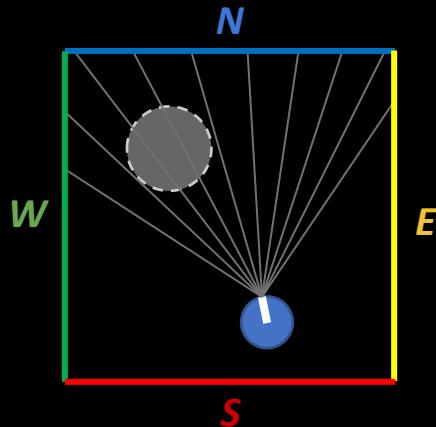


Fundamental Visual Navigation Algorithms: Indirect Sequential, Biased Diffusive, & Direct Pathing



*Patrick Govoni
Romanczuk Lab
Humboldt-Universität zu Berlin*

Computational
Task + Environment

~ biology

Algorithmic
Behavior + Circuit Mechanisms

Training

representational
convergence

Implementational
Input/Output Format + Neural Architecture

~ biology + sufficiently expressive

Computational
Task + Environment

Navigation to a known destination
(not search)

Algorithmic
Behavior + Circuit Mechanisms

Training

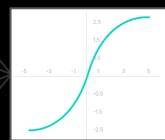
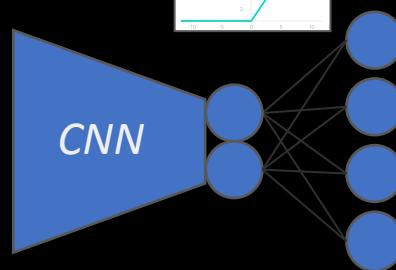
Implementational
Input/Output Format + Neural Architecture

min(assumptions)

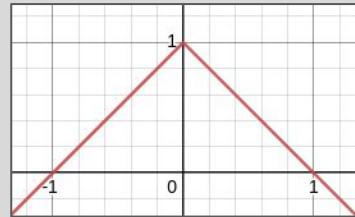
max(generalizability)

Vision (CNN)

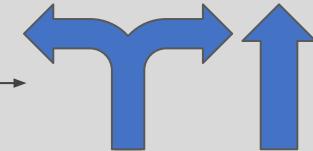
Action Conversion



speed



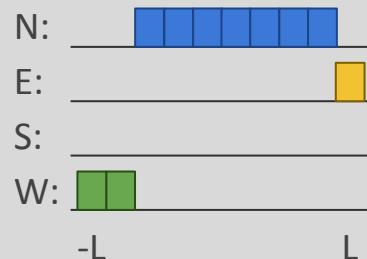
NN output \rightarrow turn angle



Distance Scaling
(if $\sigma > 0$)

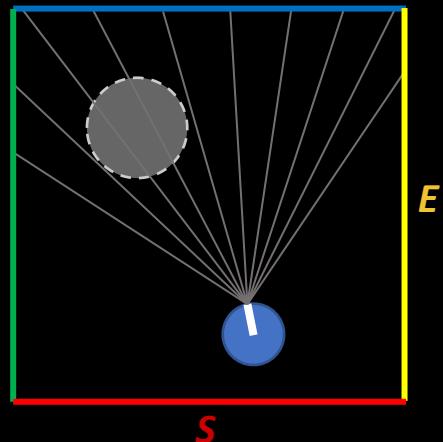


Boundary Encoding



Visual Encoding

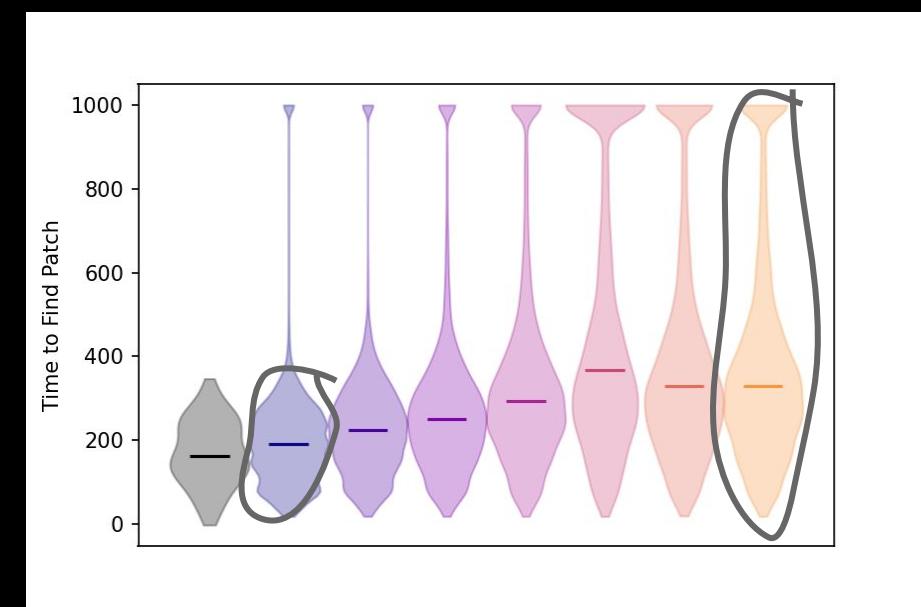
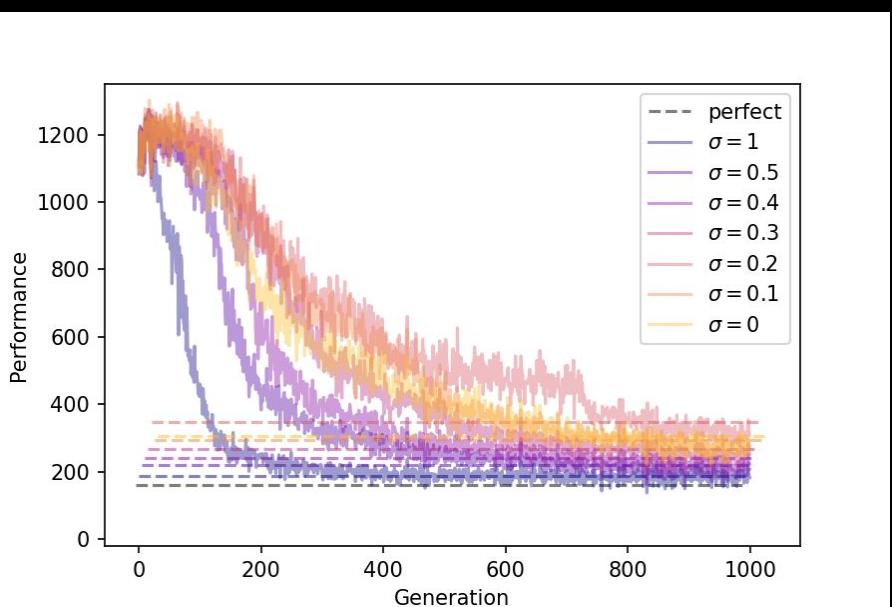
$$y = -\frac{1}{k(\sigma)} \ln(x) + m(\sigma)$$



Trained via Evolutionary Strategies

- Performance =
time to patch +
remaining distance

visual input =
angles ($\sigma = 0$) → learns task
+ distance ($\sigma > 0$) → near perfect



Local behavior
Angles Only

Navigational
Class
1

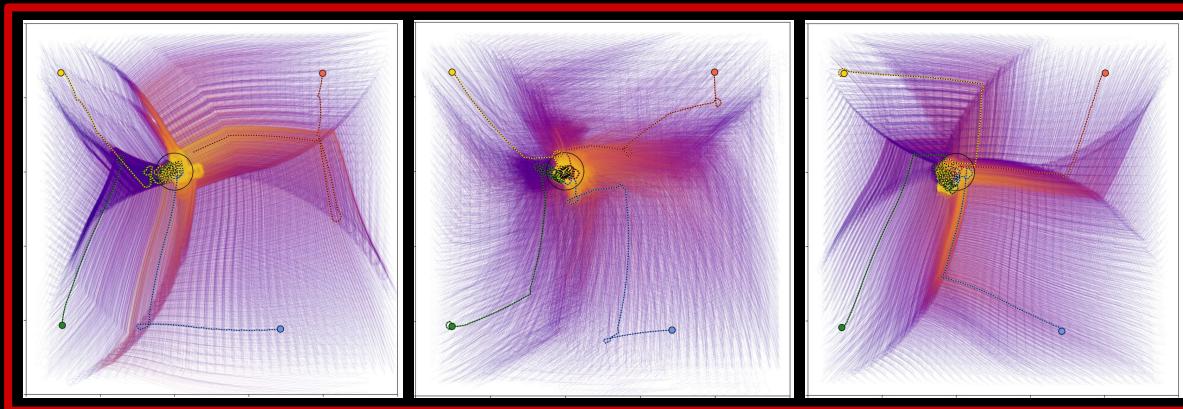
“Indirect
Sequential”

Local behavior
Angles Only

Navigational
Class
2

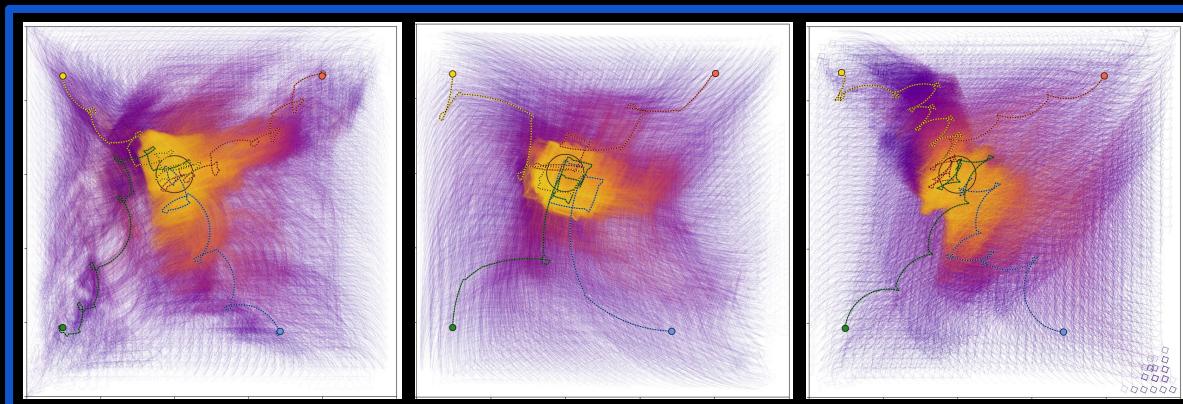
“Biased Diffusive”

Global behavior Angles Only



Indirect Sequential

- Grid-like trajectories
- Compositional route segments
- Elliptical decision manifolds



Biased Diffusive

- Spin/ratchet-like trajectories
- Directed towards patch

Local behavior

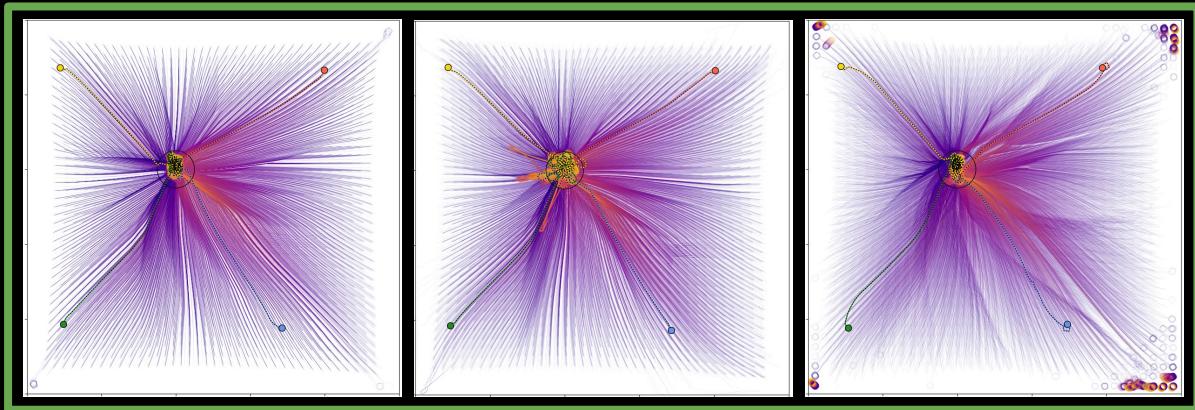
Angles + Distance

Navigational
Class

3

“Direct Pathing”

