Universidade Federal de Pernambuco

# las4s e pelados

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# Contest (1)

template.cpp 8 lines

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
using ll = long long;
using pii = pair<int,int>;
```

.bashrc 2 lines

```
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \
  -fsanitize=undefined,address'
```

hash.sh 2 lines

```
# bash hash.sh file.cpp l1 l2
sed -n $2','$3' p' $1 | sed '/^#w/d' | cpp -dD -P -
    fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

troubleshoot.txt 52 lines

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
```

# Mathematics (2)

## 2.1 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \cdots - c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots $r$ become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

## 2.2 Trigonometry

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where $V, W$ are lengths of sides opposite angles $v, w$.

$$a\cos x + b\sin x = r\cos(x-\phi)$$
$$a\sin x + b\cos x = r\sin(x+\phi)$$

where $r = \sqrt{a^2+b^2}, \phi = \operatorname{atan2}(b, a)$.

## 2.3 Geometry

### 2.3.1 Triangles

Side lengths: $a, b, c$

Semiperimeter: $p = \dfrac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1 - \left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

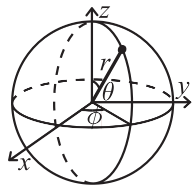
### 2.3.2 Quadrilaterals

With side lengths $a, b, c, d$, diagonals $e, f$, diagonals angle $\theta$, area $A$ and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180^\circ$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

### 2.3.3 Spherical coordinates




$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \operatorname{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z &= r\cos\theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

### 2.3.4 Pick's Theorem

The area of a simple polygon whose vertices have integer coordinates is:

$$A = I + \frac{B}{2} - 1$$

where $I$ is the number of interior integer points, and $B$ is the number of integer points in the border of the polygon.

### 2.3.5 Two Ears Theorem

Every simple polygon with more than 3 vertices has at least two non-overlapping ears (a ear is a vertex whose diagonal induced by its neighbors which lies strictly inside the polygon). Equivalently, every simple polygon can be triangulated.

## 2.4 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan x = 1+\tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\operatorname{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

## 2.5 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1}-c^a}{c-1},\ c \neq 1$$

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$\sum_{i=0}^{n} ic^i = \frac{nc^{n+2}-(n+1)c^{n+1}+c}{(c-1)^2},\ c \neq 1$$

$$g_k(n) = \sum_{i=1}^{n} i^k = \frac{1}{k+1}\left(n^{k+1} + \sum_{j=1}^{k}\binom{k+1}{j+1}(-1)^{j+1}g_{k-j}(n)\right)$$

## 2.6 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots,\ (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots,\ (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots,\ (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots,\ (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots,\ (-\infty < x < \infty)$$

$$\sum_{i=0}^{\infty} ic^i = \frac{c}{(1-c)^2}, \quad |c| < 1$$

## 2.7 Probability theory

Let $X$ be a discrete random variable with probability $p_X(x)$ of assuming the value $x$. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x-\mathbb{E}(X))^2 p_X(x)$ where $\sigma$ is the standard deviation. If $X$ is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX+bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent $X$ and $Y$,

$$V(aX+bY) = a^2V(X) + b^2V(Y).$$

### 2.7.1 Discrete distributions

#### Binomial distribution

The number of successes in $n$ independent yes/no experiments, each which yields success with probability $p$ is $\mathrm{Bin}(n,p)$, $n = 1, 2, \ldots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n,p)$ is approximately $\mathrm{Po}(np)$ for small $p$.

#### First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability $p$ is $\mathrm{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

#### Poisson distribution

The number of events occurring in a fixed period of time $t$ if these events occur with a known average rate $\kappa$ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

# Data structures (3)

Bit.h

**Description:** *lower_bound* works the same as on vectors

**Time:** $\mathcal{O}(\log N)$

d41d8c, 23 lines

```
d41 struct Bit {
d41   vector<ll> bit;
d41   Bit(int n) : bit(n + 1) {}
d41   void update(int i, ll v) {
d41     for (i++; i < sz(bit); i += i & -i) bit[i] += v;
d41   }
d41   ll query(int i) {
d41     ll ret = 0;
d41     for (i++; i > 0; i -= i & -i) ret += bit[i];
d41     return ret;
d41   }
d41   int lower_bound(ll v){ // min pos st sum[0, pos] >= v
d41     int pos = 0;
d41     for(int j=(1 << 23); j >= 1; j/=2){
d41       if(pos+j < sz(bit) && bit[pos + j] < v){
d41         pos += j;
d41         v -= bit[pos];
d41       }
d41     }
d41     return pos;
d41   }
d41 };
```

Bit2d.h

**Description:** Points called on the update function NEED to be on the *pts* vector parameter on build.

**Time:** $\mathcal{O}\left((\log N)^2\right)$

"Bit.h" d41d8c, 37 lines

```
d41 struct Bit2d {
d41   vector<vector<int>> ys;
d41   vector<Bit> bit;
d41   vector<int> cmp_x;
d41   Bit2d(){}
d41   void put(int x, int y) {
d41     for (x++; x < sz(ys); x += x & -x) ys[x].push_back(y);
d41   }
d41   int id(const vector<int> &v, int y) {
d41     return (upper_bound(all(v), y) - v.begin()) - 1;
d41   }
d41   void build(vector<pii> pts) {
d41     sort(all(pts));
d41     for(auto p : pts) cmp_x.push_back(p.first);
d41     cmp_x.erase(unique(all(cmp_x)), cmp_x.end());
d41     ys.resize(cmp_x.size() + 1);
d41     for(auto p : pts) put(id(cmp_x, p.first), p.second);
d41     for(auto &v:ys)sort(all(v)), bit.emplace_back(sz(v));
d41   }
d41   void update(int x, int y, int val){
d41     x = id(cmp_x, x);
```

```
d41      for(x++; x < sz(ys); x+= x&-x)
d41        bit[x].update(id(ys[x], y), val);
d41    }
d41    int query(int x, int y){
d41      x = id(cmp_x, x);
d41      int ret = 0;
d41      for(x++; x > 0; x-= x&-x)
d41        ret += bit[x].query(id(ys[x], y));
d41      return ret;
d41    }
d41    int query(int x1, int y1, int x2, int y2){
d41      int a = query(x2, y2)-query(x2, y1-1);
d41      return a-query(x1-1, y2)+query(x1-1, y1-1);
d41    }
d41  };
```

## LineContainer.h

**Description:** Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
**Time:** $\mathcal{O}(\log N)$

d41d8c, 32 lines

```
d41  struct Line {
d41   mutable ll k, m, p;
d41   bool operator<(const Line& o) const { return k < o.k; }
d41   bool operator<(ll x) const { return p < x; }
d41  };

d41  struct LineContainer : multiset<Line, less<>> {
       // (for doubles, use inf = 1/.0, div(a,b) = a/b)
d41   static const ll inf = LLONG_MAX;
d41   ll div(ll a, ll b) { // floored division
d41     return a / b - ((a ^ b) < 0 && a % b); }
d41   bool isect(iterator x, iterator y) {
d41     if (y == end()) return x->p = inf, 0;
d41     if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
d41     else x->p = div(y->m - x->m, x->k - y->k);
d41     return x->p >= y->p;
d41   }
d41   void add(ll k, ll m) {
d41     auto z = insert({k, m, 0}), y = z++, x = y;
d41     while (isect(y, z)) z = erase(z);
d41     if (x != begin() && isect(--x, y))
d41       isect(x, y = erase(y));
d41     while ((y = x) != begin() && (--x)->p >= y->p)
d41       isect(x, erase(y));
d41   }
d41   ll query(ll x) {
d41     assert(!empty());
d41     auto l = *lower_bound(x);
d41     return l.k * x + l.m;
d41   }
d41  };
```

## Mo.h

**Description:** For subtree queries, perform an Euler tour and map each node u to the interval $[tin[u], tin[u] + subtree\_size[u] - 1]$. A subtree query becomes a range query over this interval.
For path queries between nodes U and V, Let U be the closest to the root. If V lies in U's subtree, the path corresponds to the interval $[tin[U], tin[V]]$. Otherwise, the path corresponds to the interval $[min(tout[U], tout[V]), max(tin[U], tin[V])]$.
In both cases, nodes on the U–V path appear exactly once in the interval, while all other nodes appear either 0 or 2 times.
**Usage:** `queries.push(Query(l, r, index of query))`, intervals are `[l, r]`
**Time:** $\mathcal{O}\left(N\sqrt(Q)\right)$

d41d8c, 44 lines

```
d41  inline int64_t hilOrd(int x, int y, int pow, int rot) {
d41    if (pow == 0) return 0;
d41    int hpow = 1 << (pow - 1);
d41    int seg = (x < hpow) ? ((y < hpow) ? 0 : 3) : ((y < hpow
     ) ? 1 : 2);
d41    seg = (seg + rot) & 3;
d41    const int rotDelta[4] = { 3, 0, 0, 1 };
d41    int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
d41    int nrot = (rot + rotDelta[seg]) & 3;
d41    int64_t sub = int64_t(1) << (2 * pow - 2);
d41    int64_t ans = seg * sub;
d41    int64_t add = hilOrd(nx, ny, pow - 1, nrot);
d41    ans += (seg == 1 || seg == 2) ? add : (sub - add - 1);
d41    return ans;
d41  }

d41  struct Query {
d41    int l, r, idx;
d41    int64_t ord;
d41    Query(int l, int r, int idx) : l(l), r(r), idx(idx) {
d41      ord = hilOrd(l, r, 21, 0);
d41    }
d41    bool operator < (const Query& other) const {
d41      return ord < other.ord;
d41    }
d41  };

d41  vector<Query> queries;
d41  int ans[ms];
d41  void put(int x) {} // F
d41  void remove(int x) {} // F
d41  int getAns() {}

d41  void Mo() {
d41    int l = 0, r = -1;
d41    sort(queries.begin(), queries.end());
d41    for (Query q : queries) {
d41      while (l > q.l) put(--l);
d41      while (r < q.r) put(++r);
d41      while (l < q.l) remove(l++);
d41      while (r > q.r) remove(r--);
d41      ans[q.idx] = getAns();
d41    }
d41  }
```

## MoUpdate.h

**Description:** Block size should be around $(2 * N * N)^{\frac{1}{3}}$
**Usage:** `intervals are [l, r], addQuery(l, r, number of updates happened before this query, index of query), addUpdate(index of updated position, value before update, value after update)`
**Time:** $\mathcal{O}\left(Q * (2 * N * N)^{\frac{1}{3}} * F\right)$

d41d8c, 55 lines

```
d41  const int B = 2700;
d41  struct MoUpdate {
d41    struct Query {
d41      int l, r, t, idx;
d41      Query(int l, int r, int t, int idx)
d41        : l(l), r(r), t(t), idx(idx) {}
d41      bool operator < (const Query& p) const {
d41        if (l / B != p.l / B) return l < p.l;
d41        if (r / B != p.r / B) return r < p.r;
d41        return t < p.t;
d41      }
d41    };
d41    struct Upd {
d41      int i, old, now;
d41      Upd(int i, int old, int now): i(i),old(old),now(now){}
d41    };
d41    vector<Query> queries;
d41    vector<Upd> updates;

d41    void addQuery(int l, int r, int t, int idx) {
d41      queries.push_back(Query(l, r, t, idx)); }
d41    void addUpdate(int i, int old, int now) {
d41      updates.push_back(Upd(i, old, now)); }

d41    void add(int x) {} // F
d41    void rem(int x) {} // F
d41    int getAns() {}
d41    void update(int novo, int idx, int l, int r) {
d41      if (l <= idx && idx <= r) rem(idx);
d41      arr[idx] = novo;
d41      if (l <= idx && idx <= r) add(idx);
d41    }

d41    void solve() {
d41      int l = 0, r = -1, t = 0;
d41      sort(queries.begin(), queries.end());
d41      for (Query q : queries) {
d41        while (l > q.l) add(--l);
d41        while (r < q.r) add(++r);
d41        while (l < q.l) rem(l++);
d41        while (r > q.r) rem(r--);
d41        while (t < q.t) {
d41          auto u = updates[t++];
d41          update(u.now, u.i, l, r);
d41        }
d41        while (t > q.t) {
d41          auto u = updates[--t];
d41          update(u.old, u.i, l, r);
d41        }
d41        ans[q.idx] = getAns();
d41      }
d41    }
d41  };
```

## SegmentTree.h

**Description:** Zero-indexed max-tree. Bounds are inclusive to the left and inclusive to the right. Can be changed by modifying T, f and unit.
**Time:** $\mathcal{O}(\log N)$

d41d8c, 21 lines

```
d41  struct Tree {
d41   typedef int T;
d41   static constexpr T unit = INT_MIN;
d41   T f(T a, T b) { return max(a, b); } // (any associative
     fn)
d41   vector<T> s; int n;
d41   Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {}
d41   void update(int pos, T val) {
d41     for (s[pos += n] = val; pos /= 2;)
d41       s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
d41   }
d41   T query(int b, int e) { // query [b, e]
d41     e++;
d41     T ra = unit, rb = unit;
d41     for (b += n, e += n; b < e; b /= 2, e /= 2) {
d41       if (b % 2) ra = f(ra, s[b++]);
d41       if (e % 2) rb = f(s[--e], rb);
d41     }
d41     return f(ra, rb);
d41   }
d41  };
```

### OrderStatisticTree.h

**Description:** A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
**Time:** $\mathcal{O}(\log N)$

d41d8c, 17 lines

```
d41 #include <bits/extc++.h>
d41 using namespace __gnu_pbds;

d41 template<class T>
d41 using Tree = tree<T, null_type, less<T>, rb_tree_tag,
d41     tree_order_statistics_node_update>;

d41 void example() {
d41  Tree<int> t, t2; t.insert(8);
d41  auto it = t.insert(10).first;
d41  assert(it == t.lower_bound(9));
d41  assert(t.order_of_key(10) == 1);
d41  assert(t.order_of_key(11) == 2);
d41  assert(*t.find_by_order(0) == 8);
d41  t.join(t2); // merge t2 into t
d41 }
```

### PersistentSegTree.h

**Usage:** `SegP(size of the segtree, number of updates)`

```
roots = {0}, newRoot = update(roots.back(),...),
roots.push(newRoot)
```

d41d8c, 42 lines

```
d41 struct SegP {
d41   static constexpr ll neut = 0;
d41   struct Node {
d41     ll v; // start with neutral value
d41     int l, r;
d41     Node(ll v=neut, int l=0, int r=0) : v(v), l(l), r(r){}
d41   };
d41   vector<Node> seg;
d41   int n, CNT;
d41   SegP(int _n, int upd): seg(20*(upd+_n)), n(_n), CNT(1){}
d41   ll merge(ll a, ll b) { return a + b; }
d41   int update(int root, int pos, int val, int l, int r) {
d41     int p = CNT++;
d41     seg[p] = seg[root];
d41     if (l == r) {
d41       seg[p].v += val;
d41       return p;
d41     }
d41     int mid = (l + r) / 2;
d41     if (pos <= mid){
d41       seg[p].l = update(seg[p].l, pos, val, l, mid);
d41     }else seg[p].r = update(seg[p].r,pos,val,mid+1,r);

d41     seg[p].v=merge(seg[seg[p].l].v, seg[seg[p].r].v);
d41     return p;
d41   }
d41   int query(int p, int L, int R, int l, int r) {
d41     if (l > R || r < L) return neut;
d41     if (L <= l && r <= R) return seg[p].v;
d41     int mid = (l + r) / 2;
d41     int left = query(seg[p].l, L, R, l, mid);
d41     int right = query(seg[p].r, L, R, mid + 1, r);
d41     return merge(left, right);
d41   }
d41   int update(int root, int pos, int val) {
d41     return update(root, pos, val, 0, n - 1);
d41   }
d41   int query(int root, int L, int R) {
d41     return query(root, L, R, 0, n - 1);
d41   }
d41 };
```

### SegBeats.h

**Description:** In Segment Tree Beats, 'lazy' does NOT mean "updates still missing here". The node already reflects all previous updates. Instead, 'lazy' stores what must be propagated to the children before recursing. Always call 'apply(l,r,p)' before descending. This node layout supports range add, range chmin and range chmax operations. Beats conditions:
break: MIN x: mx1 <= x ; MAX x: mi1 >= x
tag: MIN x: x > mx2 ; MAX x: x < mi2
**Time:** amortized $\mathcal{O}(\log^2 N)$, without range add $\mathcal{O}(\log N)$

d41d8c, 47 lines

