

3+ Sample testing with continuous outcomes

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Skipping around

3+ Sample testing
with continuous
outcomes

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Reminder, if you are following the textbook, to stick with continuous outcomes, we are going to skip ahead to chapter 24 (ANOVA) and chapter 23(regression) and bring in some outside information about non-parametric testing.

Summary of Continuous outcomes so far - the flavors of T

In R, the `t.test` function will allow you to conduct any of the t-tests we have covered so far

- ▶ One sample T test comparing a sample mean to a hypothesized null (if we know σ and have a large sample we could also consider a Z test)
`t.test(data, mu=value)`
- ▶ Two sample T test comparing samples from independent populations
`t.test(continuousvar~categoricalvar)` or `t.test(data1, data2)`
- ▶ Two sample T test comparing paired (non-independent) groups of observations
`t.test(continuousvar~categoricalvar, paired=TRUE)` or
`t.test(data1, data2, paired=TRUE)`

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVAExample: Cannabis to treat
brain cancerAfter a significant
ANOVA...

Three sample testing

Now that we have looked at how to compare one sample to a value and means between two samples, let's extend this to a case where we have 3 samples or 3 groups that we are interested in comparing.

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Today's lecture

3+ Sample testing
with continuous
outcomes

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

- ▶ Introducing ANOVA - what is the null hypothesis of this test?
- ▶ Visualizing 3 sample data
- ▶ Using ANOVA to test for a difference
- ▶ Example

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

ANOVA

ANOVA

ANOVA or analysis of variance can be thought of as an extension of the two-sample t-test to three or more samples.

If we had 3 groups to compare, we could do this by using the two-sample t test multiple times. This would result in $\binom{3}{2}$ comparisons:

$$\mu_1 \neq \mu_2 \quad \mu_2 \neq \mu_3 \quad \mu_1 \neq \mu_3$$

The problem with that approach is that we would end up with 3 p-values, one for each test performed. That doesn't tell us how likely it is that three sample means are spread apart as far as they are. If we are comparing more than 3 groups, this problem is compounded by creating even more comparisons.

We need a method that allows us to have an overall measure of confidence in all our conclusions about comparisons. This is a common problem of multiple comparisons.

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using ANOVA

Example: Cannabis to treat brain cancer

After a significant ANOVA...

ANOVA

Here we are testing a null hypothesis that all the means are the same, for 3 samples this would be

$$H_0: \mu_1 = \mu_2 = \mu_3$$

Our alternative hypothesis is that at least one of the means is not equal to the others

Even though your hypothesis involves means, the test compares the variability between groups to the variability within groups

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

ANOVA

Analysis of variance (ANOVA) is also referred to as the F test.

The ANOVA is based on two kinds of variability:

- The variability among sample means or how much the individual group means vary around the overall mean
- The variability within groups, how much do individual observation values vary around the group mean

If the variability within the k different populations is small relative to the variability among their respective means, this suggests that the population means are in fact different.

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using ANOVA

Example: Cannabis to treat brain cancer

After a significant ANOVA...

Loosely expressed:

$$F = \left(\frac{\text{variation among sample means}}{\text{variation among observations in the same sample}} \right)$$

Note: You will **NOT** need to know the full formula for the F-test, you **will** need to know how to do one in R

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

What would the data look like in a data frame?

What would the data look like in a data frame?

- ▶ One “grouping” variable (categorical)
- ▶ One continuous response variable

ANOVA asks if there is an association between the grouping variable and the response variable.

