Cryptography And Voting

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$\mu\Pi\lambda\forall$

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Overview

- Motivation
- Cryptography Building Blocks
- Mixnets: A solution to voter privacy
- Verifiably Correct Mixnets
- Almost Verifiably Correct Mixnets
- Conclusion

Motivation

Election Systems: High Level properties

- **Integrity**
  - Votes are cast as intended
  - Votes are counted as cast
- **Ballot Secrecy** Nobody can figure out how you voted (privacy), even if you try to prove it (selling/coercion).
- **Authentication and Authorisation**
  - Only authorised voters can vote
  - a specified number of times as stated in election law
- **Enfranchisement** All voters must have the opportunity and be encouraged to vote
- **Availability** (Voting,Tallying)
- **Efficiency** (Cost,Time)

Remark: Authentication vs Secrecy vs Enfranchisement

Election Systems: Interpretation of properties in building a voting system

- **Privacy** The votes must remain secret
- **Individual Verifiability** Each voter can check that their ballot was included in the outcome (*to ensure integrity*)
- **Universal Verifiability** All voters can check that a voter's ballot was included in the outcome (*to ensure integrity*)
- **Receipt - Freeness** A voter cannot prove how she voted even if she wants to.
- **Robustness** Nobody can disrupt an election (*to ensure availability*)
- **Fairness** No partial results are known. No vote cancellation/duplication. (*to ensure availability, integrity, privacy*)

Remark: Individual Verifiability vs Receipt - Freeness

Electronic Voting For Better Elections(?) I

- Traditional Systems lack many of the properties we described earlier
  - Lack of Individual/Universal Verifiability (we cast our votes, without verifying that they are counted)
  - Trust is based on tradition and conflict of interest
- By computerising the elections we can actually improve the voting process
  - By counting faster
  - By enabling winner selection by a variety of social choice functions
  - Most importantly: by enabling some of the before mentioned properties
  - We can design the election system, from the ground up following specifications
- But computers themselves introduce many problems
  - Can we implement systems with conflicting characteristics?
  - Lack Of Transparency
  - eVoting is like voting by proxy. Can we trust another entity to vote for us?
  - For an example: Check the HBO documentary Hacking Democracy!

Electronic Voting For Better Elections(?) II

- One Solution: Open Source Voting Software
  - Open Source Code can be scrutinised by competing parties and everybody else
  - Voters can build the tallying programs themselves
  - Will surely play a role in the future of voting
  - **But:** How can we be sure of the actual bits that do the tallying?
- **The Solution: Cryptography**
  - Cryptography has been used to keep secrets
  - Cryptography can be used to build trust
  - How: By keeping secrecy and providing verification of each operation

Cryptography Building Blocks

Hash Functions

A function h that maps arbitrary size data (message) to fixed size data (hash) with the following properties

- Given the message it is easy to calculate the hash
- Given the hash it is computationally infeasible to find the message
- Given a message m it is computationally infeasible to find another message m' such that $h(m) = h(m')$
- It is computationally infeasible to find two messages m_1, m_2 such that $h(m_1) = h(m_2)$

Public Key Cryptosystems

- Enable exchange of secret messages without prior engagements
- Introduced by Diffie And Hellman in 1976
- Each user has a public and a private key
- In order to send an encrypted message
  - The public key is retrieved
  - The message is encrypted with the it
  - Upon receipt, the message is decrypted with the private key
- Three algorithms are needed (Key Generation, Encryption, Decryption)
- Security based on (conjectured) hard problems from number theory (factoring, discrete log, quadratic residuosity)
- Can be turned around to provide signatures (encrypt with the private key)

RSA Encryption (1977) I

- **Generate keys**
  - Select Randomly and Independently Two Large Primes p, q
  - Calculate product $n = p \cdot q$
  - Calculate $\phi(n) = (p-1) \cdot (q-1)$
  - Randomly select $e \in \mathbb{Z}_n^*$ st: $gcd(e, \phi(n)) = 1$
  - Calculate reverse $d = e^{-1} mod \phi(n)$
  - Public key is (e, n) and private key is (p, q, d)
- **Encrypt** Message m : Raise to the public key $c = m^e mod n$
- **Decrypt** message c : Raise to the private key $c^d mod n = m^{ed} mod n = m$
- For security: randomize encryption. Append random padding to the message.
- if n can be factored than breaking RSA is easy
- Threshold decryption
  - Break the private key into n pieces so that that t parties can decrypt it
  - Enables the distribution of trust

