Sending Secret Messages with Synchronized Chaotic Systems

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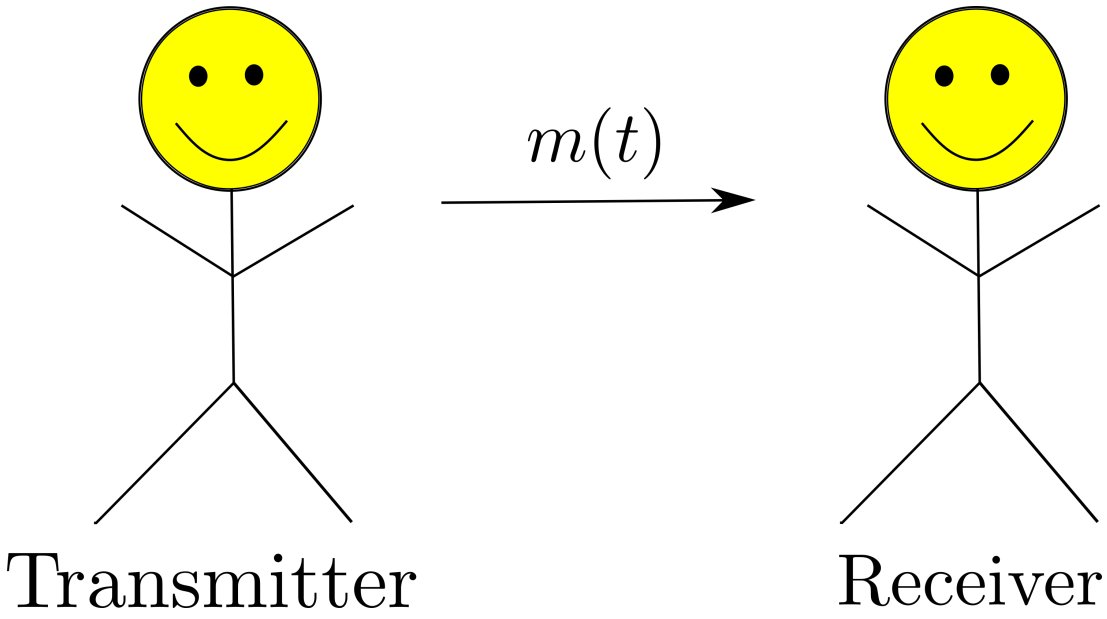
Problem



- Transmitter want to send a message $m(t)$ to Receiver.
- Don't want anyone, but the receiver to read the message.
- Goal: Encrypt the message.

Synchronized Chaotic System

Transmitter

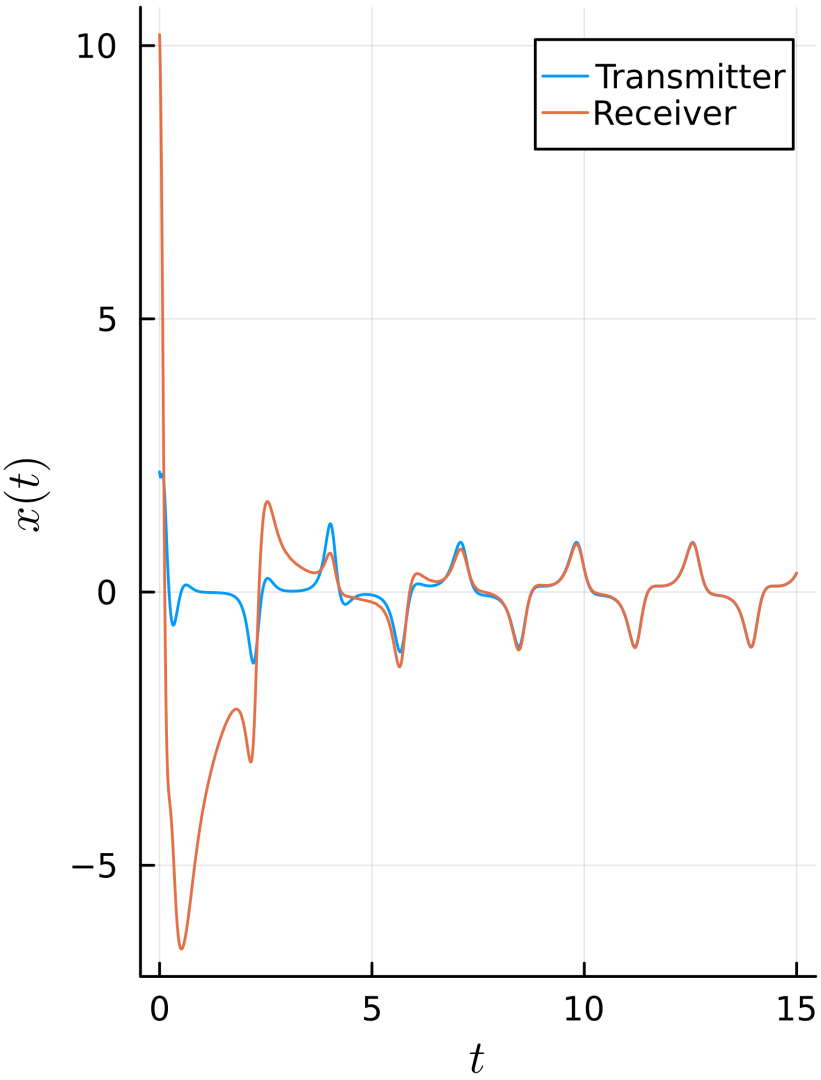
$$\dot{x_T} = \sigma(y_T - x_T)$$
$$\dot{y_T} = \rho x_T - y_T - 20 x_T z_T$$
$$\dot{z_T} = 5 x_T y_T - \beta z_T$$

