1st Students Dynamic Macro Workshop

Business Cycles Analysis

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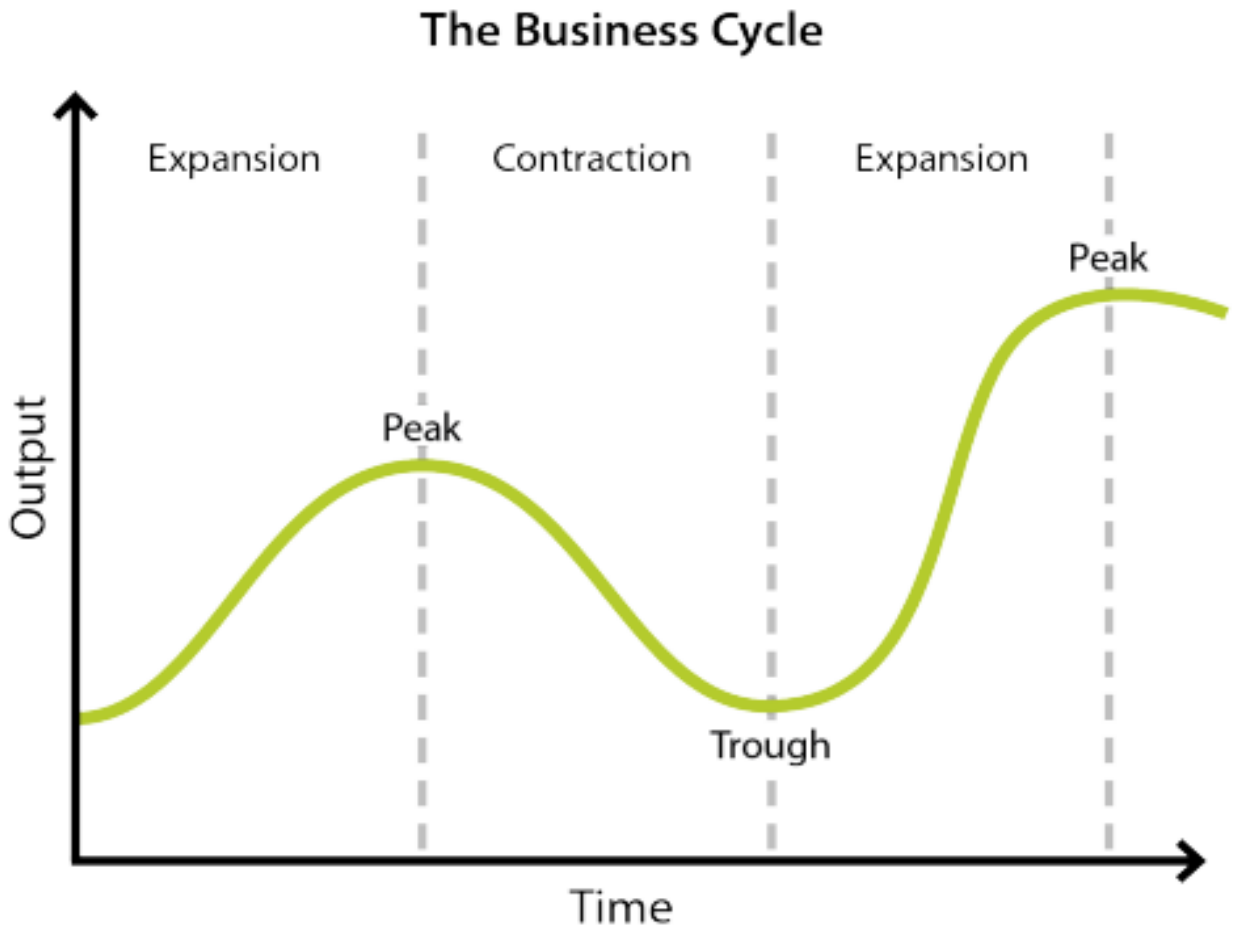
The Business Cycle

- Business cycles represent fluctuations in economic activity over the trend over time.