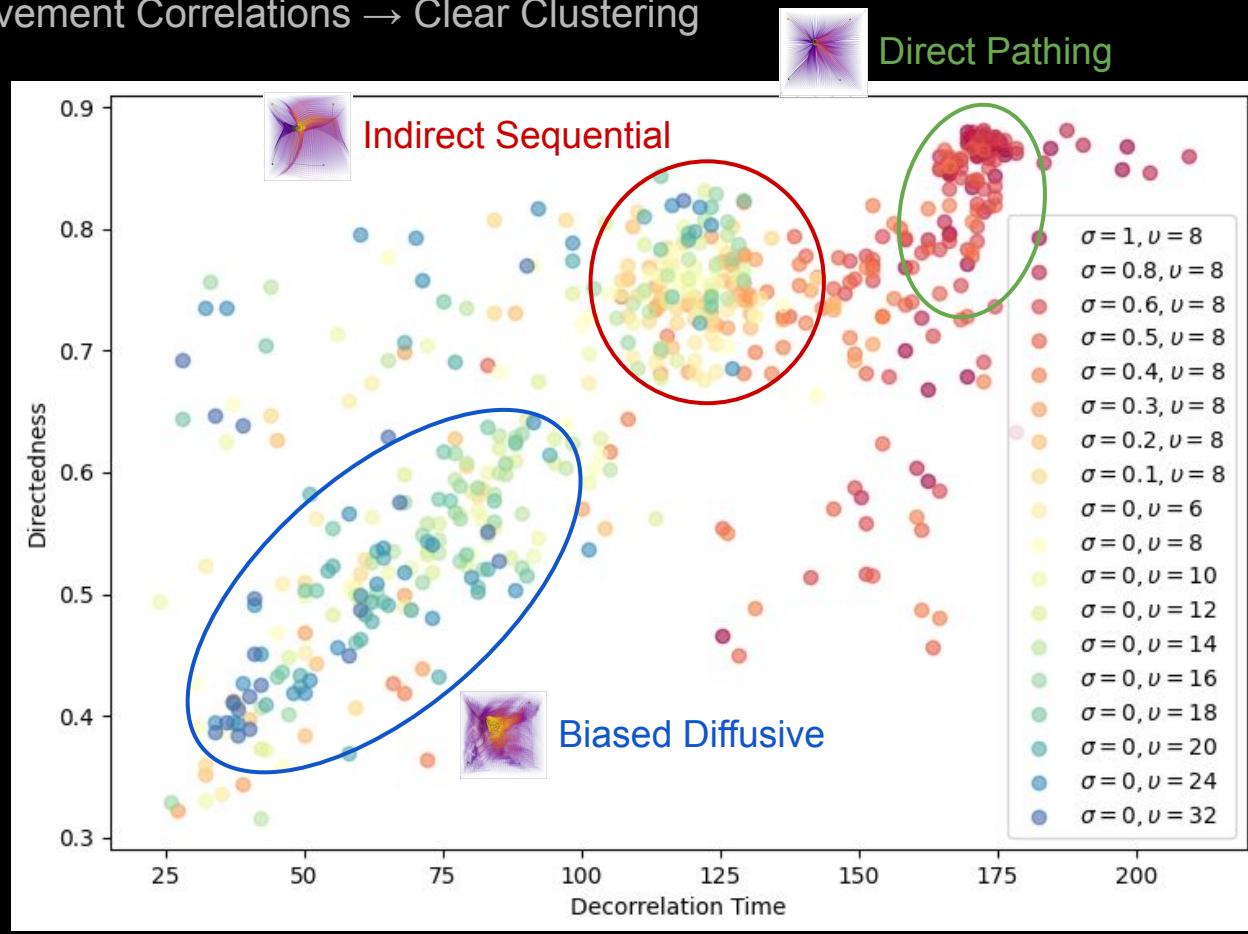
Global behavior Angles + Distance



Direct Pathing

- Strong unidirectional persistence
- Highly accurate patch estimation

Movement Correlations → Clear Clustering



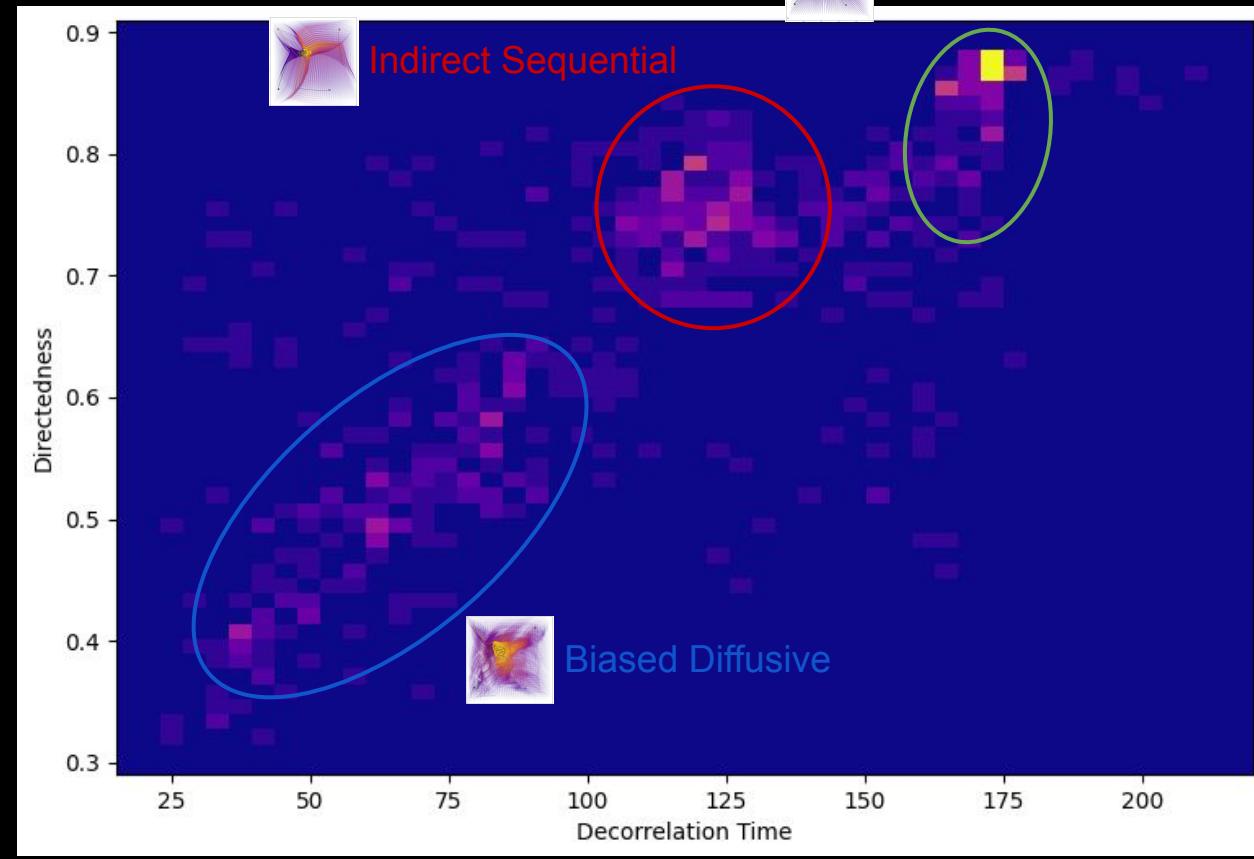
σ = distance scaling factor

v = visual resolution

Movement Correlations → Clear Clustering



Direct Pathing



Temporal Correlation

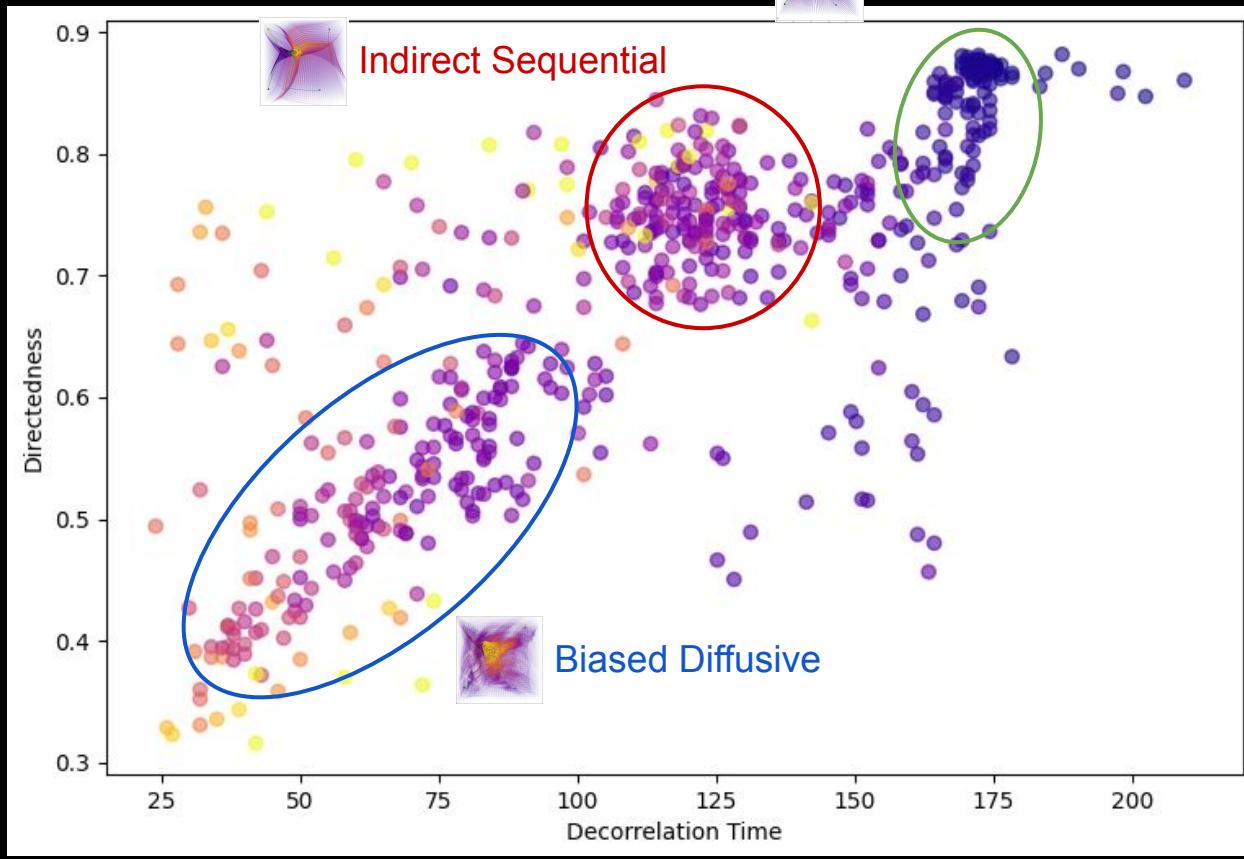
Fitnesses : DP > IS ~ BD



Direct Pathing

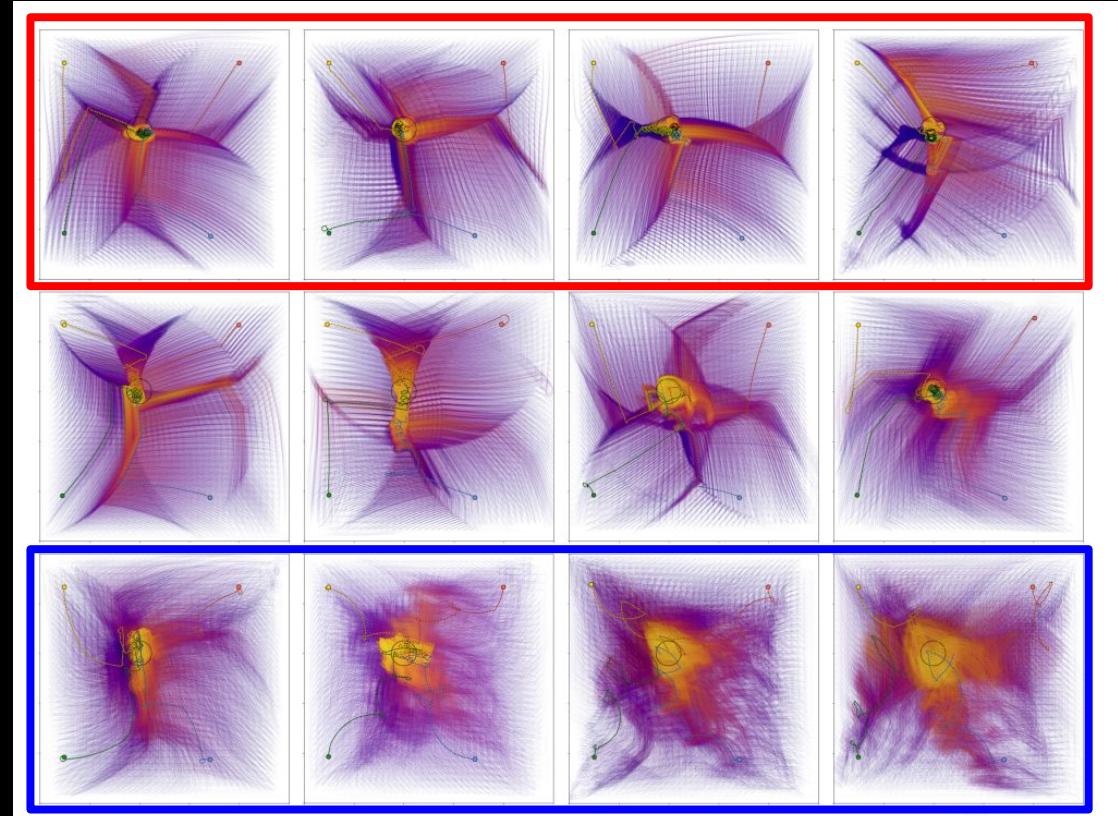
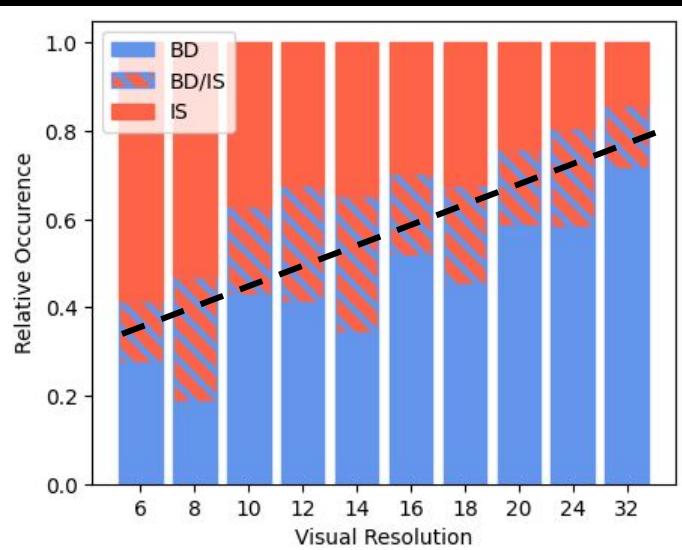
Indirect Sequential

Spatial Correlation

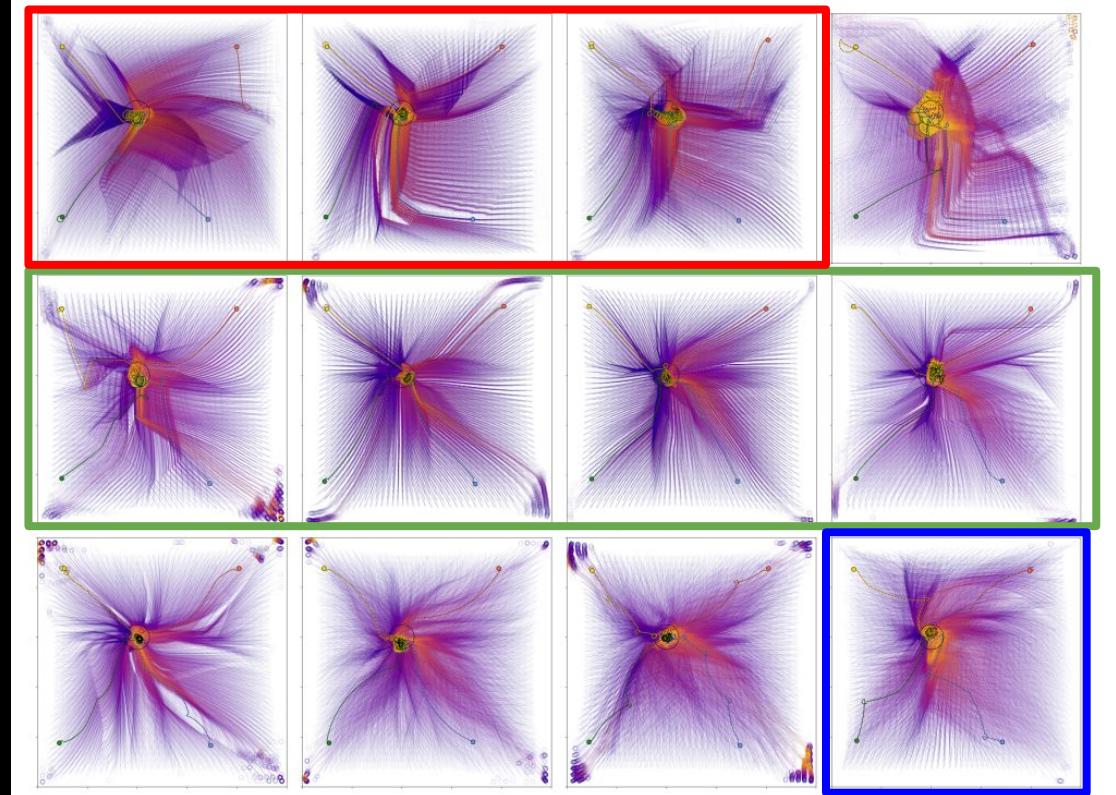
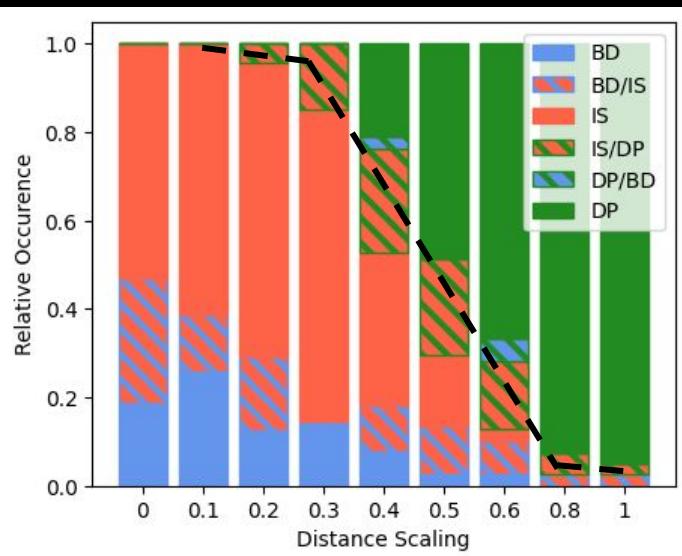


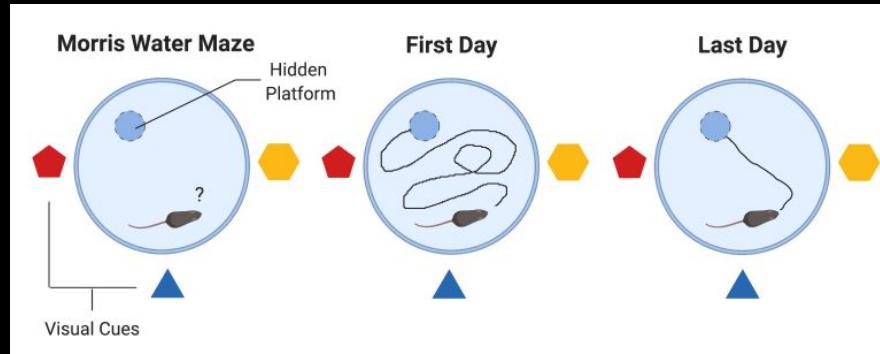
Temporal Correlation

Visual resolution gradually modulates
BD/IS class preference/evolvability



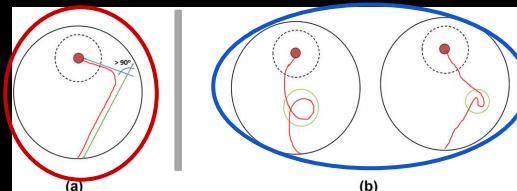
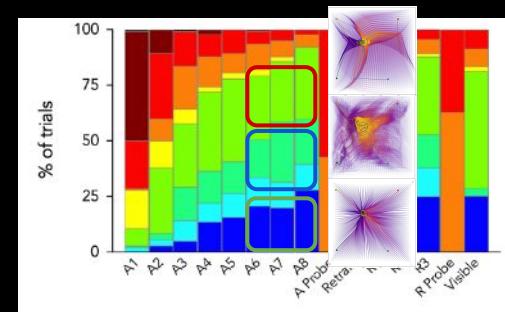
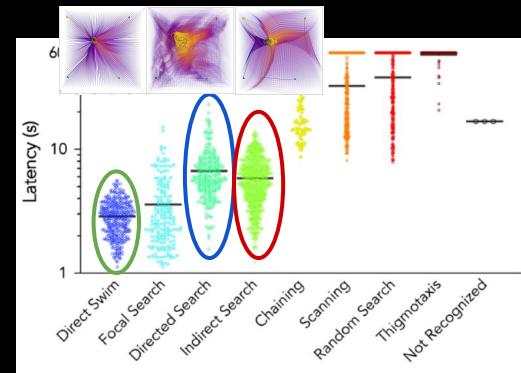
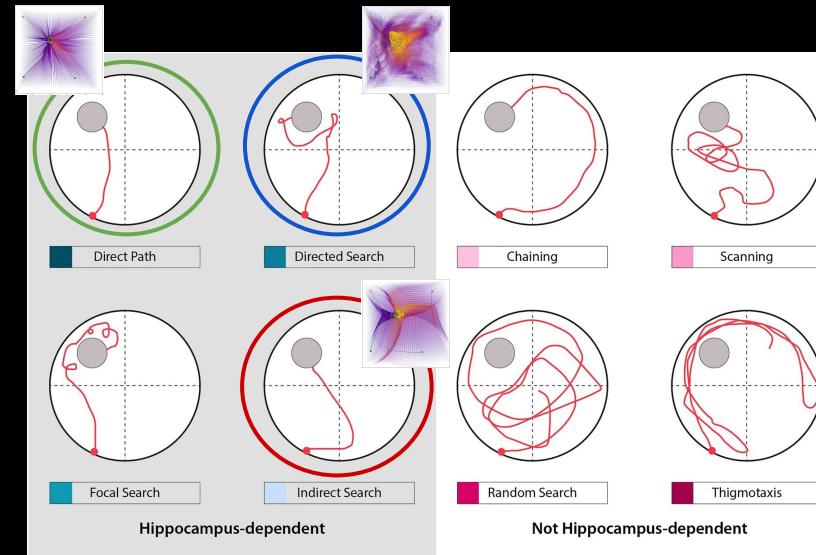
Sigmoidal appearance of DP



