

Theorem: Denote the Fibonacci sequence by f_n . Then

$$f_{n-1}f_{n+1} - f_n^2 = (-1)^n$$

Proof: We proceed by induction on n . The base case is trivial. Suppose that the claim holds for n . Then,

$$\begin{aligned} f_n f_{n+1} - f_{n+1}^2 &= f_n(f_n + f_{n+1}) - f_{n+1}^2 \\ &= f_n^2 + f_n f_{n+1} - f_{n+1}(f_n + f_{n-1}) \\ &= f_n^2 + f_n f_{n+1} - f_{n+1} f_n - f_{n+1} f_{n-1} \\ &= f_n^2 - f_{n+1} f_{n-1} \\ &= -(f_{n+1} f_{n-1} - f_n^2) \\ &= -(-1)^n && \text{(Induction Hypothesis)} \\ &= (-1)^{n+1} \end{aligned}$$

This completes the proof.