```
d41 struct node{
d41   ll mx1, mx2, sum, lazy;
d41   ll mi1, mi2;
d41   int cMax, cMin, tam;
d41   node(int x=0) : mx1(x),mx2(-inf),mi1(x),mi2(inf),
d41             cMax(1),cMin(1),tam(1),sum(x),lazy(0){}
d41   node(node a, node b){
d41     sum = a.sum+b.sum, tam = a.tam+b.tam;
d41     lazy = 0;
d41     mx1 = max(a.mx1, b.mx1);
d41     mx2 = max(a.mx2, b.mx2);
d41     if(a.mx1 != b.mx1) mx2 = max(mx2, min(a.mx1, b.mx1));
d41     cMax=(a.mx1==mx1 ? a.cMax:0)+(b.mx1==mx1 ? b.cMax:0);

d41     mi1 = min(a.mi1, b.mi1);
d41     mi2 = min(a.mi2, b.mi2);
d41     if(a.mi1 != b.mi1) mi2=min(mi2, max(a.mi1, b.mi1));
d41     cMin=(a.mi1==mi1 ? a.cMin:0)+(b.mi1==mi1 ? b.cMin:0);
d41   }
d41   void apply_sum(ll x){
d41     mx1 += x, mx2 += x,mi1 += x, mi2 += x;
d41     sum += tam*x, lazy += x;
d41   }
d41   void apply_min(ll x){
d41     if(x >= mx1) return;
d41     sum -= (mx1 - x)*cMax;
d41     if(mi1 == mx1) mi1 = x;
d41     if(mi2 == mx1) mi2 = x;
d41     mx1 = x;
d41   }
d41   void apply_max(ll x){
d41     if(x <= mi1) return;
d41     sum -= (mi1 - x)*cMin;
d41     if(mx1 == mi1) mx1 = x;
d41     if(mx2 == mi1) mx2 = x;
d41     mi1 = x;
d41   }
d41 };
d41 void apply(int l, int r, int p){
d41   for(int i=2*p+1; i<=2*p+2; i++){
d41     seg[i].apply_sum(st[p].lazy);
d41     seg[i].apply_min(st[p].mx1);
d41     seg[i].apply_max(st[p].mi1);
d41   }
d41   seg[p].lazy = 0;
d41 }
```

### RMQ.h

**Usage:** `RMQ rmq(values);`
`rmq.query(inclusive, inclusive);`
**Time:** $\mathcal{O}(|V|\log|V| + Q)$

d41d8c, 17 lines

```
d41 struct RMQ {
d41   vector<vector<int>> dp;
d41   RMQ(const vector<int>& a) : dp(1, a) {
d41     for (int i = 1, pw = 1; pw*2 <= sz(a); pw*=2, i++) {
d41       dp.emplace_back(sz(a) - pw*2 + 1);
d41       for (int j = 0; j < sz(dp[i]); j++) {
d41         dp[i][j] = min(dp[i-1][j], dp[i-1][j+pw]);
d41       }
d41     }
d41   }
d41   int query(int l, int r) {
d41     assert(l <= r);
d41     int k = 31 - __builtin_clz(r - l + 1);
d41     return min(dp[k][l], dp[k][r - (1 << k) + 1]);
d41   }
d41 };
```

# Numerical (4)

## 4.1 Polynomials and recurrences

### Polynomial.h

d41d8c, 19 lines

```
d41 struct Poly {
d41  vector<double> a;
d41  double operator()(double x) const {
d41    double val = 0;
d41    for (int i = sz(a); i--;) (val *= x) += a[i];
d41    return val;
d41  }
d41  void diff() {
d41    rep(i,1,sz(a)) a[i-1] = i*a[i];
d41    a.pop_back();
d41  }
d41  void divroot(double x0) {
d41    double b = a.back(), c; a.back() = 0;
d41    for(int i=sz(a)-1; i--;)
d41       c = a[i], a[i] = a[i+1]*x0+b, b=c;
d41    a.pop_back();
d41  }
d41 };
```

### PolyRoots.h

**Description:** Finds the real roots to a polynomial.
**Usage:** `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
**Time:** $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" d41d8c, 24 lines

```
d41 vector<double> polyRoots(Poly p, double xmin, double xmax)
     {
d41  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
d41  vector<double> ret;
d41  Poly der = p;
d41  der.diff();
d41  auto dr = polyRoots(der, xmin, xmax);
d41  dr.push_back(xmin-1);
d41  dr.push_back(xmax+1);
d41  sort(all(dr));
d41  rep(i,0,sz(dr)-1) {
d41    double l = dr[i], h = dr[i+1];
d41    bool sign = p(l) > 0;
d41    if (sign ^ (p(h) > 0)) {
d41      rep(it,0,60) { // while (h - l > 1e-8)
d41        double m = (l + h) / 2, f = p(m);
d41        if ((f <= 0) ^ sign) l = m;
d41        else h = m;
d41      }
d41      ret.push_back((l + h) / 2);
d41    }
d41  }
d41  return ret;
d41 }
```

### BerlekampMassey.h

**Description:** Recovers any $n$-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\le n$.
**Usage:** `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
**Time:** $\mathcal{O}\left(N^2\right)$ d41d8c, 21 lines

```
d41  vector<ll> berlekampMassey(vector<ll> s) {
d41   int n = sz(s), L = 0, m = 0;
d41   vector<ll> C(n), B(n), T;
d41   C[0] = B[0] = 1;

d41   ll b = 1;
d41   rep(i,0,n) { ++m;
d41     ll d = s[i] % mod;
d41     rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
d41     if (!d) continue;
d41     T = C; ll coef = d * modpow(b, mod-2) % mod;
d41     rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
d41     if (2 * L > i) continue;
d41     L = i + 1 - L; B = T; b = d; m = 0;
d41   }

d41   C.resize(L + 1); C.erase(C.begin());
d41   for (ll& x : C) x = (mod - x) % mod;
d41   return C;
d41  }
```

### LinearRecurrence.h

**Description:** Generates the $k$'th term of an $n$-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \ldots \ge n-1]$ and $tr[0 \ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
**Usage:** `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
**Time:** $\mathcal{O}\left(n^2 \log k\right)$ d41d8c, 27 lines

```
d41  using Poly = vector<ll>;
d41  ll linearRec(Poly S, Poly tr, ll k) {
d41   int n = sz(tr);

d41   auto combine = [&](Poly a, Poly b) {
d41     Poly res(n * 2 + 1);
d41     rep(i,0,n+1) rep(j,0,n+1)
d41       res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
d41     for (int i = 2 * n; i > n; --i) rep(j,0,n)
d41       res[i-1-j] = (res[i-1-j] + res[i] * tr[j]) % mod;
d41     res.resize(n + 1);
d41     return res;
d41   };

d41   Poly pol(n + 1), e(pol);
d41   pol[0] = e[1] = 1;

d41   for (++k; k; k /= 2) {
d41     if (k % 2) pol = combine(pol, e);
d41     e = combine(e, e);
d41   }

d41   ll res = 0;
d41   rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
d41   return res;
d41  }
```

## 4.2 Matrices

### SolveLinearBinary.h

**Time:** $\mathcal{O}\left(\frac{\min(n,m)\, nm}{64}\right)$ d41d8c, 33 lines

```
d41  pair<int, bitset<M>> gauss(vector<bitset<M>> eq) {
d41      int n = eq.size(), m = M - 1;
d41      vector<int> where(m, -1);
d41      for(int col = 0, row = 0; col < m && row < n; col++){
d41          for (int i = row; i < n; i++)
d41              if (eq[i][col]) {
d41                  swap(eq[i], eq[row]);
d41                  break;
d41              }
d41          if (!eq[row][col]) continue;
d41          where[col] = row;

d41          for (int i = 0; i < n; i++) {
d41              if (i != row && eq[i][col]) eq[i] ^= eq[row];
d41          }
d41          ++row;
d41      }

d41      bitset<M> ans;
d41      for (int i = 0; i < m; i++) {
d41          if (where[i] != -1) ans[i] = eq[where[i]][m];
d41      }
d41      for (int i = 0; i < n; i++) {
d41          int sum = (ans & eq[i]).count();
d41          sum %= 2;
d41          if (sum != eq[i][m]) return pair(0, bitset<M>());
d41      }
d41      for (int i = 0; i < m; i++) {
d41          if (where[i] == -1) return pair(INF, ans);
d41      }
d41      return pair(1, ans);
d41  }
```

### XorGauss.h

d41d8c, 30 lines

```
d41  struct XorGauss {
d41      int N;
d41      vector<ll> basis, who, mask;
d41      XorGauss(int N) : N(N), basis(N), who(N), mask(N) {}
         // if(ans & (1ll << j)) who[j] was used to form x
d41      bool belong(ll x){
d41          ll ans = 0;
d41          for(int i=N-1; i>=0; i--){
d41              if((x ^ basis[i]) < x){
d41                  ans ^= mask[i];
d41                  x ^= basis[i];
d41              }
d41          }
d41          return (x == 0);
d41      }
d41      void add(ll v, int idx) {
d41          ll msk = 0;
d41          for (int i = N - 1; i >= 0; i--) {
d41              if (!(v & (1ll << i))) continue;
d41              if (basis[i] == 0) {
d41                  basis[i] = v, who[i] = idx;
d41                  mask[i] = (msk | (1ll << i));
d41                  return;
d41              }
d41              msk ^= mask[i];
d41              v ^= basis[i];
d41          }
d41      }
d41  };
```

## 4.3 Fourier transforms

### FastFourierTransform.h

**Description:** fft(a) computes $\hat f(k) = \sum_x a[x]\exp(2\pi i \cdot kx/N)$ for all $k$. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9 \cdot 10^{14}$ (in practice $10^{16}$; higher for random inputs). Otherwise, use NTT/FFTMod.
**Time:** $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$) d41d8c, 44 lines

```
d41  typedef complex<double> C;

d41  void fft(vector<C>& a) {
d41    int n = a.size(), L = 31 - __builtin_clz(n);
d41    static vector<complex<long double>> R(2, 1); // 10% faster if double
d41    static vector<C> rt(2, 1);
d41    for (static int k = 2; k < n; k *= 2) {
d41      R.resize(n);
d41      rt.resize(n);
d41      auto x = polar(1.0L, acos(-1.0L) / k);
d41      rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
d41    }
d41    vector<ll> rev(n);
d41    rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
d41    rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);

d41    for (int k = 1; k < n; k *= 2) {
d41      for (int i = 0; i < n; i += 2 * k) {
d41        for (int j = 0; j < k; j++) {
d41          auto x = (double*)&rt[j + k];
d41          auto y = (double*)&a[i + j + k];
d41          C z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]);
d41          a[i + j + k] = a[i + j] - z;
d41          a[i + j] += z;
d41        }
d41      }
d41    }
d41  }

d41  vector<ll> conv(const vector<ll>& a, const vector<ll>& b){
d41    if (a.empty() || b.empty()) return {};
d41    vector<ll> res(sz(a) + sz(b) - 1);
d41    int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
d41    vector<C> in(n), out(n);
d41    copy(all(a), in.begin());
d41    rep(i,0,sz(b)) in[i].imag(b[i]);
d41    fft(in);
d41    for (C& x : in) x *= x;
d41    rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
d41    fft(out);
d41    rep(i,0,sz(res)) res[i]=round(imag(out[i]) / (4 * n));
d41    return res;
d41  }
```

### FastFourierTransformMod.h

**Description:** Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice $10^{16}$ or higher). Inputs must be in $[0, \text{mod})$.
**Time:** $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)
"FastFourierTransform.h" d41d8c, 23 lines

```
d41  typedef vector<ll> vl;
d41  template<int M> vl convMod(const vl &a, const vl &b) {
d41   if (a.empty() || b.empty()) return {};
d41   vl res(sz(a) + sz(b) - 1);
d41   int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M));
d41   vector<C> L(n), R(n), outs(n), outl(n);
d41   rep(i,0,sz(a)) L[i] =C((int)a[i] / cut, (int)a[i] % cut);
d41   rep(i,0,sz(b)) R[i] =C((int)b[i] / cut, (int)b[i] % cut);
d41   fft(L), fft(R);
d41   rep(i,0,n) {
```

```
d41     int j = -i & (n - 1);
d41     outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
d41     outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
d41   }
d41   fft(outl), fft(outs);
d41   rep(i,0,sz(res)) {
d41     ll av = ll(real(outl[i])+.5), cv =ll(imag(outs[i])+.5);
d41     ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
d41     res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
d41   }
d41   return res;
d41 }
```

### NumberTheoreticTransform.h

**Description:** ntt(a) computes $\hat{f}(k) = \sum_x a[x]g^{xk}$ for all $k$, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most $2^a$. For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).
**Time:** $\mathcal{O}(N \log N)$

d41d8c, 34 lines

```
d41 const int mod = 998244353, root = 62;
d41 typedef vector<ll> vl;
d41 void ntt(vl &a) {
d41  int n = sz(a), L = 31 - __builtin_clz(n);
d41  static vl rt(2, 1);
d41  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
d41    rt.resize(n);
d41    ll z[] = {1, modpow(root, mod >> s)};
d41    rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
d41  }
d41  vector<int> rev(n);
d41  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
d41  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
d41  for (int k = 1; k < n; k *= 2)
d41    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
d41      ll z = rt[j+k] * a[i+j+k] % mod, &ai = a[i+j];
d41      a[i + j + k] = ai - z + (z > ai ? mod : 0);
d41      ai += (ai + z >= mod ? z - mod : z);
d41    }
d41 }
d41 vl conv(const vl &a, const vl &b) {
d41  if (a.empty() || b.empty()) return {};
d41  int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s),
d41      n = 1 << B;
d41  int inv = modpow(n, mod - 2);
d41  vl L(a), R(b), out(n);
d41  L.resize(n), R.resize(n);
d41  ntt(L), ntt(R);
d41  rep(i,0,n)
d41    out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
d41  ntt(out);
d41  return {out.begin(), out.begin() + s};
d41 }
```

### FWHT.h

**Description:** Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of $a$ must be a power of two.
**Time:** $\mathcal{O}(N \log N)$

d41d8c, 20 lines