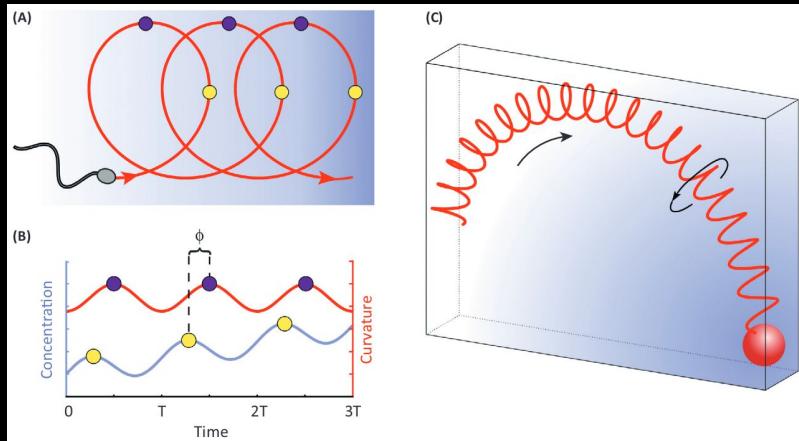


Classes ~ Morris Water Maze strategies

- Indirect Sequential ~ Indirect Search
- Biased Diffusive ~ Directed Search + Self-Orienting Behavior
- Direct Pathing ~ Direct Path/Swim
- Similar relative fitness + occurrence metrics

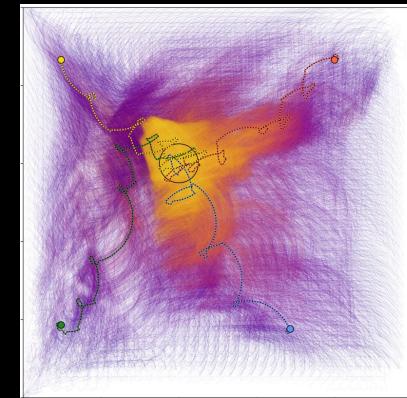
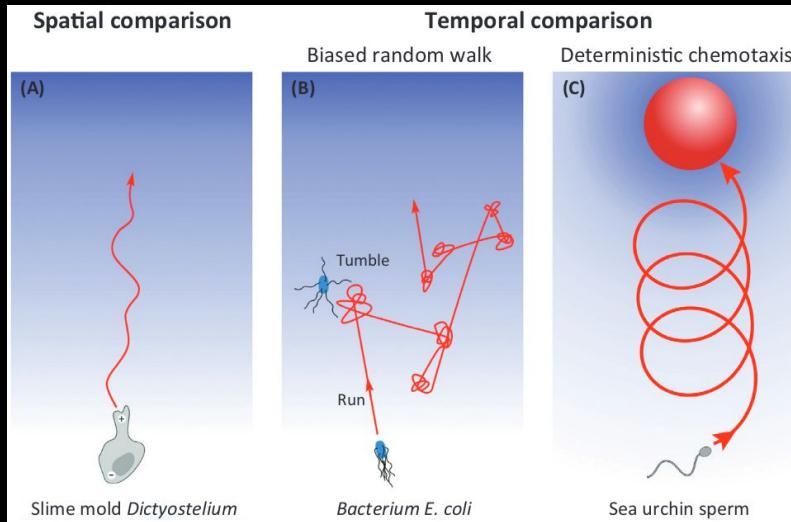


Cooke 2020
Curdt 2022
Villarreal-Silva 2022



Biased Diffusive ~ sperm chemotaxis

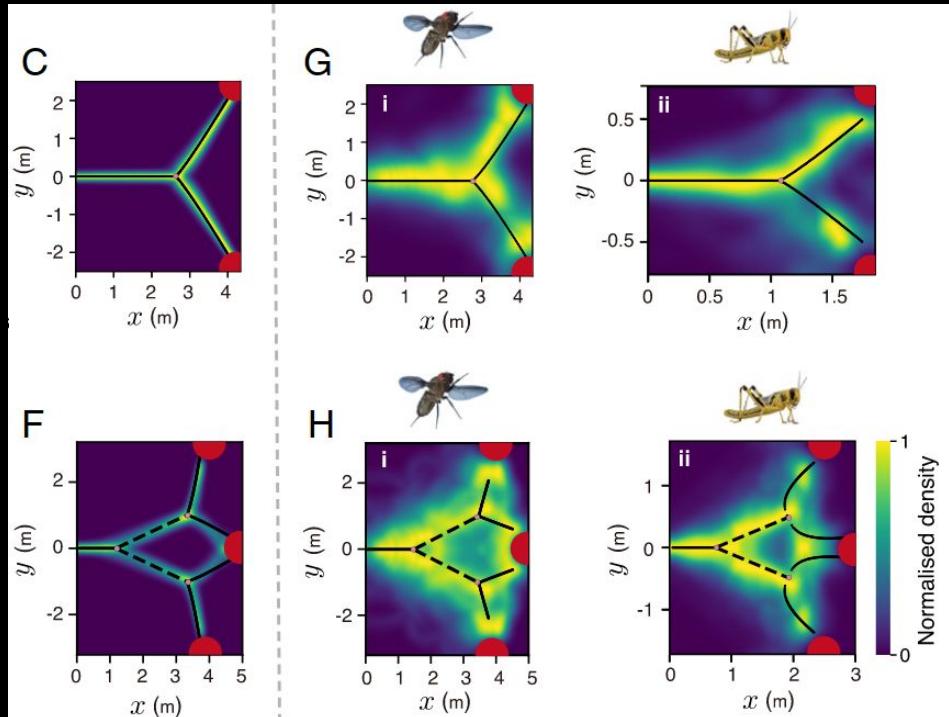
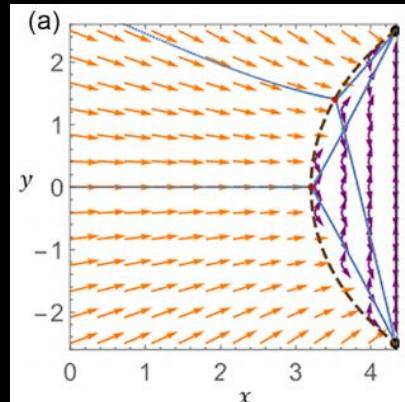
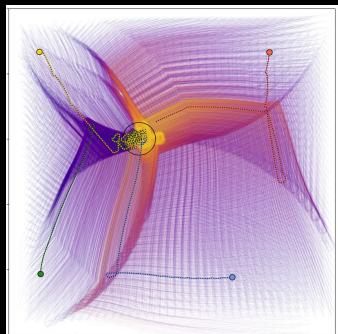
- Dynamic curvature modulation
→ up-gradient helical bias
- concentration sampling vs. landmark observations
- gradient in signal vs. indirect inference



Armon 2012
Alvarez 2014

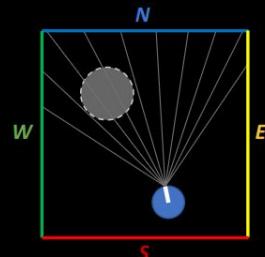
Indirect Sequential ~ decision-making thresholds from binary spatial choice
→ observed in flies + locusts + fish (+ ring attractor model / set of spin systems)

- goal choice vs. indirect spatial inference + route composition
- set of spin systems vs. CNN
- emergent vs. learned transitions



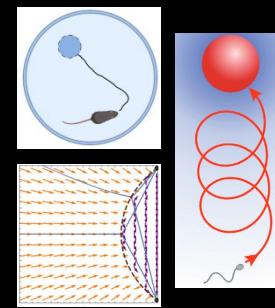
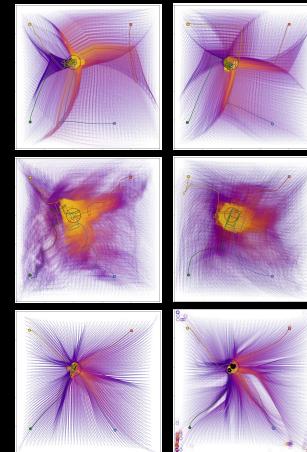
Visual space scaffolded by elliptical geometry
→ it's not all Euclidean cognitive maps

Computational
Task + Environment

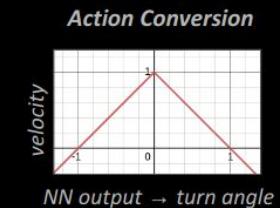
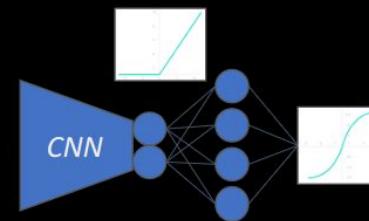


Algorithmic
Behavior + Circuit Mechanisms

Training



Implementational
Input/Output Format + Neural Architecture



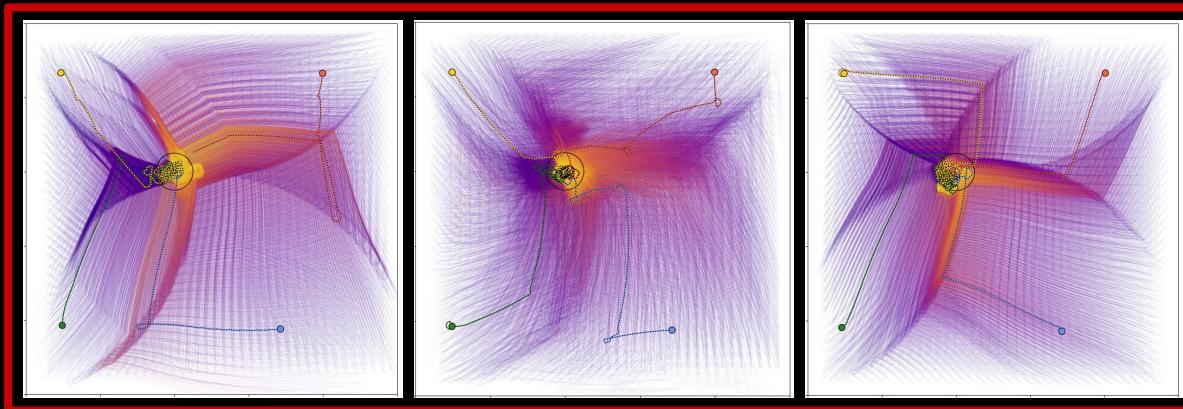
Govoni, P., & Romanczuk, P. (2024).
arXiv preprint arXiv:2407.13535.

What comes next?

- Circuit mechanisms: 🚧
- Collective navigation:
social vs. individual strategies +
metabolic/attention-based constraint

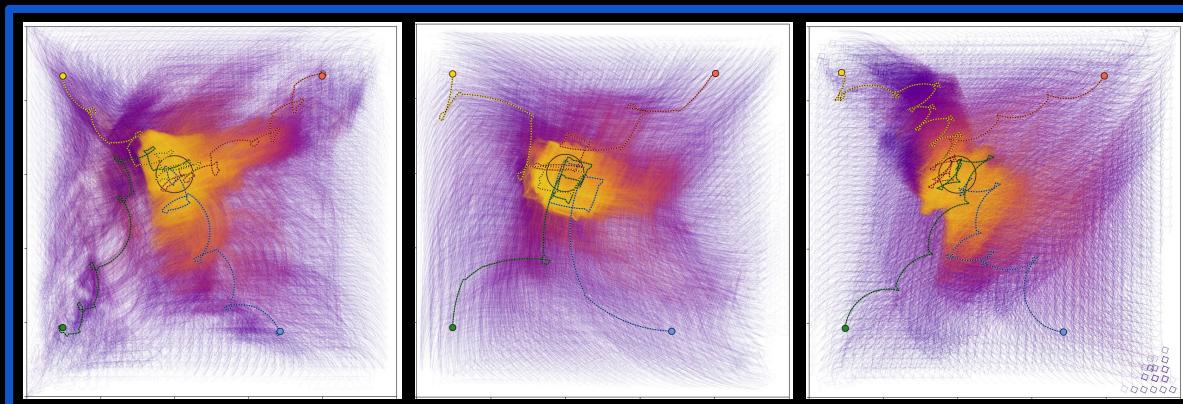


Global behavior Angles Only



Indirect Sequential

- Grid-like trajectories
- Compositional route segments
- Elliptical decision manifolds



Biased Diffusive

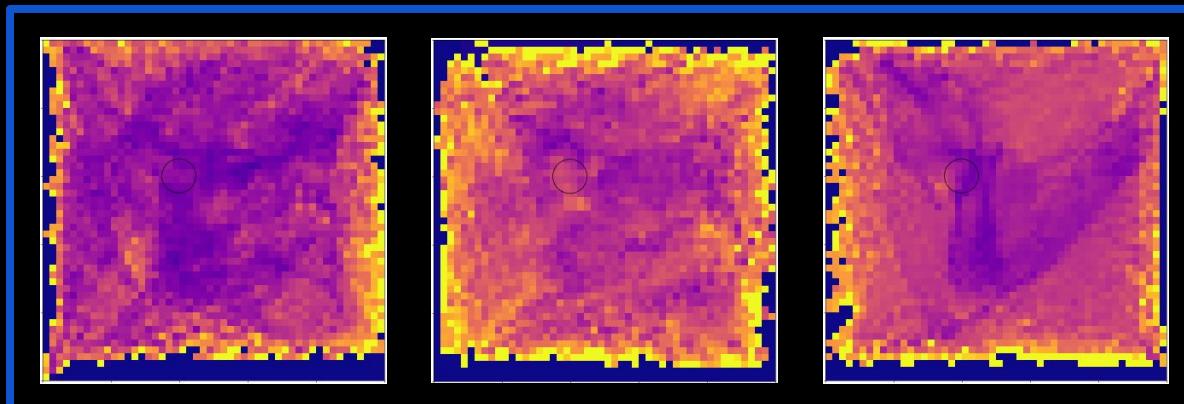
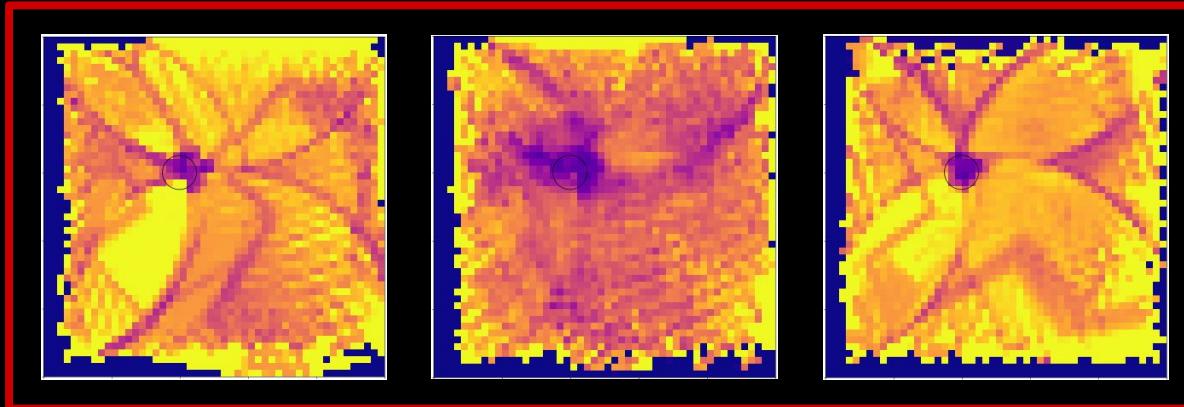
- Spin/ratchet-like trajectories
- Directed towards patch

