$$V(k_0) = \sum_{t=0}^{\infty} \left[\beta^t \ln(1 - \alpha \beta) + \beta^t \alpha \ln k_t \right]$$

$$= \ln(1 - \alpha \beta) \sum_{t=0}^{\infty} \beta^t + \alpha \sum_{t=0}^{\infty} \beta^t \left[\frac{1 - (\alpha \beta)^t}{1 - \alpha \beta} \ln \alpha \beta + \alpha^t \ln k_0 \right]$$

$$= \frac{\alpha}{1 - \alpha \beta} \ln k_0 + \frac{\ln(1 - \alpha \beta)}{1 - \beta} + \alpha \ln(\alpha \beta) \sum_{t=0}^{\infty} \left[\frac{\beta^t}{1 - \alpha} - \frac{(\alpha \beta)^t}{1 - \alpha} \right]$$

$$= \frac{\alpha}{1 - \alpha \beta} \ln k_0 + \frac{\ln(1 - \alpha \beta)}{1 - \beta} + \frac{\alpha \beta}{(1 - \beta)(1 - \alpha \beta)} \ln(\alpha \beta)$$

左边 =
$$V(k) = \frac{\alpha}{1 - \alpha\beta} \ln k + \frac{\ln(1 - \alpha\beta)}{1 - \beta} + \frac{\alpha\beta}{(1 - \beta)(1 - \alpha\beta)} \ln(\alpha\beta)$$

$$\stackrel{\triangle}{=} \frac{\alpha}{1 - \alpha\beta} \ln k + A$$

右边 =
$$\max \left\{ u(f(k) - y) + \beta V(y) \right\}$$

利用 FOC 和包络条件求解得到 $v = \alpha \beta k^{\alpha}$,代入,求右边。

右边 =
$$\max \left\{ u(f(k) - y) + \beta V(y) \right\}$$

= $u(f(k) - g(k)) + \beta \left[\frac{\alpha}{1 - \alpha \beta} \ln g(k) + A \right]$
= $\ln(k^{\alpha} - \alpha \beta k^{\alpha}) + \beta \left[\frac{\alpha}{1 - \alpha \beta} \ln \alpha \beta k^{\alpha} + A \right]$
= $\ln(1 - \alpha \beta) + \alpha \ln k + \beta \left[\frac{\alpha}{1 - \alpha \beta} \left[\ln \alpha \beta + \alpha \ln k \right] + k \right]$
= $\alpha \ln k + \frac{\alpha \beta}{1 - \alpha \beta} \alpha \ln k + \ln(1 - \alpha \beta) + \frac{\alpha \beta}{1 - \alpha \beta} \ln \alpha \beta + \beta A$
= $\frac{\alpha}{1 - \alpha \beta} \ln k + \ln(1 - \alpha \beta) + \frac{\alpha \beta}{1 - \alpha \beta} \ln \alpha \beta + \beta A$
= $\frac{\alpha}{1 - \alpha \beta} \ln k + (1 - \beta) A + \beta A$
= $\frac{\alpha}{1 - \alpha \beta} \ln k + A$

所以, 左边 = 右边, 证毕。