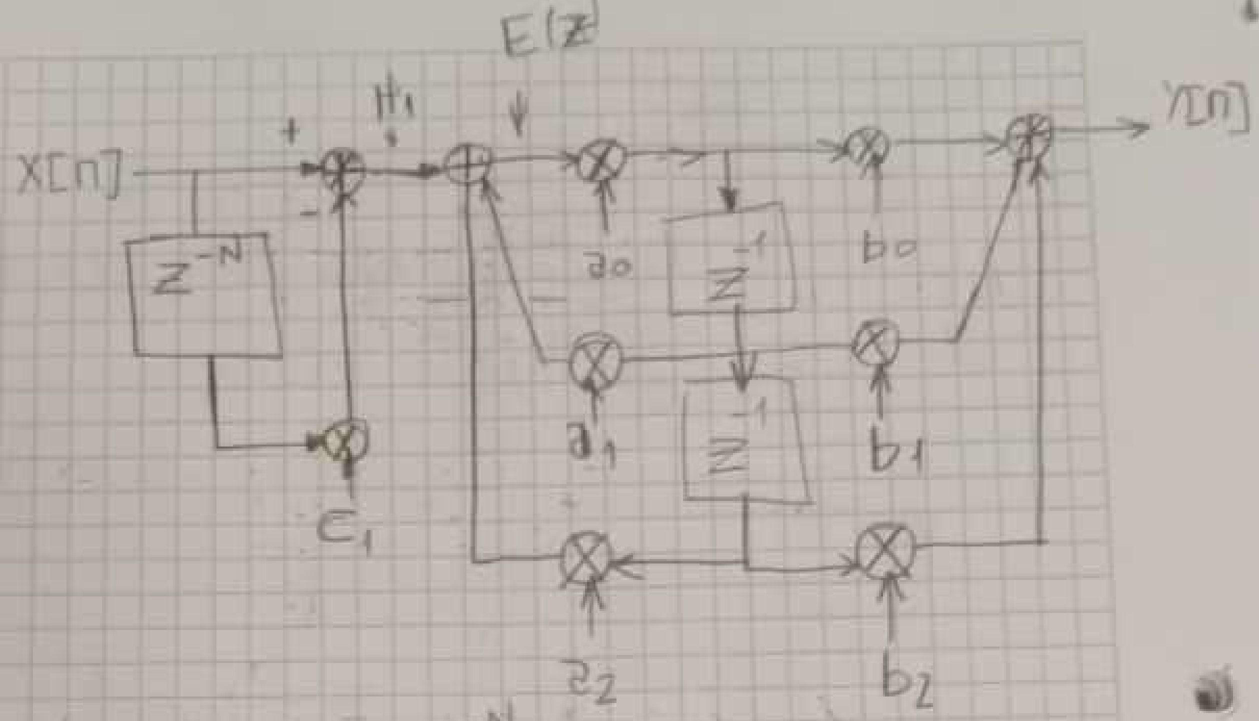
1)



$$H_1(z) = X(z) - z^{-N} c_1 X(z)$$

$$H_1(z) = X(z)(1 - z^{-N} c_1)$$

$$Y(z) = E(z) a_0 \left[b_0 + z^{-1} b_1 + z^{-2} b_2 \right]$$

$$E(z) = H_1(z) + E(z) a_0 \left[a_1 z^{-1} + a_2 z^{-2} \right]$$

$$E(z) = \frac{H_1(z)}{1 - a_0 a_1 z^{-1} - a_2 z^{-2}}$$

$$Y(z) = H_1(z) \frac{a_0 \left[b_0 + z^{-1} b_1 + z^{-2} b_2 \right]}{1 - a_0 a_1 z^{-1} - a_2 a_0 z^{-2}}$$

NOTA

$$Y(z) = H_1(z)\,\frac{b_0 + z^{-1} b_1 + z^{-2} b_2}{\frac{1}{a_0} - a_1 z^{-1} - a_2 z^{-2}}$$

$$Y(z) = X(z)\left(1 - z^{-N} C_1\right)\frac{b_0 + z^{-1} b_1 + z^{-2} b_2}{\frac{1}{a_0} - a_1 z^{-1} - a_2 z^{-2}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + z^{-1} b_1 + z^{-2} b_2}{\frac{1}{a_0} - a_1 z^{-1} - a_2 z^{-2}}\left(1 - z^{-N} C_1\right)$$

lqqd (Lo que queda demostrado)

