0) return 0;
5c6   assert(modpow(a, (p-1)/2, p) == 1); // else no solution
a75   if (p % 4 == 3) return modpow(a, (p+1)/4, p);
      // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
b94   ll s = p - 1, n = 2;
ee5   int r = 0, m;
084   while (s % 2 == 0)
082     ++r, s /= 2;
eaa   while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
0c3   ll x = modpow(a, (s + 1) / 2, p);
b74   ll b = modpow(a, s, p), g = modpow(n, s, p);
1af   for (;; r = m) {
4fd     ll t = b;
713     for (m = 0; m < r && t != 1; ++m)
c58       t = t * t % p;
ae0     if (m == 0) return x;
20e     ll gs = modpow(g, 1LL << (r - m - 1), p);
fba     g = gs * gs % p;
4fb     x = x * gs % p;
c5c     b = b * g % p;
e3a   }
19a }
```

DiscreteLog.h

Description: Returns the smallest x such that $a^x \bmod m = b \bmod m$. If no such x exists, returns -1.
Time: O(sqrt(m) * log(sqrt(m))

2f126b, 32 lines

```
758 int solve(int a, int b, int m) {
a6e     a %= m, b %= m;
ec4     if (a == 0) return (b ? -1 : 1);
        // caso gcd(a, m) > 1
6af     int k = 1, add = 0, g;
553     while ((g = gcd(a, m)) > 1) {
d90         if (b == k) return add;
642         if (b % g) return -1;
92a         b /= g, m /= g, ++add;
803         k = (k * 1ll * a / g) % m;
8a0     }

16c     int sq = sqrt(m) + 1;
b51     int big = 1;
4e1     for (int i = 0; i < sq; i++) big = (1ll * big * a) % m
    ;

053     vector<pii> vals;
3c2     for (int q = 0, cur = b; q <= sq; q++) {
b53         vals.push_back({ cur, q });
b50         cur = (1ll * cur * a) % m;
837     }
62b     sort(all(vals));
90c     for (int p = 1, cur = k; p <= sq; p++) {
5d3         cur = (1ll * cur * big) % m;
958         auto it = lower_bound(all(vals), pair(cur, INF));
721         if (it != vals.begin() && (--it)->first == cur) {
a30             return sq * p - it->second + add;
6fe         }
f22     }
daa     return -1;
2f1 }
```

DiscreteRoot.h

Description: Returns x such that $x^k \bmod m = a \bmod m$. If no such x exists, returns -1.
Time: O(sqrt(m) * log(sqrt(m))

"PrimitiveRoot.h", "DiscreteLog.h" 1d582e, 11 lines

```
    // Discrete Root

27c ll discreteRoot(ll k, ll a, ll m) {
738     ll g = primitiveRoot(m);
58b     ll y = discreteLog(fexp(g, k, m), a, m);
f31     if (y == -1) return y;
a58     return fexp(g, y, m);
1d5 }
```

5.2 Primality

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" 66fe73, 13 lines

```
da4 bool isPrime(ull n) {
c16   if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
062   ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 17952650
    22};
ae0   ull s = __builtin_ctzll(n-1), d = n >> s;
e80   for (ull a : A) {   // ^ count trailing zeroes
6b4     ull p = modpow(a%n, d, n), i = s;
274     while (p != 1 && p != n - 1 && a % n && i--)
c77       p = modmul(p, p, n);
e28     if (p != n-1 && i != s) return 0;
edf   }
6a5   return 1;
66f }
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" da0c7c, 19 lines

```
7eb ull pollard(ull n) {
222   ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
5f5   auto f = [&](ull x) { return modmul(x, x, n) + i; };
f51   while (t++ % 40 || gcd(prd, n) == 1) {
be9     if (x == y) x = ++i, y = f(x);
70f     if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
b78     x = f(x), y = f(f(y));
bf8   }
002   return gcd(prd, n);
d1b }
591 vector<ull> factor(ull n) {
1b9   if (n == 1) return {};
6b5   if (isPrime(n)) return {n};
bc6   ull x = pollard(n);
52a   auto l = factor(x), r = factor(n / x);
7af   l.insert(l.end(), all(r));
792   return l;
d54 }
```

PrimitiveRoot.h

18a01e, 15 lines

```
    //is n primitive root of p ?
ad0 bool test(ll x, ll p) {
a56     ll m = p - 1;
845     for (ll i = 2; i * i <= m; ++i) if (!(m % i)) {
e64         if (modpow(x, i, p) == 1) return false;
599         if (modpow(x, m / i, p) == 1) return false;
53a     }
8a6     return true;
c4e }
    //find the smallest primitive root for p
220 ll search(ll p) {
1bf     for (ll i = 2; i < p; i++) if (test(i, p)) return i;
daa     return -1;
a3c }
```

5.3 Divisibility

Euclid.h

Description: Find x, y such that $Ax + By = \gcd(A, B)$. If $\gcd(A, B) = 1$, then $x = A^{-1} \pmod B$ and $y = B^{-1} \pmod A$.
Time: $\mathcal{O}(\log)$

33ba8f, 6 lines

```
c22 ll euclid(ll a, ll b, ll &x, ll &y) {
1ee   if (!b) return x = 1, y = 0, a;
e3d   ll d = euclid(b, a % b, y, x);
0a4   return y -= a/b * x, d;
33b }
```

CRT.h

ba1a4a, 25 lines

```
bc9 ll modinverse(ll a, ll b, ll s0 = 1, ll s1 = 0) {
a76   return !b ? s0 : modinverse(b, a % b, s1, s0 - s1 * (a /
    b)); }

d8b ll mul(ll a, ll b, ll m) {
a6f   return (((__int128_t)a*b)%m + m)%m;
0bc }

28d struct Equation {
4c5   ll mod, ans;
08f   bool valid;
145   Equation(ll a, ll m) { mod = m, ans = a, valid = true; }
0fc   Equation() { valid = false; }
4d3   Equation(Equation a, Equation b) {
515     valid = false;
1a0     if (!a.valid || !b.valid) return;
85c     ll g = gcd(a.mod, b.mod);
44d     if ((a.ans - b.ans) % g != 0) return;
af0     valid = true;
b98     mod = a.mod * (b.mod / g);
81a     ll x = mul(a.mod, modinverse(a.mod, b.mod), mod);
38a     ans = a.ans + mul(x, (b.ans - a.ans) / g, mod);
c4c     ans = (ans % mod + mod) % mod;
6f5   }
f48 };
```

DivisionTrick.h

02aebb, 15 lines

```
7f1 void floor_ranges(int n) {
79c     for (int l = 1, r; l <= n; l = r + 1) {
746         r = n / (n / l);
            // floor(n/y) has the same value for y in [l..r]
5bf     }
eee }
678 void ceil_ranges(int n) {
79c     for (int l = 1, r; l <= n; l = r + 1) {
```

```
d47           int x = (n + l - 1) / l;
374           if (x == 1) r = n;
21b           else r = (n - 1) / (x - 1);
              // ceil(n/y) has the same value for y in [l..r]
06c       }
57c   }
```

Phi.h

Description: *Euler's* ϕ function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} ... (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$.
Euler's thm (generalized): a, m arbitrary, $n \geq \log_2 m \Rightarrow a^n \equiv a^{\phi(m) + (n \bmod \phi(m))} \pmod{m}$.

e58bf0, 6 lines

```
d08  void calculatePhi() {
265   for(int i=0; i<LIM; i++) phi[i] = i&1 ? i : i/2;
c83   for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
dc2     for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
e58  }
```

Combinatorial (6)

PartitionSolver.h

e50fb7, 61 lines

```
d38  template<const int N>
182  struct PartitionSolver {
4ce    vector<vector<int>> part, to, from;
621    PartitionSolver() {
a9d      vector<int> a;
1ed      part.push_back(a);
77f      gen(1, N, a);
796      sort(all(part));
ed4      to.assign(sz(part), vector<int>(N + 1, -1));
9a5      from = to;
ddd      for (int i = 0; i < sz(part); i++) {
a93        int sum = 0;
87f        auto arr = part[i];
bca        for (auto x : arr) sum += x;
4fa        to[i][0] = i;
615        from[i][0] = i;
afc        for (int j = 1; j + sum <= N; j++) {
123          arr = part[i];
9d6          arr.push_back(j);
ceb          sort(all(arr));
d02          to[i][j] = getIndex(arr);
942          from[to[i][j]][j] = i;
20d        }
bef      }
283    }

810    int size() const { return sz(part); }
9ee    int getIndex(const vector<int>& arr) const {
168      return lower_bound(all(part), arr) - part.begin(); }
b49    int add(int id, int num) const { return to[id][num]; }
944    int rem(int id, int num) const { return from[id][num]; }
168    vector<int> getPartition(int id) const {
37b      return part[id]; }

1ba    void gen(int i, int sum, vector<int>& a) {
a05      if (i > sum) { return; }
726      a.push_back(i);
1ed      part.push_back(a);
278      gen(i, sum - i, a);
468      a.pop_back();
48f      gen(i + 1, sum, a);
537    }
f4f  };

     // Number of partitions for all integers <= n
75c  vector<ll> partitionNumber(int n) {
d9c    vector<ll> ans(n + 1, 0);
82f    ans[0] = 1;
78a    for (int i = 1; i <= n; i++) {
87f      for (int j = 1; j * (3 * j + 1) / 2 <= i; j++) {
b6b        ll here = ans[i - j * (3 * j + 1) / 2];
c91        ans[i] = (ans[i] + (j & 1 ? here : -here));
365      }
7c6      for (int j = 1; j * (3 * j - 1) / 2 <= i; j++) {
a1a        ll here = ans[i - j * (3 * j - 1) / 2];
c91        ans[i] = (ans[i] + (j & 1 ? here : -here));
162      }
4a3    }
ba7    return ans;
08b  }
```

Graph (7)

7.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max |w_i| < \sim 2^{63}$.
Time: $\mathcal{O}(VE)$

529834, 24 lines

```
f5e  const ll inf = LLONG_MAX;
83a  struct Ed { int a, b, w, s() { return a < b ? a : -a; }};
9ac  struct Node { ll dist = inf; int prev = -1; };

6fc  void bell(vector<Node>& nodes, vector<Ed>& eds, int s) {
97b   nodes[s].dist = 0;
eb9   sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s(); });

74e   int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled
      vertices
c5a   rep(i,0,lim) for (Ed ed : eds) {
905     Node cur = nodes[ed.a], &dest = nodes[ed.b];
d7d     if (abs(cur.dist) == inf) continue;
6ab     ll d = cur.dist + ed.w;
6ec     if (d < dest.dist) {
956       dest.prev = ed.a;
4c2       dest.dist = (i < lim-1 ? d : -inf);
452     }
75a   }
ced   rep(i,0,lim) for (Ed e : eds) {
3ab     if (nodes[e.a].dist == -inf)
5ff       nodes[e.b].dist = -inf;
1d7   }
166  }
```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m, where $m[i][j] =$ inf if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, inf if no path, or -inf if the path goes through a negative-weight cycle.
Time: $\mathcal{O}(N^3)$

531245, 13 lines

```
964  const ll inf = 1LL << 62;
914  void floydWarshall(vector<vector<ll>>& m) {
e9d   int n = sz(m);
831   rep(i,0,n) m[i][i] = min(m[i][i], 0LL);
99d   rep(k,0,n) rep(i,0,n) rep(j,0,n)
19b     if (m[i][k] != inf && m[k][j] != inf) {
6e8       auto newDist = max(m[i][k] + m[k][j], -inf);
e89       m[i][j] = min(m[i][j], newDist);
f38     }
a69   rep(k,0,n) if (m[k][k] < 0) rep(i,0,n) rep(j,0,n)
ffd     if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
f12  }
```

7.2 Network flow and Matching

Dinic.h

Time: - $\mathcal{O}(\min(m \cdot \text{max\_flow},\ n^2 m))$.
- For graphs with unit capacities: $\mathcal{O}\left(\min(m\sqrt{m},\ mn^{2/3})\right)$.
- If every vertex has in-degree 1 or out-degree 1: $\mathcal{O}(m\sqrt{n})$.
- With capacity scaling: $\mathcal{O}(nm \log(\text{MAXCAP}))$ with high constant factor.

