Universidade Federal de Pernambuco

las4s e pelados

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Contest (1)

template.cpp

9 lines

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
using ll = long long;
using pii = pair<int,int>;
using vi = vector<int>;
```

.bashrc

2 lines

```
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \
  -fsanitize=undefined,address'
```

hash.sh

2 lines

```
# bash hash.sh file.cpp l1 l2
sed -n $2','$3' p' $1 | sed '/^#w/d' | cpp -dD -P -
    fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

stressTest.sh

20 lines

```
P=code  #mude pro filename do codigo
Q=brute #mude pro filename do brute [correto]
g++ ${P}.cpp -o sol -O2 || exit 1
g++ ${Q}.cpp -o brt -O2 || exit 1
g++ gen.cpp  -o gen -O2 || exit 1
for ((i = 1; ; i++)) do
  echo $i
  ./gen $i > in
  ./sol < in > out
  ./brt < in > out2
  if (! cmp -s out out2) then
    echo "--> entrada:"
    cat in
    echo "--> saida code:"
    cat out
    echo "--> saida brute:"
    cat out2
    break;
  fi
done
```

paperStress.py

26 lines

```
927 import random
a1a import subprocess
5c9 MAX_N = 100
b5d def gen_case() -> str:
c7e     return f"1\n"

94a random.seed((1 << 9) | 31)

a22 for i in range(100):
d19     print(), print()
a3f     case = gen_case()
266     print(f"Test #{i+1}: ")
ce5     print(case)
d41     # test bruteforce
f60     bf = subprocess.run(['out/b'], input=case, encoding='
    ascii', capture_output=True)
d41     # test solution
37c     sol = subprocess.run(['out/m'], input=case, encoding='
    ascii', capture_output=True)
d55     bf_res = bf.stdout
af9     sol_res = sol.stdout
6b6     print(f"bruteforce {bf_res}, solution {sol_res}")
508     if bf_res == sol_res:
dd4         print("accepted")
f68     else:
ef2         print("WA")
1cb         break
```

troubleshoot.txt

52 lines

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
```

Theoretical (2)

2.1 Mathematics

2.1.1 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \cdots - c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.1.2 Trigonometry

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

2.1.3 Geometry

Triangles

Side lengths: a, b, c

Semiperimeter: $p = \frac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \frac{abc}{4A}$

Inradius: $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1 - \left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = \frac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\frac{a+b}{a-b} = \frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

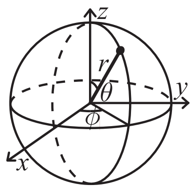
Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

Spherical coordinates



$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \text{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z &= r\cos\theta & \phi &= \text{atan2}(y, x) \end{aligned}$$

Pick's Theorem

The area of a simple polygon whose vertices have integer coordinates is:

$$A = I + \frac{B}{2} - 1$$

where I is the number of interior integer points, and B is the number of integer points in the border of the polygon.

Two Ears Theorem

Every simple polygon with more than 3 vertices has at least two non-overlapping ears (a ear is a vertex whose diagonal induced by its neighbors which lies strictly inside the polygon). Equivalently, every simple polygon can be triangulated.

2.1.4 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\text{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.1.5 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, \ c \neq 1$$

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$\sum_{i=0}^{n} ic^i = \frac{nc^{n+2} - (n+1)c^{n+1} + c}{(c-1)^2}, \ c \neq 1$$

$$g_k(n) = \sum_{i=1}^{n} i^k = \frac{1}{k+1}\left(n^{k+1} + \sum_{j=1}^{k}\binom{k+1}{j+1}(-1)^{j+1}g_{k-j}(n)\right)$$

2.1.6 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, \ (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, \ (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, \ (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, \ (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, \ (-\infty < x < \infty)$$

$$\sum_{i=0}^{\infty} ic^i = \frac{c}{(1-c)^2}, \quad |c| < 1$$

$$(1+x)^n = \sum_{i=0}^{n}\binom{n}{i}x^i$$

$$\frac{1}{1-x} = \sum_{i=0}^{\infty} x^i, \ (-1 < x < 1)$$

$$\frac{1}{(1-x)^n} = \sum_{i=0}^{\infty}\binom{n+i-1}{n-1}x^i, \ (-1 < x < 1)$$

2.1.7 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\text{Bin}(n, p)$, $n = 1, 2, \ldots, \ 0 \leq p \leq 1$.

$$p(k) = \binom{n}{k}p^k(1-p)^{n-k}$$

$$\mu = np, \ \sigma^2 = np(1-p)$$

$\text{Bin}(n, p)$ is approximately $\text{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\text{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1}, \ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p}, \ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \dots$$

$$\mu = \lambda, \sigma^2 = \lambda$$

2.2 Combinatorial

2.2.1 Binomial Identities

$$\binom{n-1}{k} - \binom{n-1}{k-1} = \frac{n-2k}{k}\binom{n}{k} \qquad \binom{n}{h}\binom{n-h}{k} = \binom{n}{k}\binom{n-k}{h}$$

$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1} \qquad \sum_{k=0}^{n} k^2\binom{n}{k} = (n+n^2)2^{n-2}$$

$$\sum_{j=0}^{k} \binom{m}{j}\binom{n-m}{k-j} = \binom{n}{k} \qquad \sum_{j=0}^{m} \binom{m}{j}^2 = \binom{2m}{m}$$

$$\sum_{m=0}^{n} \binom{m}{j}\binom{n-m}{k-j} = \binom{n+1}{k+1} \qquad \sum_{m=0}^{n} \binom{m}{k} = \binom{n+1}{k+1}$$

$$\sum_{r=0}^{m} \binom{n+r}{r} = \binom{n+m+1}{m} \qquad \sum_{k=0}^{n} \binom{n-k}{k} = \mathrm{Fib}(n+1)$$

$$\sum_{k=0}^{n} \binom{r}{k}\binom{s}{n-k} = \binom{r+s}{n}$$

2.2.2 Permutations

Factorial

n	1 2 3	4	5	6	7	8	9	10
$n!$	1 2 6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17	
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14	
n	20	25	30	40	50	100	150	171
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX

Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

Burnside's lemma

Counts the number of distinct colorings of an object under symmetry.

$$\frac{1}{|G|}\sum_{g\in G} k^{\mathrm{cyc}(g)},$$

where G is the symmetry group, k the number of colors, and $\mathrm{cyc}(g)$ the number of cycles induced by g.

Example: number of ways to color a necklace with n beads using k colors (rotations only):

$$g(n) = \frac{1}{n}\sum_{i=0}^{n-1} k^{(\gcd(n,i))}$$

where rotation i shifts the necklace by i positions.

2.2.3 Partitions and subsets

Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k}\binom{n_i}{m_i} \pmod p$.

2.2.4 Sum of Binomials (FFT)

Goal: Given freq. array C, compute $Ans[k] = \sum_i C[i]\binom{i}{k}$ for all k. Rewrite: $Ans[k] = \frac{1}{k!}\sum_i (C[i]\cdot i!)\frac{1}{(i-k)!}$.

- Construct P where $P[i] = C[i]\cdot i!$
- Construct Q where $Q[i] = (i!)^{-1}$
- Reverse Q (to handle the $i-k$ subtraction).
- Multiply $R = NTT(P, Q)$.
- Result: $Ans[k] = R[k + |Q| - 1]\cdot\frac{1}{k!}$.

2.2.5 General purpose numbers

Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able). $B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k\cdot(n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\dots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$

Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

Labeled unrooted trees

- on n vertices: n^{n-2}
- on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
- with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

2.3 Number Theory

2.3.1 Bézout's identity

For $a \neq$, $b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

2.3.2 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^{\times}$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

2.3.3 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 6700 for $n < 1e12$, 200 000 for $n < 1e19$.

2.3.4 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \le m \le n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \le m \le n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

2.3.5 Theorems

Goldbach's conjecture: Every even integer $n > 2$ can be written as $n = a + b$ with a, b prime.

Legendre's conjecture: There is always at least one prime between n^2 and $(n+1)^2$.

Lagrange's four-square theorem: Every positive integer can be written as

$$n = a^2 + b^2 + c^2 + d^2.$$

Zeckendorf's theorem: Every integer $n \geq 1$ has a unique representation as a sum of non-consecutive Fibonacci numbers:

$$n = F_{i_1} + F_{i_2} + \cdots + F_{i_k}, \quad i_j - i_{j+1} \geq 2.$$

Euclid's formula (primitive Pythagorean triples): The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2),\ \ b = k \cdot (2mn),\ \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

Wilson's theorem: n is prime iff

$$(n-1)! \equiv -1 \pmod{n}.$$

Chicken McNugget theorem: For coprime n, m, the largest integer not representable as $an + bm$ (with $a, b \geq 0$) is

$$nm - n - m.$$

There are $\frac{(n-1)(m-1)}{2}$ non-representable integers, and for each pair $(k,\ nm - n - m - k)$ exactly one is representable.

2.4 Graphs

2.4.1 Flows and Matching

Hall's Theorem

In bipartite graphs, there exists a perfect matching covering the entire side X if and only if for every subset $Y \subseteq X$,

$$|Y| \leq |N(Y)|,$$

where $N(Y)$ denotes the set of neighbors of Y.

Kőnig's Theorem

In a bipartite graph, the size of a Minimum Vertex Cover is equal to the size of a Maximum Matching. A Minimum Vertex Cover is a minimum set of vertices such that every edge of the graph has at least one endpoint in the set.

As a consequence,

$$n - \text{Maximum Matching} = \text{Maximum Independent Set},$$

where a Maximum Independent Set is the largest set of vertices with no edges between them.

Recovering the Minimum Vertex Cover Given a maximum matching in a bipartite graph (X, Y):

- Construct the residual graph by orienting:
  - non-matching edges from X to Y;
  - matching edges from Y to X.
- Perform a BFS or DFS starting from all free (unmatched) vertices in X.
- Let Z_X be the set of reachable vertices in X, and Z_Y the set of reachable vertices in Y.

The Minimum Vertex Cover is given by:

$$(X \setminus Z_X) \cup Z_Y.$$

Node-Disjoint Path Cover

A node-disjoint path cover is a set of paths such that each vertex belongs to exactly one path.

In a directed acyclic graph (DAG),

$$\text{Minimum Node-Disjoint Path Cover} = n - \text{Maximum Matching}.$$

The construction is as follows: for each vertex u, create a copy u'. Add an edge $u \to v'$ if there exists an edge $u \to v$ in the original graph.

Recovering the Paths

- Vertices that do not appear as destinations in the matching are starting points of paths.
- Each matching edge $u \to v'$ corresponds to an edge $u \to v$ in the original DAG.
- Following these edges reconstructs all paths of the path cover.

General Path Cover

A general path cover is a path cover where a vertex may belong to more than one path.

In a DAG, the construction is similar to the node-disjoint case, but an edge $u \to v'$ exists if there is a path from u to v in the original graph.

Recovering the Cover The vertices can be grouped according to the edges used in the matching to form the path cover.

Dilworth's Theorem

An antichain is a set of vertices such that there is no path between any pair of vertices in the set.

In a directed acyclic graph,

$$\text{Minimum General Path Cover} = \text{Maximum Antichain}.$$

Recovering a Maximum Antichain Given a minimum general path cover, selecting one vertex from each path produces a maximum antichain.

2.4.2 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--`, `mat[b][b]++` (and `mat[b][a]--`, `mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

2.4.3 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

2.4.4 Planar Graphs

If G has k connected components, then $n - m + f = k + 1$.

2.5 Optimization tricks

2.5.1 Bit hacks

- `for (int x = m; x; x = (x - 1)&m ) { ... }` loops over all subset masks of `m` (except 0).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
  `if (i & 1 << b) D[i] += D[i^(1 << b)];`
  computes all sums of subsets.

2.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC target("bmi,bmi2,popcnt,lzcnt")` improve bit operations.
- `#pragma GCC optimize("unroll-loops")` self explanatory.

2.6 Various

2.6.1 Master Theorem (Simple)

$T(n) = aT(n/b) + O(n^d)$. Compare a vs b^d:

- $a > b^d \implies O(n^{\log_b a})$ (Work at leaves dominates)
- $a = b^d \implies O(n^d \log n)$ (Work is uniform)
- $a < b^d \implies O(n^d)$ (Work at root dominates)

Data structures (3)

Bit.h

Description: *lower_bound* works the same as on vectors
Time: $\mathcal{O}(\log N)$

5891da, 23 lines

```
8eb struct Bit {
406   vector<ll> bit;
1dd   Bit(int n) : bit(n + 1) {}
265   void update(int i, ll v) {
c38     for (i++; i < sz(bit); i += i & -i) bit[i] += v;
f21   }
74a   ll query(int i) {
b73     ll ret = 0;
71c     for (i++; i > 0; i -= i & -i) ret += bit[i];
edf     return ret;
e40   }
dc8   int lower_bound(ll v){ // min pos st sum[0, pos] >= v
bec     int pos = 0;
a40     for(int j=(1 << 23); j >= 1; j/=2){
3b1       if(pos+j < sz(bit) && bit[pos + j] < v){
b4e         pos += j;
18d         v -= bit[pos];
f6c       }
156     }
d75     return pos;
37b   }
589 };
```

Bit2d.h

Description: Points called on the update function NEED to be on the *pts* vector parameter on build.
Time: $\mathcal{O}\left((\log N)^2\right)$

"Bit.h" 5a98ac, 37 lines

```
9c0 struct Bit2d {
a37   vector<vector<int>> ys;
fe8   vector<Bit> bit;
543   vector<int> cmp_x;
425   Bit2d(){}
521   void put(int x, int y) {
005     for (x++; x < sz(ys); x += x & -x) ys[x].push_back(y);
f3c   }
ce0   int id(const vector<int> &v, int y) {
1e9     return (upper_bound(all(v), y) - v.begin()) - 1;
19a   }
7ff   void build(vector<pii> pts) {
3cb     sort(all(pts));
f99     for(auto p : pts) cmp_x.push_back(p.first);
9a7     cmp_x.erase(unique(all(cmp_x)), cmp_x.end());
f82     ys.resize(cmp_x.size() + 1);
94d     for(auto p : pts) put(id(cmp_x, p.first), p.second);
310     for(auto &v:ys)sort(all(v)), bit.emplace_back(sz(v));
a01   }
767   void update(int x, int y, int val){
f3f     x = id(cmp_x, x);
681     for(x++; x < sz(ys); x+= x&-x)
507       bit[x].update(id(ys[x], y), val);
c88   }
d95   int query(int x, int y){
f3f     x = id(cmp_x, x);
7c9     int ret = 0;
f32     for(x++; x > 0; x-= x&-x)
ea8       ret += bit[x].query(id(ys[x], y));
edf     return ret;
8f7   }
251   int query(int x1, int y1, int x2, int y2){
e4d     int a = query(x2, y2)-query(x2, y1-1);
7d1     return a-query(x1-1, y2)+query(x1-1, y1-1);
c33   }
5a9 };
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$

8ec1c7, 32 lines

```
72c struct Line {
3e2  mutable ll k, m, p;
ca5  bool operator<(const Line& o) const { return k < o.k; }
abf  bool operator<(ll x) const { return p < x; }
7e3 };

781 struct LineContainer : multiset<Line, less<>> {
     // (for doubles, use inf = 1/.0, div(a,b) = a/b)
fd2  static const ll inf = LLONG_MAX;
33a  ll div(ll a, ll b) { // floored division
10f    return a / b - ((a ^ b) < 0 && a % b); }
a1c  bool isect(iterator x, iterator y) {
a95    if (y == end()) return x->p = inf, 0;
9cb    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
591    else x->p = div(y->m - x->m, x->k - y->k);
870    return x->p >= y->p;
2fa  }
a0c  void add(ll k, ll m) {
116    auto z = insert({k, m, 0}), y = z++, x = y;
7b1    while (isect(y, z)) z = erase(z);
d94    if (x != begin() && isect(--x, y))
c07      isect(x, y = erase(y));
57d    while ((y = x) != begin() && (--x)->p >= y->p)
774      isect(x, erase(y));
086  }
4ad  ll query(ll x) {
229    assert(!empty());
7d1    auto l = *lower_bound(x);
96a    return l.k * x + l.m;
d21  }
577 };
```

Mo.h

Description: For subtree queries, perform an Euler tour and map each node u to the interval $[tin[u], tin[u] + subtree\_size[u] - 1]$. A subtree query becomes a range query over this interval.
For path queries between nodes U and V, Let U be the closest to the root. If V lies in U's subtree, the path corresponds to the interval $[tin[U], tin[V]]$. Otherwise, the path corresponds to the interval $[min(tout[U], tout[V]), max(tin[U], tin[V])]$.
In both cases, nodes on the U–V path appear exactly once in the interval, while all other nodes appear either 0 or 2 times.
Usage: `queries.push(Query(l, r, index of query))`, intervals are `[l, r]`
Time: $\mathcal{O}\left(N\sqrt(Q)\right)$ fb7161, 44 lines

```
626 inline int64_t hilOrd(int x, int y, int pow, int rot) {
51a   if (pow == 0) return 0;
a6e   int hpow = 1 << (pow - 1);
01f   int seg = (x < hpow) ? ((y < hpow) ? 0 : 3) : ((y < hpow
      ) ? 1 : 2);
e08   seg = (seg + rot) & 3;
669   const int rotDelta[4] = { 3, 0, 0, 1 };
d0b   int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
115   int nrot = (rot + rotDelta[seg]) & 3;
fba   int64_t sub = int64_t(1) << (2 * pow - 2);
65b   int64_t ans = seg * sub;
1ae   int64_t add = hilOrd(nx, ny, pow - 1, nrot);
ff7   ans += (seg == 1 || seg == 2) ? add : (sub - add - 1);
ba7   return ans;
ec4 }

670 struct Query {
738   int l, r, idx;
ce8   int64_t ord;
36f   Query(int l, int r, int idx) : l(l), r(r), idx(idx) {
6c4     ord = hilOrd(l, r, 21, 0);
926   }
847   bool operator < (const Query& other) const {
328     return ord < other.ord;
e05   }
315 };

240 vector<Query> queries;
4d5 int ans[ms];
566 void put(int x) {} // F
c29 void remove(int x) {} // F
64b int getAns() {}

1c1 void Mo() {
3d9   int l = 0, r = -1;
bfa   sort(queries.begin(), queries.end());
275   for (Query q : queries) {
482     while (l > q.l) put(--l);
fec     while (r < q.r) put(++r);
5b8     while (l < q.l) remove(l++);
9b5     while (r > q.r) remove(r--);
745     ans[q.idx] = getAns();
5a4   }
2a4 }
```

