Homework #3

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Exercise 1

$(A \wedge B) \vee (B \wedge C)$

The models that make this sentence true are: 6

- A, B, C, D
- $A, B, C, \neg D$
- $A, B, \neg C, D$
- $A, B, \neg C, \neg D$
- $\neg A, B, C, D$
- $\neg A, B, C, \neg D$

$(A \vee B)$
The models that make this sentence true are: 12

- $\neg A, B, C, D$
- $A, \neg B, C, D$
- A, B, C, D
- $\neg A, B, \neg C, D$
- $A, \neg B, \neg C, D$
- $A, B, \neg C, D$
- $\neg A, B, C, \neg D$
- $A, \neg B, C, \neg D$
- $A, B, C, \neg D$

- $\neg A, B, \neg C, \neg D$
- $A, \neg B, \neg C, \neg D$
- $A, B, \neg C, \neg D$

$(A \iff B) \iff C$
The models that make this sentence true are: 8

- A, B, C, D
- $\neg A, \neg B, \neg C, D$
- $A, B, C, \neg D$
- $\neg A, \neg B, \neg C, \neg D$
- $A, \neg B, \neg C, D$
- $\neg A, B, \neg C, D$
- $A, \neg B, \neg C, \neg D$
- $\neg A, B, \neg C, \neg D$

Exercise 2

$Smoke \implies Smoke$
Possible Interpretations:
$True \implies True$, which is $True$
$False \implies False$, which is $True$
Therefore the formula is valid and satisfiable.

$(Smoke \implies Fire) \implies (\neg Smoke \implies \neg Fire)$
Possible Interpretations:
$(True \implies True) \implies (False \implies False)$, which is $True$
$(True \implies False) \implies (False \implies True)$, which is $True$
$(False \implies True) \implies (True \implies False)$, which is $False$
$(False \implies False) \implies (True \implies True)$, which is $True$
We found an Interpretation that is not a Model of the formula, therefore the formula is satisfiable but not valid.

$((Smoke \wedge Heat) \implies Fire) \iff ((Smoke \implies Fire) \vee (Heat \implies Fire))$
Possible Interpretations:
$((True \wedge True) \implies True) \iff ((True \implies True) \vee (True \implies True))$, which is $True$
$((True \wedge True) \implies False) \iff ((True \implies False) \vee (True \implies False))$,

which is $True$
$((True \wedge False) \implies True) \iff ((True \implies True) \vee (False \implies True))$, which is $True$
$((True \wedge False) \implies False) \iff ((True \implies False) \vee (False \implies False))$, which is $True$
$((False \wedge True) \implies True) \iff ((False \implies True) \vee (True \implies True))$, which is $True$
$((False \wedge True) \implies False) \iff ((False \implies False) \vee (True \implies False))$, which is $True$
$((False \wedge False) \implies True) \iff ((False \implies True) \vee (False \implies True))$, which is $True$
$((False \wedge False) \implies False) \iff ((False \implies False) \vee (False \implies False))$, which is $True$
Therefore the formula is valid and satisfiable.

Exercise 3

$$
\begin{aligned}
&((A \wedge B) \vee (B \wedge C)) \implies (D \wedge E) \\
&\neg((A \wedge B) \vee (B \wedge C)) \vee (D \wedge E) \\
&(\neg(A \wedge B) \wedge \neg(B \wedge C)) \vee (D \wedge E) \\
&((\neg A \vee \neg B) \wedge (\neg B \vee \neg C)) \vee (D \wedge E) \\
&(D \wedge E) \vee ((\neg A \vee \neg B) \wedge (\neg B \vee \neg C)) \\
&(D \vee ((\neg A \vee \neg B) \wedge (\neg B \vee \neg C)) \wedge (E \vee ((\neg A \vee \neg B) \wedge (\neg B \vee \neg C)) \\
&(\neg A \vee \neg B \vee D) \wedge (\neg B \vee \neg C \vee D) \wedge (\neg A \vee \neg B \vee E) \wedge (\neg B \vee \neg C \vee E)
\end{aligned} \tag{1}
$$