892d6e, 56 lines

```
14d  struct Dinic {
61f    const bool scaling = false;
206    int lim;
670    struct edge {
c63      int to, rev;
a14      ll cap, flow;
7f9      bool res;
6dd      edge(int to_, ll cap_, int rev_, bool res_)
a94        : to(to_), cap(cap_), rev(rev_), flow(0), res(res_){}
477    };

002    vector<vector<edge>> g;
216    vector<int> lev, beg;
a71    ll F;
63f    Dinic(int n) : g(n), lev(n), beg(n), F(0) {}

0c5    void add(int a, int b, ll c, ll other = 0) {
de2      g[a].emplace_back(b, c, sz(g[b]), false);
fa5      g[b].emplace_back(a, other, sz(g[a])-1, true);
14f    }
123    bool bfs(int s, int t) {
e59      fill(all(lev), -1);
4e7      fill(all(beg), 0);
0a4          lev[s] = 0;
8b2      queue<int> q; q.push(s);
647      while (sz(q)) {
be1        int u = q.front(); q.pop();
bd9        for (auto& i : g[u]) {
dbc          if (lev[i.to] != -1 or (i.flow == i.cap)) continue;
b4f          if (scaling and i.cap - i.flow < lim) continue;
185          lev[i.to] = lev[u] + 1;
8ca          q.push(i.to);
f97        }
b1b      }
0de      return lev[t] != -1;
310    }
1dc    ll dfs(int v, int s, ll f = INF) {
50b      if (!f or v == s) return f;
84d      for (int& i = beg[v]; i < sz(g[v]); i++) {
027        auto& e = g[v][i];
206        if (lev[e.to] != lev[v] + 1) continue;
a30        ll foi = dfs(e.to, s, min(f, e.cap - e.flow));
749        if (!foi) continue;
3c5        e.flow += foi, g[e.to][e.rev].flow -= foi;
45c        return foi;
e08      }
bb3      return 0;
b98    }
2b4    ll maxFlow(int s, int t) {
a86      for (lim = scaling ? (1<<30) : 1; lim; lim /= 2)
69c        while (bfs(s, t)) while (ll ff = dfs(s, t)) F += ff;
4ff      return F;
```

```
6c8   }
0fe       bool inCut(int u){ return lev[u] != -1; }
892   };
```

LowerBoundFlow.h

Description: Calculates maximum flow with lower/upper bounds on edges. Returns -1 if no feasible flow exists. `add(a, b, l, r)` adds edge $a \to b$ where flow f must satisfy $l \le f \le r$. `add(a, b, c)` adds edge $a \to b$ with capacity c (implies $0 \le f \le c$). Same complexity as Dinic.

"Dinic.h" 756539, 36 lines

```
0ca struct lb_max_flow : Dinic {
96f   vector<ll> d;
be9   lb_max_flow(int n) : Dinic(n + 2), d(n, 0) {}
b12   void add(int a, int b, int l, int r) {
c97     d[a] -= l;
f1b     d[b] += l;
cb6     Dinic::add(a, b, r - l);
989   }
087   void add(int a, int b, int c) {
610     Dinic::add(a, b, c);
330   }
7a1   bool has_circulation() {
ac0     int n = sz(d);
854     ll cost = 0;
fea     rep(i, 0, n){
c69       if (d[i] > 0) {
f56         cost += d[i];
4f6         Dinic::add(n, i, d[i]);
551       } else if (d[i] < 0) {
bd2         Dinic::add(i, n+1, -d[i]);
bd9       }
a13     }

9f2     return (Dinic::maxFlow(n, n+1) == cost);
cc6   }
7bd   bool has_flow(int src, int snk) {
eda     Dinic::add(snk, src, INF);
e40     return has_circulation();
4aa   }
4eb   ll max_flow(int src, int snk) {
ee8     if (!has_flow(src, snk)) return -1;
99c     Dinic::F = 0;
703     return Dinic::maxFlow(src, snk);
0bb   }
756 };
```

MinCost.h

Description: Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only. If graph is a DAG pi can be calculated with DP instead of Bellman ford.

Time: $\mathcal{O}(FE\log(V))$ where F is max flow. $\mathcal{O}(VE)$ for setpi. 6f4fae, 95 lines

```
c4d #include <bits/extc++.h>

9f4 const ll INF = numeric_limits<ll>::max() / 4;

6f3 struct MCMF {
670   struct edge {
ede     int from, to, rev;
e20     ll cap, cost, flow;
092   };
060   int N;
091   vector<vector<edge>> ed;
a83   vector<int> seen, vis;
0ec   vector<ll> dist, pi;
c45   vector<edge*> par;

2cc   MCMF(int N) : N(N), ed(N), seen(N), vis(N),
dc7     dist(N), pi(N), par(N) {}

6f3   void addEdge(int from, int to, ll cap, ll cost) {
ad8     if (from == to || cap == 0) return;
1af     ed[from].push_back(edge{from,to,sz(ed[to]),cap,cost,0
    });
700     ed[to].push_back(edge{to,from,sz(ed[from])-1,0,-cost,0
    });
dad   }

975   void path(int s) {
7d4     fill(all(seen), 0);
04e     fill(all(dist), INF);
a93     dist[s] = 0;
841     ll di;
937     __gnu_pbds::priority_queue<pair<ll, int>> q;
9fb     vector<decltype(q)::point_iterator> its(N);
23b     q.push({ 0, s });

14d     while (!q.empty()) {
eda       s = q.top().second; q.pop();
2af       seen[s] = 1; di = dist[s] + pi[s];
6bd       for (edge& e : ed[s]) {
d20         if (!seen[e.to]) {
f1f           ll val = di - pi[e.to] + e.cost;
f3c           if(e.cap - e.flow > 0 && val < dist[e.to]){
0c7             dist[e.to] = val;
fb6             par[e.to] = &e;
22d             if (its[e.to] == q.end()) {
aac               its[e.to] = q.push({-dist[e.to], e.to});
388             }
6f8             else q.modify(its[e.to], {-dist[e.to], e.to});
80b           }
fce         }
013       }
e16     }
faa     for (int i = 0; i < N; i++) {
0ef       pi[i] = min(pi[i] + dist[i], INF);
ded     }
17b   }

310   pair<ll, ll> maxflow(int s, int t) {
923     setpi(s, t);
3d3     ll totflow = 0, totcost = 0;
8dd     while (path(s), seen[t]) {
535       ll fl = INF;
733       for (edge* x = par[t]; x; x = par[x->from]) {
8ed         fl = min(fl, x->cap - x->flow);
ddf       }
f9f       totflow += fl;
733       for (edge* x = par[t]; x; x = par[x->from]) {
10b         x->flow += fl;
e58         ed[x->to][x->rev].flow -= fl;
3bf       }
219     }
faa     for (int i = 0; i < N; i++) {
a18       for (edge& e : ed[i]) {
7a0         totcost += e.cost * e.flow;
774       }
a06     }
17e     return { totflow, totcost / 2 };
411   }

      // If some costs can be negative, call this before
          maxflow:
eda   void setpi(int s, int t) {
3ef     fill(all(pi), INF);
156     pi[s] = 0;
45c     int it = N, ch = 1;
aa3     ll v;
5e8     while (ch-- && it--) {
faa       for (int i = 0; i < N; i++) {
c9b         if (pi[i] != INF)
fb0           for (edge& e : ed[i]) if (e.cap)
257             if((v= pi[i] + e.cost)< pi[e.to])
a43               pi[e.to] = v, ch = 1;
d0b       }
250     }
38b     assert(it >= 0); // negative cost cycle
545   }
f1d };
```

PushRelabel.h

Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

Time: $\mathcal{O}\left(V^2\sqrt{E}\right)$ a7bbd5, 55 lines

```
49f struct PushRelabel {
e9b   struct Edge {
548     int dest, back;
e00     ll f, c;
571   };
ed3   vector<vector<Edge>> g;
51c   vector<ll> ec;
658   vector<Edge*> cur;
b08   vector<vector<int>> hs;
4d4   vector<int> H;
4e1   PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n){}

b1c   void addEdge(int s, int t, ll cap, ll rcap=0) {
50b     if (s == t) return;
cc8     g[s].push_back({t, sz(g[t]), 0, cap});
2aa     g[t].push_back({s, sz(g[s])-1, 0, rcap});
817   }

359   void addFlow(Edge& e, ll f) {
759     Edge &back = g[e.dest][e.back];
f7e     if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
d2e     e.f += f; e.c -= f; ec[e.dest] += f;
c47     back.f -= f; back.c += f; ec[back.dest] -= f;
340   }
0e0   ll calc(int s, int t) {
f00     int v = sz(g); H[s] = v; ec[t] = 1;
fbb     vector<int> co(2*v); co[0] = v-1;
e20     for(int i=0; i<v; i++) cur[i] = g[i].data();
8c2     for (Edge& e : g[s]) addFlow(e, e.c);

604     for (int hi = 0;;) {
ae9       while (hs[hi].empty()) if (!hi--) return -ec[s];
c6f       int u = hs[hi].back(); hs[hi].pop_back();
a3e       while (ec[u] > 0)  // discharge u
457         if (cur[u] == g[u].data() + sz(g[u])) {
e94           H[u] = 1e9;
5fa           for (Edge& e : g[u]){
256             if (e.c && H[u] > H[e.dest]+1)
740               H[u] = H[e.dest]+1, cur[u] = &e;
88f           }
f04           if (++co[H[u]], !--co[hi] && hi < v){
10d             for(int i=0; i<v; i++){
4be               if (hi < H[i] && H[i] < v)
021                 --co[H[i]], H[i] = v + 1;
a21             }
cc1           }
3a2           hi = H[u];
b6b         } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1){
779           addFlow(*cur[u], min(ec[u], cur[u]->c));
e91         }else ++cur[u];
```

```
4d7       }
b65     }
385     bool inCut(int a) { return H[a] >= sz(g); }
a7b   };
```

Blossom.h

Description: Max matching on general Graph. $mate[i]$ = match of i
Time: $\mathcal{O}\left(N^3\right)$

21cc7b, 56 lines

```
40f vector<int> Blossom(vector<vector<int>>& g) {
10a     int n = sz(g), timer = -1;
f55     vector<int> mate(n, -1),label(n),par(n),orig(n),aux(n,
      -1),q;

060     auto lca = [&](int x, int y) {
7b8         for (timer++; ; swap(x, y)) {
583             if (x == -1) continue;
4be             if (aux[x] == timer) return x;
90d             aux[x] = timer;
fb4             x=(mate[x] == -1 ? -1 : orig[par[mate[x]]]);
f6a         }
aba     };
be4     auto blossom = [&](int v, int w, int a) {
509         while (orig[v] != a) {
721             par[v] = w; w = mate[v];
1e2             if(label[w] == 1) label[w] = 0,q.push_back(w);
8c7             orig[v] = orig[w] = a;
3d0             v = par[w];
eae         }
068     };
a0f     auto aug = [&](int v) {
8c8         while (v != -1) {
86a             int pv = par[v], nv = mate[pv];
941             mate[v] = pv; mate[pv] = v; v = nv;
ba8         }
54c     };
9f9     auto bfs = [&](int root) {
be5         fill(all(label), -1);
652         iota(all(orig), 0);
4b6         q.clear();
594         label[root] = 0; q.push_back(root);
a43         rep(i, 0, sz(q)) {
4c1             int v = q[i];
5aa             for (auto x : g[v]) {
464                 if (label[x] == -1) {
73a                     label[x] = 1; par[x] = v;
1bd                     if (mate[x] == -1) return aug(x), 1;
8d9                     label[mate[x]] = 0;
de3                     q.push_back(mate[x]);
641                 }
018                 else if(!label[x] && orig[v] != orig[x]){
37f                     int a = lca(orig[v], orig[x]);
f12                     blossom(x, v, a);
183                     blossom(v, x, a);
405                 }
ab5             }
9e2         }
bb3         return 0;
139     };
        // Time halves if you start with (any) maximal
            matching.
fea     rep(i, 0, n) {
698         if (mate[i] == -1) bfs(i);
7b5     }
568     return mate;
21c }
```

HopcroftKarp.h

Description: *ans* is the size of the max matching.
The match of x is $l[x]$
Usage: `HopcroftKarp(|X|, |Y|, edges(x, y))`
Time: $\mathcal{O}\left(\sqrt{V}E\right)$

c4f2f2, 46 lines

```
725 struct HopcroftKarp {
e40     vector<int> g, l, r;
959     int ans;
b82     HopcroftKarp(int n, int m, vector<pii> e)
aa0           : g(sz(e)), l(n, -1), r(m, -1), ans(0) {
bb0         shuffle(all(e), rng);
322         vector<int> deg(n + 1);
235         for (auto& [x, y] : e) deg[x]++;
b4a         rep(i, 1, n+1) deg[i] += deg[i - 1];
85a         for (auto& [x, y] : e) g[--deg[x]] = y;

5ae         vector<int> q(n);
667         while (true) {
661             vector<int> a(n, -1), p(n, -1);
6bb             int t = 0;
fea             rep(i, 0, n){
4b1                 if (l[i] == -1) {
b53                     q[t++] = a[i] = p[i] = i;
4b6                 }
62e             }
a15             bool match = false;
edb             rep(i, 0, t){
912                 int x = q[i];
08c                 if (~l[a[x]]) continue;
0ba                 rep(j, deg[x], deg[x+1]) {
360                     int y = g[j];
89a                     if (r[y] == -1) {
d3b                         while (~y) {
ee7                             r[y] = x;
dbb                             swap(l[x], y);
2a5                             x = p[x];
ebf                         }
6aa                         match = true, ans++;
c2b                         break;
b54                     }
f06                     if (p[r[y]] == -1) {
a74                         q[t++] = y = r[y];
d11                         p[y] = x, a[y] = a[x];
9ef                     }
e8a                 }
0ab             }
984             if (!match) break;
bc5         }
6ec     }
c4f };
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \leq M$.
Time: $\mathcal{O}\left(N^2M\right)$

4a75d2, 41 lines

