Universidade Federal de Pernambuco

las4s e pelados

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Contest (1)

template.cpp

8 lines

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
using ll = long long;
using pii = pair<int,int>;
```

.bashrc

2 lines

```
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \
  -fsanitize=undefined,address'
```

hash.sh

2 lines

```
# bash hash.sh file.cpp l1 l2
sed -n $2','$3' p' $1 | sed '/^#w/d' | cpp -dD -P -
    fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

troubleshoot.txt

52 lines

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
```

Mathematics (2)

2.1 Recurrences

If $a_n = c_1 a_{n-1} + \dots + c_k a_{n-k}$, and $r_1, \dots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \dots - c_k$, there are $d_1, \dots, d_k$ s.t.

$$a_n = d_1 r_1^n + \dots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.2 Trigonometry

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

2.3 Geometry

2.3.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1 - \left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\dfrac{\sin \alpha}{a} = \dfrac{\sin \beta}{b} = \dfrac{\sin \gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan \dfrac{\alpha+\beta}{2}}{\tan \dfrac{\alpha-\beta}{2}}$

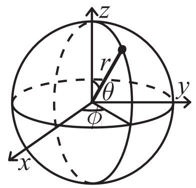
2.3.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F \tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180°, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.3.3 Spherical coordinates



$$x = r \sin\theta \cos\phi \qquad r = \sqrt{x^2 + y^2 + z^2}$$
$$y = r \sin\theta \sin\phi \qquad \theta = \operatorname{acos}(z/\sqrt{x^2+y^2+z^2})$$
$$z = r \cos\theta \qquad \phi = \operatorname{atan2}(y, x)$$

2.3.4 Pick's Theorem

The area of a simple polygon whose vertices have integer coordinates is:

$$A = I + \frac{B}{2} - 1$$

where I is the number of interior integer points, and B is the number of integer points in the border of the polygon.

2.3.5 Two Ears Theorem

Every simple polygon with more than 3 vertices has at least two non-overlapping ears (a ear is a vertex whose diagonal induced by its neighbors which lies strictly inside the polygon). Equivalently, every simple polygon can be triangulated.

2.4 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x \sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$

$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\operatorname{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.5 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1},\ c \neq 1$$

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$\sum_{i=0}^{n} ic^i = \frac{nc^{n+2} - (n+1)c^{n+1} + c}{(c-1)^2},\ c \neq 1$$

$$g_k(n) = \sum_{i=1}^{n} i^k = \frac{1}{k+1}\left(n^{k+1} + \sum_{j=1}^{k}\binom{k+1}{j+1}(-1)^{j+1}g_{k-j}(n)\right)$$

2.6 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots,\ (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots,\ (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots,\ (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots,\ (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots,\ (-\infty < x < \infty)$$

$$\sum_{i=0}^{\infty} ic^i = \frac{c}{(1-c)^2}, \quad |c| < 1$$

2.7 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.7.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n,p)$, $n = 1, 2, \ldots,\ 0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n,p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\mathrm{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

Data structures (3)

Bit.h

Description: *lower_bound* works the same as on vectors

Time: $\mathcal{O}(\log N)$

d41d8c, 23 lines

```
d41 struct Bit {
d41   vector<ll> bit;
d41   Bit(int n) : bit(n + 1) {}
d41   void update(int i, ll v) {
d41     for (i++; i < sz(bit); i += i & -i) bit[i] += v;
d41   }
d41   ll query(int i) {
d41     ll ret = 0;
d41     for (i++; i > 0; i -= i & -i) ret += bit[i];
d41     return ret;
d41   }
d41   int lower_bound(ll v){ // min pos st sum[0, pos] >= v
d41     int pos = 0;
d41     for(int j=(1 << 23); j >= 1; j/=2){
d41       if(pos+j < sz(bit) && bit[pos + j] < v){
d41         pos += j;
d41         v -= bit[pos];
d41       }
d41     }
d41     return pos;
d41   }
d41 };
```

Bit2d.h

Description: Points called on the update function NEED to be on the *pts* vector parameter on build.

Time: $\mathcal{O}\left((\log N)^2\right)$

"Bit.h" d41d8c, 37 lines

```
d41 struct Bit2d {
d41   vector<vector<int>> ys;
d41   vector<Bit> bit;
d41   vector<int> cmp_x;
d41   Bit2d(){}
d41   void put(int x, int y) {
d41     for (x++; x < sz(ys); x += x & -x) ys[x].push_back(y);
d41   }
d41   int id(const vector<int> &v, int y) {
d41     return (upper_bound(all(v), y) - v.begin()) - 1;
d41   }
d41   void build(vector<pii> pts) {
d41     sort(all(pts));
d41     for(auto p : pts) cmp_x.push_back(p.first);
d41     cmp_x.erase(unique(all(cmp_x)), cmp_x.end());
d41     ys.resize(cmp_x.size() + 1);
d41     for(auto p : pts) put(id(cmp_x, p.first), p.second);
d41     for(auto &v:ys)sort(all(v)), bit.emplace_back(sz(v));
d41   }
d41   void update(int x, int y, int val){
d41     x = id(cmp_x, x);
```

```
    for(x++; x < sz(ys); x+= x&-x)
      bit[x].update(id(ys[x], y), val);
  }
  int query(int x, int y){
    x = id(cmp_x, x);
    int ret = 0;
    for(x++; x > 0; x-= x&-x)
      ret += bit[x].query(id(ys[x], y));
    return ret;
  }
  int query(int x1, int y1, int x2, int y2){
    int a = query(x2, y2)-query(x2, y1-1);
    return a-query(x1-1, y2)+query(x1-1, y1-1);
  }
};
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$

d41d8c, 32 lines

```
struct Line {
 mutable ll k, m, p;
 bool operator<(const Line& o) const { return k < o.k; }
 bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
 // (for doubles, use inf = 1/.0, div(a,b) = a/b)
 static const ll inf = LLONG_MAX;
 ll div(ll a, ll b) { // floored division
   return a / b - ((a ^ b) < 0 && a % b); }
 bool isect(iterator x, iterator y) {
   if (y == end()) return x->p = inf, 0;
   if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
   else x->p = div(y->m - x->m, x->k - y->k);
   return x->p >= y->p;
 }
 void add(ll k, ll m) {
   auto z = insert({k, m, 0}), y = z++, x = y;
   while (isect(y, z)) z = erase(z);
   if (x != begin() && isect(--x, y))
     isect(x, y = erase(y));
   while ((y = x) != begin() && (--x)->p >= y->p)
     isect(x, erase(y));
 }
 ll query(ll x) {
   assert(!empty());
   auto l = *lower_bound(x);
   return l.k * x + l.m;
 }
};
```

Mo.h

Description: For subtree queries, perform an Euler tour and map each node u to the interval $[tin[u], tin[u] + subtree\_size[u] - 1]$. A subtree query becomes a range query over this interval.
For path queries between nodes U and V, Let U be the closest to the root. If V lies in U's subtree, the path corresponds to the interval $[tin[U], tin[V]]$. Otherwise, the path corresponds to the interval $[min(tout[U], tout[V]), max(tin[U], tin[V])]$.
In both cases, nodes on the U–V path appear exactly once in the interval, while all other nodes appear either 0 or 2 times.
Usage: `queries.push(Query(l, r, index of query)), intervals are [l, r]`
Time: $\mathcal{O}\left(N\sqrt(Q)\right)$

d41d8c, 44 lines

```
inline int64_t hilOrd(int x, int y, int pow, int rot) {
  if (pow == 0) return 0;
  int hpow = 1 << (pow - 1);
  int seg = (x < hpow) ? ((y < hpow) ? 0 : 3) : ((y < hpow
) ? 1 : 2);
  seg = (seg + rot) & 3;
  const int rotDelta[4] = { 3, 0, 0, 1 };
  int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
  int nrot = (rot + rotDelta[seg]) & 3;
  int64_t sub = int64_t(1) << (2 * pow - 2);
  int64_t ans = seg * sub;
  int64_t add = hilOrd(nx, ny, pow - 1, nrot);
  ans += (seg == 1 || seg == 2) ? add : (sub - add - 1);
  return ans;
}

struct Query {
  int l, r, idx;
  int64_t ord;
  Query(int l, int r, int idx) : l(l), r(r), idx(idx) {
    ord = hilOrd(l, r, 21, 0);
  }
  bool operator < (const Query& other) const {
    return ord < other.ord;
  }
};

vector<Query> queries;
int ans[ms];
void put(int x) {} // F
void remove(int x) {} // F
int getAns() {}

void Mo() {
  int l = 0, r = -1;
  sort(queries.begin(), queries.end());
  for (Query q : queries) {
    while (l > q.l) put(--l);
    while (r < q.r) put(++r);
    while (l < q.l) remove(l++);
    while (r > q.r) remove(r--);
    ans[q.idx] = getAns();
  }
}
```

MoUpdate.h

Description: Block size should be around $(2*N*N)^{\frac{1}{3}}$
Usage: `intervals are [l, r], addQuery(l, r, number of updates happened before this query, index of query), addUpdate(index of updated position, value before update, value after update)`
Time: $\mathcal{O}\left(Q*(2*N*N)^{\frac{1}{3}}*F\right)$

d41d8c, 55 lines

```
const int B = 2700;
struct MoUpdate {
  struct Query {
    int l, r, t, idx;
    Query(int l, int r, int t, int idx)
      : l(l), r(r), t(t), idx(idx) {}
    bool operator < (const Query& p) const {
      if (l / B != p.l / B) return l < p.l;
      if (r / B != p.r / B) return r < p.r;
      return t < p.t;
    }
  };
  struct Upd {
    int i, old, now;
    Upd(int i, int old, int now): i(i),old(old),now(now){}
  };

  vector<Query> queries;
  vector<Upd> updates;

  void addQuery(int l, int r, int t, int idx) {
    queries.push_back(Query(l, r, t, idx)); }
  void addUpdate(int i, int old, int now) {
    updates.push_back(Upd(i, old, now)); }

  void add(int x) {} // F
  void rem(int x) {} // F
  int getAns() {}
  void update(int novo, int idx, int l, int r) {
    if (l <= idx && idx <= r) rem(idx);
    arr[idx] = novo;
    if (l <= idx && idx <= r) add(idx);
  }

  void solve() {
    int l = 0, r = -1, t = 0;
    sort(queries.begin(), queries.end());
    for (Query q : queries) {
      while (l > q.l) add(--l);
      while (r < q.r) add(++r);
      while (l < q.l) rem(l++);
      while (r > q.r) rem(r--);
      while (t < q.t) {
        auto u = updates[t++];
        update(u.now, u.i, l, r);
      }
      while (t > q.t) {
        auto u = updates[--t];
        update(u.old, u.i, l, r);
      }
      ans[q.idx] = getAns();
    }
  }
};
```

SegmentTree.h

Description: Zero-indexed max-tree. Bounds are inclusive to the left and inclusive to the right. Can be changed by modifying T, f and unit.
Time: $\mathcal{O}(\log N)$

d41d8c, 21 lines

```
struct Tree {
 typedef int T;
 static constexpr T unit = INT_MIN;
 T f(T a, T b) { return max(a, b); } // (any associative
fn)
 vector<T> s; int n;
 Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {}
 void update(int pos, T val) {
    for (s[pos += n] = val; pos /= 2;)
      s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
 }
 T query(int b, int e) { // query [b, e]
    e++;
   T ra = unit, rb = unit;
   for (b += n, e += n; b < e; b /= 2, e /= 2) {
     if (b % 2) ra = f(ra, s[b++]);
     if (e % 2) rb = f(s[--e], rb);
   }
   return f(ra, rb);
 }
};
```

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
Time: $\mathcal{O}(\log N)$

d41d8c, 17 lines

```
d41 #include <bits/extc++.h>
d41 using namespace __gnu_pbds;

d41 template<class T>
d41 using Tree = tree<T, null_type, less<T>, rb_tree_tag,
d41     tree_order_statistics_node_update>;

d41 void example() {
d41  Tree<int> t, t2; t.insert(8);
d41  auto it = t.insert(10).first;
d41  assert(it == t.lower_bound(9));
d41  assert(t.order_of_key(10) == 1);
d41  assert(t.order_of_key(11) == 2);
d41  assert(*t.find_by_order(0) == 8);
d41  t.join(t2); // merge t2 into t
d41 }
```

PersistentSegTree.h

Usage: `SegP(size of the segtree, number of updates)`

`roots = {0}, newRoot = update(roots.back(),...),`
`roots.push(newRoot)`

d41d8c, 42 lines

```
d41 struct SegP {
d41   static constexpr ll neut = 0;
d41   struct Node {
d41     ll v; // start with neutral value
d41     int l, r;
d41     Node(ll v=neut, int l=0, int r=0) : v(v), l(l), r(r){}
d41   };
d41   vector<Node> seg;
d41   int n, CNT;
d41   SegP(int _n, int upd): seg(20*(upd+_n)), n(_n), CNT(1){}
d41   ll merge(ll a, ll b) { return a + b; }
d41   int update(int root, int pos, int val, int l, int r) {
d41     int p = CNT++;
d41     seg[p] = seg[root];
d41     if (l == r) {
d41       seg[p].v += val;
d41       return p;
d41     }
d41     int mid = (l + r) / 2;
d41     if (pos <= mid){
d41       seg[p].l = update(seg[p].l, pos, val, l, mid);
d41     }else seg[p].r = update(seg[p].r,pos,val,mid+1,r);

d41     seg[p].v=merge(seg[seg[p].l].v, seg[seg[p].r].v);
d41     return p;
d41   }
d41   int query(int p, int L, int R, int l, int r) {
d41     if (l > R || r < L) return neut;
d41     if (L <= l && r <= R) return seg[p].v;
d41     int mid = (l + r) / 2;
d41     int left = query(seg[p].l, L, R, l, mid);
d41     int right = query(seg[p].r, L, R, mid + 1, r);
d41     return merge(left, right);
d41   }
d41   int update(int root, int pos, int val) {
d41     return update(root, pos, val, 0, n - 1);
d41   }
d41   int query(int root, int L, int R) {
d41     return query(root, L, R, 0, n - 1);
d41   }
d41 };
```

