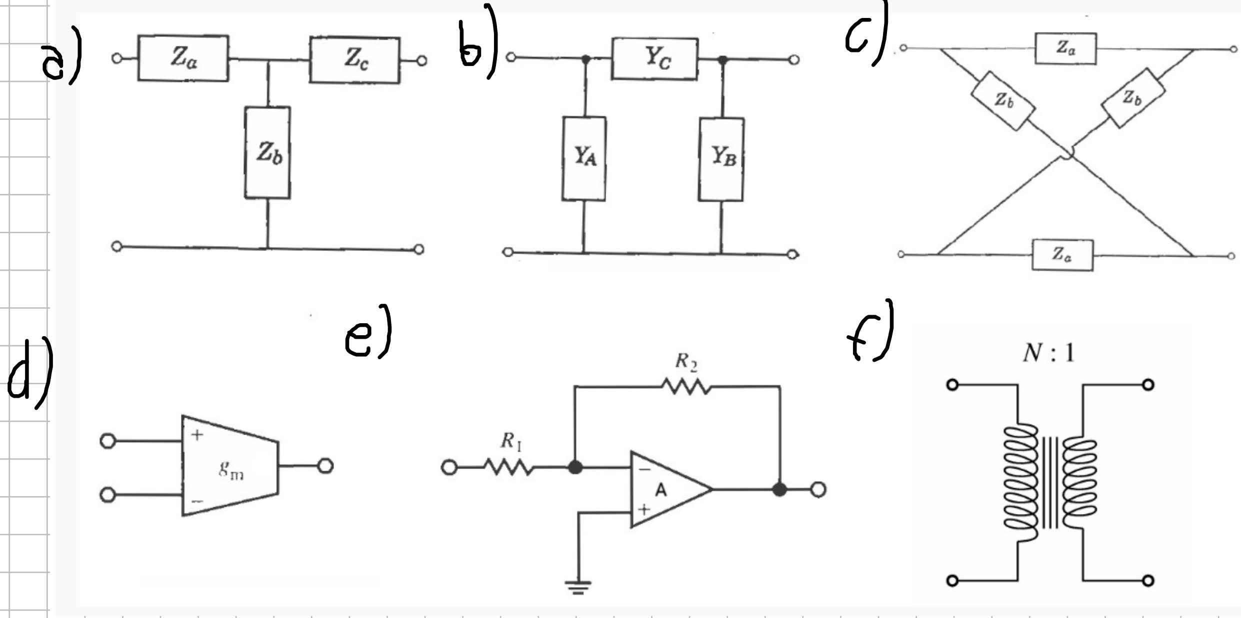
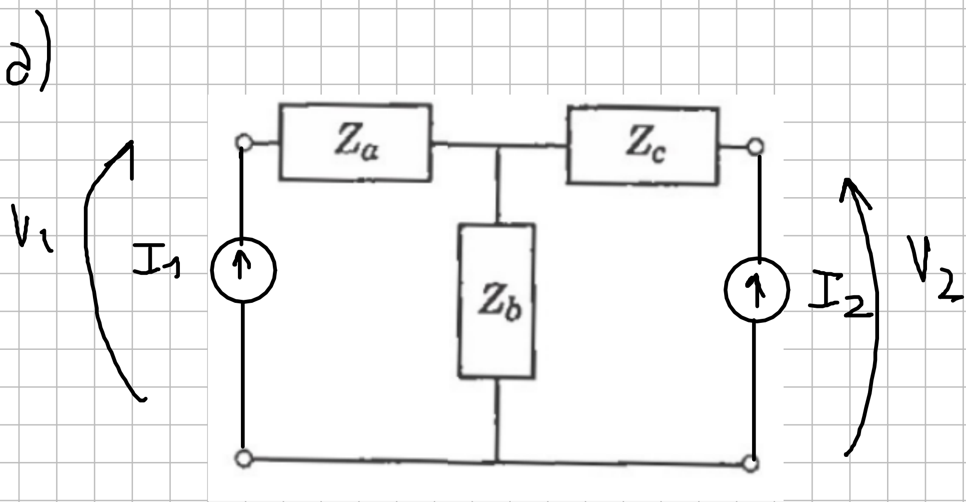
1)

