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Nonlinear Interaction between a Frequency Signal and Neighboring Data Channels in a Commercial Optical Fiber Communication System

by

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ABSTRACT

Title of dissertation: NONLINEAR INTERACTION BETWEEN A FREQUENCY SIGNAL AND NEIGHBORING DATA CHANNELS IN A COMMERCIAL OPTICAL FIBER COMMUNICATION SYSTEM

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We theoretically investigate the feasibility of transmitting a frequency signal in an interstice of the data channels in a commercial wavelength division multiplexed optical fiber communications system. We will give an overview of some different measures used for frequency stability. We also list the typical optical impairments that affect light propagating in an optical fiber and how the impairments can induce phase noise in a frequency signal. The phase noise on the frequency signal due to the optical impairments can be limited by restricting the optical power, bandwidth, and center frequency of the signal. The primary source of phase noise is cross-phase modulation (XPM) between the frequency signal and its neighboring data channels. We calculate the first order structure functions and Allan deviation of the phase noise resulting from XPM as the averaging time varies using typical commercial system parameters. We find that the instability added by this effect is comparable to experimentally observed instabilities in research networks, suggesting that frequency transfer over commercial networks without occupying an entire data channel should be feasible.

A frequency source can be represented as

$$u_c(t) = [U_0 + \epsilon(t)] \sin[\omega_0 t + \phi(t)] \tag{2.1}$$

where U_0 is the amplitude and $\epsilon(t)$ is amplitude fluctuation. The quantity $\omega_0 = 2\pi f_0$ is the nominal angular frequency, and $\phi(t)$ is the phase fluctuation. The amplitude fluctuation must be much less than the nominal amplitude, $|\epsilon(t)| \ll |U_0|$; similarly, the frequency fluctuation, given by the time derivative of the phase, $\dot{\phi} \equiv d\phi/dt$, must be much less than the nominal angular frequency, $|\dot{\phi}| \ll |\omega_0|$. Otherwise, the frequency signal is too heavily distorted to be useful.

No clock can give a meaningful time until it is compared to another clock; for instance, a wall clock should match in some way with the position of the sun (which we can consider to be a natural clock). If a clock is fast or slow, it is only in reference to another clock used as a standard. Any attempt to measure the stability of a clock can only be done with respect to some reference source. The readings of many different clocks can be collected and averaged to provide such a reference [21, 22].

Consider two different ~~clocks~~ clocks with internal frequency sources ~~frequency sources~~ that produce sinusoidally oscillating signals proportional to

$$\sin[2\pi f_1 t + \phi_1(t)] \quad \text{and} \quad \sin[2\pi f_2 t + \phi_2(t)], \tag{2.2}$$

with phase terms.

$$\psi_1(t) = 2\pi f_1 t + \phi_1(t), \quad \psi_2(t) = 2\pi f_2 t + \phi_2(t), \tag{2.3}$$

The time t is a derived quantity ~~determined~~ that is calculated by counting every instance that the phase term equals a particular value Ψ. ~~Then~~ There are two times t_{n1} and t_{n2} that correspond to ~~the~~ n counts of Ψ for each phase term, $\psi_j(t_{nj}) = n\Psi$. The time difference between the two is

that is produced by each clock

$$t_{n1} - t_{n2} = \frac{n\Psi}{2\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right] - \left[\frac{\phi_1(t_{n1})}{2\pi f_1} + \frac{\phi_2(t_{n2})}{2\pi f_2}\right]. \tag{2.4}$$

Suppose one clock is ~~sufficiently~~ more accurate and precise than the others, so that we can ~~consider~~ treat it ~~to be as~~ ~~We split the error between the two clocks in the form of ϕ_1 and ϕ_2, but we can consider one of the clocks as~~ a reference, and attribute all of the error to the other. Consider the first clock as the reference. The terms in the right brackets represent a phase drift, so that if we suppose that any long-term drift in the system is compensated, then we ~~have~~ have only a short-term phase instability, which we assume has zero mean. Averaging over several of these matched phase terms, we have

② ~~Of course,~~ This procedure does not work for the most accurate and precise clocks, since there is no ~~reference~~ "better" clock that can be used a reference. For the best clocks that are used as ~~frequency standards~~ ~~standard reference~~ time and frequency standards, it is necessary to compare times from several clocks of the same type. It is then possible to statistically infer the fractional frequency uncertainty uncertainty [reference]

$$\Delta t = \langle t_{n1} - t_{n2} \rangle = \frac{n\Psi}{2\pi f_1}\frac{f_2 - f_1}{f_2} = \frac{\Delta f}{f} T, \tag{2.5}$$

where Δt is the time deviation, $T = n\Psi/(2\pi f_1)$ is a total time, and $\Delta f/f$ is the fractional frequency uncertainty. ② ~~For example,~~ A typical atomic clock based on the cesium standard has a fractional frequency uncertainty of about 3×10^{-15}, corresponding to a time deviation of ± 1 sec in 10 million years [23].