Homomorphic Encryption

- Computation with encrypted data.
- $E(m_1) \otimes E(m_2) = E(m1 \oplus m2)$
- Apply a function to the ciphertexts that corresponds to another function on the plaintexts. The result can be obtained by one decryption.
- For simple tallying we would require to evaluate a function on ciphertexts that corresponds to adding the plaintexts (additive homomorphism)
- For other social choice functions we would require computation of arbitrary functions on encrypted data.
- It can be done ... in theory (Gentry, 2010)

ElGamal Encryption (1984)

- Randomised Public Key Encryption From Diffie-Hellman Key Exchange
- Key Generation
  - Select 2 large primes p, q st $q \mid (p-1)$ and a generator g
  - Randomly select $x \in_R \mathbb{Z}_q$
  - Calculate $y = g^x modp$
  - Return $(pk = y, sk = x)$
- Encrypt Message m: Multiply with randomisation of public key
  - Randomly select $r \in_R \mathbb{Z}_q$
  - Calculate $G = g^r\, modp$
  - Calculate $M = m \cdot y^r\, modp$
  - Return $c = (G, M)$
- Decrypt message (G, M) with secret key x
  - return $M/G^x\, modp$
- Security based on difficulty of computing discrete logs

Useful Elgamal properties I

- **Reencryption**
  - Change ciphertext without affecting decryption

$$ReEnc(c, r') = c \cdot Enc(1, r') = (g^{r+r'}, m \cdot (g^x)^{r+r'})$$

  - No knowledge of secret key is required.
  - Without the secret key or the re randomisation factor it is infeasible to show that two messages are reencryptions of each other.
- **Multiplicative Homomorphism**
  - Let m_1, m_2 plaintexts. Then $Enc(m_1) \cdot Enc(m_2) = Enc(m_1 \cdot m_2)$
  - In elections we would desire **additive** homomorphism
  - ElGamal Solution: encrypt message m as $(G, M) = (g^r mod p, g^m \cdot y mod p)$
  - Problem:Need to solve DLOG, too many exponentiations
  - Other cryptosystems provide additive homomorphism

Commitments

- Commitment Schemes
  - Commit to a value
  - without revealing it (*hiding* property)
  - and without being able to change it (*binding* property)
- An application: Coin flipping over the telephone
  - Alice and Bob are in different locations but want to flip a coin
  - Alice select head/tails
  - Bob flips the coin
  - Bob doesn't have to flip the coin, he can just announce that he wins

Solution

- Commit to heads or tails
- Flip the coin and announce the result
- Reveal the commitment
- Check the result

Zero Knowledge Proofs (Goldwasser, Micali, Rackoff - 1985) I

- Interactive protocol between 2 parties (prover, verifier)
- **Objective:** The prover wants to convince the verifier about the knowledge of a secret, without disclosing (any part) of it
- Properties:
  - Completeness: Honest prover convinces honest verifier with overwhelming probability
  - Soundness: Dishonest prover cannot succeed with overwhelming probability
  - Zero Knowledge: The verifier cannot learn anything from the protocol

