Relationships between 2 variables

Recap of chapters 1 and 2

Mostly looking at a single variable:

- Graphs to explore the distribution of single variables (histograms, bar charts)
- Summary numbers to describe our distributions:
  - Measures of central tendency (mean, median)
  - Measures of spread (standard deviation, IQR)

One example of two variables:

- Time plots to examine what happens to a variable over time

Learning objectives for today

- Explore the relationship between two quantitative variables
  - Directionality
  - Association vs causation
- Make scatter plots to look at relationships visually
  - using `geom_point()`
- Use the correlation coefficient to quantify the strength of linear relationships
  - calculate correlations using `cor()`

Relationships between two quantitative variables

Explanatory (X) and response (Y) variables

Bi-directional:

- "X predicts Y", or "Y predicts X"
- "X is associated with Y", or "Y is associated with X"

Unidirectional:

- "X causes Y"

Which variable is x and which is y?

In prediction we generally use X to denote the variable we are using to predict the variable of interest (Y)

In causation we generally use X to denote the explanatory (independent) and Y to denote the response (dependent)

Graphically the X variable is on the X (horizontal) axis and the Y variable is the Y(vertical) axis

Which variable is x and which is y?

1. Each hospital's rate of hospital-acquired infections, and whether the hospital has implemented a hand-washing intervention as part of a cluster randomized trial.
2. The weight in kilograms and height in centimeters of a person
3. Inches of rain in the growing season and the yield of corn in bushels per day
4. A person's leg length and arm length, in centimeters

How to investigate causation?

- Randomized controlled trials (RCTs) to randomize individuals to different levels
- Observational study that is *designed* to investigate causation and reduce the risk of bias

Looking at relationships visually: Scatterplots

Scatterplots

- Scatterplots are a good way to visualize a relationship between two variables
- When we look at a scatterplot we want to evaluate:
  - The overall Pattern of the dots
  - Any notable exceptions to the pattern
  - Direction (positive or negative)
  - Form (straight line or curved)
  - Strength (how closely the points follow a line)
  - Are there any obvious outliers

Scatterplot Syntax in R

name of plot <- ggplot(data = dataset, aes(x = xvariable, y = yvariable)) +

geom_point(na.rm=TRUE) + theme_minimal(base_size = 15)+

labs(x = “xlabel”, y = “ylabel”, title = “Title”)

Bi-directional relationships ex: systolic and diastolic BP

Read in NHANES dataset

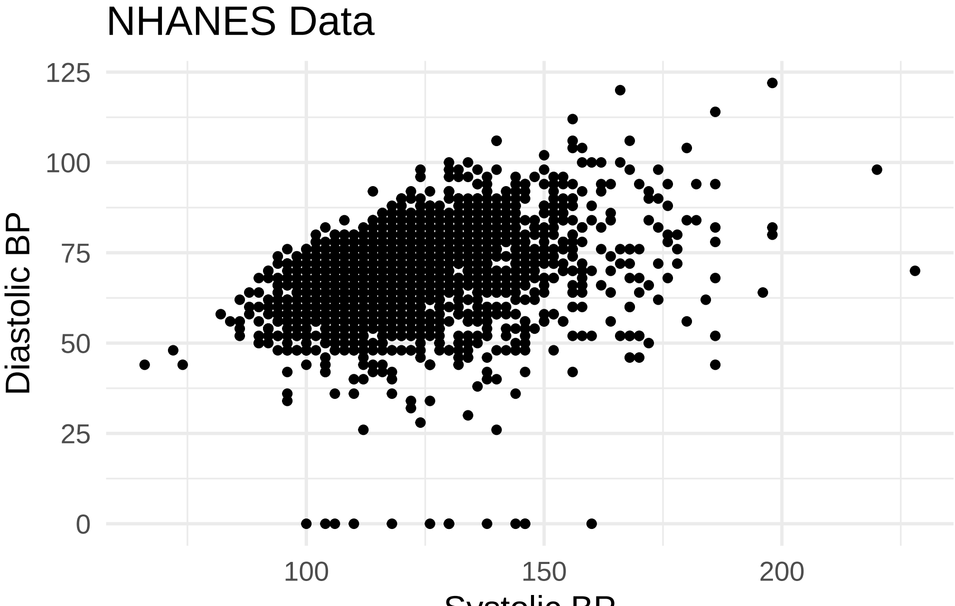
```
nhanes_dataNA <- read_csv("nhanes.csv")
nhanes_data<-nhanes_dataNA[rowSums(is.na(nhanes_dataNA[ , 15:18])) == 0,]
names(nhanes_data)
```

```
##  [1] "ridageyr"   "agegroup"   "gender"     "military"   "born"       "citizen"
##  [7] "drinks"     "drinkscat"  "bmxwt"      "bmxht"      "bmxbmi"     "bmicat"
## [13] "bpxpls"     "bpxsy1"     "bpxsy2"     "sys1d"      "sys2d"      "bpxdi1"
## [19] "bpxdi2"     "dias1d"     "dias2d"     "bpcat"      "chest"      "fs1"
## [25] "fs2"        "fs3"        "lbdhdd"     "hdlcat"     "highhdl"    "hi"
## [31] "asthma"     "vwa"        "vra"        "va"         "aspirin"    "sleep"
## [37] "is"         "hs"         "lbdldl"     "highldl"
```

