

Non-Parametric testing  
Wilcoxon two-sample tests  
Wilcoxon sign rank  
Non-parametric test for  
three or more samples

# Non-parametrics

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In part II so far:

- ▶ One sample comparison to a mean (one sample t)
- ▶ Two independent samples (two sample t)
- ▶ Two non-independent samples (paired t)
- ▶ Multiple samples/groups (ANOVA)
  - ▶ Bonferroni
  - ▶ Tukey's HSD

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But all of the methods we have looked at so far depend on some assumptions about the underlying distribution.

What have we assumed?

What do we do if our assumptions are violated?

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## Non-Parametric testing

# Non-Parametric Testing

From <http://biostatisticsryangoslingreturns.tumblr.com/>



## Non-parametrics

### Non-Parametric testing

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# Non-Parametric Testing

**PROS:** Non-parametric methods make very few assumptions about the variable(s) we samples or their distribution and thus rely less on parameters.

- ▶ They do not use means or standard deviations
- ▶ Use a ranking of the data instead of actual values
- ▶ Do not assume a normal distribution of the data
- ▶ Less sensitive to outliers and skewed data
- ▶ Do not need a large sample size

**CONS:** Non-parametric methods use less of the information offered in the data

- ▶ If the assumptions of for a parametric test are met and a non-parametric test is used, it will have lower power (probability of detecting a false null hypothesis)
- ▶ They are less specific in what they test
- ▶ They in essence ignore important parts of the data

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## Non-Parametric testing

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We will discuss non-parametric equivalents for:

Two sample  $t$  : Wilcoxon Rank-Sum

Paired  $t$  : Wilcoxon sign-rank

ANOVA: Kruskal Wallis



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## Wilcoxon two-sample tests

# Frank Wilcoxon



## Non-parametrics

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Non-Parametric testing

**Wilcoxon two-sample tests**

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- ▶ Sometimes also called the Mann-Whitney U test
- ▶ Non-parametric test for comparing two independent samples with a continuous outcome
- ▶ This is the non-parametric counterpart of the two sample t-test
- ▶ Assumes that the distributions have the same general shape but assumes nothing about that shape.
- ▶ Evaluates the null hypothesis that the two populations are identical.

To calculate a rank sum test

The observations are ordered from lowest to highest and assigned the **rank** of their order.

If there are “tie” values, these are assigned the average of the ranks, ie if two observations have the same value and the next lower value is rank=3 then the two observations are both given the rank of 4.5 (because they would have been ranks 4 and 5).

Then the sums of ranks belonging to group 1 are compared to the sums of ranks belonging to group 2

# Wilcoxon Rank-Sum

Values in group 1: 4,3,5,2,6

Values in group 2: 6,5,7,4,8

# Wilcoxon Rank-Sum

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Group 1	rank	Group 2	rank
4	3.5	6	7.5
3	2	5	5.5
5	5.5	7	9
2	1	4	3.5
6	7.5	8	10
sum	19.5	sum	35.5

# Wilcoxon Rank-Sum

The smaller of the two sums is called  $W$ . This is then used in the following equation to generate a  $Z$  statistic.

$$Z_w = \frac{W - \mu_w}{\sigma_w}$$

where

$$\mu_w = \frac{n_s(n_s + n_l + 1)}{2}$$

and

$$\sigma_w = \sqrt{\frac{n_s n_l (n_s + n_l + 1)}{12}}$$

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# Wilcoxon Rank-Sum

So from our example where group 1 had a rank sum of 19.5 and group 2 had a rank sum of 35.5

$$\mu_w = \frac{n_s(n_s + n_l + 1)}{2} = \frac{5(5 + 5 + 1)}{2} = 27.5$$

and

$$\sigma_w = \sqrt{\frac{n_s n_l (n_s + n_l + 1)}{12}} = \sqrt{\frac{5 * 5 (5 + 5 + 1)}{12}} = 4.8$$

$$Z_w = \frac{W - \mu_w}{\sigma_w} = \frac{19.5 - 27.5}{4.8} = -1.67$$



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The  $Z_w$  we generate follows an approximate standard normal distribution. So we can use our  $Z$  score to get a p-value in R

```
2*pnorm(-1.67)
```

```
## [1] 0.09491936
```

# Wilcoxon Rank-Sum in R

The general syntax will be:

```
wilcox.test(group1, group2, paired=F)
```

or

```
wilcox.test(outcome ~ group, paired=F)
```

remember that you can always type `help(wilcox.test)` in your console to get the full details

# Wilcoxon Rank-Sum example :phenylketonuria

Normalized mental age scores for children with phenylketonuria from Wrona, R.M., "A Clinical Epidemiologic Study of Hyperphenylalaninemia," American Journal of Public Health, volume 69, July 1979, 673-679.

