

Check Your Gap or Get Scrapped

An Investigation of a Car Following Model

Kaeshav Danesh Kevin Phan

April 27, 2022

Table of Contents

1 Introduction

2 Model

3 Implementation

4 Examining Scenarios

- Homogeneous Traffic
- Obstacle
- Multi-lane Bottleneck

5 Conclusion

Table of Contents

1 Introduction

2 Model

3 Implementation

4 Examining Scenarios

- Homogeneous Traffic
- Obstacle
- Multi-lane Bottleneck

5 Conclusion

Introduction

- Traffic flow theory deals with modeling vehicular flow.
- Focus on microscopic model which model cars as a single unit.
- Goal: Model and examine
 - Homogeneous traffic
 - Obstacles on a road
 - Multi-lane bottleneck

Table of Contents

1 Introduction

2 Model

3 Implementation

4 Examining Scenarios

- Homogeneous Traffic
- Obstacle
- Multi-lane Bottleneck

5 Conclusion

Introducing variables

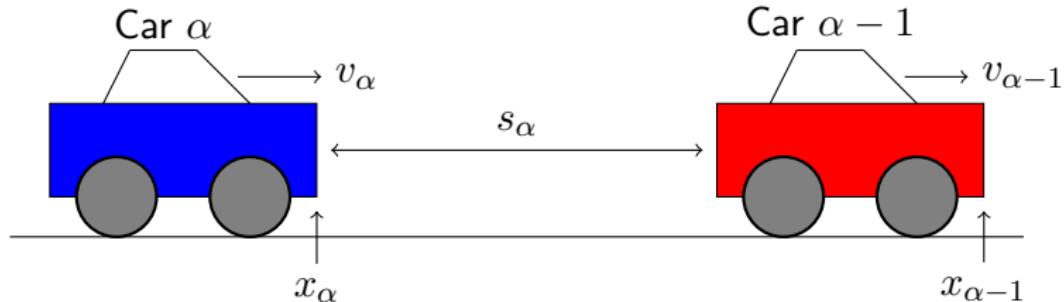


Figure 1: Defining index, position, velocity, and gap of a car.

- x_α , position of α th car.
- v_α , velocity of α th car.
- a_α , acceleration of α th car.
- s_α , gap of α th car.
- Will denote the car $\alpha - 1$ by car l .

Coupled Differential Equations

$$\begin{aligned}\frac{dx_\alpha(t)}{dt} &= v_\alpha(t), \\ \frac{dv_\alpha(t)}{dt} &= a_{\text{mic}}(s_\alpha, v_\alpha, v_l).\end{aligned}$$

Each car following model has a specific acceleration function:
 $a_{\text{mic}}(s_\alpha, v_\alpha, v_l)$.

Full Velocity Difference Model

$$a_{\text{mic}}(s_\alpha, v_\alpha, v_l) = \frac{v_{\text{opt}}(s_\alpha) - v_\alpha}{\tau} - \gamma \Delta v,$$
$$v_{\text{opt}}(s) = \max \left(0, \min \left(v_0, \frac{s - s_0}{T} \right) \right).$$

