TEOREMA MAESTRO

Para cada una de las siguientes recurrencias obtener una expresión para el tiempo de ejecución $T(n)$ mediante el Teorema Maestro; indicar aquellas en las que el TM no ha podido aplicarse.

1. $T(n) = 3T(n/2) + n^2$
2. $T(n) = 4T(n/2) + n^2$
3. $T(n) = T(n/2) + 2^n$
4. $T(n) = 2^n T(n/2) + n^n$
5. $T(n) = 16T(n/4) + n$
6. $T(n) = 2T(n/2) + n \log n$
7. $T(n) = 2T(n/2) + n/\log n$
8. $T(n) = 2T(n/4) + n^{0,51}$
9. $T(n) = 0{,}5T(n/2) + 1/n$
10. $T(n) = 16T(n/4) + n!$
11. $T(n) = \sqrt{2}T(n/2) + \log(n)$
12. $T(n) = 3T(n/2) + n$
13. $T(n) = 3T(n/3) + \sqrt{n}$
14. $T(n) = 4T(n/2) + cn$
15. $T(n) = 3T(n/4) + n \log n$
16. $T(n) = 3T(n/3) + n/2$
17. $T(n) = 6T(n/3) + n^2 \log n$
18. $T(n) = 4T(n/2) + n/\log n$
19. $T(n) = 64T(n/8) - n^2 \log n$
20. $T(n) = 7T(n/3) + n^2$
21. $T(n) = 4T(n/2) + \log n$

Soluciones:

1. $T(n) = 3T(n/2) + n^2 \quad \Rightarrow T(n) = \Theta(n^2)$ (caso 3)
2. $T(n) = 4T(n/2) + n^2 \quad \Rightarrow T(n) = \Theta(n^2 \log n)$ (caso 2)
3. $T(n) = T(n/2) + 2^n \quad \Rightarrow T(n) = \Theta(2^n)$ (caso 3)
4. $T(n) = 2^n T(n/2) + n^n \quad \Rightarrow$ No se aplica: a no es una constante
5. $T(n) = 16T(n/4) + n \quad \Rightarrow T(n) = \Theta(n^2)$ (caso 1)
6. $T(n) = 2T(n/2) + n \log n \quad \Rightarrow$ Relación entre $f(n)$ y $n^{\log_b a}$ no es polinómica
7. $T(n) = 2T(n/2) + n/\log n \quad \Rightarrow$ Relación entre $f(n)$ y $n^{\log_b a}$ no es polinómica
8. $T(n) = 2T(n/4) + n^{0,51} \quad \Rightarrow T(n) = \Theta(n^{0,51})$ (caso 3)
9. $T(n) = 0{,}5T(n/2) + 1/n \quad \Rightarrow$ No se aplica: $a < 1$
10. $T(n) = 16T(n/4) + n! \quad \Rightarrow T(n) = \Theta(n!)$ (caso 3)
11. $T(n) = \sqrt{2}T(n/2) + \log(n) \quad \Rightarrow T(n) = \Theta(\sqrt{n})$ (caso 1)
12. $T(n) = 3T(n/2) + n \quad \Rightarrow T(n) = \Theta(n^{\log 3})$ (caso 1)
13. $T(n) = 3T(n/3) + \sqrt{n} \quad \Rightarrow T(n) = \Theta(n)$ (caso 1)
14. $T(n) = 4T(n/2) + cn \quad \Rightarrow T(n) = \Theta(n^2)$ (caso 1)
15. $T(n) = 3T(n/4) + n \log n \quad \Rightarrow T(n) = \Theta(n \log n)$ (caso 3)
16. $T(n) = 3T(n/3) + n/2 \quad \Rightarrow T(n) = \Theta(n \log n)$ (caso 2)
17. $T(n) = 6T(n/3) + n^2 \log n \quad \Rightarrow T(n) = \Theta(n^2 \log n)$ (caso 3)
18. $T(n) = 4T(n/2) + n/\log n \quad \Rightarrow T(n) = \Theta(n^2)$ (caso 1)
19. $T(n) = 64T(n/8) - n^2 \log n \quad \Rightarrow$ No se aplica: $f(n)$ no es positiva
20. $T(n) = 7T(n/3) + n^2 \quad \Rightarrow T(n) = \Theta(n^2)$ (caso 3)
21. $T(n) = 4T(n/2) + \log n \quad \Rightarrow T(n) = \Theta(n^2)$ (caso 1)