2)

$$H(z) = \frac{\frac{1}{N}}{1 - z^{-1}} \left(1 - z^{-N}\right)$$

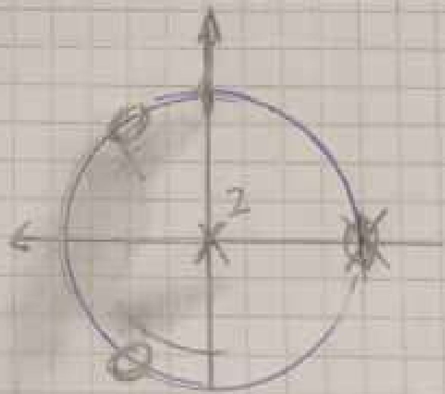
$$H(z) = \frac{1 - z^{-N}}{1 - z^{-1}} \, \frac{1}{N}$$

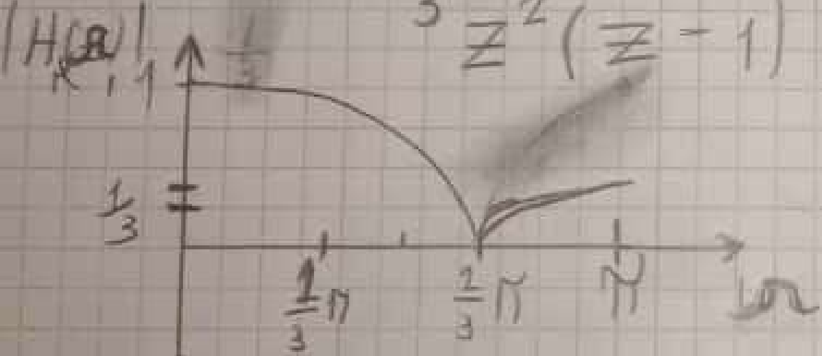
$$H(z) = \frac{\left(z^{N} - 1\right) z}{z^{N}\left(z - 1\right)} \, \frac{1}{N}$$

$$H(z) = \frac{\left(z^{N} - 1\right)}{z^{N-1}\left(z - 1\right)} \cdot \frac{1}{N}$$