```
d57 pair<ll, vector<int>> hunga(const vector<vector<ll>>& a) {
c04     if (a.empty()) return { 0, {} };
1a9     int n = sz(a) + 1, m = sz(a[0]) + 1;
fc8     vector<ll> u(n), v(m), p(m);
5bd     vector<int> ans(n - 1);
6f5     for (int i = 1; i < n; i++) {
8c9         p[0] = i;
625         int j0 = 0;
91d         vector<ll> dist(m, LLONG_MAX), pre(m, -1);
910         vector<bool> done(m + 1);
016         do {
781             done[j0] = true;
507             ll i0 = p[j0], j1 = -1, delta = LLONG_MAX;
b84             for (int j = 1; j < m; j++) {
10a                 if (!done[j]) {
ed6                     ll cur = a[i0-1][j-1] - u[i0] - v[j];
607                     if (cur < dist[j])
29f                         dist[j] = cur, pre[j] = j0;
172                     if(dist[j] < delta)
4ab                         delta = dist[j], j1 = j;
103                 }
bb2             }
891             for (int j = 0; j < m; j++) {
7a9                 if (done[j])
3bc                     u[p[j]] += delta, v[j] -= delta;
202                 else dist[j] -= delta;
11a             }
e73             assert(j1 != -1);
6d4             j0 = j1;
ac1         } while (p[j0]);
4b9         while (j0) {
196             int j1 = pre[j0];
0c1             p[j0] = p[j1], j0 = j1;
f55         }
193     }
b84     for (int j = 1; j < m; j++) {
eb3         if (p[j]) ans[p[j] - 1] = j - 1;
c9a     }
def     return { -v[0], ans }; // min cost
4a7 }
```

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}\left(V^3\right)$

8b0e19, 22 lines

```
192 pair<int, vi> globalMinCut(vector<vi> mat) {
afa   pair<int, vi> best = {INT_MAX, {}};
755   int n = sz(mat);
91d   vector<vi> co(n);
d0f   rep(i,0,n) co[i] = {i};
488   rep(ph,1,n) {
2e9     vi w = mat[0];
e44     size_t s = 0, t = 0;
694     rep(it,0,n-ph) { // O(V^2) -> O(E log V) with prio.
    queue
d6e       w[t] = INT_MIN;
a5f       s = t, t = max_element(all(w)) - w.begin();
d39       rep(i,0,n) w[i] += mat[t][i];
ec9     }
3df     best = min(best, {w[t] - mat[t][t], co[t]});
096     co[s].insert(co[s].end(), all(co[t]));
959     rep(i,0,n) mat[s][i] += mat[t][i];
984     rep(i,0,n) mat[i][s] = mat[s][i];
5dd     mat[0][t] = INT_MIN;
ca0   }
f26   return best;
8b0 }
```

7.3 DFS algorithms

Bridges.h

1fa56b, 24 lines

```
cd9 vector<int> g[ms];
9e4 int low[ms], tin[ms], vis[ms], t;

403 void dfs(int u = 0, int p = -1) {
b9c     vis[u] = true;
b4a     low[u] = tin[u] = t++;
```

```
7b9       for (auto v : g[u]) {
730           if (v == p) continue;
c84           if (vis[v]) {
34f               low[u] = min(low[u], tin[v]);
728           }
4e6           else {
95e               dfs(v, u);
ab6               low[u] = min(low[u], low[v]);
                  // if (low[v] >= tin[u] && p != -1), U is an
                       articulation point
975               if (low[v] > tin[u]) {
                      // edge from U to V is a bridge
4b8               }
                  // children++
862           }
677       }
          // if(children > 1 && p == -1) root is an articulation
              point
30c   }
```

BridgeOnline.h

Description: Maintains bridges and 2-edge-connected components (2-ECC) incrementally. `ds[0]` tracks Connected Components (CC). `ds[1]` tracks 2-ECCs. Nodes u, v are in the same 2-ECC iff `dsfind(u, 1) == dsfind(v, 1)`. `g` stores the spanning forest edges (edges that were bridges when added). An edge $(u, v) \in g$ is a current bridge iff `dsfind(u, 1) != dsfind(v, 1)`. `bridges` tracks the total count of active bridges. Use `init()` before starting.
Time: Amortized $\mathcal{O}(\log N)$

ef24c8, 75 lines

```
4dd   int bridges;
801   int ds[2][ms], sz[2][ms];
87b   int h[ms], pai[ms], old[ms];
cd9   vector<int> g[ms];

ca2   void init() {
786       bridges = 0;
f0d       rep(i, 0, ms) {
a4e           g[i].clear(), h[i] = 0;
606           ds[0][i] = ds[1][i] = i;
8f3           sz[0][i] = sz[1][i] = 1;
4a6       }
c1e   }

243   int dsfind(int j, int i) {
7fa       if(j == ds[i][j]) return ds[i][j];
db7       return ds[i][j] = dsfind(ds[i][j], i);
4a4   }

b55   void dfs(int u, int p, int l) {
40d       h[u] = l;
49e       pai[u] = p;
a32       old[u] = dsfind(u, 1);
4d5       for (int v : g[u]) {
730           if (v == p) continue;
0c5           dfs(v, u, l + 1);
11d       }
f2e   }

94c   void updateNodes(int u, int p) {
840       if (old[u] == old[p]) {
dc4           ds[1][u] = ds[1][p];
574       }
e79       else ds[1][u] = u;
4d5       for (int v : g[u]) {
730           if (v == p) continue;
01c           updateNodes(v, u);
42a       }
329   }

814   void mergeTrees(int a, int b) {
cbf       bridges++;
5cb       int iniA = a, iniB = b;
19d       a = dsfind(a, 0), b = dsfind(b, 0);
834       if (sz[0][a] < sz[0][b]) swap(a, b), swap(iniA, iniB);
e14       dfs(iniB, iniA, h[iniA] + 1);
376       old[iniA] = -1;
ee0       updateNodes(iniB, iniA);
86b       ds[0][b] = a;
013       sz[0][a] += sz[0][b];
c9a   }

416   void removeBridges(int a, int b) {
532       a = dsfind(a, 1), b = dsfind(b, 1);
984       while (a != b) {
e7a           bridges--;
54b           if (h[a] < h[b]) swap(a, b);
              // ponte entre (a, pai[a]) deixou de existir
9f6           ds[1][a] = dsfind(pai[a], 1);
e40           a = ds[1][a];
cda       }
a78   }

02b   void addEdge(int a, int b) {
7b9       if (dsfind(a, 0) == dsfind(b, 0)) {
69d           removeBridges(a, b);
221       }
4e6       else {
              // nova ponte entre (a, b)
025           g[a].push_back(b);
3e9           g[b].push_back(a);
f8e           mergeTrees(a, b);
447       }
e57   }
```

BlockCutTree.h

Description: Constructs the Block-Cut Tree, which is a bipartite graph with blocks (maximal 2-vertex-connected components) on one side and articulation points on the other. Works for disconnected graphs. Tree size is $\leq 2N$. Be careful with self loops and multi edges. `art[i]`: number of new components created by removing i (AP if ≥ 1). `blocks[i]`, `edgblocks[i]`: vertices/edges of block i. `tree[i]`: the tree node index corresponding to block i. `pos[i]`: the tree node index corresponding to vertex i.
Time: $\mathcal{O}(N + M)$

e55ab0, 66 lines

```
d10   struct block_cut_tree {
d8e    vector<vector<int>> g, blocks, tree;
43b    vector<vector<pair<int, int>>> edgblocks;
4ce    stack<int> s;
6c0    stack<pair<int, int>> s2;
2bb    vector<int> id, art, pos;

763    block_cut_tree(vector<vector<int>> g_) : g(g_) {
625      int n = sz(g);
37a      id.resize(n, -1), art.resize(n), pos.resize(n);
6f2      build();
246    }

df6    int dfs(int i, int& t, int p = -1) {
cf0      int lo = id[i] = t++;
18e      s.push(i);

827      if (p != -1) s2.emplace(i, p);
43f      for (int j : g[i])
6bf              if (j != p and id[j] != -1) s2.emplace(i, j);

cac      for (int j : g[i]) if (j != p) {
9a3        if (id[j] == -1) {
121          int val = dfs(j, t, i);
0c3          lo = min(lo, val);

588          if (val >= id[i]) {
66a            art[i]++;
483            blocks.emplace_back(1, i);
110            while (blocks.back().back() != j)
138              blocks.back().push_back(s.top()), s.pop();

128            edgblocks.emplace_back(1, s2.top()), s2.pop();
904            while (edgblocks.back().back() != pii(j, i))
bce              edgblocks.back().push_back(s2.top()), s2.pop();
041          }
38c        }
328        else lo = min(lo, id[j]);
5b6      }
924      if (p == -1) {
2db              if(art[i]) art[i]--;
4e6              else{
483                  blocks.emplace_back(1, i);
433                  edgblocks.emplace_back();
333              }
384          }
253      return lo;
6d7    }

0a8    void build() {
6bb      int t = 0;
c80        rep(i, 0, sz(g)) if(id[i] == -1) dfs(i, t, -1);
de0      tree.resize(sz(blocks));
008      rep(i, 0, sz(g)) if (art[i])
b9a        pos[i] = sz(tree), tree.emplace_back();

05c      rep(i, 0, sz(blocks)) for (int j : blocks[i]) {
403        if (!art[j]) pos[j] = i;
4e6        else {
49d                tree[i].push_back(pos[j]);
9a7                tree[pos[j]].push_back(i);
01e            }
27c      }
5a7    }
e55   };
```

DominatorTree.h

Description: Builds the Dominator Tree of a directed graph rooted at `root`. Node u dominates v if every path from `root` to v passes through u. The immediate dominator of v is the unique dominator closest to v (excluding v). Returns a vector `par` where `par[u]` is the parent of u in the tree. Roots and unreachable nodes satisfy `par[u] = u`.
Time: $\mathcal{O}(M \log N)$

8c4613, 55 lines

```
3db   struct dominator_tree {
577    int n, t;
324    vector<vector<int>> g, rg, bucket;
7f3    vector<int> arr, par, rev, sdom, dom, ds, lbl;

226    dominator_tree(int n) : n(n),t(0),g(n),rg(n),bucket(n),
7a1      arr(n,-1),par(n),rev(n),sdom(n),dom(n),ds(n),lbl(n) {}

c2b    void add_edge(int u, int v) { g[u].push_back(v); }

315    void dfs(int u) {
12e      arr[u] = t;
64f      rev[t] = u;
bad      lbl[t] = sdom[t] = ds[t] = t;
c82       t++;
6f1      for (int w : g[u]) {
0c2        if (arr[w] == -1) {
8c6          dfs(w);
81a          par[arr[w]] = arr[u];
```

```
869       }
f8e       rg[arr[w]].push_back(arr[u]);
93a     }
b04   }
792   int find(int u, int x=0) {
9fe     if (u == ds[u]) return x ? -1 : u;
41f     int v = find(ds[u], x+1);
388     if (v < 0) return u;
b30     if(sdom[lbl[ds[u]]] < sdom[lbl[u]]) lbl[u]= lbl[ds[u]];
300     ds[u] = v;
784     return x ? v : lbl[u];
a59   }

46f   vector<int> run(int root) {
14e     dfs(root);
b81     iota(all(dom), 0);
da8     for (int i=t-1; i>=0; i--) {
76c       for(int w : rg[i]) sdom[i] = min(sdom[i],sdom[find(w)
      ]);
c94       if (i) bucket[sdom[i]].push_back(i);
3b2       for (int w : bucket[i]) {
46a         int v = find(w);
ae4         if (sdom[v] == sdom[w]) dom[w] = sdom[w];
41c         else dom[w] = v;
1e6       }
fd8       if (i > 1) ds[i] = par[i];
b9e     }
e8f     rep(i, 1, t) {
7d7       if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
32d     }
af8     vector<int> par(n);
2c2     iota(all(par), 0);
533     rep(i, 0, t) par[rev[i]] = rev[dom[i]];
148     return par;
900   }
8c4 };
```

EulerPath.h

Description: Receives as input graph(node, edge index), number of edges and source. Returns list of node, index of edge he came from, if path/circuit does not exists returns empty list.

a3ed13, 27 lines