MoUpdate.h

Description: Block size should be around $(2 * N * N)^{\frac{1}{3}}$
Usage: intervals are `[l, r]`, `addQuery(l, r, number of updates happened before this query, index of query)`, `addUpdate(index of updated position, value before update, value after update)`
Time: $\mathcal{O}\left(Q * (2 * N * N)^{\frac{1}{3}} * F\right)$ f8eda8, 55 lines

```
496 const int B = 2700;
247 struct MoUpdate {
670   struct Query {
fd6     int l, r, t, idx;
fc8     Query(int l, int r, int t, int idx)
8bf       : l(l), r(r), t(t), idx(idx) {}
f51     bool operator < (const Query& p) const {
f06       if (l / B != p.l / B) return l < p.l;
e80       if (r / B != p.r / B) return r < p.r;
d0c       return t < p.t;
673     }
bc2   };
f2f   struct Upd {
f25     int i, old, now;
f23     Upd(int i, int old, int now): i(i),old(old),now(now){}
c12   };

240   vector<Query> queries;
e2b   vector<Upd> updates;

ac5   void addQuery(int l, int r, int t, int idx) {
fc9     queries.push_back(Query(l, r, t, idx)); }
968   void addUpdate(int i, int old, int now) {
936     updates.push_back(Upd(i, old, now)); }

1aa   void add(int x) {} // F
598   void rem(int x) {} // F
64b   int getAns() {}
0d2   void update(int novo, int idx, int l, int r) {
2b9     if (l <= idx && idx <= r) rem(idx);
4ce     arr[idx] = novo;
ec1     if (l <= idx && idx <= r) add(idx);
100   }

63d   void solve() {
cb1     int l = 0, r = -1, t = 0;
bfa     sort(queries.begin(), queries.end());
275     for (Query q : queries) {
a95       while (l > q.l) add(--l);
875       while (r < q.r) add(++r);
8f6       while (l < q.l) rem(l++);
a38       while (r > q.r) rem(r--);
fda       while (t < q.t) {
df3         auto u = updates[t++];
285         update(u.now, u.i, l, r);
8a4       }
32a       while (t > q.t) {
d69         auto u = updates[--t];
ce2         update(u.old, u.i, l, r);
3bf       }
745       ans[q.idx] = getAns();
f06     }
b09   }
d3e };
```

MinQueue.h

40df8d, 19 lines

```
925 struct MQueue {
fdd     int tin, tout;
375     deque<pair<int, int>> dq;
1ce     MQueue() : tin(0), tout(0) {}
619     void push(int val) {
f0d         while (!dq.empty() && min(dq.back().first, val) ==
        val) dq.pop_back();
9c6         dq.push_back(pair(val, tin++));
769     }
42d     void pop() {
            // assert(!dq.empty());
48c         if (dq.front().second == tout) dq.pop_front();
470         tout++;
b0e     }
f46     int front() {
            // assert(!dq.empty());
651         return dq.front().first;
fa2     }
40d };
```

SegmentTree.h

Description: Zero-indexed max-tree. Bounds are inclusive to the left and inclusive to the right. Can be changed by modifying T, f and unit.
Time: $\mathcal{O}(\log N)$ f609d9, 21 lines

```
5ae struct Tree {
ef4   typedef int T;
cbe   static constexpr T unit = INT_MIN;
e54   T f(T a, T b) { return max(a, b); } // (any associative
    fn)
6cd   vector<T> s; int n;
3d2   Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {}
6a3   void update(int pos, T val) {
56a     for (s[pos += n] = val; pos /= 2;)
326       s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
0e9   }
b4c   T query(int b, int e) { // query [b, e]
1a3     e++;
0f9     T ra = unit, rb = unit;
fbb     for (b += n, e += n; b < e; b /= 2, e /= 2) {
e83       if (b % 2) ra = f(ra, s[b++]);
064       if (e % 2) rb = f(s[--e], rb);
561     }
cb2     return f(ra, rb);
707   }
f60 };
```

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
Time: $\mathcal{O}(\log N)$ 782797, 17 lines

```
c4d #include <bits/extc++.h>
0d7 using namespace __gnu_pbds;

4fc template<class T>
c20 using Tree = tree<T, null_type, less<T>, rb_tree_tag,
3a1     tree_order_statistics_node_update>;

ad0 void example() {
c6f   Tree<int> t, t2; t.insert(8);
559   auto it = t.insert(10).first;
d28   assert(it == t.lower_bound(9));
969   assert(t.order_of_key(10) == 1);
d39   assert(t.order_of_key(11) == 2);
1b7   assert(*t.find_by_order(0) == 8);
a60   t.join(t2); // merge t2 into t
9ad }
```

PersistentSegTree.h

Usage: `SegP(size of the segtree, number of updates)`

`roots = {0}, newRoot = update(roots.back(),...),`
`roots.push(newRoot)` 58842f, 42 lines

```
b17 struct SegP {
709   static constexpr ll neut = 0;
bf2   struct Node {
aa3     ll v; // start with neutral value
74f     int l, r;
9ef     Node(ll v=neut, int l=0, int r=0) : v(v), l(l), r(r){}
945   };
```

```
  vector<Node> seg;
  int n, CNT;
  SegP(int _n, int upd): seg(20*(upd+_n)), n(_n), CNT(1){}
  ll merge(ll a, ll b) { return a + b; }
  int update(int root, int pos, int val, int l, int r) {
    int p = CNT++;
    seg[p] = seg[root];
    if (l == r) {
      seg[p].v += val;
      return p;
    }
    int mid = (l + r) / 2;
    if (pos <= mid){
      seg[p].l = update(seg[p].l, pos, val, l, mid);
    }else seg[p].r = update(seg[p].r,pos,val,mid+1,r);

    seg[p].v=merge(seg[seg[p].l].v, seg[seg[p].r].v);
    return p;
  }
  int query(int p, int L, int R, int l, int r) {
    if (l > R || r < L) return neut;
    if (L <= l && r <= R) return seg[p].v;
    int mid = (l + r) / 2;
    int left = query(seg[p].l, L, R, l, mid);
    int right = query(seg[p].r, L, R, mid + 1, r);
    return merge(left, right);
  }
  int update(int root, int pos, int val) {
    return update(root, pos, val, 0, n - 1);
  }
  int query(int root, int L, int R) {
    return query(root, L, R, 0, n - 1);
  }
};
```

SegBeats.h

Description: In Segment Tree Beats, 'lazy' does NOT mean "updates still missing here". The node already reflects all previous updates. Instead, 'lazy' stores what must be propagated to the children before recursing. Always call 'apply(l,r,p)' before descending. This node layout supports range add, range chmin and range chmax operations. Beats conditions:
break: MIN x: mx1 <= x ; MAX x: mi1 >= x
tag: MIN x: x > mx2 ; MAX x: x < mi2
Time: amortized $\mathcal{O}\left(\log^2 N\right)$, without range add $\mathcal{O}\left(\log N\right)$

fa8527, 47 lines

```
struct node{
  ll mx1, mx2, sum, lazy;
  ll mi1, mi2;
  int cMax, cMin, tam;
  node(int x=0) : mx1(x),mx2(-inf),mi1(x),mi2(inf),
             cMax(1),cMin(1),tam(1),sum(x),lazy(0){}
  node(node a, node b){
    sum = a.sum+b.sum, tam = a.tam+b.tam;
    lazy = 0;
    mx1 = max(a.mx1, b.mx1);
    mx2 = max(a.mx2, b.mx2);
    if(a.mx1 != b.mx1) mx2 = max(mx2, min(a.mx1, b.mx1));
    cMax=(a.mx1==mx1 ? a.cMax:0)+(b.mx1==mx1 ? b.cMax:0);

    mi1 = min(a.mi1, b.mi1);
    mi2 = min(a.mi2, b.mi2);
    if(a.mi1 != b.mi1) mi2=min(mi2, max(a.mi1, b.mi1));
    cMin=(a.mi1==mi1 ? a.cMin:0)+(b.mi1==mi1 ? b.cMin:0);
  }
  void apply_sum(ll x){
    mx1 += x, mx2 += x,mi1 += x, mi2 += x;
    sum += tam*x, lazy += x;
  }
  void apply_min(ll x){
    if(x >= mx1) return;
    sum -= (mx1 - x)*cMax;
    if(mi1 == mx1) mi1 = x;
    if(mi2 == mx1) mi2 = x;
    mx1 = x;
  }
  void apply_max(ll x){
    if(x <= mi1) return;
    sum -= (mi1 - x)*cMin;
    if(mx1 == mi1) mx1 = x;
    if(mx2 == mi1) mx2 = x;
    mi1 = x;
  }
};
void apply(int l, int r, int p){
  for(int i=2*p+1; i<=2*p+2; i++){
    seg[i].apply_sum(st[p].lazy);
    seg[i].apply_min(st[p].mx1);
    seg[i].apply_max(st[p].mi1);
  }
  seg[p].lazy = 0;
}
```

RMQ.h

Usage: `RMQ rmq(values);`
`rmq.query(inclusive, inclusive);`
Time: $\mathcal{O}\left(|V|\log|V|+Q\right)$

bca062, 17 lines

```
struct RMQ {
  vector<vector<int>> dp;
  RMQ(const vector<int>& a) : dp(1, a) {
    for (int i = 1, pw = 1; pw*2 <= sz(a); pw*=2, i++) {
      dp.emplace_back(sz(a) - pw*2 + 1);
      for (int j = 0; j < sz(dp[i]); j++) {
        dp[i][j] = min(dp[i-1][j], dp[i-1][j+pw]);
      }
    }
  }
  int query(int l, int r) {
    assert(l <= r);
    int k = 31 - __builtin_clz(r - l + 1);
    return min(dp[k][l], dp[k][r - (1 << k) + 1]);
  }
};
```

UnionFind.h

Description: Disjoint-set data structure with bipartite check

6d2739, 22 lines

```
struct Uf{
    vector<int> tam, ds, bi, c;
    Uf(int n) : tam(n, 1), ds(n), bi(n, 1), c(n){
        iota(all(ds), 0);
    }
    int find(int i){ return (i==ds[i] ? i : find(ds[i]));}
    int color(int i){
        return (i==ds[i] ? 0 : (c[i]^color(ds[i])));}
    void merge(int a, int b){
        int ca = color(a), cb = color(b);
        a = find(a), b = find(b);
        if(a == b){
            if(ca == cb) bi[a] = false;
            return;
        }
        if(tam[a] < tam[b]) swap(a, b);
        ds[b] = a, tam[a] += tam[b];
        bi[a] = (bi[a] && bi[b]);
        c[b] = (ca ^ cb ^ 1);
    }
};
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().
Usage: `int t = uf.time(); ...; uf.rollback(t);`
Time: $\mathcal{O}\left(\log(N)\right)$

d4405e, 23 lines

```
struct RollbackUF {
  vector<int> e;
    vector<pii> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return sz(st); }
  void rollback(int t) {
    for (int i = time(); i --> t;)
      e[st[i].first] = st[i].second;
    st.resize(t);
  }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.push_back({a, e[a]});
    st.push_back({b, e[b]});
    e[a] += e[b]; e[b] = a;
    return true;
  }
};
```

Numerical (4)

4.1 Polynomials and recurrences

Polynomial.h

c9b7b0, 19 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;)
        c = a[i], a[i] = a[i+1]*x0+b, b=c;
    a.pop_back();
  }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2\log(1/\epsilon)\right)$

"Polynomial.h" b00bfe, 24 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax)
  {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
```

```
423    sort(all(dr));
b98    rep(i,0,sz(dr)-1) {
d85      double l = dr[i], h = dr[i+1];
ad1      bool sign = p(l) > 0;
b41      if (sign ^ (p(h) > 0)) {
03d        rep(it,0,60) { // while (h - l > 1e-8)
761          double m = (l + h) / 2, f = p(m);
0ac          if ((f <= 0) ^ sign) l = m;
193          else h = m;
b69        }
ff5        ret.push_back((l + h) / 2);
fc2      }
d15    }
edf    return ret;
b00  }
```

PolyInverse.h

2745a7, 18 lines

```
747  vector<ll> get_inverse(vector<ll> a) {
e4d    if (a.empty()) return {};
044    int Y = sz(a) - 1, n = 32 - __builtin_clz(Y);
ba5    n = (1 << n);
711    a.resize(n);
e3e    vector<ll> inv = { modpow(a[0], mod - 2) }, f, c;
a2b    inv.reserve(n);
599    for (int tam = 2; tam <= n; tam *= 2) {
d29      while (sz(f) < tam) f.push_back(a[sz(f)]);
fec      c = conv(f, inv);
757      rep(i, 0, tam) c[i] = (c[i] == 0 ? 0 : mod - c[i]);
df6      c[0] += (c[0] + 2 >= mod ? 2 - mod : 2);
f8b      inv = conv(inv, c);
118      inv.resize(tam);
9f4    }
531    return inv;
274  }
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
Time: $\mathcal{O}\left(N^2\right)$

96548b, 21 lines

```
c10  vector<ll> berlekampMassey(vector<ll> s) {
ea1    int n = sz(s), L = 0, m = 0;
2a2    vector<ll> C(n), B(n), T;
2b3    C[0] = B[0] = 1;

d6f    ll b = 1;
36d    rep(i,0,n) { ++m;
b7f      ll d = s[i] % mod;
45a      rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
53a      if (!d) continue;
169      T = C; ll coef = d * modpow(b, mod-2) % mod;
2d1      rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
b6c      if (2 * L > i) continue;
dc3      L = i + 1 - L; B = T; b = d; m = 0;
8c2    }

51b    C.resize(L + 1); C.erase(C.begin());
e98    for (ll& x : C) x = (mod - x) % mod;
a91    return C;
965  }
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \ldots \geq n-1]$ and $tr[0 \ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}\left(n^2 \log k\right)$

547b93, 27 lines

```
437  using Poly = vector<ll>;
2ef  ll linearRec(Poly S, Poly tr, ll k) {
327    int n = sz(tr);

0e9    auto combine = [&](Poly a, Poly b) {
b1c      Poly res(n * 2 + 1);
5f7      rep(i,0,n+1) rep(j,0,n+1)
389        res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
bdc      for (int i = 2 * n; i > n; --i) rep(j,0,n)
fc3        res[i-1-j] = (res[i-1-j] + res[i] * tr[j]) % mod;
b76      res.resize(n + 1);
b50      return res;
55c    };

bf8    Poly pol(n + 1), e(pol);
997    pol[0] = e[1] = 1;

e96    for (++k; k; k /= 2) {
491      if (k % 2) pol = combine(pol, e);
0d9      e = combine(e, e);
813    }

cd2    ll res = 0;
e8d    rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
b50    return res;
594  }
```

4.2 Optimization

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a, b]$ assuming f is unimodal on the interval, i.e. has only one local minimum and no local maximum. The maximum error in the result is eps. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
Usage: `double func(double x) { return 4+x+.3*x*x; }`
`double xmin = gss(-1000,1000,func);`
Time: $\mathcal{O}\left(\log((b-a)/\epsilon)\right)$

31d45b, 15 lines

```
eb1  double gss(double a, double b, double (*f)(double)) {
97e    double r = (sqrt(5)-1)/2, eps = 1e-7;
b87    double x1 = b - r*(b-a), x2 = a + r*(b-a);
47d    double f1 = f(x1), f2 = f(x2);
708    while (b-a > eps)
f4d      if (f1 < f2) { //change to > to find maximum
da5        b = x2; x2 = x1; f2 = f1;
dfb        x1 = b - r*(b-a); f1 = f(x1);
451      } else {
d6e        a = x1; x1 = x2; f1 = f2;
815        x2 = a + r*(b-a); f2 = f(x2);
2fe      }
3f5    return a;
31d  }
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b$, $x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
Usage: `vvd A = {{1,-1}, {-1,1}, {-1,-2}};`
`vd b = {1,1,-4}, c = {-1,-1}, x;`
`T val = LPSolver(A, b, c).solve(x);`
Time: $\mathcal{O}\left(NM * \#pivots\right)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}\left(2^n\right)$ in the general case.

aa8530, 69 lines

```
943  typedef double T; // long double, Rational, double + mod<P
     >...
487  typedef vector<T> vd;
840  typedef vector<vd> vvd;

8cb  const T eps = 1e-8, inf = 1/.0;
85f  #define MP make_pair
90c  #define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s]))
     ) s=j

34b  struct LPSolver {
b5c    int m, n;
14e    vi N, B;
d5f    vvd D;

9b8    LPSolver(const vvd& A, const vd& b, const vd& c) :
f40      m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
27d        rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
f03        rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[
     i];}
59d        rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
24a        N[n] = -1; D[m+1][n] = 1;
6ff      }

333    void pivot(int r, int s) {
3cd      T *a = D[r].data(), inv = 1 / a[s];
12b      rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
449        T *b = D[i].data(), inv2 = b[s] * inv;
e07        rep(j,0,n+2) b[j] -= a[j] * inv2;
e78        b[s] = a[s] * inv2;
ca4      }
485      rep(j,0,n+2) if (j != s) D[r][j] *= inv;
3b7      rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
dbd      D[r][s] = inv;
c97      swap(B[r], N[s]);
9cd    }

24e    bool simplex(int phase) {
8b8      int x = m + phase - 1;
1de      for (;;) {
7a6        int s = -1;
aea        rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
4dc        if (D[x][s] >= -eps) return true;
56d        int r = -1;
670        rep(i,0,m) {
776          if (D[i][s] <= eps) continue;
c95          if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
133                       < MP(D[r][n+1] / D[r][s], B[r])) r = i
     ;
468        }
fbf        if (r == -1) return false;
dba        pivot(r, s);
7d8      }
f15    }

859    T solve(vd &x) {
898      int r = 0;
435      rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
32d      if (D[r][n+1] < -eps) {
f65        pivot(r, n);
fda        if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
939        rep(i,0,m) if (B[i] == -1) {
37f          int s = 0;
c66          rep(j,1,n+1) ltj(D[i]);
b98          pivot(i, s);
683        }
b65      }
203      bool ok = simplex(1); x = vd(n);
972      rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
```

```
d3a     return ok ? D[m][n+1] : inf;
396   }
c57 };
```

4.3 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}(N^3)$

bd5cec, 16 lines

```
e36 double det(vector<vector<double>>& a) {
70e   int n = sz(a); double res = 1;
fea   rep(i,0,n) {
281     int b = i;
b0b     rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
311     if (i != b) swap(a[i], a[b]), res *= -1;
9b1     res *= a[i][i];
d5c     if (res == 0) return 0;
3e3     rep(j,i+1,n) {
f15       double v = a[j][i] / a[i][i];
353       if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
4ec     }
ee1   }
b50   return res;
bd5 }
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}(N^3)$

3313dc, 19 lines

```
031 const ll mod = 12345;
38e ll det(vector<vector<ll>>& a) {
da9   int n = sz(a); ll ans = 1;
fea   rep(i,0,n) {
3e3     rep(j,i+1,n) {
f36       while (a[j][i] != 0) { // gcd step
479         ll t = a[i][i] / a[j][i];
b87         if (t) rep(k,i,n)
e5b           a[i][k] = (a[i][k] - a[j][k] * t) % mod;
332         swap(a[i], a[j]);
17c         ans *= -1;
e81       }
30d     }
a97     ans = ans * a[i][i] % mod;
f4e     if (!ans) return 0;
f39   }
d38   return (ans + mod) % mod;
5e8 }
```