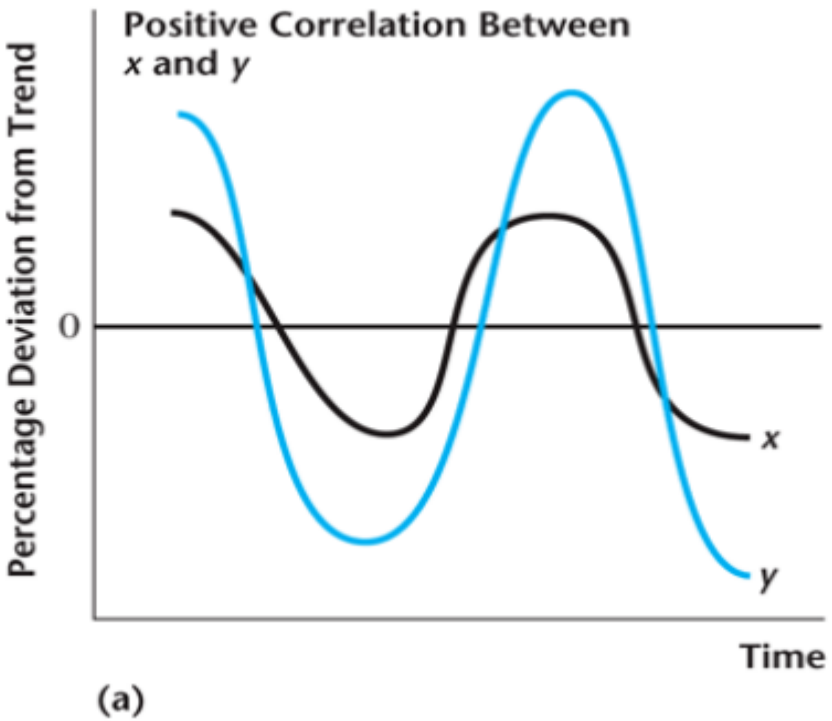


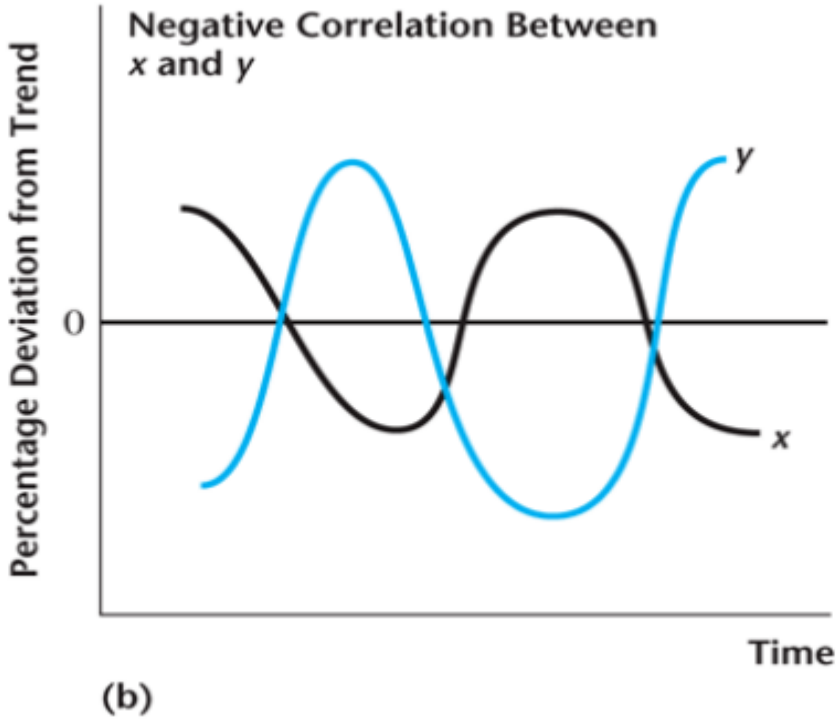
Analysis of the Cycle

We are interested in how the cycle of a specific variable behaves in relation to the cycle of real GDP:

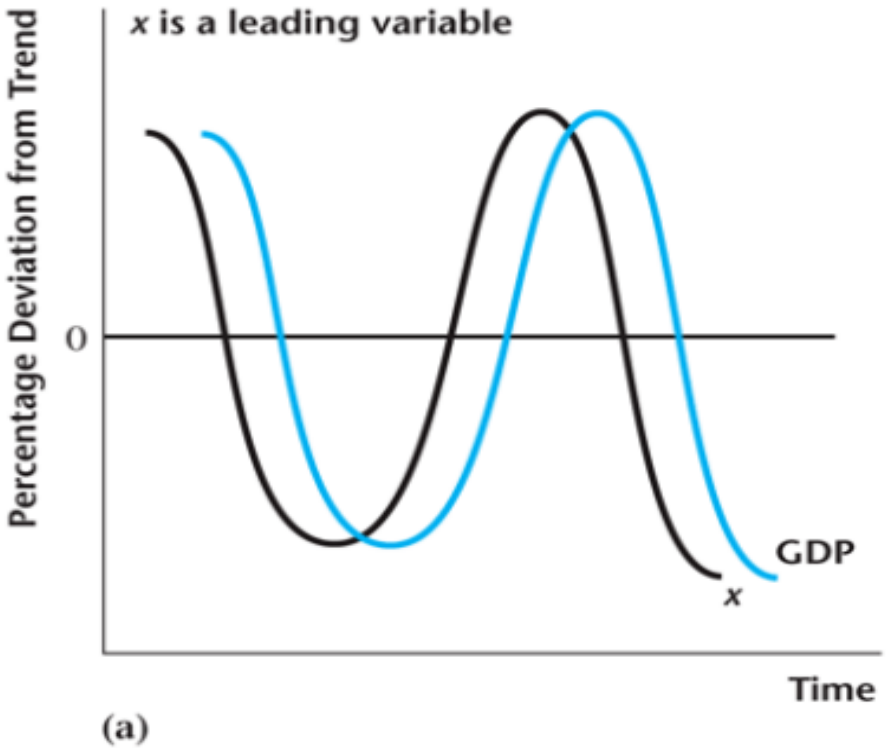
1. **Correlation**: procyclical, countercyclical, or acyclical
2. **Timing**: leading, lagging, or coincident
3. **Relative Volatility**

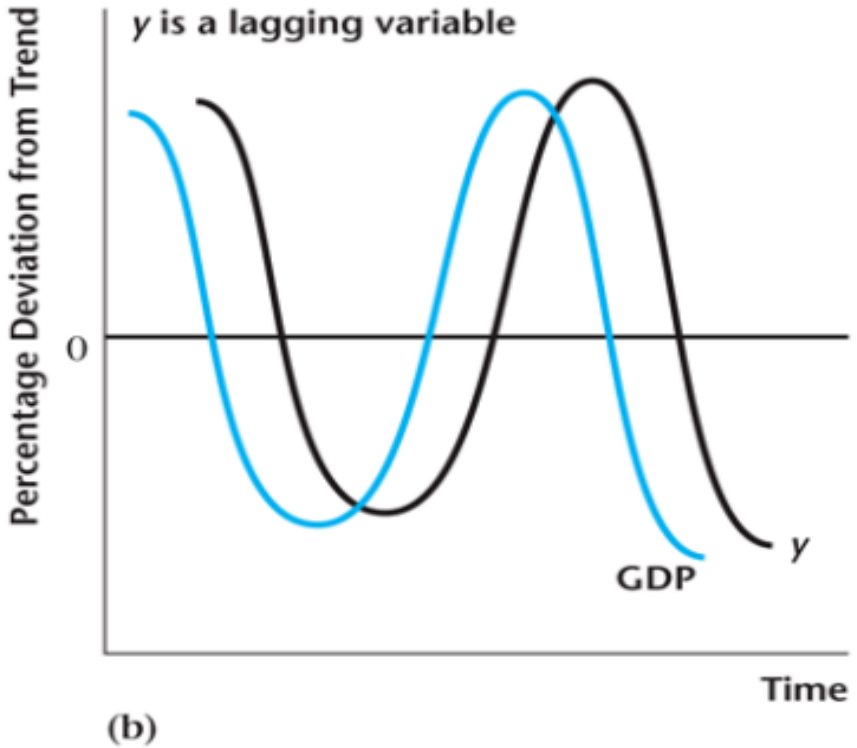
Procyclical vs Countercyclical





Leading vs Lagging





Stylized Facts

Kydland and Prescott (1990) report stylized business cycle facts for the US:

Variable	**Correlation**	**Timing**	**Relative Volatility**
Consumption	procyclical	coincident	smaller
Investment	procyclical	coincident	larger
Money Supply	procyclical	leading	smaller
Unemployment	countercyclical	lagging	smaller

Time Series Detrending

Any time series $y_t = \ln Y_t$ can be decomposed such that

$$y_t = y_t^T + y_t^C$$

where y_t^T is the trend and y_t^C is the cycle.

Thus, the cycle can be written as:

$$y_t^C = y_t - y_t^T = \ln Y_t - \ln Y_t^T$$

$$= \ln\left(\frac{Y_t}{Y_t^T}\right) = \ln\left(1 + \left(\frac{Y_t}{Y_t^T} - 1\right)\right)$$

$$= \ln\left(1 + \left(\frac{Y_t - Y_t^T}{Y_t^T}\right)\right) \approx \frac{Y_t - Y_t^T}{Y_t^T}$$

Detrend in practice

- **First differences in logs**
  - $\Delta y_t \equiv \ln Y_t - \ln Y_{t-1}$
- **Filters**
  - Hodrick–Prescott filter
  - Band-Pass filter (e.g. Baxter-King)

The Hodrick-Prescott Filter

- Originated in the engineering literature, where it is known as the Whitaker–Henderson filter.
- Very popular in the macro literature
- The idea is to generalize the linear trend and allow the slope of the "trend" to vary — but not too much.

Mathematics of the Hodrick-Prescott Filter

$$\min_{\{y_t^T\}_{t=1}^T} \sum_{t=1}^{T}(y_t - y_t^T)^2 + \lambda \sum_{t=2}^{T-1} \left((y_{t+1}^T - y_t^T) - (y_t^T - y_{t-1}^T)\right)^2$$

- The first term, $\sum_{t=1}^{T}(y_t - y_t^T)^2$, penalizes deviations of the trend Y_t from the actual data y_t, ensuring that y_t^T stays close to y_t.

- The second term, $\lambda \sum_{t=2}^{T-1} \left((y_{t+1}^T - y_t^T) - (y_t^T - y_{t-1}^T)\right)^2$, penalizes changes in the slope of y_t^T, enforcing smoothness in the trend.

  - $(y_{t+1}^T - y_t^T) \approx \frac{Y_{t+1}^T - Y_t^T}{Y_t^T}$: growth rate of the trend between periods t+1 and t

  - $(y_t^T - y_{t-1}^T) \approx \frac{Y_t^T - Y_{t-1}^T}{Y_{t-1}^T}$: growth rate of the trend between periods t and t-1