```
d41 void FST(vector<ll>& a, bool inv) {
d41   for (int n = sz(a), step = 1; step < n; step *= 2) {
d41     for (int i = 0; i < n; i += 2 * step) {
d41       for (int j = i; j < i + step; j++) {
d41         ll& u = a[j], &v = a[j + step];
d41         tie(u, v) =
d41         inv ? pair(v - u, u) : pair(v, u + v); // AND
d41         inv ? pair(v, u - v) : pair(u + v, u); // OR
d41         pair(u + v, u - v);  // XOR
d41       }
d41     }
d41   }
d41   if(inv) for(ll& x : a) x /= sz(a); // XOR only
d41 }
d41 vector<ll> conv(vector<ll> a, vector<ll> b) {
d41   FST(a, 0); FST(b, 0);
d41   for (int i = 0; i < sz(a); i++) a[i]*=b[i];
d41   FST(a, 1); return a;
d41 }
```

# Number theory (5)

## 5.1 Modular arithmetic

### ModInverse.h

**Description:** Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

d41d8c, 5 lines

```
d41 const ll mod = 1000000007, LIM = 200000;
d41 inv[1] = 1;
d41 for(int i=2; i<LIM; i++)
d41     inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

### ModMulLL.h

**Description:** Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.
**Time:** $\mathcal{O}(1)$ for `modmul`, $\mathcal{O}(\log b)$ for `modpow`

d41d8c, 12 lines

```
d41 typedef unsigned long long ull;
d41 ull modmul(ull a, ull b, ull M) {
d41  ll ret = a * b - M * ull(1.L / M * a * b);
d41  return ret + M * (ret < 0) - M * (ret >= (ll)M);
d41 }
d41 ull modpow(ull b, ull e, ull mod) {
d41  ull ans = 1;
d41  for (; e; b = modmul(b, b, mod), e /= 2)
d41    if (e & 1) ans = modmul(ans, b, mod);
d41  return ans;
d41 }
```

### DiscreteLog.h

**Description:** Returns the smallest $x$ such that $a^x \bmod m = b \bmod m$. If no such $x$ exists, returns $-1$.
**Time:** O(sqrt(m) * log(sqrt(m))

d41d8c, 32 lines

```
d41 int solve(int a, int b, int m) {
d41     a %= m, b %= m;
d41     if (a == 0) return (b ? -1 : 1);
        // caso gcd(a, m) > 1
d41     int k = 1, add = 0, g;
d41     while ((g = gcd(a, m)) > 1) {
d41         if (b == k) return add;
d41         if (b % g) return -1;
d41         b /= g, m /= g, ++add;
d41         k = (k * 1ll * a / g) % m;
d41     }

d41     int sq = sqrt(m) + 1;
d41     int big = 1;
d41     for (int i = 0; i < sq; i++) big = (1ll * big * a) % m
    ;

d41     vector<pii> vals;
d41     for (int q = 0, cur = b; q <= sq; q++) {
d41         vals.push_back({ cur, q });
d41         cur = (1ll * cur * a) % m;
d41     }
d41     sort(all(vals));
d41     for (int p = 1, cur = k; p <= sq; p++) {
d41         cur = (1ll * cur * big) % m;
d41         auto it = lower_bound(all(vals), pair(cur, INF));
d41         if (it != vals.begin() && (--it)->first == cur) {
d41             return sq * p - it->second + add;
d41         }
d41     }
d41     return -1;
d41 }
```

### DiscreteRoot.h

**Description:** Returns $x$ such that $x^k \bmod m = a \bmod m$. If no such $x$ exists, returns $-1$.
**Time:** O(sqrt(m) * log(sqrt(m))

"PrimitiveRoot.h", "DiscreteLog.h" d41d8c, 11 lines

```
    // Discrete Root

d41 ll discreteRoot(ll k, ll a, ll m) {
d41     ll g = primitiveRoot(m);
d41     ll y = discreteLog(fexp(g, k, m), a, m);
d41     if (y == -1) return y;
d41     return fexp(g, y, m);
d41 }
```

## 5.2 Primality

### MillerRabin.h

**Description:** Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
**Time:** 7 times the complexity of $a^b \bmod c$.

"ModMulLL.h" d41d8c, 13 lines

```
d41 bool isPrime(ull n) {
d41  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
d41  ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 17952650
    22};
d41  ull s = __builtin_ctzll(n-1), d = n >> s;
d41  for (ull a : A) {   // ^ count trailing zeroes
d41    ull p = modpow(a%n, d, n), i = s;
d41    while (p != 1 && p != n - 1 && a % n && i--)
d41      p = modmul(p, p, n);
d41    if (p != n-1 && i != s) return 0;
d41  }
d41  return 1;
d41 }
```

### Factor.h

**Description:** Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
**Time:** $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" d41d8c, 19 lines

```
d41 ull pollard(ull n) {
d41  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
d41  auto f = [&](ull x) { return modmul(x, x, n) + i; };
d41  while (t++ % 40 || gcd(prd, n) == 1) {
d41    if (x == y) x = ++i, y = f(x);
d41    if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
d41    x = f(x), y = f(f(y));
d41  }
d41  return gcd(prd, n);
d41 }
d41 vector<ull> factor(ull n) {
d41  if (n == 1) return {};
d41  if (isPrime(n)) return {n};
```

```
d41   ull x = pollard(n);
d41   auto l = factor(x), r = factor(n / x);
d41   l.insert(l.end(), all(r));
d41   return l;
d41  }
```

PrimitiveRoot.h

d41d8c, 15 lines

```
     //is n primitive root of p ?
d41  bool test(ll x, ll p) {
d41      ll m = p - 1;
d41      for (ll i = 2; i * i <= m; ++i) if (!(m % i)) {
d41          if (modpow(x, i, p) == 1) return false;
d41          if (modpow(x, m / i, p) == 1) return false;
d41      }
d41      return true;
d41  }
     //find the smallest primitive root for p
d41  ll search(ll p) {
d41      for (ll i = 2; i < p; i++) if (test(i, p)) return i;
d41      return -1;
d41  }
```

## 5.3 Divisibility

Euclid.h

**Description:** Find $x, y$ such that $Ax + By = \gcd(A, B)$. If $\gcd(A, B) = 1$, then $x = A^{-1} \pmod B$ and $y = B^{-1} \pmod A$.
**Time:** $\mathcal{O}(\log)$

d41d8c, 6 lines

```
d41  ll euclid(ll a, ll b, ll &x, ll &y) {
d41   if (!b) return x = 1, y = 0, a;
d41   ll d = euclid(b, a % b, y, x);
d41   return y -= a/b * x, d;
d41  }
```

CRT.h

d41d8c, 25 lines

```
d41  ll modinverse(ll a, ll b, ll s0 = 1, ll s1 = 0) {
d41    return !b ? s0 : modinverse(b, a % b, s1, s0 - s1 * (a /
      b)); }

d41  ll mul(ll a, ll b, ll m) {
d41    return (((__int128_t)a*b)%m + m)%m;
d41  }

d41  struct Equation {
d41    ll mod, ans;
d41    bool valid;
d41    Equation(ll a, ll m) { mod = m, ans = a, valid = true; }
d41    Equation() { valid = false; }
d41    Equation(Equation a, Equation b) {
d41      valid = false;
d41      if (!a.valid || !b.valid) return;
d41      ll g = gcd(a.mod, b.mod);
d41      if ((a.ans - b.ans) % g != 0) return;
d41      valid = true;
d41      mod = a.mod * (b.mod / g);
d41      ll x = mul(a.mod, modinverse(a.mod, b.mod), mod);
d41      ans = a.ans + mul(x, (b.ans - a.ans) / g, mod);
d41      ans = (ans % mod + mod) % mod;
d41    }
d41  };
```

DivisionTrick.h

d41d8c, 15 lines

```
d41  void floor_ranges(int n) {
d41      for (int l = 1, r; l <= n; l = r + 1) {
d41          r = n / (n / l);
             // floor(n/y) has the same value for y in [l..r]
d41      }
d41  }
d41  void ceil_ranges(int n) {
d41      for (int l = 1, r; l <= n; l = r + 1) {
d41          int x = (n + l - 1) / l;
d41          if (x == 1) r = n;
d41          else r = (n - 1) / (x - 1);
             // ceil(n/y) has the same value for y in [l..r]
d41      }
d41  }
```

### 5.3.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If $(x, y)$ is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

Phi.h

**Description:** *Euler's* $\phi$ function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with $n$. $\phi(1) = 1$, $p$ prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, $m, n$ coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1 - 1} ... (p_r - 1)p_r^{k_r - 1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
**Euler's thm**: $a, n$ coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.
**Euler's thm (generalized)**: $a, m$ arbitrary, $n \geq \log_2 m \Rightarrow a^n \equiv a^{\phi(m) + (n \bmod \phi(m))} \pmod m$.

d41d8c, 6 lines

```
d41  void calculatePhi() {
d41   for(int i=0; i<LIM; i++) phi[i] = i&1 ? i : i/2;
d41   for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
d41     for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
d41  }
```

## 5.4 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power $p^a$, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^{\times}$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

## 5.5 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of $n$ is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

## 5.6 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

## 5.7 Theorems

**Goldbach's conjecture:** Every even integer $n > 2$ can be written as $n = a + b$ with $a, b$ prime.

**Legendre's conjecture:** There is always at least one prime between $n^2$ and $(n+1)^2$.

**Lagrange's four-square theorem:** Every positive integer can be written as

$$n = a^2 + b^2 + c^2 + d^2.$$

**Zeckendorf's theorem:** Every integer $n \geq 1$ has a unique representation as a sum of non-consecutive Fibonacci numbers:

$$n = F_{i_1} + F_{i_2} + \cdots + F_{i_k}, \quad i_j - i_{j+1} \geq 2.$$

**Euclid's formula (primitive Pythagorean triples):** The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either $m$ or $n$ even.

**Wilson's theorem:** $n$ is prime iff

$$(n-1)! \equiv -1 \pmod n.$$

**Chicken McNugget theorem:** For coprime $n, m$, the largest integer not representable as $an + bm$ (with $a, b \geq 0$) is

$$nm - n - m.$$

There are $\frac{(n-1)(m-1)}{2}$ non-representable integers, and for each pair $(k,\ nm - n - m - k)$ exactly one is representable.

# Combinatorial (6)

## 6.1 Binomial Identities

$$\binom{n-1}{k} - \binom{n-1}{k-1} = \frac{n-2k}{k}\binom{n}{k} \qquad \binom{n}{h}\binom{n-h}{k} = \binom{n}{k}\binom{n-k}{h}$$

$$\textstyle\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1} \qquad \sum_{k=0}^{n} k^2\binom{n}{k} = (n+n^2)2^{n-2}$$

$$\textstyle\sum_{j=0}^{k} \binom{m}{j}\binom{n-m}{k-j} = \binom{n}{k} \qquad \sum_{j=0}^{m} \binom{m}{j}^2 = \binom{2m}{m}$$

$$\textstyle\sum_{m=0}^{n} \binom{m}{j}\binom{n-m}{k-j} = \binom{n+1}{k+1} \qquad \sum_{m=0}^{n} \binom{m}{k} = \binom{n+1}{k+1}$$

$$\textstyle\sum_{r=0}^{m} \binom{n+r}{r} = \binom{n+m+1}{m} \qquad \sum_{k=0}^{n} \binom{n-k}{k} = \mathrm{Fib}(n+1)$$

$$\textstyle\sum_{k=0}^{n} \binom{r}{k}\binom{s}{n-k} = \binom{r+s}{n}$$

## 6.2 Permutations

### 6.2.1 Factorial

| $n$ | 1 2 3 4 5 6 7 8 9 10 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| $n!$ | 1 | 2 | 6 | 24 | 120 | 720 | 5040 | 40320 | 362880 | 3628800 |
| $n$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | | | |
| $n!$ | 4.0e7 | 4.8e8 | 6.2e9 | 8.7e10 | 1.3e12 | 2.1e13 | 3.6e14 | | | |
| $n$ | 20 | 25 | 30 | 40 | 50 | 100 | 150 | 171 | | |
| $n!$ | 2e18 | 2e25 | 3e32 | 8e47 | 3e64 | 9e157 | 6e262 | >DBL_MAX | | |

### 6.2.2 Cycles

Let $g_S(n)$ be the number of $n$-permutations whose cycle lengths all belong to the set $S$. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

### 6.2.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor\frac{n!}{e}\right\rceil$$

### 6.2.4 Burnside's lemma

Counts the number of distinct colorings of an object under symmetry.

$$\frac{1}{|G|}\sum_{g\in G} k^{\mathrm{cyc}(g)},$$

where $G$ is the symmetry group, $k$ the number of colors, and $\mathrm{cyc}(g)$ the number of cycles induced by $g$.

Example: number of ways to color a necklace with $n$ beads using $k$ colors (rotations only):

$$g(n) = \frac{1}{n}\sum_{i=0}^{n-1} k^{(\gcd(n,i))}$$

where rotation $i$ shifts the necklace by $i$ positions.

## 6.3 Partitions and subsets

### 6.3.1 Partition function

Number of ways of writing $n$ as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n-k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 20 | 50 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p(n)$ | 1 | 1 | 2 | 3 | 5 | 7 | 11 | 15 | 22 | 30 | 627 | ~2e5 | ~2e8 |

PartitionSolver.h

d41d8c, 61 lines

```
d41 template<const int N>
d41 struct PartitionSolver {
d41   vector<vector<int>> part, to, from;
d41   PartitionSolver() {
d41     vector<int> a;
d41     part.push_back(a);
d41     gen(1, N, a);
d41     sort(all(part));
d41     to.assign(sz(part), vector<int>(N + 1, -1));
d41     from = to;
d41     for (int i = 0; i < sz(part); i++) {
d41       int sum = 0;
d41       auto arr = part[i];
d41       for (auto x : arr) sum += x;
d41       to[i][0] = i;
d41       from[i][0] = i;
d41       for (int j = 1; j + sum <= N; j++) {
d41         arr = part[i];
d41         arr.push_back(j);
d41         sort(all(arr));
d41         to[i][j] = getIndex(arr);
d41         from[to[i][j]][j] = i;
d41       }
d41     }
d41   }

d41   int size() const { return sz(part); }
d41   int getIndex(const vector<int>& arr) const {
d41     return lower_bound(all(part), arr) - part.begin(); }
d41   int add(int id, int num) const { return to[id][num]; }
d41   int rem(int id, int num) const { return from[id][num]; }
d41   vector<int> getPartition(int id) const {
d41     return part[id]; }

d41   void gen(int i, int sum, vector<int>& a) {
d41     if (i > sum) { return; }
d41     a.push_back(i);
d41     part.push_back(a);
d41     gen(i, sum - i, a);
d41     a.pop_back();
d41     gen(i + 1, sum, a);
d41   }
d41 };

    // Number of partitions for all integers <= n
d41 vector<ll> partitionNumber(int n) {
d41   vector<ll> ans(n + 1, 0);
d41   ans[0] = 1;
d41   for (int i = 1; i <= n; i++) {
d41     for (int j = 1; j * (3 * j + 1) / 2 <= i; j++) {
d41       ll here = ans[i - j * (3 * j + 1) / 2];
d41       ans[i] = (ans[i] + (j & 1 ? here : -here));
d41     }
d41     for (int j = 1; j * (3 * j - 1) / 2 <= i; j++) {
d41       ll here = ans[i - j * (3 * j - 1) / 2];
d41       ans[i] = (ans[i] + (j & 1 ? here : -here));
d41     }
d41   }
d41   return ans;
d41 }
```

### 6.3.2 Lucas' Theorem

Let $n, m$ be non-negative integers and $p$ a prime. Write $n = n_k p^k + \ldots + n_1 p + n_0$ and $m = m_k p^k + \ldots + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k}\binom{n_i}{m_i} \pmod{p}$.

