Statistics with SpaRRows

Lecture 10

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Outline

- Hypothesis testing in linear models
- Interpretation (a bit)
- Standardizing
- Reporting

Hypothesis testing in linear models

- What do we actually test?

$$y_i = b_0 + b_1 x_i + \varepsilon_i$$

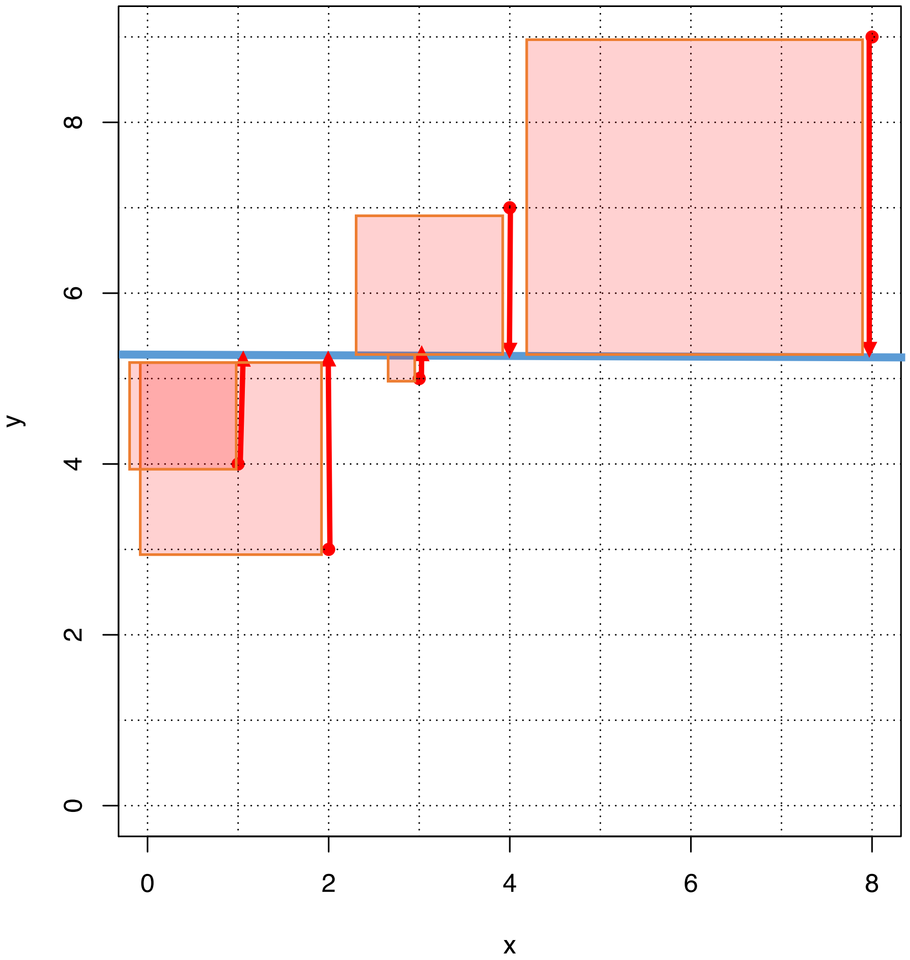
y	i
5	1
3	2
6	3
10	4
4	5

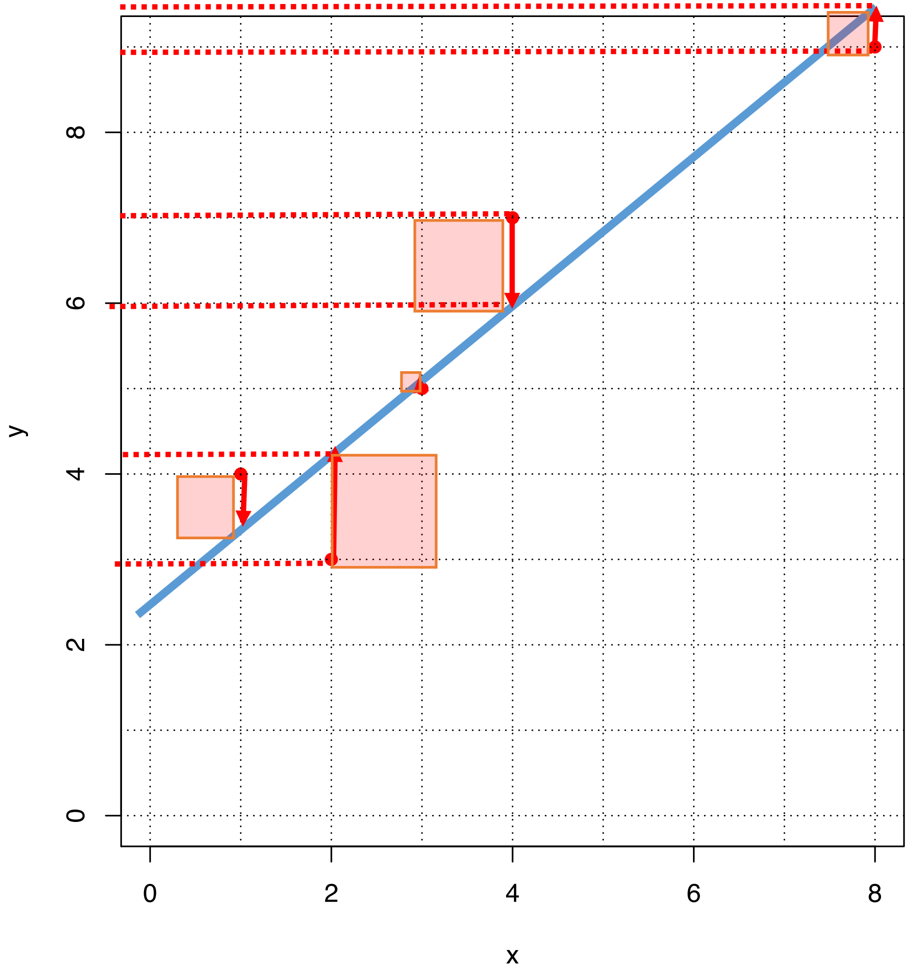
b_0 = ?

b_1 = ?

