Investigating Car Following Model

Kaeshav Danesh and Kevin Phan

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Abstract

Traffic flow dynamics is generally split into either macroscopic models or microscopic models. For this paper, we will focus on car following models which is a type of microscopic model. We first introduce the assumptions and the mathematical description car following model and the numerical scheme used to compute them. Then, we give the full velocity difference model. Finally, we examine different scenarios such as bottlenecks, phantom traffic, and lane changes and analyze the behavior of cars.

Contents

1 Introduction

How does a bottleneck affect a series of vehicles? How do a traffic jam propagate through a series of vehicles? What impact does changing a road from multiple lanes to one lane have a series of cars? To answer these questions, our paper will describe a basic microscopic car following model from traffic flow theory, implement these scenarios, and analyze them.

Traffic flow models can be categorized as microscopic or macroscopic models [[van+15]]. Microscopic models describe vehicles on the individual level by treating them as entire unit. Meanwhile, macroscopic models treat vehicles as a continuum. Further classifications can be done through the model equations such as partial differential equations, discrete equations, discrete or continuous variables, and deterministic or stochastic process. Some applications of traffic flow models include simulating traffic, optimizing traffic lights, and calculating carbon emissions from traffic.

Our paper will focus only on the microscopic model called the Full Velocity Difference model. Using this model, we will explore scenarios including bottlenecks, phantom traffic, and lane changes. From this, we will analyze state variables including position, velocity, and acceleration, and other metrics such as density and flow rate.

2 General Model

2.1 Mathematical Formulation

We follow the mathematical formulation of car following model in §10.2 of [TK13]. Suppose that there are n vehicles in the simulation using the car following model. We index the 1st vehicle by 1, the 2nd car by 2, the αth car by α, and so on. The state variables of vehicle α are position x_α, velocity v_α, and acceleration a_α. Furthermore, we are also assuming that the vehicles in the model has a length l. The position x_α of car α is defined as the front bumper of the car. Another useful variable to define is gap. We define the gap of car α by the difference in distance between the back bumper of car $\alpha - 1$ and the front bumper of car α. Mathematically, the gap s_α is defined as

$$s_\alpha = x_{\alpha-1} - l_{\alpha-1} - x_\alpha \tag{1}$$

where $x_{\alpha-1}$ is the position of car $\alpha - 1$, $l_{\alpha-1}$ is the length of car $\alpha - 1$, and x_α is the position of car α. We note that the gap is not defined for a vehicle with no vehicles in front of it.

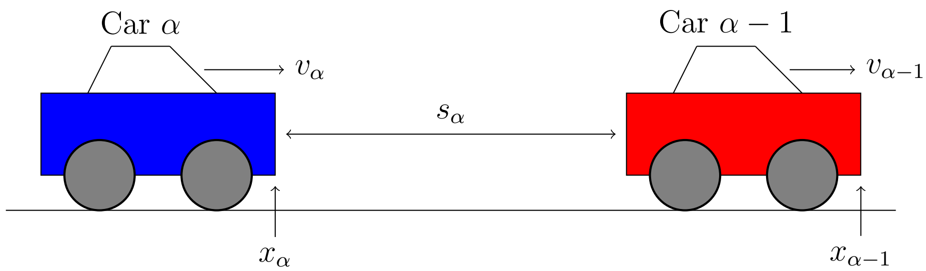


Figure 1: Defining index, position, velocity, and gap of a car.

For simpler notation, we will now refer to the vehicle in front of vehicle α by the leader vehicle l. For a single lane, car l is vehicle $\alpha - 1$. However, it is not necessarily true that vehicle l is vehicle $\alpha - 1$ for multiple lanes.

Taking the time derivatives of $x_\alpha(t)$ and $v_\alpha(t)$ lead to the general coupled differential equation describing velocity and acceleration respectively:

$$\frac{\mathrm{d}x_\alpha(t)}{\mathrm{d}t} = v_\alpha(t), \tag{2}$$

$$\frac{\mathrm{d}v_\alpha(t)}{\mathrm{d}t} = a_{\mathrm{mic}}(s_\alpha, v_\alpha, v_l). \tag{3}$$

Each car following model has a specific acceleration function: a_{mic}[1]. For the simulation, we will use the Full Velocity Difference Model (FVDM) which is described in section (3.1).

2.2 Numerical Scheme

The standard way of numerically solving a system of coupled differential equations would be to use a fourth order Runge Kutta method. However, higher order methods all assume higher orders of smoothness in the differential equation and its solution [[Ste11]]. Our model is only smooth to first

[1]We use a_{mic} for the acceleration function and a to describe the state variable acceleration.

order because suddenly changes in acceleration is common, for example, during lane changes. So, using a high order method could be worse than a simple first order method.

