Homework 10

Your name and student ID

Today's date

- Solutions released date: Tuesday, October 27
- Remember: autograder is meant as sanity check ONLY. It will not tell you if you have the correct answer. It will tell you if you are in the ball park of the answer so CHECK YOUR WORK.
- Submission process: Follow the submission instructions on the final page. Make sure you do not remove any \newpage tags or rename this file, as this will break the submission.

Helpful hints:

- Every function you need to use was taught during lecture! So you may need to revisit the lecture code to help you along by opening the relevant files on Datahub. Alternatively, you may wish to view the code in the condensed PDFs posted on the course website. Good luck!
- Knit your file early and often to minimize knitting errors! If you copy and paste code for the slides, you are bound to get an error that is hard to diagnose. Typing out the code is the way to smooth knitting! We recommend knitting your file each time after you write a few sentences/add a new code chunk, so you can detect the source of knitting errors more easily. This will save you and the GSIs from frustration! You must knit correctly before submitting.
- If your code runs off the page of the knitted PDF then you will LOSE POINTS! To avoid this, have a look at your knitted PDF and ensure all the code fits in the file (you can easily view it on Gradescope via the provided link after submitting). If it doesn't look right, go back to your .Rmd file and add spaces (new lines) using the return or enter key so that the code runs onto the next line.

You would like to conduct a survey of highschool students to determine the proportion who are current ecigarettes users. Before you conduct your survey, you need to determine how large of a sample size. Suppose that you would like the width of the 95% confidence interval to be 2.5 percentage points.

1. [1 point] Determine the most conservative sample size you would require and assign it to object p1. Recall that to do this, you need to use a p^* of 0.5.

```
p1 <- "YOUR ANSWER HERE"
# remember to remove " " if you want to store a number
# BEGIN SOLUTION
p1 \leftarrow ceiling((1.96/0.025)^2*0.5*(1-0.5))
р1
## [1] 1537
# END SOLUTION
check_problem1()
## [1] "Checkpoint 1 Passed: Your answer is numeric"
## [1] "Checkpoint 2 Passed: Correct, the answer is 1537."
##
## Problem 1
## Checkpoints Passed: 2
## Checkpoints Errored: 0
## 100% passed
## -----
## Test: PASSED
```

BEGIN SOLUTION

```
n = (z * /m)^2 p * (1 - p*)  n = (1.96/0.025)^2 \times 0.5 \times (1 - 0.5) = 1536.64 = 1537
```

Thus, we would need a sample size of 1537 high school students to obtain a margin of error of 2.5 percentage points if we assume the true prevalence is 50%. # END SOLUTION

2. [1 point] You've seen a recent publication from the Annals of Internal Medicine that estimated that 9.2% of individuals aged 18 to 24 years old are current e-cigarette users. What is the sample size estimate assuming that $p^* = 0.092$.

```
p2 <- "YOUR ANSWER HERE"
# remember to remove " " if you want to store a number
# BEGIN SOLUTION
p2 <- ceiling((1.96/0.025)^2*0.092*(1-0.092))
p2
## [1] 514
# END SOLUTION
check_problem2()
## [1] "Checkpoint 1 Passed: Your answer is numeric"
## [1] "Checkpoint 2 Passed: Correct, the answer is 514."
##
## Problem 2
## Checkpoints Passed: 2
## Checkpoints Errored: 0
## 100% passed
## -----
## Test: PASSED
```

BEGIN SOLUTION

```
n = (z * /m)^2 p * (1 - p*)  n = (1.96/0.025)^2 \times 0.092 \times (1 - 0.092) = 513.459 = 514
```

Thus, we would need a sample size of 514 high school students to obtain a margin of error of 2.5 percentage points if we assume the true prevalence is 9.2%. # END SOLUTION

3. [1 point] The recent publication referenced in the previous question only looked at adults (aged 18+), but observed that the rate of current use was inversely related to age among the population they surveyed. Because of this finding would you suppose that the sample size estimated in part (b) is too low or too high?

```
# Uncomment one of the following options:
# p3 <- "Too low"
# p3 <- "Too high"
# BEGIN SOLUTION
p3 <- "Too low"
# END SOLUTION
check_problem3()
## [1] "Checkpoint 1 Passed: Correct"
## [1] "Checkpoint 2 Passed: Correct, the sample size estimated in part (b) is too low."
##
## Problem 3
## Checkpoints Passed: 2
## Checkpoints Errored: 0
## 100% passed
## -----
## Test: PASSED
```