RMQ.h

Usage: `RMQ rmq(values);`
`rmq.query(inclusive, inclusive);`
Time: $\mathcal{O}(|V|\log|V| + Q)$

d41d8c, 17 lines

```
d41 struct RMQ {
d41   vector<vector<int>> dp;
d41   RMQ(const vector<int>& a) : dp(1, a) {
d41     for (int i = 1, pw = 1; pw*2 <= sz(a); pw*=2, i++) {
d41       dp.emplace_back(sz(a) - pw*2 + 1);
d41       for (int j = 0; j < sz(dp[i]); j++) {
d41         dp[i][j] = min(dp[i-1][j], dp[i-1][j+pw]);
d41       }
d41     }
d41   }
d41   int query(int l, int r) {
d41     assert(l <= r);
d41     int k = 31 - __builtin_clz(r - l + 1);
d41     return min(dp[k][l], dp[k][r - (1 << k) + 1]);
d41   }
d41 };
```

Numerical (4)

4.1 Polynomials and recurrences

Polynomial.h

d41d8c, 19 lines

```
d41 struct Poly {
d41  vector<double> a;
d41  double operator()(double x) const {
d41    double val = 0;
d41    for (int i = sz(a); i--;) (val *= x) += a[i];
d41    return val;
d41  }
d41  void diff() {
d41    rep(i,1,sz(a)) a[i-1] = i*a[i];
d41    a.pop_back();
d41  }
d41  void divroot(double x0) {
d41    double b = a.back(), c; a.back() = 0;
d41    for(int i=sz(a)-1; i--;)
d41        c = a[i], a[i] = a[i+1]*x0+b, b=c;
d41    a.pop_back();
d41  }
d41 };
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}(n^2 \log(1/\epsilon))$

"Polynomial.h" d41d8c, 24 lines

```
d41 vector<double> polyRoots(Poly p, double xmin, double xmax)
     {
d41  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
d41  vector<double> ret;
d41  Poly der = p;
d41  der.diff();
d41  auto dr = polyRoots(der, xmin, xmax);
d41  dr.push_back(xmin-1);
d41  dr.push_back(xmax+1);
d41  sort(all(dr));
d41  rep(i,0,sz(dr)-1) {
d41    double l = dr[i], h = dr[i+1];
d41    bool sign = p(l) > 0;
d41    if (sign ^ (p(h) > 0)) {
d41      rep(it,0,60) { // while (h - l > 1e-8)
d41        double m = (l + h) / 2, f = p(m);
d41        if ((f <= 0) ^ sign) l = m;
d41        else h = m;
d41      }
d41      ret.push_back((l + h) / 2);
d41    }
d41  }
d41  return ret;
d41 }
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
Time: $\mathcal{O}(N^2)$

d41d8c, 21 lines

```
d41 vector<ll> berlekampMassey(vector<ll> s) {
d41  int n = sz(s), L = 0, m = 0;
d41  vector<ll> C(n), B(n), T;
d41  C[0] = B[0] = 1;

d41  ll b = 1;
d41  rep(i,0,n) { ++m;
d41    ll d = s[i] % mod;
d41    rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
d41    if (!d) continue;
d41    T = C; ll coef = d * modpow(b, mod-2) % mod;
d41    rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
d41    if (2 * L > i) continue;
d41    L = i + 1 - L; B = T; b = d; m = 0;
d41  }

d41  C.resize(L + 1); C.erase(C.begin());
d41  for (ll& x : C) x = (mod - x) % mod;
d41  return C;
d41 }
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0\ldots \geq n-1]$ and $tr[0\ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}(n^2 \log k)$

d41d8c, 27 lines

```
d41 using Poly = vector<ll>;
d41 ll linearRec(Poly S, Poly tr, ll k) {
d41  int n = sz(tr);

d41  auto combine = [&](Poly a, Poly b) {
d41    Poly res(n * 2 + 1);
d41    rep(i,0,n+1) rep(j,0,n+1)
d41      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
d41    for (int i = 2 * n; i > n; --i) rep(j,0,n)
d41      res[i-1-j] = (res[i-1-j] + res[i] * tr[j]) % mod;
d41    res.resize(n + 1);
d41    return res;
d41  };

d41  Poly pol(n + 1), e(pol);
d41  pol[0] = e[1] = 1;

d41  for (++k; k; k /= 2) {
d41    if (k % 2) pol = combine(pol, e);
d41    e = combine(e, e);
d41  }

d41  ll res = 0;
d41  rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
```

```
d41   return res;
d41 }
```

4.2 Matrices

SolveLinearBinary.h

Time: $\mathcal{O}\left(\frac{\min(n,m)\,nm}{64}\right)$ d41d8c, 33 lines

```
d41 pair<int, bitset<M>> gauss(vector<bitset<M>> eq) {
d41     int n = eq.size(), m = M - 1;
d41     vector<int> where(m, -1);
d41     for(int col = 0, row = 0; col < m && row < n; col++){
d41         for (int i = row; i < n; i++)
d41             if (eq[i][col]) {
d41                 swap(eq[i], eq[row]);
d41                 break;
d41             }
d41         if (!eq[row][col]) continue;
d41         where[col] = row;

d41         for (int i = 0; i < n; i++) {
d41             if (i != row && eq[i][col]) eq[i] ^= eq[row];
d41         }
d41         ++row;
d41     }

d41     bitset<M> ans;
d41     for (int i = 0; i < m; i++) {
d41         if (where[i] != -1) ans[i] = eq[where[i]][m];
d41     }
d41     for (int i = 0; i < n; i++) {
d41         int sum = (ans & eq[i]).count();
d41         sum %= 2;
d41         if (sum != eq[i][m]) return pair(0, bitset<M>());
d41     }
d41     for (int i = 0; i < m; i++) {
d41         if (where[i] == -1) return pair(INF, ans);
d41     }
d41     return pair(1, ans);
d41 }
```

XorGauss.h

d41d8c, 30 lines

```
d41 struct XorGauss {
d41     int N;
d41     vector<ll> basis, who, mask;
d41     XorGauss(int N) : N(N), basis(N), who(N), mask(N) {}
        // if(ans & (1ll << j)) who[j] was used to form x
d41     bool belong(ll x){
d41         ll ans = 0;
d41         for(int i=N-1; i>=0; i--){
d41             if((x ^ basis[i]) < x){
d41                 ans ^= mask[i];
d41                 x ^= basis[i];
d41             }
d41         }
d41         return (x == 0);
d41     }
d41     void add(ll v, int idx) {
d41         ll msk = 0;
d41         for (int i = N - 1; i >= 0; i--) {
d41             if (!(v & (1ll << i))) continue;
d41             if (basis[i] == 0) {
d41                 basis[i] = v, who[i] = idx;
d41                 mask[i] = (msk | (1ll << i));
d41                 return;
d41             }
d41             msk ^= mask[i];
d41             v ^= basis[i];
d41         }
d41     }
d41 };
```

4.3 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.

Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ (~1s for $N = 2^{22}$) d41d8c, 44 lines

```
d41 typedef complex<double> C;

d41 void fft(vector<C>& a) {
d41   int n = a.size(), L = 31 - __builtin_clz(n);
d41   static vector<complex<long double>> R(2, 1); // 10%
      faster if double
d41   static vector<C> rt(2, 1);
d41   for (static int k = 2; k < n; k *= 2) {
d41     R.resize(n);
d41     rt.resize(n);
d41     auto x = polar(1.0L, acos(-1.0L) / k);
d41     rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
d41   }
d41   vector<ll> rev(n);
d41   rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
d41   rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);

d41   for (int k = 1; k < n; k *= 2) {
d41     for (int i = 0; i < n; i += 2 * k) {
d41       for (int j = 0; j < k; j++) {
d41         auto x = (double*)&rt[j + k];
d41         auto y = (double*)&a[i + j + k];
d41         C z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]);
d41         a[i + j + k] = a[i + j] - z;
d41         a[i + j] += z;
d41       }
d41     }
d41   }
d41 }

d41 vector<ll> conv(const vector<ll>& a, const vector<ll>& b){
d41   if (a.empty() || b.empty()) return {};
d41   vector<ll> res(sz(a) + sz(b) - 1);
d41   int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
d41   vector<C> in(n), out(n);
d41   copy(all(a), in.begin());
d41   rep(i,0,sz(b)) in[i].imag(b[i]);
d41   fft(in);
d41   for (C& x : in) x *= x;
d41   rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
d41   fft(out);
d41   rep(i,0,sz(res)) res[i]=round(imag(out[i]) / (4 * n));
d41   return res;
d41 }
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.

Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h" d41d8c, 23 lines

```
d41 typedef vector<ll> vl;
d41 template<int M> vl convMod(const vl &a, const vl &b) {
d41  if (a.empty() || b.empty()) return {};
d41   vl res(sz(a) + sz(b) - 1);
d41   int B=32-__builtin_clz(sz(res)), n=1<<B,cut=int(sqrt(M));
d41   vector<C> L(n), R(n), outs(n), outl(n);
d41   rep(i,0,sz(a)) L[i] =C((int)a[i] / cut, (int)a[i] % cut);
d41   rep(i,0,sz(b)) R[i] =C((int)b[i] / cut, (int)b[i] % cut);
d41   fft(L), fft(R);
d41   rep(i,0,n) {
d41     int j = -i & (n - 1);
d41     outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
d41     outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
d41   }
d41   fft(outl), fft(outs);
d41   rep(i,0,sz(res)) {
d41     ll av = ll(real(outl[i])+.5), cv =ll(imag(outs[i])+.5);
d41     ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
d41     res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
d41   }
d41   return res;
d41 }
```

NumberTheoreticTransform.h

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x] g^{xk}$ for all k, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).

Time: $\mathcal{O}(N \log N)$ d41d8c, 34 lines

```
d41 const int mod = 998244353, root = 62;
d41 typedef vector<ll> vl;
d41 void ntt(vl &a) {
d41  int n = sz(a), L = 31 - __builtin_clz(n);
d41  static vl rt(2, 1);
d41  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
d41    rt.resize(n);
d41    ll z[] = {1, modpow(root, mod >> s)};
d41    rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
d41  }
d41  vector<int> rev(n);
d41  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
d41  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
d41  for (int k = 1; k < n; k *= 2)
d41    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
d41      ll z = rt[j+k] * a[i+j+k] % mod, &ai = a[i+j];
d41      a[i + j + k] = ai - z + (z > ai ? mod : 0);
d41      ai += (ai + z >= mod ? z - mod : z);
d41    }
d41 }
d41 vl conv(const vl &a, const vl &b) {
d41  if (a.empty() || b.empty()) return {};
d41  int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s),
d41      n = 1 << B;
d41  int inv = modpow(n, mod - 2);
d41  vl L(a), R(b), out(n);
d41  L.resize(n), R.resize(n);
d41  ntt(L), ntt(R);
d41  rep(i,0,n)
d41    out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
d41  ntt(out);
d41  return {out.begin(), out.begin() + s};
d41 }
```

FWHT.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.

Time: $\mathcal{O}(N \log N)$ d41d8c, 20 lines

```
d41  void FST(vector<ll>& a, bool inv) {
d41    for (int n = sz(a), step = 1; step < n; step *= 2) {
d41      for (int i = 0; i < n; i += 2 * step) {
d41        for (int j = i; j < i + step; j++) {
d41          ll& u = a[j], &v = a[j + step];
d41          tie(u, v) =
d41          inv ? pair(v - u, u) : pair(v, u + v); // AND
d41          inv ? pair(v, u - v) : pair(u + v, u); // OR
d41          pair(u + v, u - v);   // XOR
d41        }
d41      }
d41    }
d41    if(inv) for(ll& x : a) x /= sz(a); // XOR only
d41  }
d41  vector<ll> conv(vector<ll> a, vector<ll> b) {
d41    FST(a, 0); FST(b, 0);
d41    for (int i = 0; i < sz(a); i++) a[i]*=b[i];
d41    FST(a, 1); return a;
d41  }
```

Number theory (5)

5.1 Modular arithmetic

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime. d41d8c, 5 lines

```
d41  const ll mod = 1000000007, LIM = 200000;
d41  inv[1] = 1;
d41  for(int i=2; i<LIM; i++)
d41      inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.
Time: $\mathcal{O}(1)$ for `modmul`, $\mathcal{O}(\log b)$ for `modpow` d41d8c, 12 lines

```
d41  typedef unsigned long long ull;
d41  ull modmul(ull a, ull b, ull M) {
d41   ll ret = a * b - M * ull(1.L / M * a * b);
d41   return ret + M * (ret < 0) - M * (ret >= (ll)M);
d41  }
d41  ull modpow(ull b, ull e, ull mod) {
d41   ull ans = 1;
d41   for (; e; b = modmul(b, b, mod), e /= 2)
d41     if (e & 1) ans = modmul(ans, b, mod);
d41   return ans;
d41  }
```

DiscreteLog.h

Description: Returns the smallest x such that $a^x \bmod m = b \bmod m$. If no such x exists, returns -1.
Time: O(sqrt(m) * log(sqrt(m))) d41d8c, 32 lines

```
d41  int solve(int a, int b, int m) {
d41      a %= m, b %= m;
d41      if (a == 0) return (b ? -1 : 1);
         // caso gcd(a, m) > 1
d41      int k = 1, add = 0, g;
d41      while ((g = gcd(a, m)) > 1) {
d41          if (b == k) return add;
d41          if (b % g) return -1;
d41          b /= g, m /= g, ++add;
d41          k = (k * 1ll * a / g) % m;
d41      }

d41      int sq = sqrt(m) + 1;
d41      int big = 1;
d41      for (int i = 0; i < sq; i++) big = (1ll * big * a) % m
     ;

d41      vector<pii> vals;
d41      for (int q = 0, cur = b; q <= sq; q++) {
d41          vals.push_back({ cur, q });
d41          cur = (1ll * cur * a) % m;
d41      }
d41      sort(all(vals));
d41      for (int p = 1, cur = k; p <= sq; p++) {
d41          cur = (1ll * cur * big) % m;
d41          auto it = lower_bound(all(vals), pair(cur, INF));
d41          if (it != vals.begin() && (--it)->first == cur) {
d41              return sq * p - it->second + add;
d41          }
d41      }
d41      return -1;
d41  }
```

DiscreteRoot.h

Description: Returns x such that $x^k \bmod m = a \bmod m$. If no such x exists, returns -1.
Time: O(sqrt(m) * log(sqrt(m)))
"PrimitiveRoot.h", "DiscreteLog.h" d41d8c, 11 lines

```
     // Discrete Root

d41  ll discreteRoot(ll k, ll a, ll m) {
d41      ll g = primitiveRoot(m);
d41      ll y = discreteLog(fexp(g, k, m), a, m);
d41      if (y == -1) return y;
d41      return fexp(g, y, m);
d41  }
```

5.2 Primality

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \bmod c$.
"ModMulLL.h" d41d8c, 13 lines