Most of the literature on time and frequency control uses the fractional fre-

For any of the quantities $w = x$, y, ϕ, or $\dot{\phi}$ the autocorrelation function is defined as

$$R_w(\tau) = \langle w(t)w(t+\tau)\rangle = \lim_{T\to\infty} \frac{1}{T}\int_0^T w(t)w(t+\tau)dt, \tag{2.7}$$

and the corresponding PSDs are the Fourier transforms of the autocorrelation functions

?

$$S_w(\omega) = 2\int_0^\infty R_w(\tau)\cos(\omega\tau)d\tau, \tag{2.8}$$

$$R_w(\tau) = \frac{1}{\pi}\int_0^\infty S_w(\omega)\cos(\omega\tau)d\omega. \tag{2.9}$$

Evaluating $R_w(\tau)$ at $\tau = 0$ yields the second moment of w,

$$R_w(0) = \langle [w(t)]^2\rangle = \int_0^\infty S_w(\omega)d\omega, \tag{2.10}$$

which we refer to as the signal power. If we compare the PSD of two different sources, then the one with the lower signal power will typically have less error.

The PSDs for each of our quantities are related. We find

$$S_{\dot{\phi}}(\omega) = \omega^2 S_\phi(\omega), \tag{2.11a}$$

$$S_y(\omega) = \frac{\omega^2}{\omega_0^2} S_\phi(\omega), \tag{2.11b}$$

$$S_x(\omega) = \frac{1}{\omega_0^2} S_\phi(\omega). \tag{2.11c}$$

The oscillator noise can typically be decomposed into a power series $S_\phi(\omega) = \sum_{k=0}^4 h_k\omega^{-k}$ [26]. The term proportional to ω^0 is referred to as white phase noise,

the term proportional to ω^{-1} is referred to as flicker phase noise, and the term proportional to ω^{-2} is referred to as random walk phase noise. The term proportional to ω^{-3} is referred to as flicker frequency noise, and the term proportional to ω^{-4} is referred to as random walk frequency noise [7, 24, 28, 29]. Since $S_{\dot{\phi}}(\omega) = \omega^2 S_{\phi}(\omega)$, the powers in the series increase by 2 for $S_{\dot{\phi}}(\omega)$. Figure 2.1 shows the different noise regions for the PSDs of the fractional frequency y and the phase noise ϕ.

2.3 Allan Variance

The distribution of the frequency errors is difficult to determine because it is typically nonstationary. Nonstationary processes may have divergent statistical measures, specifically the variance. ~~And even~~ Moreover, if there is a true convergent variance, the sample variance from finitely many measurements may not converge to the true variance of the process as the number of samples and the time over which they are measured goes to infinity. The Allan variance is the mean of sample variances calculated over an interval. The definition is based on the fractional frequency y and phase time x; however, we will express it in terms of the phase ϕ. We use the averaged quantity $\bar{y}_k$ defined in Eq. ??.

The N-sample mean is defined as

$$\mu = \frac{1}{N} \sum_{k=1}^{N} \bar{y}_k. \tag{2.12}$$

2.5.~~0.1~~ 1 Allan variance to the second-order structure function:

We now show that the Allan variance is proportional to the second-order structure function. Using the definition of Allan variance in Eq. 2.17, we obtain

$$\begin{aligned}\sigma_A^2(2,\tau) &= \frac{1}{2}\left\langle \left[\frac{\phi(t_k+2\tau)-\phi(t_k+\tau)}{\omega_0\tau} - \frac{\phi(t_k+\tau)-\phi(t_k)}{\omega_0\tau}\right]^2 \right\rangle \\ &= \frac{1}{2\omega_0^2\tau^2}\langle[\phi(t_k+2\tau)-2\phi(t_k+\tau)+\phi(t_k)]^2\rangle. \end{aligned} \tag{2.24}$$

The t_k are arbitrary when averaging over all time; so, the ensemble average is equal to the structure function $D_\phi^{(2)}(\tau)$ divided by $2\omega_0^2$.