SolveLinear.h

Description: If inv = 1, finds the inverse of the matrix eq and returns it as a flat vector
Time: $\mathcal{O}(\min(n,m)\,nm)$

2c134e, 52 lines

```
320 struct Gauss {
d6d   const double eps = 1e-9;
93d   vector<vector<double>> eq;
754   void addEquation(const vector<double>& e) {
503     eq.push_back(e); }
04f   pair<int, vector<double>> solve(int inv=0) {
214     int n = sz(eq), m = sz(eq[0]) - 1 + inv;
f9c     if(inv){
d33       rep(i, 0, n) eq[i].resize(2*n), eq[i][n+i] = 1;
2e2     }
3cb     vector<int> where(m, -1);
a73     for (int col = 0, row = 0; col < m && row < n; col++)
    {
f05       int sel = row;
53c       rep(i, row, n) {
664         if (abs(eq[i][col]) > abs(eq[sel][col])) sel = i;
e04       }
68b       if (abs(eq[sel][col]) < eps) continue;
3ad       rep(i, col, sz(eq[0])) swap(eq[sel][i], eq[row][i]);
2c3       where[col] = row;
dff       rep(i, 0, n) if (i != row) {
184         double c = eq[i][col] / eq[row][col];
7f1         rep(j, col, sz(eq[0])) eq[i][j] -= eq[row][j] * c;
17d       }
4ef       ++row;
9b8     }
f9c     if(inv){
208       vector<double> res;
fea       rep(i, 0, n) {
420         if (where[i] == -1) return {0, {}}; // Singular
3af         rep(j, n, 2*n)
f89           res.push_back(eq[where[i]][j] / eq[where[i]][i])
    ;
d81       }
3b1       return {1, res};
700     }

233     vector<double> ans(m, 0);
670     rep(i, 0, m) {
c19       if (where[i] != -1)
02c         ans[i] = eq[where[i]][m] / eq[where[i]][i];
5bb     }
fea     rep(i, 0, n) {
68c       double sum = 0;
5f8       rep(j, 0, m) {
f48         sum = sum + ans[j] * eq[i][j];
fa6       }
3c8       if(abs(sum - eq[i][m]) > eps)return {0, {}};
bf2     }
260     rep(i, 0, m) if (where[i] == -1) return {2, ans};
d4a     return {1, ans};
a95   }
2c1 };
```

SolveLinearBinary.h

Time: $\mathcal{O}\left(\frac{\min(n,m)\,nm}{64}\right)$

28c946, 32 lines

```
e81 pair<int, bitset<M>> gauss(vector<bitset<M>> eq) {
579     int n = eq.size(), m = M - 1;
3cb     vector<int> where(m, -1);
a73     for(int col = 0, row = 0; col < m && row < n; col++){
dbb         rep(i,row,n)
926             if (eq[i][col]) {
c35                 swap(eq[i], eq[row]);
c2b                 break;
177             }
f4f         if (!eq[row][col]) continue;
2c3         where[col] = row;

fea         rep(i, 0, n) {
b60             if (i != row && eq[i][col]) eq[i] ^= eq[row];
981         }
4ef         ++row;
c74     }
7eb     bitset<M> ans;
670     rep(i,0,m) {
713         if (where[i] != -1) ans[i] = eq[where[i]][m];
691     }
fea     rep(i,0,n) {
e5c         int sum = (ans & eq[i]).count();
53f         sum %= 2;
36a         if (sum != eq[i][m]) return pair(0, bitset<M>());
29e     }
670     rep(i,0,m) {
be2         if (where[i] == -1) return pair(INF, ans);
958     }
280     return pair(1, ans);
28c }
```

XorGauss.h

5a1957, 30 lines

```
b94 struct XorGauss {
060     int N;
471     vector<ll> basis, who, mask;
47b     XorGauss(int N) : N(N), basis(N), who(N), mask(N) {}
        // if(ans & (1ll << j)) who[j] was used to form x
221     bool belong(ll x){
04b         ll ans = 0;
042         for(int i=N-1; i>=0; i--){
e13             if((x ^ basis[i]) < x){
4ec                 ans ^= mask[i];
6b0                 x ^= basis[i];
254             }
2ad         }
069         return (x == 0);
c26     }
397     void add(ll v, int idx) {
a4d         ll msk = 0;
042         for (int i = N - 1; i >= 0; i--) {
80f             if (!(v & (1ll << i))) continue;
bf3             if (basis[i] == 0) {
1c7                 basis[i] = v, who[i] = idx;
940                 mask[i] = (msk | (1ll << i));
505                 return;
bc8             }
00e             msk ^= mask[i];
647             v ^= basis[i];
25b         }
fcc     }
5a1 };
```

4.4 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x]\exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ (~1s for $N = 2^{22}$)

773fcd, 44 lines

```
bcc typedef complex<double> C;

7c0 void fft(vector<C>& a) {
a5b   int n = a.size(), L = 31 - __builtin_clz(n);
f82   static vector<complex<long double>> R(2, 1); // 10%
    faster if double
991   static vector<C> rt(2, 1);
ad8   for (static int k = 2; k < n; k *= 2) {
9d9     R.resize(n);
335     rt.resize(n);
411     auto x = polar(1.0L, acos(-1.0L) / k);
cdb     rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
a8a   }
e66   vector<ll> rev(n);
dcb   rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
47b   rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);

d3f   for (int k = 1; k < n; k *= 2) {
cda     for (int i = 0; i < n; i += 2 * k) {
0c2       for (int j = 0; j < k; j++) {
30c         auto x = (double*)&rt[j + k];
ebe         auto y = (double*)&a[i + j + k];
```

```
15c         C z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]);
20a         a[i + j + k] = a[i + j] - z;
1b0         a[i + j] += z;
b5b       }
1fe     }
fa0   }
b33 }

ccc vector<ll> conv(const vector<ll>& a, const vector<ll>& b){
f88   if (a.empty() || b.empty()) return {};
920   vector<ll> res(sz(a) + sz(b) - 1);
441   int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
060   vector<C> in(n), out(n);
b1a   copy(all(a), in.begin());
fef   rep(i,0,sz(b)) in[i].imag(b[i]);
21a   fft(in);
6fb   for (C& x : in) x *= x;
4d7   rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
3d7   fft(out);
aa3   rep(i,0,sz(res)) res[i]=round(imag(out[i]) / (4 * n));
b50   return res;
7f4 }
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.
Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h" b82773, 23 lines

```
192 typedef vector<ll> vl;
3fe template<int M> vl convMod(const vl &a, const vl &b) {
f88   if (a.empty() || b.empty()) return {};
19d   vl res(sz(a) + sz(b) - 1);
a6f   int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M));
3dd   vector<C> L(n), R(n), outs(n), outl(n);
a1d   rep(i,0,sz(a)) L[i] =C((int)a[i] / cut, (int)a[i] % cut);
97d   rep(i,0,sz(b)) R[i] =C((int)b[i] / cut, (int)b[i] % cut);
5d5   fft(L), fft(R);
fea   rep(i,0,n) {
39d     int j = -i & (n - 1);
65e     outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
91a     outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
cb3   }
d08   fft(outl), fft(outs);
35e   rep(i,0,sz(res)) {
351     ll av = ll(real(outl[i])+.5), cv =ll(imag(outs[i])+.5);
988     ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
6a3     res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
58f   }
b50   return res;
c1f }
```

NumberTheoreticTransform.h

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x]g^{xk}$ for all k, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).
Time: $\mathcal{O}(N \log N)$

84c11e, 34 lines

```
376 const int mod = 998244353, root = 62;
192 typedef vector<ll> vl;
8ec void ntt(vl &a) {
6ae   int n = sz(a), L = 31 - __builtin_clz(n);
7c9   static vl rt(2, 1);
8ee   for (static int k = 2, s = 2; k < n; k *= 2, s++) {
335     rt.resize(n);
d43     ll z[] = {1, modpow(root, mod >> s)};
8e7     rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
f39   }
808   vector<int> rev(n);
dcb   rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
47b   rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
657   for (int k = 1; k < n; k *= 2)
2cb     for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
86e       ll z = rt[j+k] * a[i+j+k] % mod, &ai = a[i+j];
598       a[i + j + k] = ai - z + (z > ai ? mod : 0);
589       ai += (ai + z >= mod ? z - mod : z);
9a8     }
de9 }
08f vl conv(const vl &a, const vl &b) {
f88   if (a.empty() || b.empty()) return {};
f51   int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s),
570       n = 1 << B;
9ef   int inv = modpow(n, mod - 2);
e4c   vl L(a), R(b), out(n);
6b4   L.resize(n), R.resize(n);
d9e   ntt(L), ntt(R);
dfc   rep(i,0,n)
0db     out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
ec9   ntt(out);
c20   return {out.begin(), out.begin() + s};
387 }
```

FWHT.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

124c14, 20 lines

```
5ad void FST(vector<ll>& a, bool inv) {
a9d   for (int n = sz(a), step = 1; step < n; step *= 2) {
5bd     for (int i = 0; i < n; i += 2 * step) {
4ee       for (int j = i; j < i + step; j++) {
2fe         ll& u = a[j], &v = a[j + step];
c6f         tie(u, v) =
2d3         inv ? pair(v - u, u) : pair(v, u + v); // AND
aba         inv ? pair(v, u - v) : pair(u + v, u); // OR
a5a         pair(u + v, u - v);   // XOR
0b4       }
fb4     }
cd3   }
c9b   if(inv) for(ll& x : a) x /= sz(a); // XOR only
075 }
eb2 vector<ll> conv(vector<ll> a, vector<ll> b) {
595   FST(a, 0); FST(b, 0);
2dd   for (int i = 0; i < sz(a); i++) a[i]*=b[i];
062   FST(a, 1); return a;
7bf }
```

Number theory (5)

5.1 Modular arithmetic

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

c375f5, 5 lines

```
88a const ll mod = 1000000007, LIM = 200000;
0f2 inv[1] = 1;
379 for(int i=2; i<LIM; i++)
86c     inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.
Time: $\mathcal{O}(1)$ for `modmul`, $\mathcal{O}(\log b)$ for `modpow`

bbbd8f, 12 lines

```
f4c typedef unsigned long long ull;
f85 ull modmul(ull a, ull b, ull M) {
2dd   ll ret = a * b - M * ull(1.L / M * a * b);
964   return ret + M * (ret < 0) - M * (ret >= (ll)M);
e93 }
4f6 ull modpow(ull b, ull e, ull mod) {
c1a   ull ans = 1;
a18   for (; e; b = modmul(b, b, mod), e /= 2)
9e8     if (e & 1) ans = modmul(ans, b, mod);
ba7   return ans;
100 }
```

ModPow.h

b83e45, 9 lines

```
e2e const ll mod = 1000000007; // faster if const

9d8 ll modpow(ll b, ll e) {
d54   ll ans = 1;
36e   for (; e; b = b * b % mod, e /= 2)
b46     if (e & 1) ans = ans * b % mod;
ba7   return ans;
d1e }
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).
Time: $\mathcal{O}(\log^2 p)$ worst case, $\mathcal{O}(\log p)$ for most p

"ModPow.h" 19a793, 25 lines

```
a77 ll sqrt(ll a, ll p) {
5de   a %= p; if (a < 0) a += p;
b47   if (a == 0) return 0;
5c6   assert(modpow(a, (p-1)/2, p) == 1); // else no solution
a75   if (p % 4 == 3) return modpow(a, (p+1)/4, p);
      // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
b94   ll s = p - 1, n = 2;
ee5   int r = 0, m;
084   while (s % 2 == 0)
082     ++r, s /= 2;
eaa   while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
0c3   ll x = modpow(a, (s + 1) / 2, p);
b74   ll b = modpow(a, s, p), g = modpow(n, s, p);
1af   for (;; r = m) {
4fd     ll t = b;
713     for (m = 0; m < r && t != 1; ++m)
c58       t = t * t % p;
ae0     if (m == 0) return x;
20e     ll gs = modpow(g, 1LL << (r - m - 1), p);
fba     g = gs * gs % p;
4fb     x = x * gs % p;
c5c     b = b * g % p;
e3a   }
19a }
```

DiscreteLog.h

Description: Returns the smallest x such that $a^x \bmod m = b \bmod m$. If no such x exists, returns -1.
Time: O(sqrt(m) * log(sqrt(m))

2f126b, 32 lines

```
758 int solve(int a, int b, int m) {
a6e     a %= m, b %= m;
ec4     if (a == 0) return (b ? -1 : 1);
        // caso gcd(a, m) > 1
6af     int k = 1, add = 0, g;
553     while ((g = gcd(a, m)) > 1) {
d90         if (b == k) return add;
```

```
642          if (b % g) return -1;
92a          b /= g, m /= g, ++add;
803          k = (k * 1ll * a / g) % m;
8a0      }

16c      int sq = sqrt(m) + 1;
b51      int big = 1;
4e1      for (int i = 0; i < sq; i++) big = (1ll * big * a) % m;

053      vector<pii> vals;
3c2      for (int q = 0, cur = b; q <= sq; q++) {
b53          vals.push_back({ cur, q });
b50          cur = (1ll * cur * a) % m;
837      }
62b      sort(all(vals));
90c      for (int p = 1, cur = k; p <= sq; p++) {
5d3          cur = (1ll * cur * big) % m;
958          auto it = lower_bound(all(vals), pair(cur, INF));
721          if (it != vals.begin() && (--it)->first == cur) {
a30              return sq * p - it->second + add;
6fe          }
f22      }
daa      return -1;
2f1  }
```

DiscreteRoot.h

Description: Returns x such that $x^k \bmod m = a \bmod m$. If no such x exists, returns -1.
Time: O(sqrt(m) * log(sqrt(m))

"PrimitiveRoot.h", "DiscreteLog.h" — 1d582e, 11 lines

```
    // Discrete Root

27c  ll discreteRoot(ll k, ll a, ll m) {
738      ll g = primitiveRoot(m);
58b      ll y = discreteLog(fexp(g, k, m), a, m);
f31      if (y == -1) return y;
a58      return fexp(g, y, m);
1d5  }
```

5.2 Primality

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" — 66fe73, 13 lines

```
da4  bool isPrime(ull n) {
c16   if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
062   ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
ae0   ull s = __builtin_ctzll(n-1), d = n >> s;
e80   for (ull a : A) {   // ^ count trailing zeroes
6b4     ull p = modpow(a%n, d, n), i = s;
274     while (p != 1 && p != n - 1 && a % n && i--)
c77       p = modmul(p, p, n);
e28     if (p != n-1 && i != s) return 0;
edf   }
6a5   return 1;
66f  }
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" — da0c7c, 19 lines

```
7eb  ull pollard(ull n) {
222   ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
5f5   auto f = [&](ull x) { return modmul(x, x, n) + i; };
f51   while (t++ % 40 || gcd(prd, n) == 1) {
be9     if (x == y) x = ++i, y = f(x);
70f     if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
b78     x = f(x), y = f(f(y));
bf8   }
002   return gcd(prd, n);
d1b  }
591  vector<ull> factor(ull n) {
1b9   if (n == 1) return {};
6b5   if (isPrime(n)) return {n};
bc6   ull x = pollard(n);
52a   auto l = factor(x), r = factor(n / x);
7af   l.insert(l.end(), all(r));
792   return l;
d54  }
```

PrimitiveRoot.h

18a01e, 15 lines

```
     //is n primitive root of p ?
ad0  bool test(ll x, ll p) {
a56      ll m = p - 1;
845      for (ll i = 2; i * i <= m; ++i) if (!(m % i)) {
e64          if (modpow(x, i, p) == 1) return false;
599          if (modpow(x, m / i, p) == 1) return false;
53a      }
8a6      return true;
c4e  }
     //find the smallest primitive root for p
220  ll search(ll p) {
1bf      for (ll i = 2; i < p; i++) if (test(i, p)) return i;
daa      return -1;
a3c  }
```

5.3 Divisibility

Euclid.h

Description: Find x, y such that $Ax + By = \gcd(A, B)$. If $\gcd(A, B) = 1$, then $x = A^{-1} (\bmod B)$ and $y = B^{-1} (\bmod A)$.
Time: $\mathcal{O}(\log)$

33ba8f, 6 lines

```
c22  ll euclid(ll a, ll b, ll &x, ll &y) {
1ee   if (!b) return x = 1, y = 0, a;
e3d   ll d = euclid(b, a % b, y, x);
0a4   return y -= a/b * x, d;
33b  }
```

CRT.h

ba1a4a, 25 lines

```
bc9  ll modinverse(ll a, ll b, ll s0 = 1, ll s1 = 0) {
a76    return !b ? s0 : modinverse(b, a % b, s1, s0 - s1 * (a / b)); }

d8b  ll mul(ll a, ll b, ll m) {
a6f    return (((__int128_t)a*b)%m + m)%m;
0bc  }

28d  struct Equation {
4c5    ll mod, ans;
08f    bool valid;
145    Equation(ll a, ll m) { mod = m, ans = a, valid = true; }
0fc    Equation() { valid = false; }
4d3    Equation(Equation a, Equation b) {
515      valid = false;
1a0      if (!a.valid || !b.valid) return;
85c      ll g = gcd(a.mod, b.mod);
44d      if ((a.ans - b.ans) % g != 0) return;
af0      valid = true;
b98      mod = a.mod * (b.mod / g);
81a      ll x = mul(a.mod, modinverse(a.mod, b.mod), mod);
38a      ans = a.ans + mul(x, (b.ans - a.ans) / g, mod);
c4c      ans = (ans % mod + mod) % mod;
6f5    }
f48  };
```

DivisionTrick.h

02aebb, 15 lines

```
7f1  void floor_ranges(int n) {
79c      for (int l = 1, r; l <= n; l = r + 1) {
746          r = n / (n / l);
             // floor(n/y) has the same value for y in [l..r]
5bf      }
eee  }
678  void ceil_ranges(int n) {
79c      for (int l = 1, r; l <= n; l = r + 1) {
d47          int x = (n + l - 1) / l;
374          if (x == 1) r = n;
21b          else r = (n - 1) / (x - 1);
             // ceil(n/y) has the same value for y in [l..r]
06c      }
57c  }
```

Phi.h

Description: *Euler's* ϕ function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} ... (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$.
Euler's thm (generalized): a, m arbitrary, $n \geq \log_2 m \Rightarrow a^n \equiv a^{\phi(m)+(n \bmod \phi(m))} \pmod{m}$.

e58bf0, 6 lines

```
d08  void calculatePhi() {
265   for(int i=0; i<LIM; i++) phi[i] = i&1 ? i : i/2;
c83   for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
dc2    for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
e58  }
```

Combinatorial (6)