Group 1: "low exposure"  $< 10.0$  mg/dl

Group 2: "high exposure"  $\geq 10.0$  mg/dl

# Wilcoxon Rank-Sum :phenylketonuria

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three or more samples

##	Group	nMA
## 1	low	34.5
## 2	low	37.5
## 3	low	39.5
## 4	low	40.0
## 5	low	45.5
## 6	low	47.0

# Wilcoxon Rank-Sum :phenylketonuria

In this example there 18 High and 21 Low exposure individuals.

```
library(dplyr)
library(ggplot2)
group_by(pku, Group) %>%
  summarise(
    count = n(),
    median = median(nMA, na.rm = TRUE),
    IQR = IQR(nMA, na.rm = TRUE)
  )
```

```
## # A tibble: 2 x 4
##   Group count median   IQR
##   <chr> <int>   <dbl> <dbl>
## 1 high     18    48.2   9.12
## 2 low      21    51      7
```

Non-Parametric testing

Wilcoxon two-sample tests

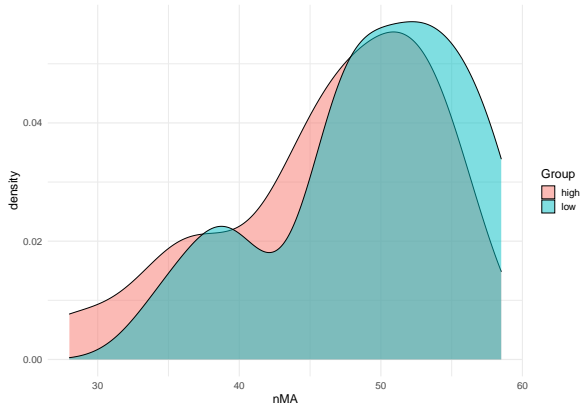
Wilcoxon sign rank

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# Wilcoxon Rank-Sum: PKU

If we graph the distributions with a density plot what do we notice?

```
ggplot(pku, aes(x = nMA)) +  
  geom_density(aes(fill = Group), alpha = 0.5) +  
  theme_minimal(base_size = 15)
```



# Wilcoxon Rank-Sum: PKU

```
wilcox.test(nMA ~ Group, data=pku,paired=F)
```

```
##
```

```
## Wilcoxon rank sum test with continuity correction
```

```
##
```

```
## data: nMA by Group
```

```
## W = 142, p-value = 0.1896
```

```
## alternative hypothesis: true location shift is not equal to 0
```

# Wilcoxon Rank-Sum vs T : NHANES example

Here I will again use the NHANES data as an example, looking at height by gender

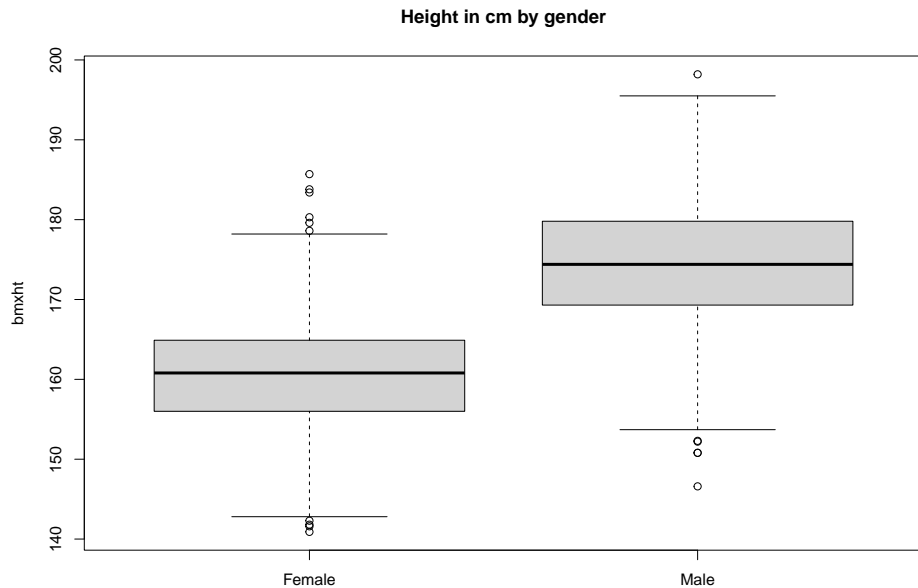
```
# Read CSV into R
```

```
nhanes <- read.csv(file="nhanes.csv", header=TRUE, sep=",")  
names(nhanes)
```

```
## [1] "ridageyr" "agegroup" "gender" "military" "born" "citizen"  
## [7] "drinks" "drinkscat" "bmxwt" "bmxht" "bmxbmi" "bmicat"  
## [13] "bpxpls" "bpxsy1" "bpxsy2" "sys1d" "sys2d" "bpxdi1"  
## [19] "bpxdi2" "dias1d" "dias2d" "bpcat" "chest" "fs1"  
## [25] "fs2" "fs3" "lbdhdd" "hdlcat" "highhdl" "hi"  
## [31] "asthma" "vwa" "vra" "va" "aspirin" "sleep"  
## [37] "is" "hs" "lbdldl" "highldl"
```