v_0 desired speed

s_0 minimum distance gap

T time gap

τ speed adaptation time

γ speed difference sensitivity

Optimal Velocity Function

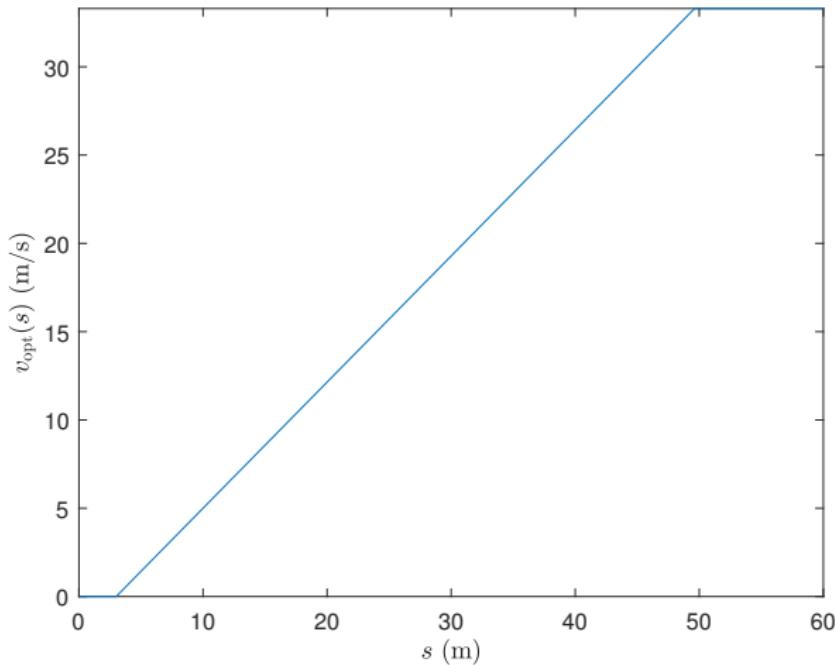


Figure 2: Graph of the optimal velocity function over a range of gaps.

Table of Contents

1 Introduction

2 Model

3 Implementation

4 Examining Scenarios

- Homogeneous Traffic
- Obstacle
- Multi-lane Bottleneck

5 Conclusion

Forwards Euler Method Scheme

During lane changes, the acceleration function is not continuous, so Euler's Method would be the most stable:

$$v_\alpha(t + \Delta t) = v_\alpha(t) + a_{\text{mic}}(s_\alpha(t), v_\alpha(t), v_l(t))\Delta t,$$

$$x_\alpha(t + \Delta t) = x_\alpha(t) + \frac{v_\alpha(t) + v_\alpha(t + \Delta t)}{2}\Delta t.$$

Lane Changes

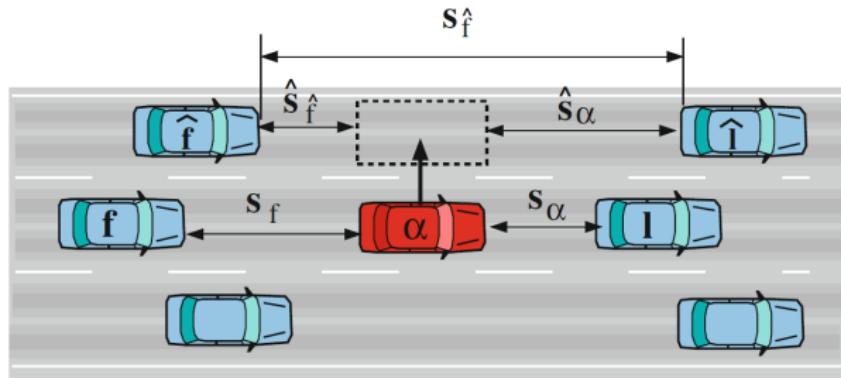


Figure 3: Multi-lane notation. From Treiber and Kesting *Traffic Flow Dynamics*.

Define minimum $\hat{s}_{\hat{f}}$ and \hat{s}_α needed to change lanes:

$$s_{\text{safe}}(v_{\hat{f}}, v_\alpha) = v_{\text{opt}}^{-1} \left[v_{\hat{f}} - \tau b_{\text{safe}} + \tau \gamma (v_{\hat{f}} - v_\alpha) \right],$$

$$s_{\text{adv}}(s_\alpha, v_l, v_{\hat{l}}) = s_\alpha + v_{\text{opt}}^{-1} \left[\tau (\Delta a + a_{\text{bias}} + \gamma (v_l - v_{\hat{l}})) \right],$$

where b_{safe} is the deceleration limit, Δa is the changing threshold, and a_{bias} is a preference for left lanes.

Pseudocode for the FVDM

Algorithm 1 Simplified algorithm for FDVM with lane changes

Require: Initial state variables for each car at $t = 0$.

