L07: Relationships between two categorical variables

Learning objectives for today

Today we will focus on how to visualize and quantify relationships between two categorical variables

- Two way tables
  - marginal vs conditional distributions
- Bar graphs
  - side by side
  - stacked
- Simpson's paradox

Reminder

Categorical variables are just that, categories.

These can be nominal (no underlying order)

or

ordinal (ordered)

Two-way tables

- Two-way, or 2X2 (for a table with two columns and two rows)
  - Used to examine the relationship between 2 categorical variables, originally for those with two levels each
- Foundational to epidemiology, because of the types of variables we are often interested in

Classic 2X2 looks like this:

Exposure group	Disease	No disease	Row total
Exposed	A	B	A+B
Not Exposed	C	D	C+D
Column total	A+C	B+D	Total # observations

Example: Lung cancer and smoking

Group	Lung Cancer	No Lung Cancer	Row total
Smoker	12	238	250
Non-smoker	7	743	750
Column total	19	981	1000

Marginal distributions

- The marginal distribution of a variable is the one that is in the margin of the table (i.e., the Row total or the Column total are the two margins of a two-way table).
- The marginal distribution is the distribution for a single categorical variable
- When we plotted categorical variables with `geom_bar()` in the earlier lectures, we were plotting marginal distributions

Marginal distributions

Group	Lung Cancer	No Lung Cancer	Row total
Smoker	12	238	250
Non-smoker	7	743	750
Column total	19	981	1000

- Overall, what % of the population has lung cancer?
- Overall, what % of the population are smokers?

Marginal distributions

Group	Lung Cancer	No Lung Cancer	Row total
Smoker	12	238	250
Non-smoker	7	743	750
Column total	19	981	1000

- Overall, what % of the population has lung cancer?
  - Answer: $19/1000 = 1.9\%$
- Overall, what % of the population are smokers?
  - Answer: 250/1000 25% smoking
- The marginal distribution of lung cancer is 1.9% lung cancer, 98.1% no lung cancer.

Conditional distributions

Group	Lung Cancer	No Lung Cancer	Row total
Smoker	12	238	250
Non-smoker	7	743	750
Column total	19	981	1000

- The conditional distribution is the distribution of one variable within or conditional on the level of a second variable
- What is the distribution of lung cancer conditional on the individuals being smokers?
- What is the conditional distribution of lung cancer given individuals are non-smoking?

Conditional distributions

Group	Lung Cancer	No Lung Cancer	Row total
Smoker	12	238	250
Non-smoker	7	743	750
Column total	19	981	1000

- The conditional distribution of lung cancer given smoking is: $12/250 = 4.8\%$ have lung cancer and $238/250 = 95.2\%$ do not
- The conditional distribution of lung cancer given non-smoking is: $7/750 = 0.9\%$ have lung cancer and $743/750 = 99.1\%$ do not

Visualizing categorical variables in R

Marginal and Conditional distributions in R

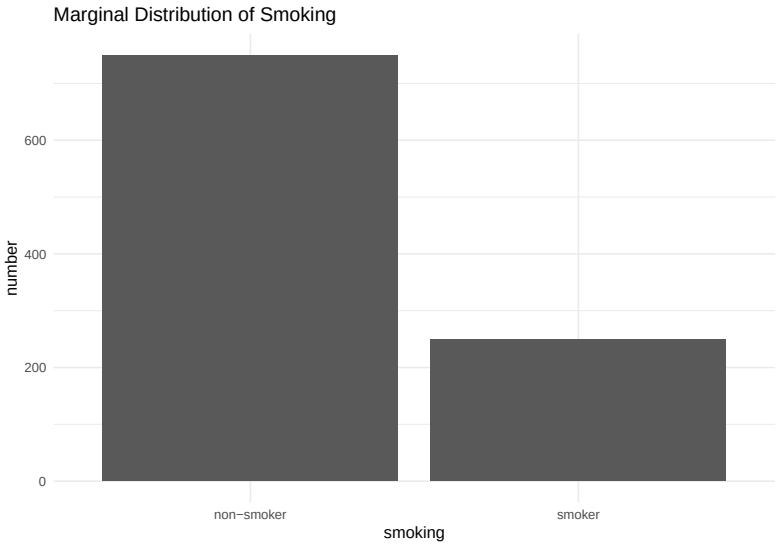
- We learned in Ch.1 how to plot marginal distributions of categorical variables using `geom_bar()`
- Can we generalize our use of `geom_bar()` to allow us to plot multiple conditional distributions? I.e., can we show the conditional distribution of lung cancer for smokers and non-smokers on the same plot?

