

# Check Your Gap or Get Scrapped

## An Investigation of a Car Following Model

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# Introduction

- Traffic flow theory deals with modeling vehicular flow.
- Focus on microscopic model which model cars as a single unit.
- Goal: Model and examine
  - ① Homogeneous traffic
  - ② Obstacles on a road
  - ③ Multi-lane bottleneck

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# Introducing variables

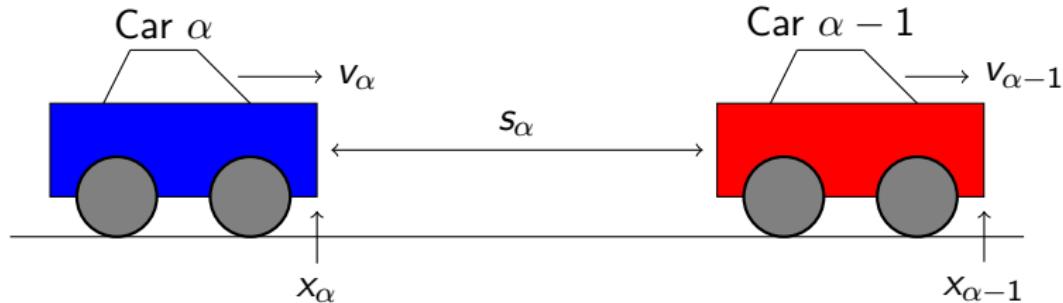


Figure 1: Defining index, position, velocity, and gap of a car.

- $x_\alpha$ , position of  $\alpha$ th car.
- $v_\alpha$ , velocity of  $\alpha$ th car.
- $a_\alpha$ , acceleration of  $\alpha$ th car.
- $s_\alpha$ , gap of  $\alpha$ th car.
- Will denote the car  $\alpha - 1$  by car  $l$ .

# Coupled Differential Equations

$$\begin{aligned}\frac{dx_\alpha(t)}{dt} &= v_\alpha(t), \\ \frac{dv_\alpha(t)}{dt} &= a_{\text{mic}}(s_\alpha, v_\alpha, v_I).\end{aligned}$$

Each car following model has a specific acceleration function:  
 $a_{\text{mic}}(s_\alpha, v_\alpha, v_I)$ .

# Full Velocity Difference Model

$$a_{\text{mic}}(s_\alpha, v_\alpha, v_l) = \frac{v_{\text{opt}}(s) - v_\alpha}{\tau} - \gamma \Delta v,$$
$$v_{\text{opt}}(s) = \max \left( 0, \min \left( v_0, \frac{s - s_0}{T} \right) \right).$$