Require: carArr, an array of cars.

```
for  $i = 1 : \text{numsteps}$  do
    for  $j = \text{length}(\text{carArr}) : -1 : 1$  do
        State variables of  $j$ th car  $\leftarrow$  Update  $j$ th car by a timestep.
        New lane of  $j$ th car  $\leftarrow \text{carArr}(j).\text{changeLane}()$ 
    end for
    sort(carArr)
end for
```

Table of Contents

1 Introduction

2 Model

3 Implementation

4 Examining Scenarios

- Homogeneous Traffic
- Obstacle
- Multi-lane Bottleneck

5 Conclusion

Homogeneous Traffic Implementation

- A line of 10 equally spaced cars on a single lane.
- Initial velocity is 0 m/s.
- Cars will begin to slow down when they reach $x_{\text{destination}} = 2000$.

Homogeneous Traffic

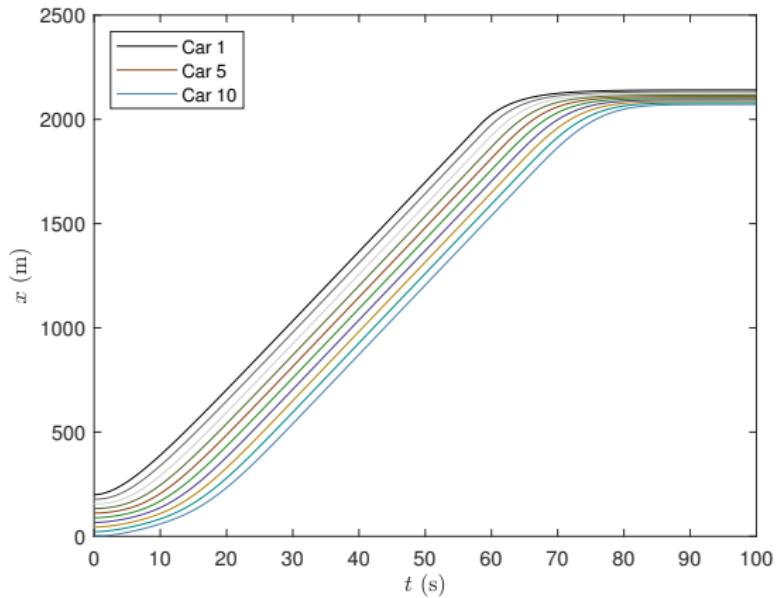


Figure 4: Position versus time.

Homogeneous Traffic

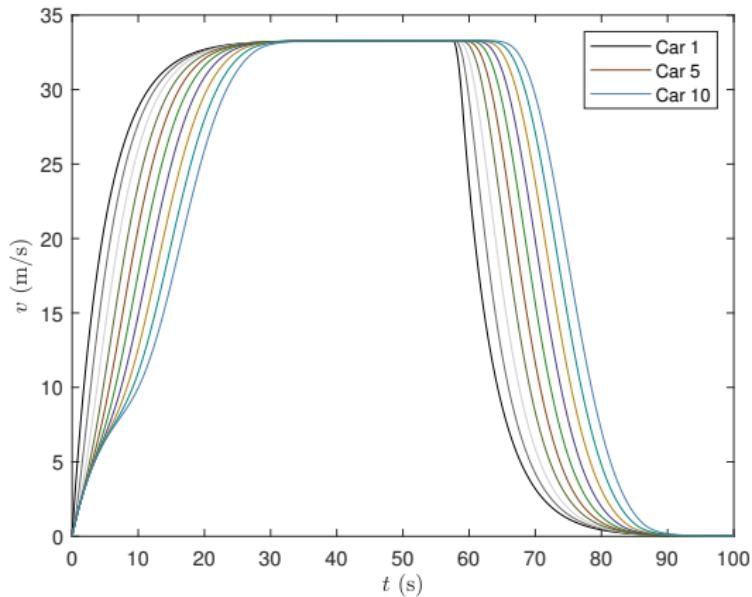


Figure 5: Velocity versus time.