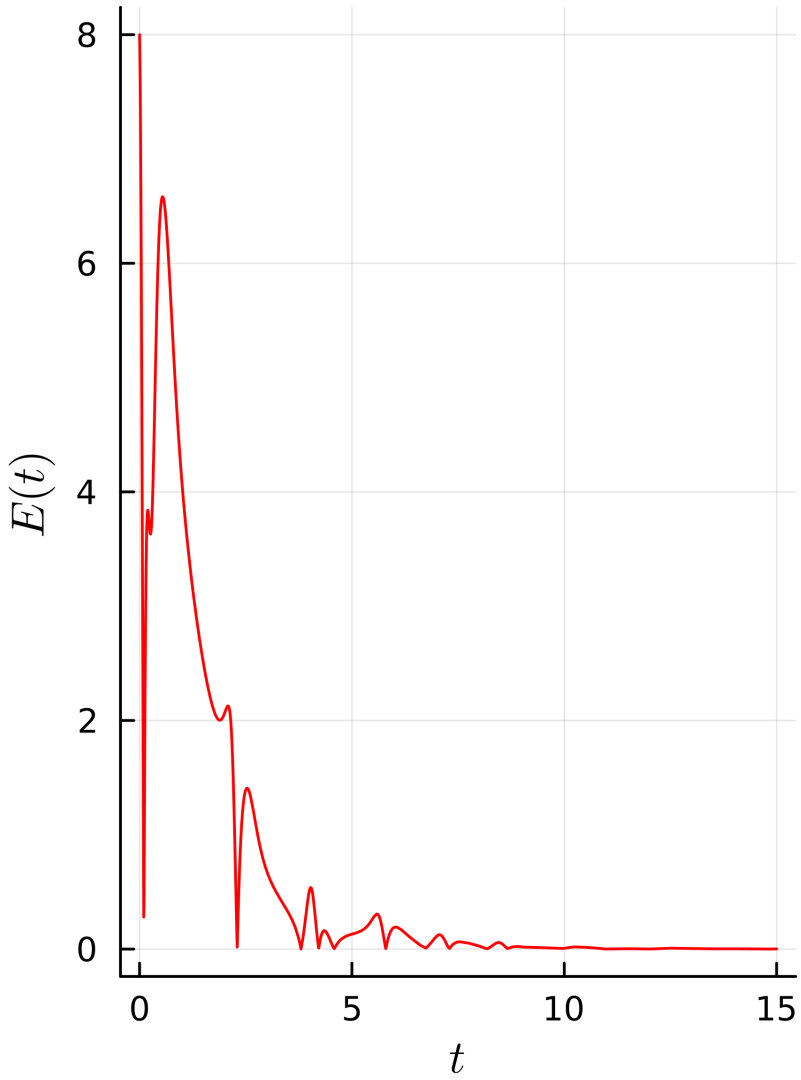
Receiver

$$\dot{x_R} = \sigma(y_R - x_R)$$
$$\dot{y_R} = \rho x_T - y_R - 20 x_T z_R$$
$$\dot{z_R} = 5 x_T y_R - \beta z_R$$

- Based on the Lorenz system.
- The "error" decreases exponentially.

Example of Synchronization





Algorithm to Send Secret Message

1. Create encrypted message $\widetilde{m}(t) = x_T(t) + m(t)$ where $||m(t)|| \ll ||x_T||$.
2. Send $\widetilde{m}(t)$ to the receiver.
3. Use the receiver's dynamical system which hopefully reproduce $x_R(t) \approx x_T(t)$.
4. Compute $\widetilde{m}(t) - x_R \approx x_T(t) + m(t) - x_T(t) = m(t)$.

How Good is It?