```
b4a   vector<pii> eulerPath(const vector<vector<pii>>& g, int
      nedges, int src) {
625       int n = sz(g);
b47       vector<int> deg(n, 0), its(n, 0), used(nedges + 1, 0);
a42       vector<pii> s = { {src, -1} };
          //deg[src]++; //to allow paths, not only circuits
a5f       vector<pii> ret;
980       while (!s.empty()) {
d0b           int u = s.back().first, &it = its[u];
c45           if (it == sz(g[u])) {
5e3               ret.push_back(s.back());
342               s.pop_back();
5e2               continue;
8e8           }
f7f           auto& [nxt, id] = g[u][it++];
b25           if (!used[id]) {
e48               deg[u]--, deg[nxt]++;
029               used[id] = 1;
e1c               s.push_back({ nxt, id });
777           }
388       }
8d8       for (int x : deg) {
518           if (x < 0 || sz(ret) != (nedges + 1)) return {};
26e       }
969       reverse(ret.begin(), ret.end());
edf       return ret;
a3e   }
```

SCC.h

Description: Kosaraju algorithm for calculating strongly connected components. Components are ordered in topological order.

008ff2, 36 lines

```
bf0   struct SCC {
dab       int n, ncomp;
0e3       vector<vector<int>> g, inv;
829       vector<int> comp, vis, stk;
8b6       SCC(){}
471       SCC(int n)
464           : n(n),ncomp(0),g(n),inv(n),comp(n, -1),vis(n){}

315       void dfs(int u) {
150           vis[u] = 1;
a35           for (int v : g[u]) if (!vis[v]) dfs(v);
967           stk.push_back(u);
37b       }
f20       void dfs_inv(int u) {
62c           comp[u] = ncomp;
3a5           for (int v : inv[u]) {
df4               if (comp[v] == -1) dfs_inv(v);
0a0           }
984       }
63d       void solve() {
603           for (int i = 0; i < n; i++) {
b65               if (!vis[i]) dfs(i);
358           }
340           reverse(all(stk));
49b           for (int u : stk) {
9ef               if (comp[u] != -1) continue;
672               dfs_inv(u);
a8f               ncomp++;
ecb           }
ef8       }
010       void add_edge(int a, int b) {
025           g[a].push_back(b);
a6a           inv[b].push_back(a);
1ec       }
008   };
```

TwoSat.h

Usage: `not A = ~A`

"SCC.h" c8b989, 37 lines

```
d9d   struct TwoSat{
1a8    int n;
3c9    SCC scc;
7c7    vector<int> value;
425    vector<pii> e;
e2c    TwoSat(int n) : n(n){}
6c0    bool solve(){
b36      value.resize(n);
8cc      scc = SCC(2*n);
1f3      for(auto &x : e) scc.add_edge(x.first, x.second);
7f9      scc.solve();
3df      for(int i=0; i<2*n; i++)
f83        if(scc.comp[i] == scc.comp[i^1]) return false;
830      for(int i=0; i<n; i++)
733        value[i] = scc.comp[id(i)] > scc.comp[id(~i)];
8a6      return true;
949    }
a0a    void atMostOne(vector<int> &li){
615      if(sz(li) <= 1) return;
da9      int cur = ~li[0];
b25      for(int i = 2; i < sz(li); i++) {
abb        int next = n++;
e0a        addOr(cur, ~li[i]);
f26        addOr(cur, next);
7ba        addOr(~li[i], next);
072        cur = ~next;
e3d      }
921      addOr(cur, ~li[1]);
bbb    }
41b    int id(int v) { return v < 0 ? (~v) * 2 ^ 1 : v * 2; }
276    void add(int a, int b) { e.push_back({id(a), id(b)}); }
bc7    void addOr(int a, int b) { add(~a, b); add(~b, a); }
671    void addImp(int a, int b) { addOr(~a, b); }
d9d    void addEqual(int a, int b){ addOr(a, ~b); addOr(~a, b);
     }
ec3    void isFalse(int a) { addImp(a, ~a); }
c8b   };
```

7.4 Coloring

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)
Time: $\mathcal{O}(NM)$

e210e2, 32 lines

```
f41   vi edgeColoring(int N, vector<pii> eds) {
727    vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
10d    for (pii e : eds) ++cc[e.first], ++cc[e.second];
e2f    int u, v, ncols = *max_element(all(cc)) + 1;
fda    vector<vi> adj(N, vi(ncols, -1));
6ec    for (pii e : eds) {
119      tie(u, v) = e;
e51      fan[0] = v;
0f4      loc.assign(ncols, 0);
696      int at = u, end = u, d, c = free[u], ind = 0, i = 0;
3b2      while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
3e1        loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
01e      cc[loc[d]] = c;
997      for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd
     ])
4ff        swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
79f      while (adj[fan[i]][d] != -1) {
a9f        int left = fan[i], right = fan[++i], e = cc[i];
99b        adj[u][e] = left;
ccb        adj[left][e] = u;
f7e        adj[right][e] = -1;
d99        free[right] = e;
316      }
dfd      adj[u][d] = fan[i];
c45      adj[fan[i]][d] = u;
0e1      for (int y : {fan[0], u, end})
3fa        for (int& z = free[y] = 0; adj[y][z] != -1; z++);
fdc    }
29d    rep(i,0,sz(eds))
961      for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i
     ];
edf    return ret;
e21   }
```

7.5 Heuristics

MaxClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.
Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

2eeaf4, 53 lines

```
db9   using vb = vector<bitset<200>>;
c7d   struct Maxclique {
24e    double limit=0.025, pk=0;
c04    struct Vertex { int i, d=0; };
547    using vv = vector<Vertex>;
d44    vb e;
```

```
df7   vv V;
e5c   vector<vector<int>> C;
497   vector<int> qmax, q, S, old;
fe3   void init(vv& r) {
fd3     for (auto& v : r) v.d = 0;
583     for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
0f1     sort(all(r), [](auto a, auto b) { return a.d > b.d; });
c43     int mxD = r[0].d;
3f8     for(int i=0; i<sz(r); i++) r[i].d = min(i, mxD) + 1;
526   }
bc8   void expand(vv& R, int lev = 1) {
ac1     S[lev] += S[lev - 1] - old[lev];
92c     old[lev] = S[lev - 1];
d18     while (sz(R)) {
3fd       if (sz(q) + R.back().d <= sz(qmax)) return;
d62       q.push_back(R.back().i);
f28       vv T;
7fb       for(auto v : R)
740         if (e[R.back().i][v.i]) T.push_back({v.i});
d21       if (sz(T)) {
eea         if (S[lev]++ / ++pk < limit) init(T);
457         int j = 0, mxk = 1, mnk = max(sz(qmax)-sz(q)+1, 1);
9bc         C[1].clear(), C[2].clear();
969         for (auto v : T) {
bfe           int k = 1;
8f5           auto f = [&](int i) { return e[v.i][i]; };
5c6           while (any_of(all(C[k]), f)) k++;
782           if (k > mxk) mxk = k, C[mxk + 1].clear();
18a           if (k < mnk) T[j++].i = v.i;
0e6           C[k].push_back(v.i);
322         }
238         if (j > 0) T[j - 1].d = 0;
d2f         for(int k=mnk; k<mxk + 1; k++){
5bf                     for (int i : C[k])
361                 T[j].i = i, T[j++].d = k;
9dc                 }
22d         expand(T, lev + 1);
61f       } else if (sz(q) > sz(qmax)) qmax = q;
c81       q.pop_back(), R.pop_back();
3e0     }
81d   }
b2d   vector<int> maxClique(){ init(V),expand(V); return qmax;}
b40   Maxclique(vb conn) : e(conn),C(sz(e)+1),S(sz(C)),old(S){
01d     for(int i=0; i<sz(e); i++) V.push_back({i});
b60   }
534 };
```

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs b0d5b1, 13 lines

```
753 typedef bitset<128> B;
044 template<class F>
6a9 void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R
    ={}) {
9bb   if (!P.any()) { if (!X.any()) f(R); return; }
a8e   auto q = (P | X)._Find_first();
cd1   auto cands = P & ~eds[q];
3d7   rep(i,0,sz(eds)) if (cands[i]) {
a75     R[i] = 1;
e78     cliques(eds, f, P & eds[i], X & eds[i], R);
bb6     R[i] = P[i] = 0; X[i] = 1;
181   }
c9d }
```

7.6 Trees

Centroid.h

Description: Call decomp(0) to solve, marked array should be initially set to zero.

Time: $\mathcal{O}(N \log N)$ b73755, 27 lines

```
6b6 int tam[ms], marked[ms];

2a1 int calc_tam(int u, int p) {
5d1   tam[u] = 1;
4d5   for (int v : g[u]) {
5ea     if (v != p && !marked[v]) tam[u] += calc_tam(v, u);
d09   }
f95   return tam[u];
d5d }

5fb int get_centroid(int u, int p, int tot) {
4d5   for (int v : g[u]) {
38c     if (v != p && !marked[v] && (tam[v] > (tot / 2)))
32c       return get_centroid(v, u, tot);
b6c   }
03f   return u;
0c7 }
    // Cent is a child of P in the centroid tree
179 void decomp(int u, int p = -1) {
308   calc_tam(u, -1);
bd4   int cent = get_centroid(u, -1, tam[u]);
83d   marked[cent] = 1;
9f1   for (int v : g[cent]) {
c6e     if (!marked[v]) decomp(v, cent);
194   }
dc1 }
```

HLD.h

Description: If values are stored on edges, set EDGE = true and store each edge's value at the endpoint farther from the root (the deeper node). rp[i] is the representative (head) of the heavy path containing node i: it is the node in that chain that is closest to the root. a129d6, 51 lines

```
5f2 template<bool EDGE> struct HLD {
577     int n, t;
789     vector<vector<int>> g;
003     vector<int> pai, rp, tam, pos, val, arr;
f1e     Seg seg;
bcf     HLD(int n, vector<vector<int>>& g, vector<int>& val)
ac9         : n(n), t(0), g(g), pai(n), rp(n), tam(n, 1),
616             pos(n), val(val), arr(n) {
f80         calc_tam(0, -1);
c91         dfs(0, -1);
d14         seg.build(arr);
a43     }

2a1     int calc_tam(int u, int p) {
49e         pai[u] = p;
704         for (int& v : g[u]) {
730             if (v == p) continue;
2e4             tam[u] += calc_tam(v, u);
2d5             if(tam[v] > tam[g[u][0]] || g[u][0] == p)
a7f                 swap(g[u][0], v);
0a3         }
f95         return tam[u];
c19     }

fb6     void dfs(int u, int p) {
4c8         pos[u] = t++;
d7b         arr[pos[u]] = val[u];
4d5         for (int v : g[u]) {
730             if (v == p) continue;
84d             rp[v] = (v == g[u][0] ? rp[u] : v);
95e             dfs(v, u);
42d         }
de1     }

4ea     int query(int a, int b) { // query on the path from a
        to b
1a4         int ans = 0; // neutral value
34d         while (rp[a] != rp[b]) {
aa1             if (pos[a] < pos[b]) swap(a, b);
9a5             ans = max(ans, seg.query(pos[rp[a]], pos[a]));
677             a = pai[rp[a]];
ebd         }
9bc         if(pos[a] > pos[b]) swap(a, b);
0f8         ans = max(ans, seg.query(pos[a] + EDGE, pos[b]));
ba7         return ans;
e8a     }

534     void update(int a, int x) {
e5e         seg.update(pos[a], x);
5db     }
a12 };
```

LCA.h

Description: LCA algorithm using binary lifting, $is\_ancestor(a,b)$ returns true if a is an ancestral of b and false otherwise.

Time: $\mathcal{O}(N \log N)$ db7791, 26 lines

```
67e int tin[MAXN], tout[MAXN], timer=0;
768 int up[MAXN][BITS];
fb6 void dfs(int u, int p){
545     tin[u] = timer++, up[u][0] = p;
532     for (int i=1; i<BITS; i++) {
88a         up[u][i] = up[up[u][i-1]][i-1];
4a0     }
712     for (int v: g[u]) if (v != p) dfs(v, u);
4f8     tout[u] = timer;
4a1 }

f31 bool is_ancestor(int u, int v){
d34     return (tin[u] <= tin[v] && tout[u] >= tout[v]);
f9f }

310 int lca(int u, int v){
bd5     if (is_ancestor(u, v)) return u;
6fc     if (is_ancestor(v, u)) return v;
3c3     for (int i=BITS-1; i>=0; i--) {
3a3         if (up[u][i] && !is_ancestor(up[u][i], v)) {
c3f             u = up[u][i];
49e         }
dc4     }
c15     return up[u][0];
001 }
```

VirtualTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S|-1$) pairwise LCA's and compressing edges. virt[u] is the adjacency list of the virtual tree: it stores pairs (v, dist), where v is a neighbor of u in the virtual tree and dist is the distance between u and v in the original tree.

Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h" 11157a, 24 lines

```
0b1 vector<pair<int, int>> virt[ms];

d0c void build_virt(vector<int>& v) {
078   auto cmp = [&](int i, int j){ return tin[i] < tin[j]; };
b84   sort(all(v), cmp);
1ee   for (int i = 0, n = sz(v);i + 1 < n; i++)
4cf     v.push_back(lca(v[i], v[i + 1]));
b84   sort(all(v), cmp);
```