PartitionSolver.h

e50fb7, 61 lines

```
d38  template<const int N>
182  struct PartitionSolver {
4ce    vector<vector<int>> part, to, from;
621    PartitionSolver() {
a9d      vector<int> a;
1ed      part.push_back(a);
77f      gen(1, N, a);
796      sort(all(part));
ed4      to.assign(sz(part), vector<int>(N + 1, -1));
9a5      from = to;
ddd      for (int i = 0; i < sz(part); i++) {
a93        int sum = 0;
87f        auto arr = part[i];
bca        for (auto x : arr) sum += x;
4fa        to[i][0] = i;
615        from[i][0] = i;
afc        for (int j = 1; j + sum <= N; j++) {
123          arr = part[i];
9d6          arr.push_back(j);
ceb          sort(all(arr));
d02          to[i][j] = getIndex(arr);
942          from[to[i][j]][j] = i;
20d        }
bef      }
```

```
283   }

810   int size() const { return sz(part); }
9ee   int getIndex(const vector<int>& arr) const {
168     return lower_bound(all(part), arr) - part.begin(); }
b49   int add(int id, int num) const { return to[id][num]; }
944   int rem(int id, int num) const { return from[id][num]; }
168   vector<int> getPartition(int id) const {
37b     return part[id]; }

1ba   void gen(int i, int sum, vector<int>& a) {
a05     if (i > sum) { return; }
726     a.push_back(i);
1ed     part.push_back(a);
278     gen(i, sum - i, a);
468     a.pop_back();
48f     gen(i + 1, sum, a);
537   }
f4f };

    // Number of partitions for all integers <= n
75c vector<ll> partitionNumber(int n) {
d9c   vector<ll> ans(n + 1, 0);
82f   ans[0] = 1;
78a   for (int i = 1; i <= n; i++) {
87f     for (int j = 1; j * (3 * j + 1) / 2 <= i; j++) {
b6b       ll here = ans[i - j * (3 * j + 1) / 2];
c91       ans[i] = (ans[i] + (j & 1 ? here : -here));
365     }
7c6     for (int j = 1; j * (3 * j - 1) / 2 <= i; j++) {
a1a       ll here = ans[i - j * (3 * j - 1) / 2];
c91       ans[i] = (ans[i] + (j & 1 ? here : -here));
162     }
4a3   }
ba7   return ans;
08b }
```

Graph (7)

7.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max|w_i| < \sim 2^{63}$.
Time: $\mathcal{O}(VE)$

529834, 24 lines

```
f5e const ll inf = LLONG_MAX;
83a struct Ed { int a, b, w, s() { return a < b ? a : -a; }};
9ac struct Node { ll dist = inf; int prev = -1; };

6fc void bell(vector<Node>& nodes, vector<Ed>& eds, int s) {
97b  nodes[s].dist = 0;
eb9  sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s(); });

74e  int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled
    vertices
c5a  rep(i,0,lim) for (Ed ed : eds) {
905    Node cur = nodes[ed.a], &dest = nodes[ed.b];
d7d    if (abs(cur.dist) == inf) continue;
6ab    ll d = cur.dist + ed.w;
6ec    if (d < dest.dist) {
956      dest.prev = ed.a;
4c2      dest.dist = (i < lim-1 ? d : -inf);
452    }
75a  }
ced  rep(i,0,lim) for (Ed e : eds) {
3ab    if (nodes[e.a].dist == -inf)
5ff      nodes[e.b].dist = -inf;
1d7  }
166 }
```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m, where $m[i][j] = \texttt{inf}$ if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, `inf` if no path, or `-inf` if the path goes through a negative-weight cycle.
Time: $\mathcal{O}(N^3)$

531245, 13 lines

```
964 const ll inf = 1LL << 62;
914 void floydWarshall(vector<vector<ll>>& m) {
e9d  int n = sz(m);
831  rep(i,0,n) m[i][i] = min(m[i][i], 0LL);
99d  rep(k,0,n) rep(i,0,n) rep(j,0,n)
19b    if (m[i][k] != inf && m[k][j] != inf) {
6e8      auto newDist = max(m[i][k] + m[k][j], -inf);
e89      m[i][j] = min(m[i][j], newDist);
f38    }
a69  rep(k,0,n) if (m[k][k] < 0) rep(i,0,n) rep(j,0,n)
ffd    if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
f12 }
```

7.2 Network flow and Matching

Dinic.h

Time: - $\mathcal{O}(\min(m \cdot \text{max\_flow},\ n^2 m))$.

- For graphs with unit capacities: $\mathcal{O}\left(\min(m\sqrt{m},\ mn^{2/3})\right)$.

- If every vertex has in-degree 1 or out-degree 1: $\mathcal{O}(m\sqrt{n})$.
- With capacity scaling: $\mathcal{O}(nm\log(\text{MAXCAP}))$ with high constant factor.

892d6e, 56 lines

```
14d struct Dinic {
61f  const bool scaling = false;
206  int lim;
670  struct edge {
c63    int to, rev;
a14    ll cap, flow;
7f9    bool res;
6dd    edge(int to_, ll cap_, int rev_, bool res_)
a94      : to(to_), cap(cap_), rev(rev_), flow(0), res(res_){}
477  };

002  vector<vector<edge>> g;
216  vector<int> lev, beg;
a71  ll F;
63f  Dinic(int n) : g(n), lev(n), beg(n), F(0) {}

0c5  void add(int a, int b, ll c, ll other = 0) {
de2    g[a].emplace_back(b, c, sz(g[b]), false);
fa5    g[b].emplace_back(a, other, sz(g[a])-1, true);
14f  }
123  bool bfs(int s, int t) {
e59    fill(all(lev), -1);
4e7    fill(all(beg), 0);
0a4         lev[s] = 0;
8b2    queue<int> q; q.push(s);
647    while (sz(q)) {
be1      int u = q.front(); q.pop();
bd9      for (auto& i : g[u]) {
dbc        if (lev[i.to] != -1 or (i.flow == i.cap)) continue;
b4f        if (scaling and i.cap - i.flow < lim) continue;
185        lev[i.to] = lev[u] + 1;
8ca        q.push(i.to);
f97      }
b1b    }
0de    return lev[t] != -1;
310  }
1dc  ll dfs(int v, int s, ll f = INF) {
50b    if (!f or v == s) return f;
84d    for (int& i = beg[v]; i < sz(g[v]); i++) {
027      auto& e = g[v][i];
206      if (lev[e.to] != lev[v] + 1) continue;
a30      ll foi = dfs(e.to, s, min(f, e.cap - e.flow));
749      if (!foi) continue;
3c5      e.flow += foi, g[e.to][e.rev].flow -= foi;
45c      return foi;
e08    }
bb3    return 0;
b98  }
2b4  ll maxFlow(int s, int t) {
a86    for (lim = scaling ? (1<<30) : 1; lim; lim /= 2)
69c      while (bfs(s, t)) while (ll ff = dfs(s, t)) F += ff;
4ff    return F;
6c8  }
0fe   bool inCut(int u){ return lev[u] != -1; }
892 };
```

LowerBoundFlow.h

Description: Calculates maximum flow with lower/upper bounds on edges. Returns -1 if no feasible flow exists. `add(a, b, l, r)` adds edge $a \to b$ where flow f must satisfy $l \leq f \leq r$. `add(a, b, c)` adds edge $a \to b$ with capacity c (implies $0 \leq f \leq c$). Same complexity as Dinic.

"Dinic.h" 756539, 36 lines

```
0ca struct lb_max_flow : Dinic {
96f  vector<ll> d;
be9  lb_max_flow(int n) : Dinic(n + 2), d(n, 0) {}
b12  void add(int a, int b, int l, int r) {
c97    d[a] -= l;
f1b    d[b] += l;
cb6    Dinic::add(a, b, r - l);
989  }
087  void add(int a, int b, int c) {
610    Dinic::add(a, b, c);
330  }
7a1  bool has_circulation() {
ac0    int n = sz(d);
854    ll cost = 0;
fea    rep(i, 0, n){
c69      if (d[i] > 0) {
f56        cost += d[i];
4f6        Dinic::add(n, i, d[i]);
551      } else if (d[i] < 0) {
bd2        Dinic::add(i, n+1, -d[i]);
bd9      }
a13    }

9f2    return (Dinic::maxFlow(n, n+1) == cost);
cc6  }
7bd  bool has_flow(int src, int snk) {
eda    Dinic::add(snk, src, INF);
e40    return has_circulation();
4aa  }
4eb  ll max_flow(int src, int snk) {
ee8    if (!has_flow(src, snk)) return -1;
99c    Dinic::F = 0;
703    return Dinic::maxFlow(src, snk);
0bb  }
756 };
```

MinCost.h

Description: Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only. If graph is a DAG pi can be calculated with DP instead of Bellman ford.
Time: $\mathcal{O}(FE\log(V))$ where F is max flow. $\mathcal{O}(VE)$ for setpi. 6f4fae, 95 lines

```
c4d #include <bits/extc++.h>

9f4 const ll INF = numeric_limits<ll>::max() / 4;

6f3 struct MCMF {
670   struct edge {
ede     int from, to, rev;
e20     ll cap, cost, flow;
092   };
060   int N;
091   vector<vector<edge>> ed;
a83   vector<int> seen, vis;
0ec   vector<ll> dist, pi;
c45   vector<edge*> par;

2cc   MCMF(int N) : N(N), ed(N), seen(N), vis(N),
dc7     dist(N), pi(N), par(N) {}

6f3   void addEdge(int from, int to, ll cap, ll cost) {
ad8     if (from == to || cap == 0) return;
1af     ed[from].push_back(edge{from,to,sz(ed[to]),cap,cost,0
 });
700     ed[to].push_back(edge{to,from,sz(ed[from])-1,0,-cost,0
 });
dad   }

975   void path(int s) {
7d4     fill(all(seen), 0);
04e     fill(all(dist), INF);
a93     dist[s] = 0;
841     ll di;
937     __gnu_pbds::priority_queue<pair<ll, int>> q;
9fb     vector<decltype(q)::point_iterator> its(N);
23b     q.push({ 0, s });

14d     while (!q.empty()) {
eda       s = q.top().second; q.pop();
2af       seen[s] = 1; di = dist[s] + pi[s];
6bd       for (edge& e : ed[s]) {
d20         if (!seen[e.to]) {
f1f           ll val = di - pi[e.to] + e.cost;
f3c           if(e.cap - e.flow > 0 && val < dist[e.to]){
0c7             dist[e.to] = val;
fb6             par[e.to] = &e;
22d             if (its[e.to] == q.end()) {
aac               its[e.to] = q.push({-dist[e.to], e.to});
388             }
6f8             else q.modify(its[e.to], {-dist[e.to], e.to});
80b           }
fce         }
013       }
e16     }
faa     for (int i = 0; i < N; i++) {
0ef       pi[i] = min(pi[i] + dist[i], INF);
ded     }
17b   }

310   pair<ll, ll> maxflow(int s, int t) {
923     setpi(s, t);
3d3     ll totflow = 0, totcost = 0;
8dd     while (path(s), seen[t]) {
535       ll fl = INF;
733       for (edge* x = par[t]; x; x = par[x->from]) {
8ed         fl = min(fl, x->cap - x->flow);
ddf       }
f9f       totflow += fl;
733       for (edge* x = par[t]; x; x = par[x->from]) {
10b         x->flow += fl;
e58         ed[x->to][x->rev].flow -= fl;
3bf       }
219     }
faa     for (int i = 0; i < N; i++) {
a18       for (edge& e : ed[i]) {
7a0         totcost += e.cost * e.flow;
774       }
a06     }
17e     return { totflow, totcost / 2 };
411   }

      // If some costs can be negative, call this before
           maxflow:
eda   void setpi(int s, int t) {
3ef     fill(all(pi), INF);
156     pi[s] = 0;
45c     int it = N, ch = 1;
aa3     ll v;
5e8     while (ch-- && it--) {
faa       for (int i = 0; i < N; i++) {
c9b         if (pi[i] != INF)
fb0           for (edge& e : ed[i]) if (e.cap)
257             if((v= pi[i] + e.cost)< pi[e.to])
a43               pi[e.to] = v, ch = 1;
d0b       }
250     }
38b     assert(it >= 0); // negative cost cycle
545   }
f1d };
```

PushRelabel.h

Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

Time: $\mathcal{O}\left(V^2\sqrt{E}\right)$

a7bbd5, 55 lines

```
49f struct PushRelabel {
e9b  struct Edge {
548    int dest, back;
e00    ll f, c;
571  };
ed3  vector<vector<Edge>> g;
51c  vector<ll> ec;
658  vector<Edge*> cur;
b08  vector<vector<int>> hs;
4d4  vector<int> H;
4e1  PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n){}

b1c  void addEdge(int s, int t, ll cap, ll rcap=0) {
50b    if (s == t) return;
cc8    g[s].push_back({t, sz(g[t]), 0, cap});
2aa    g[t].push_back({s, sz(g[s])-1, 0, rcap});
817  }

359  void addFlow(Edge& e, ll f) {
759    Edge &back = g[e.dest][e.back];
f7e    if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
d2e    e.f += f; e.c -= f; ec[e.dest] += f;
c47    back.f -= f; back.c += f; ec[back.dest] -= f;
340  }
0e0  ll calc(int s, int t) {
f00    int v = sz(g); H[s] = v; ec[t] = 1;
fbb    vector<int> co(2*v); co[0] = v-1;
e20    for(int i=0; i<v; i++) cur[i] = g[i].data();
8c2    for (Edge& e : g[s]) addFlow(e, e.c);

604    for (int hi = 0;;) {
ae9      while (hs[hi].empty()) if (!hi--) return -ec[s];
c6f      int u = hs[hi].back(); hs[hi].pop_back();
a3e      while (ec[u] > 0)  // discharge u
457        if (cur[u] == g[u].data() + sz(g[u])) {
e94          H[u] = 1e9;
5fa          for (Edge& e : g[u]){
256            if (e.c && H[u] > H[e.dest]+1)
740              H[u] = H[e.dest]+1, cur[u] = &e;
88f          }
f04          if (++co[H[u]], !--co[hi] && hi < v){
10d            for(int i=0; i<v; i++){
4be              if (hi < H[i] && H[i] < v)
021                --co[H[i]], H[i] = v + 1;
a21            }
cc1          }
3a2          hi = H[u];
b6b        } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1){
779          addFlow(*cur[u], min(ec[u], cur[u]->c));
e91        }else ++cur[u];
4d7    }
b65  }
385  bool inCut(int a) { return H[a] >= sz(g); }
a7b };
```

Blossom.h

Description: Max matching on general Graph. $mate[i]$ = match of i

Time: $\mathcal{O}\left(N^3\right)$

21cc7b, 56 lines

```
40f vector<int> Blossom(vector<vector<int>>& g) {
10a     int n = sz(g), timer = -1;
f55     vector<int> mate(n, -1),label(n),par(n),orig(n),aux(n,
     -1),q;

060     auto lca = [&](int x, int y) {
7b8         for (timer++; ; swap(x, y)) {
583             if (x == -1) continue;
4be             if (aux[x] == timer) return x;
90d             aux[x] = timer;
fb4             x=(mate[x] == -1 ? -1 : orig[par[mate[x]]]);
f6a         }
aba     };
be4     auto blossom = [&](int v, int w, int a) {
509         while (orig[v] != a) {
721             par[v] = w; w = mate[v];
1e2             if(label[w] == 1) label[w] = 0,q.push_back(w);
8c7             orig[v] = orig[w] = a;
3d0             v = par[w];
eae         }
068     };
a0f     auto aug = [&](int v) {
8c8         while (v != -1) {
86a             int pv = par[v], nv = mate[pv];
941             mate[v] = pv; mate[pv] = v; v = nv;
ba8         }
54c     };
9f9     auto bfs = [&](int root) {
be5         fill(all(label), -1);
652         iota(all(orig), 0);
4b6         q.clear();
594         label[root] = 0; q.push_back(root);
a43         rep(i, 0, sz(q)) {
4c1             int v = q[i];
5aa             for (auto x : g[v]) {
464                 if (label[x] == -1) {
73a                     label[x] = 1; par[x] = v;
1bd                     if (mate[x] == -1) return aug(x), 1;
8d9                     label[mate[x]] = 0;
de3                     q.push_back(mate[x]);
641                 }
018                 else if(!label[x] && orig[v] != orig[x]){
37f                     int a = lca(orig[v], orig[x]);
```

```
f12                 blossom(x, v, a);
183                 blossom(v, x, a);
405             }
ab5         }
9e2     }
bb3     return 0;
139 };
    // Time halves if you start with (any) maximal
       matching.
fea rep(i, 0, n) {
698     if (mate[i] == -1) bfs(i);
7b5 }
568 return mate;
21c }
```

HopcroftKarp.h

Description: *ans* is the size of the max matching.
The match of x is $l[x]$
Usage: `HopcroftKarp(|X|, |Y|, edges(x, y))`
Time: $\mathcal{O}\left(\sqrt{V}E\right)$

c4f2f2, 46 lines

```
725 struct HopcroftKarp {
e40     vector<int> g, l, r;
959     int ans;
b82     HopcroftKarp(int n, int m, vector<pii> e)
aa0         : g(sz(e)), l(n, -1), r(m, -1), ans(0) {
bb0         shuffle(all(e), rng);
322         vector<int> deg(n + 1);
235         for (auto& [x, y] : e) deg[x]++;
b4a         rep(i, 1, n+1) deg[i] += deg[i - 1];
85a         for (auto& [x, y] : e) g[--deg[x]] = y;

5ae         vector<int> q(n);
667         while (true) {
661             vector<int> a(n, -1), p(n, -1);
6bb             int t = 0;
fea             rep(i, 0, n){
4b1                 if (l[i] == -1) {
b53                     q[t++] = a[i] = p[i] = i;
4b6                 }
62e             }
a15             bool match = false;
edb             rep(i, 0, t){
912                 int x = q[i];
08c                 if (~l[a[x]]) continue;
0ba                 rep(j, deg[x], deg[x+1]) {
360                     int y = g[j];
89a                     if (r[y] == -1) {
d3b                         while (~y) {
ee7                             r[y] = x;
dbb                             swap(l[x], y);
2a5                             x = p[x];
ebf                         }
6aa                         match = true, ans++;
c2b                         break;
b54                     }
f06                     if (p[r[y]] == -1) {
a74                         q[t++] = y = r[y];
d11                         p[y] = x, a[y] = a[x];
9ef                     }
e8a                 }
0ab             }
984             if (!match) break;
bc5         }
6ec     }
c4f };
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \leq M$.
Time: $\mathcal{O}\left(N^2 M\right)$

4a75d2, 41 lines