Bi-directional relationships ex: systolic and diastolic BP

```
nhanes_scatter <- ggplot(data = nhanes_data, aes(x = bpxsy1, y = bpxdi1)) +
  geom_point(na.rm=TRUE) + theme_minimal(base_size = 15)+
  labs(x = "Systolic BP",
       y = "Diastolic BP",
       title = "NHANES Data")
```

Bi-directional relationships ex: systolic and diastolic BP



Bi-directional relationships ex: systolic and diastolic BP

What do we notice from the plot?

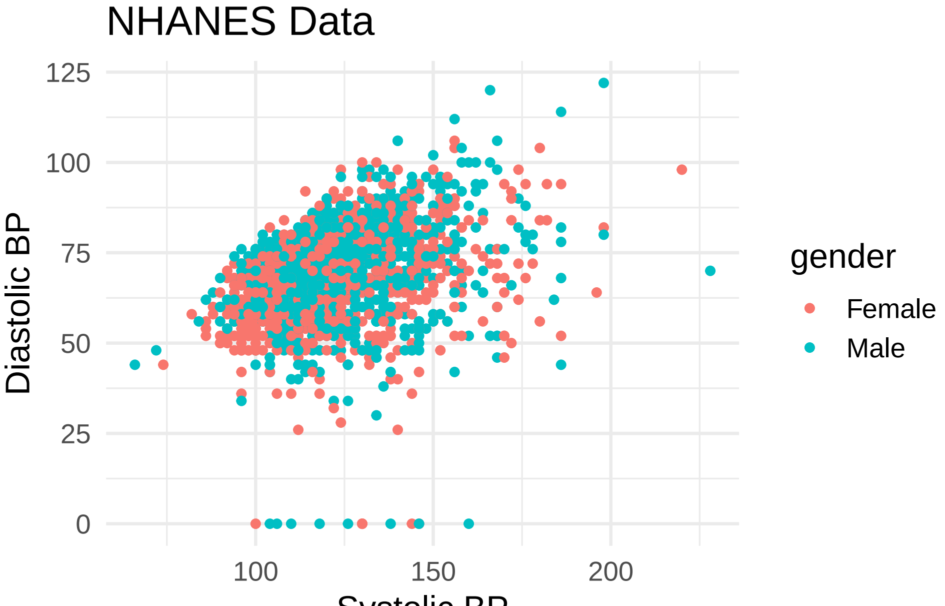
- Is there a visible association?
- Any notable exceptions to the pattern
- Direction (positive or negative)
- Form (straight line or curved)
- Strength (how closely the points follow a line)
- Are there any obvious outliers

Bi-directional relationships ex: systolic and diastolic BP

We can add a third variable to our graph by coloring the dots

```
nhanes_scatter <- ggplot(data = nhanes_data, aes(x = bpxsy1, y = bpxdi1)) +
  geom_point(aes(col=gender),na.rm=TRUE) + theme_minimal(base_size = 15)+
  labs(x = "Systolic BP",
       y = "Diastolic BP",
       title = "NHANES Data")
```

Bi-directional relationships ex: systolic and diastolic BP



Association with a plausible direction

Manatee data set from your textbook:

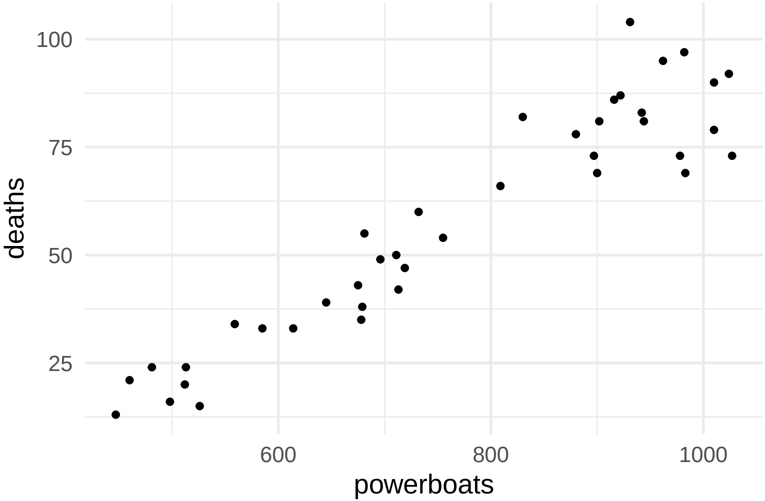
```r
mana_data <- read_csv("Ch03_Manatee-deaths.csv")
head(mana_data)
```

```
## # A tibble: 6 x 3
##    year powerboats deaths
##   <dbl>      <dbl>  <dbl>
## 1  1977        447     13
## 2  1987        645     39
## 3  1997        755     54
## 4  2007       1027     73
## 5  1978        460     21
## 6  1988        675     43
```