Entropic Directedness

- Spatial Correlation
- Allocentric Bias

$$H(\Phi, b) = - \sum_{\phi \in \Phi} p(\phi) \log(p(\phi))$$

$$D(H, \Phi, b) = \frac{H_{max}(\Phi) - H(\Phi, b)}{H_{max}(\Phi) - H_{min}(\Phi)}$$



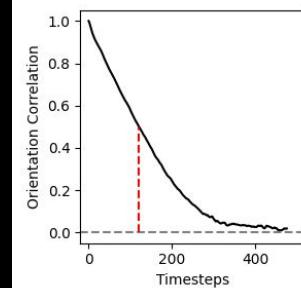
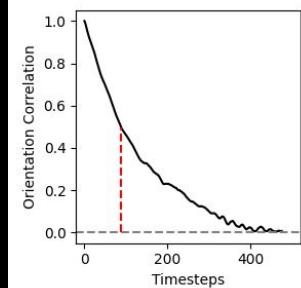
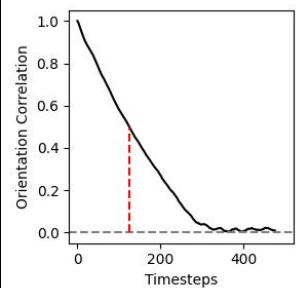
Indirect Sequential
→ Higher

Biased Diffusive
→ Lower

Decorrelation Time

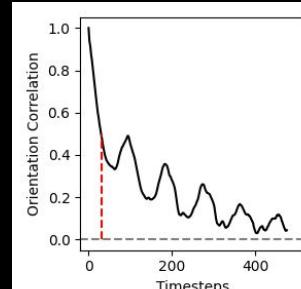
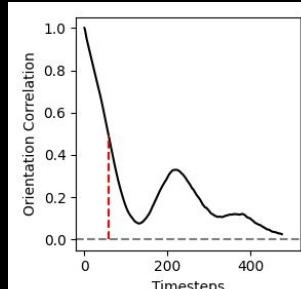
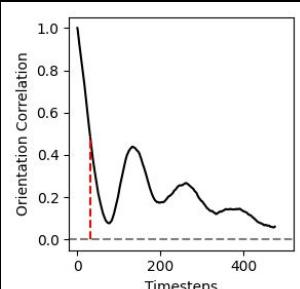
- Temporal Correlation

$$C(t) = \langle \cos[\phi(t_0 + t) - \phi(t_0)] \rangle$$



Indirect Sequential

→ Slow

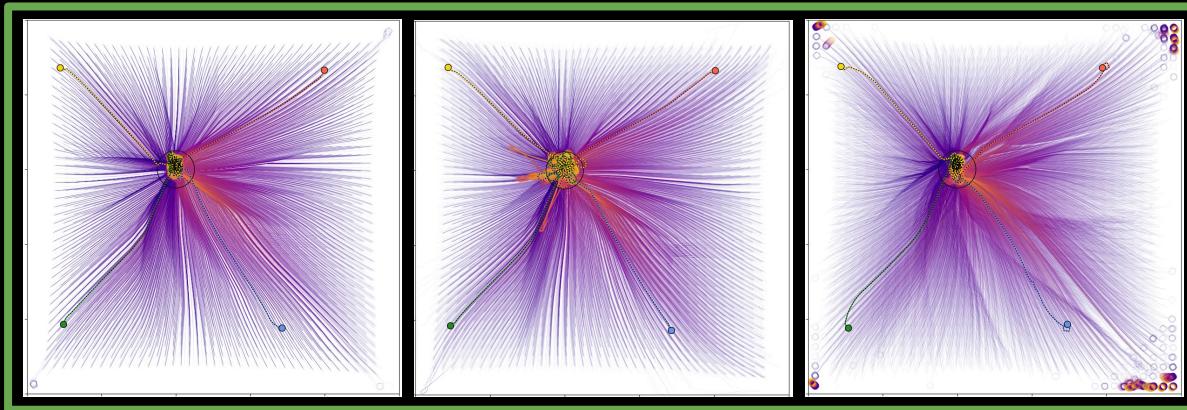


Biased Diffusive

→ Faster

+ Correlated oscillations

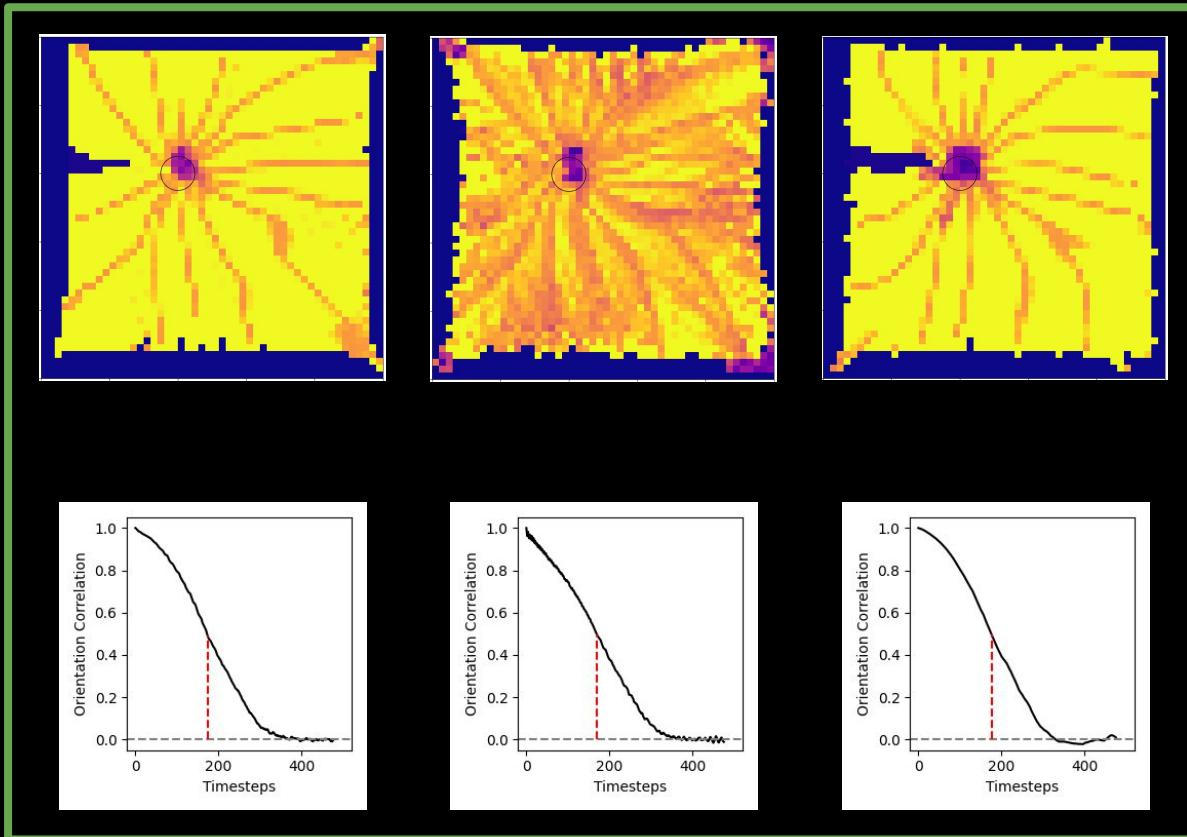
Global behavior Angles + Distance



Direct Pathing

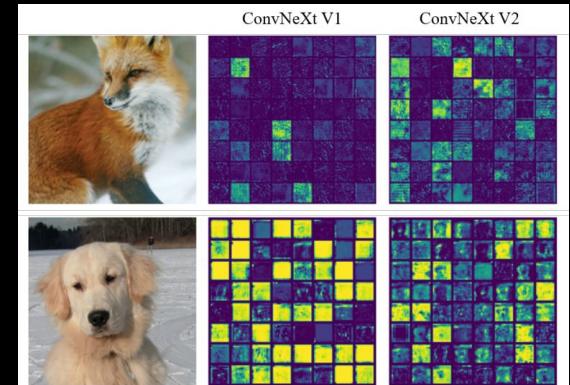
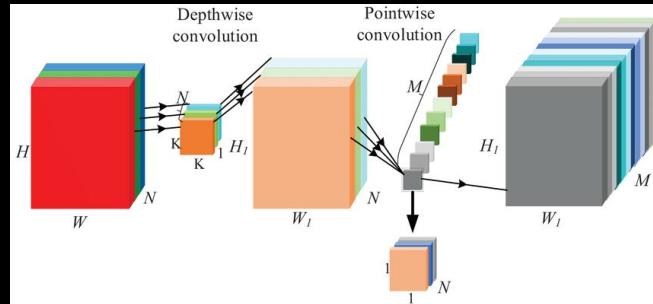
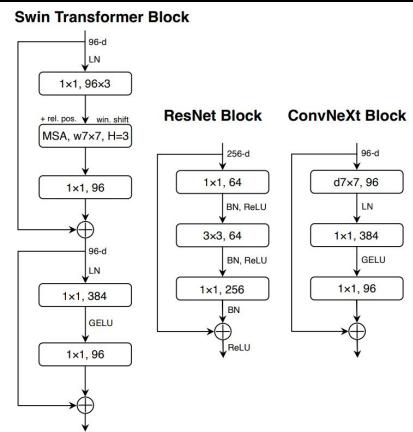
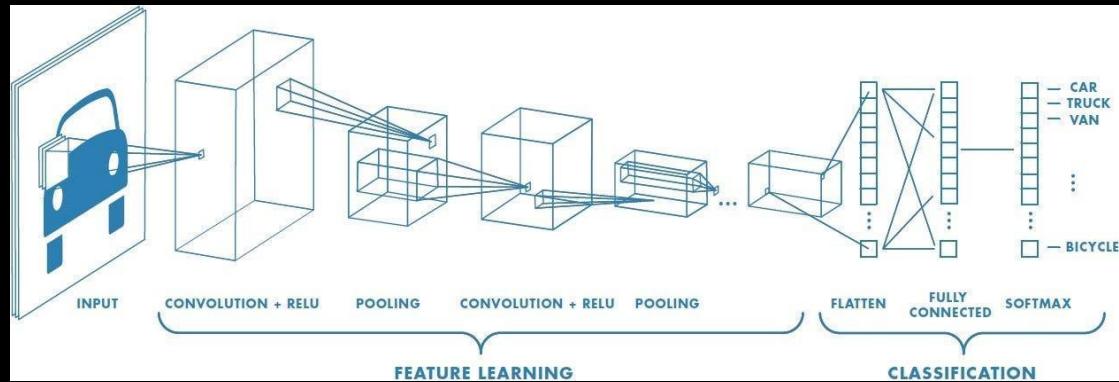
- Strong unidirectional persistence
- Highly accurate patch estimation

Global behavior Angles + Distance

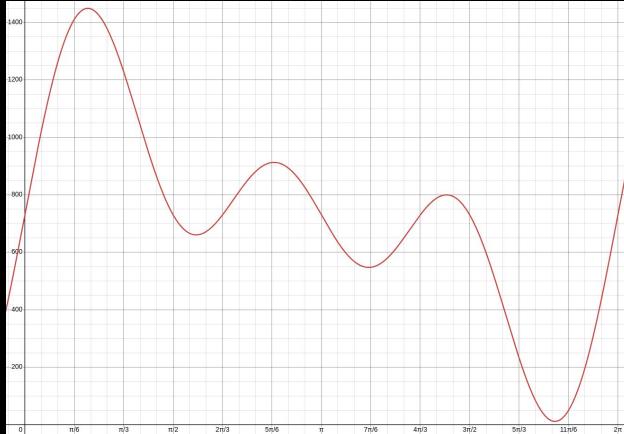


Direct Pathing

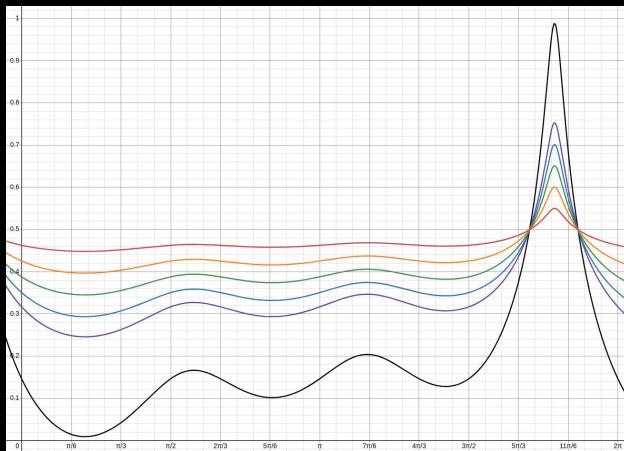
- Highest directedness
- Slowest decorrelation



*Distance
(toy data)*



Percept



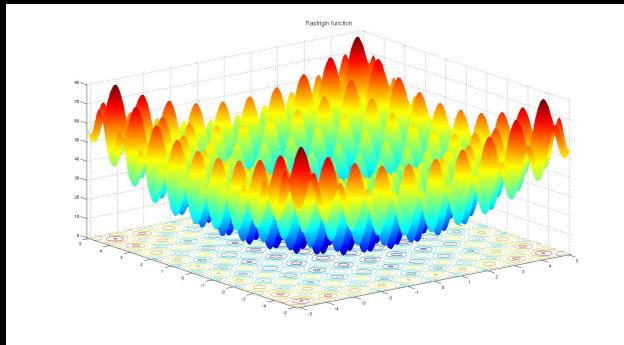
Field of View

*Distance input scaled via
Weber Fechner law*

$$y = -\ln(x) / k + m$$

→ k,m fit to min/max output ranges

Evolutionary Strategies (ES)



Less complexity/assumptions

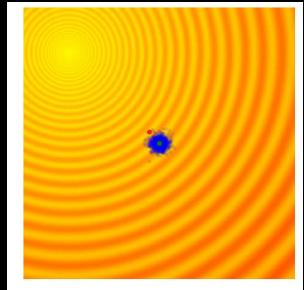
- No value function approximation
- No temporal reward dimension (no credit assignment problem)

Suitably expressive for my question

- Intrinsic exploration helps navigate rugged fitness landscapes
- Global convergence more likely

Main Issue

- Poor data efficiency (shared with RL)



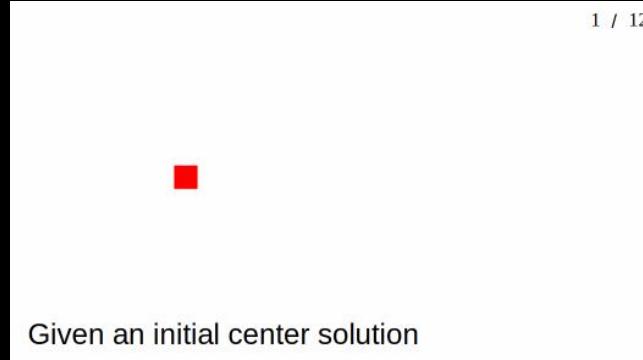
Hansen 2003

Ha 2017



CMA-ES
*(Covariance-Matrix
Adaptation)*

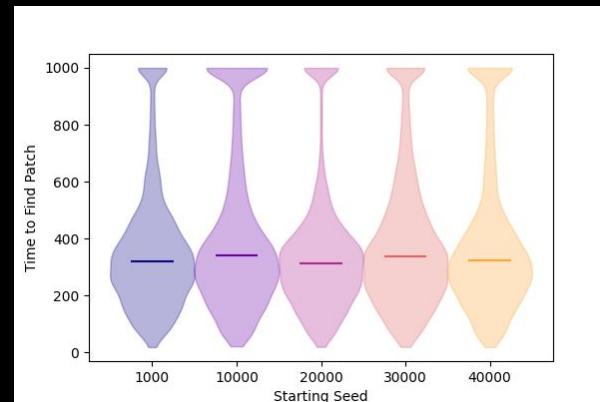
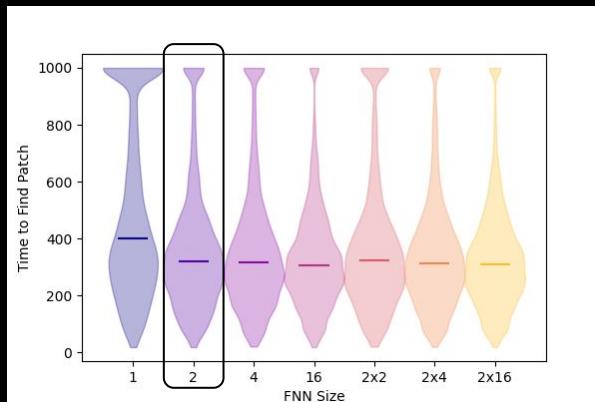
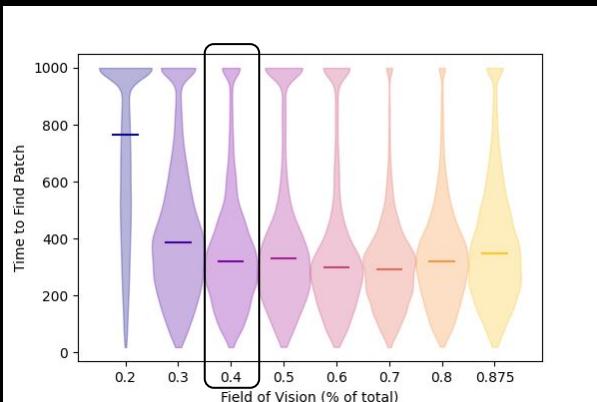
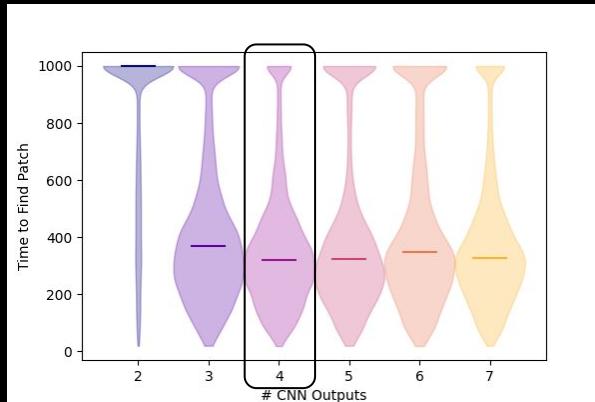
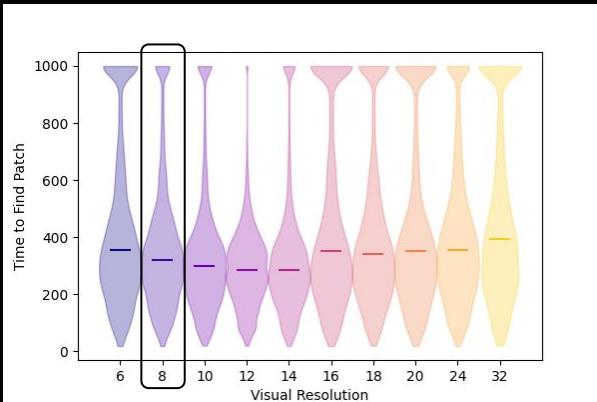
PGPE
*(Policy Gradient
Parameter Exploration)*
+
ClipUp optimizer