```
d57 pair<ll, vector<int>> hunga(const vector<vector<ll>>& a) {
c04     if (a.empty()) return { 0, {} };
1a9     int n = sz(a) + 1, m = sz(a[0]) + 1;
fc8     vector<ll> u(n), v(m), p(m);
5bd     vector<int> ans(n - 1);
6f5     for (int i = 1; i < n; i++) {
8c9         p[0] = i;
625         int j0 = 0;
91d         vector<ll> dist(m, LLONG_MAX), pre(m, -1);
910         vector<bool> done(m + 1);
016         do {
781             done[j0] = true;
507             ll i0 = p[j0], j1 = -1, delta = LLONG_MAX;
b84             for (int j = 1; j < m; j++) {
10a                 if (!done[j]) {
ed6                     ll cur = a[i0-1][j-1] - u[i0] - v[j];
607                     if (cur < dist[j])
29f                         dist[j] = cur, pre[j] = j0;
172                     if(dist[j] < delta)
4ab                         delta = dist[j], j1 = j;
103                 }
bb2             }
891             for (int j = 0; j < m; j++) {
7a9                 if (done[j])
3bc                     u[p[j]] += delta, v[j] -= delta;
202                 else dist[j] -= delta;
11a             }
e73             assert(j1 != -1);
6d4             j0 = j1;
ac1         } while (p[j0]);
4b9         while (j0) {
196             int j1 = pre[j0];
0c1             p[j0] = p[j1], j0 = j1;
f55         }
193     }
b84     for (int j = 1; j < m; j++) {
eb3         if (p[j]) ans[p[j] - 1] = j - 1;
c9a     }
def     return { -v[0], ans }; // min cost
4a7 }
```

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}\left(V^3\right)$

8b0e19, 22 lines

```
192 pair<int, vi> globalMinCut(vector<vi> mat) {
afa  pair<int, vi> best = {INT_MAX, {}};
755  int n = sz(mat);
91d  vector<vi> co(n);
d0f  rep(i,0,n) co[i] = {i};
488  rep(ph,1,n) {
2e9    vi w = mat[0];
e44    size_t s = 0, t = 0;
694    rep(it,0,n-ph) { // O(V^2) -> O(E log V) with prio.
       queue
d6e      w[t] = INT_MIN;
a5f      s = t, t = max_element(all(w)) - w.begin();
d39      rep(i,0,n) w[i] += mat[t][i];
ec9    }
3df    best = min(best, {w[t] - mat[t][t], co[t]});
096    co[s].insert(co[s].end(), all(co[t]));
959    rep(i,0,n) mat[s][i] += mat[t][i];
984    rep(i,0,n) mat[i][s] = mat[s][i];
5dd    mat[0][t] = INT_MIN;
ca0  }
f26  return best;
8b0 }
```

7.3 DFS algorithms

Bridges.h

1fa56b, 24 lines

```
cd9 vector<int> g[ms];
9e4 int low[ms], tin[ms], vis[ms], t;

403 void dfs(int u = 0, int p = -1) {
b9c     vis[u] = true;
b4a     low[u] = tin[u] = t++;
7b9     for (auto v : g[u]) {
730         if (v == p) continue;
c84         if (vis[v]) {
34f             low[u] = min(low[u], tin[v]);
728         }
4e6         else {
95e             dfs(v, u);
ab6             low[u] = min(low[u], low[v]);
                // if (low[v] >= tin[u] && p != -1), U is an
                   articulation point
975             if (low[v] > tin[u]) {
                    // edge from U to V is a bridge
4b8             }
                // children++
862         }
677     }
        // if(children > 1 && p == -1) root is an articulation
           point
30c }
```

BridgeOnline.h

Description: Maintains bridges and 2-edge-connected components (2-ECC) incrementally. `ds[0]` tracks Connected Components (CC). `ds[1]` tracks 2-ECCs. Nodes u, v are in the same 2-ECC iff `dsfind(u, 1) == dsfind(v, 1)`. `g` stores the spanning forest edges (edges that were bridges when added). An edge $(u, v) \in g$ is a current bridge iff `dsfind(u, 1) != dsfind(v, 1)`. `bridges` tracks the total count of active bridges. Use `init()` before starting.
Time: Amortized $\mathcal{O}\left(\log N\right)$

ef24c8, 75 lines

```
4dd int bridges;
801 int ds[2][ms], sz[2][ms];
87b int h[ms], pai[ms], old[ms];
cd9 vector<int> g[ms];

ca2 void init() {
786     bridges = 0;
f0d     rep(i, 0, ms) {
a4e         g[i].clear(), h[i] = 0;
606         ds[0][i] = ds[1][i] = i;
8f3         sz[0][i] = sz[1][i] = 1;
4a6     }
c1e }

243 int dsfind(int j, int i) {
7fa     if(j == ds[i][j]) return ds[i][j];
db7     return ds[i][j] = dsfind(ds[i][j], i);
4a4 }

b55 void dfs(int u, int p, int l) {
40d     h[u] = l;
49e     pai[u] = p;
a32     old[u] = dsfind(u, 1);
4d5     for (int v : g[u]) {
```

```
730         if (v == p) continue;
0c5         dfs(v, u, l + 1);
11d     }
f2e }

94c void updateNodes(int u, int p) {
840     if (old[u] == old[p]) {
dc4         ds[1][u] = ds[1][p];
574     }
e79     else ds[1][u] = u;
4d5     for (int v : g[u]) {
730         if (v == p) continue;
01c         updateNodes(v, u);
42a     }
329 }

814 void mergeTrees(int a, int b) {
cbf     bridges++;
5cb     int iniA = a, iniB = b;
19d     a = dsfind(a, 0), b = dsfind(b, 0);
834     if (sz[0][a] < sz[0][b]) swap(a, b), swap(iniA, iniB);
e14     dfs(iniB, iniA, h[iniA] + 1);
376     old[iniA] = -1;
ee0     updateNodes(iniB, iniA);
86b     ds[0][b] = a;
013     sz[0][a] += sz[0][b];
c9a }

416 void removeBridges(int a, int b) {
532     a = dsfind(a, 1), b = dsfind(b, 1);
984     while (a != b) {
e7a         bridges--;
54b         if (h[a] < h[b]) swap(a, b);
            // ponte entre (a, pai[a]) deixou de existir
9f6         ds[1][a] = dsfind(pai[a], 1);
e40         a = ds[1][a];
cda     }
a78 }

02b void addEdge(int a, int b) {
7b9     if (dsfind(a, 0) == dsfind(b, 0)) {
69d         removeBridges(a, b);
221     }
4e6     else {
            // nova ponte entre (a, b)
025         g[a].push_back(b);
3e9         g[b].push_back(a);
f8e         mergeTrees(a, b);
447     }
e57 }
```

BlockCutTree.h

Description: Constructs the Block-Cut Tree, which is a bipartite graph with blocks (maximal 2-vertex-connected components) on one side and articulation points on the other. Works for disconnected graphs. Tree size is $\leq 2N$. Be careful with self loops and multi edges. `art[i]`: number of new components created by removing i (AP if ≥ 1). `blocks[i]`, `edgblocks[i]`: vertices/edges of block i. `tree[i]`: the tree node index corresponding to block i. `pos[i]`: the tree node index corresponding to vertex i.
Time: $\mathcal{O}(N+M)$

e55ab0, 66 lines

```
d10 struct block_cut_tree {
d8e  vector<vector<int>> g, blocks, tree;
43b  vector<vector<pair<int, int>>> edgblocks;
4ce  stack<int> s;
6c0  stack<pair<int, int>> s2;
2bb  vector<int> id, art, pos;

763  block_cut_tree(vector<vector<int>> g_) : g(g_) {
625    int n = sz(g);
37a    id.resize(n, -1), art.resize(n), pos.resize(n);
6f2    build();
246  }

df6  int dfs(int i, int& t, int p = -1) {
cf0    int lo = id[i] = t++;
18e    s.push(i);

827    if (p != -1) s2.emplace(i, p);
43f    for (int j : g[i])
6bf            if (j != p and id[j] != -1) s2.emplace(i, j);

cac    for (int j : g[i]) if (j != p) {
9a3      if (id[j] == -1) {
121        int val = dfs(j, t, i);
0c3        lo = min(lo, val);

588        if (val >= id[i]) {
66a          art[i]++;
483          blocks.emplace_back(1, i);
110          while (blocks.back().back() != j)
138            blocks.back().push_back(s.top()), s.pop();

128          edgblocks.emplace_back(1, s2.top()), s2.pop();
904          while (edgblocks.back().back() != pii(j, i))
bce            edgblocks.back().push_back(s2.top()), s2.pop();
041        }
38c      }
328      else lo = min(lo, id[j]);
5b6    }
924    if (p == -1) {
2db            if(art[i]) art[i]--;
4e6            else{
483                blocks.emplace_back(1, i);
433                edgblocks.emplace_back();
333            }
384        }
253    return lo;
6d7  }

0a8  void build() {
6bb    int t = 0;
c80      rep(i, 0, sz(g)) if(id[i] == -1) dfs(i, t, -1);
de0    tree.resize(sz(blocks));
008    rep(i, 0, sz(g)) if (art[i])
b9a      pos[i] = sz(tree), tree.emplace_back();

05c    rep(i, 0, sz(blocks)) for (int j : blocks[i]) {
403      if (!art[j]) pos[j] = i;
4e6      else {
49d                tree[i].push_back(pos[j]);
9a7                tree[pos[j]].push_back(i);
01e            }
27c    }
5a7  }
e55 };
```

DominatorTree.h

Description: Builds the Dominator Tree of a directed graph rooted at `root`. Node u dominates v if every path from `root` to v passes through u. The immediate dominator of v is the unique dominator closest to v (excluding v). Returns a vector `par` where `par[u]` is the parent of u in the tree. Roots and unreachable nodes satisfy `par[u] = u`.
Time: $\mathcal{O}(M \log N)$

8c4613, 55 lines

```
3db struct dominator_tree {
577  int n, t;
324  vector<vector<int>> g, rg, bucket;
7f3  vector<int> arr, par, rev, sdom, dom, ds, lbl;

226  dominator_tree(int n) : n(n),t(0),g(n),rg(n),bucket(n),
7a1    arr(n,-1),par(n),rev(n),sdom(n),dom(n),ds(n),lbl(n) {}

c2b  void add_edge(int u, int v) { g[u].push_back(v); }

315  void dfs(int u) {
12e    arr[u] = t;
64f    rev[t] = u;
bad    lbl[t] = sdom[t] = ds[t] = t;
c82     t++;
6f1    for (int w : g[u]) {
0c2      if (arr[w] == -1) {
8c6        dfs(w);
81a        par[arr[w]] = arr[u];
869      }
f8e      rg[arr[w]].push_back(arr[u]);
93a    }
b04  }
792  int find(int u, int x=0) {
9fe    if (u == ds[u]) return x ? -1 : u;
41f    int v = find(ds[u], x+1);
388    if (v < 0) return u;
b30    if(sdom[lbl[ds[u]]] < sdom[lbl[u]]) lbl[u]= lbl[ds[u]];
300    ds[u] = v;
784    return x ? v : lbl[u];
a59  }

46f  vector<int> run(int root) {
14e    dfs(root);
b81    iota(all(dom), 0);
da8    for (int i=t-1; i>=0; i--) {
76c      for(int w : rg[i]) sdom[i] = min(sdom[i],sdom[find(w)
    ]);
c94      if (i) bucket[sdom[i]].push_back(i);
3b2      for (int w : bucket[i]) {
46a        int v = find(w);
ae4        if (sdom[v] == sdom[w]) dom[w] = sdom[w];
41c        else dom[w] = v;
1e6      }
fd8      if (i > 1) ds[i] = par[i];
b9e    }
e8f    rep(i, 1, t) {
7d7      if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
32d    }
af8    vector<int> par(n);
2c2    iota(all(par), 0);
533    rep(i, 0, t) par[rev[i]] = rev[dom[i]];
148    return par;
900  }
8c4 };
```

EulerPath.h

Description: Receives as input graph(node, edge index), number of edges and source. Returns list of node, index of edge he came from, if path/circuit does not exists returns empty list.

a3ed13, 27 lines

```
b4a vector<pii> eulerPath(const vector<vector<pii>>& g, int
     nedges, int src) {
625     int n = sz(g);
b47     vector<int> deg(n, 0), its(n, 0), used(nedges + 1, 0);
a42     vector<pii> s = { {src, -1} };
        //deg[src]++; //to allow paths, not only circuits
a5f     vector<pii> ret;
980     while (!s.empty()) {
d0b         int u = s.back().first, &it = its[u];
c45         if (it == sz(g[u])) {
5e3             ret.push_back(s.back());
```

```
342             s.pop_back();
5e2             continue;
8e8         }
f7f         auto& [nxt, id] = g[u][it++];
b25         if (!used[id]) {
e48             deg[u]--, deg[nxt]++;
029             used[id] = 1;
e1c             s.push_back({ nxt, id });
777         }
388     }
8d8     for (int x : deg) {
518         if (x < 0 || sz(ret) != (nedges + 1)) return {};
26e     }
969     reverse(ret.begin(), ret.end());
edf     return ret;
a3e }
```

SCC.h

Description: Kosaraju algorithm for calculating strongly connected components. Components are ordered in topological order.

008ff2, 36 lines

```
bf0 struct SCC {
dab     int n, ncomp;
0e3     vector<vector<int>> g, inv;
829     vector<int> comp, vis, stk;
8b6     SCC(){}
471     SCC(int n)
464         : n(n),ncomp(0),g(n),inv(n),comp(n, -1),vis(n){}

315     void dfs(int u) {
150         vis[u] = 1;
a35         for (int v : g[u]) if (!vis[v]) dfs(v);
967         stk.push_back(u);
37b     }
f20     void dfs_inv(int u) {
62c         comp[u] = ncomp;
3a5         for (int v : inv[u]) {
df4             if (comp[v] == -1) dfs_inv(v);
0a0         }
984     }
63d     void solve() {
603         for (int i = 0; i < n; i++) {
b65             if (!vis[i]) dfs(i);
358         }
340         reverse(all(stk));
49b         for (int u : stk) {
9ef             if (comp[u] != -1) continue;
672             dfs_inv(u);
a8f             ncomp++;
ecb         }
ef8     }
010     void add_edge(int a, int b) {
025         g[a].push_back(b);
a6a         inv[b].push_back(a);
1ec     }
008 };
```

TwoSat.h

Usage: not A = ~A

"SCC.h" c8b989, 37 lines

```
d9d struct TwoSat{
1a8   int n;
3c9   SCC scc;
7c7   vector<int> value;
425   vector<pii> e;
e2c   TwoSat(int n) : n(n){}
6c0   bool solve(){
b36     value.resize(n);
8cc     scc = SCC(2*n);
1f3     for(auto &x : e) scc.add_edge(x.first, x.second);
7f9     scc.solve();
3df     for(int i=0; i<2*n; i++)
f83       if(scc.comp[i] == scc.comp[i^1]) return false;
830     for(int i=0; i<n; i++)
733       value[i] = scc.comp[id(i)] > scc.comp[id(~i)];
8a6     return true;
949   }
a0a   void atMostOne(vector<int> &li){
615     if(sz(li) <= 1) return;
da9     int cur = ~li[0];
b25     for(int i = 2; i < sz(li); i++) {
abb       int next = n++;
e0a       addOr(cur, ~li[i]);
f26       addOr(cur, next);
7ba       addOr(~li[i], next);
072       cur = ~next;
e3d     }
921     addOr(cur, ~li[1]);
bbb   }
41b   int id(int v) { return v < 0 ? (~v) * 2 ^ 1 : v * 2; }
276   void add(int a, int b) { e.push_back({id(a), id(b)}); }
bc7   void addOr(int a, int b) { add(~a, b); add(~b, a); }
671   void addImp(int a, int b) { addOr(~a, b); }
d9d   void addEqual(int a, int b){ addOr(a, ~b); addOr(~a, b);
      }
ec3   void isFalse(int a) { addImp(a, ~a); }
c8b };
```

7.4 Coloring

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)

Time: $\mathcal{O}(NM)$

e210e2, 32 lines

```
f41 vi edgeColoring(int N, vector<pii> eds) {
727   vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
10d   for (pii e : eds) ++cc[e.first], ++cc[e.second];
e2f   int u, v, ncols = *max_element(all(cc)) + 1;
fda   vector<vi> adj(N, vi(ncols, -1));
6ec   for (pii e : eds) {
119     tie(u, v) = e;
e51     fan[0] = v;
0f4     loc.assign(ncols, 0);
696     int at = u, end = u, d, c = free[u], ind = 0, i = 0;
3b2     while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
3e1       loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
01e     cc[loc[d]] = c;
997     for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd
    ])
4ff       swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
79f     while (adj[fan[i]][d] != -1) {
a9f       int left = fan[i], right = fan[++i], e = cc[i];
99b       adj[u][e] = left;
ccb       adj[left][e] = u;
f7e       adj[right][e] = -1;
d99       free[right] = e;
316     }
dfd     adj[u][d] = fan[i];
c45     adj[fan[i]][d] = u;
0e1     for (int y : {fan[0], u, end})
3fa       for (int& z = free[y] = 0; adj[y][z] != -1; z++);
fdc   }
29d   rep(i,0,sz(eds))
961     for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i
    ];
edf   return ret;
e21 }
```

7.5 Heuristics

MaxClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

2eeaf4, 53 lines

```
db9 using vb = vector<bitset<200>>;
c7d struct Maxclique {
24e   double limit=0.025, pk=0;
c04   struct Vertex { int i, d=0; };
547   using vv = vector<Vertex>;
d44   vb e;
df7   vv V;
e5c   vector<vector<int>> C;
497   vector<int> qmax, q, S, old;
fe3   void init(vv& r) {
fd3     for (auto& v : r) v.d = 0;
583     for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
0f1     sort(all(r), [](auto a, auto b) { return a.d > b.d; });
c43     int mxD = r[0].d;
3f8     for(int i=0; i<sz(r); i++) r[i].d = min(i, mxD) + 1;
526   }
bc8   void expand(vv& R, int lev = 1) {
ac1     S[lev] += S[lev - 1] - old[lev];
92c     old[lev] = S[lev - 1];
d18     while (sz(R)) {
3fd       if (sz(q) + R.back().d <= sz(qmax)) return;
d62       q.push_back(R.back().i);
f28       vv T;
7fb       for(auto v : R)
740         if (e[R.back().i][v.i]) T.push_back({v.i});
d21       if (sz(T)) {
eea         if (S[lev]++ / ++pk < limit) init(T);
457         int j = 0, mxk = 1, mnk = max(sz(qmax)-sz(q)+1, 1);
9bc         C[1].clear(), C[2].clear();
969         for (auto v : T) {
bfe           int k = 1;
8f5           auto f = [&](int i) { return e[v.i][i]; };
5c6           while (any_of(all(C[k]), f)) k++;
782           if (k > mxk) mxk = k, C[mxk + 1].clear();
18a           if (k < mnk) T[j++].i = v.i;
0e6           C[k].push_back(v.i);
322         }
238         if (j > 0) T[j - 1].d = 0;
d2f         for(int k=mnk; k<mxk + 1; k++){
5bf                 for (int i : C[k])
361                     T[j].i = i, T[j++].d = k;
9dc                 }
22d         expand(T, lev + 1);
61f       } else if (sz(q) > sz(qmax)) qmax = q;
c81       q.pop_back(), R.pop_back();
3e0     }
81d   }
b2d   vector<int> maxClique(){ init(V),expand(V); return qmax;}
b40   Maxclique(vb conn) : e(conn),C(sz(e)+1),S(sz(C)),old(S){
01d     for(int i=0; i<sz(e); i++) V.push_back({i});
b60   }
534 };
```

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

b0d5b1, 13 lines

