Exploring relationships between two variables

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Administrivia

- Technical issues from Friday's lecture
  - GSIs will go over how to specify the correct working directory in R Studio
- Lectures and R Studio
  - We will explain concepts and demonstrate them with code in R Studio
  - Please focus on the concepts, and don't try to use R Studio at the same time
  - Please use the Zoom Chat to ask conceptual questions, and not technical ones

Recap of Chapters 1 and 2

- Histograms and bar charts to plot the distribution of a variable
- Measures of central tendency (e.g., mean, median) and spread (e.g., standard deviation, IQR)
- Time plots to examine the *relationship* between a variable and time

Learning objectives for today

- Explore the relationship between two quantitative variables
  - Direction, form, strength, outliers
  - Association vs. causation
- Make scatter plots to visualize bivariate relationships
  - using `geom_point()`
- Calculate the **correlation coefficient** to quantify the strength of linear relationships
  - using the `cor()` function

Readings

- Chapter 3 of Baldi and Moore
- Visual Distribution of different correlation coefficients (See section 5.7.4)
- Interpreting Correlation Coefficients (See section 5.7.5)

Explanatory (X) and response (Y) variables

Bi-directional statements:

- "X predicts Y", or "Y predicts X"
- "X is associated with Y", or "Y is associated with X"
- These statements don't comment on causation. Only that two variables are related.

Unidirectional statements:

- "X causes Y"
- This statement is stronger. Not only are X and Y related, X is a cause of Y. That is, if you change X, then Y will also change. Researchers conduct studies to investigate causal claims.

Which variable is x and which is y?

- In **prediction** modeling, X denotes the variable used to predict the variable of interest (Y)
- In **causal** modeling, X denotes the explanatory (independent) variable and Y denotes the response (dependent) variable
- Graphically, the X variable is on the X (horizontal) axis and the Y variable is the Y (vertical) axis

Which variable is x and which is y?

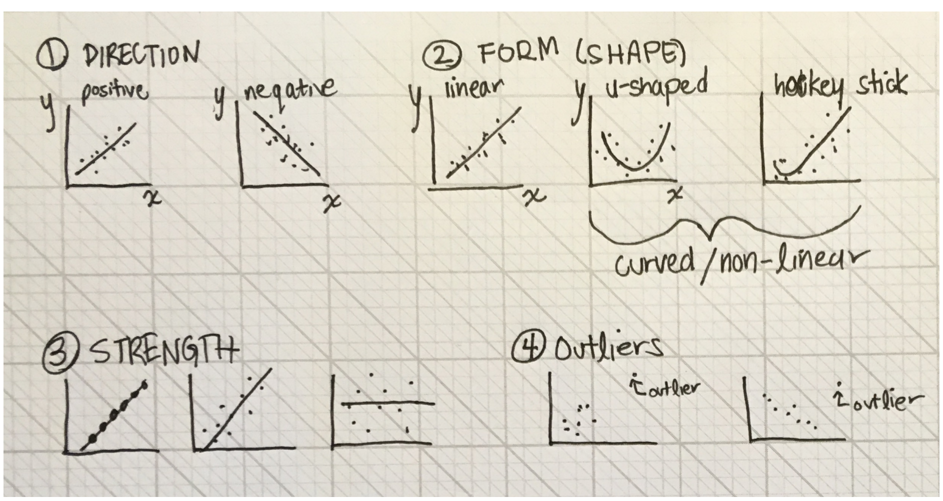
1. Each hospital's rate of hospital-acquired infections, and whether the hospital has implemented a hand-washing intervention as part of a cluster randomized trial.
2. A person's leg length and arm length, in centimeters
3. Inches of rain in the growing season and the yield of corn in bushels per day
4. The number of steps a person takes each day and a person's mental health

How to investigate causation

- Experimentally: Using a randomized controlled trial (RCT) to randomize individuals to different levels
- Observationally: Conduct an observational study that is specifically designed to investigate causation and reduce the risk of bias
- If we have time, we will talk a bit more about each of these this week. But, to know more, take a class specifically about clinical trial design or take introduction to epidemiology to learn all about conducting observational studies.
- In both settings, biostatistics is used to perform the calculations that are informed by the study design

Scatter plots

- Scatter plots are a preferred way to visualize a relationship between two variables
- They are used to evaluate:
  - **Direction**: Positive or negative?
  - **Form**: Linear or curved?
  - **Strength**: How close do the points lie to a line?
  - **Outliers**: Any individuals outside the general pattern?