Zero Knowledge Proofs (Goldwasser, Micali, Rackoff - 1985) II

An illustrating example

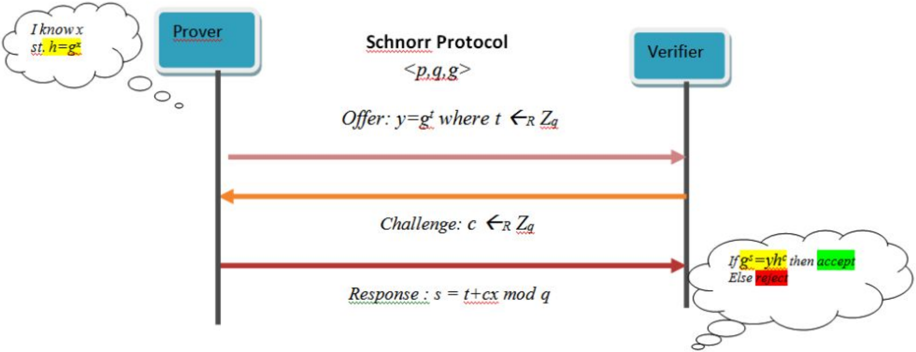
- **Prover** holds two *identical* boxes of different color
- **Verifier** is color blind
- **Prover** wants to convince the Verifier that the boxes have different color

Zero Knowledge Proofs (Goldwasser, Micali, Rackoff - 1985) III

The protocol

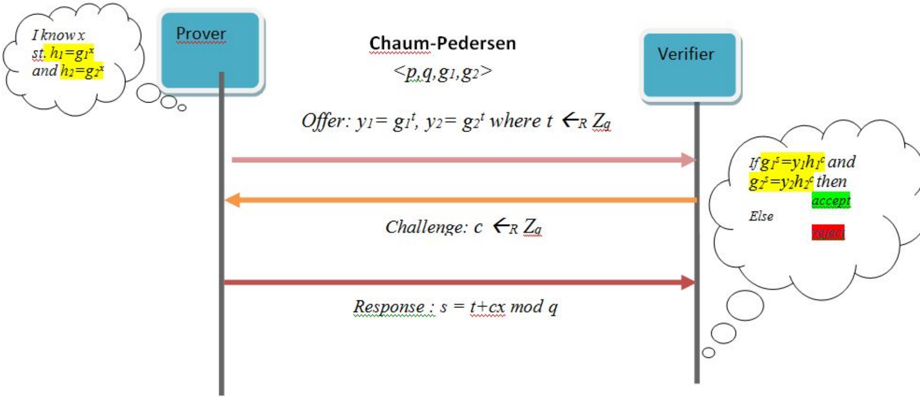
1. **Prover** gives the boxes to the verifier
2. **Verifier** hides the boxes behind her back, one box per hand
3. With probability $\frac{1}{2}$ verifier switches boxes in each hand, behind her back
4. **Verifier** reveals boxes
5. **Prover** can tell whether the verifier switched hands
6. Repeat n times to decrease cheating probability to $\frac{1}{2^n}$
7. **Verifier** is convinced, that the boxes have different color, without ever knowing what it is

Prove that you know DLOG (Schnorr, 1991)



Authentication using Zero Knowledge Proofs: Prove that you know the password, without revealing it.

Prove DLOG equality (Chaum - Pedersen protocol, 1992)



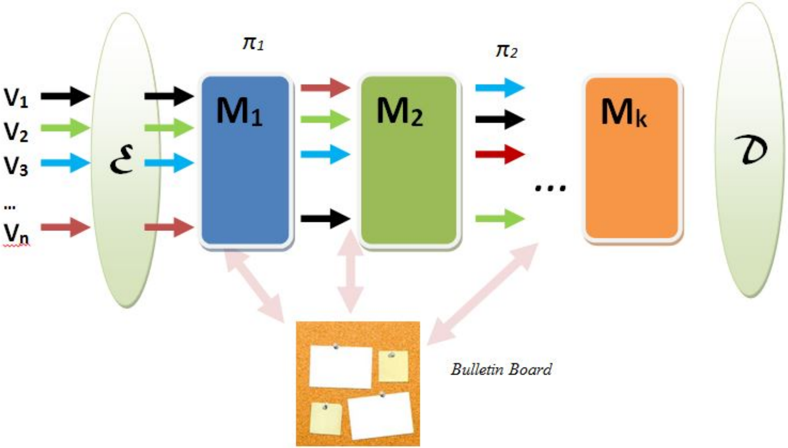
You can Prove Yourself (Fiat - Shamir heuristic, 1987) Replace verifier with hash function