Homogeneous Traffic

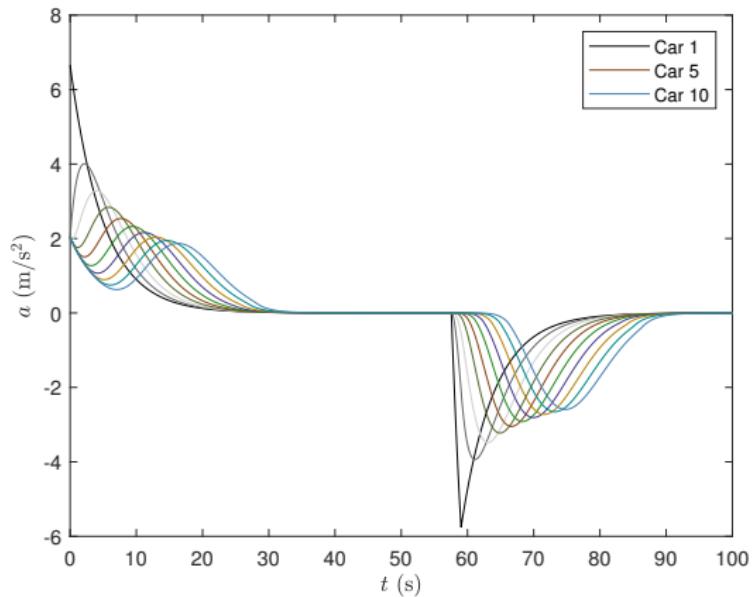


Figure 6: Acceleration versus time.

Homogeneous Traffic

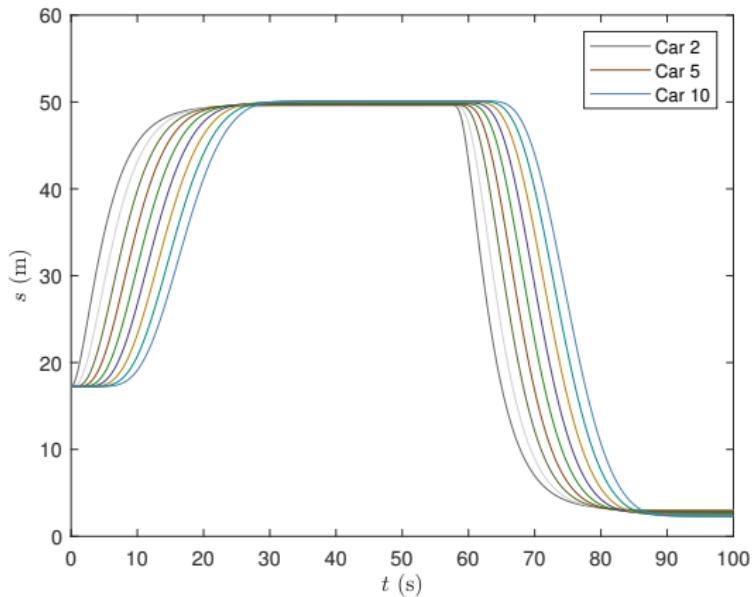


Figure 7: Gap versus time.

Obstacle Implementation

- Same conditions as in homogeneous traffic except with 20 cars.
- However, we add an obstacle at $x = 1200$ at time $t = 30$ and remove it at $t = 75$.