Bi-directional relationships ex: systolic and diastolic BP

Read in NHANES dataset

```r
library(readr)
nhanes <- read_csv("./data/BPXI_I.csv")
```

```
## New names:
## Rows: 9544 Columns: 22
## -- Column specification
## -------------------------------------------------------- Delimiter: "," dbl
## (22): ...1, SEQN, PEASCCT1, BPXCHR, BPAARM, BPACSZ, BPXPLS, BPXPULS, BPX...
## i Use `spec()` to retrieve the full column specification for this data. i
## Specify the column types or set `show_col_types = FALSE` to quiet this message.
## * `` -> `...1`
```

```r
head(nhanes)
```

```
## # A tibble: 6 x 22
##    ...1  SEQN PEASCCT1 BPXCHR BPAARM BPACSZ BPXPLS BPXPULS BPXPTY BPXML1 BPXSY1
##   <dbl> <dbl>    <dbl>  <dbl>  <dbl>  <dbl>  <dbl>   <dbl>  <dbl>  <dbl>  <dbl>
## 1     1 83732       NA     NA      1      4     76       1      1    150    128
## 2     2 83733       NA     NA      1      4     72       1      1    170    146
## 3     3 83734       NA     NA      1      4     56       1      1    160    138
## 4     4 83735       NA     NA      1      5     78       1      1    150    132
## 5     5 83736       NA     NA      1      3     76       1      1    130    100
## 6     6 83737       NA     NA      1      4     64       1      1    140    116
## # i 11 more variables: BPXDI1 <dbl>, BPAEN1 <dbl>, BPXSY2 <dbl>, BPXDI2 <dbl>,
## #   BPAEN2 <dbl>, BPXSY3 <dbl>, BPXDI3 <dbl>, BPAEN3 <dbl>, BPXSY4 <dbl>,
## #   BPXDI4 <dbl>, BPAEN4 <dbl>
```

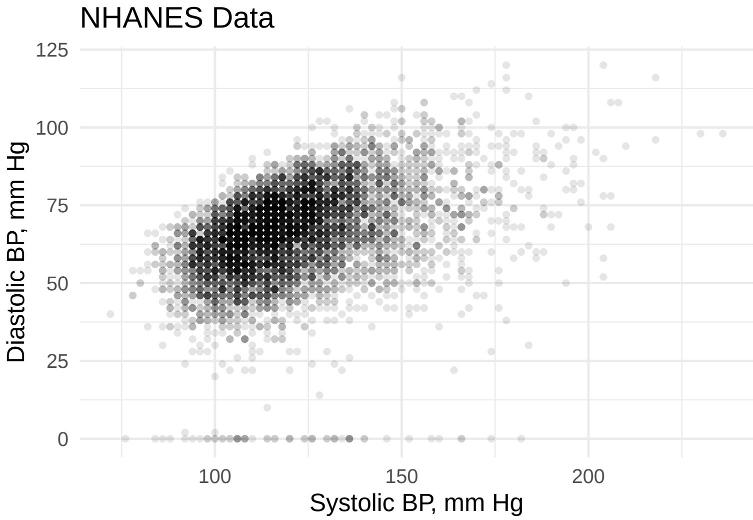
```r
# View(nhanes) #Viewer provides data labels which are very useful for picking which variables to plot
```

Bi-directional relationships ex: systolic and diastolic BP

```r
library(ggplot2)
bp_plot <- ggplot(nhanes, aes(x = BPXSY1, y = BPXDI1)) +
  geom_point(alpha = 0.1) +
  theme_minimal(base_size = 15) +
  labs(x = "Systolic BP, mm Hg",
       y = "Diastolic BP, mm Hg",
       title = "NHANES Data")
```

Bi-directional relationships ex: systolic and diastolic BP

```
## Warning: Removed 2399 rows containing missing values or values outside the scale range
## (`geom_point()`).
```



Bi-directional relationships ex: systolic and diastolic BP

What do we notice from the plot?

- **Direction**: Positive or negative?
- **Form**: Linear or curved?
- **Strength**: How close do the points lie to a line?
- **Outliers**: Any individuals outside the general pattern?

Association with a plausible direction: motor boats and manatees

Read in the manatee data set (from the text book):

