Artificial Intelligence – Homework 3

The goal of this assignment is for you to review Propositional Logic and FOL. Translating English sentences into FOL theory (representing knowledge as logical theory) will also be a part of the exercise.

We will use the following convention: strings beginning with upper case letter denote constants, predicate symbols, or function symbols; strings beginning with lower case letter denote variables.

Points for the homework: 200 points

1. Consider a vocabulary with only four propositions A, B, C, and D. Consider the following sentences:
   a. $(A \wedge B) \vee (B \wedge C)$
   b. $A \vee B$
   c. $A \Leftrightarrow B \Leftrightarrow C$

   For each of the above sentences, determine the number of models in which it is true. Provide justification for your answer. (An answer like: "a. 10" will receive zero point even if it is a correct answer!)

2. Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules (Slide 30, slide set for Chapter 7).
   a. $Smoke \Rightarrow Smoke$
   b. $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
   c. $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$

3. Convert the following sentence to CNF:

   $((A \wedge B) \vee (B \wedge C)) \Rightarrow (D \wedge E)$

   Show your step.

4. Use the resolution principle to prove that KB entails Q where KB contains the following sentences (give the steps of your proof):

   $P \Rightarrow Q$

   $L \wedge M \Rightarrow P$

   $L \wedge B \Rightarrow M$

   $A \wedge P \Rightarrow L$

   $A \wedge B \Rightarrow L$

   A

B

5. The original Stable Marriage Problem is stated as follows:

   Given *n* men and *n* women, where each person has ranked all members of the opposite sex with a unique number between 1 and *n* in order of preference, marry the men and women off such that there are no two people of opposite sex who would both rather have each other than their current partners. If there are no such people, all the marriages are "stable".

   Let us consider a modified version of this problem:

   Given *n* men and *n* women, where each person has a set of preferred people of opposite sex that they say they could marry to. Find a marriage arrangement for all people so that each person is married to one of those that he/she prefers.

Some examples for n=3 are given below. Let us denote the men by m_i and the women by w_i, respectively.

- If the preference of m_1 is $\{w_2,w_3\}$; m_2 is $\{w_2,w_3\}$; m_3 is $\{w_1\}$; w_1 is $\{m_3\}$; w_2 is $\{m_1,m_2\}$; and w_3 is $\{m_1,m_2\}$ then the arrangement (m_1,w_2), (m_2,w_3), (m_3,w_1) is a solution to the problem.
- On the other hand, if the preference of m_1 is $\{w_1\}$; m_2 is $\{w_2,w_3\}$; m_3 is $\{w_1\}$; w_1 is $\{m_3\}$; w_2 is $\{m_1,m_2\}$; and w_3 is $\{m_1,m_2\}$ then the problem has no solution.

Represent the problem as a knowledge base whose models represent the solutions of the problem. Make sure that your solution is correct by checking your representation with the two examples. Provide arguments for the correctness of your solution.

6. Given each pair of atomic sentences, find their most general unifier if it exists:
   a. P(A,B,C), P(x,y,z)
   b. Q(y, G(A,B)), Q(G(x,x), y)
   c. Older(Father(y), x), Older(Father(x), John)
   d. Knows(Father(y), y), Knows(x,x)

   Show the steps of your work.

7. Write down a logical representation for the following sentences, suitable for use with the Generalized Modus Ponens rule:
   e. Horses, cows, and pigs are animals.
   f. An offspring of a horse is a horse.
   g. Bluebeard is a horse.
   h. Bluebeard is Charlie's parent.

i. Offspring and parent are inverse relations.
j. Every mammal has a parent.
k. Every mammal is an animal

Prove that Charlie is a horse and is an offspring of Bluebeard using the GMP rule. What is the answer to the question “is Charlie a cow?”

8. How can we use the resolution algorithm to prove that a ground sentence is valid? Unsatisfiable?

Submit your written homework on Canvas by April 12, 2018.