```
64f     v.erase(unique(all(v)), v.end());
7b4     stack<int> st;
3a7     for (auto u : v) {
c53       if (st.empty()) {
4a6         st.push(u);
e82       }
4e6       else {
7eb         while(sz(st) && !is_ancestor(st.top(), u)) st.pop();
88b         int p = st.top();
bfa         virt[p].emplace_back(u, abs(lvl[u] - lvl[p]));
0a5         virt[u].emplace_back(p, abs(lvl[u] - lvl[p]));
4a6         st.push(u);
92c       }
f46     }
c83   }
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" 39e620, 61 lines

```
030 struct Edge { int a, b; ll w; };
bf2 struct Node {
25f  Edge key;
c17  Node *l, *r;
981  ll delta;
a9c  void prop() {
6f9    key.w += delta;
d2d    if (l) l->delta += delta;
d86    if (r) r->delta += delta;
978    delta = 0;
0d3  }
866  Edge top() { prop(); return key; }
ab4 };
3eb Node *merge(Node *a, Node *b) {
b9f  if (!a || !b) return a ?: b;
626  a->prop(), b->prop();
dc2  if (a->key.w > b->key.w) swap(a, b);
485  swap(a->l, (a->r = merge(b, a->r)));
3f5  return a;
c51 }
7bb void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

002 pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
8df  RollbackUF uf(n);
3f8  vector<Node*> heap(n);
563  for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e
    });
cd2  ll res = 0;
517  vi seen(n, -1), path(n), par(n);
559  seen[r] = r;
dd6  vector<Edge> Q(n), in(n, {-1,-1}), comp;
111  deque<tuple<int, int, vector<Edge>>> cycs;
328  rep(s,0,n) {
3cb    int u = s, qi = 0, w;
a0a    while (seen[u] < 0) {
572      if (!heap[u]) return {-1,{}};
ebe      Edge e = heap[u]->top();
5ed      heap[u]->delta -= e.w, pop(heap[u]);
952      Q[qi] = e, path[qi++] = u, seen[u] = s;
d56      res += e.w, u = uf.find(e.a);
9e2      if (seen[u] == s) {
28d        Node* cyc = 0;
cab        int end = qi, time = uf.time();
f38        do cyc = merge(cyc, heap[w = path[--qi]]);
4f9        while (uf.join(u, w));
562        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
c06        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
00a      }
c8f    }
068    rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
fa3  }

e41  for (auto& [u,t,comp] : cycs) { // restore sol (optional)
36c    uf.rollback(t);
1d0    Edge inEdge = in[u];
251    for (auto& e : comp) in[uf.find(e.b)] = e;
56d    in[uf.find(inEdge.b)] = inEdge;
4f9  }
427  rep(i,0,n) par[i] = in[i].a;
efb  return {res, par};
efa }
```

Geometry (8)

8.1 Geometric primitives

Point.h

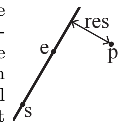
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

47ec0a, 29 lines

```
48b template <class T> int sgn(T x) { return (x > 0) - (x < 0)
    ; }
4fc template<class T>
f26 struct Point {
ea4  typedef Point P;
645  T x, y;
ea6  explicit Point(T x=0, T y=0) : x(x), y(y) {}
0d0  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y
    ); }
ec7  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y
    ); }
279  P operator+(P p) const { return P(x+p.x, y+p.y); }
40d  P operator-(P p) const { return P(x-p.x, y-p.y); }
e03  P operator*(T d) const { return P(x*d, y*d); }
0b9  P operator/(T d) const { return P(x/d, y/d); }
57b  T dot(P p) const { return x*p.x + y*p.y; }
460  T cross(P p) const { return x*p.y - y*p.x; }
b3f  T cross(P a, P b) const { return (a-*this).cross(b-*this)
    ; }
f68  T dist2() const { return x*x + y*y; }
18b  double dist() const { return sqrt((double)dist2()); }
     // angle to x-axis in interval [-pi, pi]
907  double angle() const { return atan2(y, x); }
d06  P unit() const { return *this/dist(); } // makes dist()=1
200  P perp() const { return P(-y, x); } // rotates +90
    degrees
852  P normal() const { return perp().unit(); }
     // returns point rotated 'a' radians ccw around the
         origin
f23  P rotate(double a) const {
482    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
902  friend ostream& operator<<(ostream& os, P p) {
9a9    return os << "(" << p.x << "," << p.y << ")"; }
d2d };
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

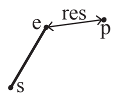


"Point.h" f6bf6b, 5 lines

```
7dc template<class P>
2ff double lineDist(const P& a, const P& b, const P& p) {
e07  return (double)(b-a).cross(p-a)/(b-a).dist();
008 }
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.



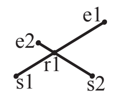
Usage: `Point<double> a, b(2,2), p(1,1);`
`bool onSegment = segDist(a,b,p) < 1e-10;`

"Point.h" 5c88f4, 7 lines

```
626 typedef Point<double> P;
929 double segDist(P& s, P& e, P& p) {
a44  if (s==e) return (p-s).dist();
f81  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)))
    ;
2c1  return ((p-s)*d-(e-s)*t).dist()/d;
ae7 }
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



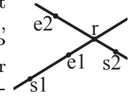
Usage: `vector<P> inter = segInter(s1,e1,s2,e2);`
`if (sz(inter)==1)`
`cout << "segments intersect at " << inter[0] << endl;`

"Point.h", "OnSegment.h" 9d57f2, 14 lines

```
dae template<class P> vector<P> segInter(P a, P b, P c, P d) {
0b6  auto oa = c.cross(d, a), ob = c.cross(d, b),
318       oc = a.cross(b, c), od = a.cross(b, d);
     // Checks if intersection is single non-endpoint point.
914  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
e5b    return {(a * ob - b * oa) / (ob - oa)};
4c1  set<P> s;
ccb  if (onSegment(c, d, a)) s.insert(a);
0ad  if (onSegment(c, d, b)) s.insert(b);
3d8  if (onSegment(a, b, c)) s.insert(c);
2fa  if (onSegment(a, b, d)) s.insert(d);
a35  return {all(s)};
9d5 }
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage: `auto res = lineInter(s1,e1,s2,e2);`
`if (res.first == 1)`
`cout << "intersection point at " << res.second << endl;`

"Point.h" a01f81, 9 lines

```
7dc template<class P>
0bf pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
14f  auto d = (e1 - s1).cross(e2 - s2);
8cc  if (d == 0) // if parallel
d99    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
f6b  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
```

```
9b8   return {1, (s1 * p + e1 * q) / d};
472 }
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.
Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h" 3af81c, 10 lines

```
7dc template<class P>
70b int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

7dc template<class P>
b5e int sideOf(const P& s, const P& e, const P& p, double eps)
    {
79e   auto a = (e-s).cross(p-s);
653   double l = (e-s).dist()*eps;
c32   return (a > l) - (a < -l);
33f }
```

OnSegment.h

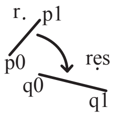
Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

"Point.h" c597e8, 4 lines

```
514 template<class P> bool onSegment(P s, P e, P p) {
5fb   return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
c59 }
```

linearTransformation.h

Description:



Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.

"Point.h" 03a306, 7 lines

```
626 typedef Point<double> P;
664 P linearTransformation(const P& p0, const P& p1,
f06     const P& q0, const P& q1, const P& r) {
99f   P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
0aa   return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist
    2();
45e }
```

LineProjectionReflection.h

Description: Projects point p onto line ab. Set refl=true to get reflection of point p across line ab instead. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h" b5562d, 6 lines

```
7dc template<class P>
981 P lineProj(P a, P b, P p, bool refl=false) {
de3   P v = b - a;
3fc   return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
4b7 }
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.
Usage: `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; } // sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i`

0f0602, 36 lines

```
755 struct Angle {
e91   int x, y;
8bd   int t;
5ac   Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
de8   Angle operator-(Angle b) const { return {x-b.x, y-b.y, t
    }; }
3cd   int half() const {
840     assert(x || y);
aa4     return y < 0 || (y == 0 && x < 0);
c93   }
dfc   Angle t90() const { return {-y, x, t + (half() && x >= 0)
    }; }
726   Angle t180() const { return {-x, -y, t + half()}; }
925   Angle t360() const { return {x, y, t + 1}; }
e25 };
a92 bool operator<(Angle a, Angle b) {
      // add a.dist2() and b.dist2() to also compare distances
ea7   return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
05f          make_tuple(b.t, b.half(), a.x * (ll)b.y);
ce5 }

    // Given two points, this calculates the smallest angle
        between
    // them, i.e., the angle that covers the defined line
        segment.
908 pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
ee4   if (b < a) swap(a, b);
423   return (b < a.t180() ?
c35           make_pair(a, b) : make_pair(b, a.t360()));
5ea }
784 Angle operator+(Angle a, Angle b) { // point a + vector b
eb1   Angle r(a.x + b.x, a.y + b.y, a.t);
8ca   if (a.t180() < r) r.t--;
d9f   return r.t180() < a ? r.t360() : r;
3d8 }
106 Angle angleDiff(Angle a, Angle b) { // angle b - angle a
125   int tu = b.t - a.t; a.t = b.t;
e63   return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a
    )};
ba3 }
```

HalfPlane.h

Description: Computes the intersection of a set of half-planes. Half-planes are sorted by angle and processed with a deque, removing redundant or conflicting constraints. Parallel half-planes are handled explicitly. Returns the convex polygon of the intersection, or empty if infeasible.
Time: $\mathcal{O}(n \log n)$

"Point.h" cf24a8, 72 lines

```
984 using ld = long double;
207 using P = Point<ld>;

533 struct Hp { // Half plane struct
      // 'p' is a passing point of the line and 'pq' is the
          direction vector of the line.
812   P p, pq;
d29   ld angle;

b93   Hp() {}
65d   Hp(const P& a, const P& b) : p(a), pq(b - a) {
0e3     angle = atan2l(pq.y, pq.x);
2ff   }
8ce   bool out(const P& r) { return pq.cross(r - p) < -eps; }
d36   bool operator < (const Hp& e) const {
1dd     return angle < e.angle;
44e   }
ea9   friend P inter(const Hp& s, const Hp& t) {
020     ld alpha = (t.p - s.p).cross(t.pq) / s.pq.cross(t.pq);
93b     return s.p + (s.pq * alpha);
825   }
b46 };
fa5 vector<P> hp_intersect(vector<Hp>& H) {
12f   P box[4] = { P(inf, inf), P(-inf, inf),
9c8                P(-inf, -inf), P(inf, -inf) };

1cd   for(int i = 0; i<4; i++) {
1a8     Hp aux(box[i], box[(i+1) % 4]);
d82     H.push_back(aux);
560   }
f1a   sort(all(H));
6c5   deque<Hp> dq;
486   int len = 0;
908   for(int i = 0; i < sz(H); i++) {
3fb     while(len>1 && H[i].out(inter(dq[len-1], dq[len-2]))){
c70       dq.pop_back();
654       --len;
a31     }
757     while (len > 1 && H[i].out(inter(dq[0], dq[1]))){
c68       dq.pop_front();
654       --len;
1eb     }
a5a     if(len && fabsl(H[i].pq.cross(dq[len-1].pq)) < eps){
25f       if (H[i].pq.dot(dq[len-1].pq) < 0.0)
282         return vector<P>();
e7b       if (H[i].out(dq[len-1].p)) {
c70         dq.pop_back();
654         --len;
2dc       }
64e       else continue;
9a0     }
fc2     dq.push_back(H[i]);
250     ++len;
8ed   }

337   while(len> 2 && dq[0].out(inter(dq[len-1], dq[len-2]))){
c70     dq.pop_back();
654     --len;
faa   }
81e   while (len > 2 && dq[len-1].out(inter(dq[0], dq[1]))){
c68     dq.pop_front();
654     --len;
694   }
1a3   if (len < 3) return vector<P>();
7e7   vector<P> ret(len);
cc7   for(int i = 0; i+1 < len; i++) {
01e     ret[i] = inter(dq[i], dq[i+1]);
00f   }
4fd   ret.back() = inter(dq[len-1], dq[0]);
edf   return ret;
deb }
```

8.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" ba7267, 12 lines

```
626 typedef Point<double> P;
27f bool circleInter(P a,P b,double r1,double r2,pair<P, P>*
    out) {
b48   if (a == b) { assert(r1 != r2); return false; }
f30   P vec = b - a;
6c8   double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2;
c28   double p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*
    d2;
5b0   if (sum*sum < d2 || dif*dif > d2) return false;
84d   P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) /
    d2);
21e   *out = {mid + per, mid - per};
```

```
8a6  return true;
170 }
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" b0153d, 14 lines