Visualization in R

First, we encode the data to read into R:

```
library(dplyr)
# students, you don't need to know how to do this
two_way_data <- tribble(~ smoking, ~ lung_cancer, ~ percent, ~number,
                        "smoker", "lung cancer", 4.8, 12,
                        "smoker", "no lung cancer", 95.2,238,
                        "non-smoker", "lung cancer", 0.9, 7,
                        "non-smoker", "no lung cancer", 99.1, 743)
```

Bar chart for the visualization of marginal distributions



Conditional distributions

A Conditional distribution shows the distribution of a variable "conditioned on" or by levels of another variable..

- This allows you to visualize the differences in the response variable for varying levels of the exposure variable.

With the lung cancer example we are asking the question, is the distribution of lung cancer different for smokers than it is for non-smokers...

Dodged bar chart for the visualization of conditional distributions

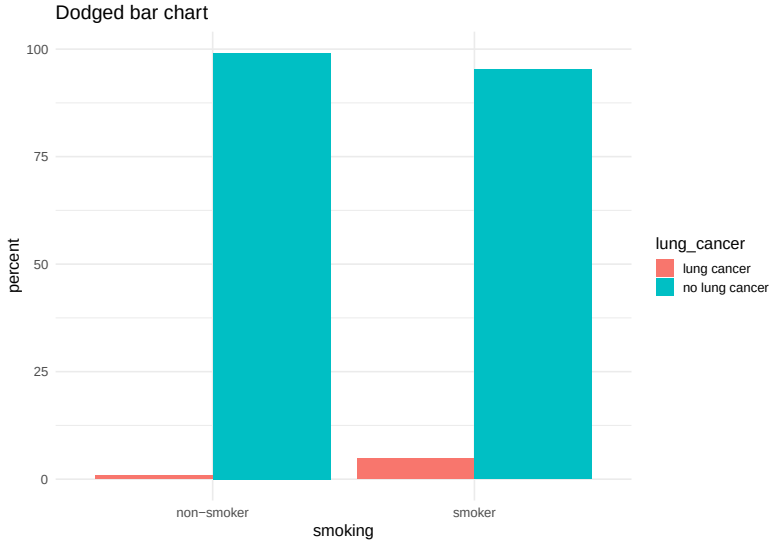
Syntax:

ggplot(two_way_data, aes(x = smoking, y = percent)) +

geom_bar(aes(fill = lung_cancer), stat = "identity", position = "dodge") +

labs(title = "Dodged bar chart") + theme_minimal(base_size = 15)

Dodged bar chart for the visualization of conditional distributions



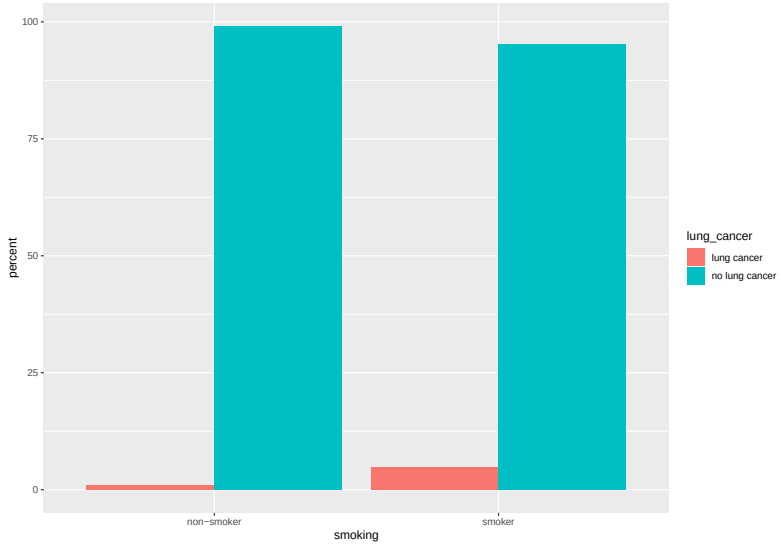
A note about which syntax is necessary vs aesthetic

