

# Phenological models on microbial growth – which model is better and why?

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## Abstract

## Introduction

Phenological models are expected to fit data trends within its biological field. Yet due to different reasons, models developed and published from one sample may not fit the others. These reasons may be due to data variabilities, confounding factors, inaccurate assumptions or models being too-specific. This project is aimed at compare and contrast published phenological models on microbial population size data, highlighting which is a better model under what conditions. The hypotheses are:

- published phenological models are better than polynomials in describing microbial population size;
- appropriate phenological model(s) can be identified through distinguishable shapes of microbial population size; and
- parameters of data under each phenological model is clustered, similar with dataset best-described by the same model but different from those described by other models.

## Methods

Experimental microbial population growth data library were divided into individual data subsets through six filters (“Temperature (in °C)”, “Microbial clade”, “growth substrate materials”, “experimental replicate number”, “population data recording unit” and “data source”). Records with data unit “OD<sub>595</sub>” were scaled into optical density percentages (i.e. data\*100)

to facilitate general analyses workflow. Independent (or explanatory) variable was “Time (hr)” and dependent (or response) variable was “population size”. Some raw data were recorded in minutes (instead of hour). This record artifact was not corrected because of two reasons: 1. shape of curves were the main concern instead of independent variable’s scale; and 2. the unit was consistent within each data subset.

## Model assessment

Six candidate models were assessed, four phenological and two polynomial equations. They were “Verhulst (classical)”<sup>1</sup>, “modified Gompertz”<sup>2</sup>, “Baranyi”<sup>3</sup>, “Buchanan”<sup>4</sup>, “quadratic” and “cubic”. NLLS was used only on the four phenological models and linear model-fitting was done on the two polynomials. Starting values selection (for phenological models only) was described below:

Initial (N0) and final (K) population sizes were selected to be the minimum and maximum values of each data subset respectively. Maximum growth rate (r.max) was selected by linear model through a recursive manner. For every iteration, population size data from the top 5% independent variable values were excluded from the linear model calculation. The data and slope would only be recorded if it was positive, higher adjusted  $R^2$  value and larger slope than the recorded “best slope” value. After scanning from the maximum side, the best slope and its respective data were taken out and screened from the minimum side. Final best slope and x-intercept were regarded as the r.max and relative time lag (t.lag) of the population (in the source experiment) respectively. Time which this linear model intersected with K was regarded as the time achieving carrying capacity (t.K). Population data was then classified into three groups (gx) according to the time:  $g1 \leq t.lag < g2 < t.K \leq g3$ . 5% was chosen as the scanning threshold because I assumed this resolution was fine enough for achieving good starting values for NLLS fitting. Inputs for phenological models were listed below (popn & time were the dependent and independent variables respectively):

Verhulst (classical):  $popn = f(N0, K, r.max, time)$

modified Gompertz:  $popn = f(N0, K, r.max, time, t.lag)$

Baranyi:  $popn = f(N0, K, r.max, time, t.lag)$

Buchanan:  $popn = f(N0, K, r.max, time, t.lag, gx)$

All test starting values were than sampled from normal distribution with mean as the estimated value and standard deviation (sd) of 1. The sd value was chosen because of different

reasons for each parameters.  $N_0$  and  $K$  were directly extracted from the raw experimental data, which could be assumed being an accurate estimate for that data subset (hence a small sd was logical).  $r_{\max}$  was a guesstimated value from fitting linear models. This process could potentially be affected by extreme values in the data and hence a large sd should be preferred. 100 trials were done as a optimal value under a trade-off between efficiency and accuracy.

Only  $AIC^{5-7}$  was used to select for optimal parameter values within each phenological model and best model between the six candidates for a data subset. Reasons would be listed in Discussion section. For models with more than one parameter sets as sharing the lowest AIC value, the first set of values from the random sampling trials were used for downstream analyses. AIC tolerance threshold was expanded to  $\min(AIC)+2^8$  to incorporate more accepted models for analyses.

## Statistical analysis

Kruskal test was used for identify the best-fit model among all included model because the count was categorical and not assumed being normally-distributed. Pairwise Nemenyi comparisons would be carried out to identify the best test if p-value of the above test was significant.

## Main Assumptions

- there was no negative population growth (i.e. starting population was always lower than carrying capacity), so negative population growth data were set to zeros;
- estimated parameter estimates would always result in a global optimal status in parameter space through the non-linear least squares method (NLLS)

## Computing tools

R (ver 3.6.0)<sup>9</sup> was used with “minpack.lm”<sup>10</sup> for computing non-linear least square statistics for model comparisons. “PMCMR”<sup>11</sup> was used for carrying out statistical analyses.