Dadas las siguientes topologías circuitales:



a)



$$V_1 = I_1 \cdot Z_{11} + I_2 \cdot Z_{12}$$

$$V_2 = I_1 \cdot Z_{21} + I_2 \cdot Z_{22}$$

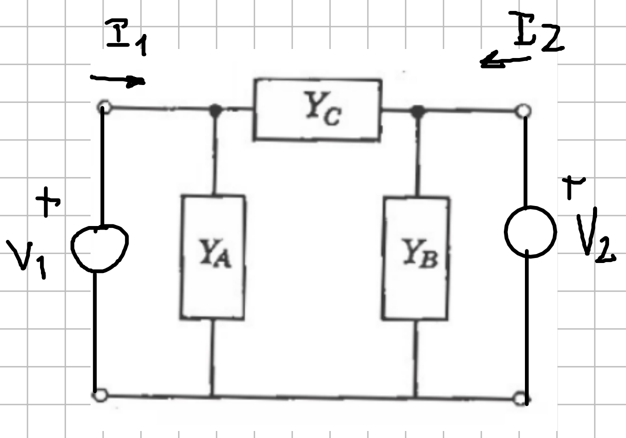
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_a + Z_b$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_b$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_b$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_c + Z_b$$

b)



$$I_1 = V_1 y_{11} + V_2 y_{12}$$
$$I_2 = V_1 y_{21} + V_2 y_{22}$$

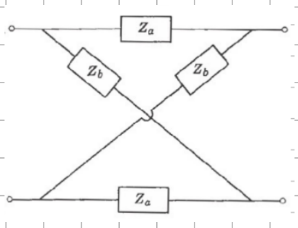
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = y_A + y_C$$

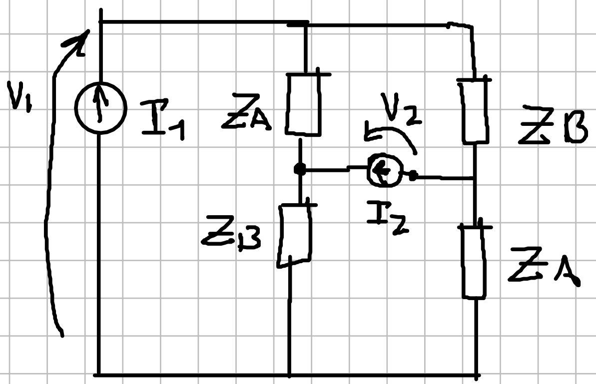
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -y_C$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -y_C$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = y_b + y_c$$

c)

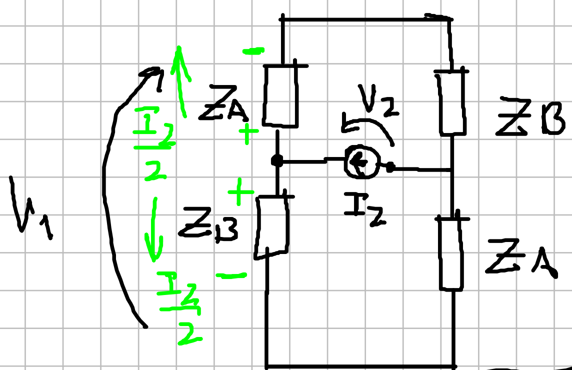




$$V_1 = I_1 \cdot Z_{11} + I_2 \cdot Z_{12}$$
$$V_2 = I_1 \cdot Z_{12} + I_2 \cdot Z_{22}$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{Z_A + Z_B}{2}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{Z_B}{2} - \frac{Z_A}{2} = \frac{Z_B - Z_A}{2}$$

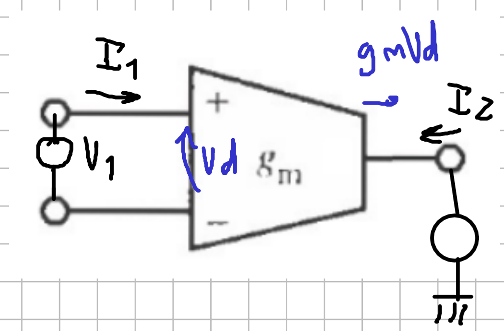


$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{Z_B - Z_A}{2}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{Z_A + Z_B}{2}$$

$$Z = \begin{pmatrix} \frac{Z_A + Z_B}{2} & \frac{Z_B - Z_A}{2} \\ \frac{Z_B - Z_A}{2} & \frac{Z_A + Z_B}{2} \end{pmatrix}$$

d)



$$I_1 = V_1 y_{11} + V_2 y_{12}$$
$$I_2 = V_1 y_{12} + V_2 y_{22}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 0$$