Syntax:

ggplot(two_way_data, aes(x = smoking, y = percent)) +

geom_bar(aes(fill = lung_cancer), stat = "identity", position = "dodge")

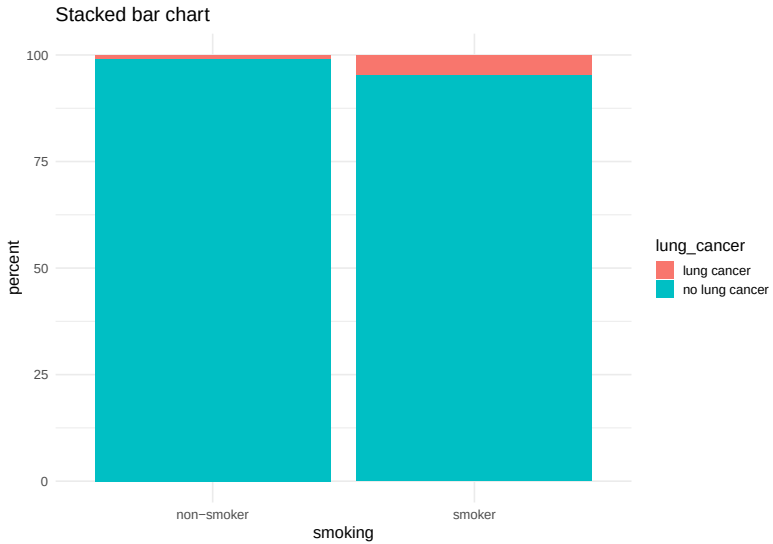
Dodged bar chart for the visualization of conditional distributions



Stacked bar chart for the visualization of conditional distributions


```
ggplot(two_way_data, aes(x = smoking, y = percent)) +
geom_bar(aes(fill = lung_cancer), stat = "identity", position = "stack") +
labs(title = "Stacked bar chart") + theme_minimal(base_size = 15)
```

Stacked bar chart for the visualization of conditional distributions



Visualization of conditional distributions: multiple levels of response variable

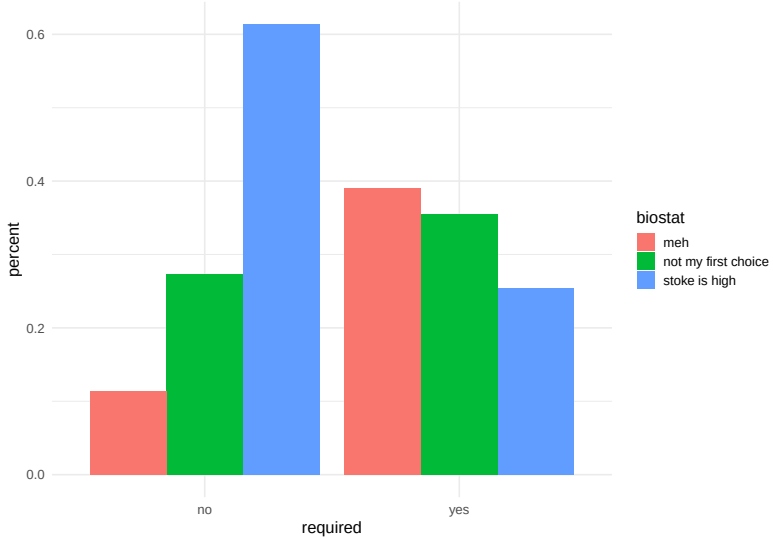
- Plots like the one previous make less sense when there are only two levels of both of the variables. This is because once you know the percent of lung cancer among smokers, you also know the percent of non-lung cancer among smokers.
- Here is another example from the start of semester survey:

	0	1	Overall
	(N=88)	(N=318)	(N=406)
factor(biostat)			
meh	24 (27.3%)	124 (39.0%)	148 (36.5%)
not my first choice of how to spend my time	10 (11.4%)	81 (25.5%)	91 (22.4%)
the stoke is high	54 (61.4%)	113 (35.5%)	167 (41.1%)

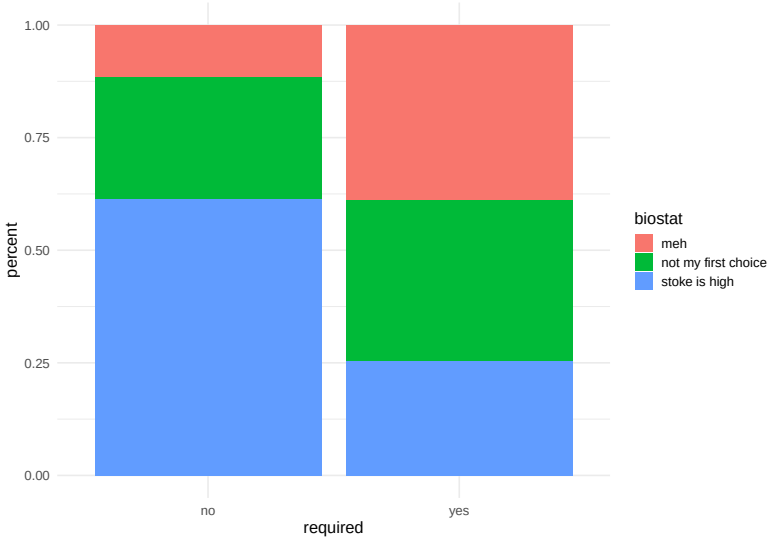
Visualization of conditional distributions: three levels of response variable

```
ggplot(survey, aes(x = required, y = percent)) +
geom_bar(stat = "identity", aes(fill = biostat), position = "dodge") +
theme_minimal(base_size = 15)
```

Visualization of conditional distributions: three levels of response variable



Visualization of conditional distributions: three levels of response variable

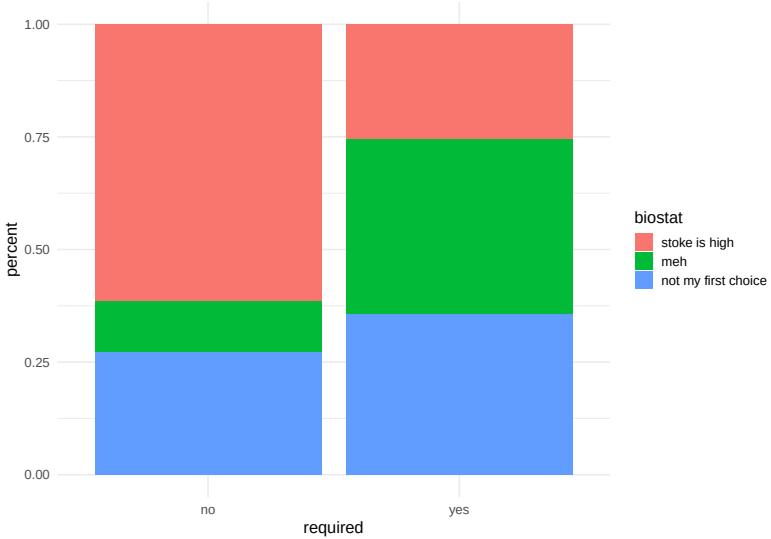


Visualization of conditional distributions: three levels of response variable

Recall from last class we learned how to reorder factor variables that affect the look of the plot:

survey <- survey %>% mutate(biostat = fct_relevel(biostat, "stoke is high", "meh", "not my first choice"))

Visualization of conditional distributions: three levels of response variable



Visualization of conditional distributions: three levels of response variable

Why might we prefer dodged plots to stacked plots?