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Visualizing first

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

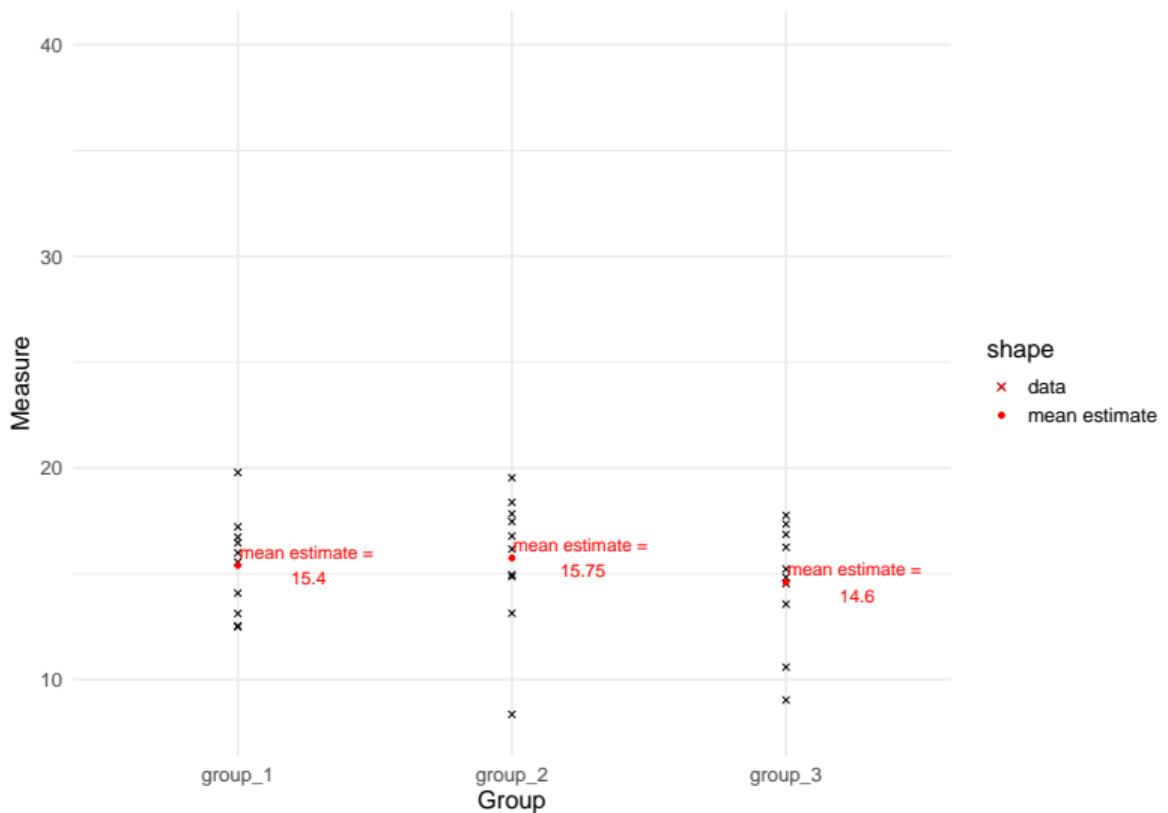
Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

What graphical strategies have we learned to look at variability within and between groups?

A) Is there a difference between these means?

Describe why you do or do not think so.



ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Summary of the plots

Plot (A)

- ▶ The means (red dots) were not very different across the groups. This means the variation **between** the group means was small.
- ▶ The distribution of the data (black Xs) was wide enough that the distribution of points for each group overlapped almost completely. This means that the variation **within** each group was relatively wide

ANOVA

Descriptive plots

Testing with ANOVA

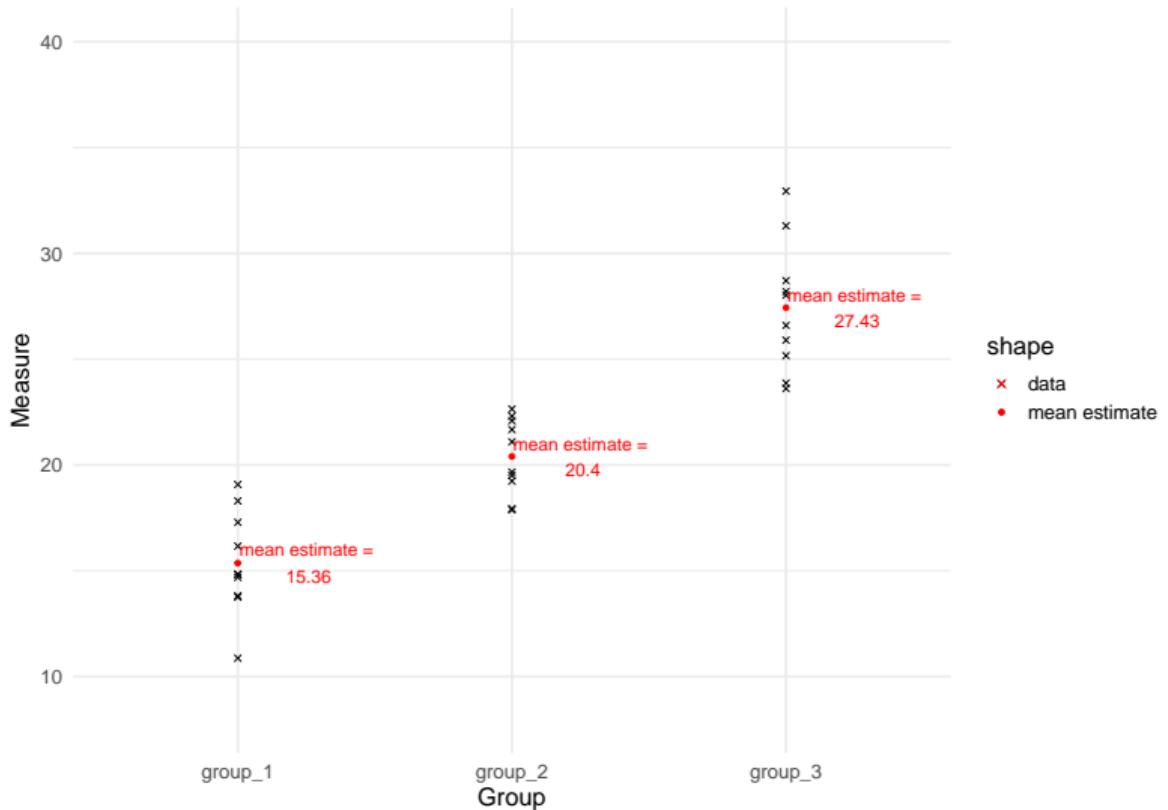
Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

