

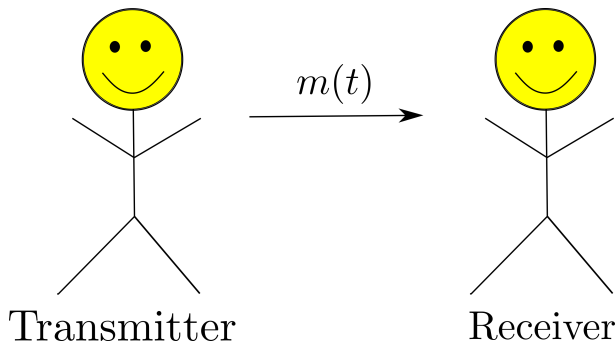
Sending Secret Messages with Synchronized Chaotic Systems

Kevin Phan

Harvey Mudd College

April 26, 2023

Problem



- Transmitter want to send a message $m(t)$ to Receiver.
- Don't want anyone, but the receiver to be able to read the message.

Synchronized Chaotic System

Transmitter

$$\dot{x}_T = \sigma(y_T - x_T)$$

$$\dot{y}_T = \rho x_T - y_T - 20x_T z_T$$

$$\dot{z}_T = 5x_T y_T - \beta z_T$$

Receiver

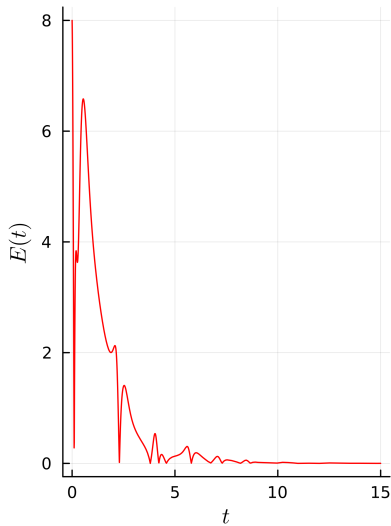
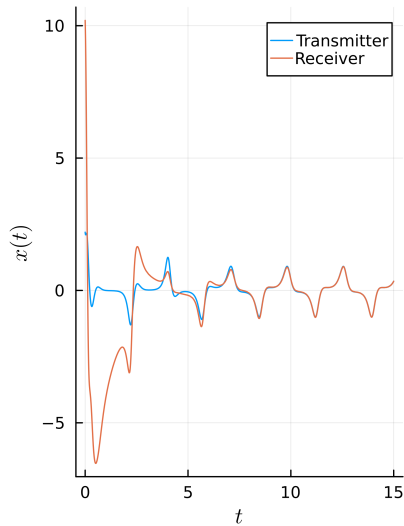
$$\dot{x}_R = \sigma(y_R - x_R)$$

$$\dot{y}_R = \rho x_T - y_R - 20x_T z_R$$

$$\dot{z}_R = 5x_T y_R - \beta z_R$$

- Based on the Lorenz system.
- The “error” decreases exponentially.

Example of Synchronization



Algorithm to Send Secret Message

- 1 Create encrypted message $\tilde{m}(t) = x_T(t) + m(t)$ where $\|m(t)\| \ll \|x_T(t)\|$.
- 2 Send $\tilde{m}(t)$ to the receiver.
- 3 Use the receiver's dynamical system which hopefully reproduce $x_R(t) \approx x_T(t)$.
- 4 Compute $\tilde{m}(t) - x_R(t) \approx x_T(t) + m(t) - x_T(t) = m(t)$.

How Good is It?