x	i
3	1
1	2
4	3
8	4
2	5

ε	i
?	1
?	2
?	3
?	4
?	5





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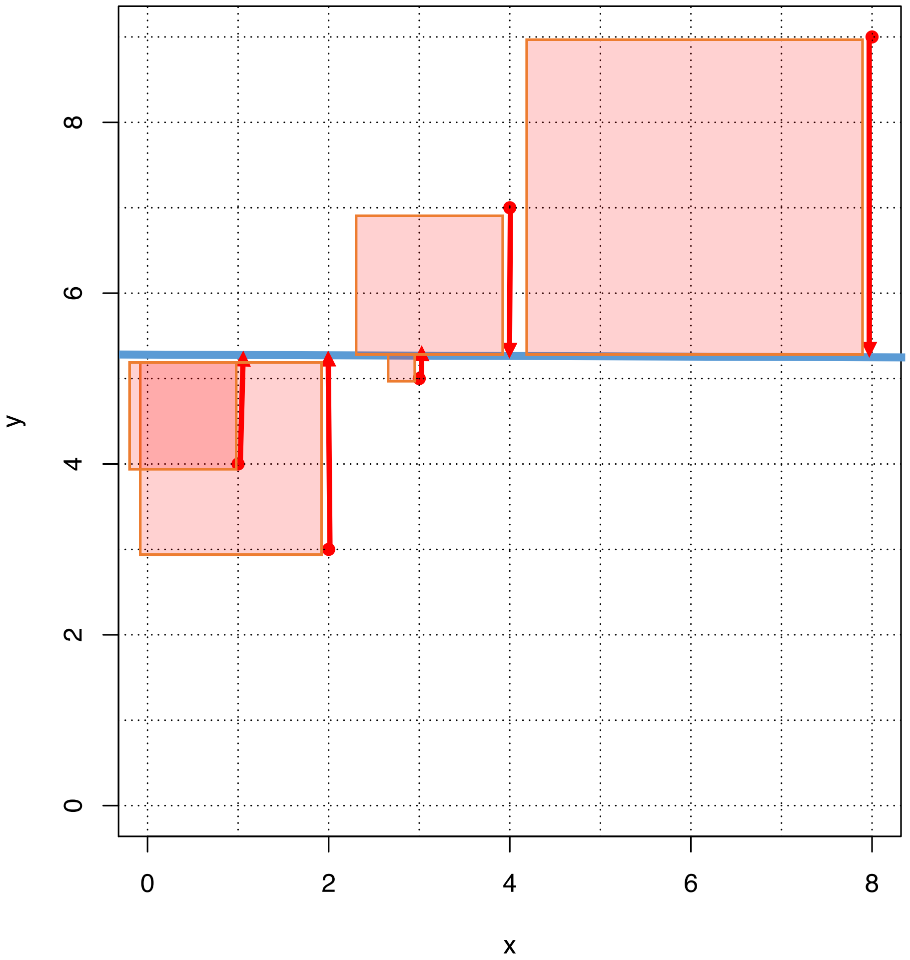

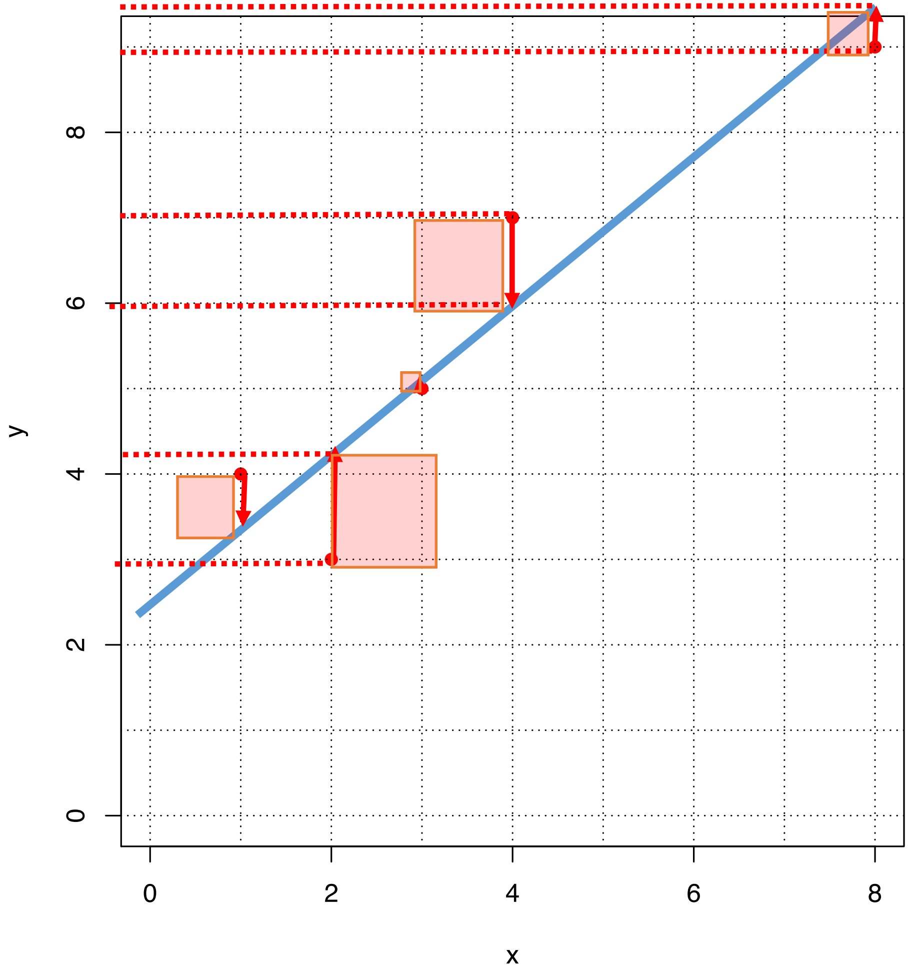

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→ Test if slope (b_1) estimate is different from 0
→ T-test!

Hypothesis testing:

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    Min      1Q  Median      3Q     Max 
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Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  5.83246    0.98195    5.94 3.48e-09 ***
Tarsus       1.18466    0.05295   22.37  < 2e-16 ***
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Tests null hypotheses that

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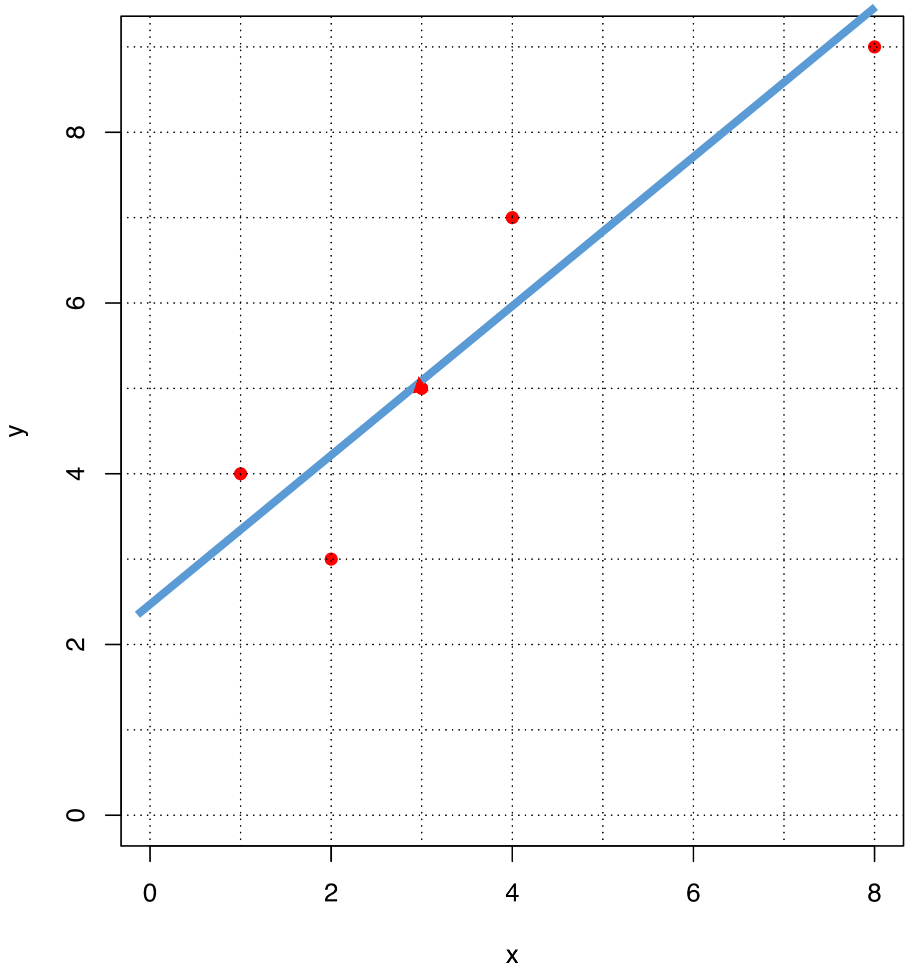

