Investigating Car Following Model

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Abstract

Traffic flow dynamics is generally split into either macroscopic models or microscopic models. For this paper, we will focus on car following models which is a type of microscopic model. We first introduce the assumptions and the mathematical description car following model and the numerical scheme used to compute them. Then, we give the full velocity difference model. Finally, we examine different scenarios such as car crashes, phantom traffic, traffic light, and lane changes and analyze the behavior of cars.

Contents

1 Introduction

Some questions to answer:

- Why are traffic models important? Applications?
- At what scale does our model operate at? Macroscopic and microscopic?
- What assumptions are we making?

2 General Model

2.1 Mathematical Formulation

We follow the mathematical formulation of car following model in [TK13]. Suppose that there are n cars in the car following model. We index the 1st car by 1, the 2nd car by 2, the αth car by α, and so on. The state variables of car α are position x_α, velocity v_α, and acceleration a_α. Furthermore, we are also assuming that the cars in the model has some kind of width. Thus, the position x_α of car α is defined as the front bumper of the car. Another useful variable to define is gap. We define the gap of car α by the difference in distance between the back bumper of car $\alpha - 1$ and the front bumper of car α. Mathematically, the gap s_α is defined as

$$s_\alpha = x_{\alpha-1} - l_{\alpha-1} - x_\alpha \tag{1}$$

where $x_{\alpha-1}$ is the position of car $\alpha - 1$, $l_{\alpha-1}$ is the length of car $\alpha - 1$, and x_α is the position of car α. We note that the gap is not defined for a car with no cars in front of it.

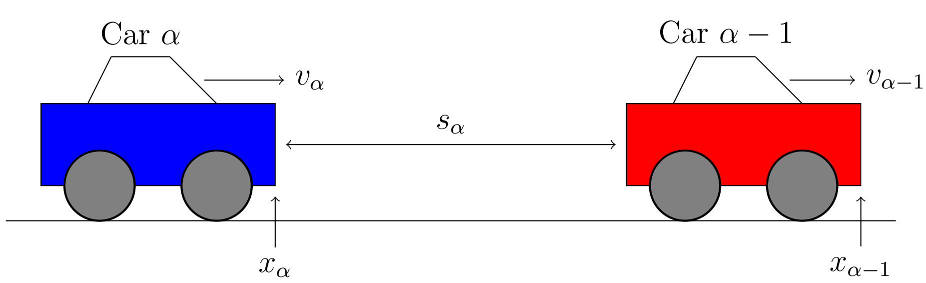

Figure 1: Defining index, position, velocity, and gap of a car.

For simpler notation, we will now refer to the car in front of car α by the leader car l. For a single lane, car l is car $\alpha-1$. However, it is not necessarily true that car l is car $\alpha - 1$ for multiple lanes.

Taking the time derivatives of $x_\alpha(t)$ and $v_\alpha(t)$ lead to the general coupled differential equation describing velocity and acceleration respectively:

$$\frac{\mathrm{d}x_\alpha(t)}{\mathrm{d}t} = v_\alpha(t), \tag{2}$$

$$\frac{\mathrm{d}v_\alpha(t)}{\mathrm{d}t} = a_{\mathrm{mic}}(s_\alpha, v_\alpha, v_l). \tag{3}$$

Each car following model has a specific acceleration function: a_{mic}. For the simulation, we will use the Full Velocity Difference Model (FVDM) which is described in section (3.1).

2.2 Numerical Scheme

TODO: Explain differential equations.

Higher order methods all assume higher orders of smoothness in the differential equation and its solution. Our model is only smooth to first order because suddenly changes in acceleration is common, for example, during lane changes. So, using a high order method like the fourth order Runge-Kutta could be worse than a simpler first order method.

Then, our options are either the forward or backwards (implicit) Euler methods. We are using the forward Euler method because for small timesteps, Δt, we can assume constant acceleration within Δt. The changes in acceleration between timesteps could be treated as a reaction time (not sure)?

Implementing the forwards Euler method scheme for a car following model gives us two coupled differential equations for each of our states:

$$v_\alpha(t+\Delta t) = v_\alpha(t) + a_{\mathrm{mic}}(s_\alpha(t), v_\alpha(t), v_l(t))\Delta t, \tag{4}$$

$$x_\alpha(t+\Delta t) = x_\alpha(t) + \frac{v_\alpha(t) + v_\alpha(t+\Delta t)}{2}\Delta t, \tag{5}$$

where a_{mic} is the acceleration function defined by the car following model used. This acceleration function is described in section(3.1).

TODO: Order we are doing the iterated numerical scheme (doing it from the last car to the first car)

To learn more, see [Ste11].

3 Car Following Model

3.1 Full Velocity Difference Model

4 Examining Different Scenarios

4.1 Homogeneous Traffic

TODO: Give basic graphs so that we can compare to in the next sections and add any pitfalls with the model (unrealistic acceleration)

4.1.1 Simulation

4.1.2 Pitfalls

4.2 Bottleneck

TODO: Add time versus position graph and label the different situations, density graph, analysis

4.2.1 Implementation

4.2.2 Simulation

4.2.3 Analysis

4.3 Phantom Traffic: Local Perturbation

4.3.1 Implementation

TODO: Look at how stopping suddenly propagate through the cars. Something about waves?

4.4 Lane Changes

TODO: Average lane speed (How much faster is one lane with and without lane changes) and bottleneck revisted (make it from 2 lanes to one lane) and how number of lanes affect flow rate. The grass is greener on the other side paradox (in the textbook)

4.4.1 Implementation

4.4.2 Simulation

4.4.3 Analysis

5 Conclusion

6 Further Remarks

List of Symbols and Constants

x	position [see ch. 2.1].
v	velocity
a	acceleration
t	time
s	gap
v_0	desired speed (typically 33 m/s) [see ch. 3.1]
τ	adaptation time (typically 0.65 s)
γ	speed difference sensitivity

References

[Ste11] David E. Stewart. *Dynamics with Inequalities*. SIAM, 2011. ISBN: 978-1-611970-70-8.

[TK13] Martin Treiber and Arne Kesting. *Traffic Flow Dynamics: Data, Models and Simulation*. Berlin Heidelberg: Springer, 2013. ISBN: 978-3-642-32459-8.