```
753 typedef bitset<128> B;
044 template<class F>
6a9 void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R
    ={}) {
9bb   if (!P.any()) { if (!X.any()) f(R); return; }
a8e   auto q = (P | X)._Find_first();
cd1   auto cands = P & ~eds[q];
3d7   rep(i,0,sz(eds)) if (cands[i]) {
a75     R[i] = 1;
e78     cliques(eds, f, P & eds[i], X & eds[i], R);
bb6     R[i] = P[i] = 0; X[i] = 1;
181   }
c9d }
```

7.6 Trees

Centroid.h

Description: Call decomp(0) to solve, marked array should be initially set to zero.
Time: $\mathcal{O}(N \log N)$

b73755, 27 lines

```
6b6 int tam[ms], marked[ms];

2a1 int calc_tam(int u, int p) {
5d1   tam[u] = 1;
4d5   for (int v : g[u]) {
5ea     if (v != p && !marked[v]) tam[u] += calc_tam(v, u);
d09   }
f95   return tam[u];
d5d }

5fb int get_centroid(int u, int p, int tot) {
4d5   for (int v : g[u]) {
38c     if (v != p && !marked[v] && (tam[v] > (tot / 2)))
32c       return get_centroid(v, u, tot);
b6c   }
03f   return u;
0c7 }
    // Cent is a child of P in the centroid tree
179 void decomp(int u, int p = -1) {
308   calc_tam(u, -1);
bd4   int cent = get_centroid(u, -1, tam[u]);
83d   marked[cent] = 1;
9f1   for (int v : g[cent]) {
c6e     if (!marked[v]) decomp(v, cent);
194   }
dc1 }
```

HLD.h

Description: If values are stored on edges, set EDGE = true and store each edge's value at the endpoint farther from the root (the deeper node). rp[i] is the representative (head) of the heavy path containing node i: it is the node in that chain that is closest to the root.

a129d6, 51 lines

```
5f2 template<bool EDGE> struct HLD {
577     int n, t;
789     vector<vector<int>> g;
003     vector<int> pai, rp, tam, pos, val, arr;
f1e     Seg seg;
bcf     HLD(int n, vector<vector<int>>& g, vector<int>& val)
ac9         : n(n), t(0), g(g), pai(n), rp(n), tam(n, 1),
616             pos(n), val(val), arr(n) {
f80         calc_tam(0, -1);
c91         dfs(0, -1);
d14         seg.build(arr);
a43     }

2a1     int calc_tam(int u, int p) {
49e         pai[u] = p;
704         for (int& v : g[u]) {
730             if (v == p) continue;
2e4             tam[u] += calc_tam(v, u);
2d5             if(tam[v] > tam[g[u][0]] || g[u][0] == p)
a7f                 swap(g[u][0], v);
0a3         }
f95         return tam[u];
c19     }

fb6     void dfs(int u, int p) {
4c8         pos[u] = t++;
d7b         arr[pos[u]] = val[u];
4d5         for (int v : g[u]) {
730             if (v == p) continue;
84d             rp[v] = (v == g[u][0] ? rp[u] : v);
95e             dfs(v, u);
42d         }
de1     }

4ea     int query(int a, int b) { // query on the path from a
    to b
1a4         int ans = 0; // neutral value
34d         while (rp[a] != rp[b]) {
aa1             if (pos[a] < pos[b]) swap(a, b);
9a5             ans = max(ans, seg.query(pos[rp[a]], pos[a]));
677             a = pai[rp[a]];
ebd         }
9bc         if(pos[a] > pos[b]) swap(a, b);
0f8         ans = max(ans, seg.query(pos[a] + EDGE, pos[b]));
ba7         return ans;
e8a     }

534     void update(int a, int x) {
e5e         seg.update(pos[a], x);
5db     }
a12 };
```

LCA.h

Description: LCA algorithm using binary lifting, $is\_ancestor(a, b)$ returns true if a is an ancestral of b and false otherwise.
Time: $\mathcal{O}(N \log N)$

db7791, 26 lines

```
67e int tin[MAXN], tout[MAXN], timer=0;
768 int up[MAXN][BITS];
fb6 void dfs(int u, int p){
545     tin[u] = timer++, up[u][0] = p;
532     for (int i=1; i<BITS; i++) {
88a         up[u][i] = up[up[u][i-1]][i-1];
4a0     }
712     for (int v: g[u]) if (v != p) dfs(v, u);
4f8     tout[u] = timer;
4a1 }

f31 bool is_ancestor(int u, int v){
d34     return (tin[u] <= tin[v] && tout[u] >= tout[v]);
f9f }

310 int lca(int u, int v){
bd5     if (is_ancestor(u, v)) return u;
6fc     if (is_ancestor(v, u)) return v;
3c3     for (int i=BITS-1; i>=0; i--) {
3a3         if (up[u][i] && !is_ancestor(up[u][i], v)) {
c3f             u = up[u][i];
49e         }
dc4     }
c15     return up[u][0];
001 }
```

VirtualTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S| - 1$) pairwise LCA's and compressing edges. virt[u] is the adjacency list of the virtual tree: it stores pairs (v, dist), where v is a neighbor of u in the virtual tree and dist is the distance between u and v in the original tree.
Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h" 11157a, 24 lines

```
0b1 vector<pair<int, int>> virt[ms];

d0c void build_virt(vector<int>& v) {
078   auto cmp = [&](int i, int j){ return tin[i] < tin[j]; };
b84   sort(all(v), cmp);
1ee   for (int i = 0, n = sz(v);i + 1 < n; i++)
4cf     v.push_back(lca(v[i], v[i + 1]));
b84   sort(all(v), cmp);
64f   v.erase(unique(all(v)), v.end());
7b4   stack<int> st;
3a7   for (auto u : v) {
c53     if (st.empty()) {
4a6       st.push(u);
e82     }
4e6     else {
7eb       while(sz(st) && !is_ancestor(st.top(), u)) st.pop();
88b       int p = st.top();
bfa       virt[p].emplace_back(u, abs(lvl[u] - lvl[p]));
0a5       virt[u].emplace_back(p, abs(lvl[u] - lvl[p]));
4a6       st.push(u);
92c     }
f46   }
c83 }
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" 39e620, 61 lines

```
030 struct Edge { int a, b; ll w; };
bf2 struct Node {
25f   Edge key;
c17   Node *l, *r;
981   ll delta;
a9c   void prop() {
6f9     key.w += delta;
d2d     if (l) l->delta += delta;
d86     if (r) r->delta += delta;
978     delta = 0;
0d3   }
866   Edge top() { prop(); return key; }
ab4 };
3eb Node *merge(Node *a, Node *b) {
b9f   if (!a || !b) return a ?: b;
626   a->prop(), b->prop();
dc2   if (a->key.w > b->key.w) swap(a, b);
485   swap(a->l, (a->r = merge(b, a->r)));
3f5   return a;
c51 }
7bb void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

002 pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
8df   RollbackUF uf(n);
3f8   vector<Node*> heap(n);
563   for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e
    });
cd2   ll res = 0;
517   vi seen(n, -1), path(n), par(n);
559   seen[r] = r;
dd6   vector<Edge> Q(n), in(n, {-1,-1}), comp;
```

```
111   deque<tuple<int, int, vector<Edge>>> cycs;
328   rep(s,0,n) {
3cb     int u = s, qi = 0, w;
a0a     while (seen[u] < 0) {
572       if (!heap[u]) return {-1,{}};
ebe       Edge e = heap[u]->top();
5ed       heap[u]->delta -= e.w, pop(heap[u]);
952       Q[qi] = e, path[qi++] = u, seen[u] = s;
d56       res += e.w, u = uf.find(e.a);
9e2       if (seen[u] == s) {
28d         Node* cyc = 0;
cab         int end = qi, time = uf.time();
f38         do cyc = merge(cyc, heap[w = path[--qi]]);
4f9         while (uf.join(u, w));
562         u = uf.find(u), heap[u] = cyc, seen[u] = -1;
c06         cycs.push_front({u, time, {&Q[qi], &Q[end]}});
00a       }
c8f     }
068     rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
fa3   }

e41   for (auto& [u,t,comp] : cycs) { // restore sol (optional)
36c     uf.rollback(t);
1d0     Edge inEdge = in[u];
251     for (auto& e : comp) in[uf.find(e.b)] = e;
56d     in[uf.find(inEdge.b)] = inEdge;
4f9   }
427   rep(i,0,n) par[i] = in[i].a;
efb   return {res, par};
efa }
```

Geometry (8)

8.1 Geometric primitives

Point.h

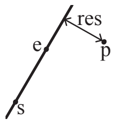
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

47ec0a, 29 lines

```
48b template <class T> int sgn(T x) { return (x > 0) - (x < 0)
    ; }
4fc template<class T>
f26 struct Point {
ea4  typedef Point P;
645  T x, y;
ea6  explicit Point(T x=0, T y=0) : x(x), y(y) {}
0d0  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y
    ); }
ec7  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y
    ); }
279  P operator+(P p) const { return P(x+p.x, y+p.y); }
40d  P operator-(P p) const { return P(x-p.x, y-p.y); }
e03  P operator*(T d) const { return P(x*d, y*d); }
0b9  P operator/(T d) const { return P(x/d, y/d); }
57b  T dot(P p) const { return x*p.x + y*p.y; }
460  T cross(P p) const { return x*p.y - y*p.x; }
b3f  T cross(P a, P b) const { return (a-*this).cross(b-*this)
    ; }
f68  T dist2() const { return x*x + y*y; }
18b  double dist() const { return sqrt((double)dist2()); }
     // angle to x-axis in interval [-pi, pi]
907  double angle() const { return atan2(y, x); }
d06  P unit() const { return *this/dist(); } // makes dist()=1
200  P perp() const { return P(-y, x); } // rotates +90
    degrees
852  P normal() const { return perp().unit(); }
     // returns point rotated 'a' radians ccw around the
          origin
f23  P rotate(double a) const {
482    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
902  friend ostream& operator<<(ostream& os, P p) {
9a9    return os << "(" << p.x << "," << p.y << ")"; }
d2d };
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

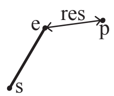


"Point.h" f6bf6b, 5 lines

```
7dc template<class P>
2ff double lineDist(const P& a, const P& b, const P& p) {
e07  return (double)(b-a).cross(p-a)/(b-a).dist();
008 }
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.



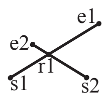
Usage: `Point<double> a, b(2,2), p(1,1);`
`bool onSegment = segDist(a,b,p) < 1e-10;`

"Point.h" 5c88f4, 7 lines

```
626 typedef Point<double> P;
929 double segDist(P& s, P& e, P& p) {
a44  if (s==e) return (p-s).dist();
f81  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)))
    ;
2c1  return ((p-s)*d-(e-s)*t).dist()/d;
ae7 }
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



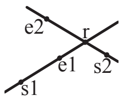
Usage: `vector<P> inter = segInter(s1,e1,s2,e2);`
`if (sz(inter)==1)`
`cout << "segments intersect at " << inter[0] << endl;`

"Point.h", "OnSegment.h" 9d57f2, 14 lines

```
dae template<class P> vector<P> segInter(P a, P b, P c, P d) {
0b6  auto oa = c.cross(d, a), ob = c.cross(d, b),
318       oc = a.cross(b, c), od = a.cross(b, d);
     // Checks if intersection is single non-endpoint point.
914  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
e5b    return {(a * ob - b * oa) / (ob - oa)};
4c1  set<P> s;
ccb  if (onSegment(c, d, a)) s.insert(a);
0ad  if (onSegment(c, d, b)) s.insert(b);
3d8  if (onSegment(a, b, c)) s.insert(c);
2fa  if (onSegment(a, b, d)) s.insert(d);
a35  return {all(s)};
9d5 }
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage: `auto res = lineInter(s1,e1,s2,e2);`
`if (res.first == 1)`
`cout << "intersection point at " << res.second << endl;`

"Point.h" a01f81, 9 lines

```
7dc template<class P>
0bf pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
14f  auto d = (e1 - s1).cross(e2 - s2);
8cc  if (d == 0) // if parallel
d99    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
f6b  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
9b8  return {1, (s1 * p + e1 * q) / d};
472 }
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h" 3af81c, 10 lines

```
7dc template<class P>
70b int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

7dc template<class P>
b5e int sideOf(const P& s, const P& e, const P& p, double eps)
     {
79e  auto a = (e-s).cross(p-s);
653  double l = (e-s).dist()*eps;
c32  return (a > l) - (a < -l);
33f }
```

OnSegment.h

Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

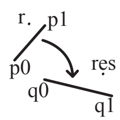
"Point.h" c597e8, 4 lines

```
514 template<class P> bool onSegment(P s, P e, P p) {
5fb  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
c59 }
```

linearTransformation.h

Description:

Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



"Point.h" 03a306, 7 lines

```
626 typedef Point<double> P;
664 P linearTransformation(const P& p0, const P& p1,
f06     const P& q0, const P& q1, const P& r) {
99f  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
0aa  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist
    2();
45e }
```

LineProjectionReflection.h

Description: Projects point p onto line ab. Set refl=true to get reflection of point p across line ab instead. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h" b5562d, 6 lines

```
7dc template<class P>
981 P lineProj(P a, P b, P p, bool refl=false) {
de3   P v = b - a;
3fc   return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
4b7 }
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage: `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
`int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }`
`// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i`

0f0602, 36 lines

```
755 struct Angle {
e91   int x, y;
8bd   int t;
5ac   Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
de8   Angle operator-(Angle b) const { return {x-b.x, y-b.y, t
      }; }
3cd   int half() const {
840     assert(x || y);
aa4     return y < 0 || (y == 0 && x < 0);
c93   }
dfc   Angle t90() const { return {-y, x, t + (half() && x >= 0)
      }; }
726   Angle t180() const { return {-x, -y, t + half()}; }
925   Angle t360() const { return {x, y, t + 1}; }
e25 };
a92 bool operator<(Angle a, Angle b) {
      // add a.dist2() and b.dist2() to also compare distances
ea7   return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
05f          make_tuple(b.t, b.half(), a.x * (ll)b.y);
ce5 }

    // Given two points, this calculates the smallest angle
        between
    // them, i.e., the angle that covers the defined line
        segment.
908 pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
ee4   if (b < a) swap(a, b);
423   return (b < a.t180() ?
c35           make_pair(a, b) : make_pair(b, a.t360()));
5ea }
784 Angle operator+(Angle a, Angle b) { // point a + vector b
eb1   Angle r(a.x + b.x, a.y + b.y, a.t);
8ca   if (a.t180() < r) r.t--;
d9f   return r.t180() < a ? r.t360() : r;
3d8 }
106 Angle angleDiff(Angle a, Angle b) { // angle b - angle a
125   int tu = b.t - a.t; a.t = b.t;
e63   return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a
      )};
ba3 }
```

HalfPlane.h

Description: Computes the intersection of a set of half-planes. Half-planes are sorted by angle and processed with a deque, removing redundant or conflicting constraints. Parallel half-planes are handled explicitly. Returns the convex polygon of the intersection, or empty if infeasible.

Time: $\mathcal{O}(n \log n)$

"Point.h" cf24a8, 72 lines

```
984 using ld = long double;
207 using P = Point<ld>;

533 struct Hp { // Half plane struct
      // 'p' is a passing point of the line and 'pq' is the
          direction vector of the line.
812   P p, pq;
d29   ld angle;

b93   Hp() {}
65d   Hp(const P& a, const P& b) : p(a), pq(b - a) {
0e3     angle = atan2l(pq.y, pq.x);
2ff   }
8ce   bool out(const P& r) { return pq.cross(r - p) < -eps; }
d36   bool operator < (const Hp& e) const {
1dd     return angle < e.angle;
44e   }
ea9   friend P inter(const Hp& s, const Hp& t) {
020     ld alpha = (t.p - s.p).cross(t.pq) / s.pq.cross(t.pq);
93b     return s.p + (s.pq * alpha);
825   }
b46 };

fa5 vector<P> hp_intersect(vector<Hp>& H) {
12f   P box[4] = { P(inf, inf), P(-inf, inf),
9c8                P(-inf, -inf), P(inf, -inf) };

1cd   for(int i = 0; i<4; i++) {
1a8     Hp aux(box[i], box[(i+1) % 4]);
d82     H.push_back(aux);
560   }
f1a   sort(all(H));
6c5   deque<Hp> dq;
486   int len = 0;
908   for(int i = 0; i < sz(H); i++) {
3fb     while(len>1 && H[i].out(inter(dq[len-1], dq[len-2]))){
c70       dq.pop_back();
654       --len;
a31     }
757     while (len > 1 && H[i].out(inter(dq[0], dq[1]))){
c68       dq.pop_front();
654       --len;
1eb     }
a5a     if(len && fabsl(H[i].pq.cross(dq[len-1].pq)) < eps){
25f       if (H[i].pq.dot(dq[len-1].pq) < 0.0)
282         return vector<P>();
e7b       if (H[i].out(dq[len-1].p)) {
c70         dq.pop_back();
654         --len;
2dc       }
64e       else continue;
9a0     }
fc2     dq.push_back(H[i]);
250     ++len;
8ed   }

337   while(len> 2 && dq[0].out(inter(dq[len-1], dq[len-2]))){
c70     dq.pop_back();
654     --len;
faa   }
81e   while (len > 2 && dq[len-1].out(inter(dq[0], dq[1]))){
c68     dq.pop_front();
654     --len;
694   }
1a3   if (len < 3) return vector<P>();
7e7   vector<P> ret(len);
cc7   for(int i = 0; i+1 < len; i++) {
01e     ret[i] = inter(dq[i], dq[i+1]);
00f   }
4fd   ret.back() = inter(dq[len-1], dq[0]);
edf   return ret;
deb }
```

8.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" ba7267, 12 lines

```
626 typedef Point<double> P;
27f bool circleInter(P a,P b,double r1,double r2,pair<P, P>*
    out) {
b48   if (a == b) { assert(r1 != r2); return false; }
f30   P vec = b - a;
6c8   double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2;
c28   double p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*
    d2;
5b0   if (sum*sum < d2 || dif*dif > d2) return false;
84d   P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) /
    d2);
21e   *out = {mid + per, mid - per};
8a6   return true;
170 }
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" b0153d, 14 lines