BEGIN SOLUTION

I would suppose that the estimated sample size is too low because the true prevalence among highschool students is likely higher than among 18-24 year olds. If that is the case, then using a higher p* in the sample size calculation would increase the sample size required. # END SOLUTION

Exclusive breastfeeding during the first six months of life is recommended for optimal infant growth and development. Suppose that you conducted a survey of randomly chosen women from California and found that 775 out of 5615 new mothers exclusively breast fed their infants.

Perform all four of the methods discussed in lecture and during lab to create a 95% confidence interval for the proportion of infants exclusively breast fed.

```
library(tidyverse)
library(tibble)
```

Store your answer to p4-p7 using the following format:

```
pX <- c(lowerbound, upperbound)
# For example, if lowerbound = 10, upperbound = 20:
pX <- c(10, 20)</pre>
```

4. [1 point] Use the large sample method of constructing a 95% CI.

```
# YOUR CODE HERE
# Replace "lowerbound" and "upperbound" with your answer
# If your answer is a number, make sure it doesn't have quotes around it
p4 <- c("lowerbound", "upperbound")
# BEGIN SOLUTION
num successes <- 775
sample_size <- 5615
p_hat <- num_successes/sample_size # estimate proportion</pre>
se <- sqrt(p_hat*(1-p_hat)/sample_size) # standard error</pre>
p4 <- c(p_hat - 1.96*se, p_hat + 1.96*se) # CI
p4
## [1] 0.1290011 0.1470452
# END SOLUTION
check_problem4()
## [1] "Checkpoint 1 Passed: Your answer is numeric."
## [1] "Checkpoint 2 Passed: Upperbound and lowerbound have been stored."
```

```
## [1] "Checkpoint 1 Passed: Your answer is numeric."
## [1] "Checkpoint 2 Passed: Upperbound and lowerbound have been stored."
## [1] "Checkpoint 3 Passed: Correct, the lowerbound is 0.1290011 and upperbound 0.1470452."
##
## Problem 4
## Checkpoints Passed: 3
## Checkpoints Errored: 0
## 100% passed
## ------
## Test: PASSED
```

5. [1 point] Use the Clopper Pearson (Exact) method of constructing a 95% CI.

```
# YOUR CODE HERE
# Replace "lowerbound" and "upperbound" with your answer
# If your answer is a number, make sure it doesn't have quotes around it
p5 <- c("lowerbound", "upperbound")
# BEGIN SOLUTION
exact_out <- binom.test(num_successes, sample_size, p=0.5)</pre>
p5 <- c(exact_out$conf.int[1], exact_out$conf.int[2])
р5
## [1] 0.1291020 0.1473222
# END SOLUTION
check_problem5()
## [1] "Checkpoint 1 Passed: Your answer is numeric."
## [1] "Checkpoint 2 Passed: Upperbound and lowerbound have been stored."
## [1] "Checkpoint 3 Passed: Correct, the lowerbound is 0.1291020 and upperbound 0.1473222."
##
## Problem 5
## Checkpoints Passed: 3
## Checkpoints Errored: 0
## 100% passed
## -----
## Test: PASSED
  6. [1 point] Use the Wilson Score method of constructing a 95% CI with a continuity correction.
# YOUR CODE HERE
# Replace "lowerbound" and "upperbound" with your answer
# If your answer is a number, make sure it doesn't have quotes around it
p6 <- c("lowerbound", "upperbound")</pre>
# BEGIN SOLUTION
wilson_out <- prop.test(num_successes, sample_size, p=0.5)</pre>
p6 <- c(wilson_out$conf.int[1], wilson_out$conf.int[2])</pre>
р6
## [1] 0.1291619 0.1473842
# END SOLUTION
check_problem6()
## [1] "Checkpoint 1 Passed: Your answer is numeric."
## [1] "Checkpoint 2 Passed: Upperbound and lowerbound have been stored."
## [1] "Checkpoint 3 Passed: Correct, the lowerbound is 0.1291619 and upperbound 0.1473842."
##
## Problem 6
## Checkpoints Passed: 3
```