Mixnets

Overview

- A solution to voter privacy - Main idea by David Chaum (1981)
- Generic primitive for anonymous channel (the original paper has 3500 citations!)
- Has been used for anonymous email, anonymous browsing, private payment system, multiparty computation and of course *elections*)
- A mixnet consists of a number of mix servers that are operated by different (mistrusting) parties
- Input: The messages to be anonymised
- Output: A permutation of the input
- To achieve anonymity: No single output item, must match an input item
- An input item is hidden by encryption and shuffling
- In theory: an honest mix server suffices to achieve privacy

Mixnets



Voting With Mixnets:Main Idea

- Voters create their ballots B_i
- Initial Encryption $C_{i0} = Enc(B_i)$
- Encrypted Ballots enter the mixnet
- Each mix server permutes and *changes* the encrypted items $C_{\pi_j(i)j} = X_j(C_{ij-1})$
- After all mixing has occured the **unencrypted ballots** are posted to the bulletin board
- The social choice function is computed
- All communication is achieved by reading from and appending to the bulletin board

Decryption Mixnets

- The original Chaum idea
- Encrypt the ballot with the public key of the mix servers in reverse order
- $C_{i0} = E_{pk1} \circ E_{pk2} \circ \ldots \circ E_{pkt}(B_i)$
- Each mix server peels of a layer of encryption (decrypts with its secret key) and performs the shuffle
- After the final stage, all ballots are decrypted
- **Remarks:**
  - Independent of underlying crypto system
  - The ciphertext size is proportional to the number of mix servers
  - A mix server can block the mixing process by refusing to decrypt
  - The last mix server knows the votes and can block the elections (share the final decryption key)

ReEncryption Mixnets - (Park, Itoh, Kurosawa 1993)

- Version 1
  - Each mix server re randomises the ballots by reencryption
  - A final decryption stage is needed
  - The decryption key must be shared to various parties
  - The decryption is jointly done by all mix servers.
- Version 2
  - Combines decryption and reencryption
  - Each mix server partially decrypts the ballots by applying its secret key.
  - Then re randomises by reencryption
  - The last mix server decrypts

A basic attack (Pfitzmann, 1995) I

Active Attack: Trace an encrypted input by injecting a correlated message

Track input m_i for participant P_i

- Initial Encryption $c_{i0} = (g^R, m_i \cdot (y_1, \cdots, y_k)^R)$
- Mix server j input: $c_{ij} = (g^{R'}, m_i \cdot (y_j, \cdots, y_k)^{R'}) = (t, u)$
- For some random x generate $c''_{ij} = (t^x, u^x) = (g^{R'x}, m_i^x \cdot (y_j, \cdots, y_k)^{R'x})$
- Output will contain both m_i^x, m_i
- Raise all output messages to the x and check for duplicates.

Reminder: El Gamal has multiplicative homomorphism $(E(m)^x = E(m^x))$

A basic attack (Pfitzmann, 1995) II

Track input $m_1, \ldots, m_s$ for participants $P_i, \ldots, P_s$

- Choose random values $x_1, \ldots, x_s$
- Calculate $c = \prod_{i=1}^{s} c_{ij}^{x_i}$
- Inject or replace with c
- Output will contain the decryption $m*$ of c
- Look for s messages such that $m* = \prod_{i=1}^{s} m_i^{x_i}$

A basic attack (Pfitzmann, 1995) III

Remarks

- Applies to both decryption and reencryption mixnets
- If there is a check on number of input items a colluding participant's message must be omitted
- Solution: Redundancy in messages in order to detect the attack
  - Increases Ciphertext Size
  - Does not work if the last mix server is corrupt, since it can replace the m_i^x with a correct looking message after the message correlation
- Something stronger is needed

Problems

Problems

- At the initial encryption stage, a different vote might be encrypted (vote changing, vote copying, vote cancelling)
- A mix server can change some of the input votes, by replacing them in the output.
- A subset of the mix servers might cooperate to break anonymity by tracing messages

Solutions must be efficient, correct and privacy respecting

Solutions

Main Idea

- Zero - knowledge proof of the contents of the vote
- Zero - knowledge proof of the correctness of the shuffle