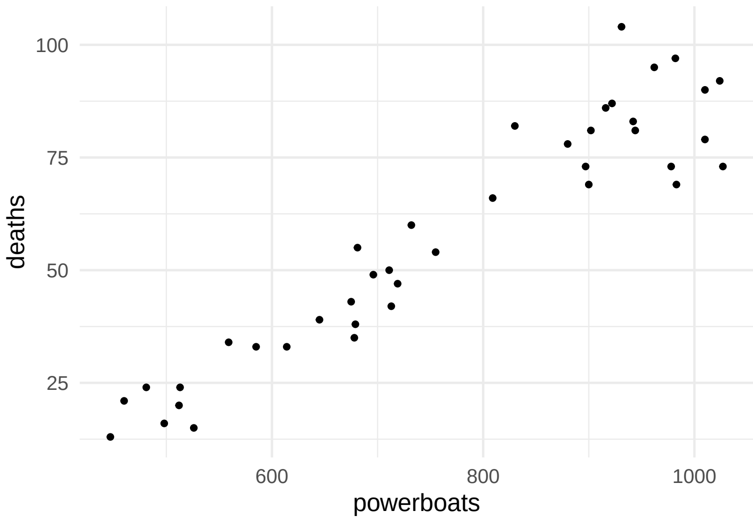
```
library(readr)
mana_data <- read_csv("./data/Ch03_Manatee-deaths.csv")
```

```
## Rows: 40 Columns: 3
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## dbl (3): year, powerboats, deaths
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

Association with a plausible direction: motor boats and manatees

```
mana_scatter <- ggplot(data = mana_data, aes(x = powerboats, y = deaths)) +
  geom_point() +
  theme_minimal(base_size = 15)

mana_scatter
```



Association with a plausible direction: motor boats and manatees

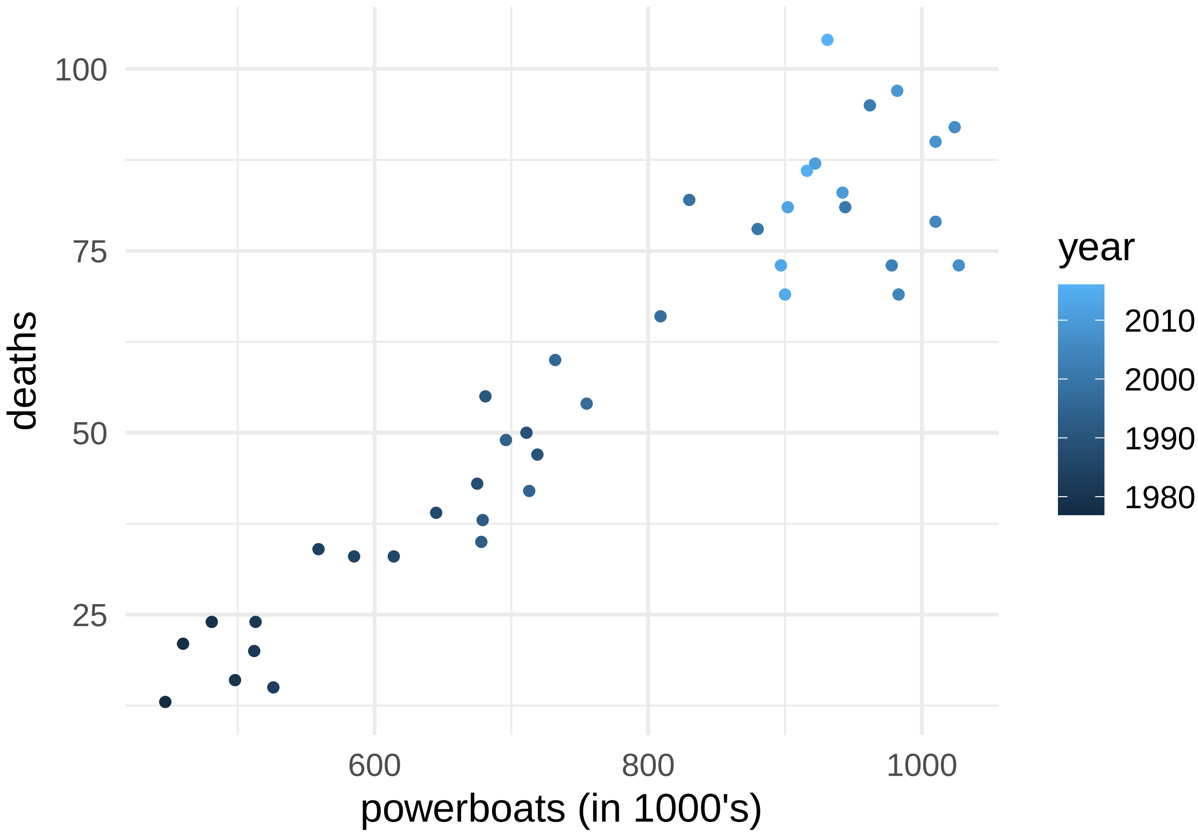
What do we notice from the plot?

- **Direction**: Positive or negative?
- **Form**: Linear or curved?
- **Strength**: How close do the points lie to a line?
- **Outliers**: Any individuals outside the general pattern?

Exercise: Power boats and Manatees

- Add (in thousands) to the x-axis title
- Change the point colour
- Is there a way to incorporate information on year into the graph?