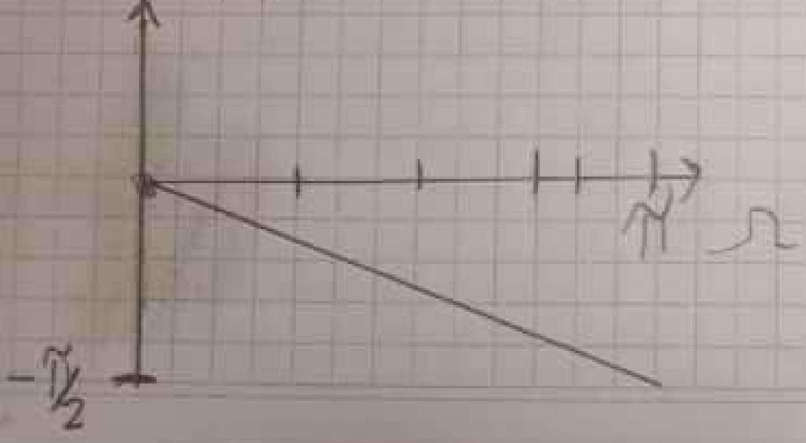
$N = 3$

$$H(z) = \frac{1}{3} \, \frac{z^{3} - 1}{z^{2}\left(z - 1\right)}$$





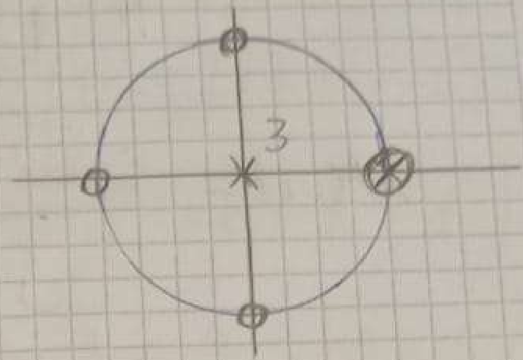
$e^{-j\frac{\Omega}{2}}$

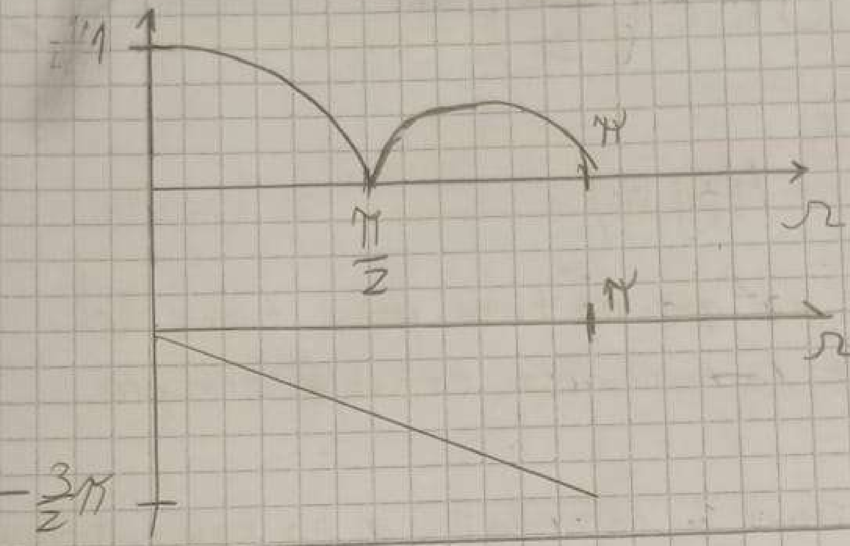


IGNORO LOS SALTOS DE FASE DE $H_R(\Omega)$

N = 4

$$H_R(z) = \frac{1}{4}\,\frac{z^4 - 1}{(z-1)z^3}$$

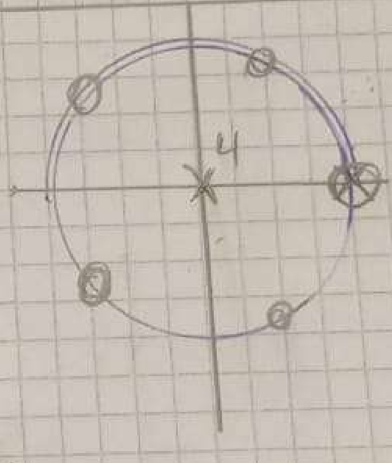


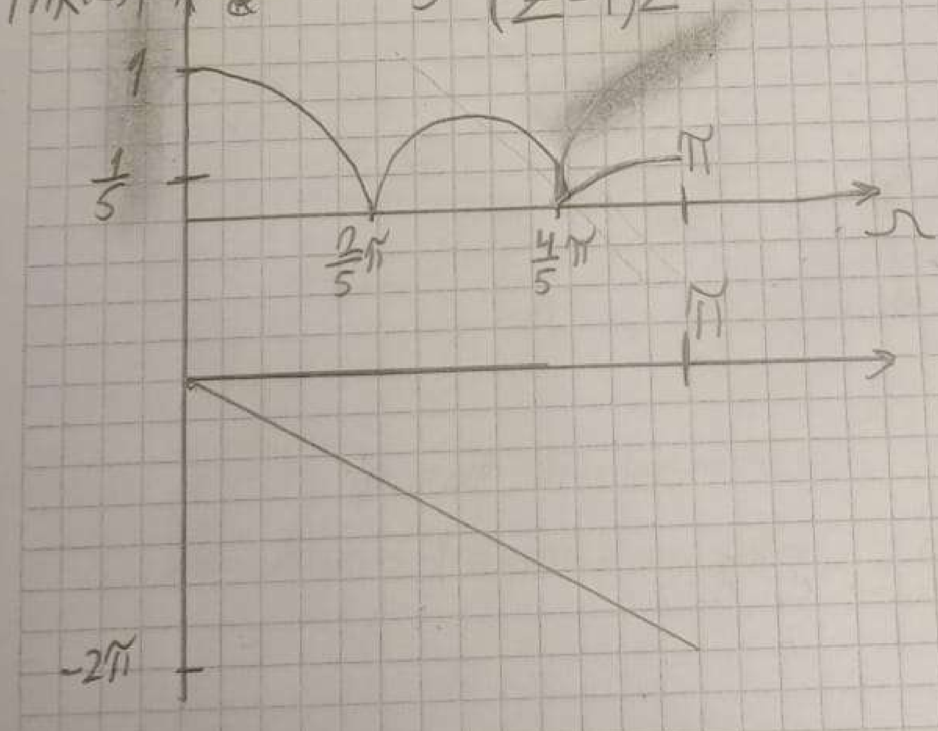


N = 5

$|H_R(z)|$

$$H_R(z) = \frac{1}{5}\,\frac{z^5 - 1}{(z-1)z^4}$$





$$H(\Omega) = \frac{1}{N}\,\frac{1-e^{-jN\Omega}}{1-e^{-j\Omega}} = \frac{1}{N}\,\frac{e^{j\frac{N\Omega}{2}}-e^{-jN\frac{\Omega}{2}}}{e^{j\frac{\Omega}{2}}-e