Check Your Gap or Get Scrapped

An Investigation of a Car Following Model

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Introduction

- Traffic flow theory deals with modeling vehicular flow.
- Focus on microscopic model which model cars as a single unit.
- Goal: Model and examine
    - Homogeneous traffic
    - Obstacles on a road
    - Multi-lane bottleneck

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Introducing variables

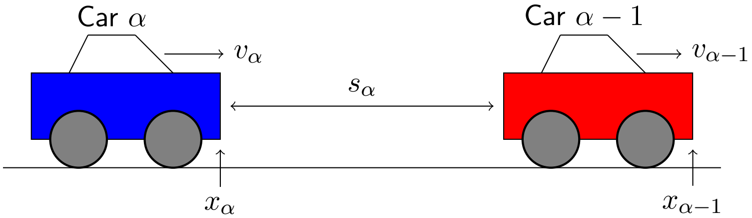

Figure 1: Defining index, position, velocity, and gap of a car.

- x_α, position of αth car.
- v_α, velocity of αth car.
- a_α, acceleration of αth car.
- s_α, gap of αth car.
- Will denote the car $\alpha - 1$ by car l.

Coupled Differential Equations

$$\frac{\mathrm{d}x_\alpha(t)}{\mathrm{d}t} = v_\alpha(t),$$
$$\frac{\mathrm{d}v_\alpha(t)}{\mathrm{d}t} = a_{\mathrm{mic}}(s_\alpha, v_\alpha, v_l).$$

Each car following model has a specific acceleration function: $a_{\mathrm{mic}}(s_\alpha, v_\alpha, v_l)$.

Full Velocity Difference Model

$$a_{\mathrm{mic}}(s_\alpha, v_\alpha, v_l) = \frac{v_{\mathrm{opt}}(s_\alpha) - v_\alpha}{\tau} - \gamma \Delta v,$$
$$v_{\mathrm{opt}}(s) = \max\left(0, \min\left(v_0, \frac{s - s_0}{T}\right)\right).$$

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v_0	desired speed
s_0	minimum distance gap
T	time gap
τ	speed adaptation time
γ	speed difference sensitivity

Optimal Velocity Function

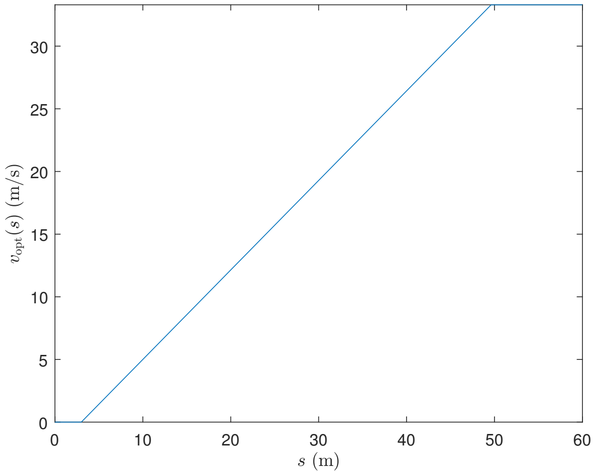

Figure 2: Graph of the optimal velocity function over a range of gaps.

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Forwards Euler Method Scheme

During lane changes, the acceleration function is not continuous, so Euler's Method would be the most stable:

$$v_\alpha(t+\Delta t) = v_\alpha(t) + a_{\text{mic}}(s_\alpha(t), v_\alpha(t), v_l(t))\Delta t,$$
$$x_\alpha(t+\Delta t) = x_\alpha(t) + \frac{v_\alpha(t) + v_\alpha(t+\Delta t)}{2}\Delta t.$$

Lane Changes

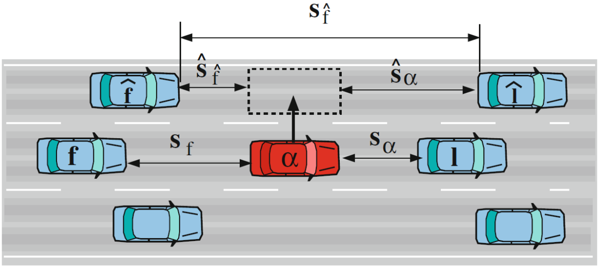

Figure 3: Multi-lane notation. From Treiber and Kesting *Traffic Flow Dynamics*.

Define minimum $\hat{s}_{\hat{f}}$ and $\hat{s}_{\alpha}$ needed to change lanes:

$$s_{\text{safe}}(v_{\hat{f}}, v_{\alpha}) = v_{\text{opt}}^{-1}\left[v_{\hat{f}} - \tau b_{\text{safe}} + \tau\gamma(v_{\hat{f}} - v_{\alpha})\right],$$

$$s_{\text{adv}}(s_{\alpha}, v_l, v_{\hat{l}}) = s_{\alpha} + v_{\text{opt}}^{-1}\left[\tau(\Delta a + a_{\text{bias}} + \gamma(v_l - v_{\hat{l}}))\right],$$

where b_{safe} is the deceleration limit, Δa is the changing threshold, and a_{bias} is a preference for left lanes.