B) Is there a difference between these means?

Describe why you do or do not think so.



ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Summary of the plots

Plot (B)

- ▶ The means are quite different across the groups. The variation **between** the group means would be larger than in plot (A)
- ▶ The distribution of the data overlaps between groups 1 and 2 and 2 and 3, but not 1 and 3. The variation **within** each group is as wide as it was in Plot (A) but doesn't mask the mean differences, especially between group 1 and 3

ANOVA

Descriptive plots

Testing with ANOVA

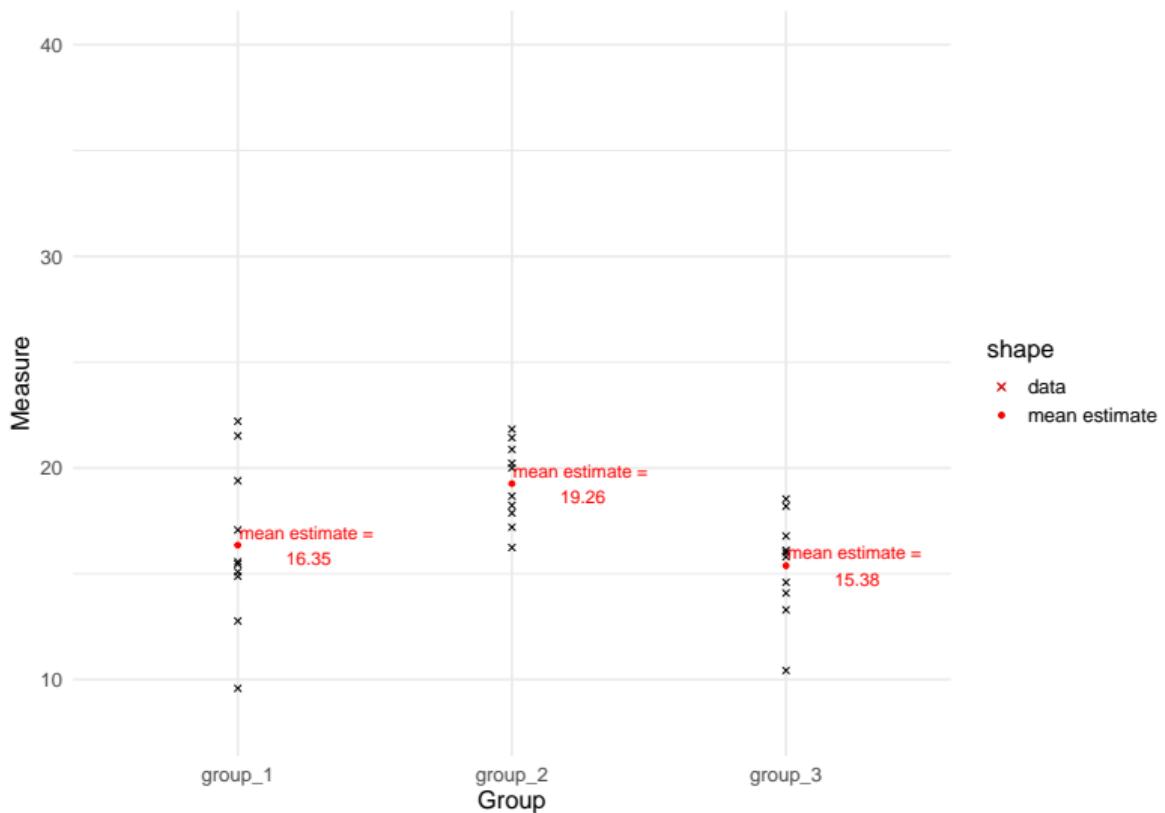
Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

C) Is there a difference between these means?

