

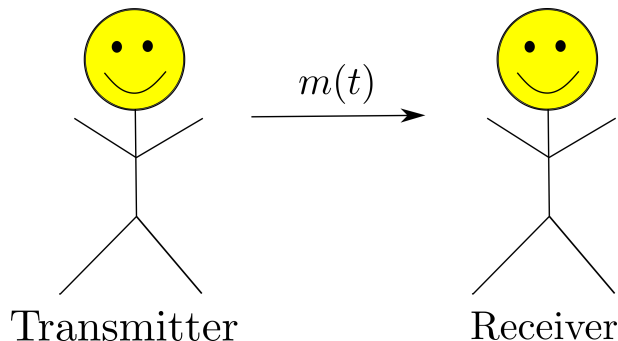
# Sending Secret Messages with Synchronized Chaotic Systems

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# Problem



- Transmitter want to send a message  $m(t)$  to Receiver.
- Don't want anyone, but the receiver to be able to read the message.

# Synchronized Chaotic System

Transmitter

$$\dot{x}_T = \sigma(y_T - x_T)$$

$$\dot{y}_T = \rho x_T - y_T - 20x_T z_T$$

$$\dot{z}_T = 5x_T y_T - \beta z_T$$

Receiver

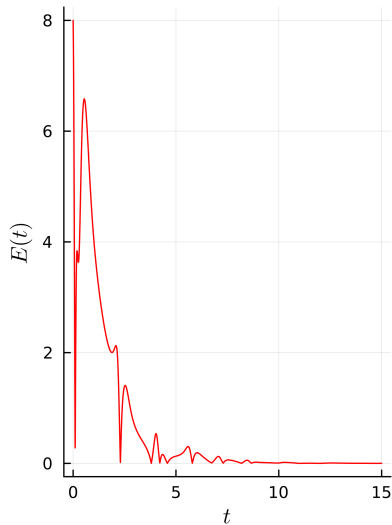
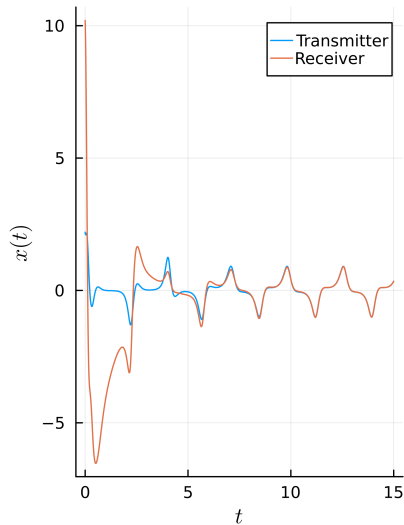
$$\dot{x}_R = \sigma(y_R - x_R)$$

$$\dot{y}_R = \rho x_T - y_R - 20x_T z_R$$

$$\dot{z}_R = 5x_T y_R - \beta z_R$$

- Based on the Lorenz system.
- The error  $E = |x_T - x_R|$  decreases exponentially.

# Example of Synchronization



# Algorithm to Send Secret Message

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- 3 Use the receiver's dynamical system to hopefully reproduce  $x_R(t) \approx x_T(t)$ .
- 4 Compute  $\tilde{m}(t) - x_R(t) \approx x_T(t) + m(t) - x_T(t) = m(t)$ .



# How Good is It?

