Assignment 7

Your name and student ID

Today's date

- Solutions released date: Tuesday, October 27

Helpful hints:

- Every function you need to use was taught during lecture! So you may need to revisit the lecture code to help you along by opening the relevant files on Datahub.

In two wards for elderly patients in a local hospital the following levels of hemoglobin (grams per liter) were found for a simple random sample of patients from each ward.:

Ward A:

```
ward_a <- c(12.2, 11.1, 14.0, 11.3, 10.8, 12.5, 12.2, 11.9, 13.6, 12.7, 13.4, 13.7)
```

Ward B:

```
ward_b <- c(11.9, 10.7, 12.3, 13.9, 11.1, 11.2, 13.3, 11.4, 12.0, 11.1)
```

1. [1 point] In one ggplot, create two box plots to compare the hemoglobin values for Ward A and Ward B. Also plot the raw data as points, overlaid on top of the box plots.

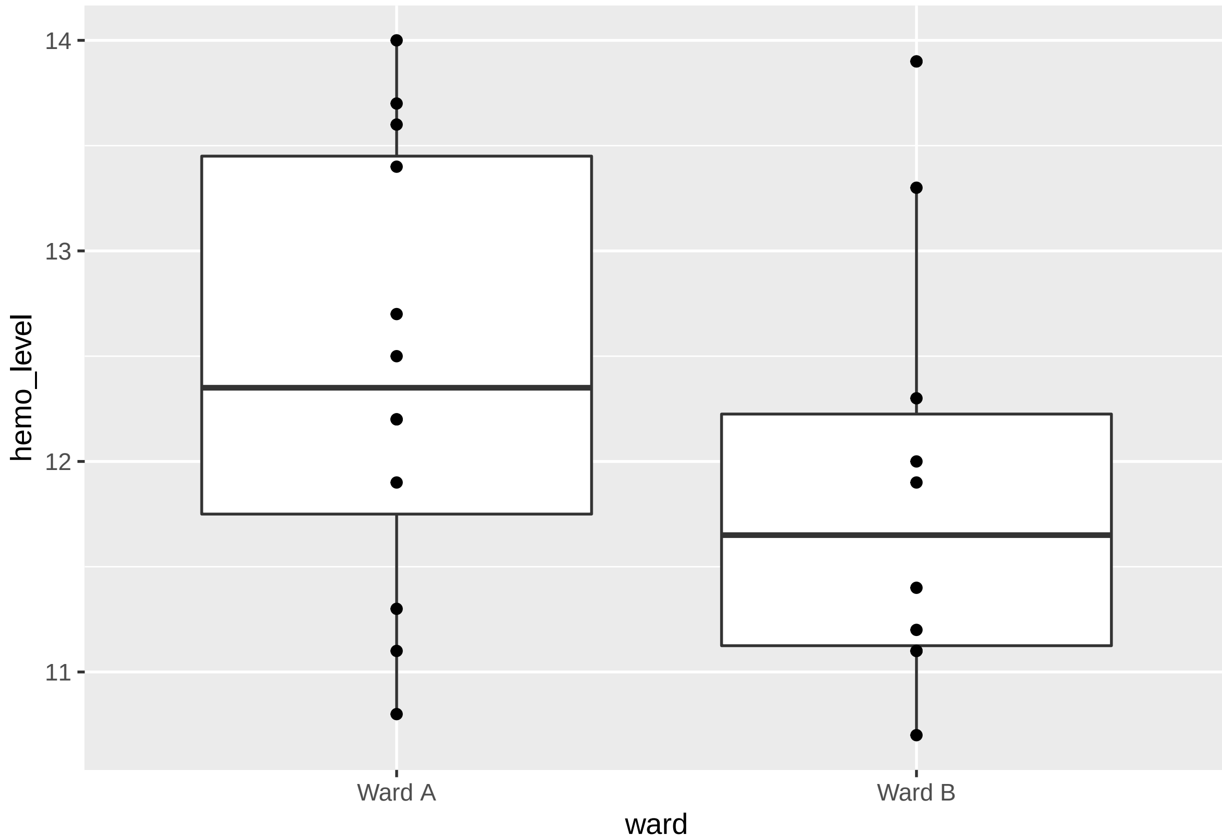
```
hemoglobin <- data.frame(hemo_level = c(ward_a, ward_b),
                         ward = c(rep("Ward A", 12), rep("Ward B", 10)))
```

```
p1 <- "YOUR ANSWER HERE"
p1
```

```
## [1] "YOUR ANSWER HERE"
```

```
# BEGIN SOLUTION
p1 <- ggplot(hemoglobin, aes(y = hemo_level, x = ward)) +
  geom_boxplot() +
  geom_point()

p1
```




```r
# END SOLUTION
check_problem1()
```

```
## [1] "Checkpoint 1 Passed: A ggplot has been defined"
## [1] "Checkpoint 2 Passed: Correct! You used hemoglobin as the data."
## [1] "Checkpoint 3 Passed: Correct! You used hemoglobin as the data!"
## [1] "Checkpoint 4 Passed: Correct! You used hemoglobin as the data!"
## [1] "Checkpoint 5 Passed: Correct! You defined a boxplot."
## [1] "Checkpoint 6 Passed: Correct! You plotted the raw data points with geom_point overlaid on top o
##
## Problem 1
## Checkpoints Passed: 6
## Checkpoints Errored: 0
## 100% passed
## --------
## Test: PASSED
```