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$v_0$  desired speed

$s_0$  minimum distance gap

$T$  time gap

$\tau$  speed adaptation time

$\gamma$  speed difference sensitivity

# Optimal Velocity Function

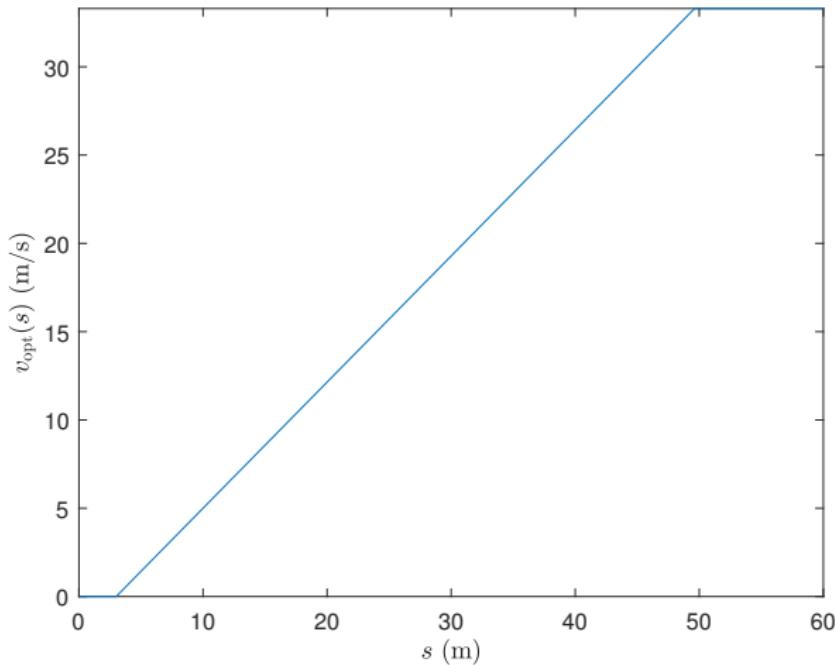


Figure 2: Graph of the optimal velocity function over a range of gaps.

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# Forwards Euler Method Scheme

During lane changes, the acceleration function is not continuous, so Euler's Method would be the most stable:

$$v_\alpha(t + \Delta t) = v_\alpha(t) + a_{\text{mic}}(s_\alpha(t), v_\alpha(t), v_l(t))\Delta t,$$
$$x_\alpha(t + \Delta t) = x_\alpha(t) + \frac{v_\alpha(t) + v_\alpha(t + \Delta t)}{2}\Delta t.$$

# Lane Changes

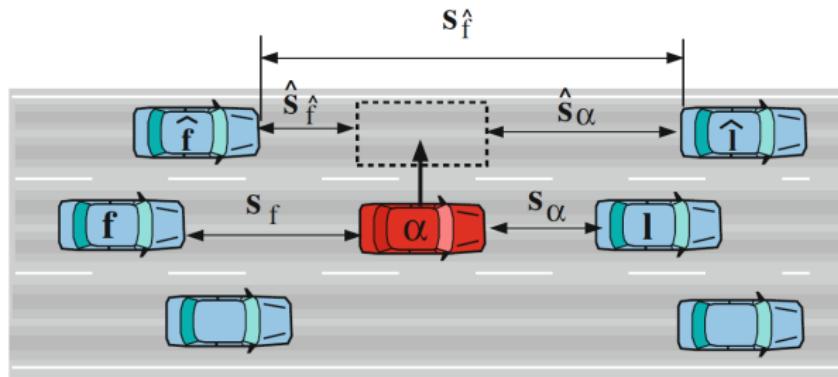


Figure 3: Multi-lane notation. From Treiber and Kesting *Traffic Flow Dynamics*

Define minimum  $\hat{s}_{\hat{f}}$  and  $\hat{s}_\alpha$  needed to change lanes:

$$s_{\text{safe}}(v_{\hat{f}}, v_\alpha) = v_{\text{opt}}^{-1} [v_{\hat{f}} - \tau b_{\text{safe}} + \tau \gamma (v_{\hat{f}} - v_\alpha)],$$

$$s_{\text{adv}} = s_\alpha + v_{\text{opt}}^{-1} [\tau (\Delta a + a_{\text{bias}} + \gamma (v_I - v_{\hat{f}}))].$$

# Pseudocode for the FVDM

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**Algorithm 1** Simplified algorithm for FDVM with lane changes

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**Require:** Initial state variables for each car at  $t = 0$ .

**Require:** carArr, an array of cars.

```
for i = 1 :numsteps do
    for j = length(carArr):-1:1 do
        State variables of jth car  $\leftarrow$  Update jth car by a timestep.
        New lane of jth car  $\leftarrow$  carArr(j).changeLane()
    end for
    sort(carArr)
end for
```

