RBC Models in Dynare

Advanced Macroeconomics Tutorials

Matteo Coronese

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Scuola Superiore Sant'Anna

INSTITUTE
OF ECONOMICS

Scuola Superiore
Sant'Anna

Planned Schedule

During last lecture, we learned how to use *Dynare*. In particular, we learned

- The basic structure of the mod file
- How to simulate a very simple RBC model
- Main commands to obtain stochastic simulation results, IRFs, variables moments and correlation
- Basic interpretation of the results

Today, we proceed by

- Simulating a slightly more complicated RBC model
- Deeper understanding of simulation results
- Experiment with different model setups

An RBC Model with Labor

Consider the following optimization problem:

$$\begin{aligned} \max_{c_t, n_t} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma} - 1}{1 - \sigma} - \chi n_t \right] \\ \text{s.t.} \quad & k_{t+1} = a_t k_t^{\alpha} n_t^{1-\alpha} - c_t + (1 - \delta) k_t \end{aligned} \tag{1}$$

With the representative consumer deriving (quasi-linear) utility from consumption c_t and leisure (that's why the minus sign in front of labor n_t). The meaning of the law of motion of capital is the usual one: $k_{t+1} = i_t + (1 - \delta)k_t$. It follows that

$$\begin{aligned} y_t =& a_t k_t^{\alpha} n_t^{1-\alpha} \\ i_t =& y_t - c_t \end{aligned} \tag{2}$$

Finally, TFP is assumed to follow a mean zero AR(1) in logarithms:

$$\ln a_t = \rho \ln a_{t-1} + \varepsilon_t \tag{3}$$

An RBC Model with Labor II

FOCs with respect to consumption and labor yield

$$c_t^{-\sigma} = \beta E_t \left[c_{t+1}^{-\sigma} \left(\alpha a_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + 1 - \delta \right) \right] \tag{4}$$

$$\chi = c_t^{-\sigma} (1-\alpha) a_t k_t^{\alpha} n_t^{-\alpha} \tag{5}$$

Eq. 4 is a standard Euler equation. The meaning of Eq. 5 is straightforward:

$$U_{n,t} = w_t U_{c,t}$$

The marginal rate of substitution between leisure and consumption equals the real wage rate w_t. In fact, labor supply, arising from consumer maximization, can be written as

$$\chi = w_t c_t^{-\sigma} \tag{6}$$

while labor demand is characterized by the equality between marginal productivity of labor MPL and its remuneration, i.e. the wage:

$$w_t = \frac{\partial y_t}{\partial n_t} = (1-\alpha) a_t k_t^{\alpha} n_t^{-\alpha} \tag{7}$$

Combining Eq. 6 and 7 yields Eq. 5, which thus characterizes labor market equilibrium.

An RBC Model with Labor III

Linear dis-utility of labor is a way to introduce **indivisible labor** (see Hansen 1985).

- In the US, about two thirds of the fluctuation in labor supply comes from changes along the extensive margin (movement into and out of employment) rather than along the intensive margin (changes in hours when employed).
- Partially fix the problem of too small fluctuations in worked ours generated by basic RBCs.
- Elasticity of labor supply is infinite (stronger effects of changes in productivity on labor supply).
- Labor supply curve is perfectly horizontal. This allows us to get a bigger increase in labor hours (and a smaller increase in wages) after the productivity shock.
- Wages becomes less pro-cyclical.