# Wilcoxon Rank-Sum vs T : NHANES example



Non-Parametric testing

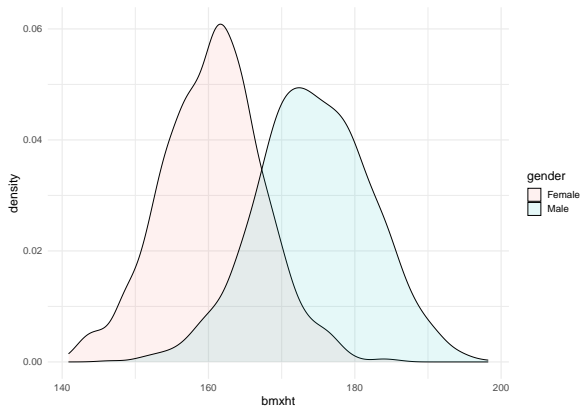
Wilcoxon two-sample tests

Wilcoxon sign rank

Non-parametric test for three or more samples

# Wilcoxon Rank-Sum vs T

```
ggplot(nhanes, aes(x = bmxht)) +  
  geom_density(aes(fill=gender), alpha=0.1) +  
  theme_minimal(base_size = 15)
```



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Non-parametric test for  
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# Wilcoxon Rank-Sum vs T

```
t.test(malesht, femalesht, paired=F)
```

```
##  
## Welch Two Sample t-test  
##  
## data: malesht and femalesht  
## t = 47.285, df = 2384, p-value < 2.2e-16  
## alternative hypothesis: true difference in means is not equal to 0  
## 95 percent confidence interval:  
## 13.37441 14.53172  
## sample estimates:  
## mean of x mean of y  
## 174.4717 160.5186
```

# Wilcoxon Rank-Sum vs T

```
wilcox.test(malesht,femalesht,paired=F)
```

```
##
```

```
## Wilcoxon rank sum test with continuity correction
```

```
##
```

```
## data: malesht and femalesht
```

```
## W = 1402065, p-value < 2.2e-16
```

```
## alternative hypothesis: true location shift is not equal to 0
```

Non-Parametric testing

**Wilcoxon two-sample tests**

Wilcoxon sign rank

Non-parametric test for  
three or more samples

When the sample size is quite large (as with these NHANES data) the assumption of approximate normality is reasonable one and the results of the hypothesis testing will generally not be different using a parametric or non-parametric approach.

Non-Parametric testing

Wilcoxon two-sample tests

**Wilcoxon sign rank**

Non-parametric test for  
three or more samples

## Wilcoxon sign rank

- ▶ Non-parametric test for comparing two non-independent (paired) sample means
- ▶ This is the non-parametric counterpart of the paired t-test
- ▶ Assumes that the distributions have the same general shape but assumes nothing about that shape.
- ▶ Evaluates the null hypothesis that the difference between the first and second measures is 0.

Steps:

- 1) Calculate the difference between each pair of observations
- 2) Rank the difference by absolute value from smallest to largest (again, tie values get the average of the ranks). Any pair where difference = 0 is thrown out.
- 3) Assign a “sign” for whether the difference was positive or negative
- 4) Take the sum of the positive ranks and the sum of the negative ranks (the smaller sum is denoted with a T).