Power boats and Manatees

```
mana_scatter <- ggplot(data = mana_data, aes(x = powerboats, y = deaths)) +
  geom_point() + theme_minimal(base_size = 15)
```

Power boats and Manatees



Power boats and Manatees

What do we notice from the plot?

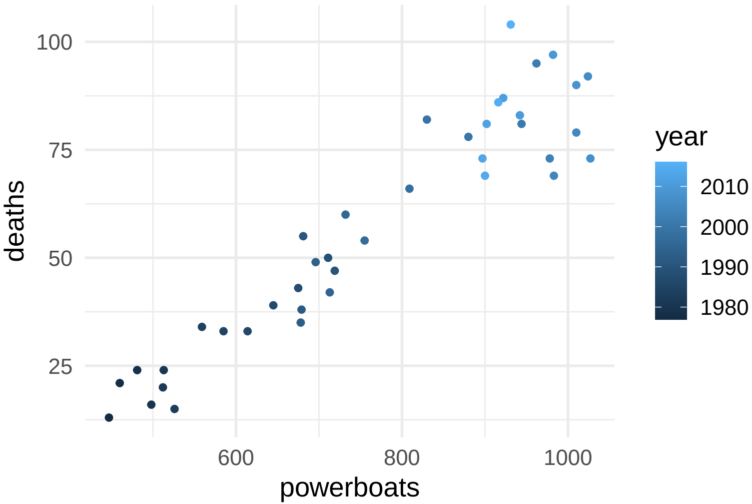
- Is there a visible association?
- Any notable exceptions to the pattern
- Direction (positive or negative)
- Form (straight line or curved)
- Strength (how closely the points follow a line)
- Are there any obvious outliers

Power boats and Manatees

What if we layer in a continuous third variable?

```
mana_scatter <- ggplot(data = mana_data, aes(x = powerboats, y = deaths)) +
  geom_point(aes(col=year)) + theme_minimal(base_size = 15)
```

Power boats and Manatees

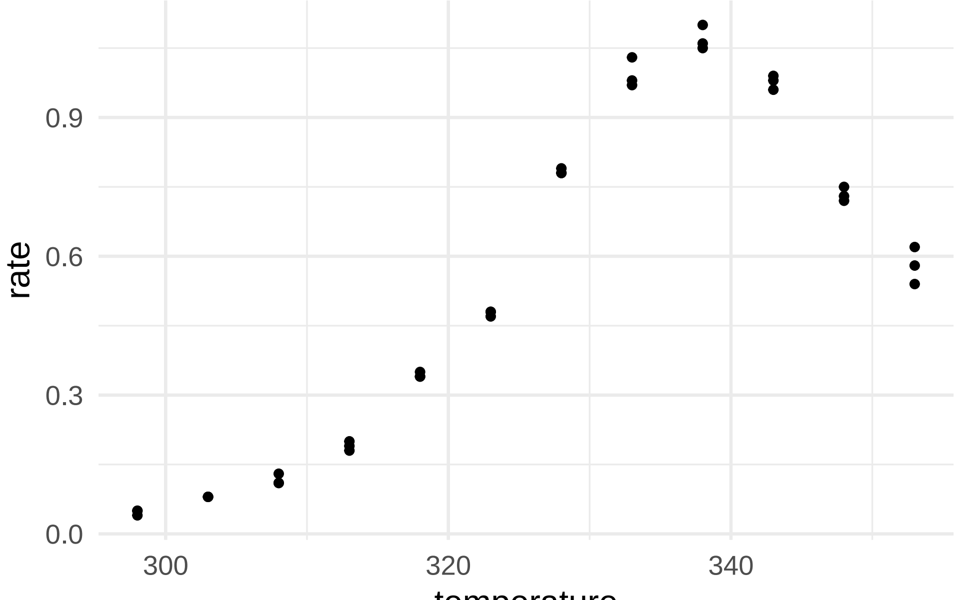


Enzyme activity and temperature

Also from your book: A study examined the activity rate (in micromoles per second) of a digestive enzyme at varying temperatures.

```
## # A tibble: 6 x 2
##   temperature  rate
##         <dbl> <dbl>
## 1         298  0.04
## 2         298  0.05
## 3         298  0.05
## 4         303  0.08
## 5         303  0.08
## 6         303  0.08
```

