Lab 7: Decimated-in-frequency FFT using the Butterfly Technique

Name: Peng Guo
GTID: 903424176

1. Tables

a) Resources

i. CLB Logic

Site Type	Used	Fixed	Available	Util%
CLB LUTs	433	0	663360	0.07
LUT as Logic	433	0	663360	0.07
LUT as Memory	0	0	293760	0.00
CLB Registers	662	0	1326720	0.05
Register as Flip Flop	662	0	1326720	0.05
Register as Latch	0	0	1326720	0.00
CARRY8	57	0	82920	0.07
F7 Muxes	0	0	331680	0.00
F8 Muxes	0	0	165840	0.00
F9 Muxes	0	0	82920	0.00

ii. IO and GT Specific

Site Type	Used	Fixed	Available	Util%
Bonded IOB	325	0	832	39.06
HPIOB	228	0	676	33.73
INPUT	50			
OUTPUT	178			
BIDIR	0			
HRIO	97	0	156	62.18
INPUT	17			
OUTPUT	80			
BIDIR	0			
HPIOBDIFFINBUF	0	0	480	0.00
HPIOBDIFFOUTBUF	0	0	480	0.00
HRIODIFFINBUF	0	0	96	0.00
HRIODIFFOUTBUF	0	0	96	0.00
BITSLICE_CONTROL	0	0	192	0.00
BITSLICE_RX_TX	0	0	1248	0.00
BITSLICE_TX	0	0	192	0.00
RIU_OR	0	0	96	0.00

iii. Primitives

Ref Name	Used	Functional Category
FDRE	662	Register
OBUF	258	I/O
LUT2	210	CLB
LUT1	74	CLB
INBUF	67	I/O
IBUFCTRL	67	Others
LUT6	66	CLB
CARRY8	57	CLB
LUT5	53	CLB
LUT3	48	CLB
LUT4	43	CLB
DSP48E2	16	Arithmetic
BUFGCE	1	Clock

b) Power

Total On-Chip Power (W)	1.529
Dynamic (W)	0.196
Device Static (W)	1.333
Effective TJA (C/W)	0.8
Max Ambient (C)	98.8
Junction Temperature (C)	26.2
Confidence Level	Low
Setting File	---
Simulation Activity File	---
Design Nets Matched	NA

c) Worst Negative Slack

Design Timing Summary

WNS(ns)	TNS(ns)	TNS Failing Endpoints	TNS Total Endpoints	WHS(ns)	THS(ns)
4.369	0.000	0	2209	0.036	0.000

THS Failing Endpoints	THS Total Endpoints	WPWS(ns)	TPWS(ns)	TPWS Failing Endpoints	TPWS Total Endpoints
0	2209	4.725	0.000	0	671

2. Question and Answer

What is the rationale behind choosing the values of the twiddle factor as given in the table? Note that the twiddle factors are signed numbers of resolution 1.8.

a) Write down the decimal fractional format of the twiddle factors.

$$W_N^{kn} = e^{-j(2\pi kn/N)}$$
$$= \cos\frac{2\pi nk}{N} - j\sin\frac{2\pi nk}{N}$$
$$W_8^0 = \cos 0 - j\sin 0 = 1$$
$$W_8^1 = \cos\frac{2\pi}{8} - j\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$$
$$W_8^2 = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -j$$
$$W_8^3 = \cos\frac{6\pi}{8} - j\sin\frac{6\pi}{8} = -\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$$

Since the twiddle factors are signed numbers of resolution 1.8, and we shift 8 bits right after each complex multiplication, we should multiply the original twiddle factors by $2^8 = 256$.

W_8^0_Real $= 1 \times 256 = 256$
W_8^0_Imag $= 0$
W_8^1_Real $= \frac{\sqrt{2}}{2} \times 256 \approx 181$
W_8^1_Imag $= -\frac{\sqrt{2}}{2} \times 256 \approx -181$
W_8^2_Real $= 0$
W_8^2_Imag $= -1 \times 256 = -256$
W_8^3_Real $= -\frac{\sqrt{2}}{2} \times 256 \approx -181$
W_8^3_Imag $= -\frac{\sqrt{2}}{2} \times 256 \approx -181$

However, the twiddle factors given in the lab are

W_8^0_Real $= 255$ W_8^0_Imag $= 0$
W_8^1_Real $= 180$ W_8^1_Imag $= -180$
W_8^2_Real $= 0$ W_8^2_Imag $= -255$
W_8^3_Real $= -180$ W_8^3_Imag $= -180$

b) Consider what is the limit of such 1.8 resolution?
It is not that precise to use 1.8 resolution, and we have no method to denote integer 1 multiplied by 256.

c) Consider for some twiddle factor(s), is the value not precise as it could have been? And why are we doing that?
Because we have to keep the modulus of the twiddle factors the same. The twiddle factor 256 is adjusted to 255, so the twiddle factor 181 is adjusted to 180 as well.