# Wilcoxon Sign rank

Under the null hypothesis that the difference is 0, we would expect the sample to have equal numbers of positive and negative ranks with equivalent sums. This expectation is tested against the statistic

$$Z_T = \frac{T - \mu_T}{\sigma_T}$$

Where

$$\mu_T = \frac{n(n+1)}{4}$$

and

$$\sigma_T = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

# Wilcoxon Sign rank: Example Pre and post test

Time 1	Time 2
65	77
87	100
77	75
90	89
70	80
84	81
92	91
83	96
85	84
91	89
68	88
72	100
81	81

Non-Parametric testing

Wilcoxon two-sample tests

**Wilcoxon sign rank**Non-parametric test for  
three or more samples

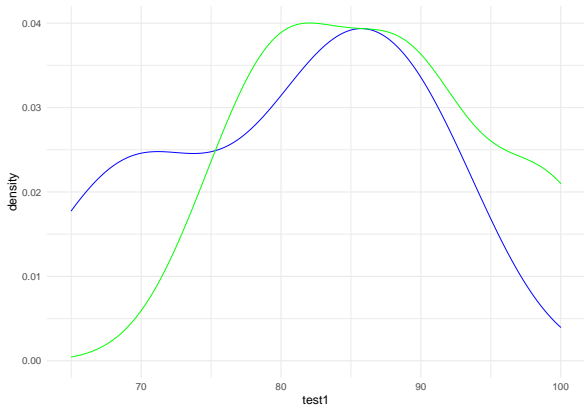
# Sign rank example

Non-Parametric testing

Wilcoxon two-sample tests

**Wilcoxon sign rank**

Non-parametric test for  
three or more samples



## Sign Rank example: calculate difference and sign

Time 1	Time 2	Difference	sign
65	77	12	+
87	100	13	+
77	75	-2	-
90	89	-1	-
70	80	10	+
84	81	-3	-
92	91	-1	-
83	96	13	+
85	84	-1	-
91	89	-2	-
68	88	20	+
72	100	28	+
81	81	0	?

Non-Parametric testing

Wilcoxon two-sample tests

**Wilcoxon sign rank**Non-parametric test for  
three or more samples

## Sign Rank example: sort by absolute value and assign rank

Time 1	Time 2	Difference	sign	rank
90	89	-1	-	2
92	91	-1	-	2
85	84	-1	-	2
77	75	-2	-	4.5
91	89	-2	-	4.5
84	81	-3	-	6
70	80	10	+	7
65	77	12	+	8
87	100	13	+	9.5
83	96	13	+	9.5
68	88	20	+	11
72	100	28	+	12
81	81	0	?	drop

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# Sign Rank example: sum the positive and negative ranks

## Negative signs

Time 1	Time 2	Difference	sign	rank
90	89	-1	-	2
92	91	-1	-	2
85	84	-1	-	2
77	75	-2	-	4.5
91	89	-2	-	4.5
84	81	-3	-	6
_____	_____	_____	_____	

Sum of Negative sign ranks is 21

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# Sign Rank example: sum the positive and negative ranks

Non-Parametric testing

Wilcoxon two-sample tests

**Wilcoxon sign rank**Non-parametric test for  
three or more samples

Time 1	Time 2	Difference	sign	rank
70	80	10	+	7
65	77	12	+	8
87	100	13	+	9.5
83	96	13	+	9.5
72	100	18	+	11
68	88	20	+	12

Sum of the positive sign ranks is 57

# Wilcoxon Sign rank: Example

Our expectation would be

$$\mu_T = \frac{n(n+1)}{4} = \frac{12(12+1)}{4} = 39$$

remember that we had 13 observations, but we dropped one because the scores at times 1 and 2 were the same and

$$\sigma_T = \sqrt{\frac{n(n+1)(2n+1)}{24}} = \sqrt{\frac{12(12+1)(2 * 12 + 1)}{24}} = 12.75$$



## Wilcoxon Sign rank: Example

And we compare our expectation to the smaller rank value (Sum of negative ranks was 21, sum of positive ranks was 57)

$$Z_T = \frac{T - \mu_T}{\sigma_T} = \frac{21 - 39}{12.75} = -1.412$$

```
2*pnorm(-1.412)
```

```
## [1] 0.15795
```

# Wilcoxon Sign Rank in R

The general syntax will be:

```
wilcox.test(group1, group2, paired=T)
```

or

```
wilcox.test(outcome ~ group, paired=T)
```

## Wilcoxon Sign rank: Example

```
wilcox.test(test1,test2,paired=T, correct=FALSE)
```

```
## Warning in wilcox.test.default(test1, test2, paired = T, correct = FALSE):
```

```
## cannot compute exact p-value with ties
```

```
## Warning in wilcox.test.default(test1, test2, paired = T, correct = FALSE):
```

```
## cannot compute exact p-value with zeroes
```

```
##
```

```
## Wilcoxon signed rank test
```

```
##
```

```
## data: test1 and test2
```

```
## V = 21, p-value = 0.157
```

```
## alternative hypothesis: true location shift is not equal to 0
```

```
wilcox.test(test1,test2, paired=T)
```

```
## Warning in wilcox.test.default(test1, test2, paired = T): cannot compute e
```

```
## p-value with ties
```