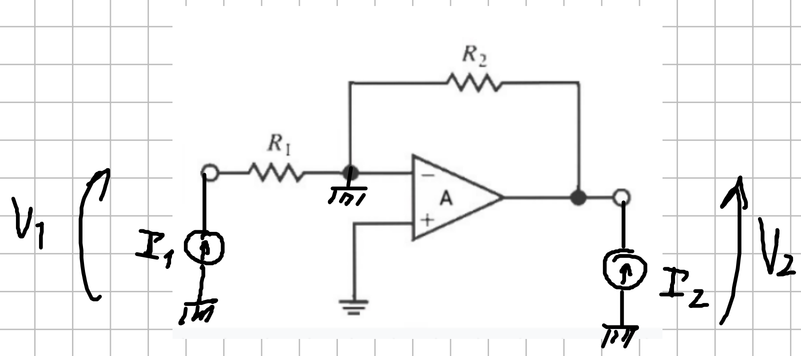
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = 0$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -gm$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = 0$$

$$y = \begin{pmatrix} 0 & 0 \\ -gm & 0 \end{pmatrix}$$

e)



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = R_1$$

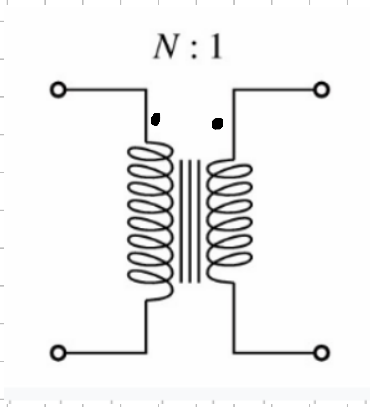
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 0$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = -R_2$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 0$$

$$Z = \begin{pmatrix} R_1 & 0 \\ -R_2 & 0 \end{pmatrix}$$

f)



$$\frac{V_1}{V_2} = N \qquad \frac{(-I_2)}{I_1} = N$$

$$V_1 = V_2 \cdot A + (-I_2)B$$
$$I_1 = V_2 \cdot C + (-I_2)D$$

$$A = \left.\frac{V_1}{V_2}\right|_{(-I_2)=0} = N$$

$$B = \left.\frac{V_1}{(-I_2)}\right|_{V_2=0} = \frac{1}{\left.\frac{(-I_2)}{V_1}\right|_{V_2=0}} = 0$$

$$C = \left.\frac{I_1}{V_2}\right|_{(-I_2)=0} = \frac{1}{\left.\frac{V_2}{I_1}\right|_{I_2=0}} = 0$$

$$D = \left.\frac{I_1}{(-I_2)}\right|_{V_2=0} = -\frac{1}{N}$$

2) a)

$$\begin{cases} V_1 = I_1 Z_{11} + I_2 Z_{12} \\ V_2 = I_1 Z_{21} + I_2 Z_{22} \end{cases}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{Z_{11}}{Z_{21}}$$

$$B = \left.\frac{V_1}{(-I_2)}\right|_{V_2=0} = \frac{\Delta Z}{Z_{21}}$$

$$\underbrace{-I_2 \frac{Z_{22}}{Z_{21}} = I_1}_{} \rightarrow V_1 = -I_2 \frac{Z_{22} Z_{11}}{Z_{21}} + I_2 Z_{12}$$

$$\frac{V_1}{(-I_2)} = \frac{Z_{22} Z_{11} - Z_{12} Z_{21}}{Z_{21}}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{Z_{21}}$$

$$D = \left.\frac{I_1}{(-I_2)}\right|_{V_2=0} = \frac{Z_{22}}{Z_{21}}$$

$$T = \frac{1}{Z_B} \begin{pmatrix} Z_A + Z_B & \Delta Z \\ 1 & Z_C + Z_B \end{pmatrix}$$

$$\Delta Z = (Z_A + Z_B)(Z_C + Z_B) - Z_B^2$$

b)