```
7dc template<class P>
3a5 vector<pair<P, P>> tangents(P c1, double r1, P c2, double
    r2) {
c0b   P d = c2 - c1;
432   double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
018   if (d2 == 0 || h2 < 0)  return {};
c14   vector<pair<P, P>> out;
092   for (double sign : {-1, 1}) {
2ad     P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
2e3     out.push_back({c1 + v * r1, c2 + v * r2});
e25   }
b21   if (h2 == 0) out.pop_back();
fe8   return out;
483 }
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" e0cfba, 10 lines

```
7dc template<class P>
195 vector<P> circleLine(P c, double r, P a, P b) {
33b   P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
55a   double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
3e4   if (h2 < 0) return {};
071   if (h2 == 0) return {p};
7cd   P h = ab.unit() * sqrt(h2);
d65   return {p - h, p + h};
59a }
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

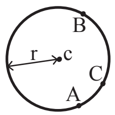
"../../content/geometry/Point.h" 19add1, 20 lines

```
626 typedef Point<double> P;
361 #define arg(p, q) atan2(p.cross(q), p.dot(q))
bb9 double circlePoly(P c, double r, vector<P> ps) {
6d1  auto tri = [&](P p, P q) {
c9c    auto r2 = r * r / 2;
291    P d = q - p;
127    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
eea    auto det = a * a - b;
691    if (det <= 0) return arg(p, q) * r2;
f43    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
aba    if (t < 0 || 1 <= s) return arg(p, q) * r2;
57f    P u = p + d * s, v = q + d * (t-1);
8c0    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
a52  };
bef  auto sum = 0.0;
8f4  rep(i,0,sz(ps))
3b7    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
e66  return sum;
f08 }
```

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" 1caa3a, 10 lines

```
626 typedef Point<double> P;
510 double ccRadius(const P& A, const P& B, const P& C) {
14b  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
f73      abs((B-A).cross(C-A))/2;
607 }
c0d P ccCenter(const P& A, const P& B, const P& C) {
28a  P b = C-A, c = B-A;
680  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
793 }
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h" 09dd0a, 18 lines

```
a28 pair<P, double> mec(vector<P> ps) {
4da  shuffle(all(ps), mt19937(time(0)));
f6a  P o = ps[0];
328  double r = 0, EPS = 1 + 1e-8;
2be  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
5cc    o = ps[i], r = 0;
4da    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
a30      o = (ps[i] + ps[j]) / 2;
6f7      r = (o - ps[i]).dist();
102      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
fa9        o = ccCenter(ps[i], ps[j], ps[k]);
6f7        r = (o - ps[i]).dist();
648      }
7b0    }
dcf  }
645  return {o, r};
09d }
```

8.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
Usage: vector<P> v = {P{4,4}, P{1,2}, P{2,1}};
bool in = inPolygon(v, P{3, 3}, false);
Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 2bf504, 12 lines

```
7dc template<class P>
0cc bool inPolygon(vector<P> &p, P a, bool strict = true) {
8b7  int cnt = 0, n = sz(p);
fea  rep(i,0,n) {
444    P q = p[(i + 1) % n];
cbd    if (onSegment(p[i], q, a)) return !strict;
       //or: if (segDist(p[i], q, a) <= eps) return !strict;
007    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
1b9  }
70a  return cnt;
c72 }
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" f12300, 7 lines

```
4fc template<class T>
a51 T polygonArea2(vector<Point<T>>& v) {
2f8  T a = v.back().cross(v[0]);
06e  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
3f5  return a;
693 }
```

PolygonCenter.h

Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

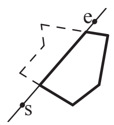
"Point.h" 9706dc, 10 lines

```
626 typedef Point<double> P;
6d9 P polygonCenter(const vector<P>& v) {
f9f  P res(0, 0); double A = 0;
70b  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
346    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
3ea    A += v[j].cross(v[i]);
307  }
33c  return res / A / 3;
0d0 }
```

PolygonCut.h

Description:

Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
Usage: vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));



"Point.h" d07181, 14 lines

```
626 typedef Point<double> P;
37d vector<P> polygonCut(const vector<P>& poly, P s, P e) {
fe2  vector<P> res;
d48  rep(i,0,sz(poly)) {
21c    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
c5f    auto a = s.cross(e, cur), b = s.cross(e, prev);
2dc    if ((a < 0) != (b < 0))
380      res.push_back(cur + (prev - cur) * (a / (a - b)));
c5c    if (a < 0)
a5f      res.push_back(cur);
757  }
b50  return res;
42c }
```

PolygonUnion.h

Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)
Time: $\mathcal{O}(N^2)$, where N is the total number of points

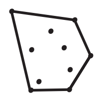
"Point.h", "sideOf.h" 3931c6, 34 lines

```
626 typedef Point<double> P;
142 double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y; }
61d double polyUnion(vector<vector<P>>& poly) {
499  double ret = 0;
9af  rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
9c8    P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
05c    vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
cbd    rep(j,0,sz(poly)) if (i != j) {
cc1      rep(u,0,sz(poly[j])) {
418        P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])];
688        int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
68b        if (sc != sd) {
295          double sa = C.cross(D, A), sb = C.cross(D, B);
e90          if (min(sc, sd) < 0)
dac            segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
cf7        } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))>0){
5b4          segs.emplace_back(rat(C - A, B - A), 1);
e96          segs.emplace_back(rat(D - A, B - A), -1);
313        }
0d1      }
fdc    }
861    sort(all(segs));
153    for (auto& s : segs) s.first = min(max(s.first, 0.0), 1.0);
68c    double sum = 0;
723    int cnt = segs[0].second;
067    rep(j,1,sz(segs)) {
081      if (!cnt) sum += segs[j].first - segs[j - 1].first;
6e9      cnt += segs[j].second;
f58    }
320    ret += A.cross(B) * sum;
191  }
ad6  return ret / 2;
6e8 }
```

ConvexHull.h

Description:

Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull. If you want to keep the collinear points in the convex hull, change the comparison to $h[t-2].cross(h[t-1], p) < 0$ and the size of the vector h to $2 * sz(pts) + 1$.
Time: $\mathcal{O}(n \log n)$



"Point.h" 310954, 14 lines

```
2c0 typedef Point<ll> P;
f16 vector<P> convexHull(vector<P> pts) {
f78  if (sz(pts) <= 1) return pts;
3cb  sort(all(pts));
abf  vector<P> h(sz(pts)+1);
573  int s = 0, t = 0;
628  for (int it = 2; it--; s = --t, reverse(all(pts)))
4eb    for (P p : pts) {
3da      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
f39      h[t++] = p;
bf0    }
```

```
036   return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
ec8 }
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).
Time: $\mathcal{O}(n)$

"Point.h" c571b8, 13 lines

```
2c0 typedef Point<ll> P;
d31 array<P, 2> hullDiameter(vector<P> S) {
e79   int n = sz(S), j = n < 2 ? 0 : 1;
354   pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
e4d   rep(i,0,j)
42e     for (;; j = (j + 1) % n) {
ca1       res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
be8       if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
c2b         break;
56c     }
3f2   return res.second;
5f7 }
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446b, 15 lines

```
2c0 typedef Point<ll> P;

2d4 bool inHull(const vector<P>& l, P p, bool strict = true) {
d44   int a = 1, b = sz(l) - 1, r = !strict;
5cc   if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
6bc   if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
456   if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
d1f     return false;
48a   while (abs(a - b) > 1) {
4f7     int c = (a + b) / 2;
ac8     (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
b26   }
06f   return sgn(l[a].cross(l[b], p)) < r;
c74 }
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log n)$

"Point.h" 7cf45b, 40 lines

```
530 #define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
f84 #define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
e7e template <class P> int extrVertex(vector<P>& poly, P dir) {
747   int n = sz(poly), lo = 0, hi = n;
fdf   if (extr(0)) return 0;
3d1   while (lo + 1 < hi) {
591     int m = (lo + hi) / 2;
855     if (extr(m)) return m;
c0c     int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
f48     (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
68a   }
253   return lo;
7f0 }

8e0 #define cmpL(i) sgn(a.cross(poly[i], b))
7dc template <class P>
ec4 array<int, 2> lineHull(P a, P b, vector<P>& poly) {
409   int endA = extrVertex(poly, (a - b).perp());
761   int endB = extrVertex(poly, (b - a).perp());
1a8   if (cmpL(endA) < 0 || cmpL(endB) > 0)
423     return {-1, -1};
649   array<int, 2> res;
f4b   rep(i,0,2) {
234     int lo = endB, hi = endA, n = sz(poly);
c2d     while ((lo + 1) % n != hi) {
57e       int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
7f6       (cmpL(m) == cmpL(endB) ? lo : hi) = m;
525     }
7dd     res[i] = (lo + !cmpL(hi)) % n;
356     swap(endA, endB);
c05   }
e00   if (res[0] == res[1]) return {res[0], -1};
3d1   if (!cmpL(res[0]) && !cmpL(res[1]))
959     switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
3f3       case 0: return {res[0], res[0]};
223       case 2: return {res[1], res[1]};
8fa     }
b50   return res;
36f }
```

Minkowski.h

Description: Computes the Minkowski sum of two convex polygons. Polygons must be convex and given in CCW order. Returns the vertices of the Minkowski sum polygon in CCW order.
Time: $\mathcal{O}(n+m)$

"Point.h" 664d67, 24 lines

```
780 using P = Point<ll>;

89f vector<P> minkowski(vector<P> p, vector<P> q) {
a8e   auto fix = [](vector<P>& A) {
bec     int pos = 0;
2bb     for (int i = 1; i < sz(A); i++) {
609       if(A[i].y < A[pos].y || (A[i].y == A[pos].y && A[i].x < A[pos].x))
e4c         pos = i;
f76     }
703     rotate(A.begin(), A.begin() + pos, A.end());
9e5     A.push_back(A[0]), A.push_back(A[1]);
236   };
889   fix(p), fix(q);
db6   vector<P> result;
692   int i = 0, j = 0;
98a   while (i < sz(p) - 2 || j < sz(q) - 2) {
942     result.push_back(p[i] + q[j]);
3bd     auto cross = (p[i + 1] - p[i]).cross(q[j + 1] - q[j]);
c3c     if (cross >= 0 && i < sz(p) - 2) i++;
f33     if (cross <= 0 && j < sz(q) - 2) j++;
801   }
dc8   return result;
2f9 }
```

Extreme.h

Description: Finds an extreme vertex of a convex polygon according to a unimodal comparator. The comparator defines a total order along the polygon (given in CCW order).
Time: $\mathcal{O}(\log n)$

"Point.h" 70b181, 26 lines

```
780 using P = Point<ll>;
c88 int extreme(vector<P> &pol, const function<bool(P, P)>& cmp) {
b1c   int n = pol.size();
4a2   auto extr = [&](int i, bool& cur_dir) {
22a     cur_dir = cmp(pol[(i+1)%n], pol[i]);
61a     return !cur_dir and !cmp(pol[(i+n-1)%n], pol[i]);
364   };
63d   bool last_dir, cur_dir;
a0d   if (extr(0, last_dir)) return 0;
993   int l = 0, r = n;
ead   while (l+1 < r) {
ee4     int m = (l+r)/2;
f29     if (extr(m, cur_dir)) return m;
44a     bool rel_dir = cmp(pol[m], pol[l]);
b18     if ((!last_dir and cur_dir) or
261         (last_dir == cur_dir and rel_dir == cur_dir)) {
8a6       l = m;
1f1       last_dir = cur_dir;
94a     } else r = m;
606   }
792   return l;
985 }
cad int max_dot(vector<P> &pol, P v) {
a98   return extreme([&](P p, P q) { return p.dot(v) > q.dot(v); });
27e }
```

Tangents.h

Description: Finds the left and right tangent points from an external point p to a convex polygon given in CCW order. A tangent point is a vertex where the segment p->v touches the polygon without intersecting its interior, defining the limits of visibility from p. Returns the indices of the left and right tangent vertices.
Time: $\mathcal{O}(\log n)$

"Point.h", "Extreme.h" dcf85f, 11 lines

```
780 using P = Point<ll>;

08d bool ccw(P p, P q, P r) {
274   return (q-p).cross(r-q) > 0;
0f3 }
826 pair<int, int> tangents(vector<P> &pol, P p) {
ae2     auto L = [&](P q, P r) { return ccw(p, r, q); };
98c     auto R = [&](P q, P r) { return ccw(p, q, r); };
861     return {extreme(pol, L), extreme(pol, R)};
3dc }
```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" ac41a6, 18 lines

```
2c0 typedef Point<ll> P;
24b pair<P, P> closest(vector<P> v) {
7f9   assert(sz(v) > 1);
7f7   set<P> S;
879   sort(all(v), [](P a, P b) { return a.y < b.y; });
571   pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
eec   int j = 0;
813   for (P p : v) {
3fb     P d{1 + (ll)sqrt(ret.first), 0};
```

```
8be     while (v[j].y <= p.y - d.x) S.erase(v[j++]);
a5a     auto lo = S.lower_bound(p - d), hi = S.upper_bound(p +
    d);
c77     for (; lo != hi; ++lo)
113       ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
8aa     S.insert(p);
5b0   }
70d   return ret.second;
bf2 }
```

ManhattanMST.h

Description: Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights w(p, q) = —p.x - q.x— + —p.y - q.y—. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
Time: $\mathcal{O}(N \log N)$

"Point.h" df6f59, 24 lines

```
bbe typedef Point<int> P;
ea9 vector<array<int, 3>> manhattanMST(vector<P> ps) {
850   vi id(sz(ps));
27c   iota(all(id), 0);
8c1   vector<array<int, 3>> edges;
8de   rep(k,0,4) {
1dd     sort(all(id), [&](int i, int j) {
02b          return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
702     map<int, int> sweep;
1e2     for (int i : id) {
84d       for (auto it = sweep.lower_bound(-ps[i].y);
904                 it != sweep.end(); sweep.erase(it++)) {
61d         int j = it->second;
6f3         P d = ps[i] - ps[j];
d18         if (d.y > d.x) break;
537         edges.push_back({d.y + d.x, i, j});
271       }
923       sweep[-ps[i].y] = i;
e69     }
4eb     for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p
    .y);
a11   }
da2   return edges;
a11 }
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" bac5b0, 64 lines

```
9a6 typedef long long T;
293 typedef Point<T> P;
305 const T INF = numeric_limits<T>::max();

173 bool on_x(const P& a, const P& b) { return a.x < b.x; }
0bd bool on_y(const P& a, const P& b) { return a.y < b.y; }

bf2 struct Node {
975   P pt; // if this is a leaf, the single point in it
877   T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
a23   Node *first = 0, *second = 0;

86a   T distance(const P& p) { // min squared distance to a
    point
28b     T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
88e     T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
d98     return (P(x,y) - p).dist2();
ca4   }

d97   Node(vector<P>&& vp) : pt(vp[0]) {
741     for (P p : vp) {
ad3       x0 = min(x0, p.x); x1 = max(x1, p.x);
e5d       y0 = min(y0, p.y); y1 = max(y1, p.y);
310     }
994     if (vp.size() > 1) {
          // split on x if width >= height (not ideal...)
9b7       sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
          // divide by taking half the array for each child (
            not
          // best performance with many duplicates in the
            middle)
0f9       int half = sz(vp)/2;
48e       first = new Node({vp.begin(), vp.begin() + half});
902       second = new Node({vp.begin() + half, vp.end()});
66e     }
204   }
a77 };

dad struct KDTree {
95f   Node* root;
c30   KDTree(const vector<P>& vp) : root(new Node({all(vp)}))
    {}

113   pair<T, P> search(Node *node, const P& p) {
ec4     if (!node->first) {
          // uncomment if we should not find the point itself:
          // if (p == node->pt) return {INF, P()};
47e       return make_pair((p - node->pt).dist2(), node->pt);
119     }

ea4     Node *f = node->first, *s = node->second;
d40     T bfirst = f->distance(p), bsec = s->distance(p);
a16     if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

        // search closest side first, other side if needed
86c     auto best = search(f, p);
314     if (bsec < best.first)
509       best = min(best, search(s, p));
f26     return best;
74c   }

      // find nearest point to a point, and its squared
        distance
      // (requires an arbitrary operator< for Point)
9b6   pair<T, P> nearest(const P& p) {
195     return search(root, p);
94c   }
6f5 };
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" eefdf5, 89 lines

```
2c0 typedef Point<ll> P;
806 typedef struct Quad* Q;
449 typedef __int128_t lll; // (can be ll if coords are < 2e4)
59b P arb(LLONG_MAX,LLONG_MAX); // not equal to any other
    point

070 struct Quad {
461   Q rot, o; P p = arb; bool mark;
b38   P& F() { return r()->p; }
23a   Q& r() { return rot->rot; }
f4f   Q prev() { return rot->o->rot; }
57e   Q next() { return r()->prev(); }
180 } *H;

d15 bool circ(P p, P a, P b, P c) { // is p in the
    circumcircle?
4b4   lll p2 = p.dist2(), A = a.dist2()-p2,
ffa       B = b.dist2()-p2, C = c.dist2()-p2;
59a   return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B >
    0;
6af }
00a Q makeEdge(P orig, P dest) {
bdf   Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
516   H = r->o; r->r()->r() = r;
2c3   rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->
    r();
ed2   r->p = orig; r->F() = dest;
4c1   return r;
b3b }
d8d void splice(Q a, Q b) {
686   swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
86c }
e92 Q connect(Q a, Q b) {
fc2   Q q = makeEdge(a->F(), b->p);
6e6   splice(q, a->next());
642   splice(q->r(), b);
bef   return q;
4a4 }

196 pair<Q,Q> rec(const vector<P>& s) {
e63   if (sz(s) <= 3) {
4a0     Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back()
    );
2ba     if (sz(s) == 2) return { a, a->r() };
19e     splice(a->r(), b);
5f8     auto side = s[0].cross(s[1], s[2]);
b9f     Q c = side ? connect(b, a) : 0;
3d8     return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
c9e   }

5ef #define H(e) e->F(), e->p
c98 #define valid(e) (e->F().cross(H(base)) > 0)
a3e   Q A, B, ra, rb;
f5e   int half = sz(s) / 2;
391   tie(ra, A) = rec({all(s) - half});
d9b   tie(B, rb) = rec({sz(s) - half + all(s)});
f80   while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
b08          (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
76d   Q base = connect(B->r(), A);
87f   if (A->p == ra->p) ra = base->r();
b58   if (B->p == rb->p) rb = base;

3e6 #define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
f02     while (circ(e->dir->F(), H(base), e->F())) { \
936       Q t = e->dir; \
6d3       splice(e, e->prev()); \
16e       splice(e->r(), e->r()->prev()); \
d47       e->o = H; H = e; e = t; \
a2e     }
1de   for (;;) {
eaa     DEL(LC, base->r(), o);  DEL(RC, base, prev());
6fa     if (!valid(LC) && !valid(RC)) break;
e09     if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
b74       base = connect(RC, base->r());
295     else
271       base = connect(base->r(), LC->r());
fcf   }
345   return { ra, rb };
7cf }

da1 vector<P> triangulate(vector<P> pts) {
af6   sort(all(pts));  assert(unique(all(pts)) == pts.end());
e00   if (sz(pts) < 2) return {};
```

```
235  Q e = rec(pts).first;
50c  vector<Q> q = {e};
6c1  int qi = 0;
7a5  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
806 #define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->
    p); \
43e  q.push_back(c->r()); c = c->next(); } while (c != e); }
9d6  ADD; pts.clear();
b58  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
a42  return pts;
a02 }
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 7 lines