Enzyme activity and temperature



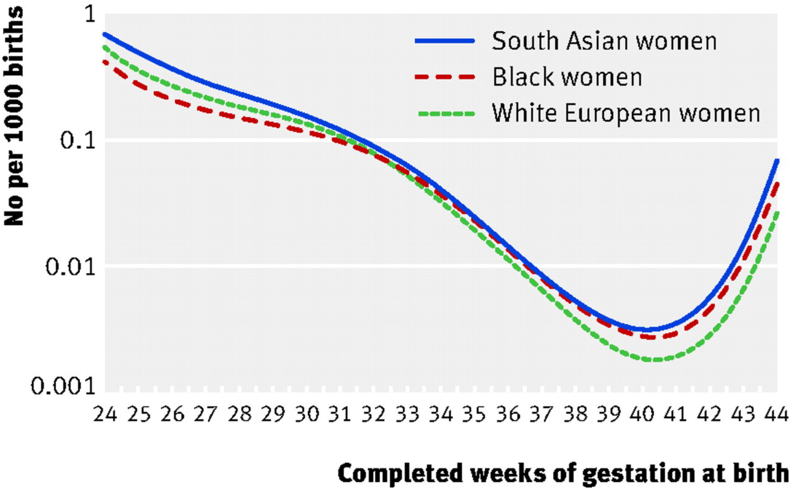
Enzyme activity and temperature

What do we notice from the plot?

- Is there a visible association?
- Any notable exceptions to the pattern
- Direction (positive or negative)
- Form (straight line or curved)
- Strength (how closely the points follow a line)
- Are there any obvious outliers

Another non-linear example

Gestational age and perinatal mortality



Source: Balchin et al. BMJ. 2007.

Exploratory analysis using scatterplots

Lean body mass and metabolic rate: Problem and Plan

Problem: Is lean body mass (person's weight after removing the fat) associated with metabolic rate (kilocalories burned in 24 hours)?

Plan: A diet study was conducted on 12 women and 7 men that measured lean body weight and metabolic rate for each individual.

Lean body mass and metabolic rate: DATA

Data: In the textbook

Subject	Sex	Mass (kg)	Rate (Cal)	Subject	Sex	Mass (kg)	Rate (Cal)
1	M	62.0	1792	11	F	40.3	1189
2	M	62.9	1666	12	F	33.1	913
3	F	36.1	995	13	M	51.9	1460
4	F	54.6	1425	14	F	42.4	1124
5	F	48.5	1396	15	F	34.5	1052
6	F	42.0	1418	16	F	51.1	1347
7	M	47.4	1362	17	F	41.2	1204
8	F	50.6	1502	18	M	51.9	1867
9	F	42.0	1256	19	M	46.9	1439
10	M	48.7	1614				

What would the corresponding data frame look like? How many variables would it have? How many rows?

Lean body mass and metabolic rate: DATA

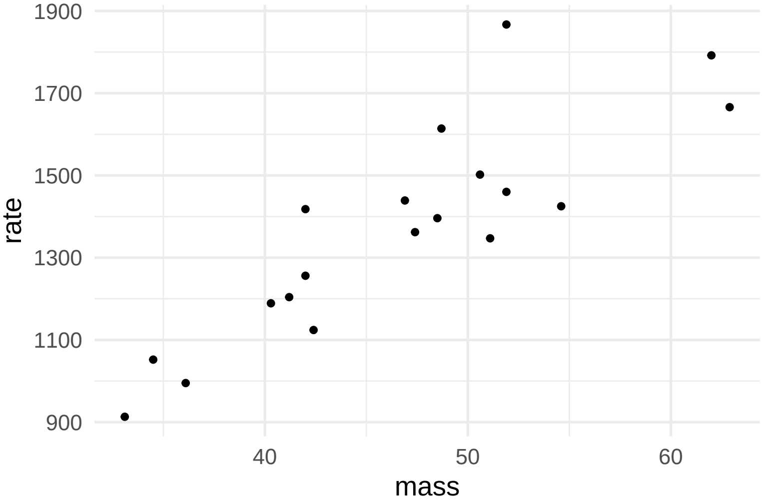
```
# Note: you won't be tested on writing code using tibble::tribble()
# Do be able to look at the code and recognize that it is creating a data set

weight_data <- tibble::tribble(
  ~subject, ~gender, ~mass, ~rate,
  1, "M", 62.0, 1792,
  2, "M", 62.9, 1666,
  3, "F", 36.1, 995,
  4, "F", 54.6, 1425,
  5, "F", 48.5, 1396,
  6, "F", 42.0, 1418,
  7, "M", 47.4, 1362,
  8, "F", 50.6, 1502,
  9, "F", 42.0, 1256,
  10, "M", 48.7, 1614,
  11, "F", 40.3, 1189,
  12, "F", 33.1, 913,
```