^{-j\frac{\Omega}{2}}}\;\frac{e^{-jN\frac{\Omega}{2}}}{e^{-j\frac{\Omega}{2}}}\;\frac{\cancel{N}\frac{\Omega}{2}}{N\frac{\Omega}{2}}$$

$$H(\Omega) = \underbrace{\frac{\operatorname{Sinc}\left(\frac{N\Omega}{2}\right)}{\operatorname{Sinc}\left(\frac{\Omega}{2}\right)}}_{H_R(\Omega)} \cdot e^{-j(N-1)\frac{\Omega}{2}}$$

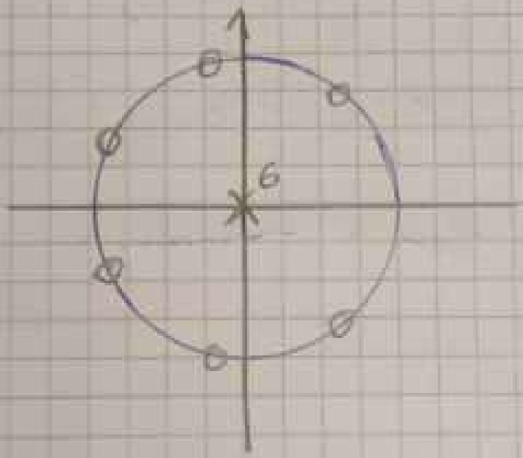
b) Es FIR ya que no tiene polos fuera del origen, por lo tanto es estable siempre

c) Son solo sumas y retardos y no hay multiplicaciones

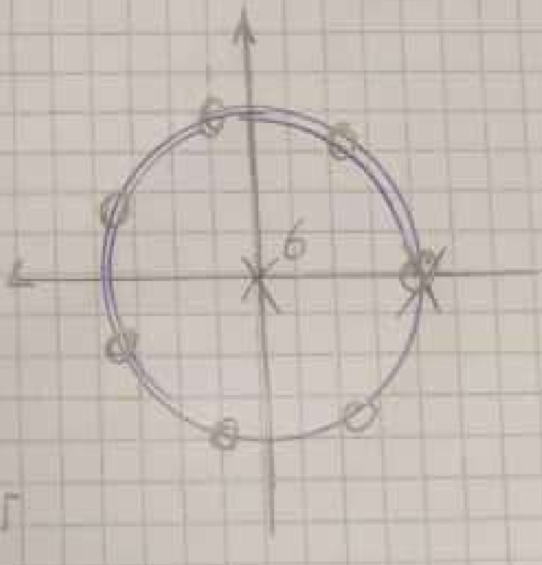
2) c) $H(k) = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$
$+ \delta[n-4] + \delta[n-5] + \delta[n-6]$