```
f9c template<class V, class L>
cb3 double signedPolyVolume(const V& p, const L& trilist) {
9e8  double v = 0;
b72  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.
    c]);
fb8  return v / 6;
fca }
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 33 lines

```
f10 template<class T> struct Point3D {
f07  typedef Point3D P;
d0e  typedef const P& R;
329  T x, y, z;
cf2  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z)
    {}
803  bool operator<(R p) const {
8ee   return tie(x, y, z) < tie(p.x, p.y, p.z); }
236  bool operator==(R p) const {
bd6   return tie(x, y, z) == tie(p.x, p.y, p.z); }
9ae  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
54a  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
743  P operator*(T d) const { return P(x*d, y*d, z*d); }
17b  P operator/(T d) const { return P(x/d, y/d, z/d); }
e49  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
270  P cross(R p) const {
923   return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
a77  }
b70  T dist2() const { return x*x + y*y + z*z; }
18b  double dist() const { return sqrt((double)dist2()); }
     //Azimuthal angle (longitude) to x-axis in interval [-pi,
         pi]
3d6  double phi() const { return atan2(y, x); }
     //Zenith angle (latitude) to the z-axis in interval [0,
         pi]
0fa  double theta() const { return atan2(sqrt(x*x+y*y),z); }
55e  P unit() const { return *this/(T)dist(); } //makes dist()
    =1
     //returns unit vector normal to *this and p
685  P normal(P p) const { return cross(p).unit(); }
     //returns point rotated 'angle' radians ccw around axis
c67  P rotate(double angle, P axis) const {
7cd   double s = sin(angle), c = cos(angle); P u = axis.unit
    ();
6b7   return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
73a  }
805 };
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.

Time: $\mathcal{O}\left(n^2\right)$

"Point3D.h" 5b45fc, 50 lines

```
b8e typedef Point3D<double> P3;

9ce struct PR {
1fc  void ins(int x) { (a == -1 ? a : b) = x; }
82f  void rem(int x) { (a == x ? a : b) = -1; }
2ad  int cnt() { return (a != -1) + (b != -1); }
ba2  int a, b;
cf7 };

5e4 struct F { P3 q; int a, b, c; };

b6d vector<F> hull3d(const vector<P3>& A) {
cd9  assert(sz(A) >= 4);
ec1  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
394 #define E(x,y) E[f.x][f.y]
afe  vector<F> FS;
9e0  auto mf = [&](int i, int j, int k, int l) {
2ce   P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
fa1   if (q.dot(A[l]) > q.dot(A[i]))
eaa    q = q * -1;
f22   F f{q, i, j, k};
ee5   E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
471   FS.push_back(f);
d73  };
30c  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
047   mf(i, j, k, 6 - i - j - k);

3ef  rep(i,4,sz(A)) {
3b5   rep(j,0,sz(FS)) {
068    F f = FS[j];
04f    if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
412     E(a,b).rem(f.c);
b61     E(a,c).rem(f.b);
e5c     E(b,c).rem(f.a);
8d5     swap(FS[j--], FS.back());
eef     FS.pop_back();
5cd    }
220   }
97f   int nw = sz(FS);
c63   rep(j,0,nw) {
068    F f = FS[j];
561 #define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i,
    f.c);
3da    C(a, b, c); C(a, c, b); C(b, c, a);
248   }
472  }
864  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
770   A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
311  return FS;
be2 };
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f07, 9 lines

```
c5f double sphericalDistance(double f1, double t1,
3e8   double f2, double t2, double radius) {
284  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
277  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
c7e  double dz = cos(t2) - cos(t1);
c09  double d = sqrt(dx*dx + dy*dy + dz*dz);
154  return radius*2*asin(d/2);
4fa }
```

Strings (9)

AhoCorasick.h

95b3e7, 46 lines

```
c2e int trie[ms][sigma], fail[ms], terminal[ms], superfail[ms
    ];
1e1 bool present[ms];
965 int z = 1;

ca3 int val(char c) { return c - 'a'; }

f97 void add(string& p) {
b3d     int cur = 0;
b4b     for (int i = 0; i < (int)p.size(); i++) {
9e4         int& nxt = trie[cur][val(p[i])];
b6e         if (nxt == 0) nxt = z++;
1bc         cur = nxt;
a92     }
c0e     present[cur] = true;
b07     terminal[cur]++;
6aa }

0a8 void build() {
26a     queue<int> q;
f47     for (q.push(0); !q.empty(); q.pop()) {
fb5         int on = q.front();
0b2         for (int i = 0; i < sigma; i++) {
df1             int& to = trie[on][i];
279             int f = (on == 0 ? 0 : trie[fail[on]][i]);
de7             int sf = (present[f] ? f : superfail[f]);
24d             if (!to) {
c4e                 to = f;
6fd             }
4e6             else {
3ef                 fail[to] = f;
b86                 superfail[to] = sf;
                    // merge infos (ex: terminal[to] +=
                        terminal[f])
91b                 q.push(to);
692             }
bff         }
e61     }
b89 }

54e void search(string& s) {
b3d     int cur = 0;
b4f     for (char c : s) {
3ba         cur = trie[cur][val(c)];
            // process infos on current node (ex: ocurrences
                += terminal[cur])
5ac     }
d1b }
```

Hash.h

Description: C can also be random, operator is $[l, r]$

79e7f5, 28 lines

```
541 using ull = uint64_t;
54d struct H {
858     ull x; H(ull x = 0) : x(x) {}
c9b     H operator+(H o) { return x + o.x + (x + o.x < x); }
5cd     H operator-(H o) { return *this + ~o.x; }
```

```
167     H operator*(H o) {
2f3         auto m = (__uint128_t)x * o.x;
540         return H((ull)m) + (ull)(m >> 64);
681     }
bf2     ull get() const { return x + !~x; }
03c     bool operator==(H o) const{ return get() == o.get();}
0ab     bool operator<(H o) const{ return get() < o.get();}
bf6 };
862 static const H C = (ll)1e11 + 3;
61c struct Hash {
2f2     vector<H> h, pw;
1df     Hash(string& str) : h(str.size()), pw(str.size()) {
9bc         pw[0] = 1, h[0] = str[0];
1c5         for (int i = 1; i < str.size(); i++) {
90a             h[i] = h[i - 1] * C + str[i];
b3c             pw[i] = pw[i - 1] * C;
57e         }
f1b     }
75e     H operator()(int l, int r) {
91f         return h[r] - (l ? h[l - 1] * pw[r - l + 1] : 0);
9cf     }
c36 };
```

KMP.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123).

c7cf15, 10 lines

```
a56 vector<int> pi(const string& s) {
627     vector<int> p(sz(s));
edb     for(int i = 1; i < sz(s); i++) {
052         int g = p[i-1];
6c0         while (g && s[i] != s[g]) g = p[g-1];
7cf         p[i] = g + (s[i] == s[g]);
a2e     }
74e     return p;
c7c }
```

KmpAutomaton.h

Description: $go[i][j]$ = length of the longest prefix of s that is a suffix of $s[0..i]$ followed by the letter j (i.e., the next matched prefix length if, at state i, we read letter j).

8833cb, 17 lines

```
ab6 int go[ms][sigma];
ca3 int val(char c) { return c - 'a'; }
8cf void automaton(string& s) {
3cc     for (int i = 0; i < sigma; i++)
48d         go[0][i] = (i == val(s[0]));

8cc     for (int i = 1, bdr = 0; i <= (int)s.size(); i++) {
782         for (int j = 0; j < sigma; j++) {
6ef             go[i][j] = go[bdr][j];
87c         }
f8d         if (i < (int)s.size()) {
02f             go[i][val(s[i])] = i + 1;
364             bdr = go[bdr][val(s[i])];
63b         }
d7e     }
0c5 }
```

Manacher.h

Description: $p[0][i+1]$ is the length of matches of even length palindrome, starting from $[i, i+1]$.
$p[1][i]$ is the length of matches of odd length palindrome, starting from $[i, i]$.
(abaxx -> p[0] = 00001)
(abaxx -> p[1] = 01000)

e7ad79, 14 lines

```
fc1 array<vi, 2> manacher(const string& s) {
f89  int n = sz(s);
f77  array<vi,2> p = {vi(n+1), vi(n)};
c9a  rep(z,0,2) for (int i=0,l=0,r=0; i<n; i++) {
24e    int t = r-i+!z;
e70    if (i<r) p[z][i] = min(t, p[z][l+t]);
fff    int L = i-p[z][i], R = i+p[z][i]-!z;
649    while (L>=1 && R+1<n && s[L-1] == s[R+1])
895      p[z][i]++, L--, R++;
f28    if (R>r) l=L, r=R;
a84  }
74e  return p;
e7a }
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(s.begin(), s.begin()+minRotation(s), s.end());`
Time: $\mathcal{O}(N)$

d07a42, 10 lines

```
5fa int minRotation(string s) {
c6c  int a=0, N=sz(s); s += s;
840  rep(b,0,N) rep(k,0,N) {
32f    if (a+k == b || s[a+k] < s[b+k]) {
873              b += max(0, k-1); break;}
068    if (s[a+k] > s[b+k]) { a = b; break; }
937  }
3f5  return a;
d07 }
```

SuffixArray.h

Description: $lcp[i]$ is the length of the longest common prefix between the suffixes $s[sa[i]..n-1]$ and $s[sa[i-1]..n-1]$.
If we concatenate multiple strings using separator characters, the separator that appears furthest to the right must be the smallest character in the alphabet.

048424, 31 lines

```
3f4 struct SuffixArray {
716  vector<int> sa, lcp;
d91  SuffixArray(string s, int lim=256) {
59b    s.push_back('$');
323    int n = sz(s), k = 0, a, b;
9f1    vector<int> x(all(s)), y(n), ws(max(n, lim));
af4    sa = lcp = y, iota(all(sa), 0);
25d    for(int j = 0, p = 0; p < n; j= max(1, j*2), lim = p) {
3cd      p = j, iota(all(y), n - j);
603      for(int i=0; i<n; i++){
071        if (sa[i] >= j) y[p++] = sa[i] - j;
cb4      }
911      fill(all(ws), 0);
483      for(int i=0; i<n; i++) ws[x[i]]++;
5d9      for(int i=1; i<lim; i++) ws[i] += ws[i - 1];
a9e      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
c7d      swap(x, y), p = 1, x[sa[0]] = 0;
6f5      for(int i=1; i<n; i++){
93f        a = sa[i - 1], b = sa[i];
ddb        x[b] = p-1;
a32        if(y[a] != y[b] || y[a+j] != y[b+j]) x[b] = p++;
1ba      }
c36    }
65b    for (int i = 0, j; i < n - 1; lcp[x[i++]] = k)
904      for (k && k--, j = sa[x[i] - 1];
262          s[i + k] == s[j + k]; k++);
68a    sa = vector<int>(sa.begin() + 1, sa.end());
5d4    lcp = vector<int>(lcp.begin() + 1, lcp.end());
4db  }
048 };
```

Zfunc.h

Description: z[i] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)

495392, 13 lines

```
572 vector<int> ZFunc(const string& s) {
d6b     int n = sz(s), a = 0, b = 0;
2b1     vector<int> z(n, 0);
29a     if (!z.empty()) z[0] = 0;
6f5     for (int i = 1; i < n; i++) {
fe0         int end = i;
98f         if (i < b) end = min(i + z[i - a], b);
65f         while (end < n && s[end] == s[end - i]) ++end;
816         z[i] = end - i; if (end > b) a = i, b = end;
253     }
070     return z;
495 }
```

Various (10)

10.1 Misc. algorithms

Dates.h

Description: dateToInt converts Gregorian date to integer (Julian day number). intToDate converts integer (Julian day number) to Gregorian date: month/day/year. intToDay converts Julian day number to day of the week

688e56, 23 lines

```
37c string day[] = { "Mon", "Tue", "Wed", "Thu", "Fri", "Sat",
     "Sun" };
fb9 int dateToInt(int m, int d, int y) {
e70     return
773         1461 * (y + 4800 + (m - 14) / 12) / 4 +
649         367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
fa0         3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
3aa         d - 32075;
a73 }
3fe void intToDate(int jd, int& m, int& d, int& y) {
ee1     int x, n, i, j;
33a     x = jd + 68569;
403     n = 4 * x / 146097;
33e     x -= (146097 * n + 3) / 4;
6fc     i = (4000 * (x + 1)) / 1461001;
b1d     x -= 1461 * i / 4 - 31;
fc9     j = 80 * x / 2447;
c8d     d = x - 2447 * j / 80;
179     x = j / 11;
335     m = j + 2 - 12 * x;
23d     y = 100 * (n - 49) + i + x;
cbb }
04e string intToDay(int jd) { return day[jd % 7]; }
```

MultisetHash.h

5648da, 8 lines

```
cdc ull hashify(ull sum) {
7b8     sum += FIXED_RANDOM;
6ec     sum += 0x9e3779b97f4a7c15;
dc6     sum = (sum ^ (sum >> 30)) * 0xbf58476d1ce4e5b9;
005     sum = (sum ^ (sum >> 27)) * 0x94d049bb133111eb;
358     return sum ^ (sum >> 31);
564 }
```

Rand.h

2de3f8, 8 lines

```
c8a mt19937 rng(chrono::steady_clock::now().time_since_epoch()
     .count());
     // _64

463 int uniform(int l, int r) { // [l, r]
a7f     uniform_int_distribution<int> uid(l, r);
f54     return uid(rng);
d9e }
```

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$

edce47, 24 lines

```
d91 set<pii>::iterator addInterval(set<pii>& is, int L, int R)
    {
bb3  if (L == R) return is.end();
d4c  auto it = is.lower_bound({L, R}), before = it;
dc6  while (it != is.end() && it->first <= R) {
164    R = max(R, it->second);
1a5    before = it = is.erase(it);
fe9  }
1af  if (it != is.begin() && (--it)->second >= L) {
3ca    L = min(L, it->first);
164    R = max(R, it->second);
861    is.erase(it);
0de  }
aa0  return is.insert(before, {L,R});
d57 }

675 void removeInterval(set<pii>& is, int L, int R) {
17b  if (L == R) return;
bef  auto it = addInterval(is, L, R);
e14  auto r2 = it->second;
655  if (it->first == L) is.erase(it);
016  else (int&)it->second = L;
ee9  if (R != r2) is.emplace(R, r2);
059 }
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$

9e9d8d, 20 lines

```
4fc template<class T>
dbe vi cover(pair<T, T> G, vector<pair<T, T>> I) {
3d5  vi S(sz(I)), R;
d00  iota(all(S), 0);
591  sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
d10  T cur = G.first;
05e  int at = 0;
336  while (cur < G.second) { // (A)
438    pair<T, int> mx = make_pair(cur, -1);
f07    while (at < sz(I) && I[S[at]].first <= cur) {
032      mx = max(mx, make_pair(I[S[at]].second, S[at]));
69a      at++;
c42    }
c54    if (mx.second == -1) return {};
953    cur = mx.first;
fbf    R.push_back(mx.second);
dd1  }
b1a  return R;
b8d }
```

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \dots < f(i) \geq \dots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the < marked with (A) to <=, and reverse the loop at (B). To minimize f, change it to >, also at (B).
Usage: `int ind = ternSearch(0,n-1);`
Time: $\mathcal{O}(\log(b-a))$

a995fb, 11 lines

```
53a int ternSearch(int a, int b) {
25b  assert(a <= b);
329  while (b - a >= 5) {
924    int mid = (a + b) / 2;
c9e    if (f(mid) < f(mid+1)) a = mid; // (A)
ceb    else b = mid+1;
ce7  }
95e  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
3f5  return a;
a99 }
```

10.2 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: $dp[i][j] = \min_{i<k<j}(dp[i][k] + dp[k][j]) + f(i, j)$, where the (minimal) optimal k increases with both i and j. This is known as Knuth DP. Sufficient criteria for this are if $f(b, c) \leq f(a, d)$ and $f(a, c) + f(b, d) \leq f(a, d) + f(b, c)$ for all $a \leq b \leq c \leq d$. Another sufficient criteria is: $opt[i][j-1] \leq opt[i][j] \leq opt[i+1][j]$
Time: $\mathcal{O}(N^2)$

fea016, 22 lines

```
7cc ll knuth(){
6a7     memset(opt, -1, sizeof opt);
45b     for(int i=n-1; i>=0; i--){
8e7         dp[i][i] = 0; // base case
b28         opt[i][i] = i;
94f         for(int j=i+1; j<n; j++){
2e2             int optL = (!j ? 0 : opt[i][j-1]);
dc4             int optR = (~opt[i+1][j] ? opt[i+1][j] : n-1);
554             ll cst = cost(i, j);
f12             dp[i][j] = INF;
3bb             optL = max(i, optL), optR = min(j-1, optR);
349             for(int k=optL; k<=optR; k++){
f8b                 ll now = dp[i][k] + dp[k+1][j] + cst;
e83                 if(now <= dp[i][j]){
960                     dp[i][j] = now;
14d                     opt[i][j] = k;
5fc                 }
114             }
4ce         }
96c     }
fea }
```

DivideAndConquerDP.h

Description: Divide and Conquer DP maintaining cost, can be used when $opt[i][j] \leq opt[i][j+1]$. In this code everything is 1-based. Memory can be optmized by keeping only the last row
Time: $\mathcal{O}(MN \log N)$

c7cb38, 42 lines

```
129 void add(int idx) {}
404 void rem(int idx) {}

749 void deC(int i, int l, int r, int optL, int optR) {
de6     if (l > r) return;
995     int j = (l + r) / 2;
d9a     for (int k = r; k > j; k--) rem(k);
c45     int opt = optL;
364     for (int k = optL; k <= min(optR, j); k++) {
            // cost = cost[k, j]
597         int val = dp[i - 1][k - 1] + cost;
532         if (val < dp[i][j]) {
482             dp[i][j] = val;
613             opt = k;
178         }
183         rem(k);
93f     }
5d9     for (int k = min(optR, j); k >= optL; k--) add(k);
446     rem(j);
ace     deC(i, l, j - 1, optL, opt);

ebd     for (int k = j; k <= r; k++) add(k);
648     for (int k = optL; k < opt; k++) rem(k);
0b6     deC(i, j + 1, r, opt, optR);
9bb     for (int k = optL; k < opt; k++) add(k);
460 }

d57 int solve(int N, int M) { // 1-based
d9f     for (int i = 0; i <= M; i++) {
138         for (int j = 0; j <= N; j++){
3db             dp[i][j] = inf; // base case
a26         }
e0f     }
c21     cost = 0; // neutral value
c62     for (int i = 1; i <= N; i++) add(i);
143     for (int i = 1; i <= M; i++) {
156         deC(i, 1, N, 1, N);
c97     }
01a     return dp[M][N];
3ab }
```