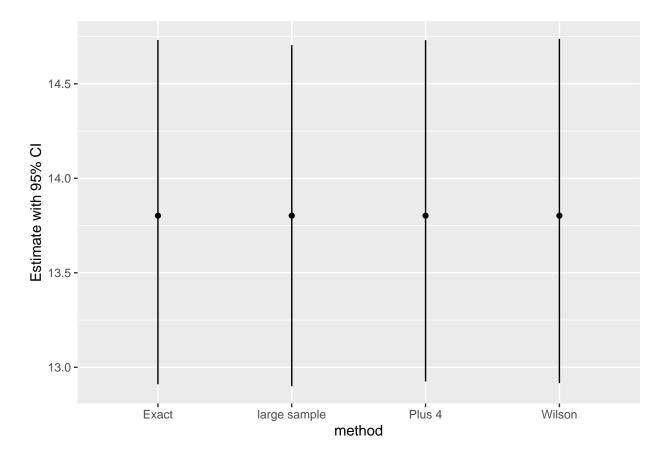
```
## Checkpoints Errored: 0
## 100% passed
## -----
## Test: PASSED
```

7. [1 point] Use the Plus Four method of constructing a 95% CI.

```
# YOUR CODE HERE
# Replace "lowerbound" and "upperbound" with your answer
# If your answer is a number, make sure it doesn't have quotes around it
p7 <- c("lowerbound", "upperbound")
# BEGIN SOLUTION
p_tilde <- (num_successes + 2)/(sample_size + 4)</pre>
se <- sqrt(p_tilde*(1 - p_tilde)/(sample_size + 4)) # standard error</pre>
p7 <- c(p_tilde - 1.96*se, p_tilde + 1.96*se) # CI
# END SOLUTION
check_problem7()
## [1] "Checkpoint 1 Passed: Your answer is numeric."
## [1] "Checkpoint 2 Passed: Upperbound and lowerbound have been stored."
## [1] "Checkpoint 3 Passed: Correct, the lowerbound is 0.1292517 and upperbound 0.1473099."
## Problem 7
## Checkpoints Passed: 3
## Checkpoints Errored: 0
## 100% passed
## -----
## Test: PASSED
```

8. [2 points] Create a plot comparing the confidence intervals. If you are stuck, refer back to the example code presented in Lab 8.

```
p8 <- "YOUR ANSWER HERE"
# BEGIN SOLUTION
breastfeed_CIs <- tibble(method = c("large sample", "Exact",</pre>
                                                                                    "Plus 4"),
                                                                     "Wilson",
                                                                     , 12.92517),
                 lower_CI = c(12.90011)
                                           , 12.91020 , 12.91619
                 upper_CI = c(14.70452)
                                           , 14.73222
                                                        , 14.73842
                                                                      , 14.73099),
                 estimate = c(p_hat*100)
                                           , p_hat*100
                                                         , p_hat*100 , p_hat*100)
p8 <- ggplot(data = breastfeed_CIs, aes(x = method, y = estimate)) +
 geom_point() +
 geom_segment(aes(x = method, xend = method, y = lower_CI, yend = upper_CI)) +
 labs(y = "Estimate with 95% CI")
р8
```



```
# END SOLUTION
check_problem8()
```

```
## [1] "Checkpoint 1 Passed: A ggplot has been defined."
## [1] "Checkpoint 2 Passed: breastfeed_CIs has been used as the data."
## [1] "Checkpoint 3 Passed: The y has been specified correctly."
```

```
## [1] "Checkpoint 4 Passed: The x has been specified correctly."
## [1] "Checkpoint 5 Passed: The point estimates have been added."
## [1] "Checkpoint 6 Passed: The CI segments have been added."
##
## Problem 8
## Checkpoints Passed: 6
## Checkpoints Errored: 0
## 100% passed
## -------
## Test: PASSED
```

9. [1 point] Do the methods produce confidence intervals that are basically the same or very different? Why?