Pseudocode for the FVDM

Algorithm 1 Simplified algorithm for FDVM with lane changes

Require: Initial state variables for each car at $t = 0$.
Require: carArr, an array of cars.

```
for i = 1 :numsteps do
    for j = length(carArr):−1:1 do
        State variables of jth car ← Update jth car by a timestep.
        New lane of jth car ← carArr(j).changeLane()
    end for
    sort(carArr)
end for
```

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Homogeneous Traffic Implementation

- A line of 10 equally spaced cars on a single lane.
- Initial velocity is 0 m/s.
- Cars will begin to slow down when they reach $x_{\text{destination}} = 2000$.

Homogeneous Traffic

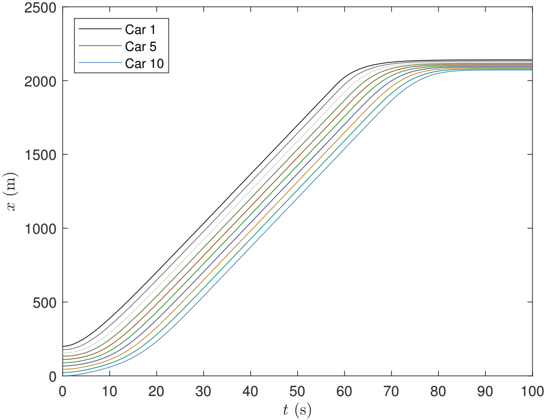

Figure 4: Position versus time.

Homogeneous Traffic

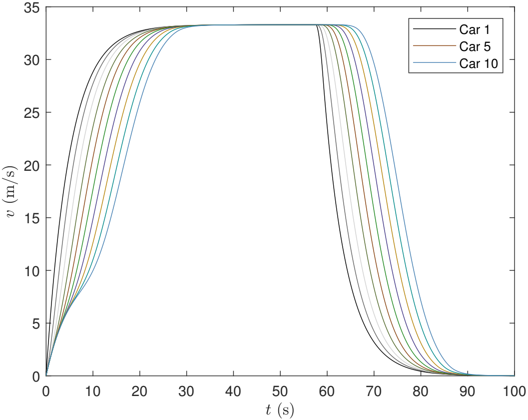

Figure 5: Velocity versus time.

Homogeneous Traffic

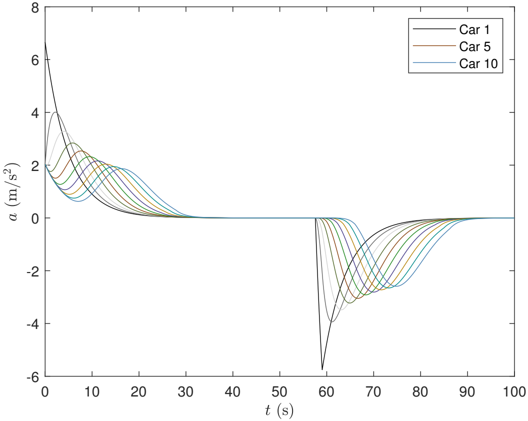

Figure 6: Acceleration versus time.

Homogeneous Traffic

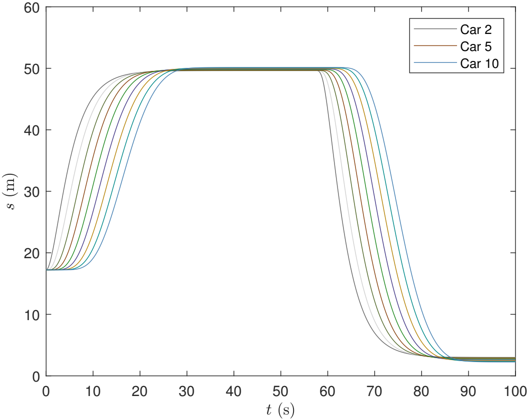

Figure 7: Gap versus time.

Obstacle Implementation

- Same conditions as in homogeneous traffic.
- However, we add an obstacle at $x = 1200$ at time $t = 30$ and remove it at $t = 75$.

Obstacle

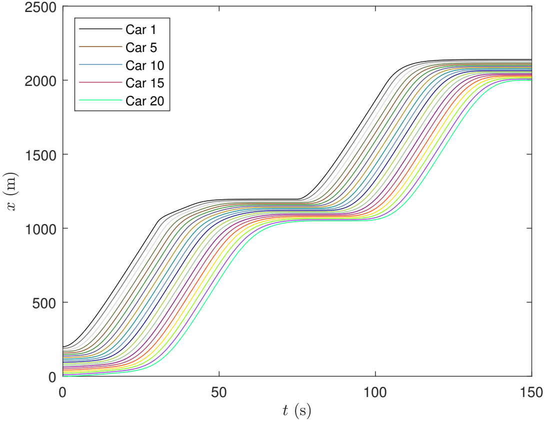

Figure 8: Position versus time.