$$H(z) = \left(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}\right)$$

$$H(z) = \frac{z^6 + z^5 + z^4 + z^3 + z^2 + z + 1}{z^6}$$



$$H(z) = \frac{z^7 - 1}{z^6 (z-1)} \quad \frac{1}{7}$$



Si, se puede implementar
~~H~~ Mismo diagrama de polos y ceros

3) $y[n] = \frac{1}{2}\left[x(n) - x(n-2)\right]$

$$Y(z) = \frac{1}{2}\left[X(z) - z^{-2}X(z)\right]$$

$$\frac{Y(z)}{X(z)} = \frac{1}{2}\left(1 - z^{-2}\right) = \frac{1}{2}\,\frac{z^{2}-1}{z^{2}}$$

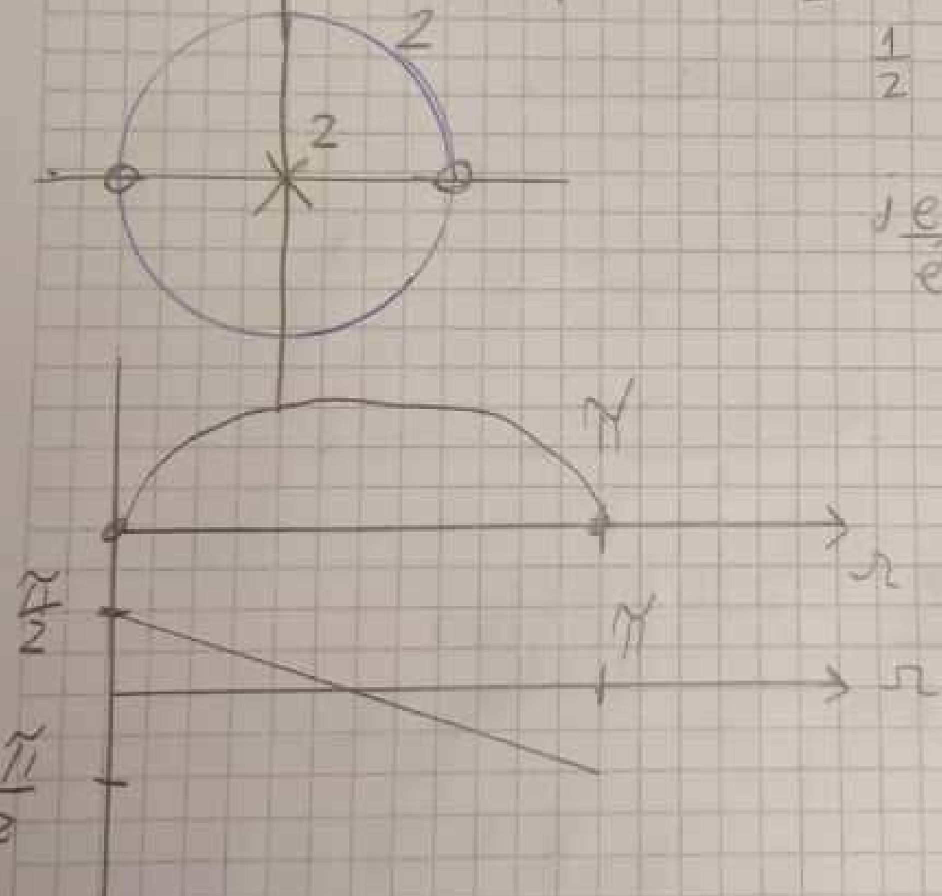
Usando solo el bloque recursivo

$a_0 = 1 \quad a_1 = 0 \quad a_2 = 0$

$b_0 = \frac{1}{2} \quad b_1 = 0 \quad b_2 = 0$

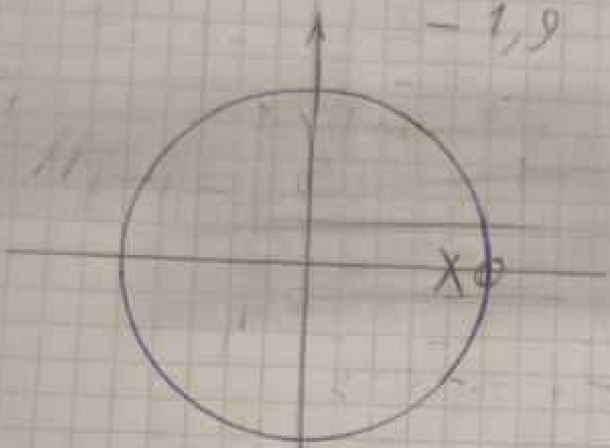
$$\frac{1}{2}\,\frac{e^{j2\Omega}-1}{e^{j2\Omega}} = \frac{1}{2}e^{j\Omega}\,\frac{e^{j\Omega}-e^{-j\Omega}}{e^{j2\Omega}}$$