2. [1 points] Comment on the similarities/differences portrayed by the plots, keeping in mind that the sample size is relatively small for these two wards.

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

There is some overlap in the middle 50% of the data form these two wards. There do not appear to be outliers in either distribution. Both samples appear to be roughly symmetric. The sample median is higher in Ward A than Ward B. # END SOLUTION

3. [2 points] What two assumptions do you need to make to use any of the t-procedures? Because each ward has a rather small sample size ($n < 12$ for both), what two characteristics of the data would you need to check for to ensure that the t-procedures can be applied?

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

- Two assumptions: SRS, normality of underlying dataset
- No outliers, data has similar shapes # END SOLUTION

4. [3 points] Using only `dplyr` and `*t` functions, create a 95% confidence interval for the mean difference between Ward A and Ward B. You can do this by using `dplyr` to calculate the inputs required to calculate the 95% CI, and then plugging these values in on a separate line of code (or using your calculator). Use a degrees of freedom of 19.515 (You don't need to calculate the degrees of freedom, you can use this value directly). Show your work and interpret the mean difference and its 95% CI. Round your solution to 3 decimal places.

```
# YOUR CODE HERE

# THEN, ASSIGN YOUR FINAL ANSWERS BELOW:
CI_lowerbound <- "YOUR ANSWER HERE"
CI_upperbound <- "YOUR ANSWER HERE"

# BEGIN SOLUTION
hemoglobin %>% group_by(ward) %>% summarise(sample_mean = mean(hemo_level),
                                             sample_var = var(hemo_level),
                                             n = length(hemo_level))
```

```
## `summarise()` ungrouping output (override with `.groups` argument)

## # A tibble: 2 x 4
##   ward   sample_mean sample_var     n
##   <chr>        <dbl>      <dbl> <int>
## 1 Ward A        12.4       1.14    12
## 2 Ward B        11.9       1.07    10
```

```
# here is how you calculate degrees of freedom
deg_free <- ( (1.140909/12) + (1.065444/10) )^2/( (1/11)*(1.140909/12)^2 + (1/9)*(1.065444/10)^2)
mean_diff <- 12.45 - 11.89
se_diff <- sqrt(1.140909/12 + 1.065444/10)
t_star <- qt(p = 0.025, df = deg_free)

CI_upperbound <- mean_diff - t_star * (se_diff)
CI_lowerbound <- mean_diff + t_star * (se_diff)
# END SOLUTION
check_problem4()
```

```
## [1] "Checkpoint 1 Passed: You got the correct CI_upperbound!"
## [1] "Checkpoint 2 Passed: You got the correct CI_lowerbound!"
##
## Problem 4
## Checkpoints Passed: 2
## Checkpoints Errored: 0
## 100% passed
## --------
## Test: PASSED
```

5. [1 points] Interpret the mean difference and its 95% CI you just calculated.

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

The sample mean difference is 0.56 and its 95% CI goes from -0.44 to 1.56. This means that if we were to repeat this procedure 100 times, we would expect that 95 of the CIs would contain the true difference. The range of the difference goes from negative to positive indicating that at the 5% level there is no evidence against the null hypothesis of no difference.