Obstacle

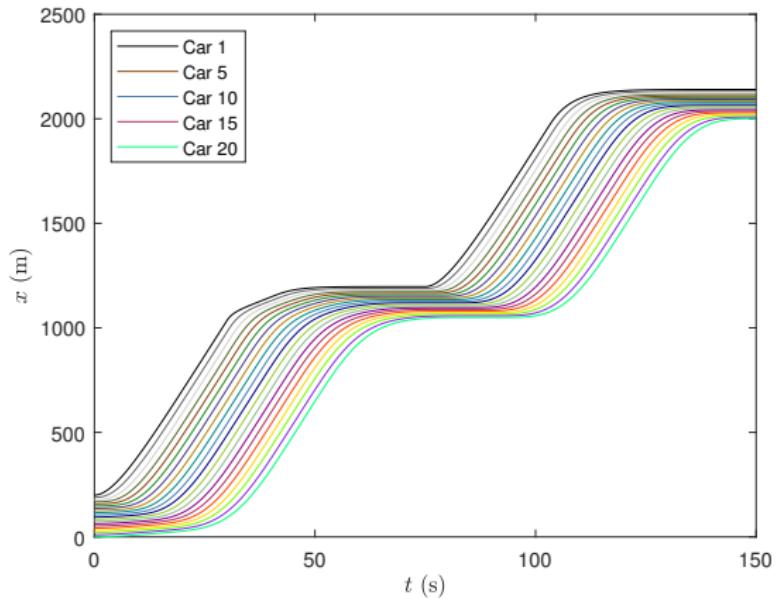


Figure 8: Position versus time.

Obstacle

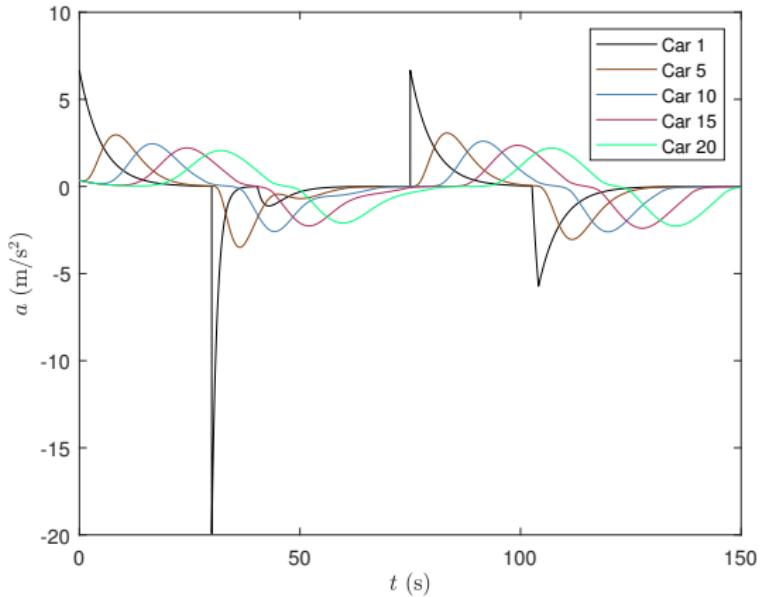


Figure 9: Acceleration versus time.

Obstacle

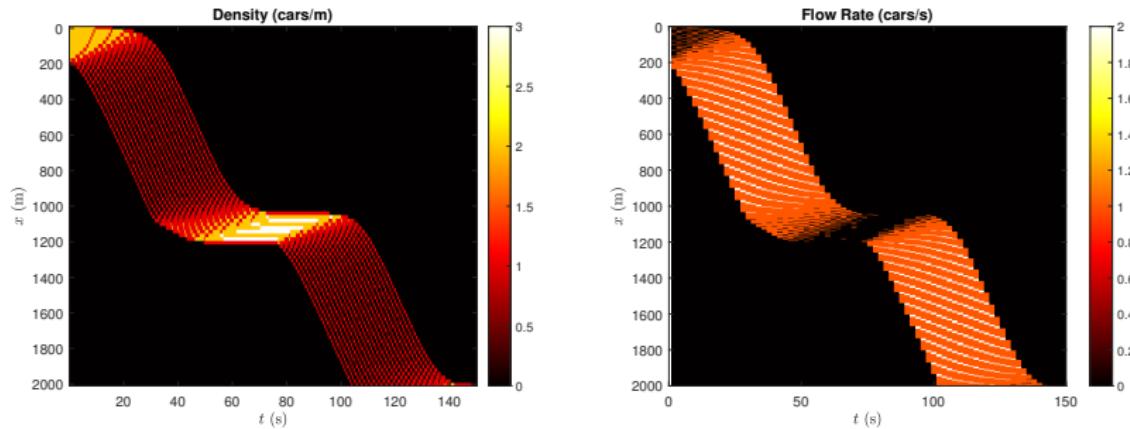


Figure 10: Left to right: density vs time and distance; flow rate vs time and distance.