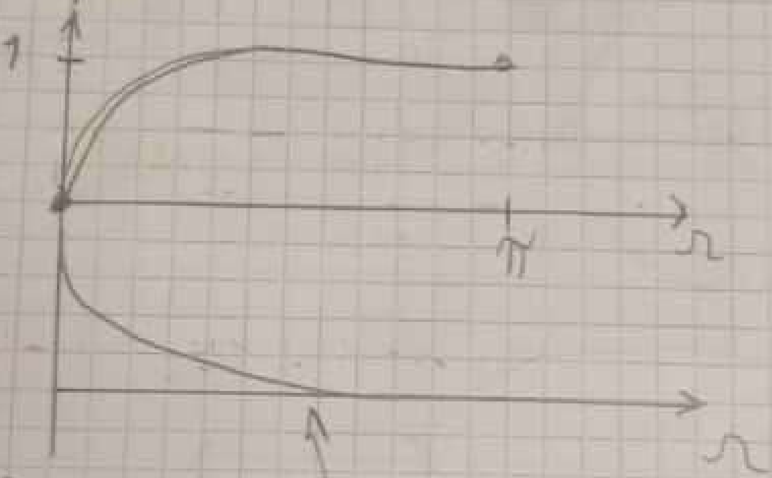
$$j\frac{e^{j\Omega}}{e^{j2\Omega}}\sin(\Omega) = \sin(\Omega)\,e^{j\left(\frac{\pi}{2}-\Omega\right)}$$



4) a) $H(z) = \dfrac{1 - z^{-1}}{1 - \beta z^{-1}} = \dfrac{z - 1}{z - \beta} = \dfrac{z - 1}{z - 0{,}9}$

$H(z=-1) = \dfrac{-2}{-1{,}9} = \dfrac{20}{19}$





$H(\Omega) = \dfrac{e^{j\Omega} - 1}{e^{j\Omega} - 0{,}9}$

python

b) $H(\Omega) = \dfrac{1-e^{-j\Omega}}{1-\beta e^{-j\Omega}}$

$$|H(\Omega)|^2 = \frac{|1-e^{-j\Omega}|^2}{|1-\beta e^{-j\Omega}|^2} = \frac{2-(e^{j\Omega}+e^{-j\Omega})}{1-\beta(e^{j\Omega}+e^{-j\Omega})+\beta^2}$$

$$|H(\Omega)|^2 = \frac{2(1-\cos(\Omega))}{1-2\beta\cos(\Omega)+\beta^2}$$

$$|H(\pi)|^2 = \frac{4}{1+2\beta+\beta^2}$$

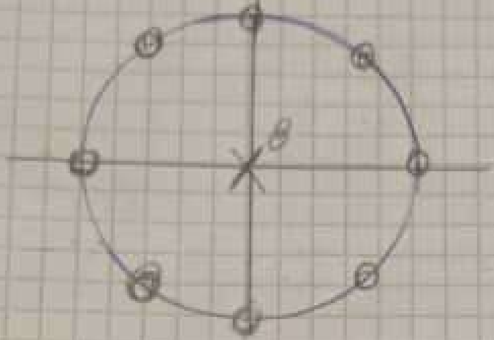
$$|H(0{,}1\pi)|^2 = \frac{0{,}09788696741}{1-2\beta\cos(0{,}1\pi)+\beta^2}$$