## Results

From Fig.1, large fluctuations between each model to be described as “best-fit” were observed. However the occurrence difference was not statistical significant. Among the counts, there were

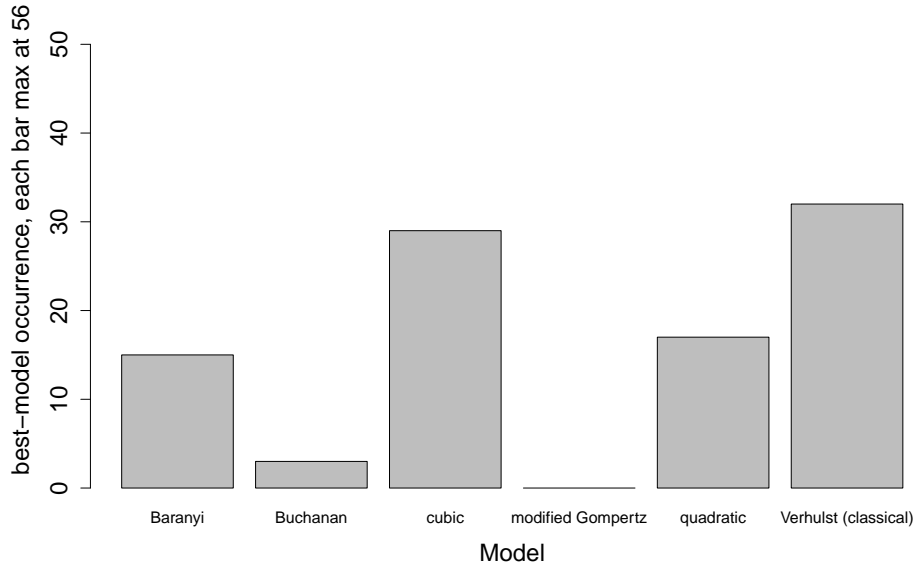


Figure 1: Barplot showing the number of “best model” identification under AIC model-selection methods with “ Kruskal-Wallis rank sum test ” statistic  $X^2 = 5$  ,  $df = 5$  ,  $p = 0.42$

40 datasets with more than one “best-fit” models. Verhulst (classical) and cubic were the top two models selected as “best-fit” for the 56 datasets ( 32 for Verhulst (classical) and 29 for cubic). There are 11 datasets calling both “best-fit” at the same trial. Between Baranyi and quadratic, the counts were 15 and 17 respectively with 6 datasets calling both models “best-fit”. The only outstanding performance was from modified Gompertz, which 0 datasets were called it as “best-fit”.

## Discussion

AIC is considered the most suitable model-selection approach within model and between models in this project. Unlike BIC, AIC are more accurate with small sample sizes<sup>12,13</sup> and sparse data<sup>13</sup>. AIC did not assume a “true model” was under examination<sup>14–16</sup>. Since candidate models were not “nested model”, BIC is not a better choice than AIC<sup>17</sup>. Hence the use of only AIC as model-selection criterion should be justified.

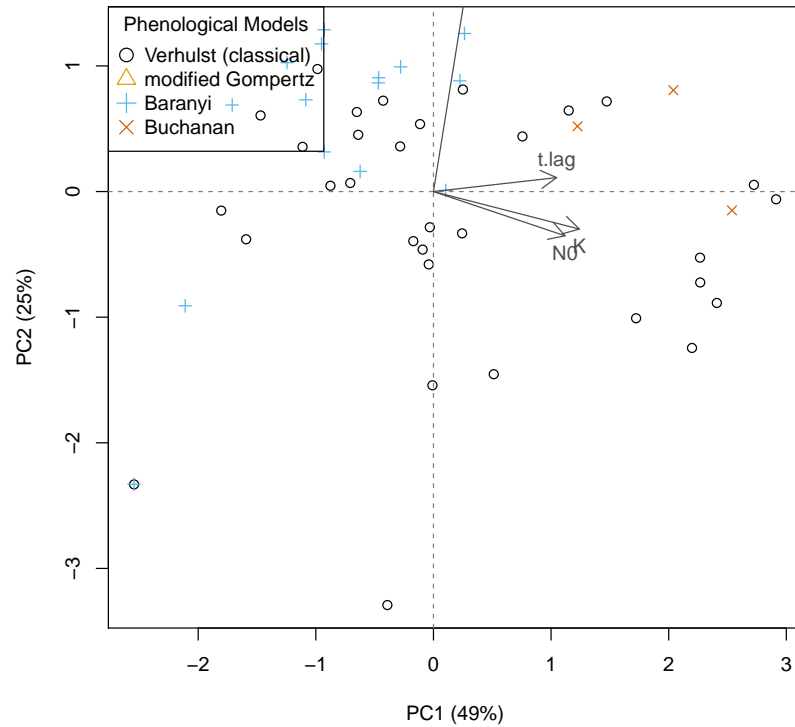


Figure 2: Biplot of Principal Component Analysis (PCA) comparing phenological models on estimated parameter values with minimal AIC evaluations.