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

The plot comparing the 4 CIs is below. They are nearly identical. This is because the sample size is large enough such that the CLT holds, implying that the large sample method is good, and so are all the other methods. When the sample size is large enough, all the CIs should agree. # END SOLUTION

10. [1 point] Suppose that in 2010, the rate of exclusive breastfeeding in California was known to be 18.6%. Based on the 95% CIs calculated in questions 4-7, is there evidence against the null hypothesis that the underlying rate is equal to 18.6% in favor of the alternative that the rate is different from 18.6%?

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

18.6% falls far above all of the CIs. Because 18.6 is outside of the range of the CIs, we can conclude that the p-value for the corresponding hypothesis test would be < 5% and conclude that yes, there is evidence in favor of the alternative hypothesis that the rate differs from 18.6% # END SOLUTION

To confirm your answer to Problem 9, perform a two-sided hypothesis test and interpret the p-value.

11. [1 point] State the null and alternative hypotheses:

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

 $H_0: \mu = 18.6\%$ vs. $H_a: \mu \neq 18.6\%$ # END SOLUTION

12. [1 point] Calculate the test statistic:

```
p12 <- "YOUR ANSWER HERE"
# BEGIN SOLUTION
n <- 5615
p0 <- 0.186
p12 <- ((p_hat - p0) / sqrt(p0 * (1-p0) / n))
p12
## [1] -9.239275
# END SOLUTION
check_problem12()
## [1] "Checkpoint 1 Passed: Your answer is numeric"
## [1] "Checkpoint 2 Passed: Correct, the answer is 9.239275."
## Problem 12
## Checkpoints Passed: 2
## Checkpoints Errored: 0
## 100% passed
## -----
## Test: PASSED
```

BEGIN SOLUTION

z-test for one-sample test for a proportion:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} = \frac{0.1380232 - .186}{\sqrt{\frac{.186(1 - .186)}{5615}}} = -9.239266$$

The test statistic is equal to -9.239266. # END SOLUTION

13. [1 point] Calculate the p-value:

14. [1 point] Interpret the p-value:

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

The p-value is very very tiny, much less than 0.0001%. This implies that the chance of seeing a proportion of 13.8% (or one even more different in magnitude) from the null value of 18.6% is < 0.0001%. Thus, there is evidence against the null hypothesis, in favor of the alternative hypothesis that the proportion differs from 18.6%. # END SOLUTION

The quadrivalent HPV vaccine was introduced to Canada in 2007, and was given to girls in Ontario, Canada who were enrolled in grade 8 (13-14 year olds). Before 2007, no girls recieved the vaccine, while in the 4 years after it was introduced nearly 40% of girls recieved the vaccine each year. One concern that some people had was that the vaccine itself would increase promiscuity if the girls felt a false sense of protection, which could thereby increase the prevalence of other sexually transmitted infections (STIs) among vaccinated girls. This paper examines this question using an advanced method called the "regression discontinuity" design which harnesses the abrupt change in vaccination status across cohorts of girls to estimate the causal effect of vaccination against HPV on the occurrence of other STIs.

Read only the abstract of the paper, and don't worry about the details because these are advanced methods. Note that the term "RD" is the difference in risk of STIs between girls exposed and unexposed to HPV vaccination. We can therefore think of this risk difference as the difference between two proportions.

15. [1 point] Interpret this result from the abstract: We identified 15 441 (5.9%) cases of pregnancy and sexually transmitted infection and found no evidence that vaccination increased the risk of this composite outcome: RD per 1000 girls -0.61 (95% confidence interval [CI] -10.71 to 9.49).

In particular, what does -0.61 estimate?

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

-0.61 is the estimated difference in the proportions of girls with an STI comparing girls who were vaccinated vs. girls who were not vaccinated. # END SOLUTION

16. [1 point] The 95% confidence interval includes 0. What can you conclude about the p-value for a two-sided test of the difference between vaccinated and unvaccinated girls and their risk of sexually transmitted diseases?

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

Given that the null value for the H_0 that there is no difference is included in the 95% CI, we know that the corresponding two-sided test of the difference between the underlying proportions would be greater than 5%. # END SOLUTION

An allergy to peanuts is increasingly common in Western countries. A randomized controlled trial enrolled infants with a diagnosed peanut sensitivity. Infants were randomized to either avoid peanuts or to consume them regularly until they reached age 5. At the end of the study, 18 out of the 51 randomized to avoid peanuts were tested to be allergic to peanuts. Only 5 out of the 47 randomized to consuming them regularly were tested to be allergic to peanuts.