```
d41  bool isPrime(ull n) {
d41   if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
d41   ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 17952650
     22};
d41   ull s = __builtin_ctzll(n-1), d = n >> s;
d41   for (ull a : A) {   // ^ count trailing zeroes
d41     ull p = modpow(a%n, d, n), i = s;
d41     while (p != 1 && p != n - 1 && a % n && i--)
d41       p = modmul(p, p, n);
d41     if (p != n-1 && i != s) return 0;
d41   }
d41   return 1;
d41  }
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.
"ModMulLL.h", "MillerRabin.h" d41d8c, 19 lines

```
d41  ull pollard(ull n) {
d41   ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
d41   auto f = [&](ull x) { return modmul(x, x, n) + i; };
d41   while (t++ % 40 || gcd(prd, n) == 1) {
d41     if (x == y) x = ++i, y = f(x);
d41     if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
d41     x = f(x), y = f(f(y));
d41   }
d41   return gcd(prd, n);
d41  }
d41  vector<ull> factor(ull n) {
d41   if (n == 1) return {};
d41   if (isPrime(n)) return {n};
d41   ull x = pollard(n);
d41   auto l = factor(x), r = factor(n / x);
d41   l.insert(l.end(), all(r));
d41   return l;
d41  }
```

PrimitiveRoot.h

d41d8c, 15 lines

```
     //is n primitive root of p ?
d41  bool test(ll x, ll p) {
d41      ll m = p - 1;
d41      for (ll i = 2; i * i <= m; ++i) if (!(m % i)) {
d41          if (modpow(x, i, p) == 1) return false;
d41          if (modpow(x, m / i, p) == 1) return false;
d41      }
d41      return true;
d41  }
     //find the smallest primitive root for p
d41  ll search(ll p) {
d41      for (ll i = 2; i < p; i++) if (test(i, p)) return i;
d41      return -1;
d41  }
```

5.3 Divisibility

Euclid.h

Description: Find x, y such that $Ax + By = \gcd(A, B)$. If $\gcd(A, B) = 1$, then $x = A^{-1} (\bmod B)$ and $y = B^{-1} (\bmod A)$.
Time: $\mathcal{O}(\log)$ d41d8c, 6 lines

```
d41  ll euclid(ll a, ll b, ll &x, ll &y) {
d41   if (!b) return x = 1, y = 0, a;
d41   ll d = euclid(b, a % b, y, x);
d41   return y -= a/b * x, d;
d41  }
```

CRT.h

d41d8c, 25 lines

```
d41  ll modinverse(ll a, ll b, ll s0 = 1, ll s1 = 0) {
d41    return !b ? s0 : modinverse(b, a % b, s1, s0 - s1 * (a /
      b)); }

d41  ll mul(ll a, ll b, ll m) {
d41    return (((__int128_t)a*b)%m + m)%m;
d41  }

d41  struct Equation {
d41    ll mod, ans;
d41    bool valid;
d41    Equation(ll a, ll m) { mod = m, ans = a, valid = true; }
d41    Equation() { valid = false; }
d41    Equation(Equation a, Equation b) {
d41      valid = false;
d41      if (!a.valid || !b.valid) return;
d41      ll g = gcd(a.mod, b.mod);
d41      if ((a.ans - b.ans) % g != 0) return;
d41      valid = true;
d41      mod = a.mod * (b.mod / g);
d41      ll x = mul(a.mod, modinverse(a.mod, b.mod), mod);
d41      ans = a.ans + mul(x, (b.ans - a.ans) / g, mod);
d41      ans = (ans % mod + mod) % mod;
d41    }
d41  };
```

DivisionTrick.h

d41d8c, 15 lines

```
d41  void floor_ranges(int n) {
d41      for (int l = 1, r; l <= n; l = r + 1) {
d41          r = n / (n / l);
             // floor(n/y) has the same value for y in [l..r]
d41      }
d41  }
d41  void ceil_ranges(int n) {
d41      for (int l = 1, r; l <= n; l = r + 1) {
d41          int x = (n + l - 1) / l;
d41          if (x == 1) r = n;
d41          else r = (n - 1) / (x - 1);
             // ceil(n/y) has the same value for y in [l..r]
d41      }
d41  }
```

5.3.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

Phi.h

Description: *Euler's* ϕ function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} ... (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$.
Euler's thm (generalized): a, m arbitrary, $n \geq \log_2 m \Rightarrow a^n \equiv a^{\phi(m)+(n \bmod \phi(m))} \pmod{m}$.

d41d8c, 6 lines

```
d41  void calculatePhi() {
d41   for(int i=0; i<LIM; i++) phi[i] = i&1 ? i : i/2;
d41   for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
d41     for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
d41  }
```

5.4 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.5 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

5.6 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

5.7 Theorems

Goldbach's conjecture: Every even integer $n > 2$ can be written as $n = a + b$ with a, b prime.

Legendre's conjecture: There is always at least one prime between n^2 and $(n+1)^2$.

Lagrange's four-square theorem: Every positive integer can be written as

$$n = a^2 + b^2 + c^2 + d^2.$$

Zeckendorf's theorem: Every integer $n \geq 1$ has a unique representation as a sum of non-consecutive Fibonacci numbers:

$$n = F_{i_1} + F_{i_2} + \cdots + F_{i_k}, \quad i_j - i_{j+1} \geq 2.$$

Euclid's formula (primitive Pythagorean triples): The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

Wilson's theorem: n is prime iff

$$(n-1)! \equiv -1 \pmod{n}.$$

Chicken McNugget theorem: For coprime n, m, the largest integer not representable as $an + bm$ (with $a, b \geq 0$) is

$$nm - n - m.$$

There are $\frac{(n-1)(m-1)}{2}$ non-representable integers, and for each pair $(k,\ nm - n - m - k)$ exactly one is representable.

Combinatorial (6)

6.1 Binomial Identities

$\binom{n-1}{k} - \binom{n-1}{k-1} = \frac{n-2k}{k}\binom{n}{k}$ $\quad$ $\binom{n}{h}\binom{n-h}{k} = \binom{n}{k}\binom{n-k}{h}$

$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$ $\quad$ $\sum_{k=0}^{n} k^2\binom{n}{k} = (n+n^2)2^{n-2}$

$\sum_{j=0}^{k} \binom{m}{j}\binom{n-m}{k-j} = \binom{n}{k}$ $\quad$ $\sum_{j=0}^{m} \binom{m}{j}^2 = \binom{2m}{m}$

$\sum_{m=0}^{n} \binom{m}{j}\binom{n-m}{k-j} = \binom{n+1}{k+1}$ $\quad$ $\sum_{m=0}^{n} \binom{m}{k} = \binom{n+1}{k+1}$

$\sum_{r=0}^{m} \binom{n+r}{r} = \binom{n+m+1}{m}$ $\quad$ $\sum_{k=0}^{n} \binom{n-k}{k} = \mathrm{Fib}(n+1)$

$\sum_{k=0}^{n} \binom{r}{k}\binom{s}{n-k} = \binom{r+s}{n}$

6.2 Permutations

6.2.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

6.2.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

6.2.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

6.2.4 Burnside's lemma

Counts the number of distinct colorings of an object under symmetry.

$$\frac{1}{|G|} \sum_{g \in G} k^{\mathrm{cyc}(g)},$$

where G is the symmetry group, k the number of colors, and $\mathrm{cyc}(g)$ the number of cycles induced by g.

Example: number of ways to color a necklace with n beads using k colors (rotations only):

$$g(n) = \frac{1}{n} \sum_{i=0}^{n-1} k^{(\gcd(n,i))}$$

where rotation i shifts the necklace by i positions.

6.3 Partitions and subsets

6.3.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	∼2e5	∼2e8

PartitionSolver.h
d41d8c, 61 lines

```
d41 template<const int N>
d41 struct PartitionSolver {
d41   vector<vector<int>> part, to, from;
d41   PartitionSolver() {
d41     vector<int> a;
d41     part.push_back(a);
d41     gen(1, N, a);
d41     sort(all(part));
d41     to.assign(sz(part), vector<int>(N + 1, -1));
d41     from = to;
d41     for (int i = 0; i < sz(part); i++) {
d41       int sum = 0;
d41       auto arr = part[i];
d41       for (auto x : arr) sum += x;
d41       to[i][0] = i;
d41       from[i][0] = i;
d41       for (int j = 1; j + sum <= N; j++) {
d41         arr = part[i];
d41         arr.push_back(j);
d41         sort(all(arr));
d41         to[i][j] = getIndex(arr);
d41         from[to[i][j]][j] = i;
d41       }
d41     }
d41   }

d41   int size() const { return sz(part); }
d41   int getIndex(const vector<int>& arr) const {
d41     return lower_bound(all(part), arr) - part.begin(); }
d41   int add(int id, int num) const { return to[id][num]; }
d41   int rem(int id, int num) const { return from[id][num]; }
d41   vector<int> getPartition(int id) const {
d41     return part[id]; }

d41   void gen(int i, int sum, vector<int>& a) {
d41     if (i > sum) { return; }
d41     a.push_back(i);
d41     part.push_back(a);
d41     gen(i, sum - i, a);
d41     a.pop_back();
d41     gen(i + 1, sum, a);
d41   }
d41 };

    // Number of partitions for all integers <= n
d41 vector<ll> partitionNumber(int n) {
d41   vector<ll> ans(n + 1, 0);
d41   ans[0] = 1;
d41   for (int i = 1; i <= n; i++) {
d41     for (int j = 1; j * (3 * j + 1) / 2 <= i; j++) {
d41       ll here = ans[i - j * (3 * j + 1) / 2];
d41       ans[i] = (ans[i] + (j & 1 ? here : -here));
d41     }
d41     for (int j = 1; j * (3 * j - 1) / 2 <= i; j++) {
d41       ll here = ans[i - j * (3 * j - 1) / 2];
d41       ans[i] = (ans[i] + (j & 1 ? here : -here));
d41     }
d41   }
d41   return ans;
d41 }
```

6.3.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod{p}$.

6.4 General purpose numbers

6.4.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able). $B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

6.4.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$

$$\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\dots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$

6.4.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

6.4.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

6.4.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

6.4.6 Labeled unrooted trees

- on n vertices: n^{n-2}
- on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
- with degrees d_i: $(n-2)!/((d_1-1)! \cdots (d_n-1)!)$

6.4.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2} C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Graph (7)

7.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max |w_i| < \sim 2^{63}$.

Time: $\mathcal{O}(VE)$

d41d8c, 24 lines

```
d41 const ll inf = LLONG_MAX;
d41 struct Ed { int a, b, w, s() { return a < b ? a : -a; }};
d41 struct Node { ll dist = inf; int prev = -1; };

d41 void bell(vector<Node>& nodes, vector<Ed>& eds, int s) {
d41  nodes[s].dist = 0;
d41  sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s(); });

d41  int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled
    vertices
d41  for(int i=0; i<lim; i++) for (Ed ed : eds) {
d41    Node cur = nodes[ed.a], &dest = nodes[ed.b];
d41    if (abs(cur.dist) == inf) continue;
d41    ll d = cur.dist + ed.w;
```

```
d41     if (d < dest.dist) {
d41       dest.prev = ed.a;
d41       dest.dist = (i < lim-1 ? d : -inf);
d41     }
d41   }
d41   for(int i=0; i<lim; i++) for (Ed e : eds) {
d41     if (nodes[e.a].dist == -inf)
d41       nodes[e.b].dist = -inf;
d41   }
d41 }
```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m, where $m[i][j] =$ `inf` if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, `inf` if no path, or `-inf` if the path goes through a negative-weight cycle.
Time: $\mathcal{O}(N^3)$ d41d8c, 20 lines

```
d41 void floydWarshall(vector<vector<ll>>& m) {
d41   int n = sz(m);
d41   for(int i=0; i<n; i++) m[i][i] = min(m[i][i], 0LL);
d41   for(int k=0; k<n; k++)
d41     for(int i=0; i<n; i++)
d41       for(int j=0; j<n; j++)
d41         if (m[i][k] != inf && m[k][j] != inf) {
d41           auto newDist = max(m[i][k] + m[k][j], -inf);
d41           m[i][j] = min(m[i][j], newDist);
d41         }

d41   for(int k=0; k<n; k++)
d41     if (m[k][k] < 0)
d41       for(int i=0; i<n; i++)
d41         for(int j=0; j<n; j++)
d41           if (m[i][k] != inf && m[k][j] != inf){
d41             m[i][j] = -inf;
d41           }
d41 }
```

7.2 Network flow and Matching

Dinic.h

Time: - $\mathcal{O}\left(\min(m \cdot \text{max\_flow},\ n^2 m)\right)$.
- For graphs with unit capacities: $\mathcal{O}\left(\min(m\sqrt{m},\ mn^{2/3})\right)$.
- If every vertex has in-degree 1 or out-degree 1: $\mathcal{O}\left(m\sqrt{n}\right)$.
- With capacity scaling: $\mathcal{O}\left(nm\log(\text{MAXCAP})\right)$ with high constant factor. d41d8c, 56 lines

```
d41 struct Dinic {
d41   const bool scaling = false;
d41   int lim;
d41   struct edge {
d41     int to, rev;
d41     ll cap, flow;
d41     bool res;
d41     edge(int to_, int cap_, int rev_, bool res_)
d41       : to(to_), cap(cap_), rev(rev_), flow(0), res(res_){}
d41   };

d41   vector<vector<edge>> g;
d41   vector<int> lev, beg;
d41   ll F;
d41   Dinic(int n) : g(n), lev(n), beg(n), F(0) {}

d41   void add(int a, int b, ll c, ll other = 0) {
d41     g[a].emplace_back(b, c, g[b].size(), false);
d41     g[b].emplace_back(a, other, g[a].size()-1, true);
d41   }
d41   bool bfs(int s, int t) {
d41     fill(all(lev), -1);
d41     fill(all(beg), 0);
d41         lev[s] = 0;
d41     queue<int> q; q.push(s);
d41     while (q.size()) {
d41       int u = q.front(); q.pop();
d41       for (auto& i : g[u]) {
d41         if (lev[i.to] != -1 or (i.flow == i.cap)) continue;
d41         if (scaling and i.cap - i.flow < lim) continue;
d41         lev[i.to] = lev[u] + 1;
d41         q.push(i.to);
d41       }
d41     }
d41     return lev[t] != -1;
d41   }
d41   ll dfs(int v, int s, ll f = INF) {
d41     if (!f or v == s) return f;
d41     for (int& i = beg[v]; i < g[v].size(); i++) {
d41       auto& e = g[v][i];
d41       if (lev[e.to] != lev[v] + 1) continue;
d41       ll foi = dfs(e.to, s, min(f, e.cap - e.flow));
d41       if (!foi) continue;
d41       e.flow += foi, g[e.to][e.rev].flow -= foi;
d41       return foi;
d41     }
d41     return 0;
d41   }
d41   ll maxFlow(int s, int t) {
d41     for (lim = scaling ? (1<<30) : 1; lim; lim /= 2)
d41       while (bfs(s, t)) while (ll ff = dfs(s, t)) F += ff;
d41     return F;
d41   }
d41     bool inCut(int u){ return lev[u] != -1; }
d41 };
```