```
7dc template<class P>
3a5 vector<pair<P, P>> tangents(P c1, double r1, P c2, double
    r2) {
c0b  P d = c2 - c1;
432  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
018  if (d2 == 0 || h2 < 0)  return {};
c14  vector<pair<P, P>> out;
092  for (double sign : {-1, 1}) {
2ad    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
2e3    out.push_back({c1 + v * r1, c2 + v * r2});
e25  }
b21  if (h2 == 0) out.pop_back();
fe8  return out;
483 }
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" e0cfba, 10 lines

```
7dc template<class P>
195 vector<P> circleLine(P c, double r, P a, P b) {
33b  P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
55a  double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
3e4  if (h2 < 0) return {};
071  if (h2 == 0) return {p};
7cd  P h = ab.unit() * sqrt(h2);
d65  return {p - h, p + h};
59a }
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

"../../content/geometry/Point.h" 19add1, 20 lines

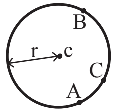
```
626 typedef Point<double> P;
361 #define arg(p, q) atan2(p.cross(q), p.dot(q))
bb9 double circlePoly(P c, double r, vector<P> ps) {
6d1  auto tri = [&](P p, P q) {
c9c    auto r2 = r * r / 2;
291    P d = q - p;
127    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist
    2();
eea    auto det = a * a - b;
691    if (det <= 0) return arg(p, q) * r2;
f43    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det
    ));
aba    if (t < 0 || 1 <= s) return arg(p, q) * r2;
57f    P u = p + d * s, v = q + d * (t-1);
8c0    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
a52  };
bef  auto sum = 0.0;
8f4  rep(i,0,sz(ps))
3b7    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
e66  return sum;
```

```
f08 }
```

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" 1caa3a, 10 lines

```
626 typedef Point<double> P;
510 double ccRadius(const P& A, const P& B, const P& C) {
14b  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
f73      abs((B-A).cross(C-A))/2;
607 }
c0d P ccCenter(const P& A, const P& B, const P& C) {
28a  P b = C-A, c = B-A;
680  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
793 }
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h" 09dd0a, 18 lines

```
a28 pair<P, double> mec(vector<P> ps) {
4da  shuffle(all(ps), mt19937(time(0)));
f6a  P o = ps[0];
328  double r = 0, EPS = 1 + 1e-8;
2be  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
5cc    o = ps[i], r = 0;
4da    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
a30      o = (ps[i] + ps[j]) / 2;
6f7      r = (o - ps[i]).dist();
102      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
fa9        o = ccCenter(ps[i], ps[j], ps[k]);
6f7        r = (o - ps[i]).dist();
648      }
7b0    }
dcf  }
645  return {o, r};
09d }
```

8.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`
Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 2bf504, 12 lines

```
7dc template<class P>
0cc bool inPolygon(vector<P> &p, P a, bool strict = true) {
8b7  int cnt = 0, n = sz(p);
fea  rep(i,0,n) {
444    P q = p[(i + 1) % n];
cbd    if (onSegment(p[i], q, a)) return !strict;
       //or: if (segDist(p[i], q, a) <= eps) return !strict;
007    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) >
    0;
1b9  }
70a  return cnt;
c72 }
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" f12300, 7 lines

```
4fc template<class T>
a51 T polygonArea2(vector<Point<T>>& v) {
2f8  T a = v.back().cross(v[0]);
06e  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
3f5  return a;
693 }
```

PolygonCenter.h

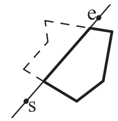
Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

"Point.h" 9706dc, 10 lines

```
626 typedef Point<double> P;
6d9 P polygonCenter(const vector<P>& v) {
f9f  P res(0, 0); double A = 0;
70b  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
346    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
3ea    A += v[j].cross(v[i]);
307  }
33c  return res / A / 3;
0d0 }
```

PolygonCut.h

Description:
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`



"Point.h" d07181, 14 lines

```
626 typedef Point<double> P;
37d vector<P> polygonCut(const vector<P>& poly, P s, P e) {
fe2  vector<P> res;
d48  rep(i,0,sz(poly)) {
21c    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
c5f    auto a = s.cross(e, cur), b = s.cross(e, prev);
2dc    if ((a < 0) != (b < 0))
380      res.push_back(cur + (prev - cur) * (a / (a - b)));
c5c    if (a < 0)
a5f      res.push_back(cur);
757  }
b50  return res;
42c }
```

PolygonUnion.h

Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)
Time: $\mathcal{O}(N^2)$, where N is the total number of points

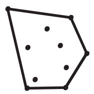
"Point.h", "sideOf.h" 3931c6, 34 lines

```
626 typedef Point<double> P;
142 double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y
    ; }
61d double polyUnion(vector<vector<P>>& poly) {
499  double ret = 0;
9af  rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
9c8    P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
05c    vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
cbd    rep(j,0,sz(poly)) if (i != j) {
cc1      rep(u,0,sz(poly[j])) {
418        P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])
    ];
688        int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
68b        if (sc != sd) {
295          double sa = C.cross(D, A), sb = C.cross(D, B);
```

```
e90         if (min(sc, sd) < 0)
dac           segs.emplace_back(sa / (sa - sb), sgn(sc - sd))
    ;
cf7       } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))
    >0){
5b4         segs.emplace_back(rat(C - A, B - A), 1);
e96         segs.emplace_back(rat(D - A, B - A), -1);
313       }
0d1     }
fdc   }
861   sort(all(segs));
153   for (auto& s : segs) s.first = min(max(s.first, 0.0), 1
    .0);
68c   double sum = 0;
723   int cnt = segs[0].second;
067   rep(j,1,sz(segs)) {
081     if (!cnt) sum += segs[j].first - segs[j - 1].first;
6e9     cnt += segs[j].second;
f58   }
320   ret += A.cross(B) * sum;
191 }
ad6 return ret / 2;
6e8 }
```

ConvexHull.h

Description:



Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull. If you want to keep the collinear points in the convex hull, change the comparison to $h[t-2].cross(h[t-1], p) < 0$ and the size of the vector h to $2 * sz(pts) + 1$.

Time: $\mathcal{O}(n \log n)$

"Point.h" 310954, 14 lines

```
2c0 typedef Point<ll> P;
f16 vector<P> convexHull(vector<P> pts) {
f78  if (sz(pts) <= 1) return pts;
3cb  sort(all(pts));
abf  vector<P> h(sz(pts)+1);
573  int s = 0, t = 0;
628  for (int it = 2; it--; s = --t, reverse(all(pts)))
4eb    for (P p : pts) {
3da      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t
    --;
f39      h[t++] = p;
bf0    }
036  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1
    ])};
ec8 }
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

Time: $\mathcal{O}(n)$

"Point.h" c571b8, 13 lines

```
2c0 typedef Point<ll> P;
d31 array<P, 2> hullDiameter(vector<P> S) {
e79  int n = sz(S), j = n < 2 ? 0 : 1;
354  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
e4d  rep(i,0,j)
42e    for (;; j = (j + 1) % n) {
ca1      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}})
    ;
be8      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >=
     0)
c2b        break;
56c    }
3f2  return res.second;
5f7 }
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446b, 15 lines

```
2c0 typedef Point<ll> P;

2d4 bool inHull(const vector<P>& l, P p, bool strict = true) {
d44  int a = 1, b = sz(l) - 1, r = !strict;
5cc  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
6bc  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
456  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<=
    -r)
d1f    return false;
48a  while (abs(a - b) > 1) {
4f7    int c = (a + b) / 2;
ac8    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
b26  }
06f  return sgn(l[a].cross(l[b], p)) < r;
c74 }
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1, -1)$ if no collision, • $(i, -1)$ if touching the corner i, • (i, i) if along side $(i, i+1)$, • (i, j) if crossing sides $(i, i+1)$ and $(j, j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i, i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.

Time: $\mathcal{O}(\log n)$

"Point.h" 7cf45b, 40 lines

```
530 #define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)
    %n]))
f84 #define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) <
    0
e7e template <class P> int extrVertex(vector<P>& poly, P dir)
    {
747  int n = sz(poly), lo = 0, hi = n;
fdf  if (extr(0)) return 0;
3d1  while (lo + 1 < hi) {
591    int m = (lo + hi) / 2;
855    if (extr(m)) return m;
c0c    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
f48    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) =
    m;
68a  }
253  return lo;
7f0 }

8e0 #define cmpL(i) sgn(a.cross(poly[i], b))
7dc template <class P>
ec4 array<int, 2> lineHull(P a, P b, vector<P>& poly) {
409  int endA = extrVertex(poly, (a - b).perp());
761  int endB = extrVertex(poly, (b - a).perp());
1a8  if (cmpL(endA) < 0 || cmpL(endB) > 0)
423    return {-1, -1};
649  array<int, 2> res;
f4b  rep(i,0,2) {
234    int lo = endB, hi = endA, n = sz(poly);
c2d    while ((lo + 1) % n != hi) {
57e      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
7f6      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
525    }
7dd    res[i] = (lo + !cmpL(hi)) % n;
356    swap(endA, endB);
c05  }
e00  if (res[0] == res[1]) return {res[0], -1};
3d1  if (!cmpL(res[0]) && !cmpL(res[1]))
959    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
3f3      case 0: return {res[0], res[0]};
223      case 2: return {res[1], res[1]};
8fa    }
b50  return res;
36f }
```

Minkowski.h

Description: Computes the Minkowski sum of two convex polygons. Polygons must be convex and given in CCW order. Returns the vertices of the Minkowski sum polygon in CCW order.

Time: $\mathcal{O}(n + m)$

"Point.h" 664d67, 24 lines

```
780 using P = Point<ll>;

89f vector<P> minkowski(vector<P> p, vector<P> q) {
a8e   auto fix = [](vector<P>& A) {
bec     int pos = 0;
2bb     for (int i = 1; i < sz(A); i++) {
609       if(A[i].y < A[pos].y || (A[i].y == A[pos].y && A[i].
    x < A[pos].x))
e4c         pos = i;
f76     }
703     rotate(A.begin(), A.begin() + pos, A.end());
9e5     A.push_back(A[0]), A.push_back(A[1]);
236   };
889   fix(p), fix(q);
db6   vector<P> result;
692   int i = 0, j = 0;
98a   while (i < sz(p) - 2 || j < sz(q) - 2) {
942     result.push_back(p[i] + q[j]);
3bd     auto cross = (p[i + 1] - p[i]).cross(q[j + 1] - q[j]);
c3c     if (cross >= 0 && i < sz(p) - 2) i++;
f33     if (cross <= 0 && j < sz(q) - 2) j++;
801   }
dc8   return result;
2f9 }
```

Extreme.h

Description: Finds an extreme vertex of a convex polygon according to a unimodal comparator. The comparator defines a total order along the polygon (given in CCW order).

Time: $\mathcal{O}(\log n)$

"Point.h" 70b181, 26 lines

```
780 using P = Point<ll>;
c88 int extreme(vector<P> &pol, const function<bool(P, P)>&
    cmp) {
b1c   int n = pol.size();
4a2   auto extr = [&](int i, bool& cur_dir) {
22a     cur_dir = cmp(pol[(i+1)%n], pol[i]);
61a     return !cur_dir and !cmp(pol[(i+n-1)%n], pol[i]);
364   };
63d   bool last_dir, cur_dir;
a0d   if (extr(0, last_dir)) return 0;
993   int l = 0, r = n;
ead   while (l+1 < r) {
ee4     int m = (l+r)/2;
f29     if (extr(m, cur_dir)) return m;
44a     bool rel_dir = cmp(pol[m], pol[l]);
b18     if ((!last_dir and cur_dir) or
261         (last_dir == cur_dir and rel_dir == cur_dir)) {
8a6       l = m;
1f1       last_dir = cur_dir;
94a     } else r = m;
```

```
606     }
792     return l;
985   }
cad   int max_dot(vector<P> &pol, P v) {
a98     return extreme([&](P p, P q) { return p.dot(v) > q.dot(v
      ); });
27e   }
```

Tangents.h

Description: Finds the left and right tangent points from an external point p to a convex polygon given in CCW order. A tangent point is a vertex where the segment p->v touches the polygon without intersecting its interior, defining the limits of visibility from p. Returns the indices of the left and right tangent vertices.
Time: $\mathcal{O}(\log n)$

"Point.h", "Extreme.h" dcf85f, 11 lines

```
780   using P = Point<ll>;

08d   bool ccw(P p, P q, P r) {
274     return (q-p).cross(r-q) > 0;
0f3   }
826   pair<int, int> tangents(vector<P> &pol, P p) {
ae2       auto L = [&](P q, P r) { return ccw(p, r, q); };
98c       auto R = [&](P q, P r) { return ccw(p, q, r); };
861       return {extreme(pol, L), extreme(pol, R)};
3dc   }
```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" ac41a6, 18 lines

```
2c0   typedef Point<ll> P;
24b   pair<P, P> closest(vector<P> v) {
7f9    assert(sz(v) > 1);
7f7    set<P> S;
879    sort(all(v), [](P a, P b) { return a.y < b.y; });
571    pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
eec    int j = 0;
813    for (P p : v) {
3fb      P d{1 + (ll)sqrt(ret.first), 0};
8be      while (v[j].y <= p.y - d.x) S.erase(v[j++]);
a5a      auto lo = S.lower_bound(p - d), hi = S.upper_bound(p +
      d);
c77      for (; lo != hi; ++lo)
113        ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
8aa      S.insert(p);
5b0    }
70d    return ret.second;
bf2   }
```