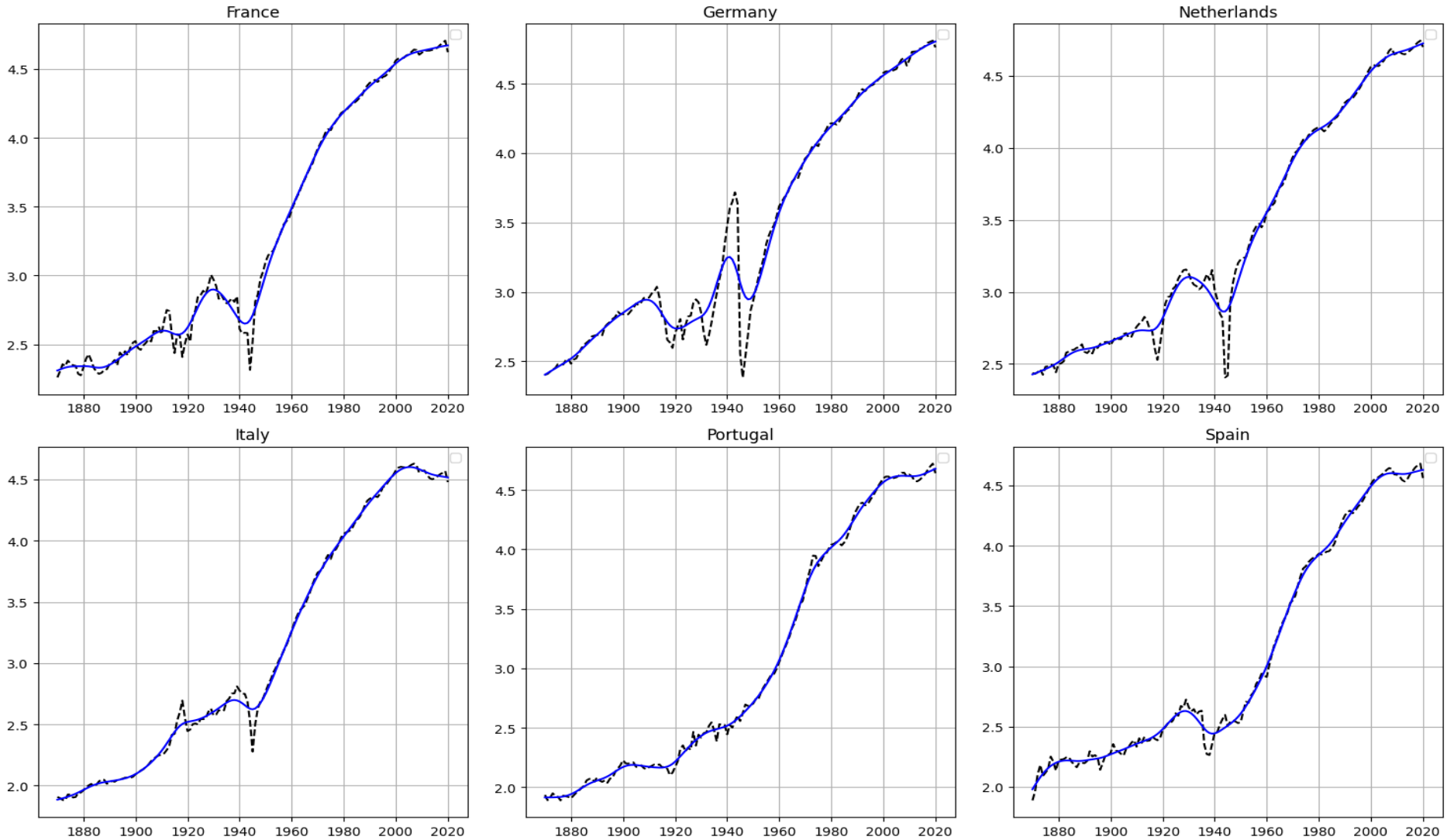
The role of λ

- λ is the **smoothing parameter**
- Measures the relative weight attached to smoothness as against close tracking.
- **Trade-off:**
  - Closeness to Data: how closely the trend T_t fits the actual data y_t.
  - Smoothness of Trend: how gradual the changes in T_t are over time.
- **High λ:** smoother trend by restricting fluctuations
  - $\lambda \to \infty$, the trend approaches a linear trend
- **Low λ:** more volatile trend
  - $\lambda = 0$, the trend is equal to the original series ($y_t = y_t^T, y_t^C = 0$)

Macro History Database

- Released by **MacroFinance and MacroHistory Lab** hosted at the **Kiel Institute for the World Economy** and affiliated with the ECONtribute Excellence Cluster.
- Covers 18 advanced economies since 1870 on an annual basis.
- Comprises 48 real and nominal variables.