MinCost.h

Description: Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only. If graph is a DAG pi can be calculated with DP instead of Bellman ford.
Time: $\mathcal{O}(FE\log(V))$ where F is max flow. $\mathcal{O}(VE)$ for setpi. d41d8c, 95 lines

```
d41 #include <bits/extc++.h>

d41 const ll INF = numeric_limits<ll>::max() / 4;

d41 struct MCMF {
d41   struct edge {
d41     int from, to, rev;
d41     ll cap, cost, flow;
d41   };
d41   int N;
d41   vector<vector<edge>> ed;
d41   vector<int> seen, vis;
d41   vector<ll> dist, pi;
d41   vector<edge*> par;

d41   MCMF(int N) : N(N), ed(N), seen(N), vis(N),
d41     dist(N), pi(N), par(N) {}

d41   void addEdge(int from, int to, ll cap, ll cost) {
d41     if (from == to || cap == 0) return;
d41     ed[from].push_back(edge{from,to,sz(ed[to]),cap,cost,0
    });
d41     ed[to].push_back(edge{to,from,sz(ed[from])-1,0,-cost,0
    });
d41    }

d41   void path(int s) {
d41     fill(all(seen), 0);
d41     fill(all(dist), INF);
d41     dist[s] = 0;
d41     ll di;
d41     __gnu_pbds::priority_queue<pair<ll, int>> q;
d41     vector<decltype(q)::point_iterator> its(N);
d41     q.push({ 0, s });

d41     while (!q.empty()) {
d41       s = q.top().second; q.pop();
d41       seen[s] = 1; di = dist[s] + pi[s];
d41       for (edge& e : ed[s]) {
d41         if (!seen[e.to]) {
d41           ll val = di - pi[e.to] + e.cost;
d41           if(e.cap - e.flow > 0 && val < dist[e.to]){
d41             dist[e.to] = val;
d41             par[e.to] = &e;
d41             if (its[e.to] == q.end()) {
d41               its[e.to] = q.push({-dist[e.to], e.to});
d41             }
d41             else q.modify(its[e.to], {-dist[e.to], e.to});
d41           }
d41         }
d41       }
d41     }
d41     for (int i = 0; i < N; i++) {
d41       pi[i] = min(pi[i] + dist[i], INF);
d41     }
d41   }

d41   pair<ll, ll> maxflow(int s, int t) {
d41     setpi(s, t);
d41     ll totflow = 0, totcost = 0;
d41     while (path(s), seen[t]) {
d41       ll fl = INF;
d41       for (edge* x = par[t]; x; x = par[x->from]) {
d41         fl = min(fl, x->cap - x->flow);
d41       }
d41       totflow += fl;
d41       for (edge* x = par[t]; x; x = par[x->from]) {
d41         x->flow += fl;
d41         ed[x->to][x->rev].flow -= fl;
d41       }
d41     }
d41     for (int i = 0; i < N; i++) {
d41       for (edge& e : ed[i]) {
d41         totcost += e.cost * e.flow;
d41       }
d41     }
d41     return { totflow, totcost / 2 };
d41   }

      // If some costs can be negative, call this before
         maxflow:
d41   void setpi(int s, int t) {
d41     fill(all(pi), INF);
d41     pi[s] = 0;
d41     int it = N, ch = 1;
d41     ll v;
d41     while (ch-- && it--) {
d41       for (int i = 0; i < N; i++) {
d41         if (pi[i] != INF)
d41           for (edge& e : ed[i]) if (e.cap)
d41             if((v= pi[i] + e.cost)< pi[e.to])
d41               pi[e.to] = v, ch = 1;
d41       }
d41     }
d41     assert(it >= 0); // negative cost cycle
d41   }
d41 };
```

PushRelabel.h

Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

Time: $\mathcal{O}\left(V^2\sqrt{E}\right)$ d41d8c, 55 lines

```
d41 struct PushRelabel {
d41   struct Edge {
d41     int dest, back;
d41     ll f, c;
d41   };
d41   vector<vector<Edge>> g;
d41   vector<ll> ec;
d41   vector<Edge*> cur;
d41   vector<vector<int>> hs;
d41   vector<int> H;
d41   PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n){}

d41   void addEdge(int s, int t, ll cap, ll rcap=0) {
d41     if (s == t) return;
d41     g[s].push_back({t, sz(g[t]), 0, cap});
d41     g[t].push_back({s, sz(g[s])-1, 0, rcap});
d41   }

d41   void addFlow(Edge& e, ll f) {
d41     Edge &back = g[e.dest][e.back];
d41     if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
d41     e.f += f; e.c -= f; ec[e.dest] += f;
d41     back.f -= f; back.c += f; ec[back.dest] -= f;
d41   }
d41   ll calc(int s, int t) {
d41     int v = sz(g); H[s] = v; ec[t] = 1;
d41     vector<int> co(2*v); co[0] = v-1;
d41     for(int i=0; i<v; i++) cur[i] = g[i].data();
d41     for (Edge& e : g[s]) addFlow(e, e.c);

d41     for (int hi = 0;;) {
d41       while (hs[hi].empty()) if (!hi--) return -ec[s];
d41       int u = hs[hi].back(); hs[hi].pop_back();
d41       while (ec[u] > 0)  // discharge u
d41         if (cur[u] == g[u].data() + sz(g[u])) {
d41           H[u] = 1e9;
d41           for (Edge& e : g[u]){
d41             if (e.c && H[u] > H[e.dest]+1)
d41               H[u] = H[e.dest]+1, cur[u] = &e;
d41           }
d41           if (++co[H[u]], !--co[hi] && hi < v){
d41             for(int i=0; i<v; i++){
d41               if (hi < H[i] && H[i] < v)
d41                 --co[H[i]], H[i] = v + 1;
d41             }
d41           }
d41           hi = H[u];
d41         } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1){
d41           addFlow(*cur[u], min(ec[u], cur[u]->c));
d41         }else ++cur[u];
d41     }
d41   }
d41   bool inCut(int a) { return H[a] >= sz(g); }
d41 };
```

Blossom.h

Description: Max matching on general Graph. $mate[i]$ = match of i

Time: $\mathcal{O}\left(N^3\right)$ d41d8c, 56 lines

```
d41 vector<int> Blossom(vector<vector<int>>& g) {
d41     int n = sz(g), timer = -1;
d41     vector<int> mate(n, -1),label(n),par(n),orig(n),aux(n,
    -1),q;
d41     auto lca = [&](int x, int y) {
d41         for (timer++; ; swap(x, y)) {
d41             if (x == -1) continue;
d41             if (aux[x] == timer) return x;
d41             aux[x] = timer;
d41             x=(mate[x] == -1 ? -1 : orig[par[mate[x]]]);
d41         }
d41     };
d41     auto blossom = [&](int v, int w, int a) {
d41         while (orig[v] != a) {
d41             par[v] = w; w = mate[v];
d41             if(label[w] == 1) label[w] = 0,q.push_back(w);
d41             orig[v] = orig[w] = a;
d41             v = par[w];
d41         }
d41     };
d41     auto aug = [&](int v) {
d41         while (v != -1) {
d41             int pv = par[v], nv = mate[pv];
d41             mate[v] = pv; mate[pv] = v; v = nv;
d41         }
d41     };
d41     auto bfs = [&](int root) {
d41         fill(all(label), -1);
d41         iota(all(orig), 0);
d41         q.clear();
d41         label[root] = 0; q.push_back(root);
d41         for (int i = 0; i < sz(q); i++) {
d41             int v = q[i];
d41             for (auto x : g[v]) {
d41                 if (label[x] == -1) {
d41                     label[x] = 1; par[x] = v;
d41                     if (mate[x] == -1) return aug(x), 1;
d41                     label[mate[x]] = 0;
d41                     q.push_back(mate[x]);
d41                 }
d41                 else if(!label[x] && orig[v] != orig[x]){
d41                     int a = lca(orig[v], orig[x]);
d41                     blossom(x, v, a);
d41                     blossom(v, x, a);
d41                 }
d41             }
d41         }
d41         return 0;
d41     };
        // Time halves if you start with (any) maximal
            matching.
d41     for (int i = 0; i < n; i++) {
d41         if (mate[i] == -1) bfs(i);
d41     }
d41     return mate;
d41 }
```

HopcroftKarp.h

Description: ans is the size of the max matching.
The match of x is $l[x]$

Usage: `HopcroftKarp(|X|, |Y|, edges(x, y))`

Time: $\mathcal{O}\left(\sqrt{V}E\right)$ d41d8c, 46 lines

```
d41 struct HopcroftKarp {
d41     vector<int> g, l, r;
d41     int ans;
d41     HopcroftKarp(int n, int m, vector<pair<int, int>> e)
d41         : g(e.size()), l(n, -1), r(m, -1), ans(0) {
d41         shuffle(e.begin(), e.end(), rng);
d41         vector<int> deg(n + 1);
d41         for (auto& [x, y] : e) deg[x]++;
d41         for (int i = 1; i <= n; i++) deg[i] += deg[i - 1];
d41         for (auto& [x, y] : e) g[--deg[x]] = y;

d41         vector<int> q(n);
d41         while (true) {
d41             vector<int> a(n, -1), p(n, -1);
d41             int t = 0;
d41             for (int i = 0; i < n; i++) {
d41                 if (l[i] == -1) {
d41                     q[t++] = a[i] = p[i] = i;
d41                 }
d41             }
d41             bool match = false;
d41             for (int i = 0; i < t; i++) {
d41                 int x = q[i];
d41                 if (~l[a[x]]) continue;
d41                 for (int j = deg[x]; j < deg[x + 1]; j++)
    {
d41                     int y = g[j];
d41                     if (r[y] == -1) {
d41                         while (~y) {
d41                             r[y] = x;
d41                             swap(l[x], y);
d41                             x = p[x];
d41                         }
d41                         match = true, ans++;
d41                         break;
d41                     }
d41                     if (p[r[y]] == -1) {
d41                         q[t++] = y = r[y];
d41                         p[y] = x, a[y] = a[x];
d41                     }
d41                 }
d41             }
d41             if (!match) break;
d41         }
d41     }
d41 };
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \leq M$.

Time: $\mathcal{O}\left(N^2M\right)$ d41d8c, 41 lines

```
d41 pair<ll, vector<int>> hunga(const vector<vector<ll>>& a) {
d41     if (a.empty()) return { 0, {} };
d41     int n = sz(a) + 1, m = sz(a[0]) + 1;
d41     vector<ll> u(n), v(m), p(m);
d41     vector<int> ans(n - 1);
d41     for (int i = 1; i < n; i++) {
d41         p[0] = i;
d41         int j0 = 0;
d41         vector<ll> dist(m, LLONG_MAX), pre(m, -1);
d41         vector<bool> done(m + 1);
d41         do {
d41             done[j0] = true;
d41             ll i0 = p[j0], j1 = -1, delta = LLONG_MAX;
d41             for (int j = 1; j < m; j++) {
d41                 if (!done[j]) {
d41                     ll cur = a[i0-1][j-1] - u[i0] - v[j];
d41                     if (cur < dist[j])
d41                         dist[j] = cur, pre[j] = j0;
d41                     if(dist[j] < delta)
d41                         delta = dist[j], j1 = j;
d41                 }
d41             }
d41             for (int j = 0; j < m; j++) {
```

```
d41                 if (done[j])
d41                     u[p[j]] += delta, v[j] -= delta;
d41                 else dist[j] -= delta;
d41             }
d41             assert(j1 != -1);
d41             j0 = j1;
d41         } while (p[j0]);
d41         while (j0) {
d41             int j1 = pre[j0];
d41             p[j0] = p[j1], j0 = j1;
d41         }
d41     }
d41     for (int j = 1; j < m; j++) {
d41         if (p[j]) ans[p[j] - 1] = j - 1;
d41     }
d41     return { -v[0], ans }; // min cost
d41 }
```

7.2.1 Hall's Theorem

In bipartite graphs, there exists a perfect matching covering the entire side X if and only if for every subset $Y \subseteq X$,

$$|Y| \leq |N(Y)|,$$

where $N(Y)$ denotes the set of neighbors of Y.

7.2.2 Kőnig's Theorem

In a bipartite graph, the size of a Minimum Vertex Cover is equal to the size of a Maximum Matching. A Minimum Vertex Cover is a minimum set of vertices such that every edge of the graph has at least one endpoint in the set.

As a consequence,

$$n - \text{Maximum Matching} = \text{Maximum Independent Set},$$

where a Maximum Independent Set is the largest set of vertices with no edges between them.

Recovering the Minimum Vertex Cover Given a maximum matching in a bipartite graph (X, Y):

- Construct the residual graph by orienting:
  - non-matching edges from X to Y;
  - matching edges from Y to X.
- Perform a BFS or DFS starting from all free (unmatched) vertices in X.
- Let Z_X be the set of reachable vertices in X, and Z_Y the set of reachable vertices in Y.

The Minimum Vertex Cover is given by:

$$(X \setminus Z_X) \cup Z_Y.$$

7.2.3 Node-Disjoint Path Cover

A node-disjoint path cover is a set of paths such that each vertex belongs to exactly one path.

In a directed acyclic graph (DAG),

$$\text{Minimum Node-Disjoint Path Cover} = n - \text{Maximum Matching}.$$

The construction is as follows: for each vertex u, create a copy u'. Add an edge $u \to v'$ if there exists an edge $u \to v$ in the original graph.

Recovering the Paths

- Vertices that do not appear as destinations in the matching are starting points of paths.
- Each matching edge $u \to v'$ corresponds to an edge $u \to v$ in the original DAG.
- Following these edges reconstructs all paths of the path cover.

7.2.4 General Path Cover

A general path cover is a path cover where a vertex may belong to more than one path.

In a DAG, the construction is similar to the node-disjoint case, but an edge $u \to v'$ exists if there is a path from u to v in the original graph.

Recovering the Cover The vertices can be grouped according to the edges used in the matching to form the path cover.

7.2.5 Dilworth's Theorem

An antichain is a set of vertices such that there is no path between any pair of vertices in the set.

In a directed acyclic graph,

$$\text{Minimum General Path Cover} = \text{Maximum Antichain}.$$

Recovering a Maximum Antichain Given a minimum general path cover, selecting one vertex from each path produces a maximum antichain.

7.3 DFS algorithms

Bridges.h

d41d8c, 24 lines