17. [1 point] Estimate the difference between the two proportions.

```
p17 <- "YOUR ANSWER HERE"
# BEGIN SOLUTION
succ1 <- 18
n1 <- 51
succ2 <- 5
n2 <- 47
p17 <- (18/51) - (5/47) # 35.3% - 10.6%
p17
## [1] 0.2465582
# END SOLUTION
check_problem17()
## [1] "Checkpoint 1 Passed: Your answer is numeric"
## [1] "Checkpoint 2 Passed: Correct, the answer is 0.2465582."
##
## Problem 17
## Checkpoints Passed: 2
## Checkpoints Errored: 0
## 100% passed
## -----
## Test: PASSED
```

18. [1 point] Use the plus four method to find a 99% confidence interval for the difference between the two groups. Store the upper and lower bounds into an object called p18.

```
# YOUR CODE HERE
# Replace "lowerbound" and "upperbound" with your answer
# If your answer is a number, make sure it doesn't have quotes around it
p18 <- c("lowerbound", "upperbound")</pre>
# BEGIN SOLUTION
p1_{tilde} \leftarrow (succ1 + 1)/(n1 + 2)
p2_{tilde} \leftarrow (succ2 + 1)/(n2 + 2)
se \leftarrow sqrt((p1_tilde*(1 - p1_tilde)/(n1 + 2)) + (p2_tilde*(1 - p2_tilde)/(n2 + 2)))
p18 <- c((p1_tilde - p2_tilde) - 2.576 * se, (p1_tilde - p2_tilde) + 2.576 * se)
p18
## [1] 0.02784538 0.44423779
# END SOLUTION
check_problem18()
## [1] "Checkpoint 1 Passed: Your answer is numeric."
## [1] "Checkpoint 2 Passed: Upperbound and lowerbound have been stored."
## [1] "Checkpoint 3 Passed: Correct, the lowerbound is 0.02784538 and upperbound 0.44423779."
## Problem 18
## Checkpoints Passed: 3
## Checkpoints Errored: 0
## 100% passed
## -----
## Test: PASSED
```

BEGIN SOLUTION

Solution: The 99% confidence interval for the difference is 2.78% to 44.4%. # END SOLUTION

19. [1 point] Why would it have been inappropriate to use the large sample method to create a 99% CI?

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

Because the number of "successes" was 5 in the group who consumed peanuts regularly. Since 5 < 10, it is not appropriate to use the large sample method. # END SOLUTION

Perform a two-sided hypothesis test for the difference between the groups. Start by stating the null and alternative hypotheses, then calculate the test statistic, the p-value, and conclude with your interpretation of the p-value.

20. [1 point] State the null and alternative hypotheses:

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

 $H_0: p_1 = p_2 \text{ vs. } H_0: p_1 \neq p_2$

END SOLUTION

21

21. [1 point] Calculate the test statistic:

```
p21 <- "YOUR ANSWER HERE"
# BEGIN SOLUTION
phat <- (succ1 + succ2)/(n1 + n2)
p21 \leftarrow (succ1/n1 - succ2/n2)/sqrt(phat*(1- phat)*(1/n1 + 1/n2))
## [1] 2.877213
# END SOLUTION
check_problem21()
## [1] "Checkpoint 1 Passed: Your answer is numeric"
## [1] "Checkpoint 2 Passed: Correct, the answer is 2.877213."
##
## Problem 21
## Checkpoints Passed: 2
## Checkpoints Errored: 0
## 100% passed
## -----
## Test: PASSED
```

BEGIN SOLUTION

First, calculate \hat{p} , the estimated probability of having a peanut allergy assuming that the proportions are the same: $\hat{p} = \frac{18+5}{51+47} = 0.2346939$

Then, the test statistic is:
$$\frac{\hat{p_1} - \hat{p_2}}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.3529412 - 0.106383}{\sqrt{0.2346939(1 - 0.2346939)\left(\frac{1}{51} + \frac{1}{47}\right)}} = 2.877213 \ \# \ \text{END SOLUTION}$$
 TION