Exercise 4

$$\frac{A, B \qquad A \wedge B \implies L}{L}\{\}$$

$$\frac{B, L \qquad L \wedge B \implies M}{M}\{\}$$

$$\frac{L, M \qquad L \wedge M \implies P}{P}\{\} \tag{2}$$

$$\frac{P \qquad P \implies Q}{Q}\{\}$$

$$Q$$

Exercise 5

The problem can be represented by the following KB:

$\forall x \exists y | \text{marriage}(x, y)$
$\forall x (\forall y | \text{marriage}(x, y) \wedge \forall z | \text{marriage}(x, z)) \implies y = z$
$\text{marriage}(x, y) \implies (\text{preference}(x, y) \wedge \text{preference}(y, x))$
$\text{marriage}(x, y) \implies \text{marriage}(y, x)$

Where the formula: $\text{marriage}(x, y)$ means that x is married with y.
The formula: $\text{preference}(x, y)$ means that x has expressed a preference in order to marry y.

Exercise 6

$P(A, B, C), P(x, y, z)$
$P(A, B, C), P(A, B, C)$
$\mathbf{mgu} = \{x/A, y/B, z/C\}$

$Q(y, G(A, B)), Q(G(x, x), y)$
$Q(G(x, x), G(A, B)), Q(G(x, x), G(x, x) \qquad \eta = \{y/G(x, x)\}$

$Q(G(A,A),G(A,B)),Q(G(A,A),G(A,A)$ $\quad \eta = \{x/A\}$
mgu doesn't exist.

$O(F(y),x),O(F(x),J)$
$O(F(x),x),O(F(x),J)$ $\quad \eta = \{y/x\}$
$O(F(J),J),O(F(J),J)$ $\quad \eta = \{x/J\}$
mgu $= \{y/x, x/J\}$

$K(F(y),y),K(x,x)$
$K(F(y),y),K(F(y),F(y))$ $\quad \eta = \{x/F(y)\}$
mgu doesn't exist.

Exercise 7

$\text{Horse}(x) \implies \text{Animal}(x)$
$\text{Cow}(x) \implies \text{Animal}(x)$
$\text{Pig}(x) \implies \text{Animal}(x)$
$\text{Horse}(x) \wedge \text{Offspring}(x,y) \implies \text{Horse}(y)$
Horse(Bluebeard)
$\text{Parent(Charlie, Bluebeard)}$
$\text{Offspring}(x,y) \iff \text{Parent}(y,x)$
$\text{Mammal}(x) \implies \exists y \ \text{Parent}(x,y)$
$\text{Mammal}(x) \implies \text{Animal}(x)$

"Charlie is an horse and is an offspring of Bluebeard":

$$\frac{\text{Parent(Charlie, Blue)}, \text{Offspring}(x,y) \iff \text{Parent}(y,x)}{}$$

$$\frac{\text{Parent(Charlie, Blue)}, \text{Offspring(Blue, Charlie)} \iff \text{Parent(Charlie, Blue)}}{\text{Offspring(Blue, Charlie)}} \eta = \{y/\text{Charlie}, x/\text{Blue}\}$$

$$\begin{gathered}\text{Offspring(Blue, Charlie)} \wedge \text{Horse(Blue)}, \ \text{Horse}(x) \wedge \text{Offspring}(x,y) \implies \text{Horse}(y) \\ \frac{\text{Offspring(Blue, Charlie)} \wedge \text{Horse(Blue)}, \ \text{Horse(Blue)} \wedge \text{Offspring(Blue, Charlie)} \implies \text{Horse(Charlie)}}{\text{Horse(Charlie)} \qquad \eta = \{y/\text{Charlie}, x/\text{Blue}\}}\end{gathered} \tag{3}$$

"Is Charlie a cow?":
Since doesn't exist any relation that states that a horse is not a cow and a cow is not a horse we cannot make any assumptions, therefore with the KB that we have we cannot answer that question.

Exercise 8

We can prove that a ground sentence is ***valid*** **or unsatisfiable** in the following way:

- Negate the desired conclusion, after converting it into CNF (in our case the ground sentence)
- Apply the resolution rule in order to derive new axioms

  If a contradiction is derived then the ground sentence is valid

  If we cannot apply the rules anymore and a contradiction has not been found, then the ground sentence is unsatisfiable