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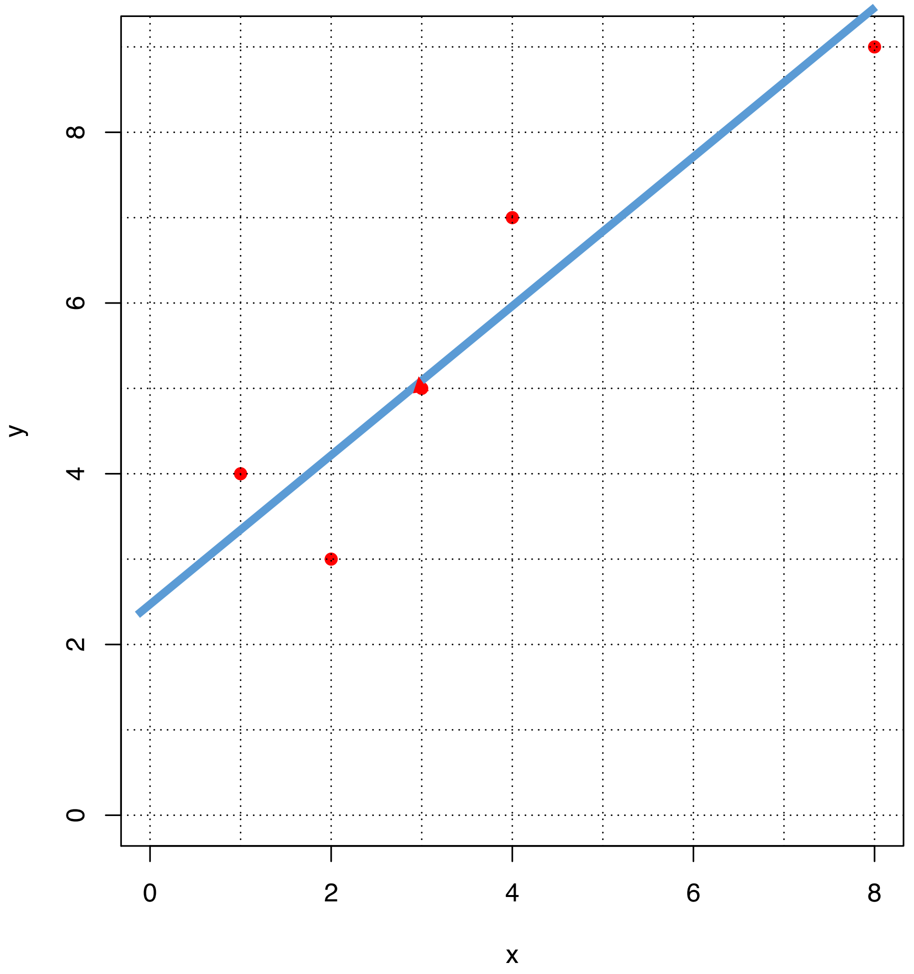
What does it mean if the null hypothesis is rejected?

Usually not much for b_0



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The slope is 5.8
For each mm longer tarsus, a sparrow weights 1.18g more.