END SOLUTION

Perform a two-sided t-test for the difference between the two samples, where the null hypothesis is that the underlying means are the same. Start by writing down the null and alternate hypotheses, then calculate the test statistic (showing your work) and p-value. Continue to assume that the degrees of freedom is 19.515. Verify the p-value by running the t-test using R's built in function. Show the output from that test. Hint: to perform the t-test using R's built in function, you need to pass the function an x and y argument, where x includes that values for Ward A and Y includes the values for Ward B. `dplyr`'s `filter()` and `pull()` functions will be your friends.

6. [1 points] Calculate the t-test statistics

```
t_statistics <- "YOUR ANSWER HERE"
t_statistics
```

```
## [1] "YOUR ANSWER HERE"
```

```
# BEGIN SOLUTION
t_statistics <- round(1.247157,2)
t_statistics
```

```
## [1] 1.25
```

```
# END SOLUTION

check_problem6()
```

```
## [1] "Checkpoint 1 Passed: Correct t statistic!"
##
## Problem 6
## Checkpoints Passed: 1
## Checkpoints Errored: 0
## 100% passed
## --------
## Test: PASSED
```

BEGIN SOLUTION

$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ $t = \frac{(12.45 - 11.89) - 0}{\sqrt{\frac{1.140909}{12} + \frac{1.065444}{10}}}$ $t = 0.56/0.4490213 = 1.247157$ # END SOLUTION

7. [1 points] We need to compare this t-statistic to a t distribution with 19.515 degrees of freedom:

```r
p_value <- "YOUR ANSWER HERE"
p_value
```

```
## [1] "YOUR ANSWER HERE"
```

```r
# BEGIN SOLUTION
p_value <- pt(1.247157, df = 19.515, lower.tail = F) * 2
p_value
```

```
## [1] 0.2271006
```

```r
# END SOLUTION

check_problem7()
```

```
## [1] "Checkpoint 1 Passed: Correct!"
##
## Problem 7
## Checkpoints Passed: 1
## Checkpoints Errored: 0
## 100% passed
## --------
## Test: PASSED
```

8.[2 points] Interpret the p value you got in the context of the this question. Are there evidence against null hypothesis?

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

Thus there is a 22.7% chance of seeing a difference of the size we saw or larger under the hypothesis of no difference. This is quite probable, so we conclude that there is no evidence against the null hypothesis. # END SOLUTION

Check this against the `t.test` output:

```
t.test(x = hemoglobin %>% filter(ward == "Ward A") %>% pull(hemo_level),
       y = hemoglobin %>% filter(ward == "Ward B") %>% pull(hemo_level),
       alternative = "two.sided")
```

```
##
##  Welch Two Sample t-test
##
## data:  hemoglobin %>% filter(ward == "Ward A") %>% pull(hemo_level) and hemoglobin %>% filter(ward ==
## t = 1.2472, df = 19.515, p-value = 0.2271
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.3781372  1.4981372
## sample estimates:
## mean of x mean of y
##     12.45     11.89
```

The time to perform open heart surgery is normally distributed. Sixteen patients (chosen as a simple random sample from a hospital) underwent open heart surgery that took the following lengths of time (in minutes).

```
op_time <- c(247.8648, 258.4343, 315.6787, 268.0563, 269.9372, 320.6821,
             280.5493, 225.3180, 243.8207, 251.5388, 304.9706, 277.3140,
             278.6247, 269.3418, 248.0131, 322.9812)
surg_data <- data.frame(op_time)
```

9. [1 point] You wish to know if the mean operating time of open heart surgeries at this hospital exceeds four hours. Set up appropriate hypotheses for investigating this issue.

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

Solution: $H_0 : \mu = 4$ hours (240 mins) $H_a : \mu > 4$ hours (240 mins)

END SOLUTION

10. [1 points] Test the hypotheses you formulated in part (a). Report the p-value. (Do not use the `t.test` function for this question)

```
p_value_10 <- "YOUR ANSWER HERE"
p_value_10
```

```
## [1] "YOUR ANSWER HERE"
```

```
# BEGIN SOLUTION
surg_data %>% summarise(mean = mean(op_time), se = sd(op_time)/sqrt(16))
```

```
##       mean       se
## 1 273.9454 7.305622
```

```
p_value_10 <- pt((273.9454-240)/7.305621, df = 15, lower.tail = F)
p_value_10
```

```
## [1] 0.0001582348
```

```
# END SOLUTION
check_problem10()
```

```
## [1] "Checkpoint 1 Passed: Correct!"
##
## Problem 10
## Checkpoints Passed: 1
## Checkpoints Errored: 0
## 100% passed
## --------
## Test: PASSED
```

11. [1 points] What are your conclusions in the context of the question?