```
ggplot(data = mana_data, aes(x = powerboats, y = deaths)) +
  geom_point(aes(col=year)) +
  theme_minimal(base_size = 15)+labs(x="powerboats (in 1000's)")
```



Example 3: Enzyme activity and temperature

- A study examined the activity rate (in micromoles per second) of a digestive enzyme at varying temperatures.

```
# this dataset was provided in Baldi and Moore Ed#4 Apply your knowledge 3.4
enzyme_data <- read_csv("./data/Ch03_Enzyme-data.csv")
```

```
## Rows: 36 Columns: 2
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## dbl (2): temperature, rate
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

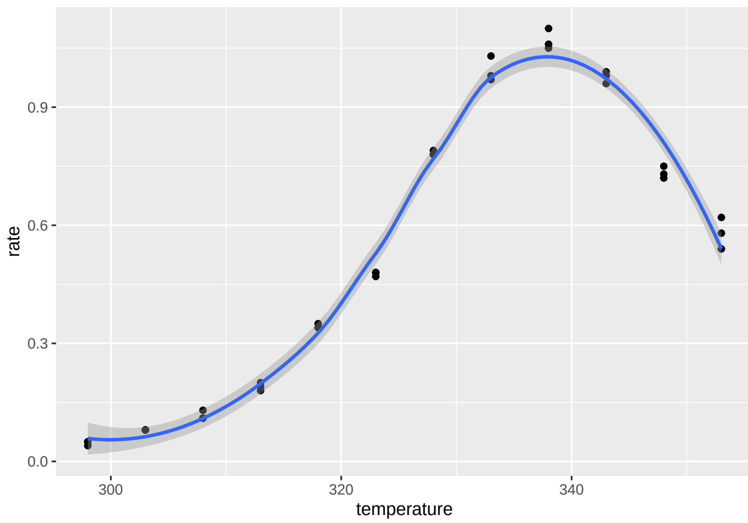
```
head(enzyme_data)
```

```
## # A tibble: 6 x 2
##   temperature  rate
##         <dbl> <dbl>
## 1         298  0.04
## 2         298  0.05
## 3         298  0.05
## 4         303  0.08
## 5         303  0.08
## 6         303  0.08
```

Scatter plot for enzyme data

```
ggplot(enzyme_data, aes(x = temperature, y = rate)) +
  geom_point() +
  geom_smooth()
```

```
## `geom_smooth()` using method = 'loess' and formula = 'y ~ x'
```



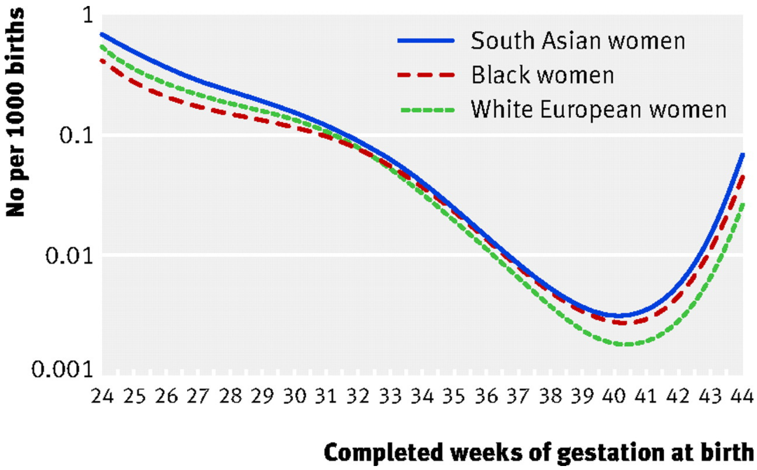
Direction:

Form:

Strength:

Outliers:

Example 4: Gestational age and perinatal mortality



Source: Balchin et al. BMJ. 2007.

Example 5: Lean body mass and metabolic rate

Problem: Is lean body mass (person's weight after removing the fat) associated with metabolic rate (kilocalories burned in 24 hours)?

Plan: A diet study was conducted on 12 women and 7 men that measured lean body weight and metabolic rate for each individual.

Lean body mass and metabolic rate

Data:

Subject	Sex	Mass (kg)	Rate (Cal)	Subject	Sex	Mass (kg)	Rate (Cal)
1	M	62.0	1792	11	F	40.3	1189
2	M	62.9	1666	12	F	33.1	913
3	F	36.1	995	13	M	51.9	1460
4	F	54.6	1425	14	F	42.4	1124
5	F	48.5	1396	15	F	34.5	1052
6	F	42.0	1418	16	F	51.1	1347
7	M	47.4	1362	17	F	41.2	1204
8	F	50.6	1502	18	M	51.9	1867
9	F	42.0	1256	19	M	46.9	1439
10	M	48.7	1614				

- What would the corresponding data frame look like in R?
- How many variables does it have?
- How many rows?

