Solution: We prove this combinatorically using the definition of a 'word' from the first lecture. Denote X as the letter for "go up" and Y as the letter for "go right", then a word of length $2n$ formed by X and Y in which X and Y occur equally is a solution to the above problem. There are $\binom{2n}{n}$ such words. This is exactly the n-th Catalan number C_n.