- Correctness of initial encryption $\Leftrightarrow$ Prove that you know DLOG
- Correctness of Shuffling $\Leftrightarrow$
  Prove that each encrypted vote in the input, appears in the output in a valid reencrypted form

Verifiable Mixnets

Killian-Sako Verifiable Mixnet (1995) I

- The first universally verifiable mixnet
- Elgamal Reencryption based Mixnet
- Verification based on cut and choose protocol

Cut - And - Choose Proof Of Correct Reencryption

- Prove knowledge of secret key and partial decryption.
- Perform secondary reencryption with new randomisation factor
- Reveal secondary reencryption or difference between primary and secondary reencryption

Killian-Sako Verifiable Mixnet (1995) II

Cut - And - Choose Proof Of Correct Shuffle - Main idea

- Each mix server generates another permutation and randomisation values
- Perform reencryption and shuffling according to them (secondary shuffle)
- Reveal secondary shuffle or the difference between primary and secondary shuffle

Mixnets based on permutation networks - 1999 I

- Each mix server M_i simulates a sorting network.
- Reencrypts then sorts the inputs
- The mix server is built in a bottom up by combining smaller comparator functions
- **Proof Of Correctness:** Prove for a 2×2 sorting network using the Chaum-Pedersen Protocol 4 times and generalise for $n \times n$ sorting network

Furukawa and Sako Mixing - 2001 I

- Mixing is represented as matrix multiplication.
- Permutation matrix

$$A_{ij} = \begin{cases} 1, \pi(i) = j \\ 0, \text{otherwise} \end{cases}$$

- For example the permutation $\pi(1,2,3) = (2,3,1)$ can be represented using the matrix:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

- Shuffling and re encryption can be represented using a permutation matrix.
- Prove that r_i and $[A_{ij}]$ exists based on the key observation that a matrix $[A_{ij}]$ is a permutation matrix iff the dot product of two columns is 0 (if they are different) and 1 (if they are the same)

Neff Verifiable Mixnet (2001, 2003) I

- Mix inputs and outputs are represented as polynomials P_{in}, P_{out}
- Key Property: A polynomial is unaffected by permutation of the roots. $\prod_{i=1}^{n}(m_i - x) = \prod_{i=1}^{n}(m_{\pi(i)} - x)$
- Verifier chooses a random $t \in Z_q$
- Evaluates both input and output polynomials
- The results match with very high probability

Verifiable Mixnets: Performance

Cost = Total Number of Exponentiations For

- Initial Encryption
- Proof of Reencryption and Shuffling
- Decryption

Performance for n voters and k mix servers:

- **Sako - Killian** $642nk$
- **Sorting Networks** $7nlogn(2k-1)$
- **Furukawa Sako** $18n(2k-1)$
- **Neff** $8n(2k-1)$

In practice: Neff Mixnet: 10^6 votes $\Rightarrow 20$ hours to mix and verify.

Lesson:Zero Knowledge Proofs Are Computationally Expensive.

Almost Verifiable Mixnets

Randomised Partial Checking I

- RPC - Jakobsson, Juels, Rivest - 2001
- Create an efficient verifiable mixnet out of any cryptosystem
- **Idea**: Give up the expensive notion of **proof**
- Provide **strong evidence** that the mix server has operated correctly (ie. the output is a processed permutation of the input)
- Strong Evidence = Probabilistic Verification
- Partial Revelation of the input/output correspondence.
- For n items, choose randomly $\frac{n}{2}$ and reveal the input/output relation.
- The mix server has no control of which items are revealed.
- **Objective:** A cheater cannot get away with altering too many votes
- **Tradeoff:** Privacy and Correctness