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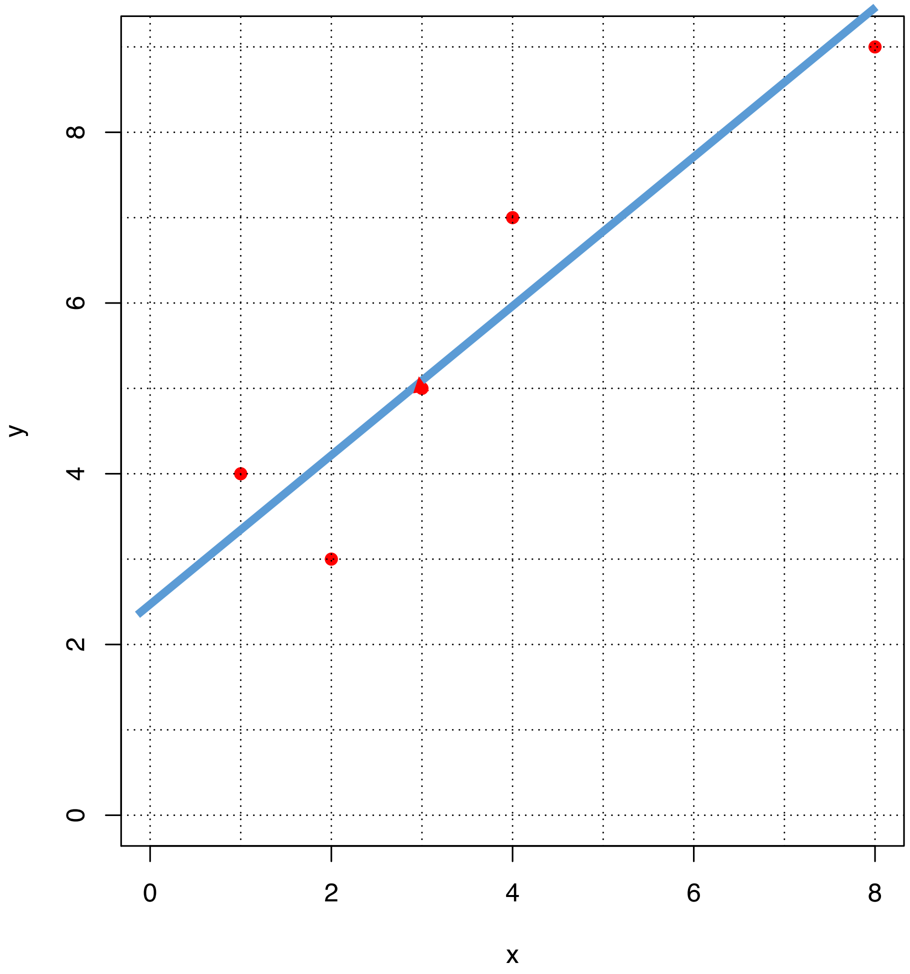
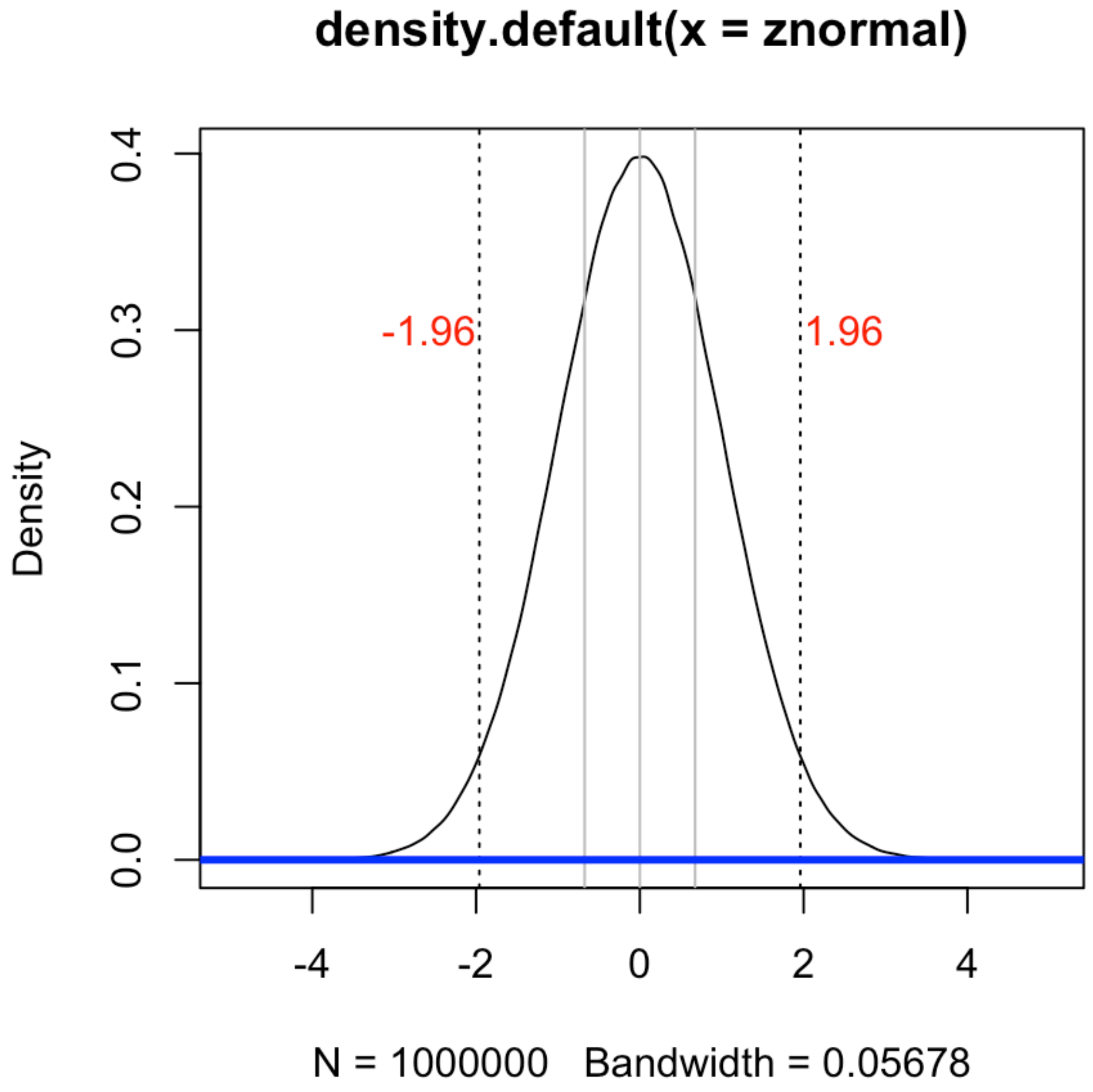
What does it mean if the null hypothesis is rejected?

Usually not much for b_0
b_1 is more interesting biologically speaking



Why standardize data?

- z-scores:
- Normal distributed
- Mean of 0
- Sd of 1



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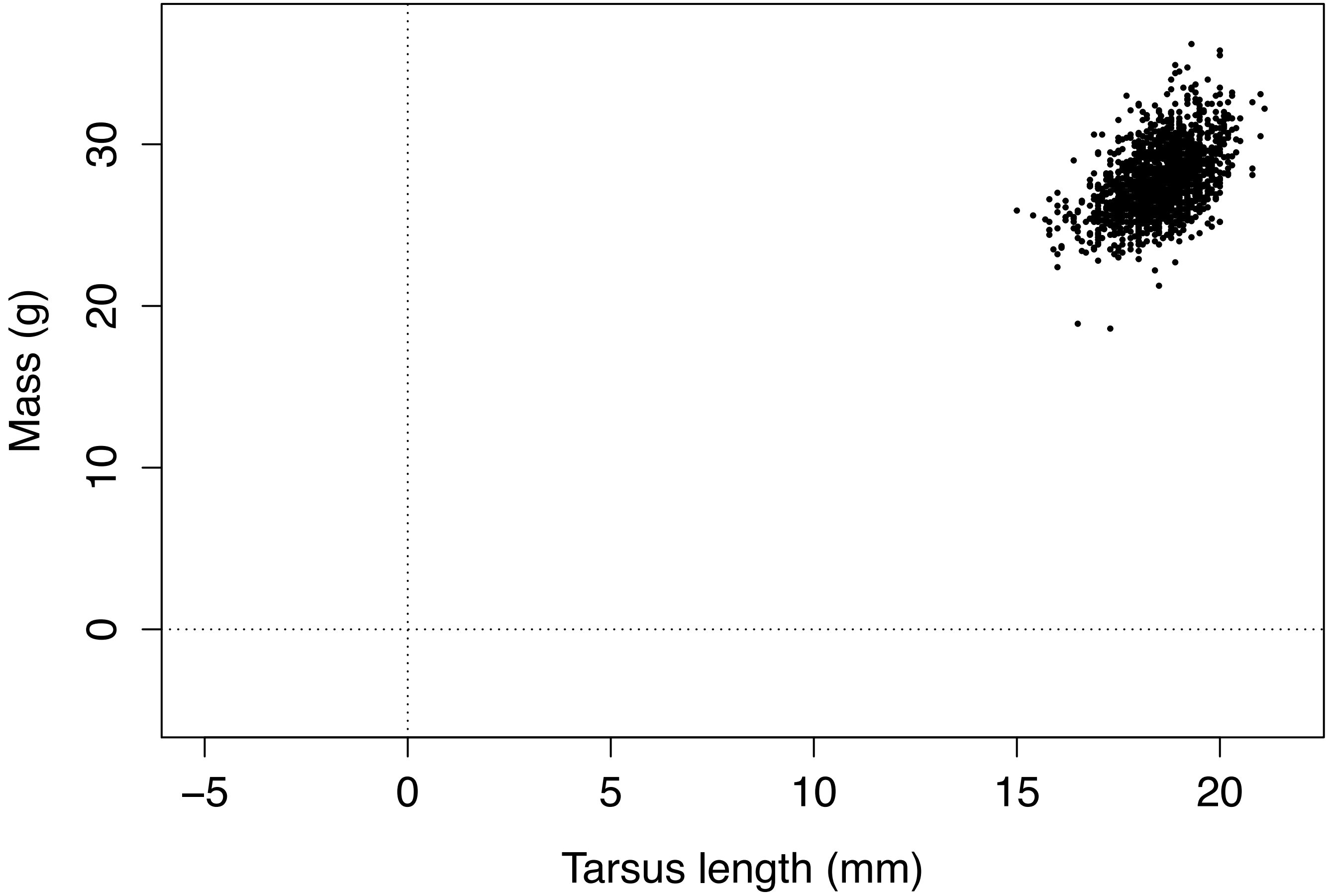