Obstacle

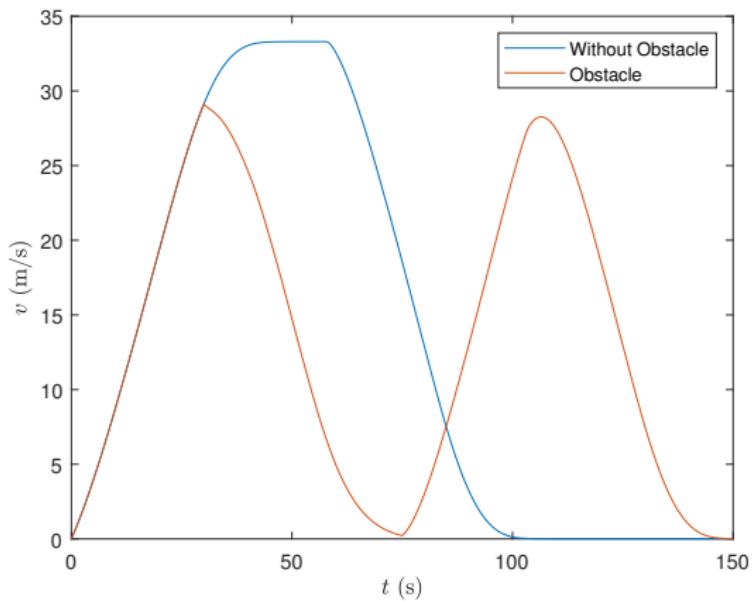


Figure 11: Mean velocity of traffic with and without an obstacle.

Multi-lane Bottleneck Implementation

- Same conditions as in homogeneous traffic except with three lanes and 20 cars.
- Two lanes of a three-lane highway are closed.

Multi-lane Bottleneck

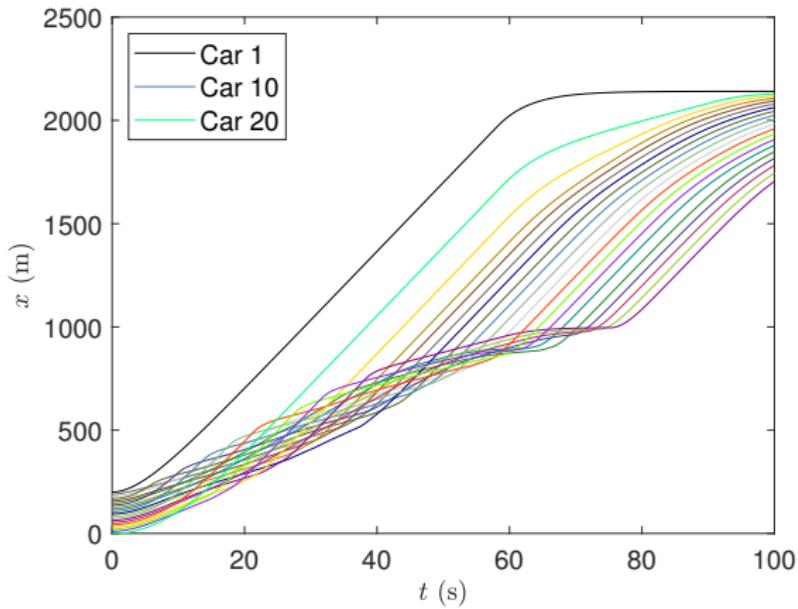


Figure 12: Position versus time.