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# Homogeneous Traffic

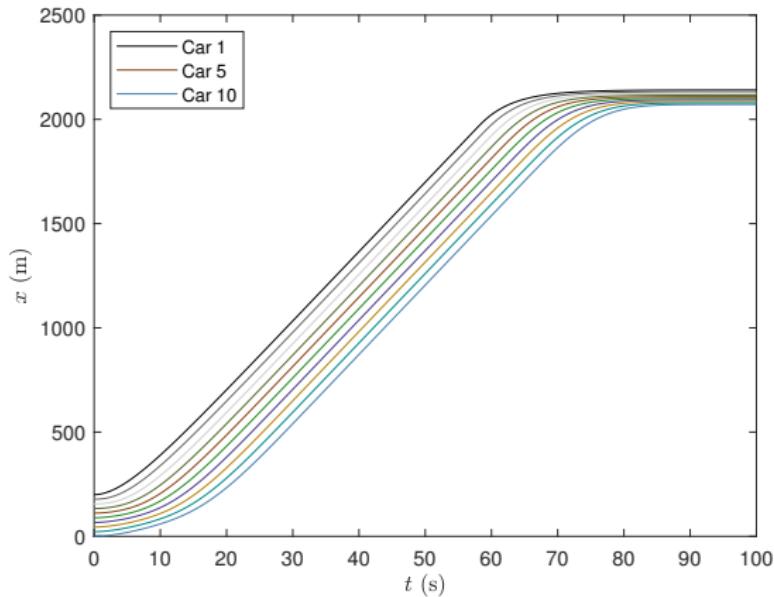


Figure 4: Position versus time

# Homogeneous Traffic

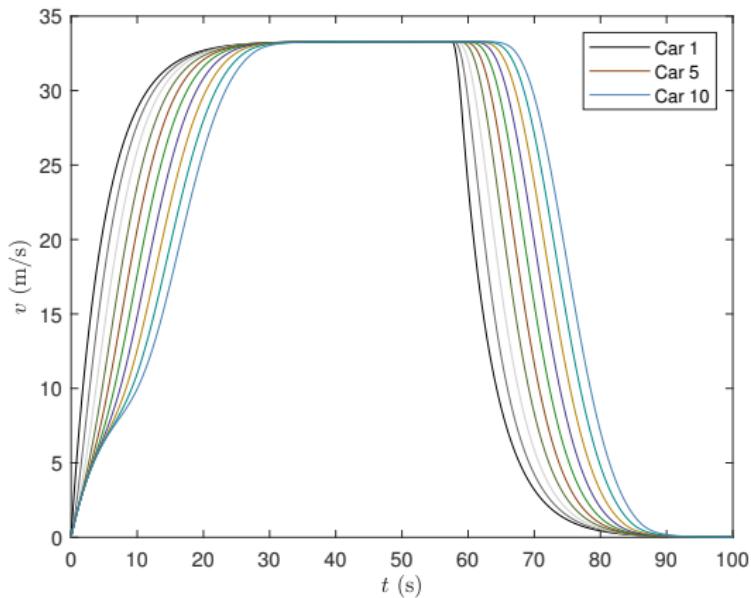


Figure 5: Velocity versus time