```
d41 vector<int> g[ms];
d41 int low[ms], tin[ms], vis[ms], t;

d41 void dfs(int u = 0, int p = -1) {
d41     vis[u] = true;
d41     low[u] = tin[u] = t++;
d41     for (auto v : g[u]) {
d41         if (v == p) continue;
d41         if (vis[v]) {
d41             low[u] = min(low[u], tin[v]);
d41         }
d41         else {
d41             dfs(v, u);
d41             low[u] = min(low[u], low[v]);
                // if (low[v] >= tin[u] && p != -1), U is an
                    articulation point
d41             if (low[v] > tin[u]) {
                    // edge from U to V is a bridge
d41             }
                // children++
d41         }
d41     }
        // if(children > 1 && p == -1) root is an articulation
            point
d41 }
```

EulerPath.h

Description: Receives as input graph(node, edge index), number of edges and source. Returns list of node, index of edge he came from, if path/circuit does not exists returns empty list.

d41d8c, 27 lines

```
d41 vector<pii> eulerPath(const vector<vector<pii>>& g, int
    nedges, int src) {
d41     int n = sz(g);
d41     vector<int> deg(n, 0), its(n, 0), used(nedges + 1, 0);
d41     vector<pii> s = { {src, -1} };
        //deg[src]++; //to allow paths, not only circuits
d41     vector<pii> ret;
d41     while (!s.empty()) {
d41         int u = s.back().first, &it = its[u];
d41         if (it == sz(g[u])) {
d41             ret.push_back(s.back());
d41             s.pop_back();
d41             continue;
d41         }
d41         auto& [nxt, id] = g[u][it++];
d41         if (!used[id]) {
d41             deg[u]--, deg[nxt]++;
d41             used[id] = 1;
d41             s.push_back({ nxt, id });
d41         }
d41     }
d41     for (int x : deg) {
d41         if (x < 0 || sz(ret) != (nedges + 1)) return {};
d41     }
d41     reverse(ret.begin(), ret.end());
d41     return ret;
d41 }
```

SCC.h

Description: Kosaraju algorithm for calculating strongly connected components. Components are ordered in topological order.

d41d8c, 36 lines

```
d41 struct SCC {
d41     int n, ncomp;
d41     vector<vector<int>> g, inv;
d41     vector<int> comp, vis, stk;
d41     SCC(){}
d41     SCC(int n)
d41         : n(n),ncomp(0),g(n),inv(n),comp(n, -1),vis(n){}

d41     void dfs(int u) {
d41         vis[u] = 1;
d41         for (int v : g[u]) if (!vis[v]) dfs(v);
d41         stk.push_back(u);
d41     }
d41     void dfs_inv(int u) {
```

```
d41         comp[u] = ncomp;
d41         for (int v : inv[u]) {
d41             if (comp[v] == -1) dfs_inv(v);
d41         }
d41     }
d41     void solve() {
d41         for (int i = 0; i < n; i++) {
d41             if (!vis[i]) dfs(i);
d41         }
d41         reverse(all(stk));
d41         for (int u : stk) {
d41             if (comp[u] != -1) continue;
d41             dfs_inv(u);
d41             ncomp++;
d41         }
d41     }
d41     void add_edge(int a, int b) {
d41         g[a].push_back(b);
d41         inv[b].push_back(a);
d41     }
d41 };
```

Twosat.h

Usage: `not A = ~A`

"SCC.h" d41d8c, 37 lines

```
d41 struct TwoSat{
d41  int n;
d41  SCC scc;
d41  vector<int> value;
d41  vector<pii> e;
d41  TwoSat(int n) : n(n){}
d41  bool solve(){
d41    value.resize(n);
d41    scc = SCC(2*n);
d41    for(auto &x : e) scc.add_edge(x.first, x.second);
d41    scc.solve();
d41    for(int i=0; i<2*n; i++)
d41      if(scc.comp[i] == scc.comp[i^1]) return false;
d41    for(int i=0; i<n; i++)
d41      value[i] = scc.comp[id(i)] > scc.comp[id(~i)];
d41    return true;
d41  }
d41  void atMostOne(vector<int> &li){
d41    if(sz(li) <= 1) return;
d41    int cur = ~li[0];
d41    for(int i = 2; i < sz(li); i++) {
d41      int next = n++;
d41      addOr(cur, ~li[i]);
d41      addOr(cur, next);
d41      addOr(~li[i], next);
d41      cur = ~next;
d41    }
d41    addOr(cur, ~li[1]);
d41  }
d41  int id(int v) { return v < 0 ? (~v) * 2 ^ 1 : v * 2; }
d41  void add(int a, int b) { e.push_back({id(a), id(b)}); }
d41  void addOr(int a, int b) { add(~a, b); add(~b, a); }
d41  void addImp(int a, int b) { addOr(~a, b); }
d41  void addEqual(int a, int b){ addOr(a, ~b); addOr(~a, b);
    }
d41  void isFalse(int a) { addImp(a, ~a); }
d41 };
```

7.4 Heuristics

MaxClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

d41d8c, 53 lines

```
d41 using vb = vector<bitset<200>>;
d41 struct Maxclique {
d41  double limit=0.025, pk=0;
d41  struct Vertex { int i, d=0; };
d41  using vv = vector<Vertex>;
d41  vb e;
d41  vv V;
d41  vector<vector<int>> C;
d41  vector<int> qmax, q, S, old;
d41  void init(vv& r) {
d41    for (auto& v : r) v.d = 0;
d41    for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
d41    sort(all(r), [](auto a, auto b) { return a.d > b.d; });
d41    int mxD = r[0].d;
d41    for(int i=0; i<sz(r); i++) r[i].d = min(i, mxD) + 1;
d41  }
d41  void expand(vv& R, int lev = 1) {
d41    S[lev] += S[lev - 1] - old[lev];
d41    old[lev] = S[lev - 1];
d41    while (sz(R)) {
d41      if (sz(q) + R.back().d <= sz(qmax)) return;
d41      q.push_back(R.back().i);
d41      vv T;
d41      for(auto v : R)
d41        if (e[R.back().i][v.i]) T.push_back({v.i});
d41      if (sz(T)) {
d41        if (S[lev]++ / ++pk < limit) init(T);
d41        int j = 0, mxk = 1, mnk = max(sz(qmax)-sz(q)+1, 1);
d41        C[1].clear(), C[2].clear();
d41        for (auto v : T) {
d41          int k = 1;
d41          auto f = [&](int i) { return e[v.i][i]; };
d41          while (any_of(all(C[k]), f)) k++;
d41          if (k > mxk) mxk = k, C[mxk + 1].clear();
d41          if (k < mnk) T[j++].i = v.i;
d41          C[k].push_back(v.i);
d41        }
d41        if (j > 0) T[j - 1].d = 0;
d41        for(int k=mnk; k<mxk + 1; k++){
d41                      for (int i : C[k])
d41              T[j].i = i, T[j++].d = k;
d41                    }
d41        expand(T, lev + 1);
d41      } else if (sz(q) > sz(qmax)) qmax = q;
d41      q.pop_back(), R.pop_back();
d41    }
d41  }
d41  vector<int> maxClique(){ init(V),expand(V); return qmax;}
d41  Maxclique(vb conn) : e(conn),C(sz(e)+1),S(sz(C)),old(S){
d41    for(int i=0; i<sz(e); i++) V.push_back({i});
d41  }
d41 };
```

7.5 Trees

Centroid.h

Description: Call decomp(0) to solve, marked array should be initially set to zero.

Time: $\mathcal{O}(N \log N)$

d41d8c, 27 lines

```
d41 int tam[ms], marked[ms];

d41 int calc_tam(int u, int p) {
d41   tam[u] = 1;
d41   for (int v : g[u]) {
d41     if (v != p && !marked[v]) tam[u] += calc_tam(v, u);
d41   }
d41   return tam[u];
d41 }

d41 int get_centroid(int u, int p, int tot) {
d41   for (int v : g[u]) {
d41     if (v != p && !marked[v] && (tam[v] > (tot / 2)))
d41       return get_centroid(v, u, tot);
d41   }
d41   return u;
d41 }
    // Cent is a child of P in the centroid tree
d41 void decomp(int u, int p = -1) {
d41   calc_tam(u, -1);
d41   int cent = get_centroid(u, -1, tam[u]);
d41   marked[cent] = 1;
d41   for (int v : g[cent]) {
d41     if (!marked[v]) decomp(v, cent);
d41   }
d41 }
```

HLD.h

Description: If values are stored on edges, set EDGE = true and store each edge's value at the endpoint farther from the root (the deeper node). rp[i] is the representative (head) of the heavy path containing node i: it is the node in that chain that is closest to the root.

d41d8c, 51 lines

```
d41 template<bool EDGE> struct HLD {
d41     int n, t;
d41     vector<vector<int>> g;
d41     vector<int> pai, rp, tam, pos, val, arr;
d41     Seg seg;
d41     HLD(int n, vector<vector<int>>& g, vector<int>& val)
d41         : n(n), t(0), g(g), pai(n), rp(n), tam(n, 1),
d41           pos(n), val(val), arr(n) {
d41         calc_tam(0, -1);
d41         dfs(0, -1);
d41         seg.build(arr);
d41     }

d41     int calc_tam(int u, int p) {
d41         pai[u] = p;
d41         for (int& v : g[u]) {
d41             if (v == p) continue;
d41             tam[u] += calc_tam(v, u);
d41             if(tam[v] > tam[g[u][0]] || g[u][0] == p)
d41                 swap(g[u][0], v);
d41         }
d41         return tam[u];
d41     }

d41     void dfs(int u, int p) {
d41         pos[u] = t++;
d41         arr[pos[u]] = val[u];
d41         for (int v : g[u]) {
d41             if (v == p) continue;
d41             rp[v] = (v == g[u][0] ? rp[u] : v);
d41             dfs(v, u);
d41         }
d41     }

d41     int query(int a, int b) { // query on the path from a
    to b
d41         int ans = 0; // neutral value
d41         while (rp[a] != rp[b]) {
d41             if (pos[a] < pos[b]) swap(a, b);
d41             ans = max(ans, seg.query(pos[rp[a]], pos[a]));
d41             a = pai[rp[a]];
d41         }
d41         if(pos[a] > pos[b]) swap(a, b);
```

```
d41         ans = max(ans, seg.query(pos[a] + EDGE, pos[b]));
d41         return ans;
d41     }

d41     void update(int a, int x) {
d41         seg.update(pos[a], x);
d41     }
d41 };
```

LCA.h

Description: LCA algorithm using binary lifting, $is\_ancestor(a, b)$ returns true if a is an ancestral of b and false otherwise.
Time: $\mathcal{O}(N \log N)$

d41d8c, 26 lines

```
d41 int tin[MAXN], tout[MAXN], timer=0;
d41 int up[MAXN][BITS];
d41 void dfs(int u, int p){
d41     tin[u] = timer++, up[u][0] = p;
d41     for (int i=1; i<BITS; i++) {
d41         up[u][i] = up[up[u][i-1]][i-1];
d41     }
d41     for (int v: g[u]) if (v != p) dfs(v, u);
d41     tout[u] = timer;
d41 }

d41 bool is_ancestor(int u, int v){
d41     return (tin[u] <= tin[v] && tout[u] >= tout[v]);
d41 }

d41 int lca(int u, int v){
d41     if (is_ancestor(u, v)) return u;
d41     if (is_ancestor(v, u)) return v;
d41     for (int i=BITS-1; i>=0; i--) {
d41         if (up[u][i] && !is_ancestor(up[u][i], v)) {
d41             u = up[u][i];
d41         }
d41     }
d41     return up[u][0];
d41 }
```

VirtualTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S|-1$) pairwise LCA's and compressing edges. virt[u] is the adjacency list of the virtual tree: it stores pairs (v, dist), where v is a neighbor of u in the virtual tree and dist is the distance between u and v in the original tree.
Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h" d41d8c, 24 lines

```
d41 vector<pair<int, int>> virt[ms];

d41 void build_virt(vector<int>& v) {
d41   auto cmp = [&](int i, int j){ return tin[i] < tin[j]; };
d41   sort(all(v), cmp);
d41   for (int i = 0, n = sz(v);i + 1 < n; i++)
d41     v.push_back(lca(v[i], v[i + 1]));
d41   sort(all(v), cmp);
d41   v.erase(unique(all(v)), v.end());
d41   stack<int> st;
d41   for (auto u : v) {
d41     if (st.empty()) {
d41       st.push(u);
d41     }
d41     else {
d41       while(sz(st) && !is_ancestor(st.top(), u)) st.pop();
d41       int p = st.top();
d41       virt[p].emplace_back(u, abs(lvl[u] - lvl[p]));
d41       virt[u].emplace_back(p, abs(lvl[u] - lvl[p]));
d41       st.push(u);
d41     }
d41   }
d41 }
```

7.6 Math

7.6.1 Number of Spanning Trees

Create an $N \times N$ matrix mat, and for each edge $a \to b \in G$, do mat[a][b]--, mat[b][b]++ (and mat[b][a]--, mat[a][a]++ if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

7.6.2 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

7.7 Planar Graphs

If G has k connected components, then $n - m + f = k + 1$.

Geometry (8)

8.1 Geometric primitives

Point.h

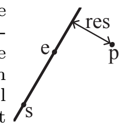
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

d41d8c, 29 lines