## 6.4 General purpose numbers

### 6.4.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able). $B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

### 6.4.2 Stirling numbers of the first kind

Number of permutations on $n$ items with $k$ cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$

$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

### 6.4.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly $k$ elements are greater than the previous element. $k$ $j$:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ $j$:s s.t. $\pi(j) \geq j$, $k$ $j$:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

### 6.4.4 Stirling numbers of the second kind

Partitions of $n$ distinct elements into exactly $k$ groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

### 6.4.5 Bell numbers

Total number of partitions of $n$ distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For $p$ prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

### 6.4.6 Labeled unrooted trees

- on $n$ vertices: $n^{n-2}$
- on $k$ existing trees of size $n_i$: $n_1 n_2 \cdots n_k n^{k-2}$
- with degrees $d_i$: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

### 6.4.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with $n$ pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

# Graph (7)

## 7.1 Fundamentals

### BellmanFord.h

**Description:** Calculates shortest paths from $s$ in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max|w_i| < \sim 2^{63}$.
**Time:** $\mathcal{O}(VE)$

d41d8c, 24 lines

```
d41 const ll inf = LLONG_MAX;
d41 struct Ed { int a, b, w, s() { return a < b ? a : -a; }};
d41 struct Node { ll dist = inf; int prev = -1; };

d41 void bell(vector<Node>& nodes, vector<Ed>& eds, int s) {
d41  nodes[s].dist = 0;
d41  sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s(); });

d41  int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled vertices
d41  for(int i=0; i<lim; i++) for (Ed ed : eds) {
d41    Node cur = nodes[ed.a], &dest = nodes[ed.b];
d41    if (abs(cur.dist) == inf) continue;
d41    ll d = cur.dist + ed.w;
d41    if (d < dest.dist) {
d41      dest.prev = ed.a;
d41      dest.dist = (i < lim-1 ? d : -inf);
d41    }
d41  }
d41  for(int i=0; i<lim; i++) for (Ed e : eds) {
d41    if (nodes[e.a].dist == -inf)
d41      nodes[e.b].dist = -inf;
d41  }
d41 }
```

### FloydWarshall.h

**Description:** Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix $m$, where $m[i][j] =$ inf if $i$ and $j$ are not adjacent. As output, $m[i][j]$ is set to the shortest distance between $i$ and $j$, inf if no path, or -inf if the path goes through a negative-weight cycle.
**Time:** $\mathcal{O}(N^3)$

d41d8c, 20 lines

```
d41 void floydWarshall(vector<vector<ll>>& m) {
d41  int n = sz(m);
d41  for(int i=0; i<n; i++) m[i][i] = min(m[i][i], 0LL);
d41  for(int k=0; k<n; k++)
d41    for(int i=0; i<n; i++)
d41      for(int j=0; j<n; j++)
d41        if (m[i][k] != inf && m[k][j] != inf) {
d41          auto newDist = max(m[i][k] + m[k][j], -inf);
d41          m[i][j] = min(m[i][j], newDist);
d41        }

d41  for(int k=0; k<n; k++)
d41    if (m[k][k] < 0)
d41      for(int i=0; i<n; i++)
d41        for(int j=0; j<n; j++)
d41          if (m[i][k] != inf && m[k][j] != inf){
d41            m[i][j] = -inf;
d41          }
d41 }
```

## 7.2 Network flow and Matching

### Dinic.h

**Time:** - $\mathcal{O}(\min(m \cdot \text{max\_flow},\ n^2 m))$.
- For graphs with unit capacities: $\mathcal{O}\left(\min(m\sqrt{m},\ mn^{2/3})\right)$.
- If every vertex has in-degree 1 or out-degree 1: $\mathcal{O}(m\sqrt{n})$.
- With capacity scaling: $\mathcal{O}(nm\log(\text{MAXCAP}))$ with high constant factor.

d41d8c, 56 lines

```
d41 struct Dinic {
d41  const bool scaling = false;
d41  int lim;
d41  struct edge {
d41    int to, rev;
d41    ll cap, flow;
d41    bool res;
d41    edge(int to_, int cap_, int rev_, bool res_)
d41      : to(to_), cap(cap_), rev(rev_), flow(0), res(res_){}
d41  };

d41  vector<vector<edge>> g;
d41  vector<int> lev, beg;
d41  ll F;
d41  Dinic(int n) : g(n), lev(n), beg(n), F(0) {}

d41  void add(int a, int b, ll c, ll other = 0) {
d41    g[a].emplace_back(b, c, g[b].size(), false);
d41    g[b].emplace_back(a, other, g[a].size()-1, true);
d41  }
d41  bool bfs(int s, int t) {
d41    fill(all(lev), -1);
d41    fill(all(beg), 0);
d41         lev[s] = 0;
d41    queue<int> q; q.push(s);
d41    while (q.size()) {
d41      int u = q.front(); q.pop();
d41      for (auto& i : g[u]) {
d41        if (lev[i.to] != -1 or (i.flow == i.cap)) continue;
d41        if (scaling and i.cap - i.flow < lim) continue;
d41        lev[i.to] = lev[u] + 1;
d41        q.push(i.to);
d41      }
d41    }
d41    return lev[t] != -1;
d41  }
d41  ll dfs(int v, int s, ll f = INF) {
d41    if (!f or v == s) return f;
d41    for (int& i = beg[v]; i < g[v].size(); i++) {
d41      auto& e = g[v][i];
d41      if (lev[e.to] != lev[v] + 1) continue;
d41      ll foi = dfs(e.to, s, min(f, e.cap - e.flow));
d41      if (!foi) continue;
d41      e.flow += foi, g[e.to][e.rev].flow -= foi;
d41      return foi;
d41    }
d41    return 0;
d41  }
d41  ll maxFlow(int s, int t) {
d41    for (lim = scaling ? (1<<30) : 1; lim; lim /= 2)
d41      while (bfs(s, t)) while (ll ff = dfs(s, t)) F += ff;
d41    return F;
d41  }
d41     bool inCut(int u){ return lev[u] != -1; }
d41 };
```

### MinCost.h

**Description:** Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only. If graph is a DAG pi can be calculated with DP instead of Bellman ford.
**Time:** $\mathcal{O}(FE\log(V))$ where F is max flow. $\mathcal{O}(VE)$ for setpi.

d41d8c, 95 lines

```
d41 #include <bits/extc++.h>

d41 const ll INF = numeric_limits<ll>::max() / 4;

d41 struct MCMF {
d41   struct edge {
d41     int from, to, rev;
d41     ll cap, cost, flow;
d41   };
d41   int N;
d41   vector<vector<edge>> ed;
d41   vector<int> seen, vis;
d41   vector<ll> dist, pi;
d41   vector<edge*> par;

d41   MCMF(int N) : N(N), ed(N), seen(N), vis(N),
d41     dist(N), pi(N), par(N) {}

d41   void addEdge(int from, int to, ll cap, ll cost) {
d41     if (from == to || cap == 0) return;
d41     ed[from].push_back(edge{from,to,sz(ed[to]),cap,cost,0});
d41     ed[to].push_back(edge{to,from,sz(ed[from])-1,0,-cost,0});
d41   }

d41   void path(int s) {
d41     fill(all(seen), 0);
```

```
d41     fill(all(dist), INF);
d41     dist[s] = 0;
d41     ll di;
d41     __gnu_pbds::priority_queue<pair<ll, int>> q;
d41     vector<decltype(q)::point_iterator> its(N);
d41     q.push({ 0, s });

d41     while (!q.empty()) {
d41       s = q.top().second; q.pop();
d41       seen[s] = 1; di = dist[s] + pi[s];
d41       for (edge& e : ed[s]) {
d41         if (!seen[e.to]) {
d41           ll val = di - pi[e.to] + e.cost;
d41           if(e.cap - e.flow > 0 && val < dist[e.to]){
d41             dist[e.to] = val;
d41             par[e.to] = &e;
d41             if (its[e.to] == q.end()) {
d41               its[e.to] = q.push({-dist[e.to], e.to});
d41             }
d41             else q.modify(its[e.to], {-dist[e.to], e.to});
d41           }
d41         }
d41       }
d41     }
d41     for (int i = 0; i < N; i++) {
d41       pi[i] = min(pi[i] + dist[i], INF);
d41     }
d41   }

d41   pair<ll, ll> maxflow(int s, int t) {
d41     setpi(s, t);
d41     ll totflow = 0, totcost = 0;
d41     while (path(s), seen[t]) {
d41       ll fl = INF;
d41       for (edge* x = par[t]; x; x = par[x->from]) {
d41         fl = min(fl, x->cap - x->flow);
d41       }
d41       totflow += fl;
d41       for (edge* x = par[t]; x; x = par[x->from]) {
d41         x->flow += fl;
d41         ed[x->to][x->rev].flow -= fl;
d41       }
d41     }
d41     for (int i = 0; i < N; i++) {
d41       for (edge& e : ed[i]) {
d41         totcost += e.cost * e.flow;
d41       }
d41     }
d41     return { totflow, totcost / 2 };
d41   }

      // If some costs can be negative, call this before
          maxflow:
d41   void setpi(int s, int t) {
d41     fill(all(pi), INF);
d41     pi[s] = 0;
d41     int it = N, ch = 1;
d41     ll v;
d41     while (ch-- && it--) {
d41       for (int i = 0; i < N; i++) {
d41         if (pi[i] != INF)
d41           for (edge& e : ed[i]) if (e.cap)
d41             if((v= pi[i] + e.cost)< pi[e.to])
d41               pi[e.to] = v, ch = 1;
d41       }
d41     }
d41     assert(it >= 0); // negative cost cycle
d41   }
d41 };
```

## PushRelabel.h

**Description:** Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

**Time:** $\mathcal{O}\left(V^2\sqrt{E}\right)$

d41d8c, 55 lines

```
d41 struct PushRelabel {
d41  struct Edge {
d41    int dest, back;
d41    ll f, c;
d41  };
d41  vector<vector<Edge>> g;
d41  vector<ll> ec;
d41  vector<Edge*> cur;
d41  vector<vector<int>> hs;
d41  vector<int> H;
d41  PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n){}

d41  void addEdge(int s, int t, ll cap, ll rcap=0) {
d41    if (s == t) return;
d41    g[s].push_back({t, sz(g[t]), 0, cap});
d41    g[t].push_back({s, sz(g[s])-1, 0, rcap});
d41  }

d41  void addFlow(Edge& e, ll f) {
d41    Edge &back = g[e.dest][e.back];
d41    if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
d41    e.f += f; e.c -= f; ec[e.dest] += f;
d41    back.f -= f; back.c += f; ec[back.dest] -= f;
d41  }
d41  ll calc(int s, int t) {
d41    int v = sz(g); H[s] = v; ec[t] = 1;
d41    vector<int> co(2*v); co[0] = v-1;
d41    for(int i=0; i<v; i++) cur[i] = g[i].data();
d41    for (Edge& e : g[s]) addFlow(e, e.c);

d41    for (int hi = 0;;) {
d41      while (hs[hi].empty()) if (!hi--) return -ec[s];
d41      int u = hs[hi].back(); hs[hi].pop_back();
d41      while (ec[u] > 0)  // discharge u
d41        if (cur[u] == g[u].data() + sz(g[u])) {
d41          H[u] = 1e9;
d41          for (Edge& e : g[u]){
d41            if (e.c && H[u] > H[e.dest]+1)
d41              H[u] = H[e.dest]+1, cur[u] = &e;
d41          }
d41          if (++co[H[u]], !--co[hi] && hi < v){
d41            for(int i=0; i<v; i++){
d41              if (hi < H[i] && H[i] < v)
d41                --co[H[i]], H[i] = v + 1;
d41            }
d41          }
d41          hi = H[u];
d41        } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1){
d41          addFlow(*cur[u], min(ec[u], cur[u]->c));
d41        }else ++cur[u];
d41    }
d41  }
d41  bool inCut(int a) { return H[a] >= sz(g); }
d41 };
```

## Blossom.h

**Description:** Max matching on general Graph. $mate[i]$ = match of $i$

**Time:** $\mathcal{O}\left(N^3\right)$

d41d8c, 56 lines

```
d41 vector<int> Blossom(vector<vector<int>>& g) {
d41     int n = sz(g), timer = -1;
d41     vector<int> mate(n, -1),label(n),par(n),orig(n),aux(n,
      -1),q;
d41     auto lca = [&](int x, int y) {
d41         for (timer++; ; swap(x, y)) {
d41             if (x == -1) continue;
d41             if (aux[x] == timer) return x;
d41             aux[x] = timer;
d41             x=(mate[x] == -1 ? -1 : orig[par[mate[x]]]);
d41         }
d41     };
d41     auto blossom = [&](int v, int w, int a) {
d41         while (orig[v] != a) {
d41             par[v] = w; w = mate[v];
d41             if(label[w] == 1) label[w] = 0,q.push_back(w);
d41             orig[v] = orig[w] = a;
d41             v = par[w];
d41         }
d41     };
d41     auto aug = [&](int v) {
d41         while (v != -1) {
d41             int pv = par[v], nv = mate[pv];
d41             mate[v] = pv; mate[pv] = v; v = nv;
d41         }
d41     };
d41     auto bfs = [&](int root) {
d41         fill(all(label), -1);
d41         iota(all(orig), 0);
d41         q.clear();
d41         label[root] = 0; q.push_back(root);
d41         for (int i = 0; i < sz(q); i++) {
d41             int v = q[i];
d41             for (auto x : g[v]) {
d41                 if (label[x] == -1) {
d41                     label[x] = 1; par[x] = v;
d41                     if (mate[x] == -1) return aug(x), 1;
d41                     label[mate[x]] = 0;
d41                     q.push_back(mate[x]);
d41                 }
d41                 else if(!label[x] && orig[v] != orig[x]){
d41                     int a = lca(orig[v], orig[x]);
d41                     blossom(x, v, a);
d41                     blossom(v, x, a);
d41                 }
d41             }
d41         }
d41         return 0;
d41     };
        // Time halves if you start with (any) maximal
            matching.
d41     for (int i = 0; i < n; i++) {
d41         if (mate[i] == -1) bfs(i);
d41     }
d41     return mate;
d41 }
```

## HopcroftKarp.h

**Description:** $ans$ is the size of the max matching. The match of x is $l[x]$

**Usage:** `HopcroftKarp(|X|, |Y|, edges(x, y))`

**Time:** $\mathcal{O}\left(\sqrt{V}E\right)$

d41d8c, 46 lines

```
d41 struct HopcroftKarp {
d41     vector<int> g, l, r;
d41     int ans;
d41     HopcroftKarp(int n, int m, vector<pair<int, int>> e)
d41         : g(e.size()), l(n, -1), r(m, -1), ans(0) {
d41         shuffle(e.begin(), e.end(), rng);
d41         vector<int> deg(n + 1);
d41         for (auto& [x, y] : e) deg[x]++;
d41         for (int i = 1; i <= n; i++) deg[i] += deg[i - 1];
```

```
        for (auto& [x, y] : e) g[--deg[x]] = y;

        vector<int> q(n);
        while (true) {
            vector<int> a(n, -1), p(n, -1);
            int t = 0;
            for (int i = 0; i < n; i++) {
                if (l[i] == -1) {
                    q[t++] = a[i] = p[i] = i;
                }
            }
            bool match = false;
            for (int i = 0; i < t; i++) {
                int x = q[i];
                if (~l[a[x]]) continue;
                for (int j = deg[x]; j < deg[x + 1]; j++)
    {
                    int y = g[j];
                    if (r[y] == -1) {
                        while (~y) {
                            r[y] = x;
                            swap(l[x], y);
                            x = p[x];
                        }
                        match = true, ans++;
                        break;
                    }
                    if (p[r[y]] == -1) {
                        q[t++] = y = r[y];
                        p[y] = x, a[y] = a[x];
                    }
                }
            }
            if (!match) break;
        }
    }
};
```