Among first order methods, our options are either the forward or backwards (implicit) Euler methods. The main difference between these methods is that the backwards Euler method involves solving an implicit equation and is generally more stable.

We are using the forward Euler method because we expect our solution to be stable and solving an implicit equation every timestep would add extra complexity to our implementation.

Implementing the forwards Euler method scheme for a car following model gives us two coupled differential equations for each of our states:

$$v_\alpha(t+\Delta t) = v_\alpha(t) + a_{\text{mic}}(s_\alpha(t), v_\alpha(t), v_l(t))\Delta t, \tag{4}$$

$$x_\alpha(t+\Delta t) = x_\alpha(t) + \frac{v_\alpha(t) + v_\alpha(t+\Delta t)}{2}\Delta t, \tag{5}$$

where Δt is the time step and a_{mic} is the acceleration function defined by the car following model used. This acceleration function is described in section(3.1).

When solving the equations above numerically, we must also consider the interactions between cars. The velocity equation uses $v_l(t)$, the velocity of the leading car at time t. So, when calculating the state of a car at each timestep, it would be easier to start from the backmost car and work up. This allows us to use the most recent velocity value from the leading car.

3 Car Following Model

3.1 Full Velocity Difference Model

We followed the details of the Full Velocity Difference Model in §10.6 and §10.7 of [[TK13]]. The Full Velocity Difference Model (FVDM) is given by the acceleration function:

$$a_{\text{mic}}(s_\alpha, v_\alpha, v_l) = \frac{v_{\text{opt}}(s) - v_\alpha}{\tau} - \gamma\Delta v \tag{6}$$

where v_{opt} is the optimal velocity function, τ is the speed adaptation time, γ is the speed difference sensitivity, and $\Delta v = v_\alpha - v_l$ is the difference between in velocities of the car α and the leader car.

A simple choice for v_{opt} is

$$v_{\text{opt}}(s) = \max\left(0, \min\left(v_0, \frac{s - s_0}{T}\right)\right) \tag{7}$$

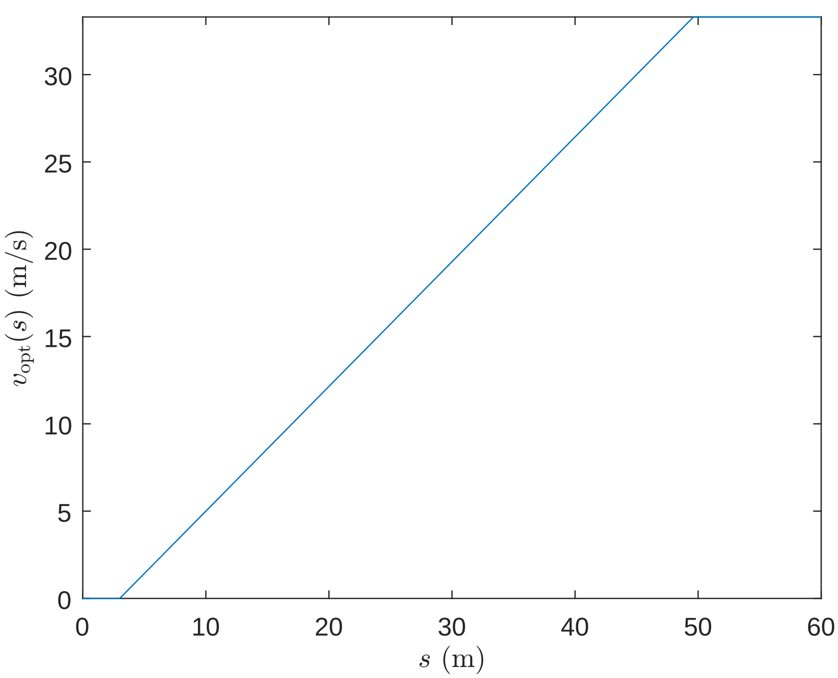
where v_0 is the desired speed, s_0 is the minimum distance gap, and T is the time gap.

Typical parameters for highway traffic for the FVDM is given in the table below.

Parameter	Value
v_0, desired speed	$33.\bar{3}$ m/s
s_0, minimum distance gap	3 m
T, time gap	1.4 s
τ, speed adaptation time	5 s
γ, speed difference sensitivity	0.6 s^{-1}

Figure 2: Typical parameters for highway traffic for the FVDM.

We now examine how different parameters of the optimal velocity function affect the graph.