Lean body mass and metabolic rate

```
# Note: you won't be tested on writing code using tibble::tribble()
# **Do** know how to look at this code and recognize that it is creating a data set

weight_data <- tibble::tribble(
  ~subject, ~gender, ~mass, ~rate,
  1, "M", 62.0, 1792,
  2, "M", 62.9, 1666,
  3, "F", 36.1, 995,
```

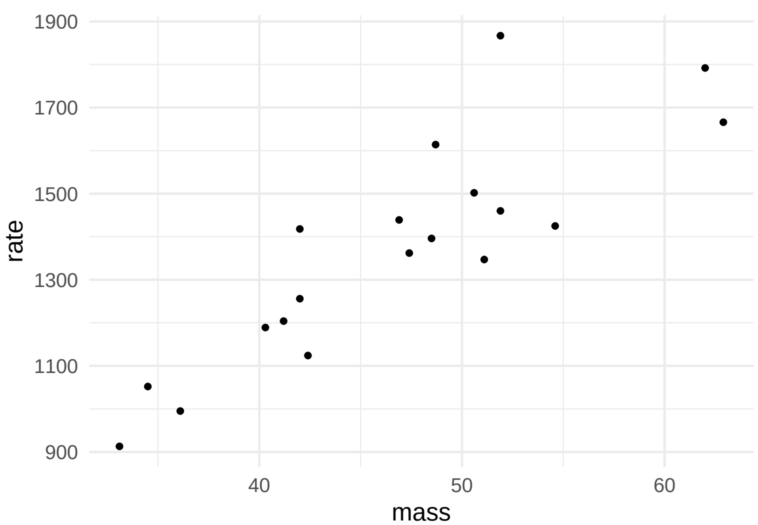
```
  4, "F", 54.6, 1425,
  5, "F", 48.5, 1396,
  6, "F", 42.0, 1418,
  7, "M", 47.4, 1362,
  8, "F", 50.6, 1502,
  9, "F", 42.0, 1256,
  10, "M", 48.7, 1614,
  11, "F", 40.3, 1189,
  12, "F", 33.1, 913,
  13, "M", 51.9, 1460,
  14, "F", 42.4, 1124,
  15, "F", 34.5, 1052,
  16, "F", 51.1, 1347,
  17, "F", 41.2, 1204,
  18, "M", 51.9, 1867,
  19, "M", 46.9, 1439
)
```

Analysis

Exploratory data analysis using scatter plots

```
weight_scatter <- ggplot(weight_data, aes(x = mass, y = rate)) +
  geom_point() +
  theme_minimal(base_size = 15)

weight_scatter
```



Analysis: Colour the points by gender

```
#Fill in during class
```

Analysis: Create separate plots for men and women

```
#Fill in during class
```

Let's test our knowledge!

Direction:

Form:

Strength:

Outliers:

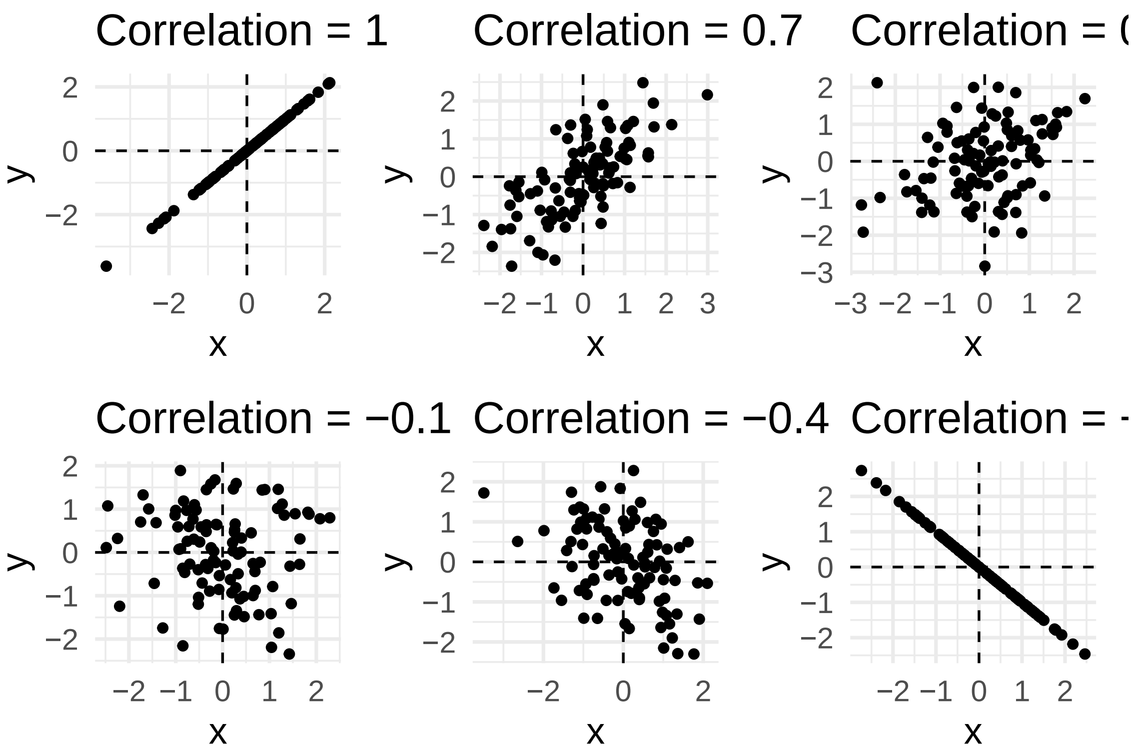
Pearson's correlation

Using just our eyes, we can often say something about whether an association between two variables is weak or strong.

```
Attaching package: 'patchwork'

The following object is masked from 'package:MASS':

    area
```



Pearson's correlation

- For **linear** associations, we can use **Pearson's correlation coefficient** (denoted by r) to **quantify the strength** of a linear relationship between two variables.
- The correlation between x and y is:

$$r = \frac{1}{n-1}\sum_{i=1}^{n}(\frac{x_i - \bar{x}}{s_x})(\frac{y_i - \bar{y}}{s_y})$$

Intuition about Pearson's correlation

To understand this formula, first only consider the numerators of the fractions (i.e., $x_i - \bar{x}$ and $y_i - \bar{y}$). If you imagine a scatter plot of x and y, we can also add a dashed line at the mean x value of $\bar{x}$ and a dashed line line at the mean y value ($\bar{y}$):

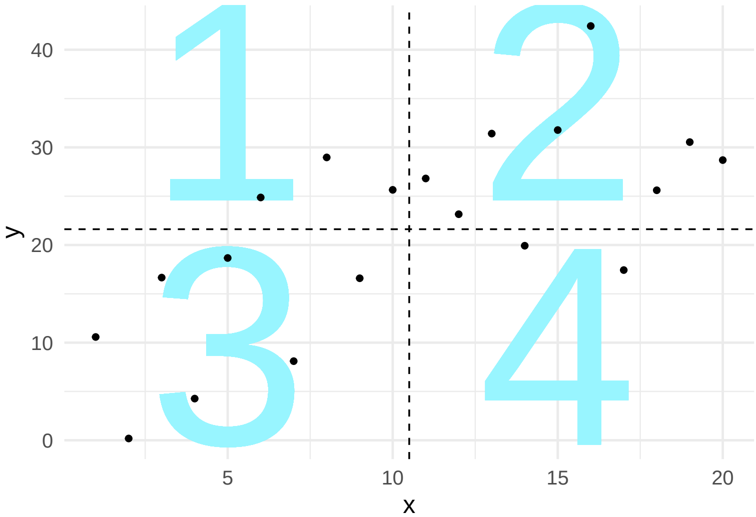
```
## Warning in geom_text(aes(x = 5, y = 35), label = 1, size = 60, col = "cadetblue1"): All aesthetics ha
## i Please consider using `annotate()` or provide this layer with data containing
##   a single row.

## Warning in geom_text(aes(x = 15, y = 35), label = 2, size = 60, col = "cadetblue1"): All aesthetics h
```

```
## i Please consider using `annotate()` or provide this layer with data containing
##   a single row.
```

```
## Warning in geom_text(aes(x = 5, y = 10), label = 3, size = 60, col = "cadetblue1"): All aesthetics ha
## i Please consider using `annotate()` or provide this layer with data containing
##   a single row.
```

```
## Warning in geom_text(aes(x = 15, y = 10), label = 4, size = 60, col = "cadetblue1"): All aesthetics h
## i Please consider using `annotate()` or provide this layer with data containing
##   a single row.
```