Simpson's Paradox

Simpson's Paradox: Example from Baldi and Moore

- We will use an example of communit mortality that is presented in your book to illustrate Simpson's paradox.
- This dataset has 4 variables, age group, community, deaths and population

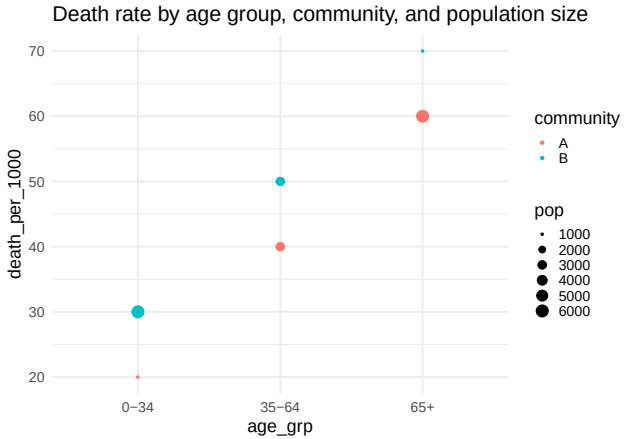
```
## # A tibble: 6 x 5
##   age_grp community deaths   pop death_per_1000
##   <chr>   <chr>      <dbl> <dbl>          <dbl>
## 1 0-34    A             20  1000             20
## 2 35-64   A            120  3000             40
## 3 65+     A            360  6000             60
## 4 all     A            500 10000             50
## 5 0-34    B            180  6000             30
## 6 35-64   B            150  3000             50
```

Simpson's Paradox Example: Only Conditional data

Plot the mortality rates according to age group and community, linking size of dot to population size

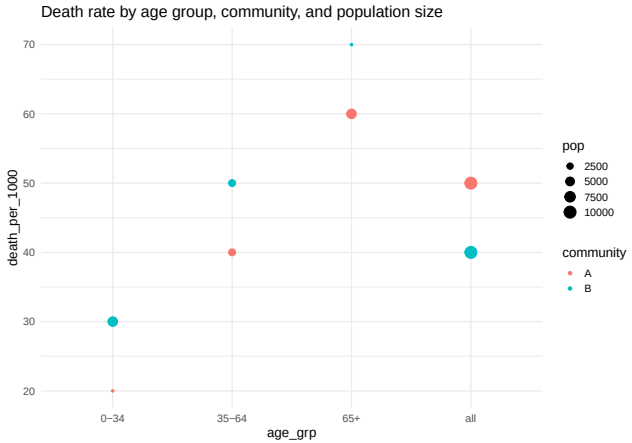
```
ggplot(simp_data_no_all, aes(x = age_grp, y = death_per_1000)) +
geom_point(aes(col = community, size = pop)) +
labs(title = "Death rate by age group, community, and population size") +
theme_minimal(base_size = 15)
```

Simpson's Paradox Example: Only Conditional data



- What do you notice here? If someone ask you which community has higher mortality, what would you say?

Simpson's Paradox Example: with marginal data



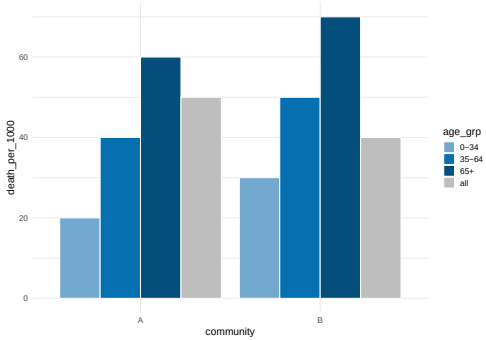
- Notice that the mortality rates for the communities overall show community A having a higher rate than community B. Why is that?

Simpson's Paradox

"An association or comparison that holds for all of several groups can reverse direction when the data are combined to form a single group. This reversal is called Simpson's Paradox"

Simpson's Paradox

- Here are the same data shown using a bar chart



Which visualization gives you more information?

Simpson's Paradox Berkeley example

A famous example of Simpson's paradox related to admissions to Berkeley by gender:

Watch: https://www.youtube.com/watch?v=E_ME4P9fQbo

Recap: Code and concepts

1. `geom_bar(aes(col = var), stat = "identity", position = "dodge")`
2. `geom_bar(aes(col = var), stat = "identity", position = "stack")`
3. Marginal vs conditional distributions
4. Simpson's Paradox

Comic Relief