2.5.~~0.2~~ 2 PSD to structure functions:

The relation between the PSD and the structure function depends on the long-term frequency drift of the oscillator and whether the M-th difference equation is stationary [27]. If we suppose that the drift is compensated or $M > N$, where N is the highest-order polynomial term for the drift, then we find

$$D_\phi^{(M)}(\tau) = 2^{2M}\int_{-\infty}^{\infty} \sin^{2M}\left(\frac{\omega\tau}{2}\right) S_\phi(\omega)d\omega. \tag{2.25}$$

2.5.~~0.3~~ 3 PSD to Allan variance:

Since we demonstrated that the Allan variance is proportional to a structure function in Eq. 2.24, we combine the results from the last two sections, and we obtain

$$\sigma_A^2(2,\tau) = \frac{2^2}{\omega_0^2\tau^2}\int_{-\infty}^{\infty} \sin^4\left(\frac{\omega\tau}{2}\right) S_\phi(\omega)d\omega. \tag{2.26}$$

2.6 Chapter remarks

We will be using the structure functions, specifically $D_\phi^{(1)}(\tau)$, as our preferred measure of instability. The reason for this choice is that it requires fewer samples to compute the flicker noise, and it is simple to implement and interpret. We will also use the Allan deviation to characterize the instability because it is a common measure of frequency instability in the oscillator community, and we can obtain it from the structure function $D_\phi^{(2)}(\tau)$. The phase noise PSD is preferred for experimental measurements.

modulation of the frequency signal

$$\omega_{\text{mod}} = \Delta\omega \sin(\omega_m t + \phi_m), \tag{3.9}$$

where ω_m is the modulation frequency, $\Delta\omega$ is the modulation depth, and ϕ_m is the phase difference between the modulation and light fields [15]. A modulation frequency between 1 kHz and 10 kHz with a large modulation depth of 10 MHz is optimal [15]. Hence, the frequency signal should have a bandwidth of about 10 MHz.

We give further details on scattering limitations later in Sec. 3.5.

3.5 Optical Impairments

In the previous sections, we showed how the Kerr effect leads to multiple nonlinear terms. In this section, we describe how each of the terms relates to an optical impairment and how that impairment affects the frequency signal. In our analysis, we discuss conditions under which many of the optical impairments become negligible.

Larger type to be consistent with earlier sub-sections 2.5.1, 2.5.2.

3.5.1 Attenuation and Amplified Spontaneous Emission (ASE) Noise

Attenuation appears in Eq. 3.8b as

$$\frac{\partial u_f}{\partial z} = -\frac{\alpha}{2} u_f. \tag{3.10}$$

Laser type

3.5.2 Chromatic Dispersion

The dispersion appears in Eq. 3.8b as

$$\frac{\partial u_f}{\partial z} = -\frac{i\beta_2}{2}\frac{\partial^2 u_f}{\partial T^2}. \tag{3.12}$$

This impairment leads to pulse spreading of the optical signal because its frequency components travel at different velocities. The time spread due to dispersion is [13]

$$\tau_{\text{disp}} = -\frac{2\pi c}{\lambda^2}\beta_{2f} L \Delta\lambda, \tag{3.13}$$

where L is the length of the fiber, and $\Delta\lambda$ is the range of wavelengths. For the optical communication system in the previous example and a frequency signal centered at a wavelength $\lambda = 1.5\ \mu$m, we find $\tau_{\text{disp}} = 1$ ps. By contrast, the time slot of a single bit at 10 Gbps occupies 100 ps; so, dispersion can be neglected for the frequency signal. In this respect, the frequency signal differs significantly from a data signal, which typically has a bandwidth on the order of 10 GHz.