Intuition about Pearson's correlation

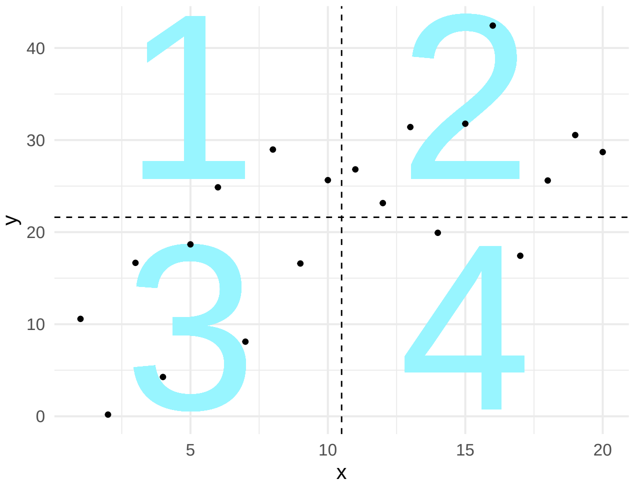
$r = \frac{1}{n-1}\sum_{i=1}^{n}(\frac{x_i-\bar{x}}{s_x})(\frac{y_i-\bar{y}}{s_y})$

```
## Warning in geom_text(aes(x = 5, y = 35), label = 1, size = 60, col = "cadetblue1"): All aesthetics ha
## i Please consider using `annotate()` or provide this layer with data containing
##   a single row.
```

```
## Warning in geom_text(aes(x = 15, y = 35), label = 2, size = 60, col = "cadetblue1"): All aesthetics h
## i Please consider using `annotate()` or provide this layer with data containing
##   a single row.
```

```
## Warning in geom_text(aes(x = 5, y = 10), label = 3, size = 60, col = "cadetblue1"): All aesthetics ha
## i Please consider using `annotate()` or provide this layer with data containing
##   a single row.
```

```
## Warning in geom_text(aes(x = 15, y = 10), label = 4, size = 60, col = "cadetblue1"): All aesthetics h
## i Please consider using `annotate()` or provide this layer with data containing
##   a single row.
```



- Points in Q2 and Q3 contribute positive products to r
- Points in Q1 and Q4 contribute negative products to r
- The more there are points in Q2 and Q3 vs. Q1 and Q4, the more the value of the correlation coefficient will be higher and positive
- If you want even more of an explanation see the response to this stack overflow post or take an intermediate statistics class!

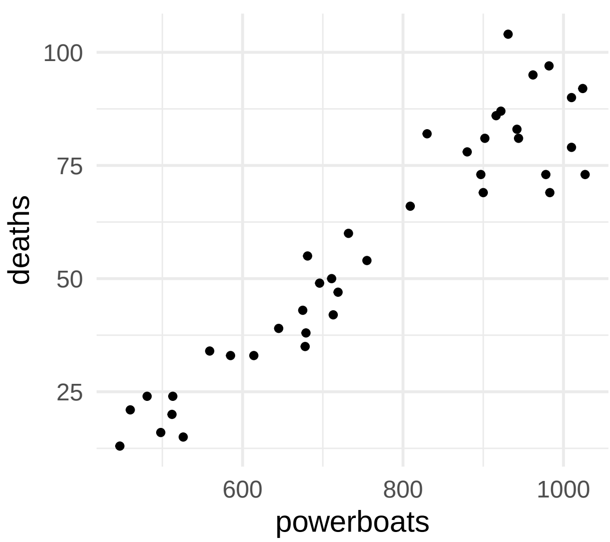
Syntax: Pearson's correlation using `cor()`

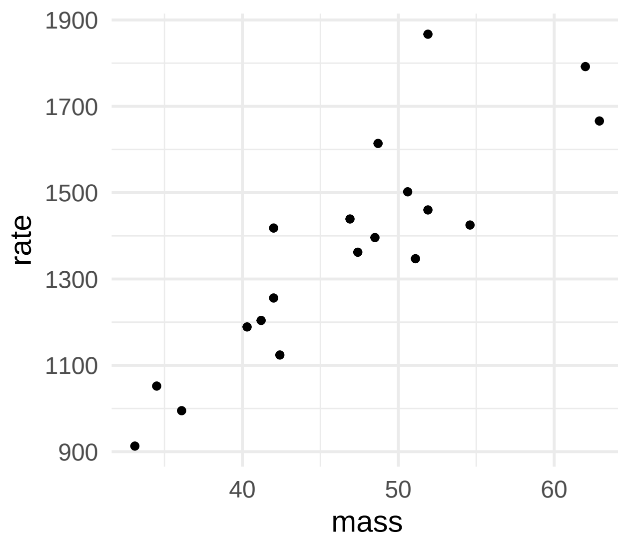
```
# Students, if you copy this code chunk, you need to set eval = T in the code chunk header for the code

correlation_coeff <- dataset %>%
  summarize(new_var = cor(x_variable, y_variable))
```

Syntax: Pearson's correlation using `cor()`

Remember the manatee plot and the weight plot:





Syntax: Pearson's correlation using `cor()`

Now, calculate the correlations between X and Y for manatees:

```r
library(dplyr)
```

```
##
## Attaching package: 'dplyr'

## The following object is masked from 'package:MASS':
##
##     select

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```r
mana_cor <- mana_data %>%
  summarize(corr_mana = cor(powerboats, deaths))

mana_cor
```

```
## # A tibble: 1 x 1
##   corr_mana
##       <dbl>
## 1     0.945
```

Syntax: Pearson's correlation using `cor()`

And for the weight data:

```r
weight_cor <- weight_data %>%
  summarize(corr_weight = cor(mass, rate))
weight_cor
```

```
## # A tibble: 1 x 1
##   corr_weight
##         <dbl>
## 1       0.865
```

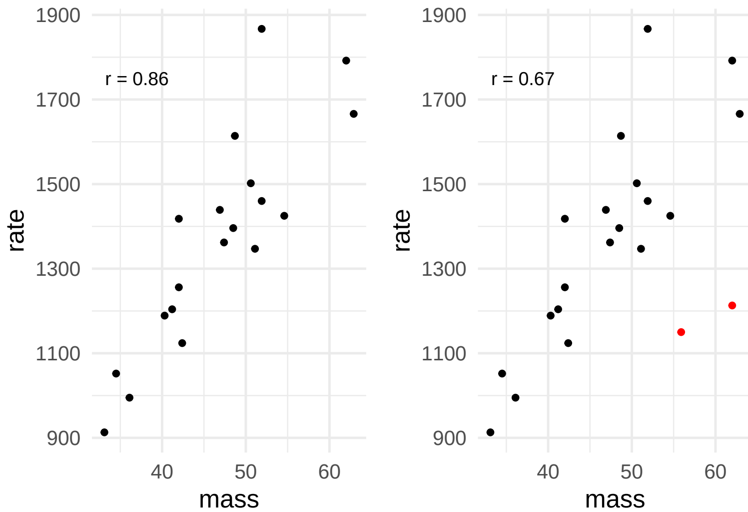
Properties of the correlation coefficient

- Always a number between -1 and 1.
  - -1: A perfect, negative linear association
  - 1: A perfect, positive linear association
  - 0: No linear association
- Don't confuse the correlation coefficient with the slope of the linear association!
- Measures association *not* causation. Even a very strong association doesn't mean that one variable causes the other.
- Is used to measure the association between two *quantitative* variables.
- Only useful for *linear* associations!

Properties of the correlation coefficient

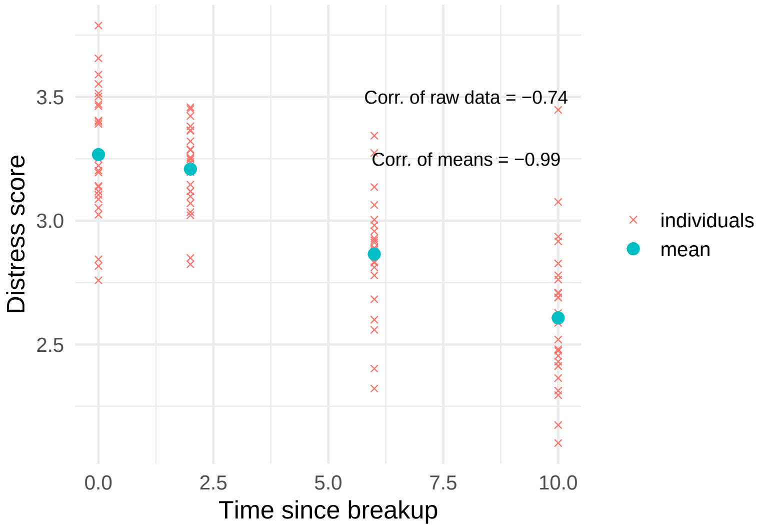
- The correlation coefficient is not resistant to outliers
- E.g., I added two outliers (in red) to the `weight_data` and recalculated correlation. How much did the correlation change? (It is labeled on each plot.)

```
## Warning: The `guide` argument in `scale_*()` cannot be `FALSE`. This was deprecated in
## ggplot2 3.3.4.
## i Please use "none" instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.
```



Properties of the correlation coefficient

- Correlations for average measures are typically stronger than correlations for individual data



Recap: What functions did we use?

- `geom_scatter()`, `aes(col = gender)` to color points by levels of `gender`
- `summarize()` to calculate correlation using `cor(var1, var2)`

Important concepts

- Determine which variable is explanatory and which is response, or when there is a bidirectional relationship (e.g., associated)
- Describe the relationship between two variables (e.g., form, direction, strength, and outliers)
- Formula for and properties of the correlation coefficient r