Since we are taking the minimum of v_0 and $(s - s_0)/T$, v_{opt} attain a maximum of v_0. This means that the optimal velocity of a vehicle is the desired speed v_0. Furthermore, v_{opt} is 0 on the interval $0 \leq s \leq s_0$ which means that a vehicle will not move if the vehicle's gap is s_0 or less. This prevents car crashes from occurring for most cases. Lastly, the time gap T determine the slope of the line. Higher values of T means that the car's optimal velocity will be reached for higher value of s.

Analyzing equation (6), $v_{\mathrm{opt}}(s) - v_\alpha$ is positive if $v_{\mathrm{opt}}(s) > v_\alpha$. The car has not reached its optimal velocity and so, acceleration is positive. Similarly, if $v_{\mathrm{opt}}(s) < v_\alpha$, $v_{\mathrm{opt}}(s) - v_\alpha$ is negative and so, the car is decelerating to reach its optimal velocity. If $v_{\mathrm{opt}}(s) = v_\alpha$, then $a_{\mathrm{mic}} = 0$. The speed adaptation time τ determine how fast the car accelerate or decelerate and thus, affect the time it takes for the car to reach its optimal velocity. The term $-\gamma \Delta v$ is positive if $v_\alpha < v_l$ and negative if $v_\alpha > v_l$. This leads to more acceleration if the car is trying to catch to the leader car and less acceleration if the car is trying to slow down due to the leader car's slower velocity.

3.2 Implementation

We first initialize initial conditions for the state variables of the vehicles. We store the vehicles in an array and iterate through them. As described in section (2.2), we iterate through the vehicles starting with the last vehicle with respect to position and go through in ascending order. We update the state variables of the cars using equations (4), (5), and (6). We repeat this process for the number of iterations required.

Algorithm 1 Simplified algorithm for FDVM

Require: Initial state variables for each car at $t = 0$.
Require: carArr, an array of cars.
  for $i = 1$:numsteps **do**
    for $j =$ length(carArr):-1:1 **do**
      State variables of jth car $\leftarrow$ Update jth car by a timestep.
    end for
  end for

We still have not addressed the situation with the first car when updating its state variables. Equation (6) need values for the car's gap s_α we are updating and the velocity of the leader car v_l. However, the first car does not have a leader car. To resolve this, we impose a destination for the cars. We use this value in the calculation of gap s_α. Secondly, we set $v_l = 0$. This resolve the problem of updating the state variables of the first car.

4 Examining Different Scenarios

4.1 Homogeneous Traffic

TODO: Give basic graphs so that we can compare to in the next sections and add any pitfalls with the model (unrealistic acceleration), Look at the pitfalls (since gap is used, stuff from far away can affect it and unrealistic acceleration which can be included in section 4.1 maybe?)

4.1.1 Simulation

4.1.2 Pitfalls

4.2 Bottleneck

TODO: Add time versus position graph and label the different situations, density graph, analysis, page 11 is a good source of questions to ask for this, a graph of car versus difference in time to get there (use the previous section)

4.2.1 Implementation

4.2.2 Simulation

4.2.3 Analysis

4.3 Phantom Traffic: Local Perturbation

4.3.1 Implementation

TODO: Look at how stopping suddenly propagate through the cars. Something about waves?

4.4 Lane Changes

TODO: Average lane speed (How much faster is one lane with and without lane changes) and bottleneck revisted (make it from 2 lanes to one lane) and how number of lanes affect flow rate. The grass is greener on the other side paradox (in the textbook)

4.4.1 Implementation

4.5 Multi-lanes Bottleneck

4.5.1 Implementation

4.5.2 Simulation

4.5.3 Analysis

5 Conclusion

6 Further Remarks

List of Symbols and Constants

x	position [see sec. 2.1].
v	velocity [see sec. 2.1].
a	acceleration [see sec. 2.1].
t	time [see sec. 2.1].
s	gap [see sec. 2.1].
v_0	desired speed (typically 33 m/s) [see ch. 3.1]
s_0	desired speed (typically 3 m) [see sec. 3.1]
T	desired speed (typically 1.4 s) [see sec. 3.1]
τ	adaptation time (typically 5 s^{-1})
γ	speed difference sensitivity (typically 0.6 s^{-1})

References

[Ste11] David E. Stewart. *Dynamics with Inequalities*. SIAM, 2011. ISBN: 978-1-611970-70-8.

[TK13] Martin Treiber and Arne Kesting. *Traffic Flow Dynamics: Data, Models and Simulation*. Berlin Heidelberg: Springer, 2013. ISBN: 978-3-642-32459-8.

[van+15] Femke van Wageningen-Kessels et al. "Genealogy of traffic flow models". In: *EURO Journal on Transportation and Logistics* 4 (2015), pp. 445–473. DOI: `https://doi.org/10.1007/s13676-014-0045-5`.