## Conclusion

Published phenological models were data-specific, which none of them were found significantly performing better than the others in general.

## Code and Data Availability

All scripts and data used for this report were publicly available at GitHub.

## References

- McKendrick, A. & Pai, M. K. XLV.—the rate of multiplication of micro-organisms: a mathematical study. *Proceedings of the Royal Society of Edinburgh* **31**, 649–653 (1912).
- Gil, M. M., Brandão, T. R. & Silva, C. L. A modified Gompertz model to predict microbial inactivation under time-varying temperature conditions. *Journal of Food Engineering* **76**.

- Bugdeath, 89–94. ISSN: 0260-8774. <http://www.sciencedirect.com/science/article/pii/S0260877405003389> (2006).
3. Baranyi, J., McClure, P., Sutherland, J. & Roberts, T. Modeling bacterial growth responses. *Journal of industrial microbiology* **12**, 190–194 (1993).
  4. Buchanan, R., Golden, M. & Whiting, R. Differentiation of the effects of pH and lactic or acetic acid concentration on the kinetics of *Listeria monocytogenes* inactivation. *Journal of Food Protection* **56**, 474–478 (1993).
  5. Johnson, J. B. & Omland, K. S. Model selection in ecology and evolution. *Trends in ecology & evolution* **19**, 101–108 (2004).
  6. Akaike, H. in *Selected papers of hirotugu akaike* 199–213 (Springer, 1998).
  7. Burnham, K. & Anderson, D. Model selection and multimodel inference: a practical information-theoretic approach. *Ecological Modelling*.
  8. Burnham, K. P. & Anderson, D. R. Multimodel inference: understanding AIC and BIC in model selection. *Sociological methods & research* **33**, 261–304 (2004).
  9. R Core Team. *R: A Language and Environment for Statistical Computing* R Foundation for Statistical Computing (Vienna, Austria, 2019). <https://www.R-project.org/>.
  10. Elzhov, T. V., Mullen, K. M., Spiess, A.-N. & Bolker, B. *minpack.lm: R Interface to the Levenberg-Marquardt Nonlinear Least-Squares Algorithm Found in MINPACK, Plus Support for Bounds* R package version 1.2-1 (2016). <https://CRAN.R-project.org/package=minpack.lm>.
  11. Pohlert, T. *The Pairwise Multiple Comparison of Mean Ranks Package (PMCMR)* R package (2014). <https://CRAN.R-project.org/package=PMCMR>.
  12. Acquah, H. D.-G. Comparison of Akaike information criterion (AIC) and Bayesian information criterion (BIC) in selection of an asymmetric price relationship. *Journal of Development and Agricultural Economics* **2**, 001–006 (2010).
  13. Kuha, J. AIC and BIC: Comparisons of assumptions and performance. *Sociological methods & research* **33**, 188–229 (2004).
  14. Aho, K., Derryberry, D. & Peterson, T. Model selection for ecologists: the worldviews of AIC and BIC. *Ecology* **95**, 631–636 (2014).

- 140 15. Vrieze, S. I. Model selection and psychological theory: a discussion of the differences  
141 between the Akaike information criterion (AIC) and the Bayesian information criterion  
142 (BIC). *Psychological methods* **17**, 228 (2012).
- 143 16. Yang, Y. Can the strengths of AIC and BIC be shared? A conflict between model inden-  
144 tification and regression estimation. *Biometrika* **92**, 937–950 (2005).
- 145 17. Wang, Y. & Liu, Q. Comparison of Akaike information criterion (AIC) and Bayesian infor-  
146 mation criterion (BIC) in selection of stock–recruitment relationships. *Fisheries Research*  
147 **77**, 220–225 (2006).
- 148 18. Schwarz, G. Estimating the dimension of a model. *Ann. Stat.* **6**, 461–464 (1978).
- 149 19. Kelley, C. T. *Iterative methods for optimization* (SIAM, 1999).
- 150 20. Turchin, P. *Complex population dynamics: a theoretical/empirical synthesis* (Princeton  
151 university press, 2003).