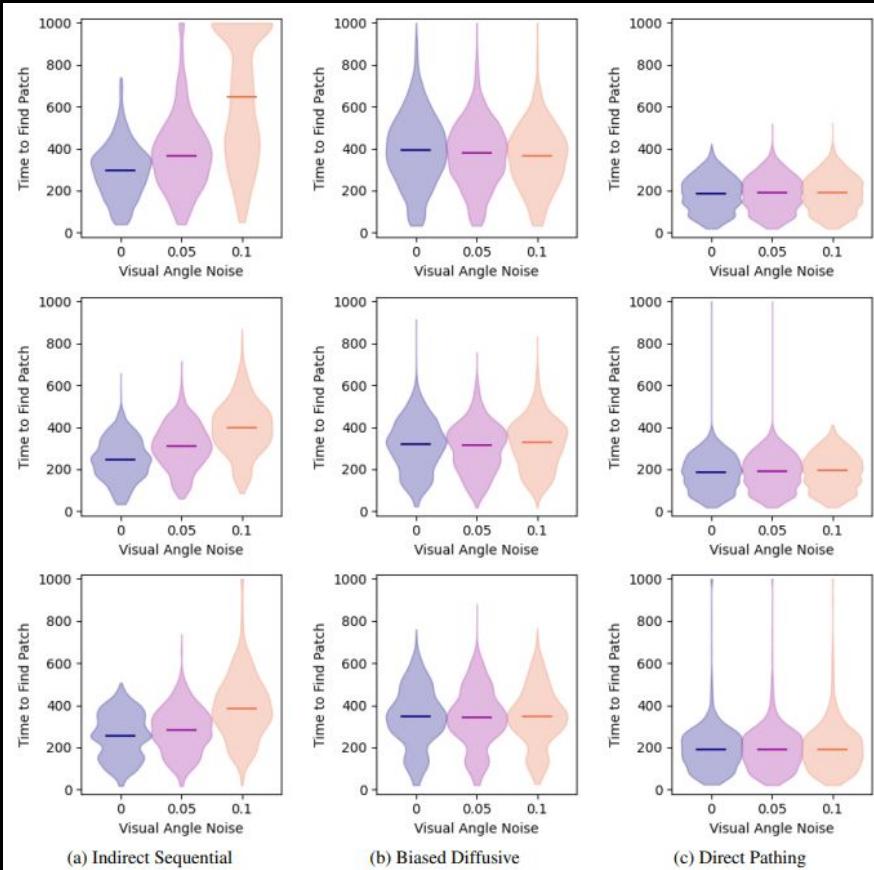
Given an initial center solution

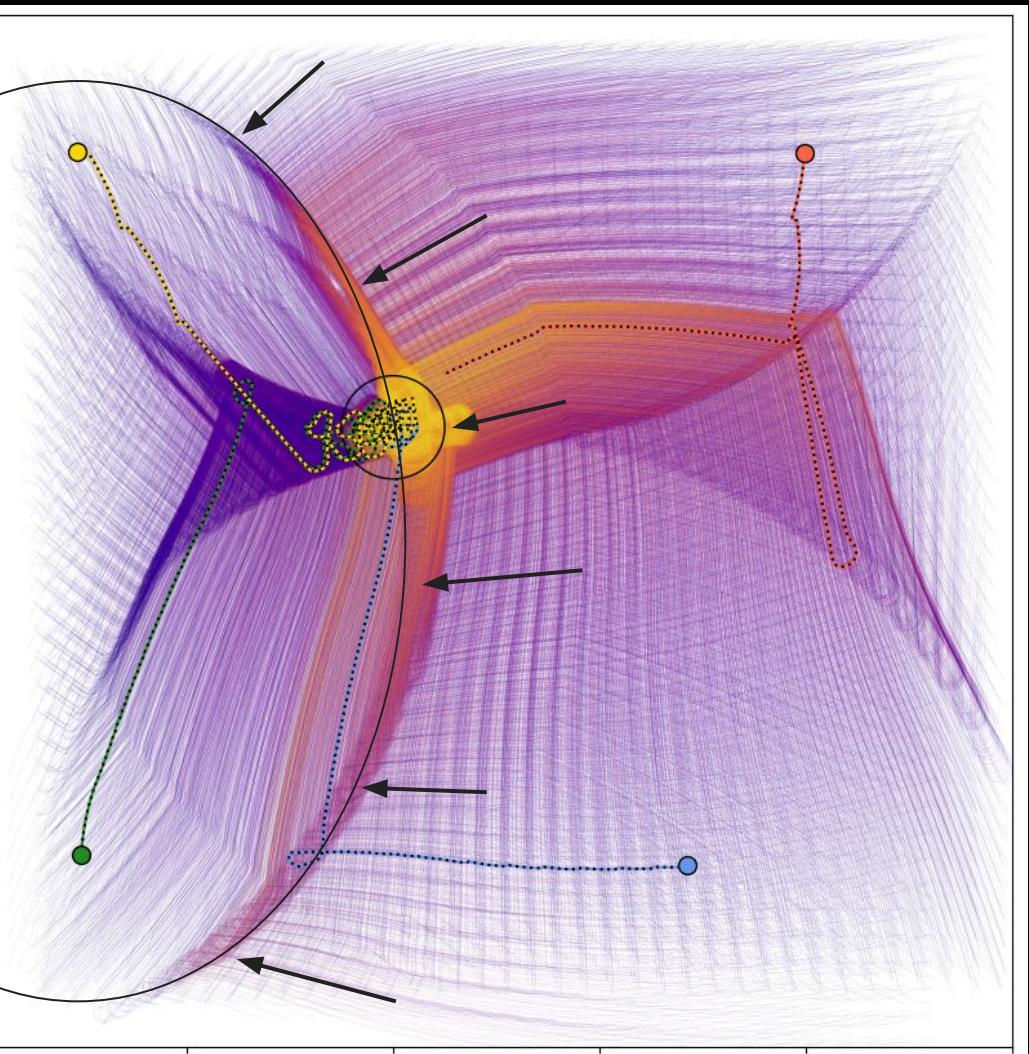
Sehnke 2010
Toklu 2020

Performance distributions robust to perceptual/cognitive hyperparameters + seed



Noise to visual angle → neg affect to IS, BD/DP robust



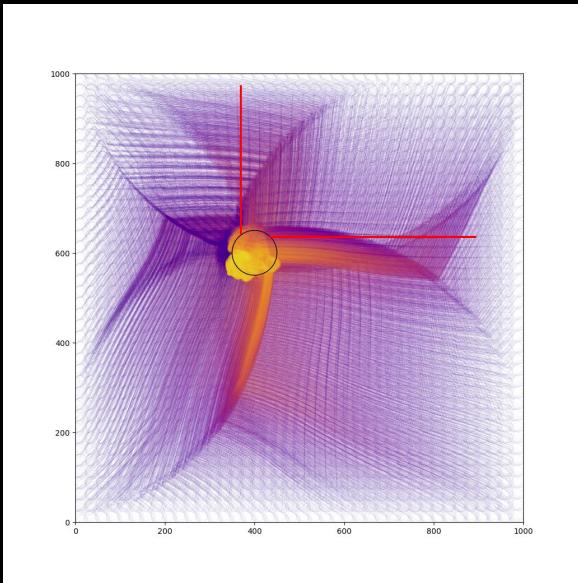


How are IS ellipses calculated?

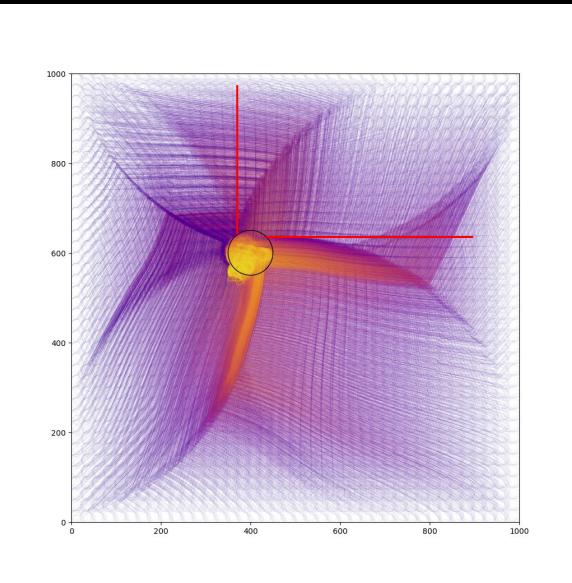
- Artifact from discretized vision?
- From CNN processing?
- Inherent geometrical affordance?

Ellipses : fn (FOV)

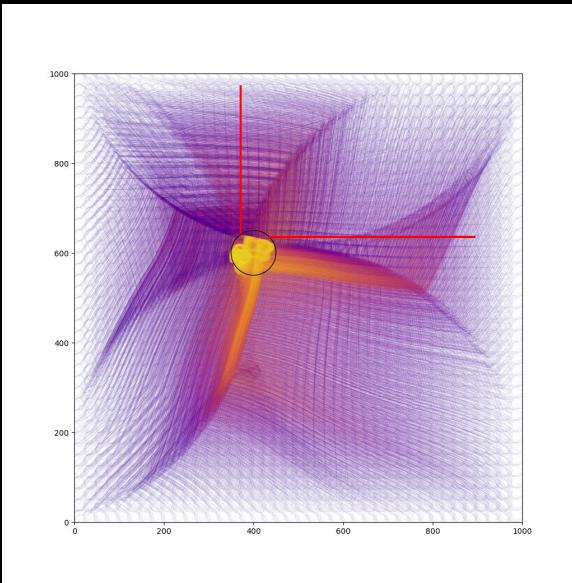
FOV = 0.39



FOV = 0.4

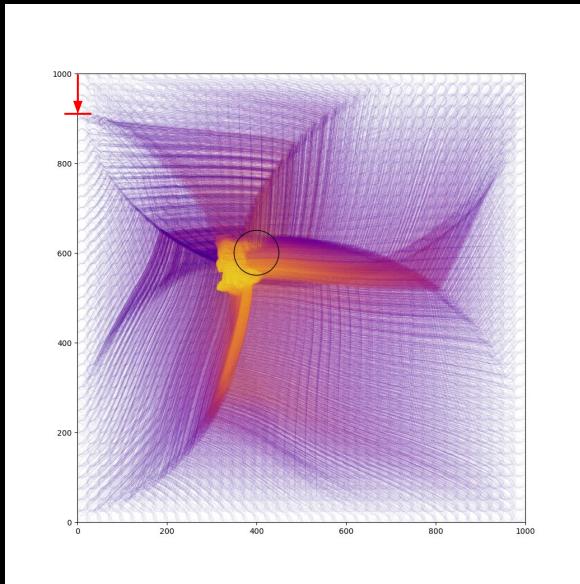


FOV = 0.41

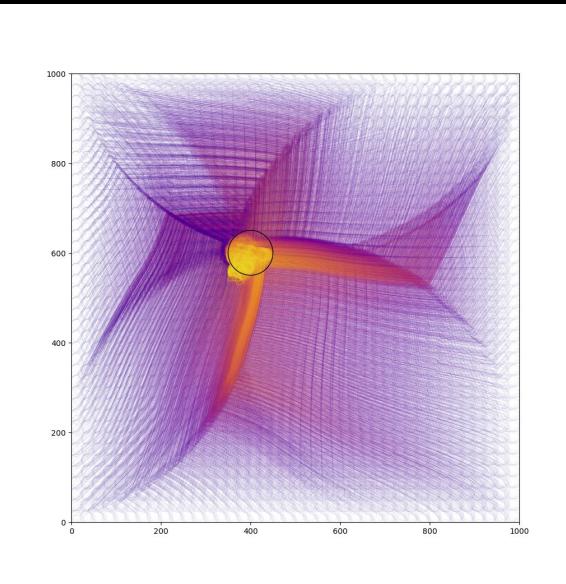


Ellipses : fn (FOV, corner locations)

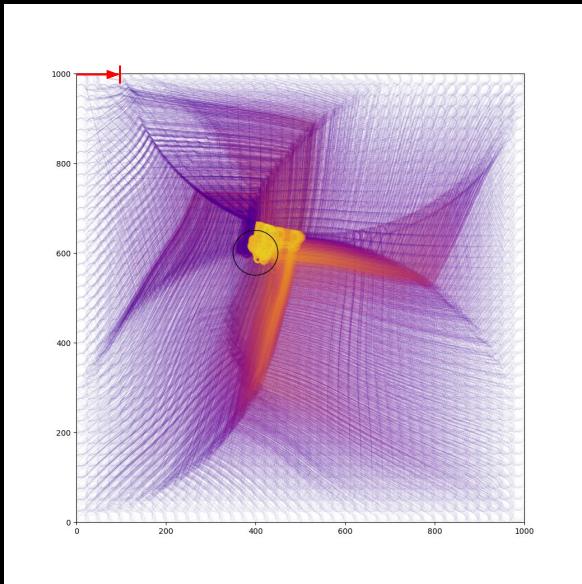
$y(TL) - 100$



unperturbed

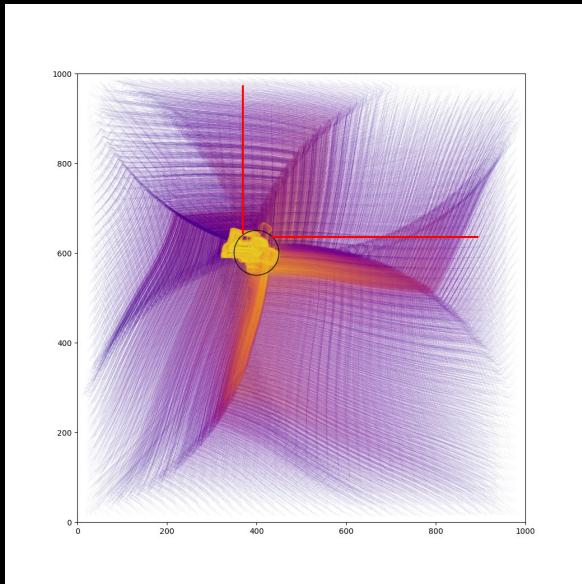


$x(TL) + 100$

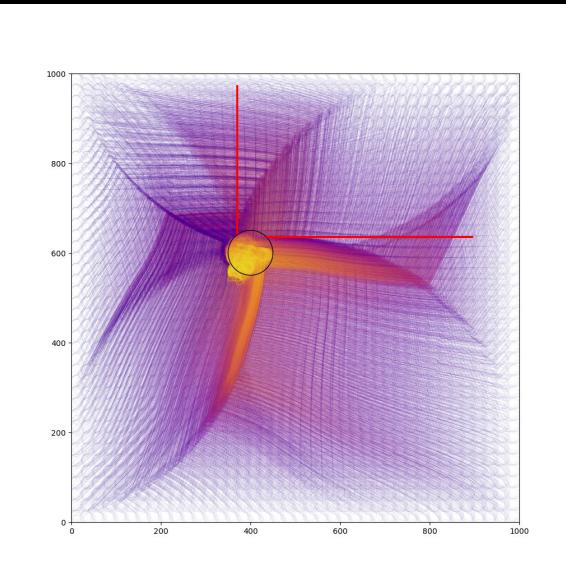


Ellipses : fn (FOV, corner locations, speed)

