Theorem: Denote the Fibonacci sequence by f_n. Then

$$f_{n-1}f_{n+1} - f_n^2 = (-1)^n$$

Proof: we proceed by induction on n. The base case is trivial. Suppose that the claim holds for n. Then,

$$\begin{aligned}
f_n f_{n+1} - f_{n+1}^2 &= f_n(f_n + f_{n+1}) - f_{n+1}^2 \\
&= f_n^2 + f_n f_{n+1} - f_{n+1}(f_n + f_{n-1}) \\
&= f_n^2 + f_n f_{n+1} - f_{n+1} f_n - f_{n+1} f_{n-1} \\
&= f_n^2 - f_{n+1} f_{n-1} \\
&= -(f_{n+1} f_{n-1} - f_n^2) \\
&= -(-1)^n && \text{Induction Hypothesis} \\
&= (-1)^{n+1}
\end{aligned}$$

This completes the proof.