An RBC Model with Labor IV

The full model can thus be written as a system of five equations in five variables:

$$c_t^{-\sigma} = \beta E_t \left[c_{t+1}^{-\sigma} (\alpha a_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + 1 - \delta) \right] \quad \text{Euler Equation}$$

$$\chi = c_t^{-\sigma} (1-\alpha) a_t k_t^{\alpha} n_t^{-\alpha} \quad \text{Labor Equation}$$

$$k_{t+1} = y_t - c_t + (1-\delta) k_t \quad \text{Capital Accumulation}$$

$$y_t = a_t k_t^{\alpha} n_t^{1-\alpha} \quad \text{Production Function}$$

$$\ln a_t = \rho \ln a_{t-1} + \varepsilon_t \quad \text{Exogenous Process}$$

Which can be plugged into *Dynare*:

```
model;
exp(c)^-sigma=beta*exp(c(+1))^-sigma*(alpha*exp(a(+1))*exp(k)^(alpha-1)*exp(n(+1))^(1-alpha)+1-delta);
chi = exp(c)^-sigma*(1-alpha)*exp(a)*exp(k(-1))^alpha*exp(n)^-alpha;
exp(c)+exp(k) = exp(a)*exp(k(-1))^alpha*exp(n)^(1-alpha)+(1-delta)*exp(k(-1));
exp(y) = exp(a)*exp(k(-1))^alpha*exp(n)^(1-alpha);
a = rho*a(-1)+z;
end;
```

Solving the Model - Details

Note that, as seen in the previous lecture, we lagged capital in our code. *Dynare* assumes as predetermined variables those variables which are “stocked are the end of the period”. Why is the distinction between predetermined and control variables relevant?

The linearized model can be expressed, in the so-called **space form**, as a system of forward looking difference equations:

$$\Pi_0 E_t \mathbf{y_{t+1}} = \Pi_1 \ \mathbf{y_t} + \Pi_2 \mathbf{Z_t} \tag{8}$$

There are several ways to solve this type of rational expectation models. We use the Blanchard-Kahn (1980) method, which leverages on the distinction between predetermined and control variables.

Forward Looking Solution

The non-predetermined variables depend on the past only through its effect on the current predetermined variables

Blanchard-Kahn (1980)

Predetermined and Control Variables

So we aim at explaining the evolution of the model in terms of predetermined variables and current shocks. What's the difference between predetermined (or state) variables S_t and non-predetermined variables or control) variables x_t?

Recall

Rational expectations: $E_t X_{t+1} = E(X_{t+1}|\Omega_t)$ where Ω_t is the information set at t. Agents make their best forecast (mathematical expectation) given the available information.

The difference between predetermined and non-predetermined variables is extremely important. A predetermined variable is a function only of variables known at time t, that is of variables in Ω_t, so that $S_{t+1} = E_t S_{t+1}$ whatever the realization of the variables in Ω_{t+1}. A non-predetermined variable X_{t+1}, can be a function of any variable in Ω_{t+1}, so that we can conclude that $X_{t+1}, = E_t X_{t+1}$ only if the realizations of all variables in Ω_{t+1} are equal to their expectations conditional on Ω_t

Blanchard-Kahn (1980)

In other words, predetermined variables do not respond to $S_t - E_{t-1} S_t$.

Predetermined and Control Variables

Rewriting the space form partitioning between predetermined (or state) variables S_t and non-predetermined (or control) variables X_t:

$$\Pi_0 \begin{bmatrix} S_{t+1} \\ E_t X_{t+1} \end{bmatrix} = \Pi_1 \begin{bmatrix} S_t \\ X_t \end{bmatrix} + \Pi_2 Z_t$$

if Π_1 is invertible, we can write

$$\begin{bmatrix} S_{t+1} \\ E_t X_{t+1} \end{bmatrix} = \Phi_1 \begin{bmatrix} S_t \\ X_t \end{bmatrix} + \Phi_2 Z_t$$

with $\Phi_1 = \Pi_0^{-1}\Pi_1$ and $\Phi_2 = \Pi_0^{-1}\Pi_2$.

Necessary Condition

If the number of eigenvalues of Φ_1 lying outside the unit circle (unstable roots) are equal the number of non-predetermined variables (jumpers), then a unique solution exists.

If too many unstable roots, then no solution. If too few, then infinite number of solutions. In *Dynare*, such condition is checked through the command check.