# Homogeneous Traffic

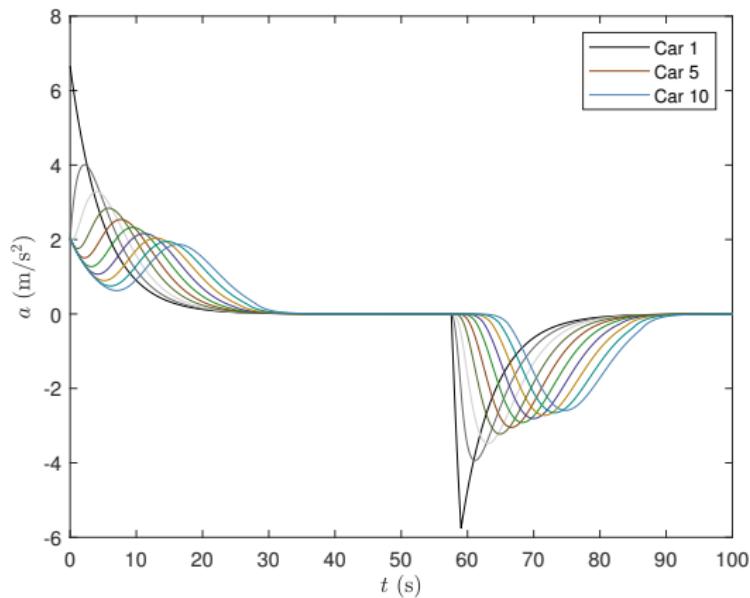


Figure 6: Acceleration versus time

# Homogeneous Traffic

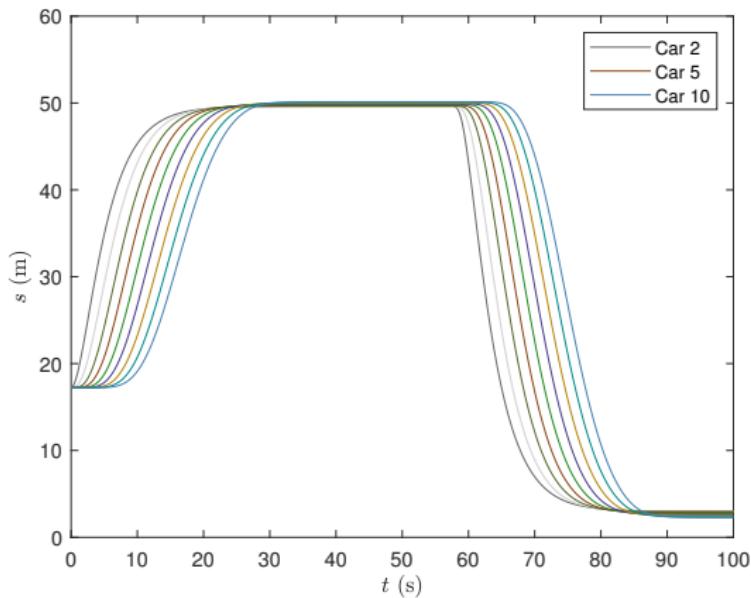


Figure 7: Gap versus time

# Obstacle

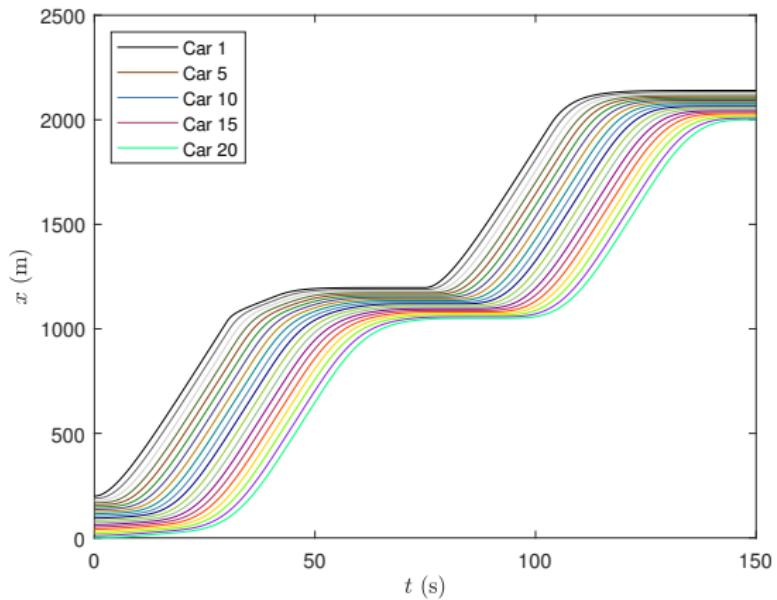
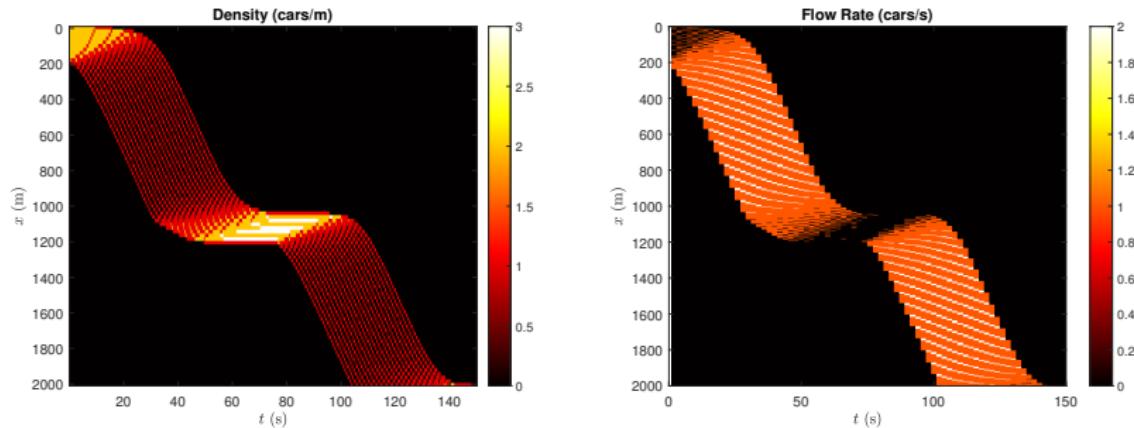