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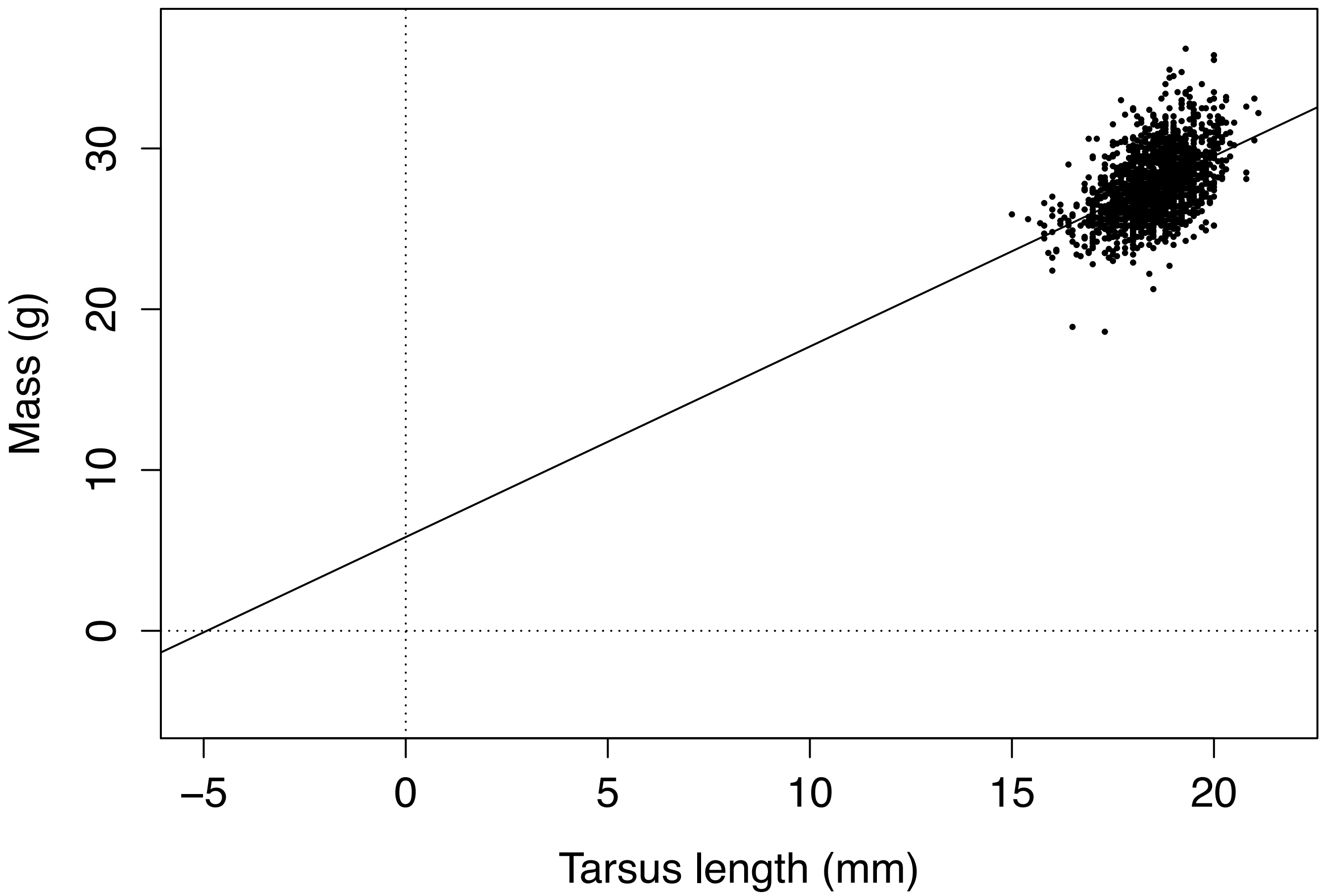

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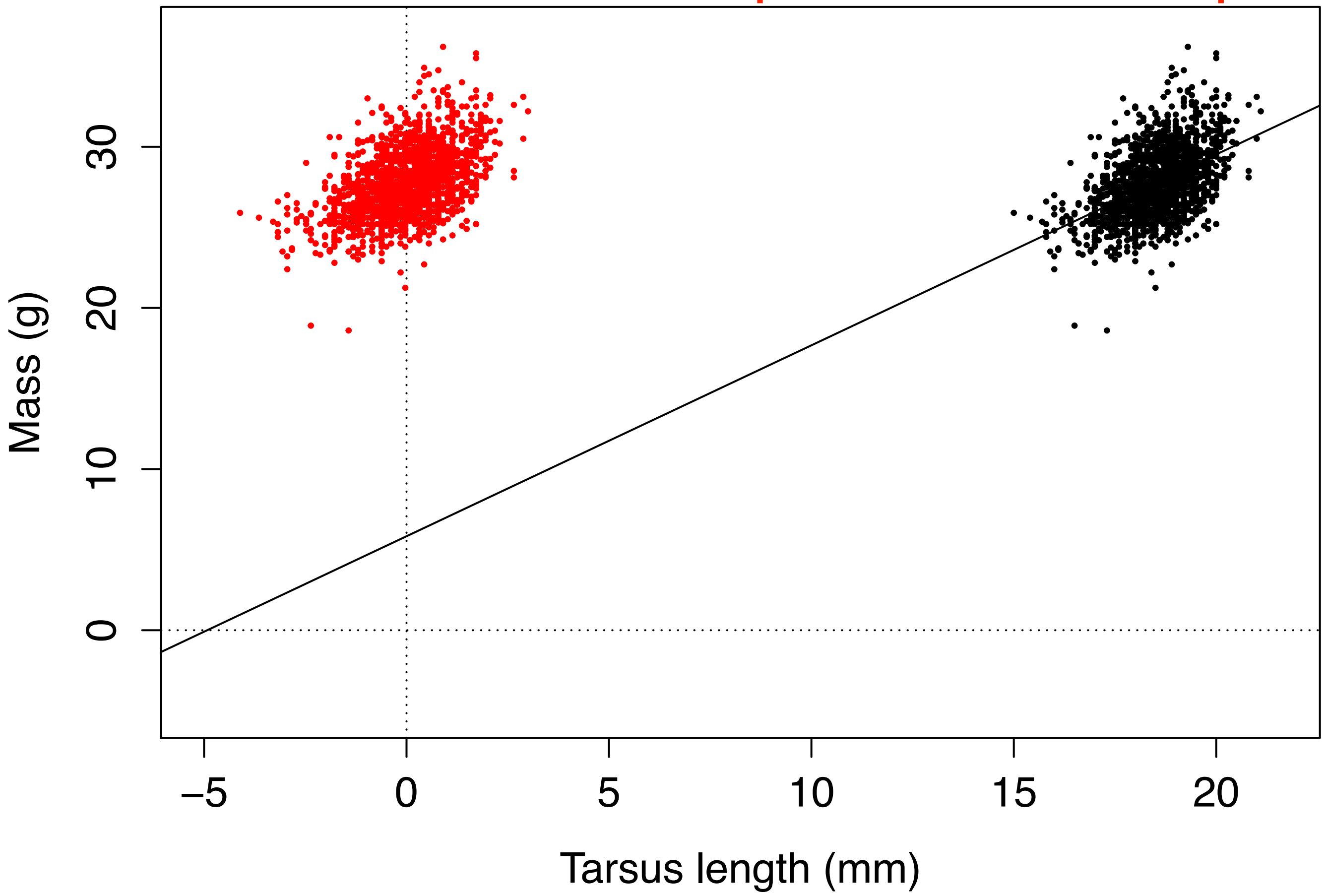
standardized intercept = mean of response variable