```
d41 template <class T> int sgn(T x) { return (x > 0) - (x < 0)
    ; }
d41 template<class T>
d41 struct Point {
d41  typedef Point P;
d41  T x, y;
d41  explicit Point(T x=0, T y=0) : x(x), y(y) {}
d41  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y
    ); }
d41  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y
    ); }
d41  P operator+(P p) const { return P(x+p.x, y+p.y); }
d41  P operator-(P p) const { return P(x-p.x, y-p.y); }
d41  P operator*(T d) const { return P(x*d, y*d); }
d41  P operator/(T d) const { return P(x/d, y/d); }
d41  T dot(P p) const { return x*p.x + y*p.y; }
d41  T cross(P p) const { return x*p.y - y*p.x; }
d41  T cross(P a, P b) const { return (a-*this).cross(b-*this)
    ; }
d41  T dist2() const { return x*x + y*y; }
d41  double dist() const { return sqrt((double)dist2()); }
     // angle to x-axis in interval [-pi, pi]
d41  double angle() const { return atan2(y, x); }
d41  P unit() const { return *this/dist(); } // makes dist()=1
d41  P perp() const { return P(-y, x); } // rotates +90
    degrees
d41  P normal() const { return perp().unit(); }
     // returns point rotated 'a' radians ccw around the
         origin
d41  P rotate(double a) const {
d41    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
d41  friend ostream& operator<<(ostream& os, P p) {
d41    return os << "(" << p.x << "," << p.y << ")"; }
d41 };
```

lineDistance.h

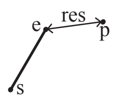
Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.



"Point.h" d41d8c, 5 lines

```
d41 template<class P>
d41 double lineDist(const P& a, const P& b, const P& p) {
d41  return (double)(b-a).cross(p-a)/(b-a).dist();
d41 }
```

SegmentDistance.h



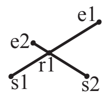
Description:
Returns the shortest distance between point p and the line segment from point s to e.
Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;

"Point.h" d41d8c, 7 lines

```
d41 typedef Point<double> P;
d41 double segDist(P& s, P& e, P& p) {
d41  if (s==e) return (p-s).dist();
d41  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)))
    ;
d41  return ((p-s)*d-(e-s)*t).dist()/d;
d41 }
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



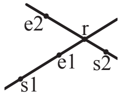
Usage: vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;

"Point.h", "OnSegment.h" d41d8c, 14 lines

```
d41 template<class P> vector<P> segInter(P a, P b, P c, P d) {
d41  auto oa = c.cross(d, a), ob = c.cross(d, b),
d41       oc = a.cross(b, c), od = a.cross(b, d);
     // Checks if intersection is single non-endpoint point.
d41  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
d41    return {(a * ob - b * oa) / (ob - oa)};
d41  set<P> s;
d41  if (onSegment(c, d, a)) s.insert(a);
d41  if (onSegment(c, d, b)) s.insert(b);
d41  if (onSegment(a, b, c)) s.insert(c);
d41  if (onSegment(a, b, d)) s.insert(d);
d41  return {all(s)};
d41 }
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage: `auto res = lineInter(s1,e1,s2,e2);`
`if (res.first == 1)`
`cout << "intersection point at " << res.second << endl;`

"Point.h" d41d8c, 9 lines

```cpp
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
 auto d = (e1 - s1).cross(e2 - s2);
 if (d == 0) // if parallel
   return {-(s1.cross(e1, s2) == 0), P(0, 0)};
 auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
 return {1, (s1 * p + e1 * q) / d};
}
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument *eps* is given 0 is returned if p is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.
Usage: `bool left = sideOf(p1,p2,q)==1;`

"Point.h" d41d8c, 10 lines

```cpp
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps)
 {
 auto a = (e-s).cross(p-s);
 double l = (e-s).dist()*eps;
 return (a > l) - (a < -l);
}
```

OnSegment.h

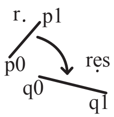
Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

"Point.h" d41d8c, 4 lines

```cpp
template<class P> bool onSegment(P s, P e, P p) {
 return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h

Description:



Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.

"Point.h" d41d8c, 7 lines

```cpp
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
   const P& q0, const P& q1, const P& r) {
 P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
 return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist
2();
}
```

LineProjectionReflection.h

Description: Projects point p onto line ab. Set refl=true to get reflection of point p across line ab instead. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h" d41d8c, 6 lines

```cpp
template<class P>
P lineProj(P a, P b, P p, bool refl=false) {
 P v = b - a;
 return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.
Usage: `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
`int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }`
`// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i`

d41d8c, 36 lines

```cpp
struct Angle {
 int x, y;
 int t;
 Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
 Angle operator-(Angle b) const { return {x-b.x, y-b.y, t
}; }
 int half() const {
   assert(x || y);
   return y < 0 || (y == 0 && x < 0);
 }
 Angle t90() const { return {-y, x, t + (half() && x >= 0)
}; }
 Angle t180() const { return {-x, -y, t + half()}; }
 Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
 // add a.dist2() and b.dist2() to also compare distances
 return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
        make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle
    between
// them, i.e., the angle that covers the defined line
    segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
 if (b < a) swap(a, b);
 return (b < a.t180() ?
         make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
 Angle r(a.x + b.x, a.y + b.y, a.t);
 if (a.t180() < r) r.t--;
 return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
 int tu = b.t - a.t; a.t = b.t;
 return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a
)};
}
```

HalfPlane.h

Description: Computes the intersection of a set of half-planes. Half-planes are sorted by angle and processed with a deque, removing redundant or conflicting constraints. Parallel half-planes are handled explicitly. Returns the convex polygon of the intersection, or empty if infeasible.
Time: $\mathcal{O}(n \log n)$

"Point.h" d41d8c, 72 lines

```cpp
using ld = long double;
using P = Point<ld>;

struct Hp { // Half plane struct
  // 'p' is a passing point of the line and 'pq' is the
      direction vector of the line.
  P p, pq;
  ld angle;

  Hp() {}
  Hp(const P& a, const P& b) : p(a), pq(b - a) {
    angle = atan2l(pq.y, pq.x);
  }
  bool out(const P& r) { return pq.cross(r - p) < -eps; }
  bool operator < (const Hp& e) const {
    return angle < e.angle;
  }
  friend P inter(const Hp& s, const Hp& t) {
    ld alpha = (t.p - s.p).cross(t.pq) / s.pq.cross(t.pq);
    return s.p + (s.pq * alpha);
  }
};

vector<P> hp_intersect(vector<Hp>& H) {
  P box[4] = { P(inf, inf), P(-inf, inf),
               P(-inf, -inf), P(inf, -inf) };

  for(int i = 0; i<4; i++) {
    Hp aux(box[i], box[(i+1) % 4]);
    H.push_back(aux);
  }
  sort(all(H));
  deque<Hp> dq;
  int len = 0;
  for(int i = 0; i < sz(H); i++) {
    while(len>1 && H[i].out(inter(dq[len-1], dq[len-2]))){
      dq.pop_back();
      --len;
    }
    while (len > 1 && H[i].out(inter(dq[0], dq[1]))){
      dq.pop_front();
      --len;
    }
    if(len && fabsl(H[i].pq.cross(dq[len-1].pq)) < eps){
      if (H[i].pq.dot(dq[len-1].pq) < 0.0)
        return vector<P>();
      if (H[i].out(dq[len-1].p)) {
        dq.pop_back();
        --len;
      }
      else continue;
    }
    dq.push_back(H[i]);
    ++len;
  }

  while(len> 2 && dq[0].out(inter(dq[len-1], dq[len-2]))){
    dq.pop_back();
    --len;
  }
  while (len > 2 && dq[len-1].out(inter(dq[0], dq[1]))){
    dq.pop_front();
    --len;
  }
  if (len < 3) return vector<P>();
  vector<P> ret(len);
  for(int i = 0; i+1 < len; i++) {
    ret[i] = inter(dq[i], dq[i+1]);
```

```
d41     }
d41     ret.back() = inter(dq[len-1], dq[0]);
d41     return ret;
d41   }
```

8.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" d41d8c, 12 lines

```
d41   typedef Point<double> P;
d41   bool circleInter(P a,P b,double r1,double r2,pair<P, P>*
      out) {
d41    if (a == b) { assert(r1 != r2); return false; }
d41    P vec = b - a;
d41    double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2;
d41    double p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*
      d2;
d41    if (sum*sum < d2 || dif*dif > d2) return false;
d41    P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) /
      d2);
d41    *out = {mid + per, mid - per};
d41    return true;
d41   }
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" d41d8c, 14 lines

```
d41   template<class P>
d41   vector<pair<P, P>> tangents(P c1, double r1, P c2, double
      r2) {
d41    P d = c2 - c1;
d41    double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
d41    if (d2 == 0 || h2 < 0)  return {};
d41    vector<pair<P, P>> out;
d41    for (double sign : {-1, 1}) {
d41      P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
d41      out.push_back({c1 + v * r1, c2 + v * r2});
d41    }
d41    if (h2 == 0) out.pop_back();
d41    return out;
d41   }
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" d41d8c, 10 lines

```
d41   template<class P>
d41   vector<P> circleLine(P c, double r, P a, P b) {
d41    P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
d41    double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
d41    if (h2 < 0) return {};
d41    if (h2 == 0) return {p};
d41    P h = ab.unit() * sqrt(h2);
d41    return {p - h, p + h};
d41   }
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

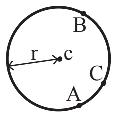
"../../content/geometry/Point.h" d41d8c, 20 lines

```
d41   typedef Point<double> P;
d41   #define arg(p, q) atan2(p.cross(q), p.dot(q))
d41   double circlePoly(P c, double r, vector<P> ps) {
d41    auto tri = [&](P p, P q) {
d41      auto r2 = r * r / 2;
d41      P d = q - p;
d41      auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist
      2();
d41      auto det = a * a - b;
d41      if (det <= 0) return arg(p, q) * r2;
d41      auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det
      ));
d41      if (t < 0 || 1 <= s) return arg(p, q) * r2;
d41      P u = p + d * s, v = q + d * (t-1);
d41      return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
d41    };
d41    auto sum = 0.0;
d41    rep(i,0,sz(ps))
d41      sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
d41    return sum;
d41   }
```

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" d41d8c, 10 lines

```
d41   typedef Point<double> P;
d41   double ccRadius(const P& A, const P& B, const P& C) {
d41    return (B-A).dist()*(C-B).dist()*(A-C).dist()/
d41        abs((B-A).cross(C-A))/2;
d41   }
d41   P ccCenter(const P& A, const P& B, const P& C) {
d41    P b = C-A, c = B-A;
d41    return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
d41   }
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h" d41d8c, 18 lines

```
d41   pair<P, double> mec(vector<P> ps) {
d41    shuffle(all(ps), mt19937(time(0)));
d41    P o = ps[0];
d41    double r = 0, EPS = 1 + 1e-8;
d41    rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
d41      o = ps[i], r = 0;
d41      rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
d41        o = (ps[i] + ps[j]) / 2;
d41        r = (o - ps[i]).dist();
d41        rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
d41          o = ccCenter(ps[i], ps[j], ps[k]);
d41          r = (o - ps[i]).dist();
d41        }
d41      }
d41    }
d41    return {o, r};
d41   }
```

8.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`
Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" d41d8c, 12 lines

```
d41   template<class P>
d41   bool inPolygon(vector<P> &p, P a, bool strict = true) {
d41    int cnt = 0, n = sz(p);
d41    rep(i,0,n) {
d41      P q = p[(i + 1) % n];
d41      if (onSegment(p[i], q, a)) return !strict;
         //or: if (segDist(p[i], q, a) <= eps) return !strict;
d41      cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) >
      0;
d41    }
d41    return cnt;
d41   }
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" d41d8c, 7 lines

```
d41   template<class T>
d41   T polygonArea2(vector<Point<T>>& v) {
d41    T a = v.back().cross(v[0]);
d41    rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
d41    return a;
d41   }
```

PolygonCenter.h

Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

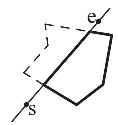
"Point.h" d41d8c, 10 lines

```
d41   typedef Point<double> P;
d41   P polygonCenter(const vector<P>& v) {
d41    P res(0, 0); double A = 0;
d41    for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
d41      res = res + (v[i] + v[j]) * v[j].cross(v[i]);
d41      A += v[j].cross(v[i]);
d41    }
d41    return res / A / 3;
d41   }
```

PolygonCut.h

Description:

Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.



Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h" d41d8c, 14 lines

```
d41   typedef Point<double> P;
d41   vector<P> polygonCut(const vector<P>& poly, P s, P e) {
d41    vector<P> res;
d41    rep(i,0,sz(poly)) {
d41      P cur = poly[i], prev = i ? poly[i-1] : poly.back();
d41      auto a = s.cross(e, cur), b = s.cross(e, prev);
d41      if ((a < 0) != (b < 0))
d41        res.push_back(cur + (prev - cur) * (a / (a - b)));
d41      if (a < 0)
d41        res.push_back(cur);
d41    }
d41    return res;
d41   }
```

PolygonUnion.h

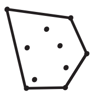
Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)
Time: $\mathcal{O}\left(N^2\right)$, where N is the total number of points

"Point.h", "sideOf.h" d41d8c, 34 lines

```
d41 typedef Point<double> P;
d41 double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y; }
d41 double polyUnion(vector<vector<P>>& poly) {
d41   double ret = 0;
d41   rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
d41     P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
d41     vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
d41     rep(j,0,sz(poly)) if (i != j) {
d41       rep(u,0,sz(poly[j])) {
d41         P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])];
d41         int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
d41         if (sc != sd) {
d41           double sa = C.cross(D, A), sb = C.cross(D, B);
d41           if (min(sc, sd) < 0)
d41             segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
d41         } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))>0){
d41           segs.emplace_back(rat(C - A, B - A), 1);
d41           segs.emplace_back(rat(D - A, B - A), -1);
d41         }
d41       }
d41     }
d41     sort(all(segs));
d41     for (auto& s : segs) s.first = min(max(s.first, 0.0), 1.0);
d41     double sum = 0;
d41     int cnt = segs[0].second;
d41     rep(j,1,sz(segs)) {
d41       if (!cnt) sum += segs[j].first - segs[j - 1].first;
d41       cnt += segs[j].second;
d41     }
d41     ret += A.cross(B) * sum;
d41   }
d41   return ret / 2;
d41 }
```

ConvexHull.h

Description:
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull. If you want to keep the collinear points in the convex hull, change the comparison to $h[t-2].cross(h[t-1], p) < 0$ and the size of the vector h to $2 * sz(pts) + 1$.



Time: $\mathcal{O}\left(n \log n\right)$

"Point.h" d41d8c, 14 lines

```
d41 typedef Point<ll> P;
d41 vector<P> convexHull(vector<P> pts) {
d41   if (sz(pts) <= 1) return pts;
d41   sort(all(pts));
d41   vector<P> h(sz(pts)+1);
d41   int s = 0, t = 0;
d41   for (int it = 2; it--; s = --t, reverse(all(pts)))
d41     for (P p : pts) {
d41       while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
d41       h[t++] = p;
d41     }
d41   return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
d41 }
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).
Time: $\mathcal{O}\left(n\right)$