$$I_1 = V_1\, y_{11} + V_2\, y_{12}$$

$$I_2 = V_1\, y_{21} + V_2\, y_{22}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = -\frac{y_{22}}{y_{21}}$$

$$B = \left.\frac{V_1}{(-I_2)}\right|_{V_2=0} = -\frac{1}{y_{21}}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{-\Delta y}{y_{21}}$$

$$V_1 = -V_2 \frac{y_{22}}{y_{21}}$$

$$I_1 = V_2\left(-\frac{y_{22}}{y_{21}}\, y_{11} + y_{12}\right) = \frac{-\Delta y}{y_{21}} \cdot V_2$$

$$D = \left.\frac{I_1}{(-I_2)}\right|_{V_2=0} = \frac{-y_{11}}{y_{21}}$$

$$T = \begin{pmatrix} -(y_A + y_C) & -1 \\ -\Delta y & -(y_A + y_B) \end{pmatrix} \cdot \frac{1}{-y_C}$$

$$\Delta y = (y_A + y_C)(y_A + y_B) - y_C^2$$

c)

$$Z = \begin{pmatrix} \frac{Z_A + Z_B}{2} & \frac{Z_B - Z_A}{2} \\ \frac{Z_B - Z_A}{2} & \frac{Z_A + Z_B}{2} \end{pmatrix}$$

$$\Delta Z = \frac{1}{4}\left[(Z_A + Z_B)^2 - (Z_B - Z_A)^2\right]$$

$$\boxed{T = \begin{pmatrix} \frac{Z_A + Z_B}{Z_B - Z_A} & \frac{\Delta Z \cdot 2}{Z_B - Z_A} \\ \frac{2}{Z_B - Z_A} & \frac{Z_A + Z_B}{Z_B - Z_A} \end{pmatrix}}$$

d)

$$Y = \begin{pmatrix} 0 & 0 \\ -g_m & 0 \end{pmatrix}$$

$$T = \begin{pmatrix} 0 & \frac{1}{g_m} \\ 0 & 0 \end{pmatrix}$$

e)

$$\underline{Z} = \begin{pmatrix} R_1 & 0 \\ -R_2 & 0 \end{pmatrix}$$

$$\boxed{T = \begin{pmatrix} -\frac{R_1}{R_2} & 0 \\ \frac{1}{R_2} & 0 \end{pmatrix}}$$

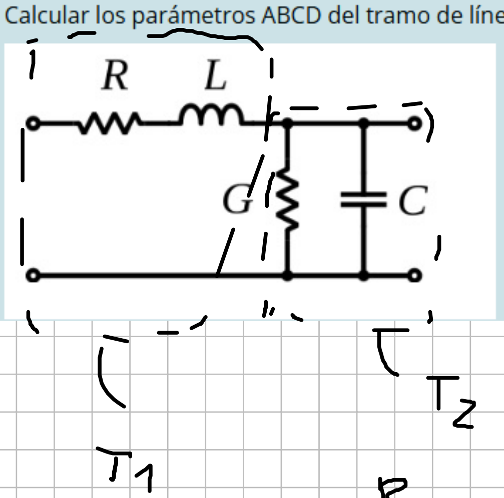
f)

$$T = \begin{pmatrix} N & 0 \\ 0 & -\frac{1}{N} \end{pmatrix}$$

No se pueden obtener parametros Z o Y debido a que es un transformador ideal y no estan definidos

Bonus 1.

Calcular los parámetros ABCD del tramo de línea de transmisión normalizada (R, G, C y L unitatios)