Figure 8: Position versus time

# Obstacle



**Figure 9:** Left to right: density vs time and distance; flow rate vs time and distance

# Obstacle

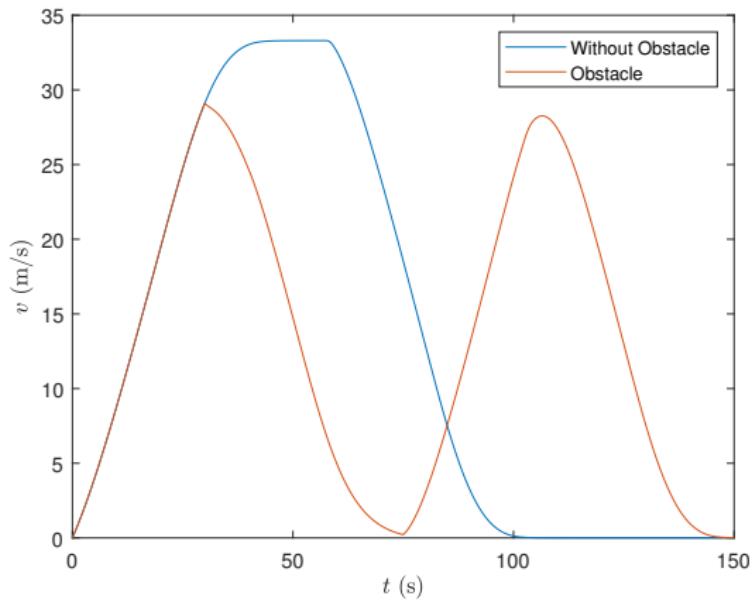
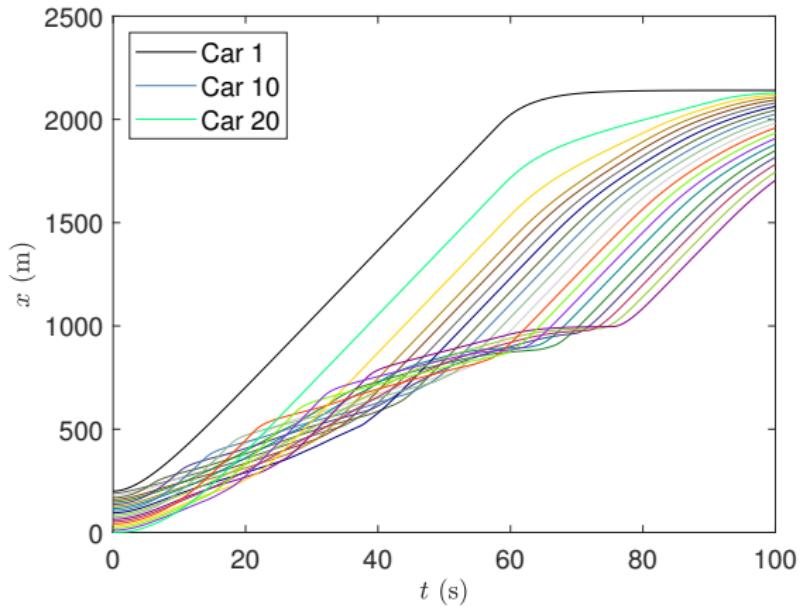


Figure 10: Mean velocity of traffic with and without an obstacle.