"Point.h" d41d8c, 13 lines

```
d41 typedef Point<ll> P;
d41 array<P, 2> hullDiameter(vector<P> S) {
d41   int n = sz(S), j = n < 2 ? 0 : 1;
d41   pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
d41   rep(i,0,j)
d41     for (;; j = (j + 1) % n) {
d41       res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
d41       if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
d41         break;
d41     }
d41   return res.second;
d41 }
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}\left(\log N\right)$

"Point.h", "sideOf.h", "OnSegment.h" d41d8c, 15 lines

```
d41 typedef Point<ll> P;

d41 bool inHull(const vector<P>& l, P p, bool strict = true) {
d41   int a = 1, b = sz(l) - 1, r = !strict;
d41   if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
d41   if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
d41   if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
d41     return false;
d41   while (abs(a - b) > 1) {
d41     int c = (a + b) / 2;
d41     (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
d41   }
d41   return sgn(l[a].cross(l[b], p)) < r;
d41 }
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}\left(\log n\right)$

"Point.h" d41d8c, 40 lines

```
d41 #define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
d41 #define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
d41 template <class P> int extrVertex(vector<P>& poly, P dir) {
d41   int n = sz(poly), lo = 0, hi = n;
d41   if (extr(0)) return 0;
d41   while (lo + 1 < hi) {
d41     int m = (lo + hi) / 2;
d41     if (extr(m)) return m;
d41     int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
d41     (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
d41   }
d41   return lo;
d41 }

d41 #define cmpL(i) sgn(a.cross(poly[i], b))
d41 template <class P>
d41 array<int, 2> lineHull(P a, P b, vector<P>& poly) {
d41   int endA = extrVertex(poly, (a - b).perp());
d41   int endB = extrVertex(poly, (b - a).perp());
d41   if (cmpL(endA) < 0 || cmpL(endB) > 0)
d41     return {-1, -1};
d41   array<int, 2> res;
d41   rep(i,0,2) {
d41     int lo = endB, hi = endA, n = sz(poly);
d41     while ((lo + 1) % n != hi) {
d41       int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
d41       (cmpL(m) == cmpL(endB) ? lo : hi) = m;
d41     }
d41     res[i] = (lo + !cmpL(hi)) % n;
d41     swap(endA, endB);
d41   }
d41   if (res[0] == res[1]) return {res[0], -1};
d41   if (!cmpL(res[0]) && !cmpL(res[1]))
d41     switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
d41       case 0: return {res[0], res[0]};
d41       case 2: return {res[1], res[1]};
d41     }
d41   return res;
d41 }
```

Minkowski.h

Description: Computes the Minkowski sum of two convex polygons. Polygons must be convex and given in CCW order. Returns the vertices of the Minkowski sum polygon in CCW order.
Time: $\mathcal{O}\left(n+m\right)$

"Point.h" d41d8c, 24 lines

```
d41 using P = Point<ll>;

d41 vector<P> minkowski(vector<P> p, vector<P> q) {
d41   auto fix = [](vector<P>& A) {
d41     int pos = 0;
d41     for (int i = 1; i < sz(A); i++) {
d41       if(A[i].y < A[pos].y || (A[i].y == A[pos].y && A[i].x < A[pos].x))
d41         pos = i;
d41     }
d41     rotate(A.begin(), A.begin() + pos, A.end());
d41     A.push_back(A[0]), A.push_back(A[1]);
d41   };
d41   fix(p), fix(q);
d41   vector<P> result;
d41   int i = 0, j = 0;
d41   while (i < sz(p) - 2 || j < sz(q) - 2) {
d41     result.push_back(p[i] + q[j]);
d41     auto cross = (p[i + 1] - p[i]).cross(q[j + 1] - q[j]);
d41     if (cross >= 0 && i < sz(p) - 2) i++;
d41     if (cross <= 0 && j < sz(q) - 2) j++;
d41   }
d41   return result;
d41 }
```

Extreme.h

Description: Finds an extreme vertex of a convex polygon according to a unimodal comparator. The comparator defines a total order along the polygon (given in CCW order).
Time: $\mathcal{O}(\log n)$

"Point.h" d41d8c, 26 lines

```
d41 using P = Point<ll>;
d41 int extreme(vector<P> &pol, const function<bool(P, P)>& cmp) {
d41   int n = pol.size();
d41   auto extr = [&](int i, bool& cur_dir) {
d41     cur_dir = cmp(pol[(i+1)%n], pol[i]);
d41     return !cur_dir and !cmp(pol[(i+n-1)%n], pol[i]);
d41   };
d41   bool last_dir, cur_dir;
d41   if (extr(0, last_dir)) return 0;
d41   int l = 0, r = n;
d41   while (l+1 < r) {
d41     int m = (l+r)/2;
d41     if (extr(m, cur_dir)) return m;
d41     bool rel_dir = cmp(pol[m], pol[l]);
d41     if ((!last_dir and cur_dir) or
d41         (last_dir == cur_dir and rel_dir == cur_dir)) {
d41       l = m;
d41       last_dir = cur_dir;
d41     } else r = m;
d41   }
d41   return l;
d41 }
d41 int max_dot(vector<P> &pol, P v) {
d41   return extreme([&](P p, P q) { return p.dot(v) > q.dot(v); });
d41 }
```

Tangents.h

Description: Finds the left and right tangent points from an external point p to a convex polygon given in CCW order. A tangent point is a vertex where the segment p->v touches the polygon without intersecting its interior, defining the limits of visibility from p. Returns the indices of the left and right tangent vertices.
Time: $\mathcal{O}(\log n)$

"Point.h", "Extreme.h" d41d8c, 11 lines

```
d41 using P = Point<ll>;

d41 bool ccw(P p, P q, P r) {
d41  return (q-p).cross(r-q) > 0;
d41 }
d41 pair<int, int> tangents(vector<P> &pol, P p) {
d41     auto L = [&](P q, P r) { return ccw(p, r, q); };
d41     auto R = [&](P q, P r) { return ccw(p, q, r); };
d41     return {extreme(pol, L), extreme(pol, R)};
d41 }
```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" d41d8c, 18 lines

```
d41 typedef Point<ll> P;
d41 pair<P, P> closest(vector<P> v) {
d41  assert(sz(v) > 1);
d41  set<P> S;
d41  sort(all(v), [](P a, P b) { return a.y < b.y; });
d41  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
d41  int j = 0;
d41  for (P p : v) {
d41    P d{1 + (ll)sqrt(ret.first), 0};
d41    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
d41    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
d41    for (; lo != hi; ++lo)
d41      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
d41    S.insert(p);
d41  }
d41  return ret.second;
d41 }
```

ManhattanMST.h

Description: Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights w(p, q) = —p.x - q.x— + —p.y - q.y—. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
Time: $\mathcal{O}(N \log N)$

"Point.h" d41d8c, 24 lines

```
d41 typedef Point<int> P;
d41 vector<array<int, 3>> manhattanMST(vector<P> ps) {
d41  vi id(sz(ps));
d41  iota(all(id), 0);
d41  vector<array<int, 3>> edges;
d41  rep(k,0,4) {
d41    sort(all(id), [&](int i, int j) {
d41        return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
d41    map<int, int> sweep;
d41    for (int i : id) {
d41      for (auto it = sweep.lower_bound(-ps[i].y);
d41                it != sweep.end(); sweep.erase(it++)) {
d41        int j = it->second;
d41        P d = ps[i] - ps[j];
d41        if (d.y > d.x) break;
d41        edges.push_back({d.y + d.x, i, j});
d41      }
d41      sweep[-ps[i].y] = i;
d41    }
d41    for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p.y);
d41  }
d41  return edges;
d41 }
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" d41d8c, 64 lines

```
d41 typedef long long T;
d41 typedef Point<T> P;
d41 const T INF = numeric_limits<T>::max();

d41 bool on_x(const P& a, const P& b) { return a.x < b.x; }
d41 bool on_y(const P& a, const P& b) { return a.y < b.y; }

d41 struct Node {
d41  P pt; // if this is a leaf, the single point in it
d41  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
d41  Node *first = 0, *second = 0;

d41  T distance(const P& p) { // min squared distance to a point
d41    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
d41    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
d41    return (P(x,y) - p).dist2();
d41  }

d41  Node(vector<P>&& vp) : pt(vp[0]) {
d41    for (P p : vp) {
d41      x0 = min(x0, p.x); x1 = max(x1, p.x);
d41      y0 = min(y0, p.y); y1 = max(y1, p.y);
d41    }
d41    if (vp.size() > 1) {
         // split on x if width >= height (not ideal...)
d41      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
         // divide by taking half the array for each child (not
         // best performance with many duplicates in the middle)
d41      int half = sz(vp)/2;
d41      first = new Node({vp.begin(), vp.begin() + half});
d41      second = new Node({vp.begin() + half, vp.end()});
d41    }
d41  }
d41 };

d41 struct KDTree {
d41  Node* root;
d41  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

d41  pair<T, P> search(Node *node, const P& p) {
d41    if (!node->first) {
         // uncomment if we should not find the point itself:
         // if (p == node->pt) return {INF, P()};
d41      return make_pair((p - node->pt).dist2(), node->pt);
d41    }

d41    Node *f = node->first, *s = node->second;
d41    T bfirst = f->distance(p), bsec = s->distance(p);
d41    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

       // search closest side first, other side if needed
d41    auto best = search(f, p);
d41    if (bsec < best.first)
d41      best = min(best, search(s, p));
d41    return best;
d41  }

   // find nearest point to a point, and its squared distance
   // (requires an arbitrary operator< for Point)
d41  pair<T, P> nearest(const P& p) {
d41    return search(root, p);
d41  }
d41 };
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], . . . }, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" d41d8c, 89 lines

```
d41 typedef Point<ll> P;
d41 typedef struct Quad* Q;
d41 typedef __int128_t lll; // (can be ll if coords are < 2e4)
d41 P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

d41 struct Quad {
d41  Q rot, o; P p = arb; bool mark;
d41  P& F() { return r()->p; }
d41  Q& r() { return rot->rot; }
d41  Q prev() { return rot->o->rot; }
d41  Q next() { return r()->prev(); }
d41 } *H;
```

```
d41 bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
d41  lll p2 = p.dist2(), A = a.dist2()-p2,
d41      B = b.dist2()-p2, C = c.dist2()-p2;
d41  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
d41 }
d41 Q makeEdge(P orig, P dest) {
d41  Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
d41  H = r->o; r->r()->r() = r;
d41  rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
d41  r->p = orig; r->F() = dest;
d41  return r;
d41 }
d41 void splice(Q a, Q b) {
d41  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
d41 }
d41 Q connect(Q a, Q b) {
d41  Q q = makeEdge(a->F(), b->p);
d41  splice(q, a->next());
d41  splice(q->r(), b);
d41  return q;
d41 }

d41 pair<Q,Q> rec(const vector<P>& s) {
d41  if (sz(s) <= 3) {
d41    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
d41    if (sz(s) == 2) return { a, a->r() };
d41    splice(a->r(), b);
d41    auto side = s[0].cross(s[1], s[2]);
d41    Q c = side ? connect(b, a) : 0;
d41    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
d41  }

d41 #define H(e) e->F(), e->p
d41 #define valid(e) (e->F().cross(H(base)) > 0)
d41  Q A, B, ra, rb;
d41  int half = sz(s) / 2;
d41  tie(ra, A) = rec({all(s) - half});
d41  tie(B, rb) = rec({sz(s) - half + all(s)});
d41  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
d41         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
d41  Q base = connect(B->r(), A);
d41  if (A->p == ra->p) ra = base->r();
d41  if (B->p == rb->p) rb = base;

d41 #define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
d41    while (circ(e->dir->F(), H(base), e->F())) { \
d41      Q t = e->dir; \
d41      splice(e, e->prev()); \
d41      splice(e->r(), e->r()->prev()); \
d41      e->o = H; H = e; e = t; \
d41    }
d41  for (;;) {
d41    DEL(LC, base->r(), o);  DEL(RC, base, prev());
d41    if (!valid(LC) && !valid(RC)) break;
d41    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
d41      base = connect(RC, base->r());
d41    else
d41      base = connect(base->r(), LC->r());
d41  }
d41  return { ra, rb };
d41 }

d41 vector<P> triangulate(vector<P> pts) {
d41  sort(all(pts));  assert(unique(all(pts)) == pts.end());
d41  if (sz(pts) < 2) return {};
d41  Q e = rec(pts).first;
d41  vector<Q> q = {e};
d41  int qi = 0;
d41  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
d41 #define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
d41  q.push_back(c->r()); c = c->next(); } while (c != e); }
d41  ADD; pts.clear();
d41  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
d41  return pts;
d41 }
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

d41d8c, 7 lines

```
d41 template<class V, class L>
d41 double signedPolyVolume(const V& p, const L& trilist) {
d41  double v = 0;
d41  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
d41  return v / 6;
d41 }
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

d41d8c, 33 lines

```
d41 template<class T> struct Point3D {
d41  typedef Point3D P;
d41  typedef const P& R;
d41  T x, y, z;
d41  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
d41  bool operator<(R p) const {
d41    return tie(x, y, z) < tie(p.x, p.y, p.z); }
d41  bool operator==(R p) const {
d41    return tie(x, y, z) == tie(p.x, p.y, p.z); }
d41  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
d41  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
d41  P operator*(T d) const { return P(x*d, y*d, z*d); }
d41  P operator/(T d) const { return P(x/d, y/d, z/d); }
d41  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
d41  P cross(R p) const {
d41    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
d41  }
d41  T dist2() const { return x*x + y*y + z*z; }
d41  double dist() const { return sqrt((double)dist2()); }
     //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
d41  double phi() const { return atan2(y, x); }
     //Zenith angle (latitude) to the z-axis in interval [0, pi]
d41  double theta() const { return atan2(sqrt(x*x+y*y),z); }
d41  P unit() const { return *this/(T)dist(); } //makes dist()=1
     //returns unit vector normal to *this and p
d41  P normal(P p) const { return cross(p).unit(); }
     //returns point rotated 'angle' radians ccw around axis
d41  P rotate(double angle, P axis) const {
d41    double s = sin(angle), c = cos(angle); P u = axis.unit();
d41    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
d41  }
d41 };
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.

Time: $\mathcal{O}(n^2)$

"Point3D.h" d41d8c, 50 lines