$$3\,dB = 10\log\left(\frac{|H(\pi)|^2}{|H(0{,}1\pi)|^2}\right)$$

Resuelto numericamente con la calculadora

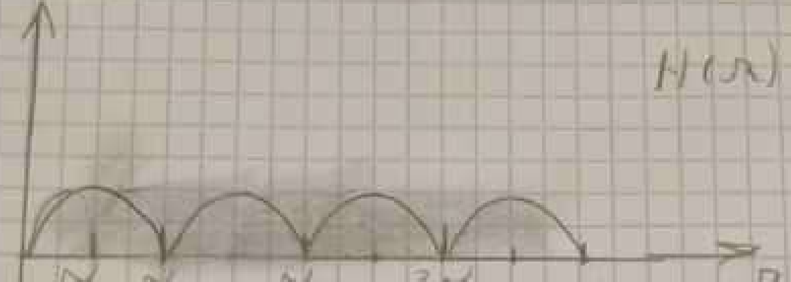
$$\beta = 0{,}7271025814$$

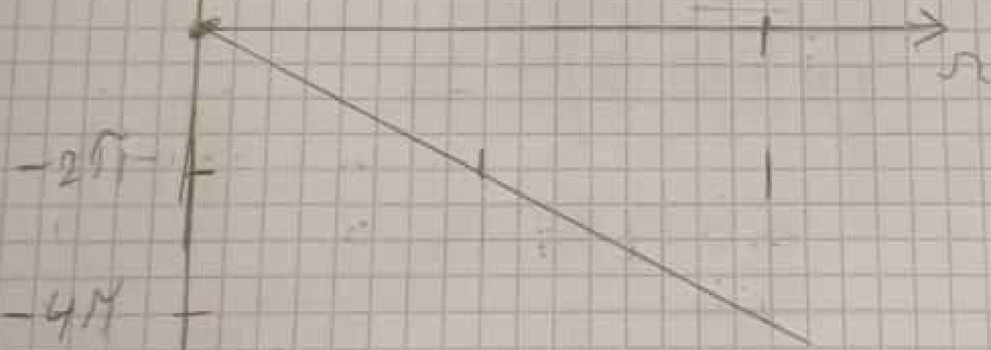
5) $H(z) = 1 + z^{-8} = \dfrac{z^{8}+1}{z^{8}}$



$H(\Omega) = \dfrac{e^{j\Omega 8}+1}{e^{j\Omega 8}}$

$H(\Omega) = \dfrac{e^{4j\Omega}\cos(4\Omega)\cdot 2}{e^{j\Omega 8}}$





b) $f_s = 8KHz \rightarrow \pi \rightarrow \dfrac{F_s}{2}$

$\dfrac{\pi}{4}\cdot m \rightarrow \underbrace{\dfrac{F_s}{8}\cdot m}_{\text{NULOS}}$ $(m = 0,1,2,3,4)$

$\pi \rightarrow f_s/2$

$\dfrac{\pi}{8}\cdot(2m+1) \rightarrow \underbrace{\dfrac{F_s}{16}(2m+1)}_{\text{MAXIMOS}}$ $(m = 0,1,2,3)$

Solo faltaría reemplazar numericamente.

Bonus #2

$$H(s) = \frac{s-\alpha}{s+\alpha} \longrightarrow \frac{K\frac{z-1}{z+1} - \alpha}{K\frac{z-1}{z+1} + \alpha}$$

$$s = K\,\frac{z-1}{z+1}$$

$$H(z) = \frac{K(z-1) - \alpha(z+1)}{K(z-1) + \alpha(z+1)}$$

$$H(z) = \frac{(K-\alpha)z - (K+\alpha)}{(K+\alpha)z + (\alpha - K)}$$

$$\boxed{H(z) = \frac{K-\alpha}{K+\alpha}\;\frac{z - \frac{K+\alpha}{K-\alpha}}{z + \frac{\alpha-K}{K+\alpha}}}$$

$K = 2fs$