WeightedMatching.h

**Description:** Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \leq M$.
**Time:** $\mathcal{O}\left(N^2M\right)$

d41d8c, 41 lines

```
pair<ll, vector<int>> hunga(const vector<vector<ll>>& a) {
    if (a.empty()) return { 0, {} };
    int n = sz(a) + 1, m = sz(a[0]) + 1;
    vector<ll> u(n), v(m), p(m);
    vector<int> ans(n - 1);
    for (int i = 1; i < n; i++) {
        p[0] = i;
        int j0 = 0;
        vector<ll> dist(m, LLONG_MAX), pre(m, -1);
        vector<bool> done(m + 1);
        do {
            done[j0] = true;
            ll i0 = p[j0], j1 = -1, delta = LLONG_MAX;
            for (int j = 1; j < m; j++) {
                if (!done[j]) {
                    ll cur = a[i0-1][j-1] - u[i0] - v[j];
                    if (cur < dist[j])
                        dist[j] = cur, pre[j] = j0;
                    if(dist[j] < delta)
                        delta = dist[j], j1 = j;
                }
            }
            for (int j = 0; j < m; j++) {
                if (done[j])
                    u[p[j]] += delta, v[j] -= delta;
                else dist[j] -= delta;
            }
            assert(j1 != -1);
            j0 = j1;
        } while (p[j0]);
        while (j0) {
            int j1 = pre[j0];
            p[j0] = p[j1], j0 = j1;
        }
    }
    for (int j = 1; j < m; j++) {
        if (p[j]) ans[p[j] - 1] = j - 1;
    }
    return { -v[0], ans }; // min cost
}
```

### 7.2.1 Hall's Theorem

In bipartite graphs, there exists a perfect matching covering the entire side $X$ if and only if for every subset $Y \subseteq X$,

$$|Y| \leq |N(Y)|,$$

where $N(Y)$ denotes the set of neighbors of $Y$.

### 7.2.2 Kőnig's Theorem

In a bipartite graph, the size of a Minimum Vertex Cover is equal to the size of a Maximum Matching. A Minimum Vertex Cover is a minimum set of vertices such that every edge of the graph has at least one endpoint in the set.

As a consequence,

$$n - \text{Maximum Matching} = \text{Maximum Independent Set},$$

where a Maximum Independent Set is the largest set of vertices with no edges between them.

**Recovering the Minimum Vertex Cover** Given a maximum matching in a bipartite graph $(X, Y)$:

- Construct the residual graph by orienting:
  - non-matching edges from $X$ to $Y$;
  - matching edges from $Y$ to $X$.
- Perform a BFS or DFS starting from all free (unmatched) vertices in $X$.
- Let $Z_X$ be the set of reachable vertices in $X$, and $Z_Y$ the set of reachable vertices in $Y$.

The Minimum Vertex Cover is given by:

$$(X \setminus Z_X) \cup Z_Y.$$

### 7.2.3 Node-Disjoint Path Cover

A node-disjoint path cover is a set of paths such that each vertex belongs to exactly one path.

In a directed acyclic graph (DAG),

$$\text{Minimum Node-Disjoint Path Cover} = n - \text{Maximum Matching}.$$

The construction is as follows: for each vertex $u$, create a copy $u'$. Add an edge $u \to v'$ if there exists an edge $u \to v$ in the original graph.

**Recovering the Paths**

- Vertices that do not appear as destinations in the matching are starting points of paths.
- Each matching edge $u \to v'$ corresponds to an edge $u \to v$ in the original DAG.
- Following these edges reconstructs all paths of the path cover.

### 7.2.4 General Path Cover

A general path cover is a path cover where a vertex may belong to more than one path.

In a DAG, the construction is similar to the node-disjoint case, but an edge $u \to v'$ exists if there is a path from $u$ to $v$ in the original graph.

**Recovering the Cover** The vertices can be grouped according to the edges used in the matching to form the path cover.

### 7.2.5 Dilworth's Theorem

An antichain is a set of vertices such that there is no path between any pair of vertices in the set.

In a directed acyclic graph,

$$\text{Minimum General Path Cover} = \text{Maximum Antichain}.$$

**Recovering a Maximum Antichain** Given a minimum general path cover, selecting one vertex from each path produces a maximum antichain.

## 7.3 DFS algorithms

Bridges.h

d41d8c, 24 lines

```
vector<int> g[ms];
int low[ms], tin[ms], vis[ms], t;

void dfs(int u = 0, int p = -1) {
    vis[u] = true;
    low[u] = tin[u] = t++;
    for (auto v : g[u]) {
        if (v == p) continue;
```

```
d41         if (vis[v]) {
d41             low[u] = min(low[u], tin[v]);
d41         }
d41         else {
d41             dfs(v, u);
d41             low[u] = min(low[u], low[v]);
                // if (low[v] >= tin[u] && p != -1), U is an
                    articulation point
d41             if (low[v] > tin[u]) {
                    // edge from U to V is a bridge
d41             }
                // children++
d41         }
d41     }
        // if(children > 1 && p == -1) root is an articulation
            point
d41 }
```

## EulerPath.h

**Description:** Receives as input graph(node, edge index), number of edges and source. Returns list of node, index of edge he came from, if path/circuit does not exists returns empty list.

d41d8c, 27 lines

```
d41 vector<pii> eulerPath(const vector<vector<pii>>& g, int
    nedges, int src) {
d41     int n = sz(g);
d41     vector<int> deg(n, 0), its(n, 0), used(nedges + 1, 0);
d41     vector<pii> s = { {src, -1} };
        //deg[src]++; //to allow paths, not only circuits
d41     vector<pii> ret;
d41     while (!s.empty()) {
d41         int u = s.back().first, &it = its[u];
d41         if (it == sz(g[u])) {
d41             ret.push_back(s.back());
d41             s.pop_back();
d41             continue;
d41         }
d41         auto& [nxt, id] = g[u][it++];
d41         if (!used[id]) {
d41             deg[u]--, deg[nxt]++;
d41             used[id] = 1;
d41             s.push_back({ nxt, id });
d41         }
d41     }
d41     for (int x : deg) {
d41         if (x < 0 || sz(ret) != (nedges + 1)) return {};
d41     }
d41     reverse(ret.begin(), ret.end());
d41     return ret;
d41 }
```

## SCC.h

**Description:** Kosaraju algorithm for calculating strongly connected components. Components are ordered in topological order.

d41d8c, 36 lines

```
d41 struct SCC {
d41     int n, ncomp;
d41     vector<vector<int>> g, inv;
d41     vector<int> comp, vis, stk;
d41     SCC(){}
d41     SCC(int n)
d41         : n(n),ncomp(0),g(n),inv(n),comp(n, -1),vis(n){}

d41     void dfs(int u) {
d41         vis[u] = 1;
d41         for (int v : g[u]) if (!vis[v]) dfs(v);
d41         stk.push_back(u);
d41     }
d41     void dfs_inv(int u) {
d41         comp[u] = ncomp;
d41         for (int v : inv[u]) {
d41             if (comp[v] == -1) dfs_inv(v);
d41         }
d41     }
d41     void solve() {
d41         for (int i = 0; i < n; i++) {
d41             if (!vis[i]) dfs(i);
d41         }
d41         reverse(all(stk));
d41         for (int u : stk) {
d41             if (comp[u] != -1) continue;
d41             dfs_inv(u);
d41             ncomp++;
d41         }
d41     }
d41     void add_edge(int a, int b) {
d41         g[a].push_back(b);
d41         inv[b].push_back(a);
d41     }
d41 };
```

## Twosat.h

**Usage:** not A = ~A

"SCC.h" d41d8c, 37 lines

```
d41 struct TwoSat{
d41  int n;
d41  SCC scc;
d41  vector<int> value;
d41  vector<pii> e;
d41  TwoSat(int n) : n(n){}
d41  bool solve(){
d41    value.resize(n);
d41    scc = SCC(2*n);
d41    for(auto &x : e) scc.add_edge(x.first, x.second);
d41    scc.solve();
d41    for(int i=0; i<2*n; i++)
d41      if(scc.comp[i] == scc.comp[i^1]) return false;
d41    for(int i=0; i<n; i++)
d41      value[i] = scc.comp[id(i)] > scc.comp[id(~i)];
d41    return true;
d41  }
d41  void atMostOne(vector<int> &li){
d41    if(sz(li) <= 1) return;
d41    int cur = ~li[0];
d41    for(int i = 2; i < sz(li); i++) {
d41      int next = n++;
d41      addOr(cur, ~li[i]);
d41      addOr(cur, next);
d41      addOr(~li[i], next);
d41      cur = ~next;
d41    }
d41    addOr(cur, ~li[1]);
d41  }
d41  int id(int v) { return v < 0 ? (~v) * 2 ^ 1 : v * 2; }
d41  void add(int a, int b) { e.push_back({id(a), id(b)}); }
d41  void addOr(int a, int b) { add(~a, b); add(~b, a); }
d41  void addImp(int a, int b) { addOr(~a, b); }
d41  void addEqual(int a, int b){ addOr(a, ~b); addOr(~a, b);
    }
d41  void isFalse(int a) { addImp(a, ~a); }
d41 };
```

# 7.4 Heuristics

## MaxClique.h

**Description:** Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

**Time:** Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

d41d8c, 53 lines

```
d41 using vb = vector<bitset<200>>;
d41 struct Maxclique {
d41  double limit=0.025, pk=0;
d41  struct Vertex { int i, d=0; };
d41  using vv = vector<Vertex>;
d41  vb e;
d41  vv V;
d41  vector<vector<int>> C;
d41  vector<int> qmax, q, S, old;
d41  void init(vv& r) {
d41    for (auto& v : r) v.d = 0;
d41    for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
d41    sort(all(r), [](auto a, auto b) { return a.d > b.d; });
d41    int mxD = r[0].d;
d41    for(int i=0; i<sz(r); i++) r[i].d = min(i, mxD) + 1;
d41  }
d41  void expand(vv& R, int lev = 1) {
d41    S[lev] += S[lev - 1] - old[lev];
d41    old[lev] = S[lev - 1];
d41    while (sz(R)) {
d41      if (sz(q) + R.back().d <= sz(qmax)) return;
d41      q.push_back(R.back().i);
d41      vv T;
d41      for(auto v : R)
d41        if (e[R.back().i][v.i]) T.push_back({v.i});
d41      if (sz(T)) {
d41        if (S[lev]++ / ++pk < limit) init(T);
d41        int j = 0, mxk = 1, mnk = max(sz(qmax)-sz(q)+1, 1);
d41        C[1].clear(), C[2].clear();
d41        for (auto v : T) {
d41          int k = 1;
d41          auto f = [&](int i) { return e[v.i][i]; };
d41          while (any_of(all(C[k]), f)) k++;
d41          if (k > mxk) mxk = k, C[mxk + 1].clear();
d41          if (k < mnk) T[j++].i = v.i;
d41          C[k].push_back(v.i);
d41        }
d41        if (j > 0) T[j - 1].d = 0;
d41        for(int k=mnk; k<mxk + 1; k++){
d41                    for (int i : C[k])
d41              T[j].i = i, T[j++].d = k;
d41                  }
d41        expand(T, lev + 1);
d41      } else if (sz(q) > sz(qmax)) qmax = q;
d41      q.pop_back(), R.pop_back();
d41    }
d41  }
d41  vector<int> maxClique(){ init(V),expand(V); return qmax;}
d41  Maxclique(vb conn) : e(conn),C(sz(e)+1),S(sz(C)),old(S){
d41    for(int i=0; i<sz(e); i++) V.push_back({i});
d41  }
d41 };
```

# 7.5 Trees

## Centroid.h

**Description:** Call decomp(0) to solve, marked array should be initially set to zero.

**Time:** $\mathcal{O}(N \log N)$

d41d8c, 27 lines

```
d41 int tam[ms], marked[ms];

d41 int calc_tam(int u, int p) {
d41   tam[u] = 1;
d41   for (int v : g[u]) {
d41     if (v != p && !marked[v]) tam[u] += calc_tam(v, u);
d41   }
```

```
d41     return tam[u];
d41   }

d41   int get_centroid(int u, int p, int tot) {
d41     for (int v : g[u]) {
d41       if (v != p && !marked[v] && (tam[v] > (tot / 2)))
d41         return get_centroid(v, u, tot);
d41     }
d41     return u;
d41   }
      // Cent is a child of P in the centroid tree
d41   void decomp(int u, int p = -1) {
d41     calc_tam(u, -1);
d41     int cent = get_centroid(u, -1, tam[u]);
d41     marked[cent] = 1;
d41     for (int v : g[cent]) {
d41       if (!marked[v]) decomp(v, cent);
d41     }
d41   }
```

### HLD.h

**Description:** If values are stored on edges, set EDGE = true and store each edge's value at the endpoint farther from the root (the deeper node). rp[i] is the representative (head) of the heavy path containing node i: it is the node in that chain that is closest to the root.

d41d8c, 51 lines

```
d41   template<bool EDGE> struct HLD {
d41       int n, t;
d41       vector<vector<int>> g;
d41       vector<int> pai, rp, tam, pos, val, arr;
d41       Seg seg;
d41       HLD(int n, vector<vector<int>>& g, vector<int>& val)
d41           : n(n), t(0), g(g), pai(n), rp(n), tam(n, 1),
d41             pos(n), val(val), arr(n) {
d41           calc_tam(0, -1);
d41           dfs(0, -1);
d41           seg.build(arr);
d41       }

d41       int calc_tam(int u, int p) {
d41           pai[u] = p;
d41           for (int& v : g[u]) {
d41               if (v == p) continue;
d41               tam[u] += calc_tam(v, u);
d41               if(tam[v] > tam[g[u][0]] || g[u][0] == p)
d41                   swap(g[u][0], v);
d41           }
d41           return tam[u];
d41       }

d41       void dfs(int u, int p) {
d41           pos[u] = t++;
d41           arr[pos[u]] = val[u];
d41           for (int v : g[u]) {
d41               if (v == p) continue;
d41               rp[v] = (v == g[u][0] ? rp[u] : v);
d41               dfs(v, u);
d41           }
d41       }

d41       int query(int a, int b) { // query on the path from a to b
d41           int ans = 0; // neutral value
d41           while (rp[a] != rp[b]) {
d41               if (pos[a] < pos[b]) swap(a, b);
d41               ans = max(ans, seg.query(pos[rp[a]], pos[a]));
d41               a = pai[rp[a]];
d41           }
d41           if(pos[a] > pos[b]) swap(a, b);
d41           ans = max(ans, seg.query(pos[a] + EDGE, pos[b]));
d41           return ans;
d41       }

d41       void update(int a, int x) {
d41           seg.update(pos[a], x);
d41       }
d41   };
```