Multi-lane Bottleneck

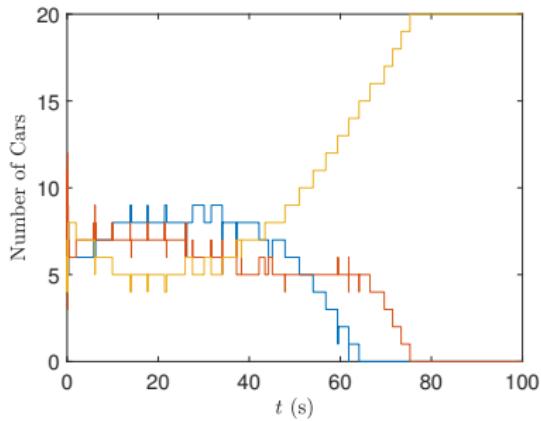
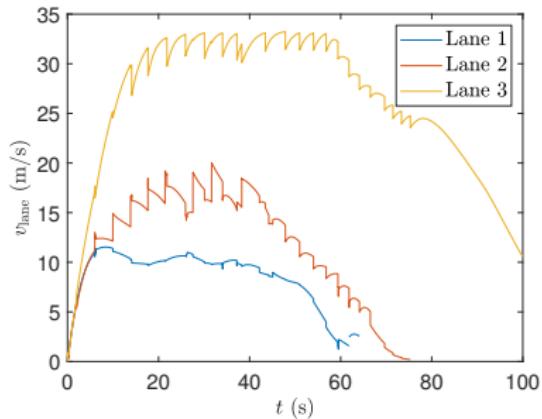


Figure 13: Left to right: mean velocity in each lane vs time; number of cars in each lane vs time.

Multi-lane Bottleneck

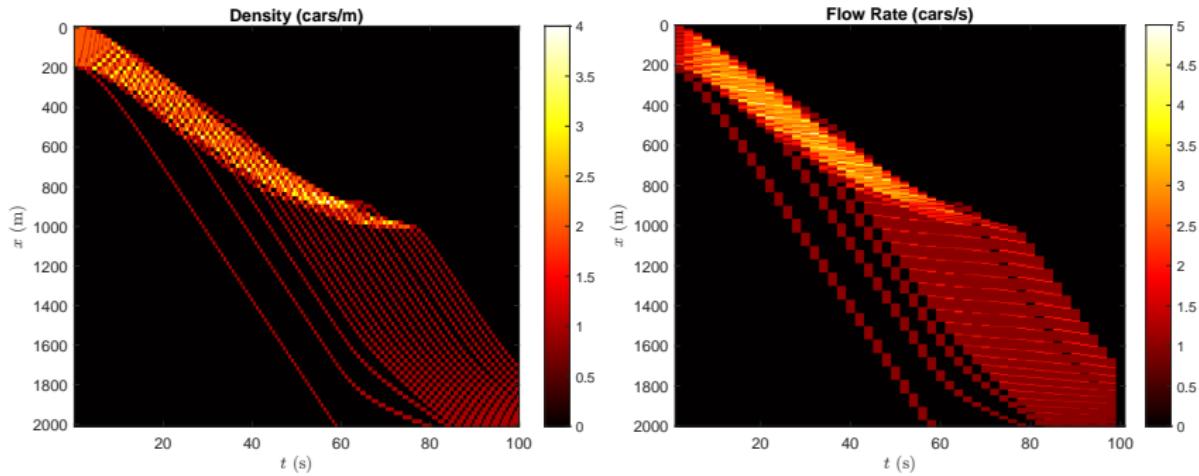


Figure 14: Left to right: density vs time and distance; flow rate vs time and distance.

Table of Contents

1 Introduction

2 Model

3 Implementation

4 Examining Scenarios

- Homogeneous Traffic
- Obstacle
- Multi-lane Bottleneck

5 Conclusion

Conclusion

- The FVDM with lane changes can model traffic flow and lets us examine how it can be interrupted by obstacles and bottlenecks.
- Pitfalls in the model include unrealistic accelerations and erratic lane changes.
- Future work could model priority lanes and roundabouts.

References

- David E. Stewart. *Dynamics with Inequalities*. SIAM, 2011.
- Martin Treiber and Arne Kesting. *Traffic Flow Dynamics: Data, Models and Simulation*. Berlin Heidelberg: Springer, 2013.