Lean body mass and metabolic rate: Analysis

Exploratory data analysis using scatter plots

```
weight_scatter <- ggplot(weight_data, aes(x = mass, y = rate)) +
  geom_point() +
  theme_minimal(base_size = 15)
```

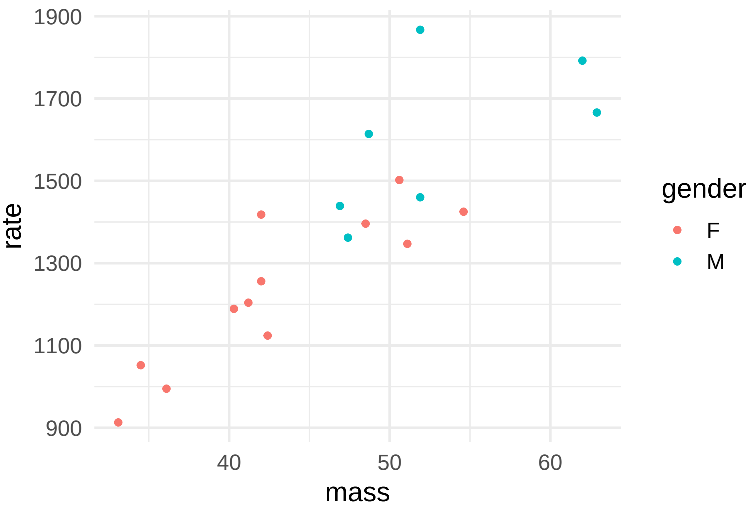
Lean body mass and metabolic rate: Analysis



Analysis: Colour the points by gender

```
weight_scatter <- ggplot(weight_data, aes(x = mass, y = rate)) +
  geom_point(aes(col=gender)) +
  theme_minimal(base_size = 15)
```

Analysis: Colour the points by gender

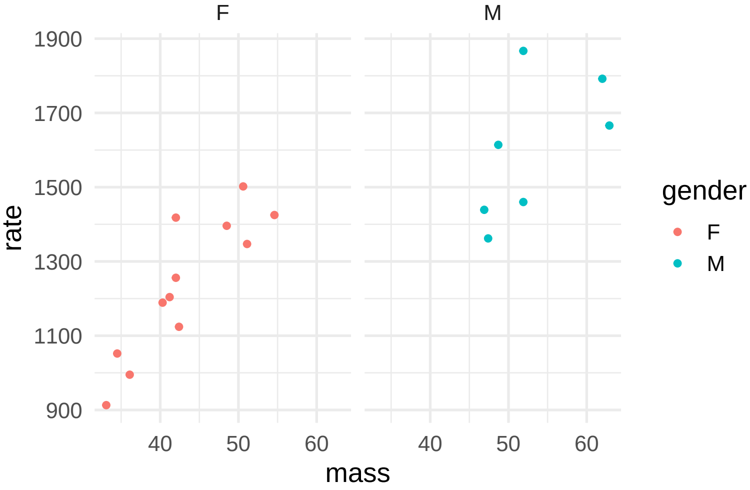


Analysis: Create separate plots for men and women

```
weight_scatter <- ggplot(weight_data, aes(x = mass, y = rate)) +
  geom_point(aes(col=gender)) +
  theme_minimal(base_size = 15)+
  facet_wrap(~ gender)
```

Analysis: Create separate plots for men and women

```
weight_scatter
```



Assessing a relationship between two variables with a number: Pearson's correlation

Pearson's correlation

Using just our eyes, we can often say something about whether an association between two variables is weak or strong.