### LCA.h

**Description:** LCA algorithm using binary lifting, $is\_ancestor(a, b)$ returns true if $a$ is an ancestral of $b$ and false otherwise.
**Time:** $\mathcal{O}(N \log N)$

d41d8c, 26 lines

```
d41   int tin[MAXN], tout[MAXN], timer=0;
d41   int up[MAXN][BITS];
d41   void dfs(int u, int p){
d41       tin[u] = timer++, up[u][0] = p;
d41       for (int i=1; i<BITS; i++) {
d41           up[u][i] = up[up[u][i-1]][i-1];
d41       }
d41       for (int v: g[u]) if (v != p) dfs(v, u);
d41       tout[u] = timer;
d41   }

d41   bool is_ancestor(int u, int v){
d41       return (tin[u] <= tin[v] && tout[u] >= tout[v]);
d41   }

d41   int lca(int u, int v){
d41       if (is_ancestor(u, v)) return u;
d41       if (is_ancestor(v, u)) return v;
d41       for (int i=BITS-1; i>=0; i--) {
d41           if (up[u][i] && !is_ancestor(up[u][i], v)) {
d41               u = up[u][i];
d41           }
d41       }
d41       return up[u][0];
d41   }
```

### VirtualTree.h

**Description:** Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S| - 1$) pairwise LCA's and compressing edges. virt[u] is the adjacency list of the virtual tree: it stores pairs (v, dist), where v is a neighbor of u in the virtual tree and dist is the distance between u and v in the original tree.
**Time:** $\mathcal{O}(|S| \log |S|)$

"LCA.h" d41d8c, 24 lines

```
d41   vector<pair<int, int>> virt[ms];

d41   void build_virt(vector<int>& v) {
d41     auto cmp = [&](int i, int j){ return tin[i] < tin[j]; };
d41     sort(all(v), cmp);
d41     for (int i = 0, n = sz(v);i + 1 < n; i++)
d41       v.push_back(lca(v[i], v[i + 1]));
d41     sort(all(v), cmp);
d41     v.erase(unique(all(v)), v.end());
d41     stack<int> st;
d41     for (auto u : v) {
d41       if (st.empty()) {
d41         st.push(u);
d41       }
d41       else {
d41         while(sz(st) && !is_ancestor(st.top(), u)) st.pop();
d41         int p = st.top();
d41         virt[p].emplace_back(u, abs(lvl[u] - lvl[p]));
d41         virt[u].emplace_back(p, abs(lvl[u] - lvl[p]));
d41         st.push(u);
d41       }
d41     }
d41   }
```

## 7.6 Math

### 7.6.1 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--`, `mat[b][b]++` (and `mat[b][a]--`, `mat[a][a]++` if $G$ is undirected). Remove the $i$th row and column and take the determinant; this yields the number of directed spanning trees rooted at $i$ (if $G$ is undirected, remove any row/column).

### 7.6.2 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

## 7.7 Planar Graphs

If $G$ has $k$ connected components, then $n - m + f = k + 1$.

# Geometry (8)

## 8.1 Geometric primitives

### Point.h

**Description:** Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

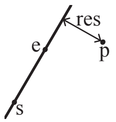
d41d8c, 29 lines

```
d41   template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
d41   template<class T>
d41   struct Point {
d41    typedef Point P;
d41    T x, y;
d41    explicit Point(T x=0, T y=0) : x(x), y(y) {}
d41    bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
d41    bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
d41    P operator+(P p) const { return P(x+p.x, y+p.y); }
d41    P operator-(P p) const { return P(x-p.x, y-p.y); }
d41    P operator*(T d) const { return P(x*d, y*d); }
d41    P operator/(T d) const { return P(x/d, y/d); }
d41    T dot(P p) const { return x*p.x + y*p.y; }
d41    T cross(P p) const { return x*p.y - y*p.x; }
d41    T cross(P a, P b) const { return (a-*this).cross(b-*this); }
d41    T dist2() const { return x*x + y*y; }
d41    double dist() const { return sqrt((double)dist2()); }
       // angle to x-axis in interval [-pi, pi]
d41    double angle() const { return atan2(y, x); }
d41    P unit() const { return *this/dist(); } // makes dist()=1
d41    P perp() const { return P(-y, x); } // rotates +90 degrees
d41    P normal() const { return perp().unit(); }
       // returns point rotated 'a' radians ccw around the origin
d41    P rotate(double a) const {
d41      return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
```

```
d41   friend ostream& operator<<(ostream& os, P p) {
d41     return os << "(" << p.x << "," << p.y << ")"; }
d41   };
```

## lineDistance.h

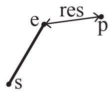
**Description:**
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.




"Point.h" d41d8c, 5 lines

```
d41   template<class P>
d41   double lineDist(const P& a, const P& b, const P& p) {
d41    return (double)(b-a).cross(p-a)/(b-a).dist();
d41   }
```

## SegmentDistance.h



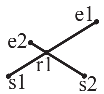

**Description:**
Returns the shortest distance between point p and the line segment from point s to e.
**Usage:** `Point<double> a, b(2,2), p(1,1);`
`bool onSegment = segDist(a,b,p) < 1e-10;`

"Point.h" d41d8c, 7 lines

```
d41   typedef Point<double> P;
d41   double segDist(P& s, P& e, P& p) {
d41    if (s==e) return (p-s).dist();
d41    auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)))
      ;
d41    return ((p-s)*d-(e-s)*t).dist()/d;
d41   }
```

## SegmentIntersection.h

**Description:**
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



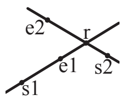

**Usage:** `vector<P> inter = segInter(s1,e1,s2,e2);`
`if (sz(inter)==1)`
`cout << "segments intersect at " << inter[0] << endl;`

"Point.h", "OnSegment.h" d41d8c, 14 lines

```
d41   template<class P> vector<P> segInter(P a, P b, P c, P d) {
d41    auto oa = c.cross(d, a), ob = c.cross(d, b),
d41         oc = a.cross(b, c), od = a.cross(b, d);
       // Checks if intersection is single non-endpoint point.
d41    if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
d41      return {(a * ob - b * oa) / (ob - oa)};
d41    set<P> s;
d41    if (onSegment(c, d, a)) s.insert(a);
d41    if (onSegment(c, d, b)) s.insert(b);
d41    if (onSegment(a, b, c)) s.insert(c);
d41    if (onSegment(a, b, d)) s.insert(d);
d41    return {all(s)};
d41   }
```

## lineIntersection.h

**Description:**
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.




**Usage:** `auto res = lineInter(s1,e1,s2,e2);`
`if (res.first == 1)`
`cout << "intersection point at " << res.second << endl;`

"Point.h" d41d8c, 9 lines

```
d41   template<class P>
d41   pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
d41    auto d = (e1 - s1).cross(e2 - s2);
d41    if (d == 0) // if parallel
d41      return {-(s1.cross(e1, s2) == 0), P(0, 0)};
d41    auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
d41    return {1, (s1 * p + e1 * q) / d};
d41   }
```

## sideOf.h

**Description:** Returns where $p$ is as seen from $s$ towards $e$. 1/0/-1 ⇔ left/on line/right. If the optional argument $eps$ is given 0 is returned if $p$ is within distance $eps$ from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.
**Usage:** `bool left = sideOf(p1,p2,q)==1;`

"Point.h" d41d8c, 10 lines

```
d41   template<class P>
d41   int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

d41   template<class P>
d41   int sideOf(const P& s, const P& e, const P& p, double eps)
       {
d41    auto a = (e-s).cross(p-s);
d41    double l = (e-s).dist()*eps;
d41    return (a > l) - (a < -l);
d41   }
```

## OnSegment.h

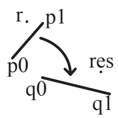
**Description:** Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

"Point.h" d41d8c, 4 lines

```
d41   template<class P> bool onSegment(P s, P e, P p) {
d41    return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
d41   }
```

## linearTransformation.h

**Description:**
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.




"Point.h" d41d8c, 7 lines

```
d41   typedef Point<double> P;
d41   P linearTransformation(const P& p0, const P& p1,
d41      const P& q0, const P& q1, const P& r) {
d41    P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
d41    return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist
       2();
d41   }
```

## LineProjectionReflection.h

**Description:** Projects point p onto line ab. Set refl=true to get reflection of point p across line ab instead. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h" d41d8c, 6 lines

```
d41   template<class P>
d41   P lineProj(P a, P b, P p, bool refl=false) {
d41    P v = b - a;
d41    return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
d41   }
```

## Angle.h

**Description:** A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.
**Usage:** `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
`int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }`
`// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i`

d41d8c, 36 lines

```
d41   struct Angle {
d41    int x, y;
d41    int t;
d41    Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
d41    Angle operator-(Angle b) const { return {x-b.x, y-b.y, t
      }; }
d41    int half() const {
d41      assert(x || y);
d41      return y < 0 || (y == 0 && x < 0);
d41    }
d41    Angle t90() const { return {-y, x, t + (half() && x >= 0)
      }; }
d41    Angle t180() const { return {-x, -y, t + half()}; }
d41    Angle t360() const { return {x, y, t + 1}; }
d41   };
d41   bool operator<(Angle a, Angle b) {
       // add a.dist2() and b.dist2() to also compare distances
d41    return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
d41           make_tuple(b.t, b.half(), a.x * (ll)b.y);
d41   }

      // Given two points, this calculates the smallest angle
          between
      // them, i.e., the angle that covers the defined line
          segment.
d41   pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
d41    if (b < a) swap(a, b);
d41    return (b < a.t180() ?
d41            make_pair(a, b) : make_pair(b, a.t360()));
d41   }
d41   Angle operator+(Angle a, Angle b) { // point a + vector b
d41    Angle r(a.x + b.x, a.y + b.y, a.t);
d41    if (a.t180() < r) r.t--;
d41    return r.t180() < a ? r.t360() : r;
d41   }
d41   Angle angleDiff(Angle a, Angle b) { // angle b - angle a
d41    int tu = b.t - a.t; a.t = b.t;
d41    return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a
      )};
d41   }
```

## HalfPlane.h

**Description:** Computes the intersection of a set of half-planes. Half-planes are sorted by angle and processed with a deque, removing redundant or conflicting constraints. Parallel half-planes are handled explicitly. Returns the convex polygon of the intersection, or empty if infeasible.
**Time:** $\mathcal{O}(n \log n)$

"Point.h" d41d8c, 72 lines

```
d41 using ld = long double;
d41 using P = Point<ld>;

d41 struct Hp { // Half plane struct
      // 'p' is a passing point of the line and 'pq' is the
           direction vector of the line.
d41   P p, pq;
d41   ld angle;

d41   Hp() {}
d41   Hp(const P& a, const P& b) : p(a), pq(b - a) {
d41     angle = atan2l(pq.y, pq.x);
d41   }
d41   bool out(const P& r) { return pq.cross(r - p) < -eps; }
d41   bool operator < (const Hp& e) const {
d41     return angle < e.angle;
d41   }
d41   friend P inter(const Hp& s, const Hp& t) {
d41     ld alpha = (t.p - s.p).cross(t.pq) / s.pq.cross(t.pq);
d41     return s.p + (s.pq * alpha);
d41   }
d41 };

d41 vector<P> hp_intersect(vector<Hp>& H) {
d41   P box[4] = { P(inf, inf), P(-inf, inf),
d41                P(-inf, -inf), P(inf, -inf) };

d41   for(int i = 0; i<4; i++) {
d41     Hp aux(box[i], box[(i+1) % 4]);
d41     H.push_back(aux);
d41   }
d41   sort(all(H));
d41   deque<Hp> dq;
d41   int len = 0;
d41   for(int i = 0; i < sz(H); i++) {
d41     while(len>1 && H[i].out(inter(dq[len-1], dq[len-2]))){
d41       dq.pop_back();
d41       --len;
d41     }
d41     while (len > 1 && H[i].out(inter(dq[0], dq[1]))){
d41       dq.pop_front();
d41       --len;
d41     }
d41     if(len && fabsl(H[i].pq.cross(dq[len-1].pq)) < eps){
d41       if (H[i].pq.dot(dq[len-1].pq) < 0.0)
d41         return vector<P>();
d41       if (H[i].out(dq[len-1].p)) {
d41         dq.pop_back();
d41         --len;
d41       }
d41       else continue;
d41     }
d41     dq.push_back(H[i]);
d41     ++len;
d41   }

d41   while(len> 2 && dq[0].out(inter(dq[len-1], dq[len-2]))){
d41     dq.pop_back();
d41     --len;
d41   }
d41   while (len > 2 && dq[len-1].out(inter(dq[0], dq[1]))){
d41     dq.pop_front();
d41     --len;
d41   }
d41   if (len < 3) return vector<P>();
d41   vector<P> ret(len);
d41   for(int i = 0; i+1 < len; i++) {
d41     ret[i] = inter(dq[i], dq[i+1]);
d41   }
d41   ret.back() = inter(dq[len-1], dq[0]);
d41   return ret;
d41 }
```

## 8.2 Circles

### CircleIntersection.h

**Description:** Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" d41d8c, 12 lines

```
d41 typedef Point<double> P;
d41 bool circleInter(P a,P b,double r1,double r2,pair<P, P>*
      out) {
d41  if (a == b) { assert(r1 != r2); return false; }
d41  P vec = b - a;
d41  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2;
d41  double p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*
      d2;
d41  if (sum*sum < d2 || dif*dif > d2) return false;
d41  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) /
      d2);
d41  *out = {mid + per, mid - per};
d41  return true;
d41 }
```

### CircleTangents.h

**Description:** Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" d41d8c, 14 lines

```
d41 template<class P>
d41 vector<pair<P, P>> tangents(P c1, double r1, P c2, double
      r2) {
d41  P d = c2 - c1;
d41  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
d41  if (d2 == 0 || h2 < 0)  return {};
d41  vector<pair<P, P>> out;
d41  for (double sign : {-1, 1}) {
d41    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
d41    out.push_back({c1 + v * r1, c2 + v * r2});
d41  }
d41  if (h2 == 0) out.pop_back();
d41  return out;
d41 }
```

### CircleLine.h

**Description:** Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" d41d8c, 10 lines

```
d41 template<class P>
d41 vector<P> circleLine(P c, double r, P a, P b) {
d41  P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
d41  double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
d41  if (h2 < 0) return {};
d41  if (h2 == 0) return {p};
d41  P h = ab.unit() * sqrt(h2);
d41  return {p - h, p + h};
d41 }
```

### CirclePolygonIntersection.h

**Description:** Returns the area of the intersection of a circle with a ccw polygon.

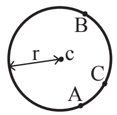
**Time:** $\mathcal{O}(n)$

"../../content/geometry/Point.h" d41d8c, 20 lines

```
d41 typedef Point<double> P;
d41 #define arg(p, q) atan2(p.cross(q), p.dot(q))
d41 double circlePoly(P c, double r, vector<P> ps) {
d41  auto tri = [&](P p, P q) {
d41    auto r2 = r * r / 2;
d41    P d = q - p;
d41    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist
      2();
d41    auto det = a * a - b;
d41    if (det <= 0) return arg(p, q) * r2;
d41    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det
      ));
d41    if (t < 0 || 1 <= s) return arg(p, q) * r2;
d41    P u = p + d * s, v = q + d * (t-1);
d41    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
d41  };
d41  auto sum = 0.0;
d41  rep(i,0,sz(ps))
d41    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
d41  return sum;
d41 }
```

### circumcircle.h

**Description:**




The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.