Randomised Partial Checking II

Operation

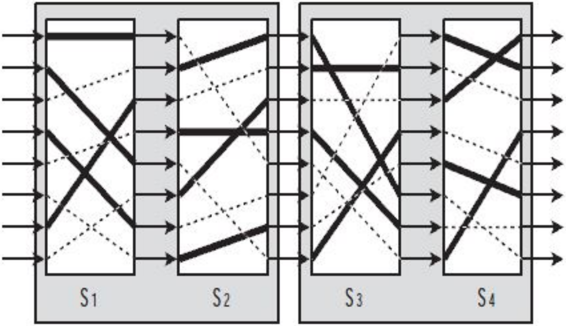
- X_j is a cryptographic operation that transforms ciphertext c to c' (reencryption, decryption)
- Mix server M_j randomly selects a permutation π_j
- Commit to the permutation by publishing to the bulletin board
  - A commitment Γ_j^{in} that input i maps to output $\pi_j(i)$, if j is odd
  - A commitment Γ_j^{out} that output i came from $\pi_j^{-1}(i)$, if j is even
- Proof of correct operation: **Partial** Revelation
  - Reveal information that allows anyone to verify that $c_{ij} = X_j(c_{kj-1})$
  - What to reveal: randomness, i, k
  - Also reveal the commitments
- Verifier validates the transformation

Randomised Partial Checking III

What about privacy

- Main idea:Pair the servers so that we never reveal the same correspondence twice.
- Only the inter-pair correspondences are revealed
- Let j odd and (M_j, M_{j+1}) a server pair . Then
  - $P_{IN}(Q_j, k) = false$
  - $P_{OUT}(Q_j, i) = true$ with probability $\frac{1}{2}$
  - $P_{IN}(Q_{j+1}, i) = 1 - P_{OUT}(Q_j, i)$
  - $P_{IN}(Q_{j+1}, m) = false$
- At least one honest pair is needed for privacy

Randomised Partial Checking IV



Almost Entirely Correct Mixing (Boneh, Golle - 2002) I

- **Idea**: Sacrifice the validation of correctness for speed.
- An almost correct proof of mixing might suffice, if the margin of victory is high

Method

- Select a random subset S of mix server inputs
- Calculate the product π_s
- Reveal S to the mix server.
- Ask to product a set of outputs S' st. $\pi_s = \pi_{s'}$
- Honest mix server: Simply apply the permutation
- Cheating mix server: **Might** be impossible to find such S'.

Optimistic Mixing (Golle, Zhong, Boneh, Jakobsson, Juels - 2002) I

- Concept based on almost entirely correct mixing
- Fast proof when all mix servers are honest
- Reminder: Product preservation might not imply absence of cheating
- Cheating might be discovered, albeit after the fact. Privacy would be exposed.
- Solution: Augment with cryptographic checksums and check both product and checksums
- If a cheating mix server is found then:
  - No output is produced
  - A correct proof is executed (Neff, Furukawa-Sako)
  - Privacy is not compromised
- **Details:**User input is encrypted twice!

Optimistic Mixing (Golle, Zhong, Boneh, Jakobsson, Juels - 2002) II

- Encrypt the vote
- Hash the encryption components
- Encrypt the hash.

- Mixing proceeds as usual with permutation and reencryption
- Verification: Decrypt once and check products and checksums
- If verification succeeds then everything is OK. Decrypt the vote and tally.
- If cheating is discovered then the cause of cheating is sought for the triples that do not checksum.
- How:
  - Starting from the end each server reveals (input, randomisation) for the triples in question.
  - If the first server is reached than the mix worked correctly.

Optimistic Mixing (Golle, Zhong, Boneh, Jakobsson, Juels - 2002) III

- Cheating was due to the users. Solution: Ignore the cheating senders and count the vote.
- If a server is exposed as cheating then repeat with slower and verifiable mixnet.

- Cheating will not expose privacy, since the cheating server will occur on (single) ciphertexts

Conclusion

Conclusion

- Electronic voting **can** improve the voting process
- If implemented correctly, it allows for properties that are not found even in the traditional systems that we have grown to trust
- Cryptography can help rebuild that trust, by allowing for secrecy and verification
- Mixnets are a mature technology that has been extensively researched in the last 20 years and can be used to anonymise the votes
- There exist many protocols for mixnets that combine efficiency, correctness, privacy to some extent
- Many eVoting systems rely on mixnets (Pret A′ Voter, Verificatum)
- The building blocks are there but much work must be done in their composition, implementation and proof of security
- The goal however remains: 'Trust nothing but verify everything' (*Ben Adida - creator of Helios*)

Thank you!

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