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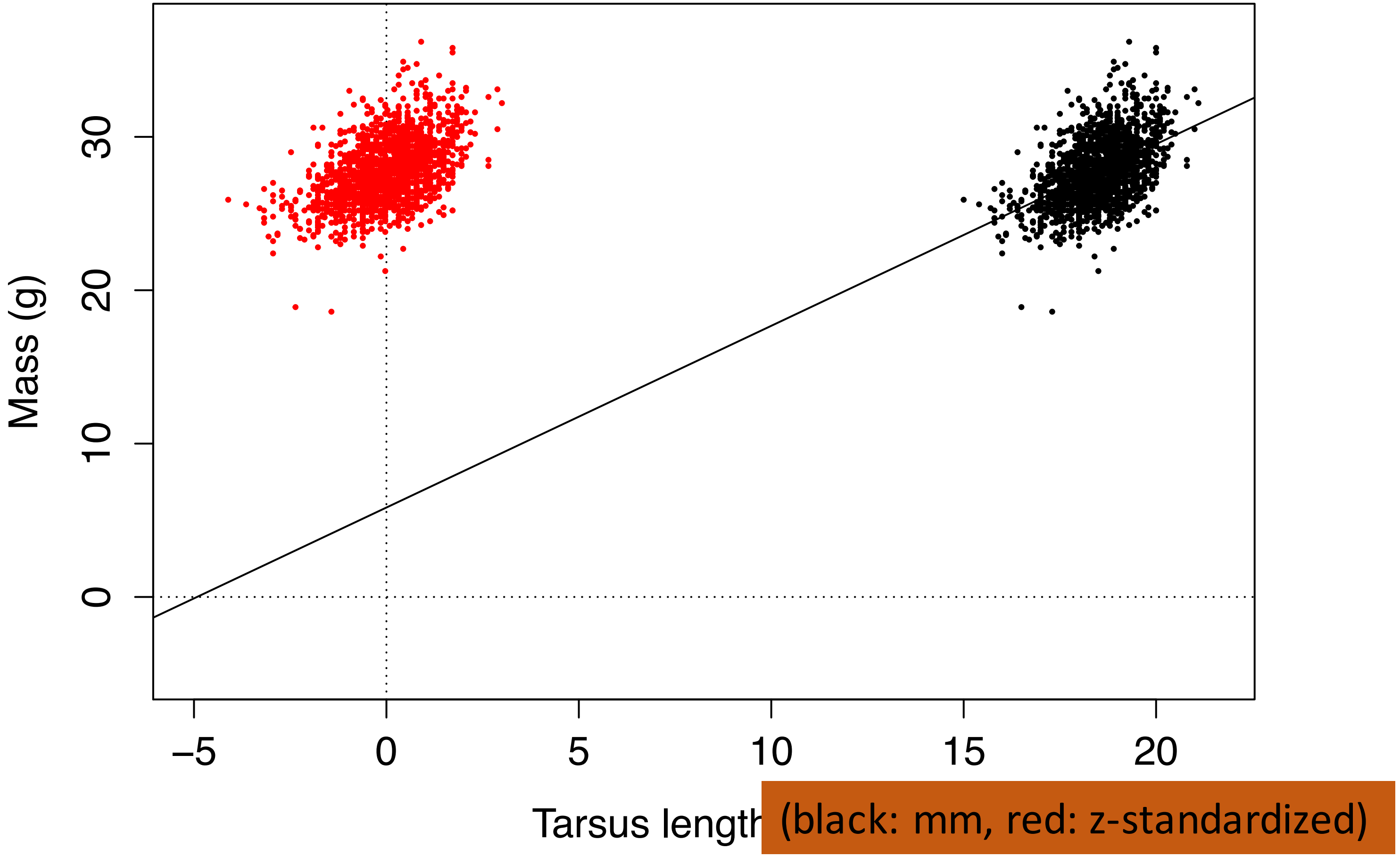