But we can also use a numeric value to describe the direciton and strength of an association

Pearson's correlation

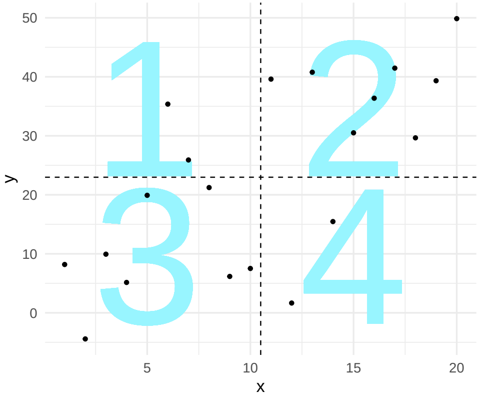
- For linear associations, we can use Pearson's correlation coefficient (denoted by r) to quantify the strength of a linear relationship between two variables.
- The correlation between x and y is:

$$r = \frac{1}{n-1}\sum_{i=1}^{n}(\frac{x_i - \bar{x}}{s_x})(\frac{y_i - \bar{y}}{s_y})$$

Notice that because we are dividing by the standard deviation the values become unitless

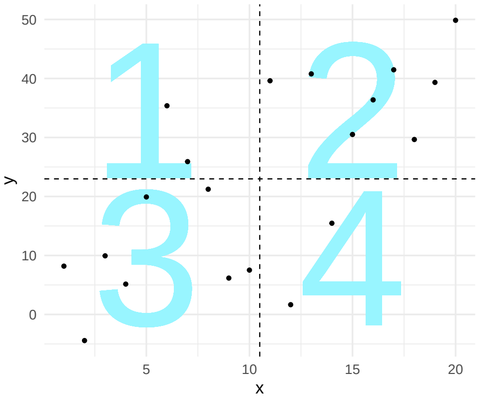
Intuition about Pearson's correlation

To understand this formula, first only consider the numerators of the fractions (i.e., $x_i - \bar{x}$ and $y_i - \bar{y}$). If you imagine a scatter plot of x and y, we can also add a dashed line at the mean x value of $\bar{x}$ and a dashed line line at the mean y value ($\bar{y}$):



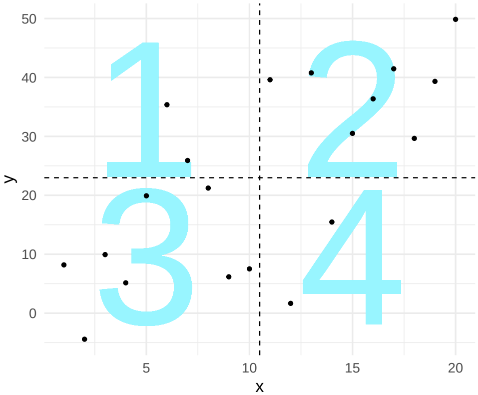
Intuition about Pearson's correlation

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$



- Points in Q2 and Q3 contribute positive products to r
- Points in Q1 and Q4 contribute negative products to r

Intuition about Pearson's correlation



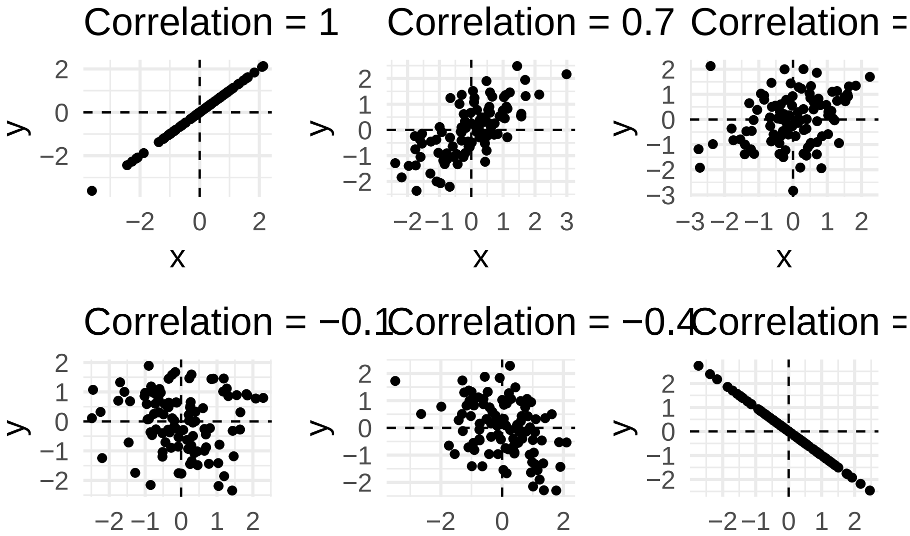
- The more there are points in Q2 and Q3 vs. Q1 and Q4, the more the value of the correlation coefficient will be higher and positive
- If you want even more of an explanation see the response to this stack overflow post

Properties of the correlation coefficient

- Always a number between -1 and 1.
  - -1: A perfect, negative linear association
  - 1: A perfect, positive linear association
  - 0: No linear association
- Is used to measure the association between two *quantitative* variables.
- Only useful for *linear* associations!