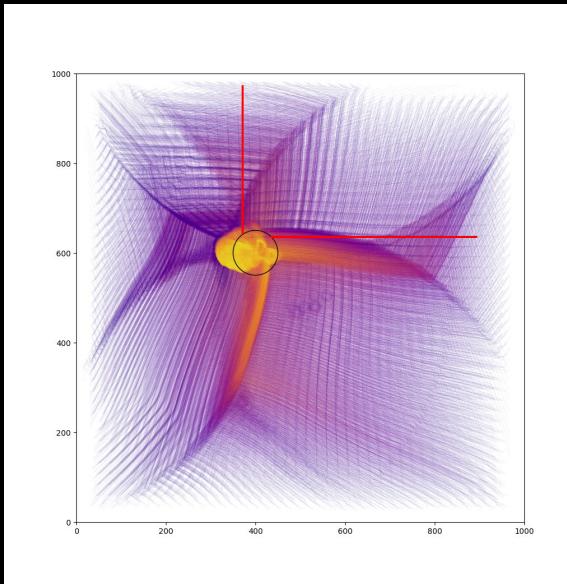
*speed * .75*



unperturbed



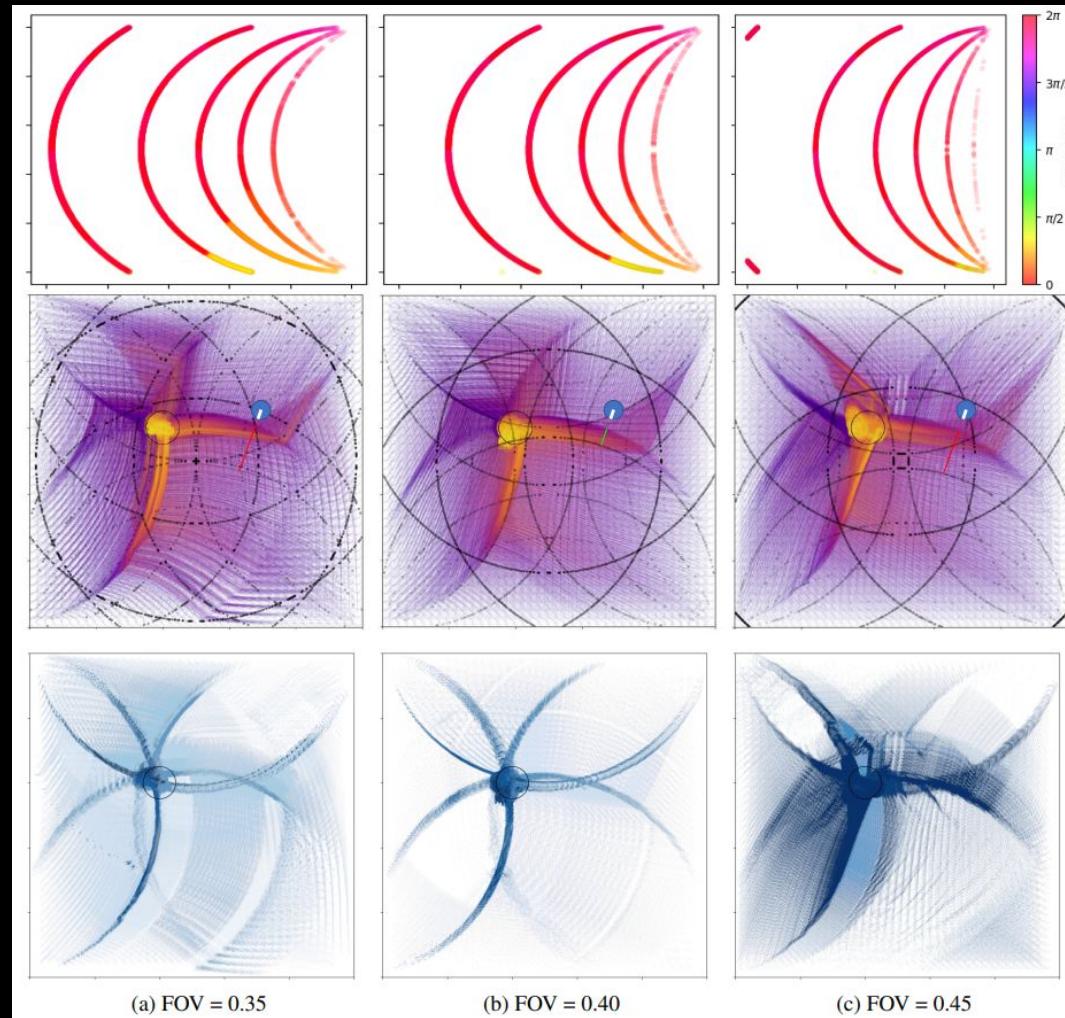
*speed * 1.25*



*Ellipses mechanism:
dual-corner detection*

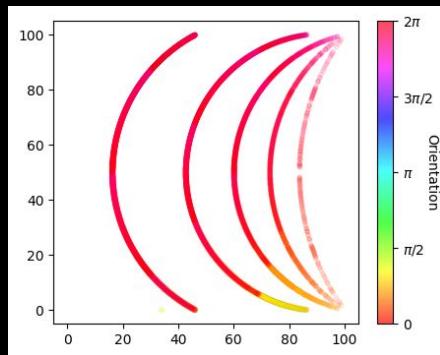
*training at different
FOV should shift
ellipses
→ they do not*

*greater turning speed
modulates ellipse
position to align with
patch position*

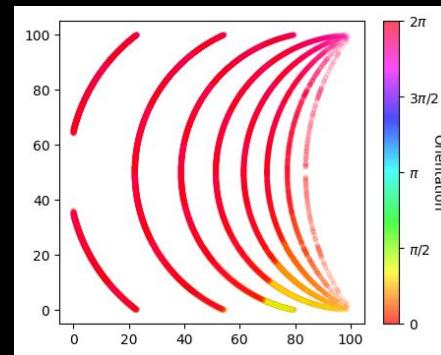


Ellipses :

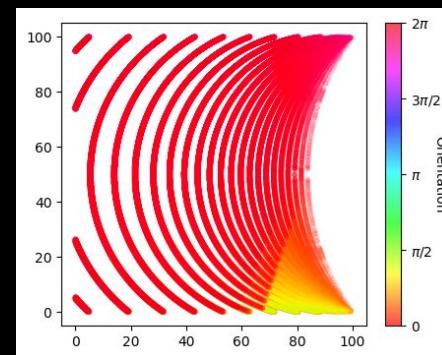
*fn (FOV, corner
locations, speed,
visual resolution)*



res = 8



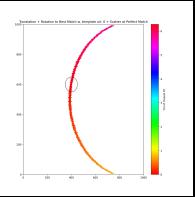
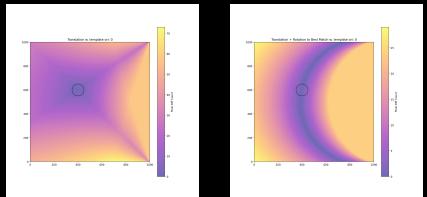
res = 12



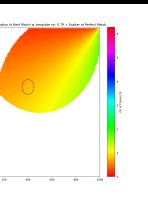
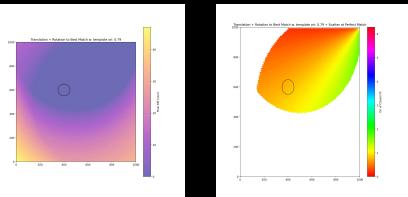
res = 32

Image Difference Relative to Patch Views

2 corners → ellipse



1 corner → leaf

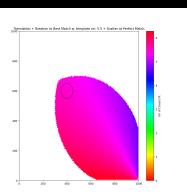
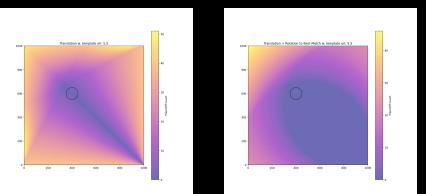
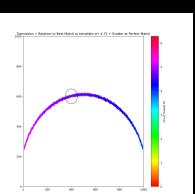
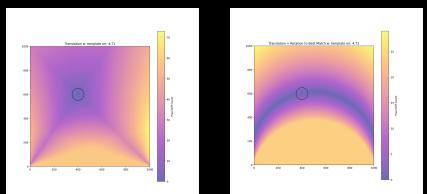
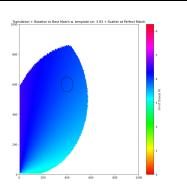
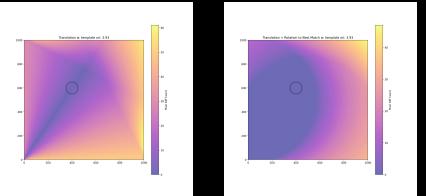
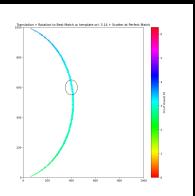
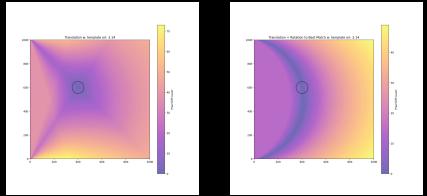


Vis res: 128
Space step: 25
Ori step: $\pi / 256$

Rot + Trans → 2 DoF

Perceiving external
geometry → degeneracy

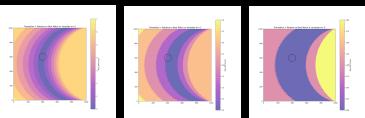
Varying vis res varies
areas only, not shapes



Trans
Trans+Rot
(# pixels match view)

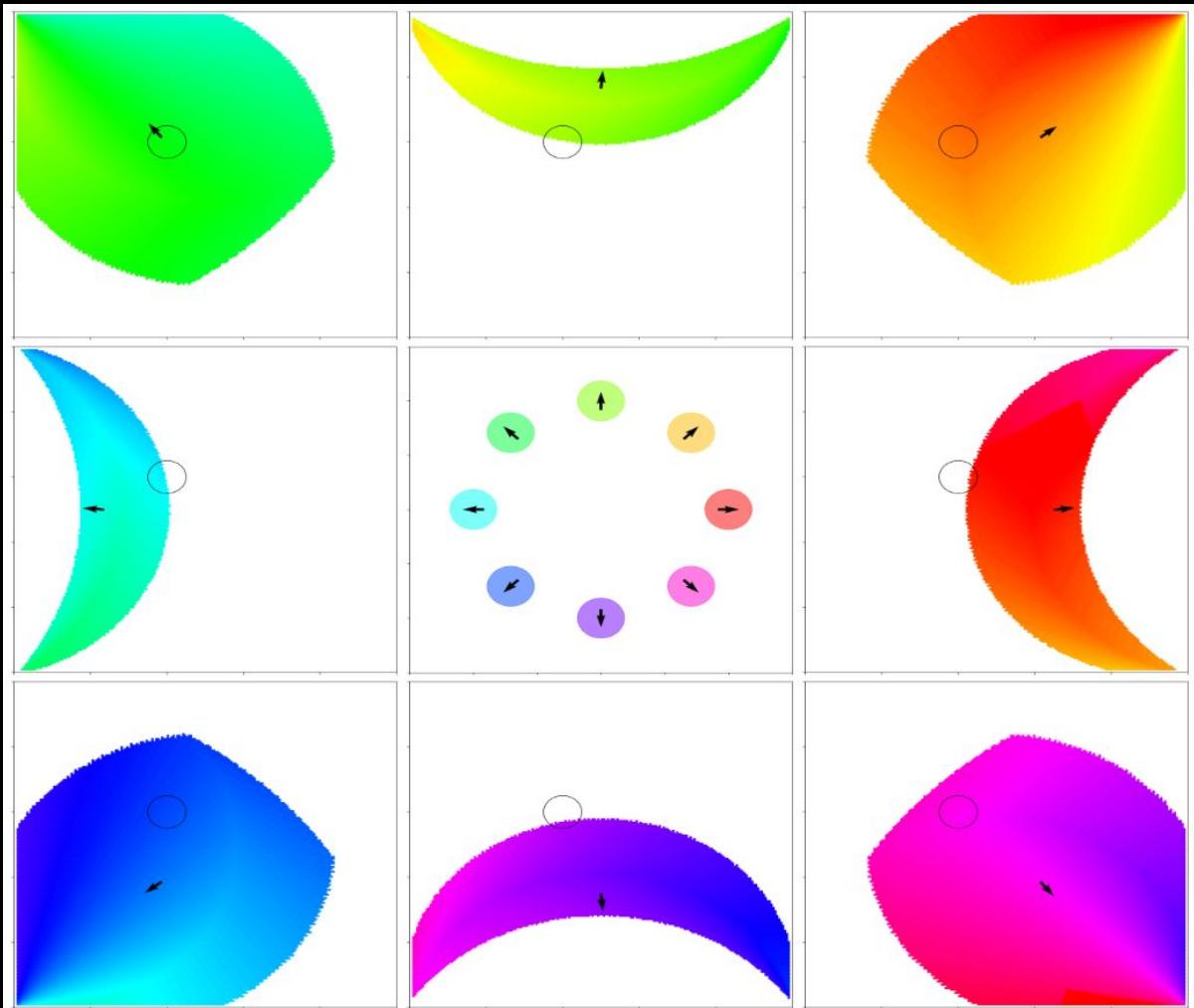
Trans+Rot
(ori of perfect match)

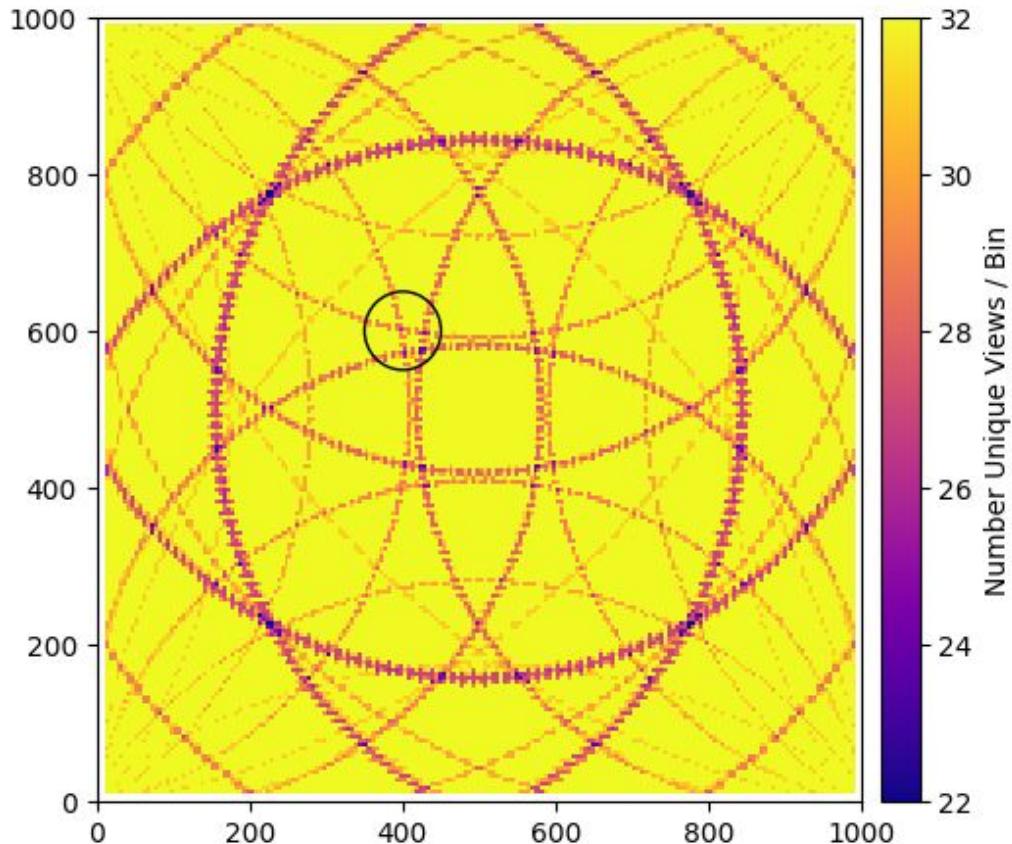
e.g... 32 16 8



9 representative views
(resolution = 8)