```
d41 typedef Point3D<double> P3;

d41 struct PR {
d41  void ins(int x) { (a == -1 ? a : b) = x; }
d41  void rem(int x) { (a == x ? a : b) = -1; }
d41  int cnt() { return (a != -1) + (b != -1); }
d41  int a, b;
d41 };

d41 struct F { P3 q; int a, b, c; };

d41 vector<F> hull3d(const vector<P3>& A) {
d41  assert(sz(A) >= 4);
d41  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
d41 #define E(x,y) E[f.x][f.y]
d41  vector<F> FS;
d41  auto mf = [&](int i, int j, int k, int l) {
d41    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
d41    if (q.dot(A[l]) > q.dot(A[i]))
d41      q = q * -1;
d41    F f{q, i, j, k};
d41    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
d41    FS.push_back(f);
d41  };
d41  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
d41    mf(i, j, k, 6 - i - j - k);

d41  rep(i,4,sz(A)) {
d41    rep(j,0,sz(FS)) {
d41      F f = FS[j];
d41      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
d41        E(a,b).rem(f.c);
d41        E(a,c).rem(f.b);
d41        E(b,c).rem(f.a);
d41        swap(FS[j--], FS.back());
d41        FS.pop_back();
d41      }
d41    }
d41    int nw = sz(FS);
d41    rep(j,0,nw) {
d41      F f = FS[j];
d41 #define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
d41      C(a, b, c); C(a, c, b); C(b, c, a);
d41    }
d41  }
d41  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
d41    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
d41  return FS;
d41 };
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis ($0 =$ north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

d41d8c, 9 lines

```
d41 double sphericalDistance(double f1, double t1,
d41    double f2, double t2, double radius) {
```

```
d41   double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
d41   double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
d41   double dz = cos(t2) - cos(t1);
d41   double d = sqrt(dx*dx + dy*dy + dz*dz);
d41   return radius*2*asin(d/2);
d41 }
```

Strings (9)

AhoCorasick.h

d41d8c, 46 lines

```
d41 int trie[ms][sigma], fail[ms], terminal[ms], superfail[ms
    ];
d41 bool present[ms];
d41 int z = 1;

d41 int val(char c) { return c - 'a'; }

d41 void add(string& p) {
d41     int cur = 0;
d41     for (int i = 0; i < (int)p.size(); i++) {
d41         int& nxt = trie[cur][val(p[i])];
d41         if (nxt == 0) nxt = z++;
d41         cur = nxt;
d41     }
d41     present[cur] = true;
d41     terminal[cur]++;
d41 }

d41 void build() {
d41     queue<int> q;
d41     for (q.push(0); !q.empty(); q.pop()) {
d41         int on = q.front();
d41         for (int i = 0; i < sigma; i++) {
d41             int& to = trie[on][i];
d41             int f = (on == 0 ? 0 : trie[fail[on]][i]);
d41             int sf = (present[f] ? f : superfail[f]);
d41             if (!to) {
d41                 to = f;
d41             }
d41             else {
d41                 fail[to] = f;
d41                 superfail[to] = sf;
                    // merge infos (ex: terminal[to] +=
                        terminal[f])
d41                 q.push(to);
d41             }
d41         }
d41     }
d41 }

d41 void search(string& s) {
d41     int cur = 0;
d41     for (char c : s) {
d41         cur = trie[cur][val(c)];
            // process infos on current node (ex: ocurrences
                += terminal[cur])
d41     }
d41 }
```

Hash.h

Description: C can also be random, operator is $[l, r]$

d41d8c, 28 lines

```
d41 using ull = uint64_t;
d41 struct H {
d41     ull x; H(ull x = 0) : x(x) {}
d41     H operator+(H o) { return x + o.x + (x + o.x < x); }
d41     H operator-(H o) { return *this + ~o.x; }
d41     H operator*(H o) {
d41         auto m = (__uint128_t)x * o.x;
d41         return H((ull)m) + (ull)(m >> 64);
d41     }
d41     ull get() const { return x + !~x; }
d41     bool operator==(H o) const{ return get() == o.get();}
d41     bool operator<(H o) const{ return get() < o.get();}
d41 };
d41 static const H C = (ll)1e11 + 3;
d41 struct Hash {
d41     vector<H> h, pw;
d41     Hash(string& str) : h(str.size()), pw(str.size()) {
d41         pw[0] = 1, h[0] = str[0];
d41         for (int i = 1; i < str.size(); i++) {
d41             h[i] = h[i - 1] * C + str[i];
d41             pw[i] = pw[i - 1] * C;
d41         }
d41     }
d41     H operator()(int l, int r) {
d41         return h[r] - (l ? h[l - 1] * pw[r - l + 1] : 0);
d41     }
d41 };
```

Kmp.h

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123).

d41d8c, 10 lines

```
d41 vector<int> pi(const string& s) {
d41     vector<int> p(sz(s));
d41     for(int i = 1; i < sz(s); i++) {
d41         int g = p[i-1];
d41         while (g && s[i] != s[g]) g = p[g-1];
d41         p[i] = g + (s[i] == s[g]);
d41     }
d41     return p;
d41 }
```

KmpAutomaton.h

Description: $go[i][j]$ = length of the longest prefix of s that is a suffix of $s[0..i]$ followed by the letter j (i.e., the next matched prefix length if, at state i, we read letter j).

d41d8c, 17 lines

```
d41 int go[ms][sigma];
d41 int val(char c) { return c - 'a'; }
d41 void automaton(string& s) {
d41     for (int i = 0; i < sigma; i++)
d41         go[0][i] = (i == val(s[0]));

d41     for (int i = 1, bdr = 0; i <= (int)s.size(); i++) {
d41         for (int j = 0; j < sigma; j++) {
d41             go[i][j] = go[bdr][j];
d41         }
d41         if (i < (int)s.size()) {
d41             go[i][val(s[i])] = i + 1;
d41             bdr = go[bdr][val(s[i])];
d41         }
d41     }
d41 }
```

Manacher.h

Description: $p[0][i+1]$ is the length of matches of even length palindrome, starting from $[i, i+1]$.
$p[1][i]$ is the length of matches of odd length palindrome, starting from $[i, i]$.
(abaxx -> p[0] = 00001)
(abaxx -> p[1] = 01000)

d41d8c, 17 lines

```
d41 array<vector<int>, 2> manacher(const string& s) {
d41     int n = sz(s);
d41     array<vector<int>,2> p={vector<int>(n+1),vector<int>(n
        )};
d41     for (int z = 0; z < 2; z++) {
d41         for (int i = 0, l = 0, r = 0; i < n; i++) {
d41             int t = r - i + !z;
d41             if (i < r) p[z][i] = min(t, p[z][l + t]);
d41             int L = i - p[z][i], R = i + p[z][i] - !z;
d41             while(L >= 1 && R+1 < n && s[L-1] == s[R+1]){
d41                 p[z][i]++, L--, R++;
d41             }
d41             if (R > r) l = L, r = R;
d41         }
d41     }
d41     return p;
d41 }
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(s.begin(), s.begin()+minRotation(s), s.end());`
Time: $\mathcal{O}(N)$

d41d8c, 14 lines

```
d41 int minRotation(string s) {
d41     int a = 0, N = s.size(); s += s;
d41     for (int b = 0; b < N; b++) {
d41         for (int k = 0; k < N; k++) {
d41             if (a+k == b || s[a+k] < s[b+k]) {
d41                 b += max(0, k-1);
d41                 break;
d41             }
d41             if (s[a+k] > s[b+k]) { a = b; break; }
d41         }
d41     }
d41     return a;
d41 }
```

SuffixArray.h

Description: $lcp[i]$ is the length of the longest common prefix between the suffixes $s[sa[i]..n-1]$ and $s[sa[i-1]..n-1]$.
If we concatenate multiple strings using separator characters, the separator that appears furthest to the right must be the smallest character in the alphabet.

d41d8c, 31 lines

```
d41 struct SuffixArray {
d41  vector<int> sa, lcp;
d41  SuffixArray(string s, int lim=256) {
d41    s.push_back('$');
d41    int n = sz(s), k = 0, a, b;
d41    vector<int> x(all(s)), y(n), ws(max(n, lim));
d41    sa = lcp = y, iota(all(sa), 0);
d41    for(int j = 0, p = 0; p < n; j= max(1, j*2), lim = p) {
d41      p = j, iota(all(y), n - j);
d41      for(int i=0; i<n; i++){
d41        if (sa[i] >= j) y[p++] = sa[i] - j;
d41      }
d41      fill(all(ws), 0);
d41      for(int i=0; i<n; i++) ws[x[i]]++;
d41      for(int i=1; i<lim; i++) ws[i] += ws[i - 1];
d41      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
d41      swap(x, y), p = 1, x[sa[0]] = 0;
d41      for(int i=1; i<n; i++){
d41        a = sa[i - 1], b = sa[i];
d41        x[b] = p-1;
d41        if(y[a] != y[b] || y[a+j] != y[b+j]) x[b] = p++;
d41      }
d41    }
d41    for (int i = 0, j; i < n - 1; lcp[x[i++]] = k)
d41      for (k && k--, j = sa[x[i] - 1];
d41          s[i + k] == s[j + k]; k++);
d41    sa = vector<int>(sa.begin() + 1, sa.end());
```

```
    lcp = vector<int>(lcp.begin() + 1, lcp.end());
  }
};
```

Zfunc.h

Description: z[i] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)

d41d8c, 13 lines

```
vector<int> ZFunc(const string& s) {
    int n = sz(s), a = 0, b = 0;
    vector<int> z(n, 0);
    if (!z.empty()) z[0] = 0;
    for (int i = 1; i < n; i++) {
        int end = i;
        if (i < b) end = min(i + z[i - a], b);
        while (end < n && s[end] == s[end - i]) ++end;
        z[i] = end - i; if (end > b) a = i, b = end;
    }
    return z;
}
```

Various (10)

10.1 Misc. algorithms

Dates.h

Description: dateToInt converts Gregorian date to integer (Julian day number). intToDate converts integer (Julian day number) to Gregorian date: month/day/year. intToDay converts Julian day number to day of the week

d41d8c, 23 lines

```
string day[] = { "Mon", "Tue", "Wed", "Thu", "Fri", "Sat",
  "Sun" };
int dateToInt(int m, int d, int y) {
    return
        1461 * (y + 4800 + (m - 14) / 12) / 4 +
        367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
        3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
        d - 32075;
}
void intToDate(int jd, int& m, int& d, int& y) {
    int x, n, i, j;
    x = jd + 68569;
    n = 4 * x / 146097;
    x -= (146097 * n + 3) / 4;
    i = (4000 * (x + 1)) / 1461001;
    x -= 1461 * i / 4 - 31;
    j = 80 * x / 2447;
    d = x - 2447 * j / 80;
    x = j / 11;
    m = j + 2 - 12 * x;
    y = 100 * (n - 49) + i + x;
}
string intToDay(int jd) { return day[jd % 7]; }
```

MultisetHash.h

d41d8c, 8 lines

```
ull hashify(ull sum) {
    sum += FIXED_RANDOM;
    sum += 0x9e3779b97f4a7c15;
    sum = (sum ^ (sum >> 30)) * 0xbf58476d1ce4e5b9;
    sum = (sum ^ (sum >> 27)) * 0x94d049bb133111eb;
    return sum ^ (sum >> 31);
}
```

Rand.h

d41d8c, 8 lines

```
mt19937 rng(chrono::steady_clock::now().time_since_epoch()
  .count());
// _64

int uniform(int l, int r) { // [l, r]
    uniform_int_distribution<int> uid(l, r);
    return uid(rng);
}
```

10.2 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: $dp[i][j] = \min_{i<k<j}(dp[i][k] + dp[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \le f(a,d) + f(b,c)$ for all $a \le b \le c \le d$. Another sufficient criteria is: $opt[i][j-1] \le opt[i][j] \le opt[i+1][j]$

Time: $\mathcal{O}(N^2)$

d41d8c, 22 lines

```
ll knuth(){
    memset(opt, -1, sizeof opt);
    for(int i=n-1; i>=0; i--){
        dp[i][i] = 0; // base case
        opt[i][i] = i;
        for(int j=i+1; j<n; j++){
            int optL = (!j ? 0 : opt[i][j-1]);
            int optR = (~opt[i+1][j] ? opt[i+1][j] : n-1);
            ll cst = cost(i, j);
            dp[i][j] = INF;
            optL = max(i, optL), optR = min(j-1, optR);
            for(int k=optL; k<=optR; k++){
                ll now = dp[i][k] + dp[k+1][j] + cst;
                if(now <= dp[i][j]){
                    dp[i][j] = now;
                    opt[i][j] = k;
                }
            }
        }
    }
}
```

DivideAndConquerDP.h

Description: Divide and Conquer DP maintaining cost, can be used when $opt[i][j] \le opt[i][j+1]$. In this code everything is 1-based. Memory can be optmized by keeping only the last row

Time: $\mathcal{O}(MN \log N)$

d41d8c, 42 lines

```
void add(int idx) {}
void rem(int idx) {}

void deC(int i, int l, int r, int optL, int optR) {
    if (l > r) return;
    int j = (l + r) / 2;
    for (int k = r; k > j; k--) rem(k);
    int opt = optL;
    for (int k = optL; k <= min(optR, j); k++) {
        // cost = cost[k, j]
        int val = dp[i - 1][k - 1] + cost;
        if (val < dp[i][j]) {
            dp[i][j] = val;
            opt = k;
        }
        rem(k);
    }
    for (int k = min(optR, j); k >= optL; k--) add(k);
    rem(j);
    deC(i, l, j - 1, optL, opt);

    for (int k = j; k <= r; k++) add(k);
    for (int k = optL; k < opt; k++) rem(k);
    deC(i, j + 1, r, opt, optR);
    for (int k = optL; k < opt; k++) add(k);
}

int solve(int N, int M) { // 1-based
    for (int i = 0; i <= M; i++) {
        for (int j = 0; j <= N; j++){
            dp[i][j] = inf; // base case
        }
    }
    cost = 0; // neutral value
    for (int i = 1; i <= N; i++) add(i);
    for (int i = 1; i <= M; i++) {
        deC(i, 1, N, 1, N);
    }
    return dp[M][N];
}
```

10.3 Optimization tricks

10.3.1 Bit hacks

- `for (int x = m; x; x = (x - 1)&m ) { ... }` loops over all subset masks of `m` (except 0).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
  `if (i & 1 << b) D[i] += D[i^(1 << b)];`
  computes all sums of subsets.

10.3.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC target("bmi,bmi2,popcnt,lzcnt")` improve bit operations.
- `#pragma GCC optimize("unroll-loops")` self explanatory.