"Point.h" d41d8c, 10 lines

```
d41 typedef Point<double> P;
d41 double ccRadius(const P& A, const P& B, const P& C) {
d41  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
d41      abs((B-A).cross(C-A))/2;
d41 }
d41 P ccCenter(const P& A, const P& B, const P& C) {
d41  P b = C-A, c = B-A;
d41  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
d41 }
```

### MinimumEnclosingCircle.h

**Description:** Computes the minimum circle that encloses a set of points.

**Time:** expected $\mathcal{O}(n)$

"circumcircle.h" d41d8c, 18 lines

```
d41 pair<P, double> mec(vector<P> ps) {
d41  shuffle(all(ps), mt19937(time(0)));
d41  P o = ps[0];
d41  double r = 0, EPS = 1 + 1e-8;
d41  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
d41    o = ps[i], r = 0;
d41    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
d41      o = (ps[i] + ps[j]) / 2;
d41      r = (o - ps[i]).dist();
d41      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
d41        o = ccCenter(ps[i], ps[j], ps[k]);
d41        r = (o - ps[i]).dist();
d41      }
d41    }
d41  }
d41  return {o, r};
d41 }
```

## 8.3 Polygons

### InsidePolygon.h

**Description:** Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

**Usage:** `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`

**Time:** $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" d41d8c, 12 lines

```
d41  template<class P>
d41  bool inPolygon(vector<P> &p, P a, bool strict = true) {
d41   int cnt = 0, n = sz(p);
d41   rep(i,0,n) {
d41     P q = p[(i + 1) % n];
d41     if (onSegment(p[i], q, a)) return !strict;
        //or: if (segDist(p[i], q, a) <= eps) return !strict;
d41     cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) >
      0;
d41   }
d41   return cnt;
d41  }
```

### PolygonArea.h

**Description:** Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" d41d8c, 7 lines

```
d41  template<class T>
d41  T polygonArea2(vector<Point<T>>& v) {
d41   T a = v.back().cross(v[0]);
d41   rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
d41   return a;
d41  }
```

### PolygonCenter.h

**Description:** Returns the center of mass for a polygon.

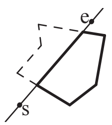
**Time:** $\mathcal{O}(n)$

"Point.h" d41d8c, 10 lines

```
d41  typedef Point<double> P;
d41  P polygonCenter(const vector<P>& v) {
d41   P res(0, 0); double A = 0;
d41   for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
d41     res = res + (v[i] + v[j]) * v[j].cross(v[i]);
d41     A += v[j].cross(v[i]);
d41   }
d41   return res / A / 3;
d41  }
```

### PolygonCut.h

**Description:**
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.




**Usage:** `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h" d41d8c, 14 lines

```
d41  typedef Point<double> P;
d41  vector<P> polygonCut(const vector<P>& poly, P s, P e) {
d41   vector<P> res;
d41   rep(i,0,sz(poly)) {
d41     P cur = poly[i], prev = i ? poly[i-1] : poly.back();
d41     auto a = s.cross(e, cur), b = s.cross(e, prev);
d41     if ((a < 0) != (b < 0))
d41       res.push_back(cur + (prev - cur) * (a / (a - b)));
d41     if (a < 0)
d41       res.push_back(cur);
d41   }
d41   return res;
d41  }
```

### PolygonUnion.h

**Description:** Calculates the area of the union of $n$ polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)

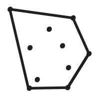
**Time:** $\mathcal{O}(N^2)$, where $N$ is the total number of points

"Point.h", "sideOf.h" d41d8c, 34 lines

```
d41  typedef Point<double> P;
d41  double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y
      ; }
d41  double polyUnion(vector<vector<P>>& poly) {
d41   double ret = 0;
d41   rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
d41     P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
d41     vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
d41     rep(j,0,sz(poly)) if (i != j) {
d41       rep(u,0,sz(poly[j])) {
d41         P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])
      ];
d41         int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
d41         if (sc != sd) {
d41           double sa = C.cross(D, A), sb = C.cross(D, B);
d41           if (min(sc, sd) < 0)
d41             segs.emplace_back(sa / (sa - sb), sgn(sc - sd))
      ;
d41         } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))
      >0){
d41           segs.emplace_back(rat(C - A, B - A), 1);
d41           segs.emplace_back(rat(D - A, B - A), -1);
d41         }
d41       }
d41     }
d41     sort(all(segs));
d41     for (auto& s : segs) s.first = min(max(s.first, 0.0), 1
      .0);
d41     double sum = 0;
d41     int cnt = segs[0].second;
d41     rep(j,1,sz(segs)) {
d41       if (!cnt) sum += segs[j].first - segs[j - 1].first;
d41       cnt += segs[j].second;
d41     }
d41     ret += A.cross(B) * sum;
d41   }
d41   return ret / 2;
d41  }
```

### ConvexHull.h

**Description:**
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull. If you want to keep the collinear points in the convex hull, change the comparison to $h[t-2].cross(h[t-1], p) < 0$ and the size of the vector $h$ to $2 * sz(pts) + 1$.



**Time:** $\mathcal{O}(n \log n)$

"Point.h" d41d8c, 14 lines

```
d41  typedef Point<ll> P;
d41  vector<P> convexHull(vector<P> pts) {
d41   if (sz(pts) <= 1) return pts;
d41   sort(all(pts));
d41   vector<P> h(sz(pts)+1);
d41   int s = 0, t = 0;
d41   for (int it = 2; it--; s = --t, reverse(all(pts)))
d41     for (P p : pts) {
d41       while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t
      --;
d41       h[t++] = p;
d41     }
d41   return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1
      ])};
d41  }
```

### HullDiameter.h

**Description:** Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

**Time:** $\mathcal{O}(n)$

"Point.h" d41d8c, 13 lines

```
d41  typedef Point<ll> P;
d41  array<P, 2> hullDiameter(vector<P> S) {
d41   int n = sz(S), j = n < 2 ? 0 : 1;
d41   pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
d41   rep(i,0,j)
d41     for (;; j = (j + 1) % n) {
d41       res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}})
      ;
d41       if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >=
        0)
d41         break;
d41     }
d41   return res.second;
d41  }
```

### PointInsideHull.h

**Description:** Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

**Time:** $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" d41d8c, 15 lines

```
d41  typedef Point<ll> P;

d41  bool inHull(const vector<P>& l, P p, bool strict = true) {
d41   int a = 1, b = sz(l) - 1, r = !strict;
d41   if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
d41   if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
d41   if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<=
      -r)
d41     return false;
d41   while (abs(a - b) > 1) {
d41     int c = (a + b) / 2;
d41     (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
d41   }
d41   return sgn(l[a].cross(l[b], p)) < r;
d41  }
```

### LineHullIntersection.h

**Description:** Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner $i$, • $(i,i)$ if along side $(i,i+1)$, • $(i,j)$ if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner $i$ is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.

**Time:** $\mathcal{O}(\log n)$

"Point.h" d41d8c, 40 lines

```
d41  #define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)
      %n]))
d41  #define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) <
      0
d41  template <class P> int extrVertex(vector<P>& poly, P dir)
      {
d41   int n = sz(poly), lo = 0, hi = n;
d41   if (extr(0)) return 0;
d41   while (lo + 1 < hi) {
d41     int m = (lo + hi) / 2;
```

```
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) =
  m;
  }
  return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
 int endA = extrVertex(poly, (a - b).perp());
 int endB = extrVertex(poly, (b - a).perp());
 if (cmpL(endA) < 0 || cmpL(endB) > 0)
   return {-1, -1};
 array<int, 2> res;
 rep(i,0,2) {
   int lo = endB, hi = endA, n = sz(poly);
   while ((lo + 1) % n != hi) {
     int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
     (cmpL(m) == cmpL(endB) ? lo : hi) = m;
   }
   res[i] = (lo + !cmpL(hi)) % n;
   swap(endA, endB);
 }
 if (res[0] == res[1]) return {res[0], -1};
 if (!cmpL(res[0]) && !cmpL(res[1]))
   switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
     case 0: return {res[0], res[0]};
     case 2: return {res[1], res[1]};
   }
 return res;
}
```

## Minkowski.h

**Description:** Computes the Minkowski sum of two convex polygons. Polygons must be convex and given in CCW order. Returns the vertices of the Minkowski sum polygon in CCW order.
**Time:** $\mathcal{O}(n+m)$

"Point.h" d41d8c, 24 lines

```
using P = Point<ll>;

vector<P> minkowski(vector<P> p, vector<P> q) {
  auto fix = [](vector<P>& A) {
    int pos = 0;
    for (int i = 1; i < sz(A); i++) {
      if(A[i].y < A[pos].y || (A[i].y == A[pos].y && A[i].
  x < A[pos].x))
        pos = i;
    }
    rotate(A.begin(), A.begin() + pos, A.end());
    A.push_back(A[0]), A.push_back(A[1]);
  };
  fix(p), fix(q);
  vector<P> result;
  int i = 0, j = 0;
  while (i < sz(p) - 2 || j < sz(q) - 2) {
    result.push_back(p[i] + q[j]);
    auto cross = (p[i + 1] - p[i]).cross(q[j + 1] - q[j]);
    if (cross >= 0 && i < sz(p) - 2) i++;
    if (cross <= 0 && j < sz(q) - 2) j++;
  }
  return result;
}
```

## Extreme.h

**Description:** Finds an extreme vertex of a convex polygon according to a unimodal comparator. The comparator defines a total order along the polygon (given in CCW order).
**Time:** $\mathcal{O}(\log n)$

"Point.h" d41d8c, 26 lines

```
using P = Point<ll>;
int extreme(vector<P> &pol, const function<bool(P, P)>&
cmp) {
  int n = pol.size();
  auto extr = [&](int i, bool& cur_dir) {
    cur_dir = cmp(pol[(i+1)%n], pol[i]);
    return !cur_dir and !cmp(pol[(i+n-1)%n], pol[i]);
  };
  bool last_dir, cur_dir;
  if (extr(0, last_dir)) return 0;
  int l = 0, r = n;
  while (l+1 < r) {
    int m = (l+r)/2;
    if (extr(m, cur_dir)) return m;
    bool rel_dir = cmp(pol[m], pol[l]);
    if ((!last_dir and cur_dir) or
        (last_dir == cur_dir and rel_dir == cur_dir)) {
      l = m;
      last_dir = cur_dir;
    } else r = m;
  }
  return l;
}
int max_dot(vector<P> &pol, P v) {
  return extreme([&](P p, P q) { return p.dot(v) > q.dot(v
); });
}
```

## Tangents.h

**Description:** Finds the left and right tangent points from an external point p to a convex polygon given in CCW order. A tangent point is a vertex where the segment p->v touches the polygon without intersecting its interior, defining the limits of visibility from p. Returns the indices of the left and right tangent vertices.
**Time:** $\mathcal{O}(\log n)$

"Point.h", "Extreme.h" d41d8c, 11 lines

```
using P = Point<ll>;

bool ccw(P p, P q, P r) {
 return (q-p).cross(r-q) > 0;
}
pair<int, int> tangents(vector<P> &pol, P p) {
    auto L = [&](P q, P r) { return ccw(p, r, q); };
    auto R = [&](P q, P r) { return ccw(p, q, r); };
    return {extreme(pol, L), extreme(pol, R)};
}
```

# 8.4 Misc. Point Set Problems

## ClosestPair.h

**Description:** Finds the closest pair of points.
**Time:** $\mathcal{O}(n \log n)$

"Point.h" d41d8c, 18 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
 assert(sz(v) > 1);
 set<P> S;
 sort(all(v), [](P a, P b) { return a.y < b.y; });
 pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
 int j = 0;
 for (P p : v) {
   P d{1 + (ll)sqrt(ret.first), 0};
   while (v[j].y <= p.y - d.x) S.erase(v[j++]);
   auto lo = S.lower_bound(p - d), hi = S.upper_bound(p +
 d);
   for (; lo != hi; ++lo)
     ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
   S.insert(p);
 }
 return ret.second;
}
```

## ManhattanMST.h

**Description:** Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights w(p, q) = —p.x - q.x— + —p.y - q.y—. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
**Time:** $\mathcal{O}(N \log N)$

"Point.h" d41d8c, 24 lines

```
typedef Point<int> P;
vector<array<int, 3>> manhattanMST(vector<P> ps) {
 vi id(sz(ps));
 iota(all(id), 0);
 vector<array<int, 3>> edges;
 rep(k,0,4) {
   sort(all(id), [&](int i, int j) {
        return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
   map<int, int> sweep;
   for (int i : id) {
     for (auto it = sweep.lower_bound(-ps[i].y);
               it != sweep.end(); sweep.erase(it++)) {
       int j = it->second;
       P d = ps[i] - ps[j];
       if (d.y > d.x) break;
       edges.push_back({d.y + d.x, i, j});
     }
     sweep[-ps[i].y] = i;
   }
   for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p
.y);
 }
 return edges;
}
```

## kdTree.h

**Description:** KD-tree (2d, can be extended to 3d)

"Point.h" d41d8c, 64 lines

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
 P pt; // if this is a leaf, the single point in it
 T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
 Node *first = 0, *second = 0;

 T distance(const P& p) { // min squared distance to a
point
   T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
   T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
   return (P(x,y) - p).dist2();
 }

 Node(vector<P>&& vp) : pt(vp[0]) {
   for (P p : vp) {
     x0 = min(x0, p.x); x1 = max(x1, p.x);
     y0 = min(y0, p.y); y1 = max(y1, p.y);
```

```cpp
    }
    if (vp.size() > 1) {
      // split on x if width >= height (not ideal...)
      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = sz(vp)/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    }
  }
};

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }

  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
};
```

### FastDelaunay.h

**Description:** Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], . . . }, all counter-clockwise.
**Time:** $\mathcal{O}(n \log n)$

"Point.h" d41d8c, 89 lines

```cpp
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
  Q rot, o; P p = arb; bool mark;
  P& F() { return r()->p; }
  Q& r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}
Q makeEdge(P orig, P dest) {
  Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
  H = r->o; r->r()->r() = r;
  rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
  r->p = orig; r->F() = dest;
  return r;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e->o = H; H = e; e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  }
  return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
  sort(all(pts));  assert(unique(all(pts)) == pts.end());
  if (sz(pts) < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
  q.push_back(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  return pts;
}
```

## 8.5 3D

### PolyhedronVolume.h

**Description:** Magic formula for the volume of a polyhedron. Faces should point outwards.

d41d8c, 7 lines

```cpp
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
}
```

### Point3D.h

**Description:** Class to handle points in 3D space. T can be e.g. double or long long.

d41d8c, 33 lines

```cpp
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

## 3dHull.h

**Description:** Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
**Time:** $\mathcal{O}(n^2)$

"Point3D.h" d41d8c, 50 lines