ManhattanMST.h

Description: Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights w(p, q) = —p.x - q.x— + —p.y - q.y—. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
Time: $\mathcal{O}(N \log N)$

"Point.h" df6f59, 24 lines

```
bbe   typedef Point<int> P;
ea9   vector<array<int, 3>> manhattanMST(vector<P> ps) {
850    vi id(sz(ps));
27c    iota(all(id), 0);
8c1    vector<array<int, 3>> edges;
8de    rep(k,0,4) {
1dd      sort(all(id), [&](int i, int j) {
02b           return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
702      map<int, int> sweep;
1e2      for (int i : id) {
84d        for (auto it = sweep.lower_bound(-ps[i].y);
904                  it != sweep.end(); sweep.erase(it++)) {
61d          int j = it->second;
6f3          P d = ps[i] - ps[j];
d18          if (d.y > d.x) break;
537          edges.push_back({d.y + d.x, i, j});
271        }
923        sweep[-ps[i].y] = i;
e69      }
4eb      for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p
      .y);
a11    }
da2    return edges;
a11   }
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" bac5b0, 64 lines

```
9a6   typedef long long T;
293   typedef Point<T> P;
305   const T INF = numeric_limits<T>::max();

173   bool on_x(const P& a, const P& b) { return a.x < b.x; }
0bd   bool on_y(const P& a, const P& b) { return a.y < b.y; }

bf2   struct Node {
975    P pt; // if this is a leaf, the single point in it
877    T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
a23    Node *first = 0, *second = 0;

86a    T distance(const P& p) { // min squared distance to a
      point
28b      T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
88e      T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
d98      return (P(x,y) - p).dist2();
ca4    }

d97    Node(vector<P>&& vp) : pt(vp[0]) {
741      for (P p : vp) {
ad3        x0 = min(x0, p.x); x1 = max(x1, p.x);
e5d        y0 = min(y0, p.y); y1 = max(y1, p.y);
310      }
994      if (vp.size() > 1) {
           // split on x if width >= height (not ideal...)
9b7        sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
           // divide by taking half the array for each child (
              not
           // best performance with many duplicates in the
              middle)
0f9        int half = sz(vp)/2;
48e        first = new Node({vp.begin(), vp.begin() + half});
902        second = new Node({vp.begin() + half, vp.end()});
66e      }
204    }
a77   };

dad   struct KDTree {
95f    Node* root;
c30    KDTree(const vector<P>& vp) : root(new Node({all(vp)}))
      {}

113    pair<T, P> search(Node *node, const P& p) {
ec4      if (!node->first) {
           // uncomment if we should not find the point itself:
           // if (p == node->pt) return {INF, P()};
47e        return make_pair((p - node->pt).dist2(), node->pt);
119      }
ea4      Node *f = node->first, *s = node->second;
d40      T bfirst = f->distance(p), bsec = s->distance(p);
a16      if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

         // search closest side first, other side if needed
86c      auto best = search(f, p);
314      if (bsec < best.first)
509        best = min(best, search(s, p));
f26      return best;
74c    }

       // find nearest point to a point, and its squared
          distance
       // (requires an arbitrary operator< for Point)
9b6    pair<T, P> nearest(const P& p) {
195      return search(root, p);
94c    }
6f5   };
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" eefdf5, 89 lines

```
2c0   typedef Point<ll> P;
806   typedef struct Quad* Q;
449   typedef __int128_t lll; // (can be ll if coords are < 2e4)
59b   P arb(LLONG_MAX,LLONG_MAX); // not equal to any other
      point

070   struct Quad {
461    Q rot, o; P p = arb; bool mark;
b38    P& F() { return r()->p; }
23a    Q& r() { return rot->rot; }
f4f    Q prev() { return rot->o->rot; }
57e    Q next() { return r()->prev(); }
180   } *H;

d15   bool circ(P p, P a, P b, P c) { // is p in the
      circumcircle?
4b4    lll p2 = p.dist2(), A = a.dist2()-p2,
ffa        B = b.dist2()-p2, C = c.dist2()-p2;
59a    return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B >
       0;
6af   }
00a   Q makeEdge(P orig, P dest) {
bdf    Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
516    H = r->o; r->r()->r() = r;
2c3    rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->
      r();
ed2    r->p = orig; r->F() = dest;
4c1    return r;
b3b   }
d8d   void splice(Q a, Q b) {
686    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
86c   }
e92   Q connect(Q a, Q b) {
fc2    Q q = makeEdge(a->F(), b->p);
6e6    splice(q, a->next());
642    splice(q->r(), b);
bef    return q;
4a4   }

196   pair<Q,Q> rec(const vector<P>& s) {
e63    if (sz(s) <= 3) {
```

```
4a0     Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
2ba     if (sz(s) == 2) return { a, a->r() };
19e     splice(a->r(), b);
5f8     auto side = s[0].cross(s[1], s[2]);
b9f     Q c = side ? connect(b, a) : 0;
3d8     return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
c9e   }

5ef   #define H(e) e->F(), e->p
c98   #define valid(e) (e->F().cross(H(base)) > 0)
a3e    Q A, B, ra, rb;
f5e    int half = sz(s) / 2;
391    tie(ra, A) = rec({all(s) - half});
d9b    tie(B, rb) = rec({sz(s) - half + all(s)});
f80    while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
b08           (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
76d    Q base = connect(B->r(), A);
87f    if (A->p == ra->p) ra = base->r();
b58    if (B->p == rb->p) rb = base;

3e6   #define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
f02     while (circ(e->dir->F(), H(base), e->F())) { \
936       Q t = e->dir; \
6d3       splice(e, e->prev()); \
16e       splice(e->r(), e->r()->prev()); \
d47       e->o = H; H = e; e = t; \
a2e     }
1de    for (;;) {
eaa     DEL(LC, base->r(), o);  DEL(RC, base, prev());
6fa     if (!valid(LC) && !valid(RC)) break;
e09     if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
b74       base = connect(RC, base->r());
295     else
271       base = connect(base->r(), LC->r());
fcf    }
345    return { ra, rb };
7cf   }

da1   vector<P> triangulate(vector<P> pts) {
af6    sort(all(pts));  assert(unique(all(pts)) == pts.end());
e00    if (sz(pts) < 2) return {};
235    Q e = rec(pts).first;
50c    vector<Q> q = {e};
6c1    int qi = 0;
7a5    while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
806   #define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
43e    q.push_back(c->r()); c = c->next(); } while (c != e); }
9d6    ADD; pts.clear();
b58    while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
a42    return pts;
a02   }
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 7 lines

```
f9c   template<class V, class L>
cb3   double signedPolyVolume(const V& p, const L& trilist) {
9e8    double v = 0;
b72    for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
fb8    return v / 6;
fca   }
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 33 lines

```
f10   template<class T> struct Point3D {
f07    typedef Point3D P;
d0e    typedef const P& R;
329    T x, y, z;
cf2    explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z){}
803    bool operator<(R p) const {
8ee      return tie(x, y, z) < tie(p.x, p.y, p.z); }
236    bool operator==(R p) const {
bd6      return tie(x, y, z) == tie(p.x, p.y, p.z); }
9ae    P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
54a    P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
743    P operator*(T d) const { return P(x*d, y*d, z*d); }
17b    P operator/(T d) const { return P(x/d, y/d, z/d); }
e49    T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
270    P cross(R p) const {
923      return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
a77    }
b70    T dist2() const { return x*x + y*y + z*z; }
18b    double dist() const { return sqrt((double)dist2()); }
       //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
3d6    double phi() const { return atan2(y, x); }
       //Zenith angle (latitude) to the z-axis in interval [0, pi]
0fa    double theta() const { return atan2(sqrt(x*x+y*y),z); }
55e    P unit() const { return *this/(T)dist(); } //makes dist()=1
       //returns unit vector normal to *this and p
685    P normal(P p) const { return cross(p).unit(); }
       //returns point rotated 'angle' radians ccw around axis
c67    P rotate(double angle, P axis) const {
7cd      double s = sin(angle), c = cos(angle); P u = axis.unit();
6b7      return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
73a    }
805   };
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.

Time: $\mathcal{O}(n^2)$

"Point3D.h" 5b45fc, 50 lines

```
b8e   typedef Point3D<double> P3;

9ce   struct PR {
1fc    void ins(int x) { (a == -1 ? a : b) = x; }
82f    void rem(int x) { (a == x ? a : b) = -1; }
2ad    int cnt() { return (a != -1) + (b != -1); }
ba2    int a, b;
cf7   };

5e4   struct F { P3 q; int a, b, c; };

b6d   vector<F> hull3d(const vector<P3>& A) {
cd9    assert(sz(A) >= 4);
ec1    vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
394   #define E(x,y) E[f.x][f.y]
afe    vector<F> FS;
9e0    auto mf = [&](int i, int j, int k, int l) {
2ce      P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
fa1      if (q.dot(A[l]) > q.dot(A[i]))
eaa        q = q * -1;
f22      F f{q, i, j, k};
ee5      E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
471      FS.push_back(f);
d73    };
30c    rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
047      mf(i, j, k, 6 - i - j - k);

3ef    rep(i,4,sz(A)) {
3b5      rep(j,0,sz(FS)) {
068        F f = FS[j];
04f        if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
412          E(a,b).rem(f.c);
b61          E(a,c).rem(f.b);
e5c          E(b,c).rem(f.a);
8d5          swap(FS[j--], FS.back());
eef          FS.pop_back();
5cd        }
220      }
97f      int nw = sz(FS);
c63      rep(j,0,nw) {
068        F f = FS[j];
561   #define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
3da        C(a, b, c); C(a, c, b); C(b, c, a);
248      }
472    }
864    for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
770      A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
311    return FS;
be2   };
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis ($0 =$ north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f07, 9 lines

```
c5f   double sphericalDistance(double f1, double t1,
3e8       double f2, double t2, double radius) {
284    double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
277    double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
c7e    double dz = cos(t2) - cos(t1);
c09    double d = sqrt(dx*dx + dy*dy + dz*dz);
154    return radius*2*asin(d/2);
4fa   }
```

Strings (9)

AhoCorasick.h

95b3e7, 46 lines

```
c2e   int trie[ms][sigma], fail[ms], terminal[ms], superfail[ms];
1e1   bool present[ms];
965   int z = 1;

ca3   int val(char c) { return c - 'a'; }

f97   void add(string& p) {
b3d       int cur = 0;
b4b       for (int i = 0; i < (int)p.size(); i++) {
9e4           int& nxt = trie[cur][val(p[i])];
b6e           if (nxt == 0) nxt = z++;
1bc           cur = nxt;
a92       }
c0e       present[cur] = true;
```

```
b07         terminal[cur]++;
6aa     }

0a8     void build() {
26a         queue<int> q;
f47         for (q.push(0); !q.empty(); q.pop()) {
fb5             int on = q.front();
0b2             for (int i = 0; i < sigma; i++) {
df1                 int& to = trie[on][i];
279                 int f = (on == 0 ? 0 : trie[fail[on]][i]);
de7                 int sf = (present[f] ? f : superfail[f]);
24d                 if (!to) {
c4e                     to = f;
6fd                 }
4e6                 else {
3ef                     fail[to] = f;
b86                     superfail[to] = sf;
                        // merge infos (ex: terminal[to] +=
                            terminal[f])
91b                     q.push(to);
692                 }
bff             }
e61         }
b89     }

54e     void search(string& s) {
b3d         int cur = 0;
b4f         for (char c : s) {
3ba             cur = trie[cur][val(c)];
                // process infos on current node (ex: ocurrences
                    += terminal[cur])
5ac         }
d1b     }
```

Hash.h

Description: C can also be random, operator is $[l,r]$

79e7f5, 28 lines

```
541 using ull = uint64_t;
54d struct H {
858     ull x; H(ull x = 0) : x(x) {}
c9b     H operator+(H o) { return x + o.x + (x + o.x < x); }
5cd     H operator-(H o) { return *this + ~o.x; }
167     H operator*(H o) {
2f3         auto m = (__uint128_t)x * o.x;
540         return H((ull)m) + (ull)(m >> 64);
681     }
bf2     ull get() const { return x + !~x; }
03c     bool operator==(H o) const{ return get() == o.get();}
0ab     bool operator<(H o) const{ return get() < o.get();}
bf6 };
862 static const H C = (ll)1e11 + 3;
61c struct Hash {
2f2     vector<H> h, pw;
1df     Hash(string& str) : h(str.size()), pw(str.size()) {
9bc         pw[0] = 1, h[0] = str[0];
1c5         for (int i = 1; i < str.size(); i++) {
90a             h[i] = h[i - 1] * C + str[i];
b3c             pw[i] = pw[i - 1] * C;
57e         }
f1b     }
75e     H operator()(int l, int r) {
91f         return h[r] - (l ? h[l - 1] * pw[r - l + 1] : 0);
9cf     }
c36 };
```

KMP.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123).

c7cf15, 10 lines

```
a56 vector<int> pi(const string& s) {
627     vector<int> p(sz(s));
edb     for(int i = 1; i < sz(s); i++) {
052         int g = p[i-1];
6c0         while (g && s[i] != s[g]) g = p[g-1];
7cf         p[i] = g + (s[i] == s[g]);
a2e     }
74e     return p;
c7c }
```

KmpAutomaton.h

Description: $go[i][j]$ = length of the longest prefix of s that is a suffix of $s[0..i]$ followed by the letter j (i.e., the next matched prefix length if, at state i, we read letter j).