22. [1 point] Calculate the p-value:

23. [1 point] Interpret the p-value:

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

The p-value is < 0.001. Because the p-value is so small there is evidence against the null hypothesis in favor of the alternative that there is a difference between the groups. # END SOLUTION

24. [1 point] Suppose you were testing the hypotheses $H_0: \mu_d = 0$ and $H_a: \mu_d \neq 0$ in a paired design and obtain a p-value of 0.21. Which one of the following could be a possible 95% confidence interval for μ_d ?

```
# Uncomment one of the following choices:
# p24 <- "-2.30 to -0.70"
# p24 <- "-1.20 to 0.90"
# p24 <- "1.50 to 3.80"
# p24 <- "4.50 to 6.90"
# BEGIN SOLUTION
p24 \leftarrow "-1.20 to 0.90"
# END SOLUTION
check_problem24()
## [1] "Checkpoint 1 Passed: One choice has been selected."
## [1] "Checkpoint 2 Passed: Correct"
## Problem 24
## Checkpoints Passed: 2
## Checkpoints Errored: 0
## 100% passed
## -----
## Test: PASSED
```

25. [1 point] Suppose you were testing the hypotheses $H_0: \mu_d = 0$ and $H_a: \mu_d \neq 0$ in a paired design and obtain a p-value of 0.02. Also suppose you computed confidence intervals for μ_d . Based on the p-value which of the following are true?

```
# Uncomment one of the following choices:
# p25 <- "Both a 95% CI and a 99% CI will contain 0."
# p25 <- "A 95% CI will contain O, but a 99% CI will not."
# p25 <- "A 95% CI will not contain 0, but a 99% CI will."
# p25 <- "Neither a 95% CI nor a 99% CI interval will contain O."
# BEGIN SOLUTION
p25 <- "A 95% CI will not contain 0, but a 99% CI will."
# END SOLUTION
check_problem25()
## [1] "Checkpoint 1 Passed: One choice has been selected."
## [1] "Checkpoint 2 Passed: Correct"
## Problem 25
## Checkpoints Passed: 2
## Checkpoints Errored: 0
## 100% passed
## -----
## Test: PASSED
```

Check your score

Click on the middle icon on the top right of this code chunk (with the downwards gray arrow and green bar) to run all your code in order. Then, run this chunk to check your score.

```
# Just run this chunk.
total_score()
```

##					Test	Points_Possible	Туре
##	Problem	1			PASSED	1	autograded
##	Problem	2			PASSED	1	autograded
##	Problem	3			PASSED	1	autograded
##	${\tt Problem}$	4			PASSED	1	autograded
##	${\tt Problem}$	5			PASSED	1	autograded
##	${\tt Problem}$	6			PASSED	1	autograded
##	${\tt Problem}$	7			PASSED	1	autograded
##	${\tt Problem}$	8			PASSED	2	autograded
##	${\tt Problem}$	9	NOT	YET	GRADED	1	free-response
##	${\tt Problem}$	10	NOT	YET	GRADED	1	free-response
##	${\tt Problem}$	11	NOT	YET	GRADED	1	free-response
##	${\tt Problem}$	12			PASSED	1	autograded
##	${\tt Problem}$	13			PASSED	1	autograded
##	${\tt Problem}$	14	NOT	YET	GRADED	1	free-response
##	${\tt Problem}$	15	NOT	YET	GRADED	1	free-response
##	${\tt Problem}$	16	NOT	YET	GRADED	1	free-response
##	${\tt Problem}$	17			PASSED	1	autograded
##	${\tt Problem}$	18			PASSED	1	autograded
##	${\tt Problem}$	19	NOT	YET	GRADED	1	free-response
##	${\tt Problem}$	20	NOT	YET	GRADED	1	free-response
##	${\tt Problem}$	21			PASSED	1	autograded
##	${\tt Problem}$	22			PASSED	1	autograded
##	${\tt Problem}$	23	NOT	YET	GRADED	1	free-response
##	${\tt Problem}$	24			PASSED	1	autograded
##	Problem	25			PASSED	1	autograded