```
d41 typedef Point3D<double> P3;

d41 struct PR {
d41  void ins(int x) { (a == -1 ? a : b) = x; }
d41  void rem(int x) { (a == x ? a : b) = -1; }
d41  int cnt() { return (a != -1) + (b != -1); }
d41  int a, b;
d41 };

d41 struct F { P3 q; int a, b, c; };

d41 vector<F> hull3d(const vector<P3>& A) {
d41  assert(sz(A) >= 4);
d41  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
d41 #define E(x,y) E[f.x][f.y]
d41  vector<F> FS;
d41  auto mf = [&](int i, int j, int k, int l) {
d41    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
d41    if (q.dot(A[l]) > q.dot(A[i]))
d41      q = q * -1;
d41    F f{q, i, j, k};
d41    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
d41    FS.push_back(f);
d41  };
d41  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
d41    mf(i, j, k, 6 - i - j - k);

d41  rep(i,4,sz(A)) {
d41    rep(j,0,sz(FS)) {
d41      F f = FS[j];
d41      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
d41        E(a,b).rem(f.c);
d41        E(a,c).rem(f.b);
d41        E(b,c).rem(f.a);
d41        swap(FS[j--], FS.back());
d41        FS.pop_back();
d41      }
d41    }
d41    int nw = sz(FS);
d41    rep(j,0,nw) {
d41      F f = FS[j];
d41 #define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i,
    f.c);
d41      C(a, b, c); C(a, c, b); C(b, c, a);
d41    }
d41  }
d41  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
d41    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
d41  return FS;
d41 };
```

## sphericalDistance.h

**Description:** Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 ($\phi_1$) and f2 ($\phi_2$) from x axis and zenith angles (latitude) t1 ($\theta_1$) and t2 ($\theta_2$) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

d41d8c, 9 lines

```
d41 double sphericalDistance(double f1, double t1,
d41    double f2, double t2, double radius) {
d41  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
d41  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
d41  double dz = cos(t2) - cos(t1);
d41  double d = sqrt(dx*dx + dy*dy + dz*dz);
d41  return radius*2*asin(d/2);
d41 }
```

# Strings (9)

## AhoCorasick.h

d41d8c, 46 lines

```
d41 int trie[ms][sigma], fail[ms], terminal[ms], superfail[ms
    ];
d41 bool present[ms];
d41 int z = 1;

d41 int val(char c) { return c - 'a'; }

d41 void add(string& p) {
d41     int cur = 0;
d41     for (int i = 0; i < (int)p.size(); i++) {
d41         int& nxt = trie[cur][val(p[i])];
d41         if (nxt == 0) nxt = z++;
d41         cur = nxt;
d41     }
d41     present[cur] = true;
d41     terminal[cur]++;
d41 }

d41 void build() {
d41     queue<int> q;
d41     for (q.push(0); !q.empty(); q.pop()) {
d41         int on = q.front();
d41         for (int i = 0; i < sigma; i++) {
d41             int& to = trie[on][i];
d41             int f = (on == 0 ? 0 : trie[fail[on]][i]);
d41             int sf = (present[f] ? f : superfail[f]);
d41             if (!to) {
d41                 to = f;
d41             }
d41             else {
d41                 fail[to] = f;
d41                 superfail[to] = sf;
                    // merge infos (ex: terminal[to] +=
                        terminal[f])
d41                 q.push(to);
d41             }
d41         }
d41     }
d41 }

d41 void search(string& s) {
d41     int cur = 0;
d41     for (char c : s) {
d41         cur = trie[cur][val(c)];
            // process infos on current node (ex: ocurrences
                += terminal[cur])
d41     }
d41 }
```

## Hash.h

**Description:** $C$ can also be random, operator is $[l, r]$

d41d8c, 28 lines

```
d41 using ull = uint64_t;
d41 struct H {
d41     ull x; H(ull x = 0) : x(x) {}
d41     H operator+(H o) { return x + o.x + (x + o.x < x); }
d41     H operator-(H o) { return *this + ~o.x; }
d41     H operator*(H o) {
d41         auto m = (__uint128_t)x * o.x;
d41         return H((ull)m) + (ull)(m >> 64);
d41     }
d41     ull get() const { return x + !~x; }
d41     bool operator==(H o) const{ return get() == o.get();}
d41     bool operator<(H o) const{ return get() < o.get();}
d41 };
d41 static const H C = (ll)1e11 + 3;
d41 struct Hash {
d41     vector<H> h, pw;
d41     Hash(string& str) : h(str.size()), pw(str.size()) {
d41         pw[0] = 1, h[0] = str[0];
d41         for (int i = 1; i < str.size(); i++) {
d41             h[i] = h[i - 1] * C + str[i];
d41             pw[i] = pw[i - 1] * C;
d41         }
d41     }
d41     H operator()(int l, int r) {
d41         return h[r] - (l ? h[l - 1] * pw[r - l + 1] : 0);
d41     }
d41 };
```

## Kmp.h

**Description:** pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123).

d41d8c, 10 lines

```
d41 vector<int> pi(const string& s) {
d41     vector<int> p(sz(s));
d41     for(int i = 1; i < sz(s); i++) {
d41         int g = p[i-1];
d41         while (g && s[i] != s[g]) g = p[g-1];
d41         p[i] = g + (s[i] == s[g]);
d41     }
d41     return p;
d41 }
```

## KmpAutomaton.h

**Description:** $go[i][j]$ = length of the longest prefix of $s$ that is a suffix of $s[0..i]$ followed by the letter $j$ (i.e., the next matched prefix length if, at state $i$, we read letter $j$).

d41d8c, 17 lines

```
d41 int go[ms][sigma];
d41 int val(char c) { return c - 'a'; }
d41 void automaton(string& s) {
d41     for (int i = 0; i < sigma; i++)
d41         go[0][i] = (i == val(s[0]));

d41     for (int i = 1, bdr = 0; i <= (int)s.size(); i++) {
d41         for (int j = 0; j < sigma; j++) {
d41             go[i][j] = go[bdr][j];
d41         }
d41         if (i < (int)s.size()) {
d41             go[i][val(s[i])] = i + 1;
d41             bdr = go[bdr][val(s[i])];
d41         }
d41     }
d41 }
```

## Manacher.h

**Description:** $p[0][i+1]$ is the length of matches of even length palindrome, starting from $[i, i+1]$.
$p[1][i]$ is the length of matches of odd length palindrome, starting from $[i, i]$.
(abaxx -> p[0] = 00001)
(abaxx -> p[1] = 01000)

d41d8c, 17 lines

```
d41 array<vector<int>, 2> manacher(const string& s) {
d41     int n = sz(s);
```

```
d41     array<vector<int>,2> p={vector<int>(n+1),vector<int>(n
  )};
d41     for (int z = 0; z < 2; z++) {
d41         for (int i = 0, l = 0, r = 0; i < n; i++) {
d41             int t = r - i + !z;
d41             if (i < r) p[z][i] = min(t, p[z][l + t]);
d41             int L = i - p[z][i], R = i + p[z][i] - !z;
d41             while(L >= 1 && R+1 < n && s[L-1] == s[R+1]){
d41                 p[z][i]++, L--, R++;
d41             }
d41             if (R > r) l = L, r = R;
d41         }
d41     }
d41     return p;
d41 }
```

### MinRotation.h

**Description:** Finds the lexicographically smallest rotation of a string.
**Usage:** `rotate(s.begin(), s.begin()+minRotation(s), s.end());`
**Time:** $\mathcal{O}(N)$

d41d8c, 14 lines

```
d41 int minRotation(string s) {
d41     int a = 0, N = s.size(); s += s;
d41     for (int b = 0; b < N; b++) {
d41         for (int k = 0; k < N; k++) {
d41             if (a+k == b || s[a+k] < s[b+k]) {
d41                 b += max(0, k-1);
d41                 break;
d41             }
d41             if (s[a+k] > s[b+k]) { a = b; break; }
d41         }
d41     }
d41     return a;
d41 }
```

### SuffixArray.h

**Description:** $lcp[i]$ is the length of the longest common prefix between the suffixes $s[sa[i]..n-1]$ and $s[sa[i-1]..n-1]$.
If we concatenate multiple strings using separator characters, the separator that appears furthest to the right must be the smallest character in the alphabet.

d41d8c, 31 lines

```
d41 struct SuffixArray {
d41  vector<int> sa, lcp;
d41  SuffixArray(string s, int lim=256) {
d41    s.push_back('$');
d41    int n = sz(s), k = 0, a, b;
d41    vector<int> x(all(s)), y(n), ws(max(n, lim));
d41    sa = lcp = y, iota(all(sa), 0);
d41    for(int j = 0, p = 0; p < n; j= max(1, j*2), lim = p) {
d41      p = j, iota(all(y), n - j);
d41      for(int i=0; i<n; i++){
d41        if (sa[i] >= j) y[p++] = sa[i] - j;
d41      }
d41      fill(all(ws), 0);
d41      for(int i=0; i<n; i++) ws[x[i]]++;
d41      for(int i=1; i<lim; i++) ws[i] += ws[i - 1];
d41      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
d41      swap(x, y), p = 1, x[sa[0]] = 0;
d41      for(int i=1; i<n; i++){
d41        a = sa[i - 1], b = sa[i];
d41        x[b] = p-1;
d41        if(y[a] != y[b] || y[a+j] != y[b+j]) x[b] = p++;
d41      }
d41    }
d41    for (int i = 0, j; i < n - 1; lcp[x[i++]] = k)
d41      for (k && k--, j = sa[x[i] - 1];
d41          s[i + k] == s[j + k]; k++);
d41    sa = vector<int>(sa.begin() + 1, sa.end());
d41    lcp = vector<int>(lcp.begin() + 1, lcp.end());
d41  }
d41  };
```

### Zfunc.h

**Description:** z[i] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)

d41d8c, 13 lines

```
d41 vector<int> ZFunc(const string& s) {
d41     int n = sz(s), a = 0, b = 0;
d41     vector<int> z(n, 0);
d41     if (!z.empty()) z[0] = 0;
d41     for (int i = 1; i < n; i++) {
d41         int end = i;
d41         if (i < b) end = min(i + z[i - a], b);
d41         while (end < n && s[end] == s[end - i]) ++end;
d41         z[i] = end - i; if (end > b) a = i, b = end;
d41     }
d41     return z;
d41 }
```

# Various (10)

## 10.1 Misc. algorithms

### Dates.h

**Description:** dateToInt converts Gregorian date to integer (Julian day number). intToDate converts integer (Julian day number) to Gregorian date: month/day/year. intToDay converts Julian day number to day of the week

d41d8c, 23 lines

```
d41 string day[] = { "Mon", "Tue", "Wed", "Thu", "Fri", "Sat",
     "Sun" };
d41 int dateToInt(int m, int d, int y) {
d41     return
d41         1461 * (y + 4800 + (m - 14) / 12) / 4 +
d41         367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
d41         3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
d41         d - 32075;
d41 }
d41 void intToDate(int jd, int& m, int& d, int& y) {
d41     int x, n, i, j;
d41     x = jd + 68569;
d41     n = 4 * x / 146097;
d41     x -= (146097 * n + 3) / 4;
d41     i = (4000 * (x + 1)) / 1461001;
d41     x -= 1461 * i / 4 - 31;
d41     j = 80 * x / 2447;
d41     d = x - 2447 * j / 80;
d41     x = j / 11;
d41     m = j + 2 - 12 * x;
d41     y = 100 * (n - 49) + i + x;
d41 }
d41 string intToDay(int jd) { return day[jd % 7]; }
```

### MultisetHash.h

d41d8c, 8 lines

```
d41 ull hashify(ull sum) {
d41     sum += FIXED_RANDOM;
d41     sum += 0x9e3779b97f4a7c15;
d41     sum = (sum ^ (sum >> 30)) * 0xbf58476d1ce4e5b9;
d41     sum = (sum ^ (sum >> 27)) * 0x94d049bb133111eb;
d41     return sum ^ (sum >> 31);
d41 }
```

### Rand.h

d41d8c, 8 lines

```
d41 mt19937 rng(chrono::steady_clock::now().time_since_epoch()
     .count());
     // _64

d41 int uniform(int l, int r) { // [l, r]
d41     uniform_int_distribution<int> uid(l, r);
d41     return uid(rng);
d41 }
```

## 10.2 Dynamic programming

### KnuthDP.h

**Description:** When doing DP on intervals: $dp[i][j] = \min_{i<k<j}(dp[i][k] + dp[k][j]) + f(i,j)$, where the (minimal) optimal $k$ increases with both $i$ and $j$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \le f(a,d) + f(b,c)$ for all $a \le b \le c \le d$. Another sufficient criteria is: $opt[i][j-1] \le opt[i][j] \le opt[i+1][j]$
**Time:** $\mathcal{O}(N^2)$

d41d8c, 22 lines

```
d41 ll knuth(){
d41     memset(opt, -1, sizeof opt);
d41     for(int i=n-1; i>=0; i--){
d41         dp[i][i] = 0; // base case
d41         opt[i][i] = i;
d41         for(int j=i+1; j<n; j++){
d41             int optL = (!j ? 0 : opt[i][j-1]);
d41             int optR = (~opt[i+1][j] ? opt[i+1][j] : n-1);
d41             ll cst = cost(i, j);
d41             dp[i][j] = INF;
d41             optL = max(i, optL), optR = min(j-1, optR);
d41             for(int k=optL; k<=optR; k++){
d41                 ll now = dp[i][k] + dp[k+1][j] + cst;
d41                 if(now <= dp[i][j]){
d41                     dp[i][j] = now;
d41                     opt[i][j] = k;
d41                 }
d41             }
d41         }
d41     }
d41 }
```

### DivideAndConquerDP.h

**Description:** Divide and Conquer DP maintaining cost, can be used when $opt[i][j] \le opt[i][j+1]$. In this code everything is 1-based. Memory can be optmized by keeping only the last row
**Time:** $\mathcal{O}(MN \log N)$

d41d8c, 42 lines

```
d41 void add(int idx) {}
d41 void rem(int idx) {}

d41 void deC(int i, int l, int r, int optL, int optR) {
d41     if (l > r) return;
d41     int j = (l + r) / 2;
d41     for (int k = r; k > j; k--) rem(k);
d41     int opt = optL;
d41     for (int k = optL; k <= min(optR, j); k++) {
            // cost = cost[k, j]
d41         int val = dp[i - 1][k - 1] + cost;
d41         if (val < dp[i][j]) {
d41             dp[i][j] = val;
d41             opt = k;
d41         }
d41         rem(k);
d41     }
d41     for (int k = min(optR, j); k >= optL; k--) add(k);
d41     rem(j);
d41     deC(i, l, j - 1, optL, opt);

d41     for (int k = j; k <= r; k++) add(k);
d41     for (int k = optL; k < opt; k++) rem(k);
d41     deC(i, j + 1, r, opt, optR);
```

```
d41         for (int k = optL; k < opt; k++) add(k);
d41     }

d41     int solve(int N, int M) { // 1-based
d41         for (int i = 0; i <= M; i++) {
d41             for (int j = 0; j <= N; j++){
d41                 dp[i][j] = inf; // base case
d41             }
d41         }
d41         cost = 0; // neutral value
d41         for (int i = 1; i <= N; i++) add(i);
d41         for (int i = 1; i <= M; i++) {
d41             deC(i, 1, N, 1, N);
d41         }
d41         return dp[M][N];
d41     }
```

## 10.3 Optimization tricks

### 10.3.1 Bit hacks

- `for (int x = m; x; x = (x - 1)&m ) { ... }` loops over all subset masks of `m` (except 0).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
  `  if (i & 1 << b) D[i] += D[i^(1 << b)];`
  computes all sums of subsets.

### 10.3.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC target("bmi,bmi2,popcnt,lzcnt")` improve bit operations.
- `#pragma GCC optimize("unroll-loops")` self explanatory.