$$V_1 = A_1 V_2 + (-I_2) B_1$$

$$I_1 = C_1 V_2 + D_1 (-I_2)$$

$$A_1 = \left. \frac{V_1}{V_2} \right|_{(-I_2)=0} = 1$$

$$B_1 = \left. \frac{V_1}{(-I_2)} \right|_{V_2=0} = sL + R$$

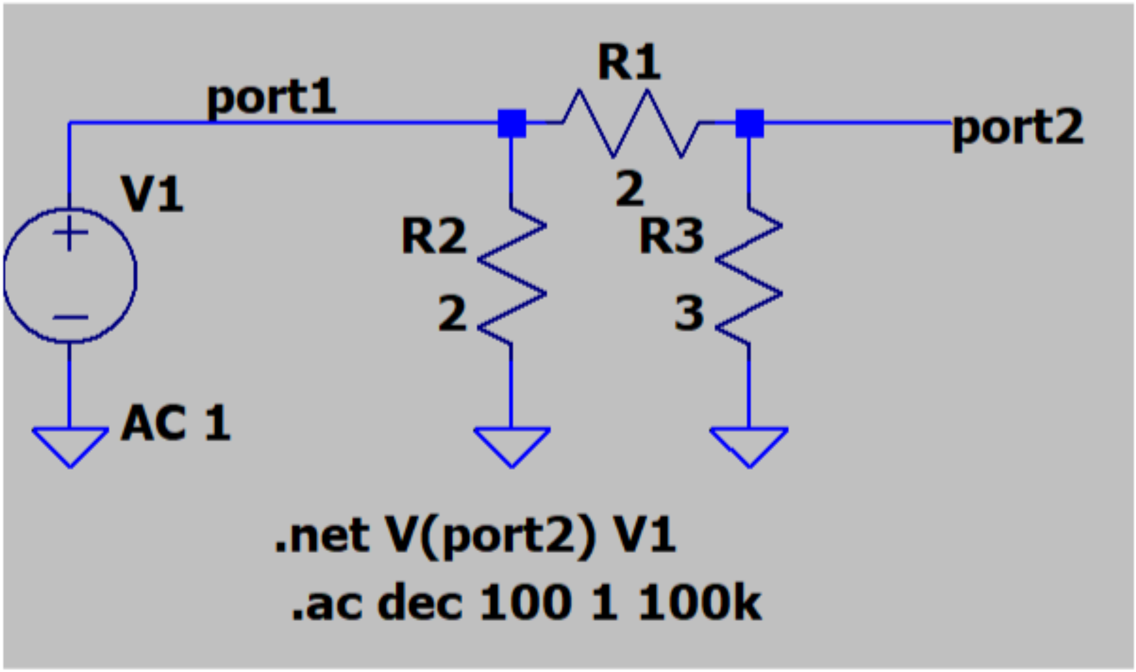
$$C_1 = \left. \frac{I_1}{V_2} \right|_{(-I_2=0)} = 0$$

$$D_1 = \left. \frac{I_1}{(-I_2)} \right|_{(V_2=0)} = 1$$

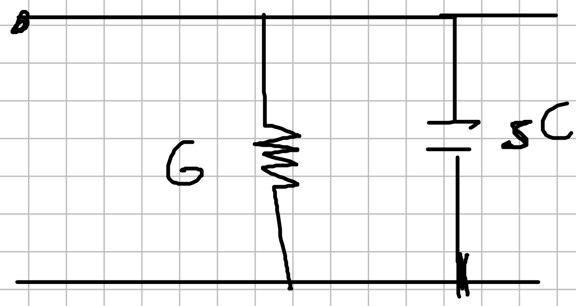
$$T_1 = \begin{pmatrix} 1 & sL+R \\ 0 & 1 \end{pmatrix}$$

Bonus #2

a)



$$Y = \begin{pmatrix} \frac{1}{2}+\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2}+\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 1 & -0{,}5 \\ -0{,}5 & \frac{5}{6} \end{pmatrix} \checkmark$$