Corellation and direction

```
Warning: package 'patchwork' was built under R version 4.0.4
```



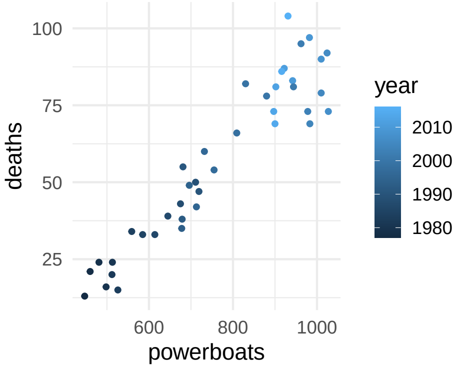
Syntax: Pearson's correlation using `cor()`

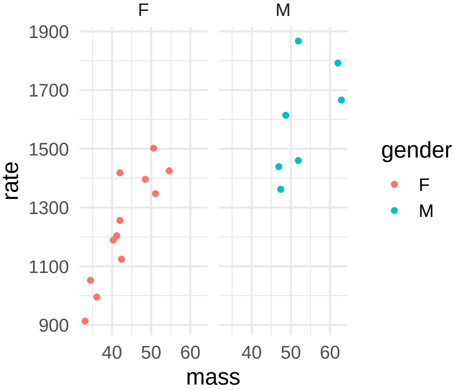
correlation coefficient <- dataset %>%

summarize(*newvar* = cor(xvar, yvar))

Syntax: Pearson's correlation using cor()

Remember the manatee plot and the weight plot:





Syntax: Pearson's correlation using cor()

Now, calculate the correlations between X and Y for manatees:

```
mana_cor <- mana_data %>%
  summarize(corr_mana = cor(powerboats, deaths))
mana_cor
```

```
## # A tibble: 1 x 1
##   corr_mana
##       <dbl>
## 1     0.945
```

Syntax: Pearson's correlation using cor()

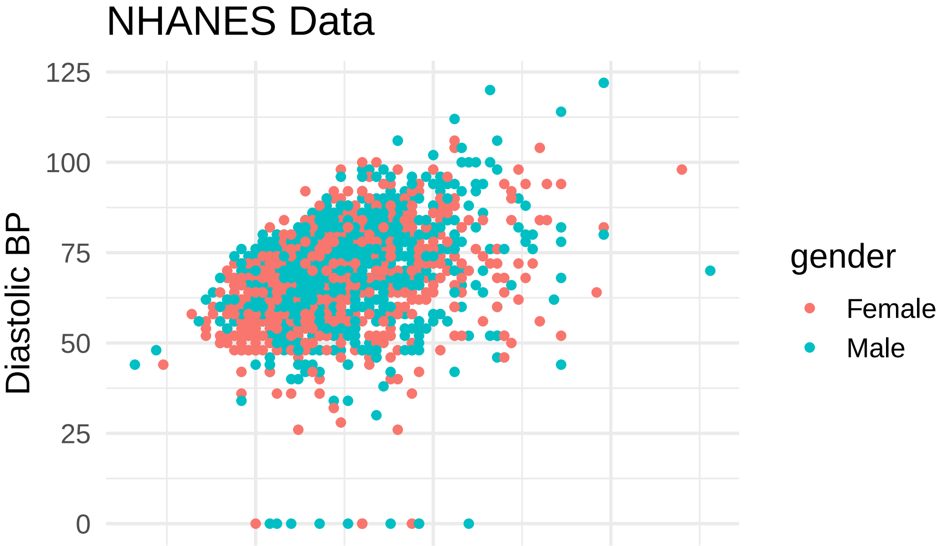
And for the weight data:

```
weight_cor <- weight_data %>%
  summarize(corr_weight = cor(mass, rate))
weight_cor
```

```
## # A tibble: 1 x 1
##   corr_weight
##         <dbl>
## 1       0.865
```

Syntax: Pearson's correlation using `cor()`

What about our blood pressure data from NHANES?



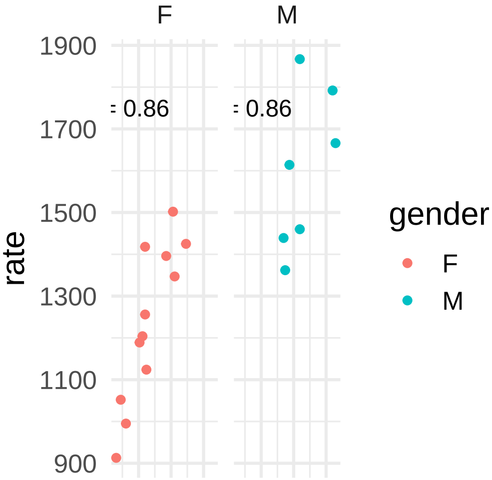
Syntax: Pearson's correlation using cor()