Obstacle

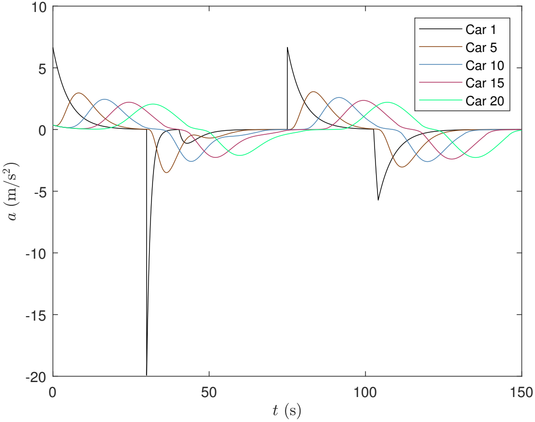

Figure 9: Acceleration versus time.

Obstacle

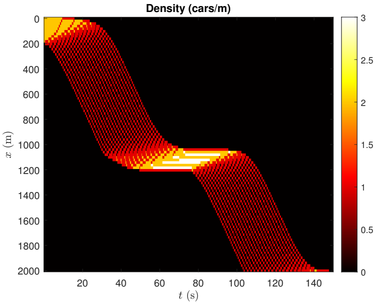

Figure 10: Left to right: density vs time and distance; flow rate vs time and distance.

Obstacle

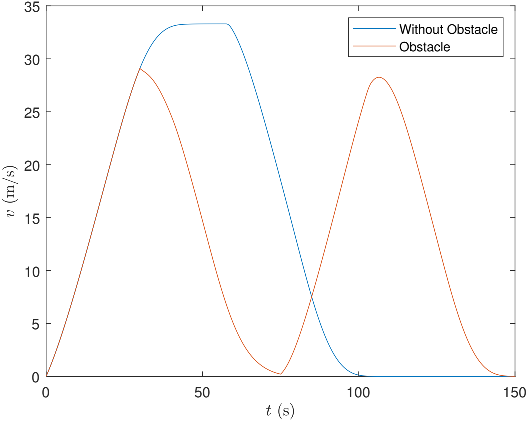

Figure 11: Mean velocity of traffic with and without an obstacle.

Multi-lane Bottleneck Implementation

- Same conditions as in homogeneous traffic expect with three lanes and 20 cars.
- Two lanes of a three-lane highway are closed.

Multi-lane Bottleneck

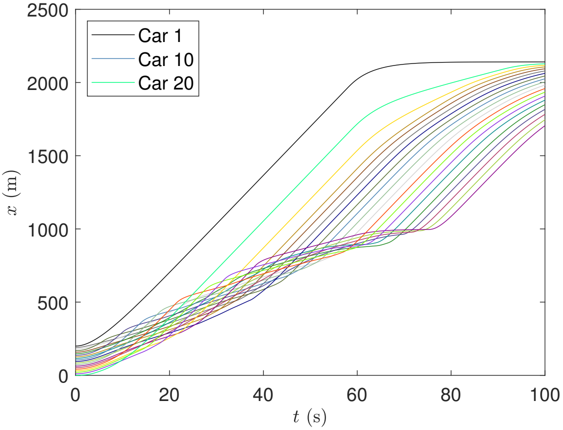

Figure 12: Position versus time.

Multi-lane Bottleneck

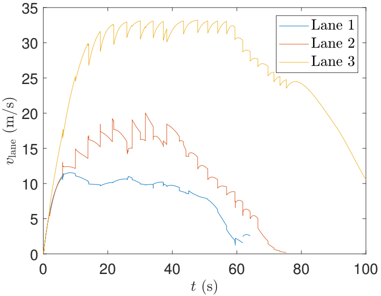

Figure 13: Left to right: mean velocity in each lane vs time; number of cars in each lane vs time.

Multi-lane Bottleneck

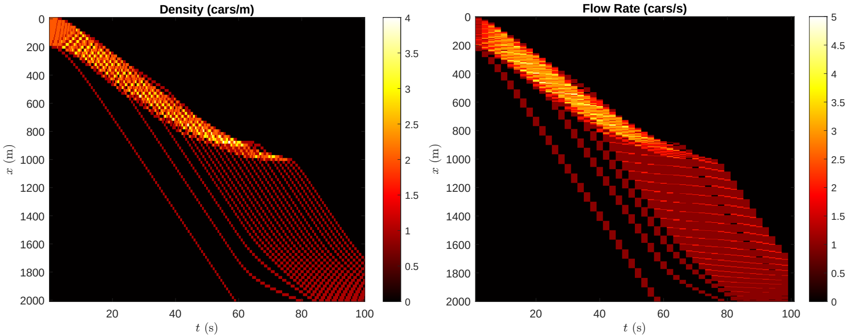

Figure 14: Left to right: density vs time and distance; flow rate vs time and distance.

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Conclusion

- The FVDM with lane changes can model traffic flow and lets us examine how it can be interrupted by obstacles and bottlenecks.
- Pitfalls in the model include unrealistic accelerations and erratic lane changes.
- Future work could model priority lanes and roundabouts.

Refrences

- David E. Stewart. *Dynamics with Inequalities.* SIAM, 2011.
- Martin Treiber and Arne Kesting. *Traffic Flow Dynamics: Data, Models and Simulation.* Berlin Heidelberg: Springer, 2013.