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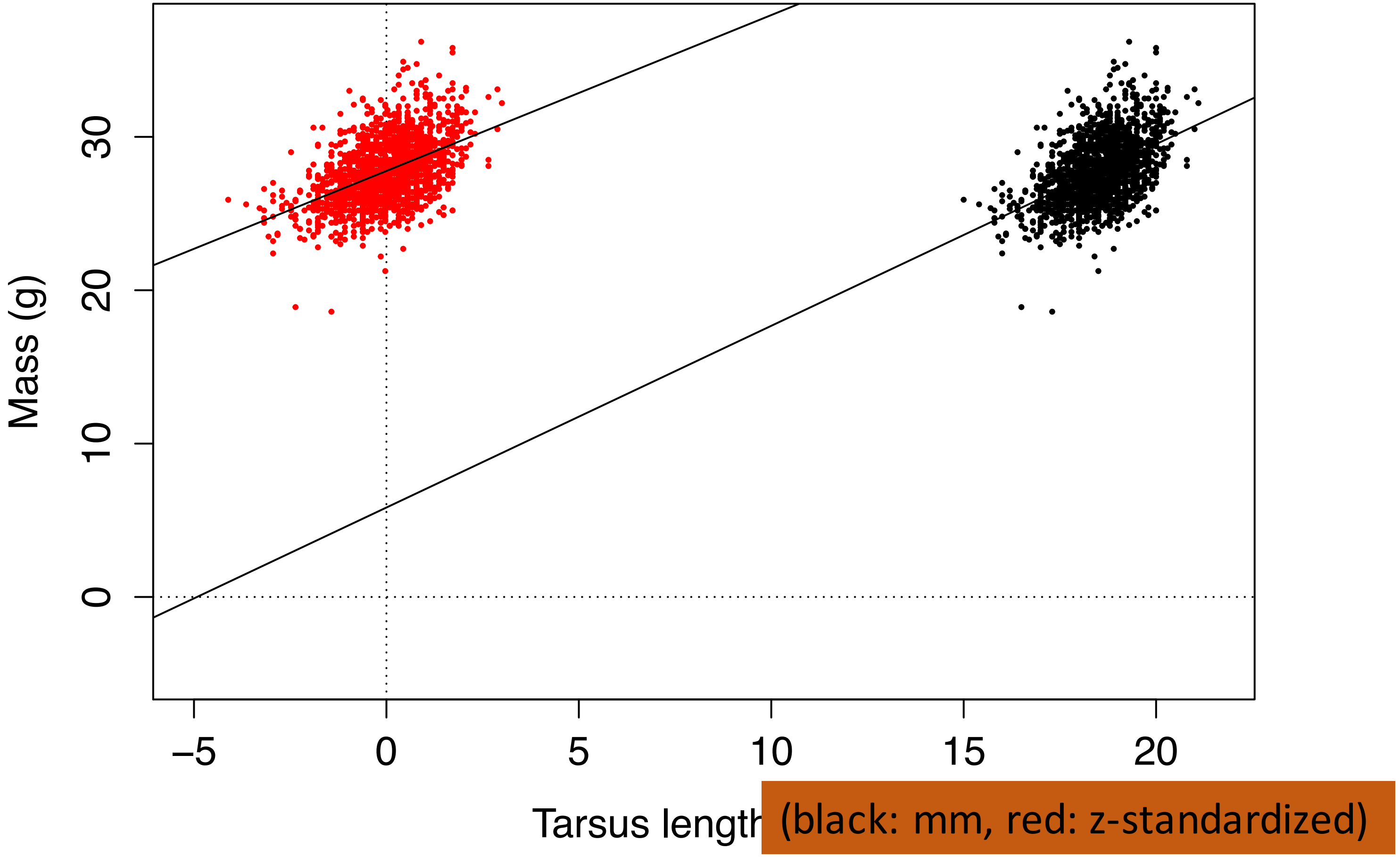
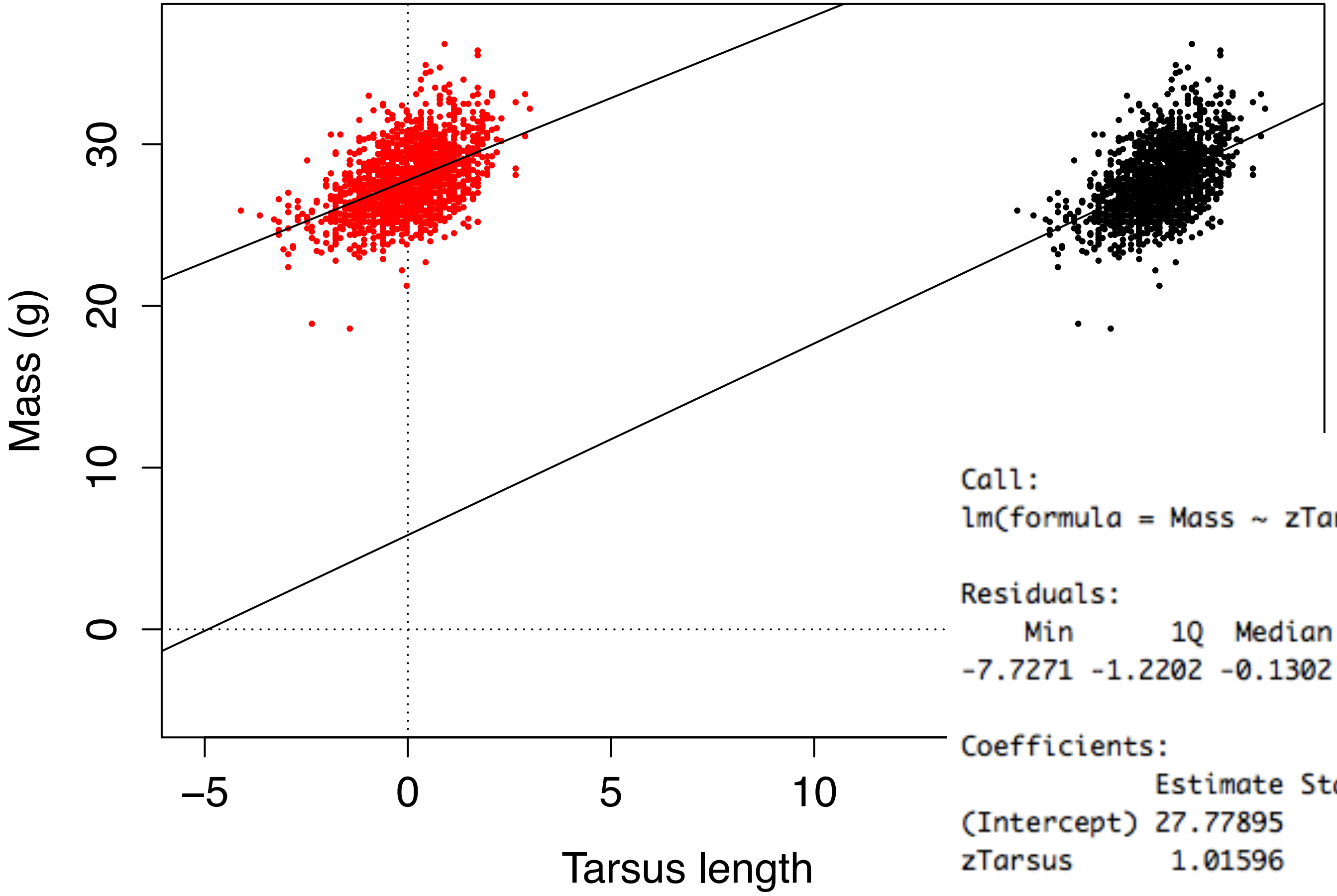


Why standardize data?

compare different data sources (e.g. elephant & sparrow)



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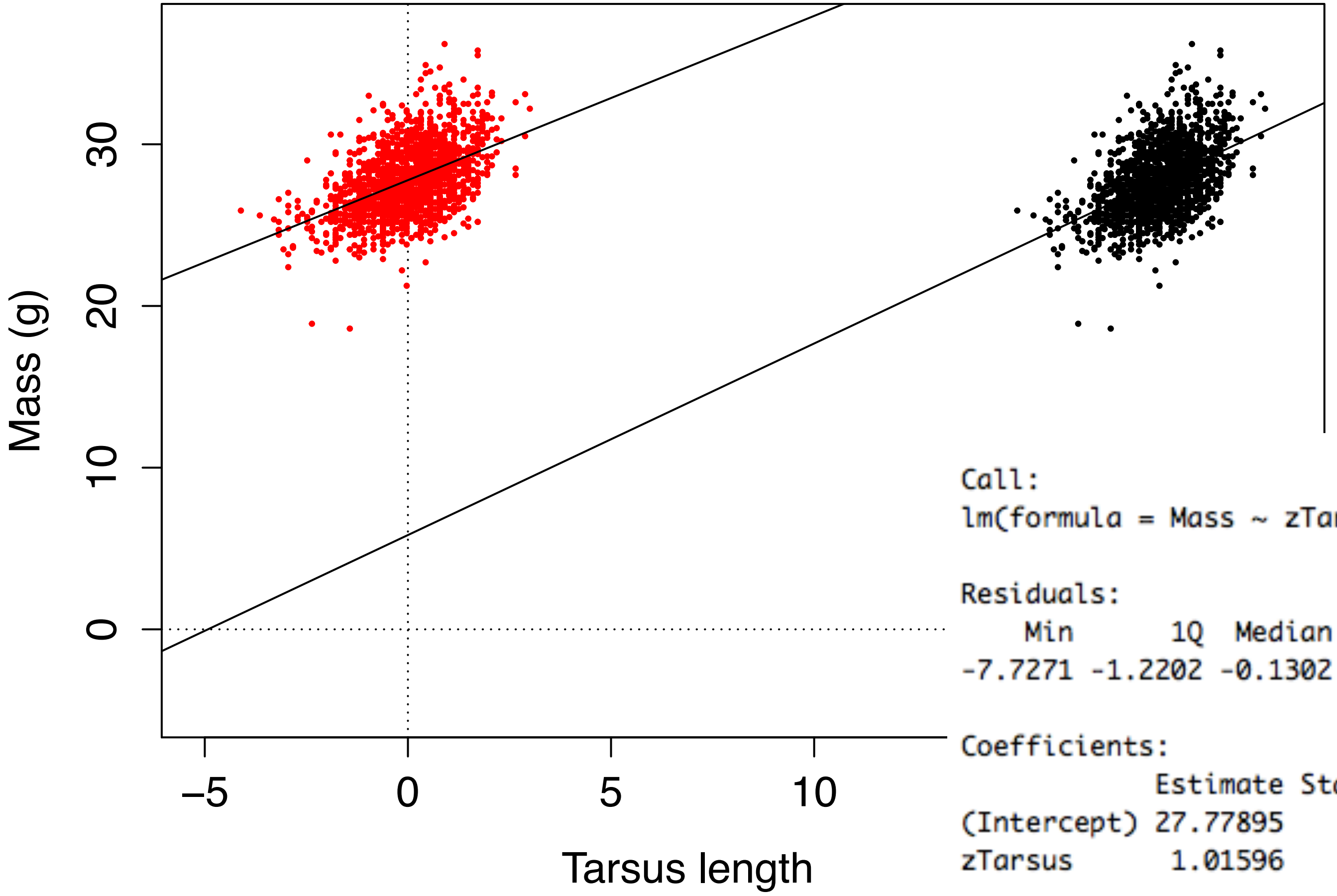