8833cb, 17 lines

```
ab6 int go[ms][sigma];
ca3 int val(char c) { return c - 'a'; }
8cf void automaton(string& s) {
3cc     for (int i = 0; i < sigma; i++)
48d         go[0][i] = (i == val(s[0]));

8cc     for (int i = 1, bdr = 0; i <= (int)s.size(); i++) {
782         for (int j = 0; j < sigma; j++) {
6ef             go[i][j] = go[bdr][j];
87c         }
f8d         if (i < (int)s.size()) {
02f             go[i][val(s[i])] = i + 1;
364             bdr = go[bdr][val(s[i])];
63b         }
d7e     }
0c5 }
```

Manacher.h

Description: $p[0][i+1]$ is the length of matches of even length palindrome, starting from $[i,i+1]$.
$p[1][i]$ is the length of matches of odd length palindrome, starting from $[i,i]$.
(abaxx -> p[0] = 00001)
(abaxx -> p[1] = 01000)

7dfe41, 17 lines

```
aa9 array<vector<int>, 2> manacher(const string& s) {
f89     int n = sz(s);
ca1     array<vector<int>,2> p={vector<int>(n+1),vector<int>(n
    )};
6b7     for (int z = 0; z < 2; z++) {
22c         for (int i = 0, l = 0, r = 0; i < n; i++) {
24e             int t = r - i + !z;
e70             if (i < r) p[z][i] = min(t, p[z][l + t]);
fff             int L = i - p[z][i], R = i + p[z][i] - !z;
40c             while(L >= 1 && R+1 < n && s[L-1] == s[R+1]){
895                 p[z][i]++, L--, R++;
48e             }
f28             if (R > r) l = L, r = R;
e05         }
7a3     }
74e     return p;
7df }
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(s.begin(), s.begin()+minRotation(s), s.end());`
Time: $\mathcal{O}(N)$

19c4ce, 14 lines

```
5fa int minRotation(string s) {
a3e     int a = 0, N = s.size(); s += s;
239     for (int b = 0; b < N; b++) {
e0d         for (int k = 0; k < N; k++) {
32f             if (a+k == b || s[a+k] < s[b+k]) {
313                 b += max(0, k-1);
c2b                 break;
873             }
068             if (s[a+k] > s[b+k]) { a = b; break; }
9b5         }
193     }
3f5     return a;
19c }
```

SuffixArray.h

Description: $lcp[i]$ is the length of the longest common prefix between the suffixes $s[sa[i]..n-1]$ and $s[sa[i-1]..n-1]$.
If we concatenate multiple strings using separator characters, the separator that appears furthest to the right must be the smallest character in the alphabet.

048424, 31 lines

```
3f4 struct SuffixArray {
716  vector<int> sa, lcp;
d91  SuffixArray(string s, int lim=256) {
59b    s.push_back('$');
323    int n = sz(s), k = 0, a, b;
9f1    vector<int> x(all(s)), y(n), ws(max(n, lim));
af4    sa = lcp = y, iota(all(sa), 0);
25d    for(int j = 0, p = 0; p < n; j= max(1, j*2), lim = p) {
3cd      p = j, iota(all(y), n - j);
603      for(int i=0; i<n; i++){
071        if (sa[i] >= j) y[p++] = sa[i] - j;
cb4      }
911      fill(all(ws), 0);
483      for(int i=0; i<n; i++) ws[x[i]]++;
5d9      for(int i=1; i<lim; i++) ws[i] += ws[i - 1];
a9e      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
c7d      swap(x, y), p = 1, x[sa[0]] = 0;
6f5      for(int i=1; i<n; i++){
93f        a = sa[i - 1], b = sa[i];
ddb        x[b] = p-1;
a32        if(y[a] != y[b] || y[a+j] != y[b+j]) x[b] = p++;
1ba      }
c36    }
65b    for (int i = 0, j; i < n - 1; lcp[x[i++]] = k)
904      for (k && k--, j = sa[x[i] - 1];
262          s[i + k] == s[j + k]; k++);
68a    sa = vector<int>(sa.begin() + 1, sa.end());
5d4    lcp = vector<int>(lcp.begin() + 1, lcp.end());
4db  }
048 };
```

Zfunc.h

Description: z[i] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)

495392, 13 lines

```
572 vector<int> ZFunc(const string& s) {
d6b     int n = sz(s), a = 0, b = 0;
2b1     vector<int> z(n, 0);
29a     if (!z.empty()) z[0] = 0;
6f5     for (int i = 1; i < n; i++) {
fe0         int end = i;
98f         if (i < b) end = min(i + z[i - a], b);
65f         while (end < n && s[end] == s[end - i]) ++end;
816         z[i] = end - i; if (end > b) a = i, b = end;
253     }
070     return z;
495 }
```

Various (10)

10.1 Misc. algorithms

Dates.h

Description: dateToInt converts Gregorian date to integer (Julian day number). intToDate converts integer (Julian day number) to Gregorian date: month/day/year. intToDay converts Julian day number to day of the week

688e56, 23 lines

```
37c string day[] = { "Mon", "Tue", "Wed", "Thu", "Fri", "Sat",
     "Sun" };
fb9 int dateToInt(int m, int d, int y) {
e70     return
773         1461 * (y + 4800 + (m - 14) / 12) / 4 +
649         367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
fa0         3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
3aa         d - 32075;
a73 }
3fe void intToDate(int jd, int& m, int& d, int& y) {
ee1     int x, n, i, j;
33a     x = jd + 68569;
403     n = 4 * x / 146097;
33e     x -= (146097 * n + 3) / 4;
6fc     i = (4000 * (x + 1)) / 1461001;
b1d     x -= 1461 * i / 4 - 31;
fc9     j = 80 * x / 2447;
c8d     d = x - 2447 * j / 80;
179     x = j / 11;
335     m = j + 2 - 12 * x;
23d     y = 100 * (n - 49) + i + x;
cbb }
04e string intToDay(int jd) { return day[jd % 7]; }
```

MultisetHash.h

5648da, 8 lines

```
cdc ull hashify(ull sum) {
7b8     sum += FIXED_RANDOM;
6ec     sum += 0x9e3779b97f4a7c15;
dc6     sum = (sum ^ (sum >> 30)) * 0xbf58476d1ce4e5b9;
005     sum = (sum ^ (sum >> 27)) * 0x94d049bb133111eb;
358     return sum ^ (sum >> 31);
564 }
```

Rand.h

2de3f8, 8 lines

```
c8a mt19937 rng(chrono::steady_clock::now().time_since_epoch()
    .count());
    // _64

463 int uniform(int l, int r) { // [l, r]
a7f     uniform_int_distribution<int> uid(l, r);
f54     return uid(rng);
d9e }
```

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).

Time: $\mathcal{O}(\log N)$

edce47, 24 lines

```
d91 set<pii>::iterator addInterval(set<pii>& is, int L, int R)
     {
bb3  if (L == R) return is.end();
d4c  auto it = is.lower_bound({L, R}), before = it;
dc6  while (it != is.end() && it->first <= R) {
164    R = max(R, it->second);
1a5    before = it = is.erase(it);
fe9  }
1af  if (it != is.begin() && (--it)->second >= L) {
3ca    L = min(L, it->first);
164    R = max(R, it->second);
861    is.erase(it);
0de  }
aa0  return is.insert(before, {L,R});
d57 }

675 void removeInterval(set<pii>& is, int L, int R) {
17b  if (L == R) return;
bef  auto it = addInterval(is, L, R);
e14  auto r2 = it->second;
655  if (it->first == L) is.erase(it);
016  else (int&)it->second = L;
ee9  if (R != r2) is.emplace(R, r2);
059 }
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).

Time: $\mathcal{O}(N \log N)$

9e9d8d, 20 lines

```
4fc template<class T>
dbe vi cover(pair<T, T> G, vector<pair<T, T>> I) {
3d5  vi S(sz(I)), R;
d00  iota(all(S), 0);
591  sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
d10  T cur = G.first;
05e  int at = 0;
336  while (cur < G.second) { // (A)
438    pair<T, int> mx = make_pair(cur, -1);
f07    while (at < sz(I) && I[S[at]].first <= cur) {
032      mx = max(mx, make_pair(I[S[at]].second, S[at]));
69a      at++;
c42    }
c54    if (mx.second == -1) return {};
953    cur = mx.first;
fbf    R.push_back(mx.second);
dd1  }
b1a  return R;
b8d }
```

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \dots < f(i) \geq \dots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the < marked with (A) to <=, and reverse the loop at (B). To minimize f, change it to >, also at (B).

Usage: `int ind = ternSearch(0,n-1);`

Time: $\mathcal{O}(\log(b-a))$

a995fb, 11 lines

```
53a int ternSearch(int a, int b) {
25b  assert(a <= b);
329  while (b - a >= 5) {
924    int mid = (a + b) / 2;
c9e    if (f(mid) < f(mid+1)) a = mid; // (A)
ceb    else b = mid+1;
ce7  }
95e  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
3f5  return a;
a99 }
```

10.2 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: $dp[i][j] = \min_{i<k<j}(dp[i][k] + dp[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \leq f(a,d)$ and $f(a,c) + f(b,d) \leq f(a,d) + f(b,c)$ for all $a \leq b \leq c \leq d$. Another sufficient criteria is: $opt[i][j-1] \leq opt[i][j] \leq opt[i+1][j]$

Time: $\mathcal{O}(N^2)$

fea016, 22 lines

```
7cc ll knuth(){
6a7     memset(opt, -1, sizeof opt);
45b     for(int i=n-1; i>=0; i--){
8e7         dp[i][i] = 0; // base case
b28         opt[i][i] = i;
94f         for(int j=i+1; j<n; j++){
2e2             int optL = (!j ? 0 : opt[i][j-1]);
dc4             int optR = (~opt[i+1][j] ? opt[i+1][j] : n-1);
554             ll cst = cost(i, j);
f12             dp[i][j] = INF;
3bb             optL = max(i, optL), optR = min(j-1, optR);
349             for(int k=optL; k<=optR; k++){
f8b                 ll now = dp[i][k] + dp[k+1][j] + cst;
e83                 if(now <= dp[i][j]){
960                     dp[i][j] = now;
14d                     opt[i][j] = k;
5fc                 }
114             }
4ce         }
96c     }
fea }
```

DivideAndConquerDP.h

Description: Divide and Conquer DP maintaining cost, can be used when $opt[i][j] \leq opt[i][j+1]$. In this code everything is 1-based. Memory can be optmized by keeping only the last row

Time: $\mathcal{O}(MN \log N)$

c7cb38, 42 lines

```
129 void add(int idx) {}
404 void rem(int idx) {}

749 void deC(int i, int l, int r, int optL, int optR) {
de6     if (l > r) return;
995     int j = (l + r) / 2;
d9a     for (int k = r; k > j; k--) rem(k);
c45     int opt = optL;
364     for (int k = optL; k <= min(optR, j); k++) {
            // cost = cost[k, j]
597         int val = dp[i - 1][k - 1] + cost;
532         if (val < dp[i][j]) {
482             dp[i][j] = val;
613             opt = k;
178         }
183         rem(k);
93f     }
5d9     for (int k = min(optR, j); k >= optL; k--) add(k);
446     rem(j);
ace     deC(i, l, j - 1, optL, opt);

ebd     for (int k = j; k <= r; k++) add(k);
648     for (int k = optL; k < opt; k++) rem(k);
0b6     deC(i, j + 1, r, opt, optR);

9bb     for (int k = optL; k < opt; k++) add(k);
460 }

d57 int solve(int N, int M) { // 1-based
d9f     for (int i = 0; i <= M; i++) {
138         for (int j = 0; j <= N; j++){
3db             dp[i][j] = inf; // base case
a26         }
e0f     }
c21     cost = 0; // neutral value
c62     for (int i = 1; i <= N; i++) add(i);
143     for (int i = 1; i <= M; i++) {
156         deC(i, 1, N, 1, N);
c97     }
01a     return dp[M][N];
```

```
3ab }
```