

Stable Matching in Bipartite Graphs

Matching in Complete Bipartite Graphs using Preference Lists

Zofia Ambrozewicz

Politechnika Łódzka

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Introduction

What is Stable Matching?

- ▶ A stable matching is a matching in which no pair of elements prefer each other over their current matches.

Real-World Applications:

- ▶ College admissions
- ▶ Job recruitment
- ▶ Students finding thesis supervisors

Problem Statement

Input:

- ▶ A complete bipartite graph with two sets of nodes:
 - ▶ Group A (e.g., students)
 - ▶ Group B (e.g., supervisors)
- ▶ Each node has a ranked preference list of nodes from the other group.

Output:

- ▶ A stable matching of nodes from Group A to Group B.

Goal:

- ▶ Ensure no pair from A and B prefers each other over their current match.

Algorithm: Gale-Shapley (Deferred Acceptance)

Step 1: Initialization

- ▶ All nodes in Group A and Group B are initially unmatched.
- ▶ Each node in Group A proposes to the highest-ranked node on its preference list.

Step 2: Proposal Phase

- ▶ Each node in Group B evaluates proposals:
 - ▶ Accepts the most preferred proposal (tentatively matched).
 - ▶ Rejects others.

Step 3: Repeat

- ▶ Rejected nodes in Group A propose to the next preferred node.
- ▶ Continue until all nodes are matched.

Step 4: Final Matching

- ▶ When no node in Group A has unmatched preferences, the algorithm terminates.

Justification of Algorithm Shape

Optimality:

- ▶ Guarantees a stable matching.
- ▶ Results in an outcome optimal for Group A (proposing group).

Termination:

- ▶ Each node in Group A proposes to each node in Group B at most once.
- ▶ Finite steps due to the bounded size of preference lists.

Fairness:

- ▶ No blocking pairs exist, ensuring stability.

Example of Operation: Students and Supervisors

Scenario:

- ▶ Group A: Students $\{S1, S2\}$
- ▶ Group B: Supervisors $\{T1, T2\}$

Preferences:

- ▶ Students:
 - ▶ S1: $T1 \succ T2$
 - ▶ S2: $T2 \succ T1$
- ▶ Supervisors:
 - ▶ T1: $S2 \succ S1$
 - ▶ T2: $S1 \succ S2$

Process:

- ▶ S1 proposes to T1, S2 proposes to T2.
- ▶ T1 and T2 tentatively accept.
- ▶ Matching: $\{(S1, T1), (S2, T2)\}$.

Stability Check:

- ▶ No blocking pairs exist.