Let C_n be the Catalan sequence. Calculate the limit

$$\lim_{n\to\infty} \frac{C_{n+1}}{C_n}$$

The n-th Catalan number can be calculated using the following formula.

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

It follows the $(n+1)$-th Catalan number.

$$C_{n+1} = \frac{1}{n+2}\binom{2(n+1)}{n+1}$$

Thus,

$$C_n = \frac{(2n)!}{n!(n+1)!}, \quad C_{n+1} = \frac{(2n+1)!}{(n+1)!(n+2)!}$$

Therefore,

$$\frac{C_{n+1}}{C_n} = \frac{\frac{1}{n+2}\binom{2(n+1)}{n+1}}{\frac{1}{n+1}\binom{2n}{n}} = \frac{\frac{(2n+2)!}{(n+1)!(n+2)!}}{\frac{(2n)!}{n!(n+1)!}}$$

$$= \frac{(2n+1)!}{(n+1)!(n+2)!}\frac{n!(n+1)!}{(2n)!} = \frac{(2n+2)!n!}{(n+2)!(2n)!}$$

$$= \frac{(2n+2)!n!}{(n+1)(n+2)n!(2n)!} = \frac{(2n+2)!}{(n+1)(n+2)(2n)!}$$

$$= \frac{(2(n+1))!}{(n+1)(n+2)(2n)!} = \frac{(2n+2)(2n+1)(2n)!}{(n+1)(n+2)(2n)!}$$

$$= \frac{(2n+2)(2n+1)}{(n+1)(n+2)} = \frac{2(n+1)(2n+1)}{(n+1)(n+2)}$$

$$= \frac{2(2n+1)}{n+2} = \frac{4n+2}{n+2} = \frac{4(n+2)-6}{n+2} = 4 - \frac{6}{n+2}$$

And therefore,

$$\lim_{n\to\infty} \frac{C_{n+1}}{C_n} = \lim_{n\to\infty}\left(4 - \frac{6}{n+2}\right) = 4$$