Understanding Transition and Policy Functions

The state space representation of the solution is thus

$$S_t = AS_{t-1} + BZ_t \tag{9}$$

$$X_t = \Gamma S_{t-1} \tag{10}$$

Dynare shows the solution in a slightly different form. Plug Eq. 9 into Eq. 10, obtaining $X_t = \Gamma AS_{t-1} + \Gamma BX_t$, defining $C = \Gamma A$ and $D = \Gamma B$.

Dynare prints results in the following form:

$$S_t = AS_{t-1} + BZ_t \tag{11}$$

$$X_t = CS_{t-1} + DZ_t \tag{12}$$

- Transition functions: how the period t values of the state variables depend on $t-1$ values of the state variables, and the shocks.
- Policy functions: how the period t values of the other variables depend on $t-1$ values of the state variables, and the shocks.

Understanding Transition and Policy Functions II

Since our two predetermined variables are capital k and TFP a (and our shock is z), this is *Dynare* output:

Table 1: Policy and Transition Functions in *Dynare*

	y	c	k	n	a
Constant	-0.257776	-0.561681	0.705661	-0.900068	0
k(-1)	0.191595	0.538937	0.821355	-0.347342	0
a(-1)	1.381119	0.662587	0.340435	0.718531	0.950000
z	1.453809	0.697460	0.358353	0.756349	1.000000

Remember that our model is in logs, so you read each column (e.g. the first one on y) as

$$\ln y_t = \ln \bar{y} + \gamma_1(\ln k_{t-1} - \ln \bar{k}) + \gamma_2(\ln a_{t-1} - \ln \bar{a}) + \gamma_3 z_t \tag{13}$$

Thus

$$\ln y_t = -0.257776 + 0.191595(\ln k_{t-1} - 0.705661) + 1.381119(\ln a_{t-1} - 0) + 1.453809 z_t \tag{14}$$

Understanding Transition and Policy Functions II

Since our two predetermined variables are capital k and TFP a (and our shock is z), this is *Dynare* output:

Table 2: Policy and Transition Functions in *Dynare*

	y	c	k	n	a
Constant	-0.257776	-0.561681	0.705661	-0.900068	0
k(-1)	0.191595	0.538937	0.821355	-0.347342	0
a(-1)	1.381119	0.662587	0.340435	0.718531	0.950000
z	1.453809	0.697460	0.358353	0.756349	1.000000

Remember that our model is in logs, so you read each column (e.g. the first one on y) as

$$\ln y_t = \ln \bar{y} + \gamma_1(\ln k_{t-1} - \ln \bar{k}) + \gamma_2(\ln a_{t-1} - \ln \bar{a}) + \gamma_3 z_t \tag{15}$$

Thus

$$\ln y_t = -0.257776 + 0.191595(\ln k_{t-1} - 0.705661) + 1.381119(\ln a_{t-1} - 0) + 1.453809 z_t \tag{16}$$

Understanding Transition and Policy Functions III

Table 3: Policy and Transition Functions in *Dynare*

	y	c	k	n	a
Constant	-0.257776	-0.561681	0.705661	-0.900068	0
k(-1)	0.191595	0.538937	0.821355	-0.347342	0
a(-1)	1.381119	0.662587	0.340435	0.718531	0.950000
z	1.453809	0.697460	0.358353	0.756349	1.000000

$$\ln y_t = -0.257776 + 0.191595(\ln k_{t-1} - 0.705661) + 1.381119 \ln a_{t-1} + 1.453809 z_t$$

$$\ln c_t = -0.561681 + 0.538937(\ln k_{t-1} - 0.705661) + 0.662587 \ln a_{t-1} + 0.697460 z_t$$

$$\ln k_t = 0.705661 + 0.821355(\ln k_{t-1} - 0.705661) + 0.340435 \ln a_{t-1} + 0.358353 z_t$$

$$\ln n_t = -0.900068 - 0.347342(\ln k_{t-1} - 0.705661) + 0.718531 \ln a_{t-1} + 0.756349 z_t$$

$$\ln a_t = 0.950000 \ln a_{t-1} + 1.000000 z_t$$

Understanding Transition and Policy Functions IV

By bringing steady state values on the LHS, it is clear that we can interpret our coefficients as **the percentage deviation from the steady state value triggered by a 1% deviation from the steady state of a certain variable** (predetermined or exogenous).