$$T_2 = \begin{pmatrix} 1 & 0 \\ G+sC & 1 \end{pmatrix}$$

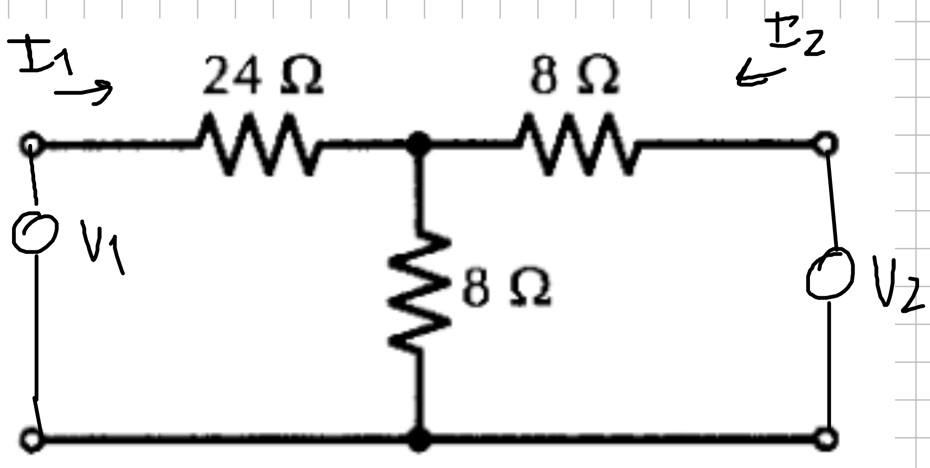
$$T_T = T_1 \cdot T_2 = \begin{pmatrix} 1 & \frac{1}{sL+R} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ G+sC & 1 \end{pmatrix}$$

$$T_T = \begin{pmatrix} (G+sC)(sL+R)+1 & sL+R \\ G+sC & 1 \end{pmatrix}$$

$$(s+1)^2+1$$

$$s^2+2s+1+1$$

$$s^2+2s+2$$

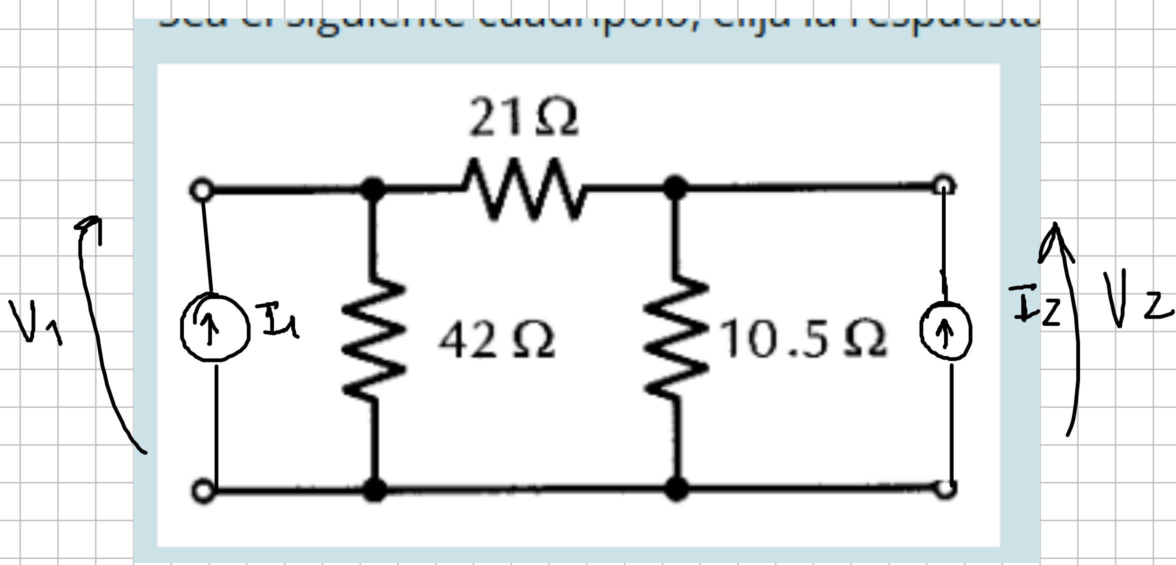


$$\frac{1}{Y_{11}} = 24\Omega + 4\Omega = 28\Omega \rightarrow Y_{11} = \frac{1}{28\Omega}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = \frac{-(8 \,//\, 24)}{8 + (8 \,//\, 24)} \cdot \frac{1}{24\Omega} = -\frac{1}{56\Omega}$$

$$\frac{1}{Y_{22}} = 8\Omega + \underset{(24 // 8)}{6}\Omega = 14\Omega \qquad Y_{22} = \frac{1}{14\Omega}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-\overset{(8 // 8)}{4}}{24+4} \cdot \frac{1}{8} = -\frac{1}{56}$$



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left(42\Omega \,//\, (21+10{,}5\Omega)\right) = 18\Omega$$