Laser type

3.5.3 Self-Phase Modulation (SPM)

The term

$$\frac{\partial u_f}{\partial z} = i\gamma |u_f|^2 u_f \tag{3.14}$$

in Eq. 3.8b corresponds to self-phase modulation. This effect leads to a phase shift that is proportional to the signal power. Thus, linear attenuation limits the effect

over some length after each amplifier. The effective length is $L_{\text{eff}} = (1/\alpha)[1 - \exp(-\alpha L)]$, so that $L_{\text{eff}} \approx 20$ km for $\alpha_{\text{dB}} = 0.2$ dB/km [14]. The maximum phase shift due to self-phase modulation between two amplifiers is [14]

$$\phi_{\text{SPM}} = \gamma P_f L_{\text{eff}}, \tag{3.15}$$

where P_f is the average power of the frequency signal after an amplifier. The total maximum phase shift is given by Eq. 3.15 multiplied by the number of amplifiers in the fiber link. For our system, we have $\gamma = 1.3$ W^{-1}km^{-1} and $L_{\text{eff}} = 20$ km with 10 amplifiers. If we impose an upper bound on ϕ_{SPM} of 1 radian, then the upper bound on the frequency signal power is 3.8 mW.

3.5.4 Four-Wave Mixing (FWM)

The term

$$i\gamma u_f^* u_{dm} u_{dn} \tag{3.16}$$

corresponds to four-wave mixing. For any two data signals centered at ω_m and ω_n with corresponding wavenumbers $\beta(\omega_m)$ and $\beta(\omega_n)$, four-wave mixing (FWM) creates a parasitic wave whenever $\omega_m + \omega_n = 2\omega_f$ and $\beta(\omega_m) + \beta(\omega_n) = 2\beta(\omega_f)$. This phase-matching condition is avoidable as long as the signals are located away from the zero-dispersion wavelength of the fiber. Frequency sources have a central wavelength that slightly wanders and a narrow bandwidth. Therefore, placing the frequency signal greater than five times its bandwidth away from the zero-dispersion wavelength prevents any portion of the spectrum of the frequency signal from phase-

matching and will eliminate this impairment [15]. We write this requirement as $\lambda - \lambda_0 > 5\Delta\lambda$, where λ is the central wavelength of the frequency signal signal, λ_0 is the zero-dispersion wavelength, and $\Delta\lambda$ is the bandwidth of the frequency signal.

3.5.5 Rayleigh Scattering

Larger type

An optical fiber has random density fluctuations that are created during the fiber's fabrication. These fluctuations are the primary source of attenuation in the fiber, on the order of 0.12–0.17 dB/km at 1.5 μm [13]. Additionally, Rayleigh scattering adds noise to a narrowband frequency signal; however, this noise can be suppressed by a frequency modulation with a bandwidth that is greater than about 10 MHz [32].

3.5.6 Brillouin and Raman Scattering

Larger type

Brillouin and Raman scattering are due to light coupling with vibrations in the crystal lattice of the optical fiber and convert the light to lower frequencies. Brillouin scattering couples with acoustic waves and Raman scattering couples with vibrational waves.

Brillouin scattering only occurs when a phase matching condition is met, $\omega_{\text{orig}} = \omega_{\text{new}} + \omega_{\text{acoustic}}$ and $\beta_{\text{orig}} = \beta_{\text{new}} + \beta_{\text{acoustic}}$, where ω_{orig} and β_{orig} are the frequency and wavenumber of the incident wave, ω_{new} and β_{new} are the frequency and wavenumber of the created optical wave, and ω_{acoustic} and β_{acoustic} are the frequency and wavenumber for the acoustic wave. The large difference between the velocities of the optical waves and the acoustic wave means that the phase matching condition only occurs when the created wave propagates in the direction opposite

3.5.7 Cross-Phase Modulation (XPM)

The remaining term,

$$\frac{\partial u_f}{\partial z} = i2\gamma|u_d|^2 u_f, \tag{3.19}$$

corresponds to cross-phase modulation (XPM), which leads to cross-talk between two signals. This effect on the frequency signal becomes negligible when the group velocity difference between the data channel and the frequency signal is large, which occurs when the frequency signal and data channel are spaced at least one data channel separation away. Therefore, the effects of XPM on the frequency signal only has to be computed for the two neighboring data channels. Since our goal is to place the frequency signal between two data channels, XPM is the primary source of frequency distortion.

The limitation on the optical power of the frequency signal ($\ll 1$ mW) implies that the effect of XPM due to the frequency signal on the data channels will be much less than a radian and can be neglected.

3.6 Phase distortion of the frequency signal due to XPM

Data signals can be modeled as random bit strings. The average behavior of two neighboring data channels on the frequency signal will be equal as long as the frequency signal is placed in the middle of the frequency gap between the data channels. We simplify our analysis of the effect of XPM on the frequency signal by replacing the effect of the two neighboring data signals with a doubling of the effect