$$
\begin{aligned}
\ln y_t + 0.257776 &= \quad 0.191595(\ln k_{t-1} - 0.705661) + \quad 1.381119 \ln a_{t-1} + \quad 1.453809 z_t \\
\ln c_t + 0.561681 &= \quad 0.538937(\ln k_{t-1} - 0.705661) + \quad 0.662587 \ln a_{t-1} + \quad 0.697460 z_t \\
\ln k_t - 0.705661 &= \quad 0.821355(\ln k_{t-1} - 0.705661) + \quad 0.340435 \ln a_{t-1} + \quad 0.358353 z_t \\
\ln n_t + 0.900068 &= -0.347342(\ln k_{t-1} - 0.705661) + \quad 0.718531 \ln a_{t-1} + \quad 0.756349 z_t \\
\ln a_t &= \quad 0.950000 \ln a_{t-1} + \quad 1.000000 z_t
\end{aligned}
$$

Suppose that the economy is in the steady state at $t = 0$. Then a shock $z_1 = 1$ arrives at $t = 1$. How above will output be with respect to its steady state at $t = 1$?

$$
\begin{aligned}
\hat{y}_1 &= 0.191595 * \hat{k}_0 \quad + 1.381119 * \hat{a}_0 \quad + 1.453809 * z_1 \\
\hat{y}_1 &= 0.191595 * 0 \quad + 1.381119 * 0 \quad + 1.453809 * 1 \\
\hat{y}_1 &= 1.453809
\end{aligned}
$$

Understanding Transition and Policy Functions IV

If we want to know what will be the value of $\hat{y}_2$, we need first to compute also the values for $\hat{k}_1$ and $\hat{a}_1$.

$$\begin{aligned}
\hat{k}_1 =& 0.821355 * \hat{k}_0 && +0.662587 * \hat{a}_0 && +0.697460 * z_1 \\
\hat{k}_1 =& 0.821355 * 0 && +0.340435 * 0 && +0.358353 * 1 \\
\hat{k}_1 =& 0.358353
\end{aligned}$$

$$\begin{aligned}
\hat{a}_1 =& 0 * \hat{k}_0 && +0.950000 * \hat{a}_0 && +1.000000 * z_1 \\
\hat{a}_1 =& && 0.950000 * 0 && +1.000000 * 1 \\
\hat{a}_1 =& 1.000000
\end{aligned}$$

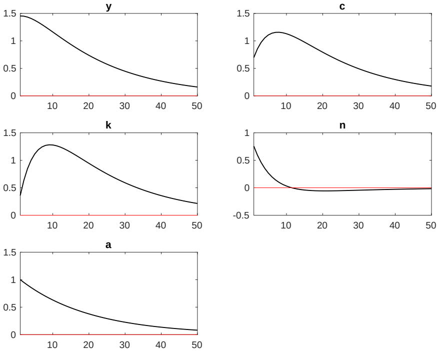
Thus we can obtain

$$\begin{aligned}
\hat{y}_2 =& 0.191595 * \hat{k}_1 && +1.381119 * \hat{a}_1 && +1.453809 * z_2 \\
\hat{y}_2 =& 0.191595 * 0.358353 && +1.381119 * 1.000000 && +1.453809 * 0 \\
\hat{y}_2 =& 1.4498
\end{aligned}$$

Impulse Response Functions

This brings us exactly to the concept of **Impulse Response Function**. Suppose your system is in equilibrium (in the steady state), and then is perturbed by an exogenous shock. How will it react?