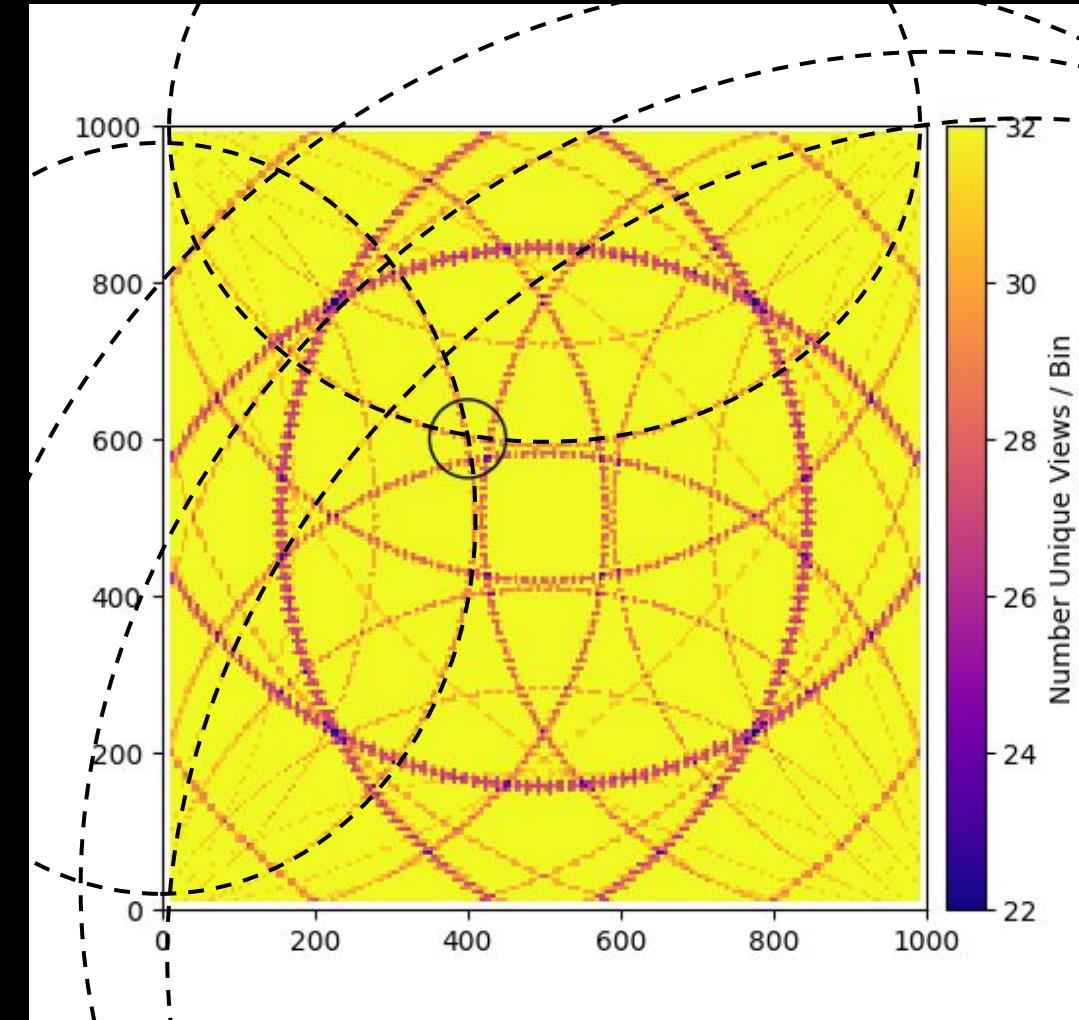
T/B/L/R : 2-corner views → ellipse
Corner : 1-corner views → leaf





Heatmap
for all perfect matches
for all views
(resolution = 8)

- thresholds : “perceptual affordance space”
- mark where agents can switch control regimes
- generated via symmetry breaking, where L+R copies collapse to one

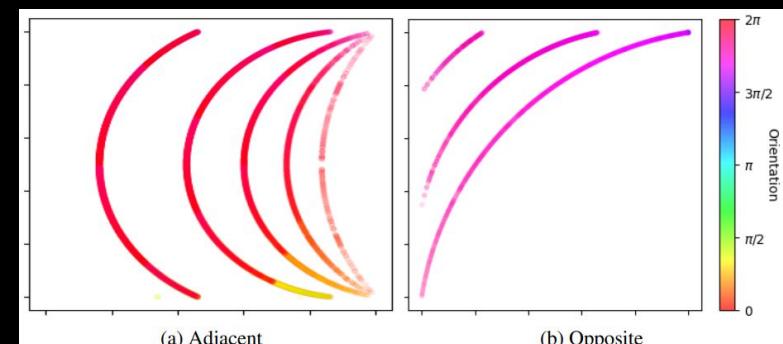


Major ellipses :

- adjacent 2-corner detection
- both predict $5 * 4$ ellipses possible

Minor diagonal ellipses:

- opposite 2-corner detection
- both predict $3 * 4$ ellipses possible

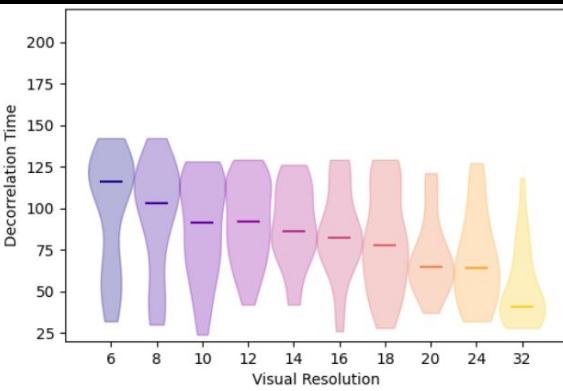


Classification parameters x (visual resolution, distance scaling factor)

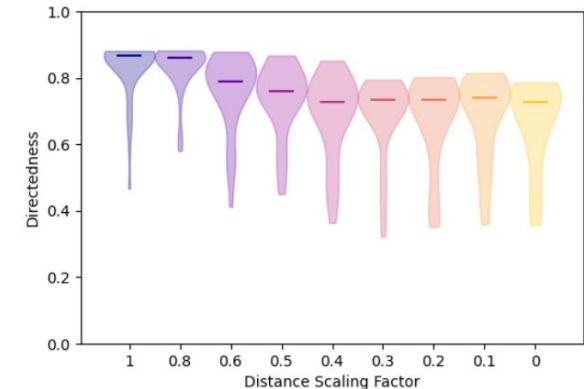
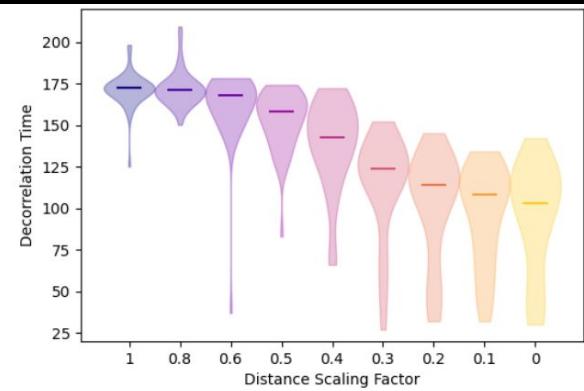
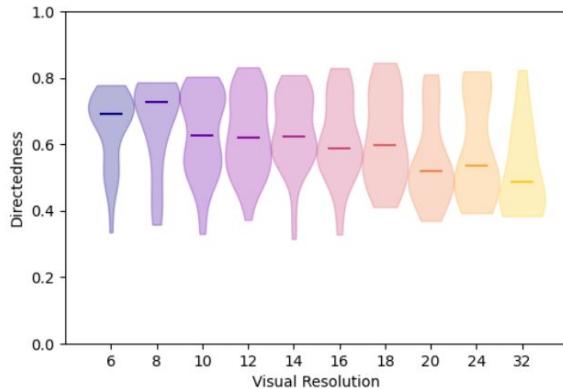
→ fairly good 1D separation

→ bimodality apparent in visual resolution (IS + BD classes)

Temporal
Correlation

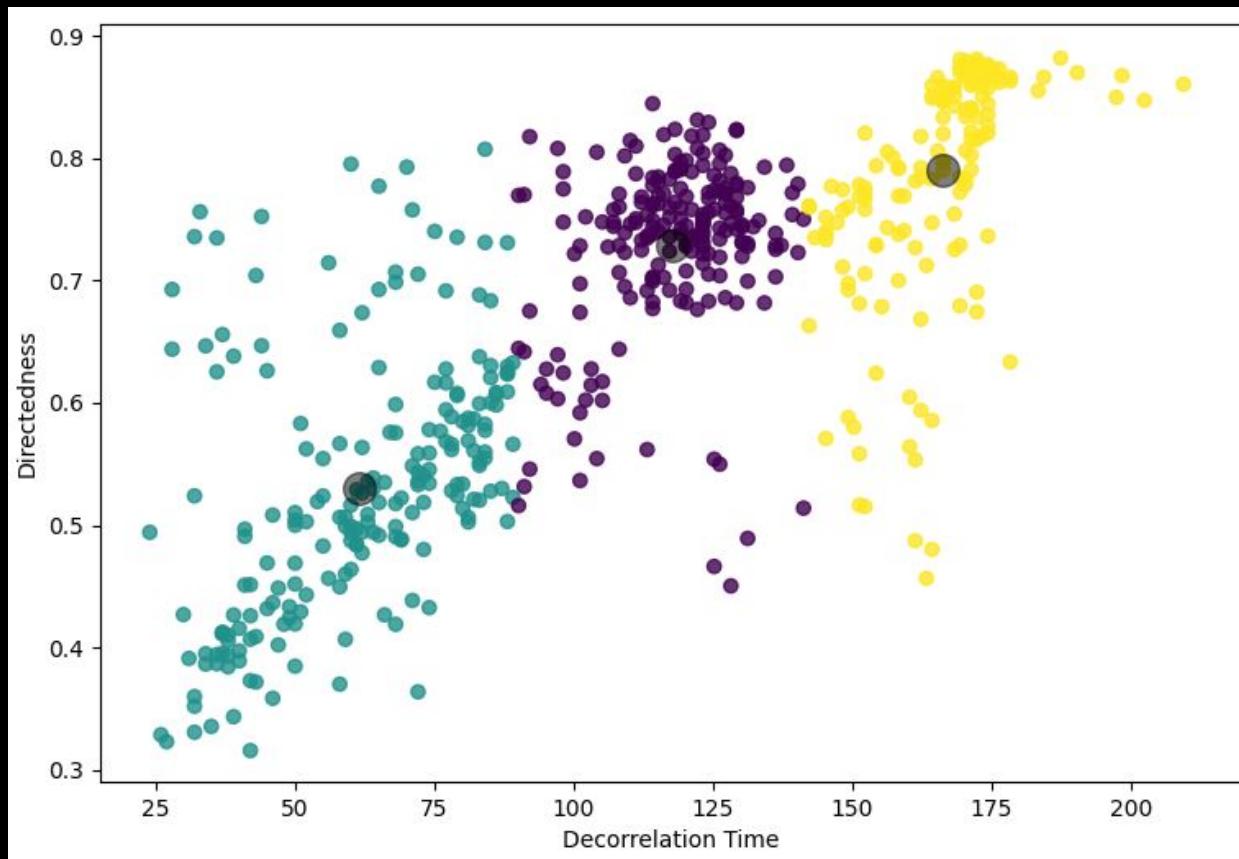


Spatial
Correlation



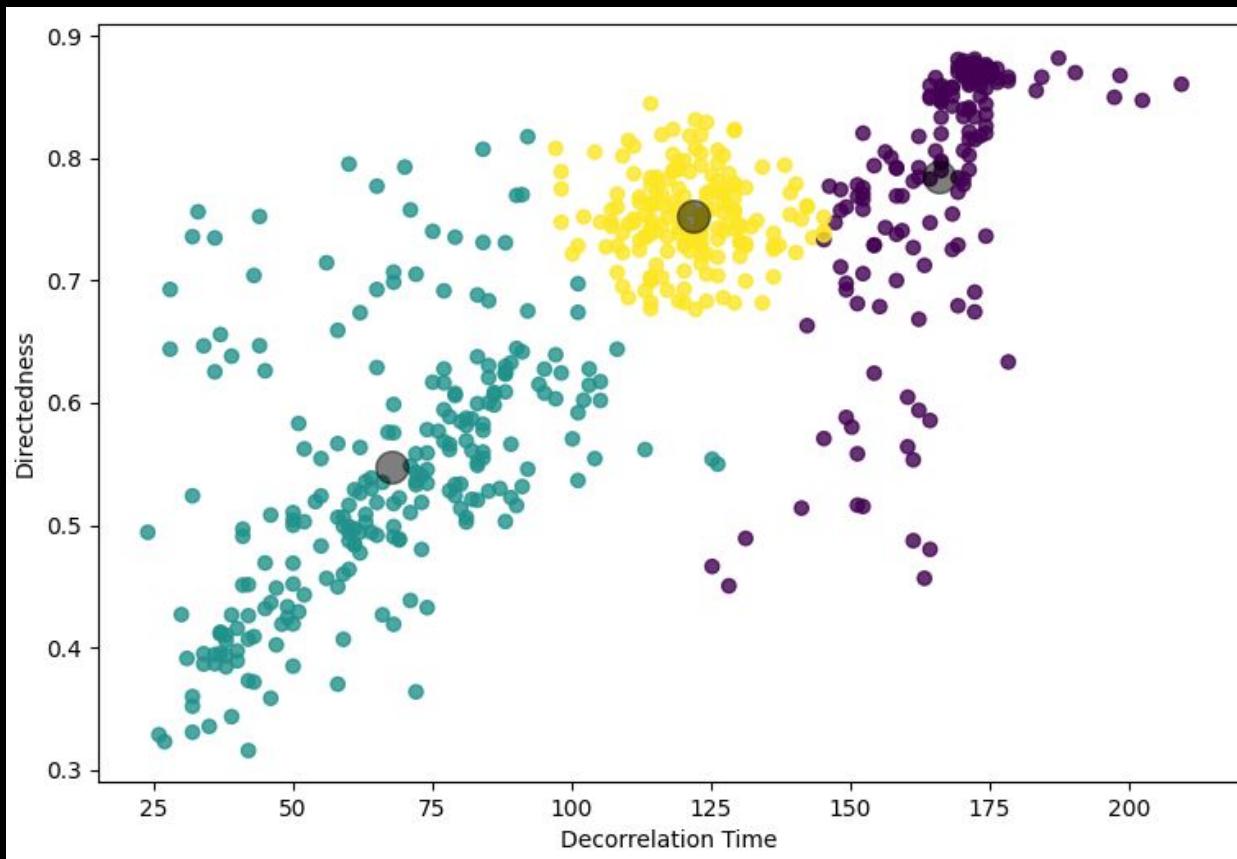
K-Means

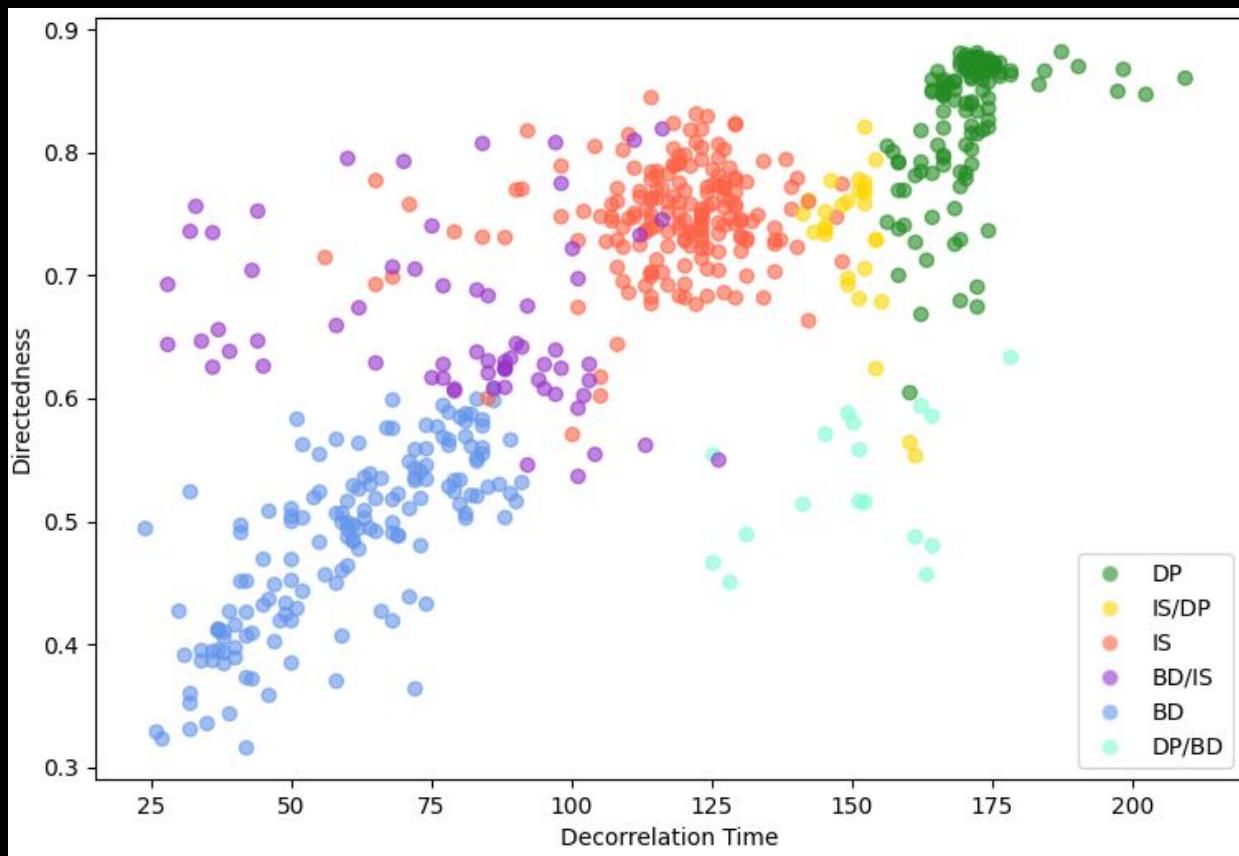
Unsupervised
clustering
works, but
solely
data-driven