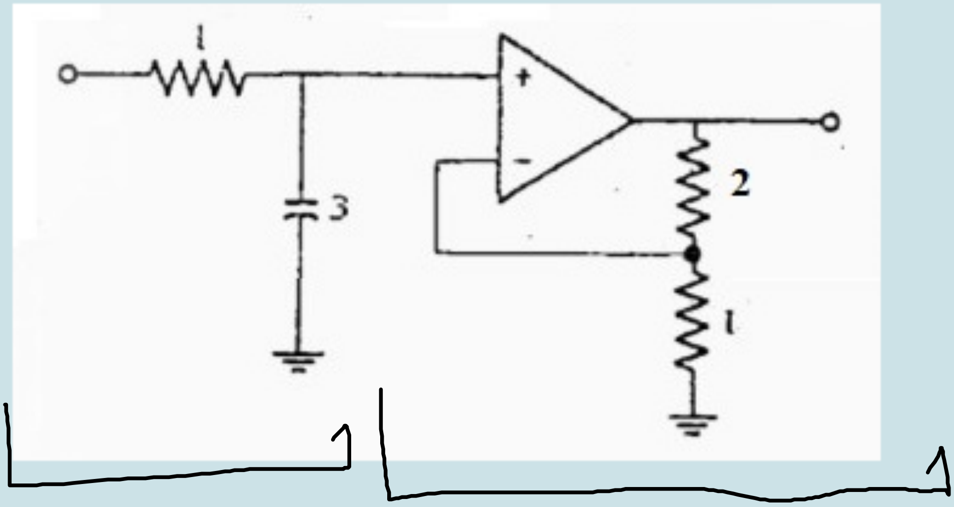
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = {\color{red}*}\ \frac{10{,}5}{10{,}5+21+42}\cdot 42 = 6\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = {\color{red}*}\ \frac{42\Omega}{21\Omega+10{,}5\Omega+42\Omega}\cdot 10{,}5\Omega = 6\Omega$$

$$Z_{22} = \left(10{,}5\Omega \,//\, (21\Omega+42\Omega)\right) = 9\Omega$$

* Divisores de corriente

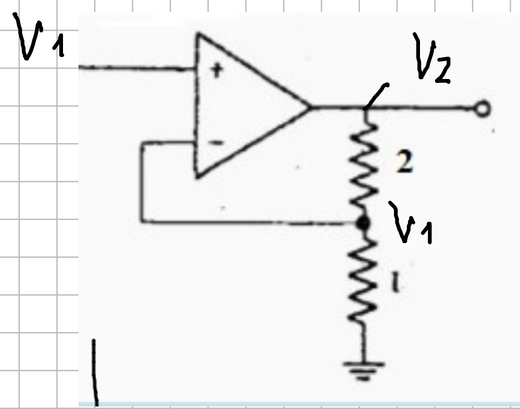
Hallar los parámetros **ABCD** del siguiente circuito activo:



T_1 T_2

$$T_1 = \begin{pmatrix} \dfrac{1 + \frac{1}{S\cdot 3}}{1/S\cdot 3} & 1 \\ \dfrac{1}{S\cdot C} & 1 \end{pmatrix}$$

$$T_1 = \begin{pmatrix} S\cdot 3 + 1 & 1 \\ \dfrac{1}{S\cdot 3} & 1 \end{pmatrix}$$



$$V_2 \cdot \frac{1}{3} = V_1$$

$$\frac{V_1}{V_2} = \frac{1}{3}$$

$$V_1 = V_2 . A + (-I_2) . B$$

$$I_1 = V_2 . C + (-I_2) . D$$

$$\left.\frac{V_1}{V_2}\right|_{(-I_2)=0} = \frac{1}{3}$$

$$\left.\frac{V_1}{(-I_2)}\right|_{V_2=0} = 0$$

$$\left.\frac{I_1}{V_2}\right|_{(-I_2=0)} = \frac{1}{\left.\frac{V_2}{I_1}\right|_{(-I_2=0)}} = 0$$

$$\left.\frac{I_1}{(-I_2)}\right|_{V_2=0} = \frac{1}{\left.\frac{(-I_2)}{I_1}\right|_{V_2=0}} = 0$$

$$T_T = \begin{pmatrix} s\cdot 3+1 & 1 \\ s\cdot 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1/3 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} (s\cdot 3+1)\frac{1}{3} & 0 \\ \frac{1}{3}(s\cdot 3) & 0 \end{pmatrix}$$