Luckily, *Dynare* computes IRFs for us automatically. You can specify the number of periods for which to compute IRFs inside the stoch_simul command: e.g. stoch_simul(irf =50).



What is going on?

We simulated a 1% positive shock in TFP. What do we see?

- Income effect decreases labor supply
- Substitution effect increases labor supply. This effect prevails.
- The transitory productivity shock temporarily raises the real wage rate: agents work more today to be able to consume more in the future when the wage is expected to be lower.
- Higher output implies higher savings and higher investment that in turns increases (gradually) the capital stock.
- Consumption increases gradually: consumption smoothing.
- Output increases more than proportionally.

Looping Over Parameters

Once the mod file is set, it is quite easy to build loops to experiment with different parameter settings.

Set up a .m Matlab script looping over an array containing different values of a certain parameter. Suppose you want to vary σ:

```
clear all;
sigmas = [1,2,4];              %specify vector of values for sigma
results = struct               %create an empty structure to store results
for i=1:length(sigmas)         %open loop
    sigma=sigmas(i);
    save sigma;                %save the current value sigmas(i) with parameter name used in .mod file
    dynare your_mod_file_here noclearall; %run Dynare
    results.(sprintf('sigma%d',i)) = oo_; %store results into substructure
end
```

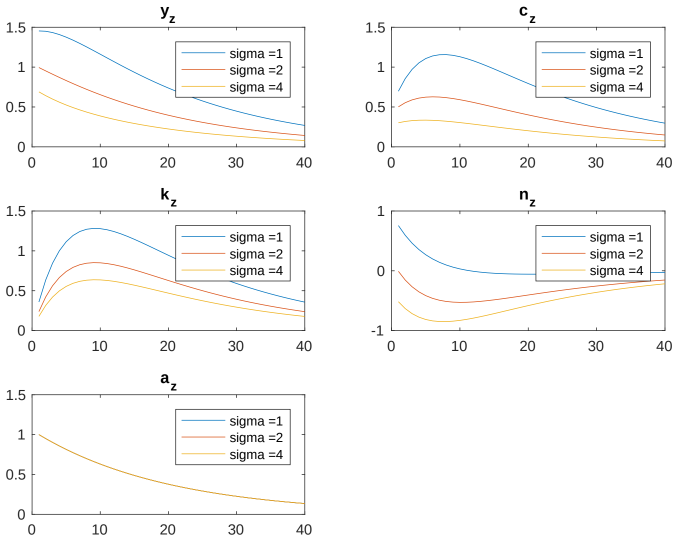
In your mod file, after your usual parameter declaration, insert two lines:

```
load sigma;                        %load previously saved parameter value
set_param_value('sigma',sigma);    %override previously specified parameter value
```

You can then access the results structure to inspect and plot your results.

When looping, use the nograph option to suppress IRF printing.

Varying σ



Varying σ II

How do we interpret these results?

- Loosely speaking, σ governs the household's desire to smooth consumption.
- If σ is very large, the household will want consumption (in expectation) to be very smooth.
- If σ is quite small then the household will be quite willing to allow consumption to not be smooth (again in expectation).
- What about labor fluctuations? Think about labor supply and demand curves.
- When TFP increases, labor demand shifts right, the amount by which is independent of σ (see Eq. 7)
- When consumption increases, labor supply shifts left. The bigger is σ, the bigger is this inward shift in labor supply, and therefore the smaller is the hours response in equilibrium and the larger is the wage response (see Eq. 5).
- That's exactly what we see in terms of IRFs: when σ is bigger, the employment jump is smaller (possibly negative), and the wage jump is larger.
- As a result, the output increases less than proportionally.

Varying ρ

How strong are propagation mechanisms for turning technology shocks into business cycles? Fluctuations appear to heavily rely on shock persistence.