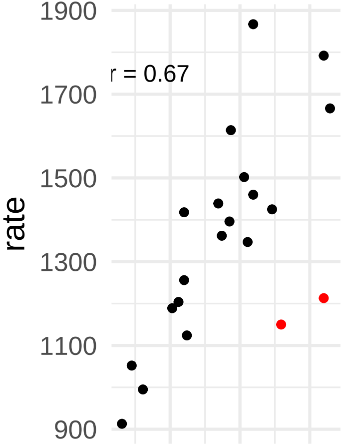
```
bp_cor <- nhanes_data %>%
  summarize(corrbp = cor(bpxsy1, bpxdi1))
bp_cor
```

```
## # A tibble: 1 x 1
##   corrbp
##    <dbl>
## 1  0.322
```

Properties of the correlation coefficient

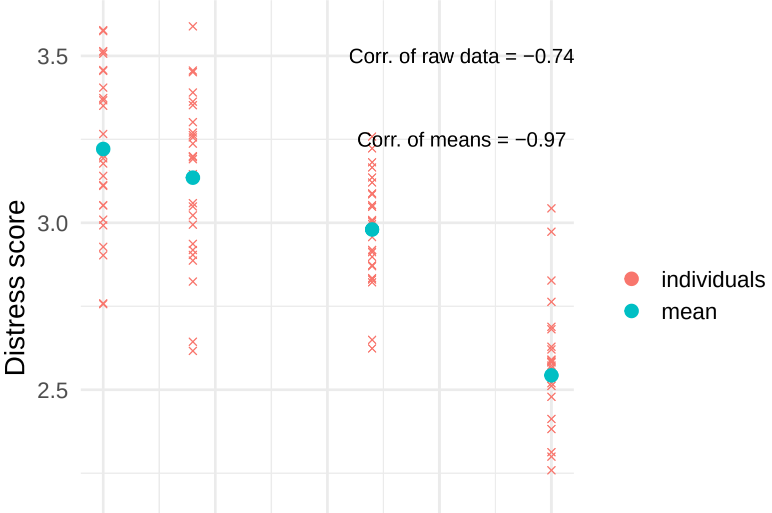
The correlation coefficient is not resistant to outliers, notice what happens when we add two outliers (in red) to the weight_data and recalculate correlation





Properties of the correlation coefficient

- Correlations for average measures is typically stronger than correlations for individual data



Important concepts

- Determine which variable is explanatory and which is response, or when it doesn't matter
- Visually describe the relationship between two variables (form, direction, strength, and outliers)
- Numerically describe the relationship with the correlation coefficient r

R Recap: What functions did we use?

- `geom_point()`,
- `aes(col = gender)` to color points by levels of `gender`
- `summarize()` to calculate correlation using `cor(var1, var2)`

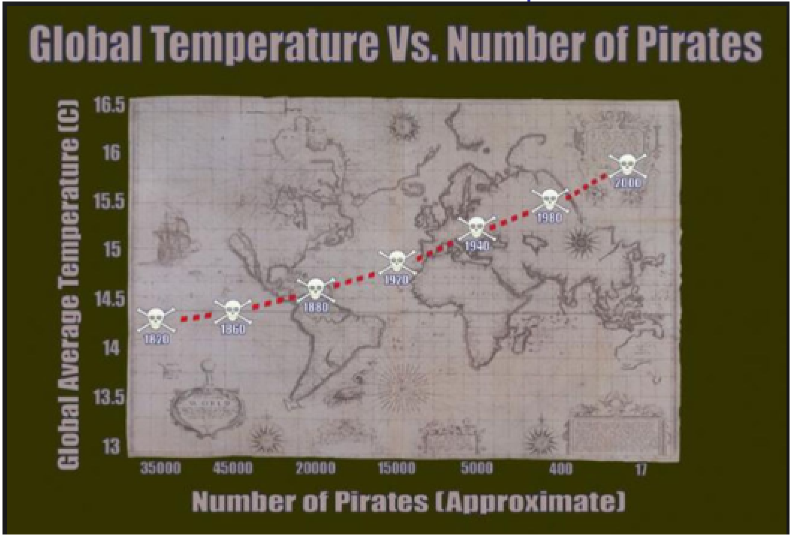
Reminder: Association does not equal causation

Remember that just because two variables are associated, does not mean there is a causal relationship

The correlation coefficient measures association *not* causation.

Even a very strong association doesn't mean that one variable causes the other.

Reminder: Association does not equal causation



This image is one from a Forbes.com article but this example pops up in lots of places