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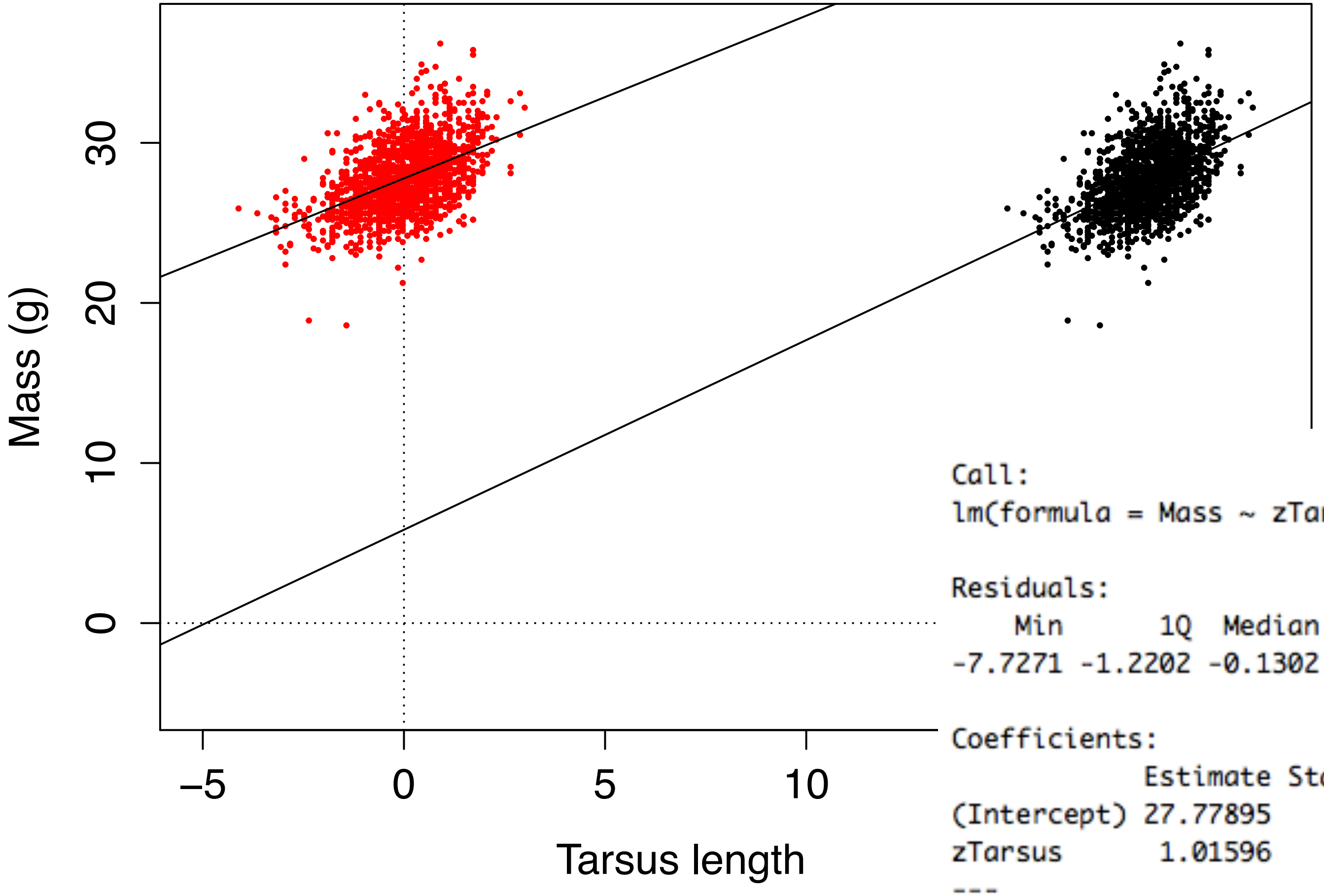

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Interpretation of statistics – take home

- Always consider units!
- Standardize to make intercept meaningful
- T-tests test for null hypothesis that parameter estimates equal –

- Always think of the biological meaning in units!

How to report?

Methods:

correlation

To test whether heavier birds also had longer tarsi, I used a linear model, where body mass (g) was the response variable, and tarsus length (mm) the explanatory variable.

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what is standardized and why

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- Table legend goes on top of tables (below figures)
- Table legend should be self-explanatory without refering to text!

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How to report – take home

- Methods:
- Always describe *all analyses you present in results. Be precise and specific. Describe what's the response variable. Say what is the explanatory variable, and say WHY you used those. Give the units. Say when you standardize, and WHY.
- Justify, justify, justify.

How to report – take home

- Results:
- Start with describing the dataset.
- Sample size, mean, range, missing values ect.
- Explain results of each analysis
- Make nice tables and think before copy/pasting values from R!
- Legends need be self-explanatory!

HO 10– DO IT NOW!

- Do HO 10! Do the excersises on HO 10!
- Run diagnostics for a model with sex as explanatory variable. Interpret the plots.
- Run a linear model, where you test the hypothesis that sparrows with bigger bills can eat more. The prediction is that the larger the bill, the heavier the sparrow.
- Write a short (1A4) report on methods and results on this last model.
- Before you go into the linear model, you should first describe your data, say how many sparrows, how many females and males, **whether there is a difference in your response between the sexes. If that difference is meaningful, you should test the sexes separately**. Write this section as you would write it for a scientific article.