Non-Parametric testing

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Non-parametric test for  
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# Wilcox Sign rank: compare to T

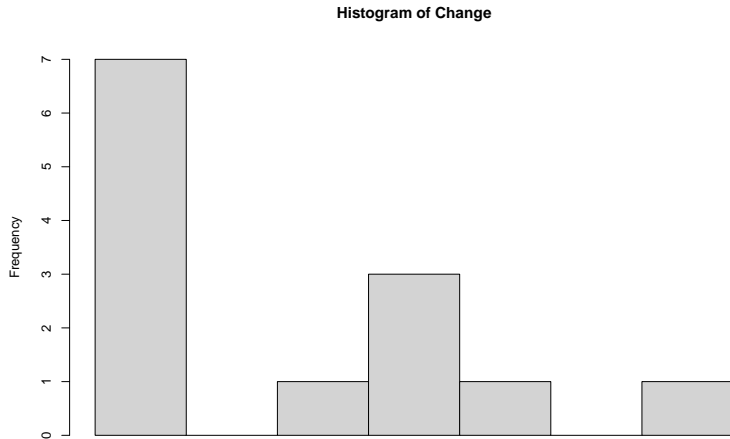
```
t.test(test1,test2,paired=TRUE)
```

```
##  
## Paired t-test  
##  
## data: test1 and test2  
## t = -2.3684, df = 12, p-value = 0.0355  
## alternative hypothesis: true difference in means is not equal to 0  
## 95 percent confidence interval:  
## -12.7011701 -0.5295991  
## sample estimates:  
## mean of the differences  
## -6.615385
```

## Wilcoxon Sign rank: Compare to T

With this study, our sample is 13 and the distribution of changes looks like this - remember that the 0 difference value gets thrown out of sign rank test:

```
hist(Change)
```



Non-Parametric testing

Wilcoxon two-sample tests

**Wilcoxon sign rank**Non-parametric test for  
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Non-Parametric testing

Wilcoxon two-sample tests

Wilcoxon sign rank

**Non-parametric test for  
three or more samples**

## Non-parametric test for three or more samples

Non-Parametric testing

Wilcoxon two-sample tests

Wilcoxon sign rank

Non-parametric test for  
three or more samples

The Kruskal Wallis test is a non-parametric alternative to the ANOVA test  
Kruskal-Wallis looks at the medians of the groups, not the means and tests if at least one is significantly different from another (but not which one) -  $H_0$ : There is no difference between the group medians -  $H_1$ : There is a statically significant difference in the group medians

Non-Parametric testing

Wilcoxon two-sample tests

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Non-parametric test for  
three or more samples

This test can be thought of as an extension of the rank sum test as it is based on the Rank-sum test. We will not do this one by hand.

In R the syntax is generally:

```
kruskal.test(outcome ~ group, dataset)
```



Non-Parametric testing

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Non-parametric test for  
three or more samples

```
##  
##  Kruskal-Wallis rank sum test  
##  
## data:  outcome by treatment  
## Kruskal-Wallis chi-squared = 13.096, df = 3, p-value = 0.004434
```

# Non parametric summary

Most parametric tests have an analagous non-parametric test We have covered the following:

Samples	Parametric	Non Parametric
Two independent samples	two sample ttest	Wilcoxon rank sum
Two paired samples	paired ttest	Wilcoxon sign rank
Three or more samples	ANOVA	Kruskal Wallis

# Non parametrics in R

Non-Parametric testing

Wilcoxon two-sample tests

Wilcoxon sign rank

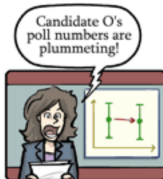
Non-parametric test for  
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Samples	test name	R function
Two independent samples	Wilcoxon rank sum	<code>wilcox.test(group1,group2,paired=F)</code>
Two paired samples	Wilcoxon sign rank	<code>wilcox.test(group1,group2,paired=T)</code>
Three or more samples	Kruskal Wallis	<code>kruskal.test(outcome ~ group)</code>

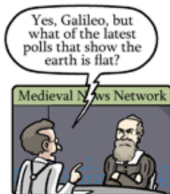
Dear News Media,

When reporting poll results, please keep  
in mind the following suggestions:

1.  
If two poll numbers  
differ by less than the  
margin of error, it's  
not a news story.



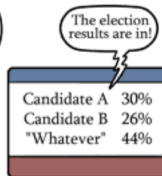
2.  
Scientific facts are  
not determined by  
public opinion polls.



3.  
A poll taken of your  
viewers/internet users  
is not a scientific poll.



4.  
What if all polls  
included the option  
"Don't care"?



Signed,

-Someone who took a  
basic statistics course.