# Multi-lane Bottleneck



# Multi-lane Bottleneck

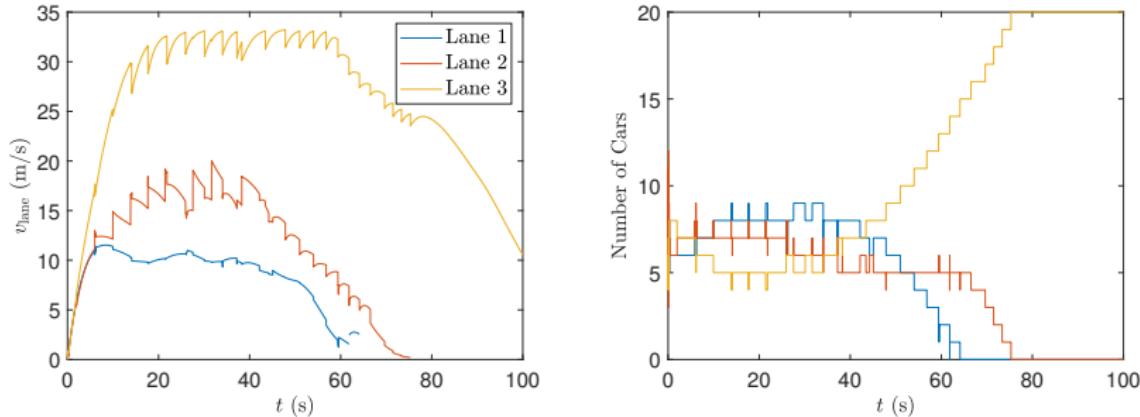


Figure 11: Left to right: Mean velocity in each lane vs time; number of cars in each lane vs time

# Multi-lane Bottleneck

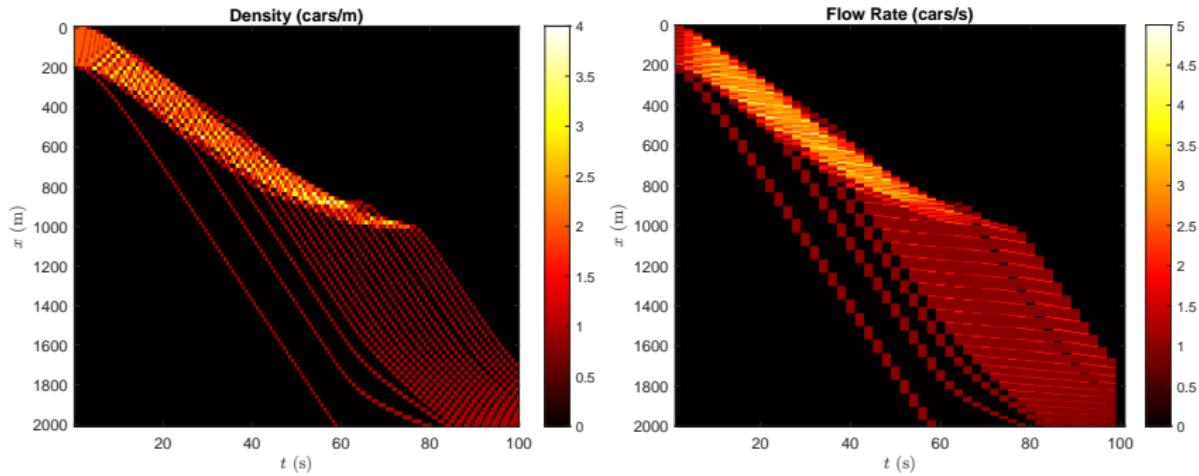


Figure 12: Left to right: density vs time and distance; flow rate vs time and distance

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# Conclusion

- The FVDM with lane changes can model traffic flow and lets us examine how it can be interrupted by obstacles and bottlenecks.
- Pitfalls in the model include unrealistic accelerations and erratic lane changes.
- Future work could model priority lanes and roundabouts