Gaussian Mix

Unsupervised
clustering
works, but
solely
data-driven

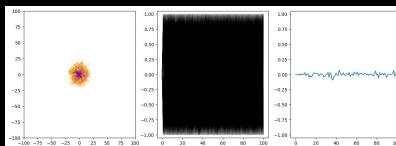
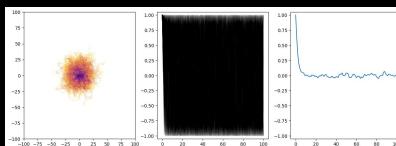
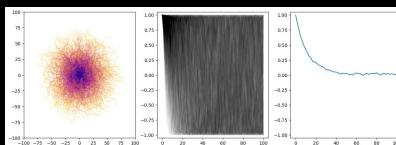
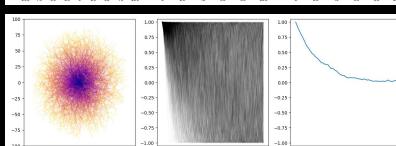
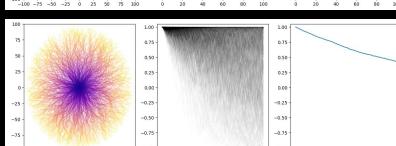
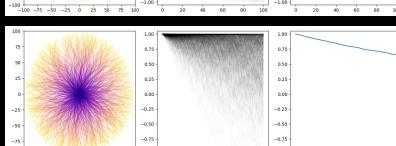
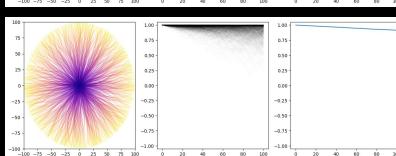




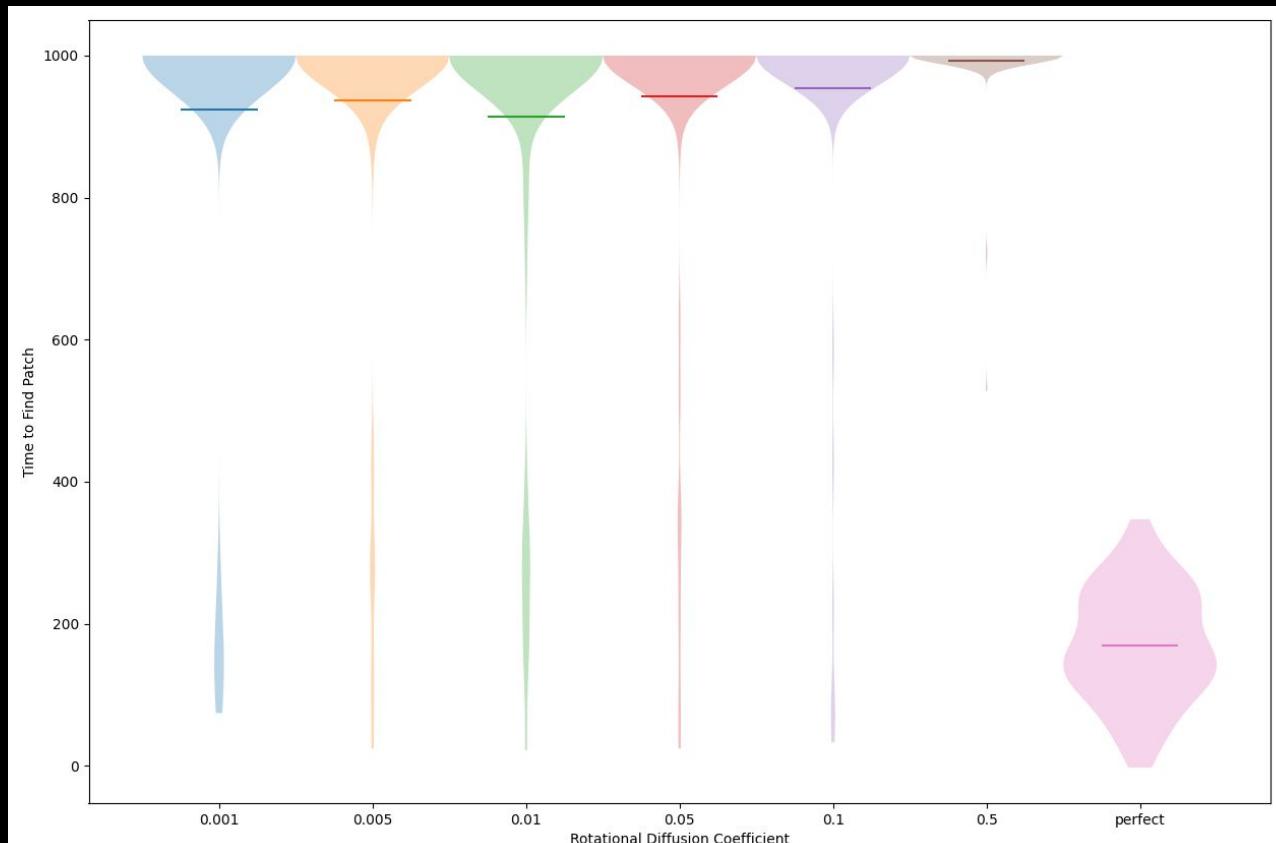
Class labels clustered via:

- heuristics based on clustering algos + heatmaps
- outliers qualitatively labeled
 - to single / hybrid class
 - some IS better labeled as hybrids

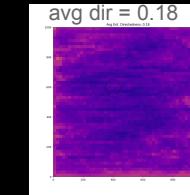
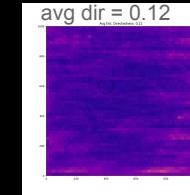
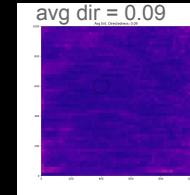
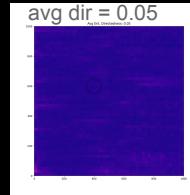
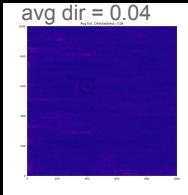
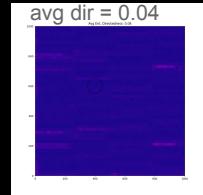
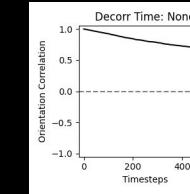
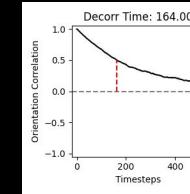
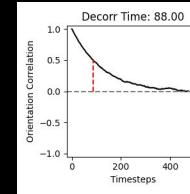
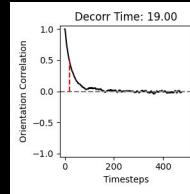
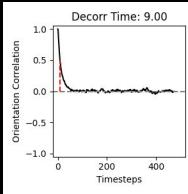
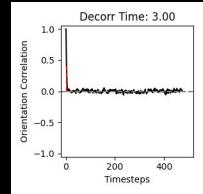
RW

PRW
 $D = 0.5$ PRW
 $D = 0.1$ PRW
 $D = 0.05$ PRW
 $D = 0.01$ PRW
 $D = 0.005$ PRW
 $D = 0.001$ 

→ Persistent random walks cannot reliably reach patch



Persistent Random Walks



D = 0.5

D = 0.1

D = 0.05

D = 0.01

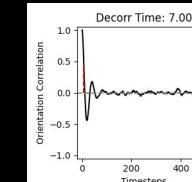
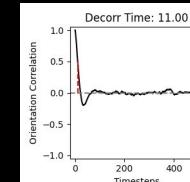
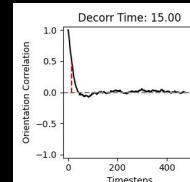
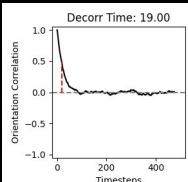
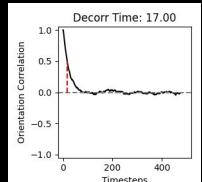
D = 0.005

D = 0.001

D =
rotation
diffusion
coefficient

Decorrelation times (Temporal)
→ encompasses observed range

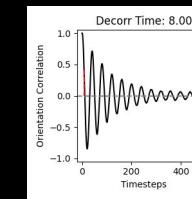
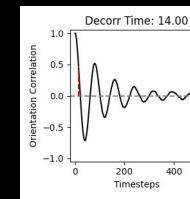
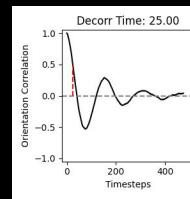
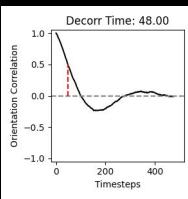
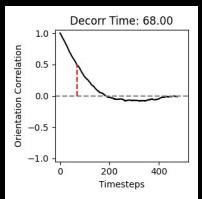
Directedness (Spatial)
→ much lower
→ random spatial direction, no allocentric bias



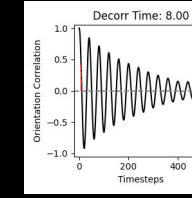
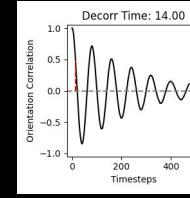
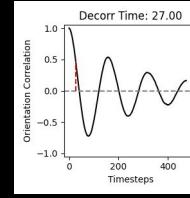
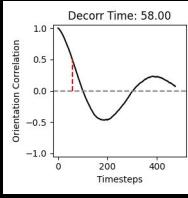
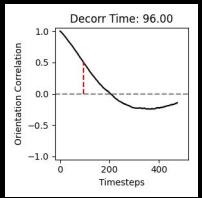
D = 0.05

Persistent Random Walks
+ Fixed Rate Spin

→
correlated oscillations

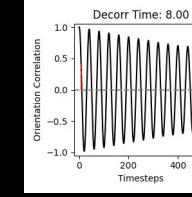
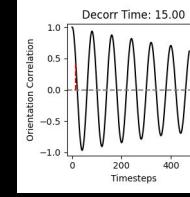
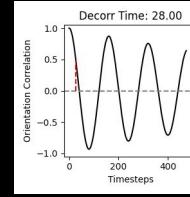
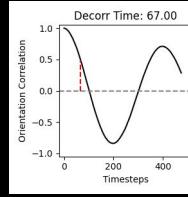
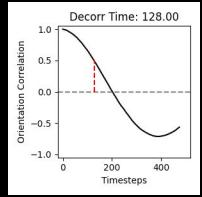


D = 0.01



D = 0.005

D =
rotation
diffusion
coefficient



D = 0.001

S =
fixed spin rate

S = 0.005

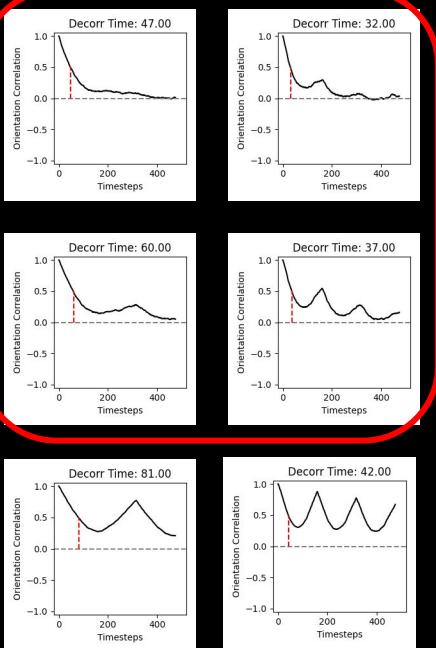
S = 0.01

S = 0.025

S = 0.05

S = 0.1

$$T = \pi * 1/2$$



$$S = 0.005$$

$$S = 0.01$$

$$S = 0.025$$

Ratcheting back once spins reach threshold
(if sufficiently low)



Sustained positive correlation



Similar plots to Biased Diffusive class
(if params in the highlighted regime)

$$D = 0.01$$

$$D = 0.005$$

$$D = 0.001$$

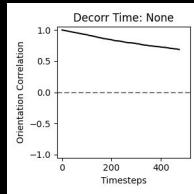
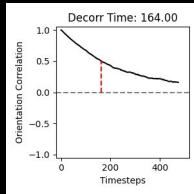
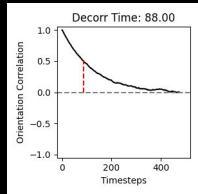
Persistent Random Walks
+ Fixed Rate Spin
+ Ratchet Threshold

$$D = \text{rotation diffusion coefficient}$$

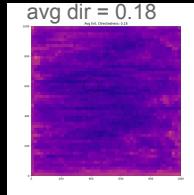
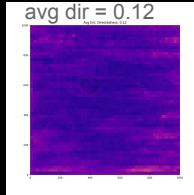
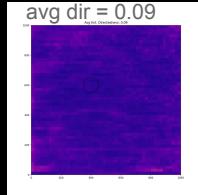
$$S = \text{fixed spin rate}$$

$$T = \text{spin threshold (before ratcheting back)}$$

Persistent Random Walks + Patch-Oriented Bias



$B = 0$

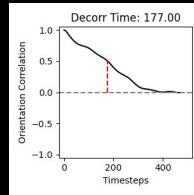
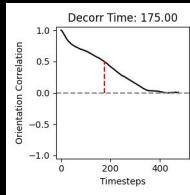
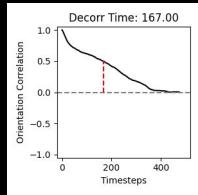


$D = 0.01$

$D = 0.005$

$D = 0.001$

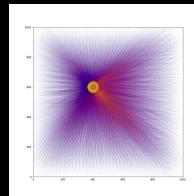
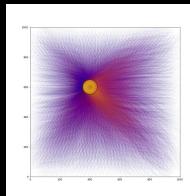
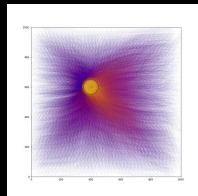
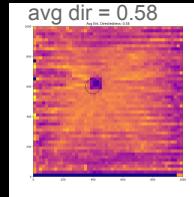
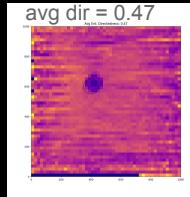
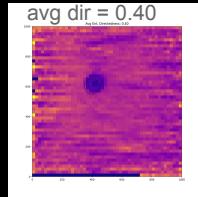
$B = \text{bias}$
(scaling factor on
relative patch
direction info)



$B = 0.1$

Persistent Random Walks + Patch-Oriented Bias

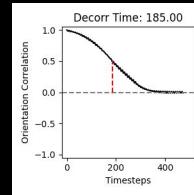
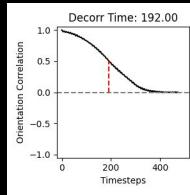
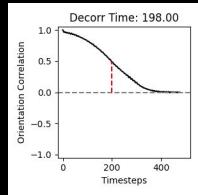
→ correlation concavity



$B = \text{bias}$
(scaling factor on
relative patch
direction info)

Decorrelation times (Temporal)
→ encompasses observed range
(for Direct Pathing agents)

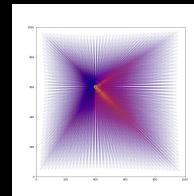
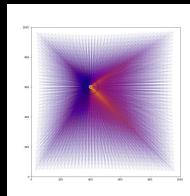
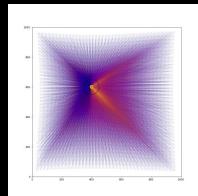
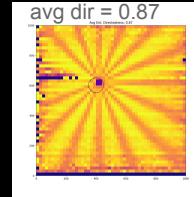
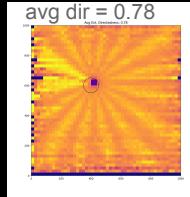
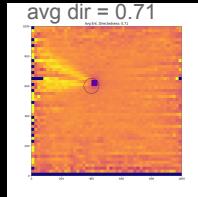
Directedness (Spatial)
→ encompasses observed range
→ allocentric bias → predictable spatial direction
→ star-like plots similar to Direct Pathing



$B = 0.75$

Persistent Random Walks
+ Patch-Oriented Bias

→ correlation concavity



$B = \text{bias}$
(scaling factor on
relative patch
direction info)

Decorrelation times (Temporal)
→ encompasses observed range
(for Direct Pathing agents)

Directedness (Spatial)
→ encompasses observed range
→ **allocentric bias** → **predictable spatial direction**
→ star-like plots similar to Direct Pathing