Figure: GDP Trend



The black dotted line is the log of real GDP per capita and the blue line is the trend obtained from the HP filter for $\lambda = 100$

Figure: GDP Cycle

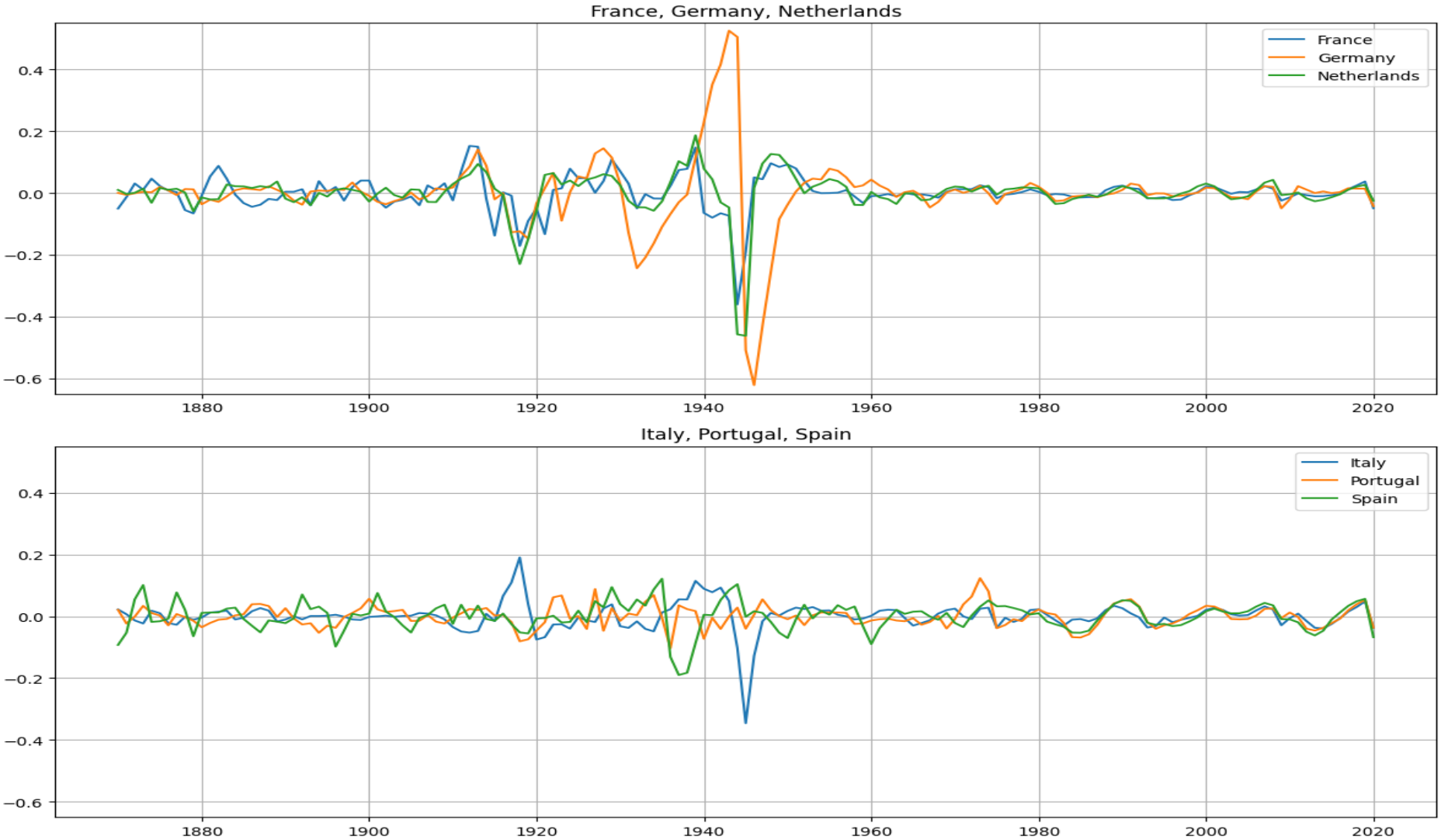
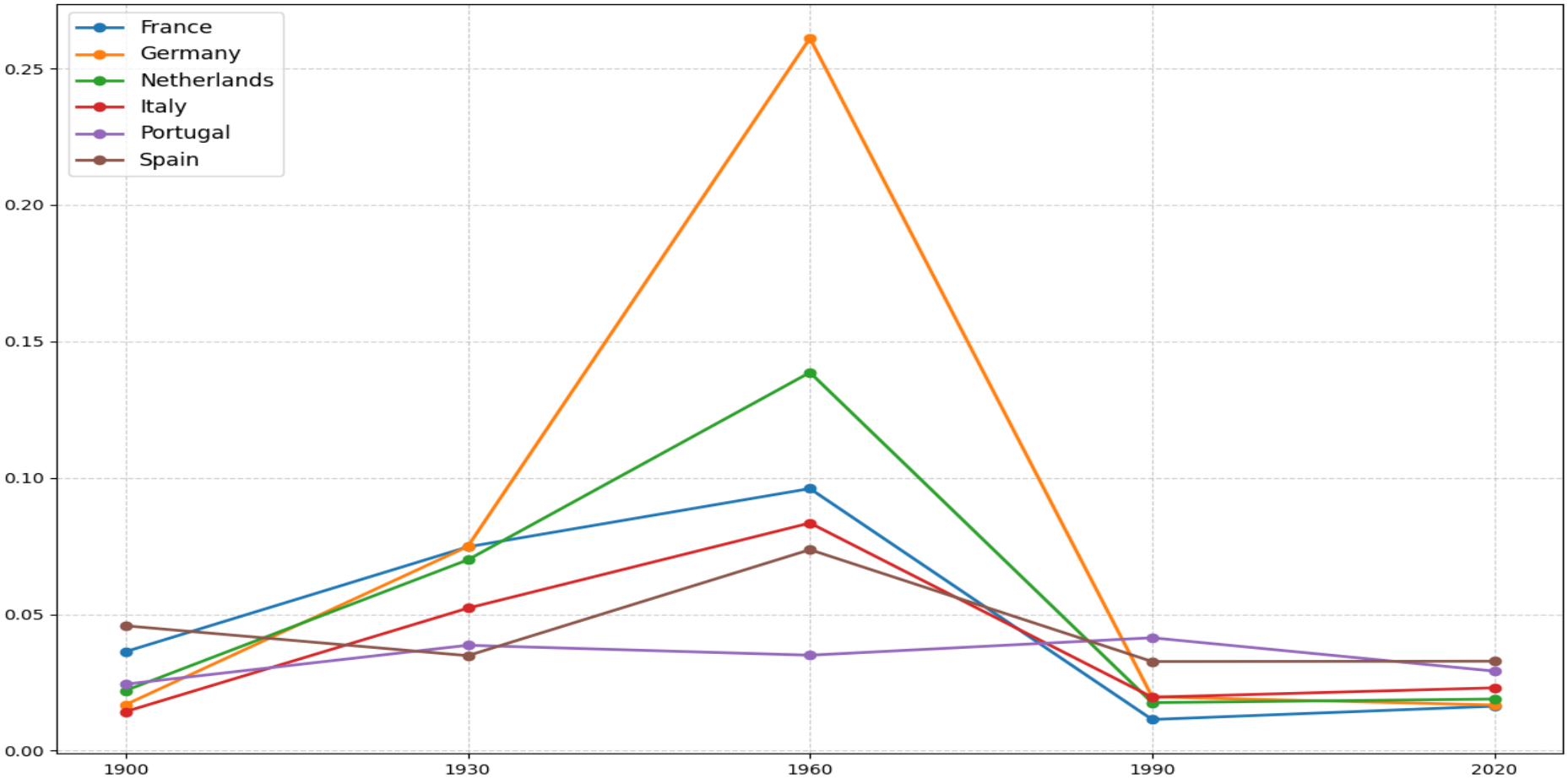


Table: Volatility of the Cycle of GDP

Country	**Volatility**
France	0.0567
Germany	0.1204
Netherlands	0.0699
Italy	0.0458
Portugal	0.0339
Spain	0.0463

Figure: GDP Cycle Volatility Per 30 Years Period



I divide 150 years (1870–2020) into five 30-year periods, using the final year of each period as x-axis ticks, and calculate cycle volatility for each.

Table: Volatility relative to GDP, 2000-2020

Country	Consumption	Investment
France	0.7494	1.7274
Germany	0.7142	1.7136
Netherlands	0.8957	2.8870
Italy	0.9893	1.4946
Portugal	1.1890	2.5844
Spain	1.0671	2.4610

References

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