[TODO: YOUR ANSWER HERE]

BEGIN SOLUTION

The p-value of 0.000158, which is very small. There is only a miniscule chance of seeing the sample mean we saw (or larger) if the null hypothesis is true. Thus we reject the null hypothesis in favor of the alternative, that the operating time exceeds 4 hours.

END SOLUTION

12. [3 points] Construct a 95% CI on the mean operating time (in hours).

```
# YOUR CODE HERE

# THEN, ASSIGN YOUR ANSWERS BELOW:
CI_lowerbound_12 <- "YOUR ANSWER HERE"
CI_upperbound_12<-"YOUR ANSWER HERE"

# BEGIN SOLUTION
CI_lowerbound_12 <- 4.31
CI_upperbound_12 <- 4.73

# END SOLUTION
check_problem12()
```

```
## [1] "Checkpoint 1 Passed: Correct!"
## [1] "Checkpoint 2 Passed: Correct"
##
## Problem 12
## Checkpoints Passed: 2
## Checkpoints Errored: 0
## 100% passed
## --------
## Test: PASSED
```

BEGIN SOLUTION

qt(p $=$ 0.975, df $=$ 15) $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$ $273.9454 \pm 2.13145 \times 7.305621 = 258.3738$ to $289.517 = 4.31$ hours to 4.73 hours

Thus, using a method that includes the null value 95 times out of 100, our 95% CI is 4.31 hours to 4.73 hours.

END SOLUTION

13. [1 point] Suppose you were testing the hypotheses $H_0 : \mu_d = 0$ and $H_a : \mu_d \neq 0$ in a paired design and obtain a p-value of 0.21. Which one of the following could be a possible 95% confidence interval for μ_d?

```r
# Uncomment one of the following choices:
# p13 <- "-2.30 to -0.70"
# p13 <- "-1.20 to 0.90"
# p13 <- "1.50 to 3.80"
# p13 <- "4.50 to 6.90"

# BEGIN SOLUTION
p13 <- "-1.20 to 0.90"
# END SOLUTION
check_problem13()
```

```
## [1] "Checkpoint 1 Passed: Correct!"
##
## Problem 13
## Checkpoints Passed: 1
## Checkpoints Errored: 0
## 100% passed
## --------
## Test: PASSED
```

14. [1 point] Suppose you were testing the hypotheses $H_0 : \mu_d = 0$ and $H_a : \mu_d \neq 0$ in a paired design and obtain a p-value of 0.02. Also suppose you computed confidence intervals for μ_d. Based on the p-value which one of the following is true?

```r
# Uncomment one of the following choices:
# p14 <- "Both a 95% CI and a 99% CI will contain 0."
# p14 <- "A 95% CI will contain 0, but a 99% CI will not."
# p14 <- "A 95% CI will not contain 0, but a 99% CI will."
# p14 <- "Neither a 95% CI nor a 99% CI interval will contain 0."

# BEGIN SOLUTION
p14 <- "A 95% CI will not contain 0, but a 99% CI will."
# END SOLUTION
check_problem14()
```

```
## [1] "Checkpoint 1 Passed: Correct!"
##
## Problem 14
## Checkpoints Passed: 1
## Checkpoints Errored: 0
## 100% passed
## --------
## Test: PASSED
```

Check your score

Click on the middle icon on the top right of this code chunk (with the downwards gray arrow and green bar) to run all your code in order. Then, run this chunk to check your score.

```r
# Just run this chunk.
total_score()
```

```
##                       Test Points_Possible          Type
## Problem 1           PASSED               1    autograded
## Problem 2  NOT YET GRADED               1 free-response
## Problem 3  NOT YET GRADED               2 free-response
## Problem 4           PASSED               3    autograded
## Problem 5  NOT YET GRADED               1 free-response
## Problem 6           PASSED               1    autograded
## Problem 7           PASSED               1    autograded
## Problem 8  NOT YET GRADED               2 free-response
## Problem 9  NOT YET GRADED               1 free-response
## Problem 10          PASSED               1    autograded
## Problem 11 NOT YET GRADED               1 free-response
## Problem 12          PASSED               3    autograded
## Problem 13          PASSED               1    autograded
## Problem 14          PASSED               1    autograded
```