Describe why you do or do not think so.



ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Summary of the plots

Plot (C)

- ▶ Here, the means for group 1 and 3 look similar, but the mean for group 2 appears a bit higher than the other two, though there is still overlap between the data from all the groups
- ▶ Is there evidence that at least one of the means is different?

ANOVA

Descriptive plots

Testing with ANOVA

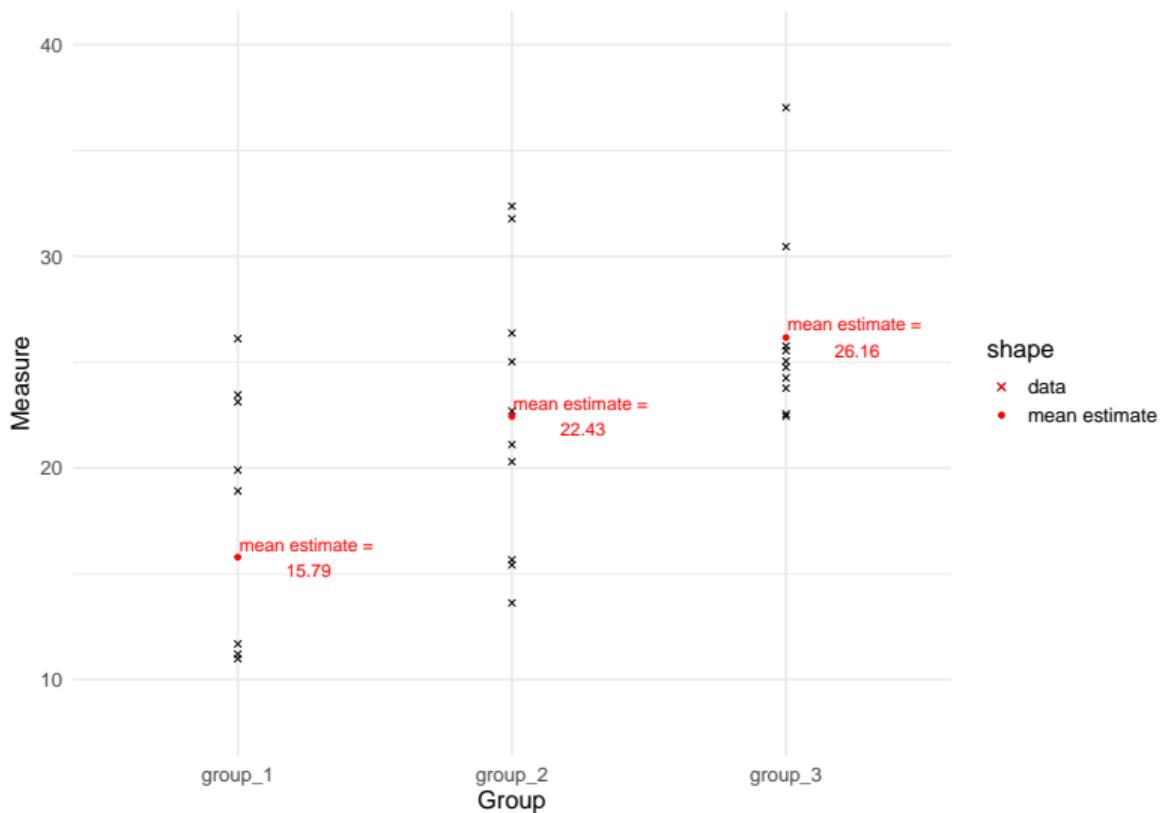
Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

D) Is there a difference between these means?

Describe why you do or do not think so.



ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Summary of the plots

Plot (D)

- ▶ Plot (D) looked like Plot (B) but with more variation **within** groups
- ▶ This variation makes the difference between the means harder to detect

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Overall summary

- ▶ What we informally did on the previous slides was compare the variation *between* group means to the variation *within* the groups
- ▶ This focus on variation is why this test is called ANOVA: an ANalysis Of VAriance
- ▶ When the ratio of between vs. within variation is large enough then we detect a difference between the groups
- ▶ When the ratio isn't large enough we don't detect the difference.
- ▶ This ratio is our test statistic, denoted by F

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using ANOVA

Example: Cannabis to treat brain cancer

After a significant ANOVA...

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Descriptive plots

Descriptive plots

3+ Sample testing
with continuous
outcomes

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

What other ways to present the data visually have we learned that might be useful before we move on to testing?

Descriptive plots

ANOVA

Descriptive plots

Testing with ANOVA

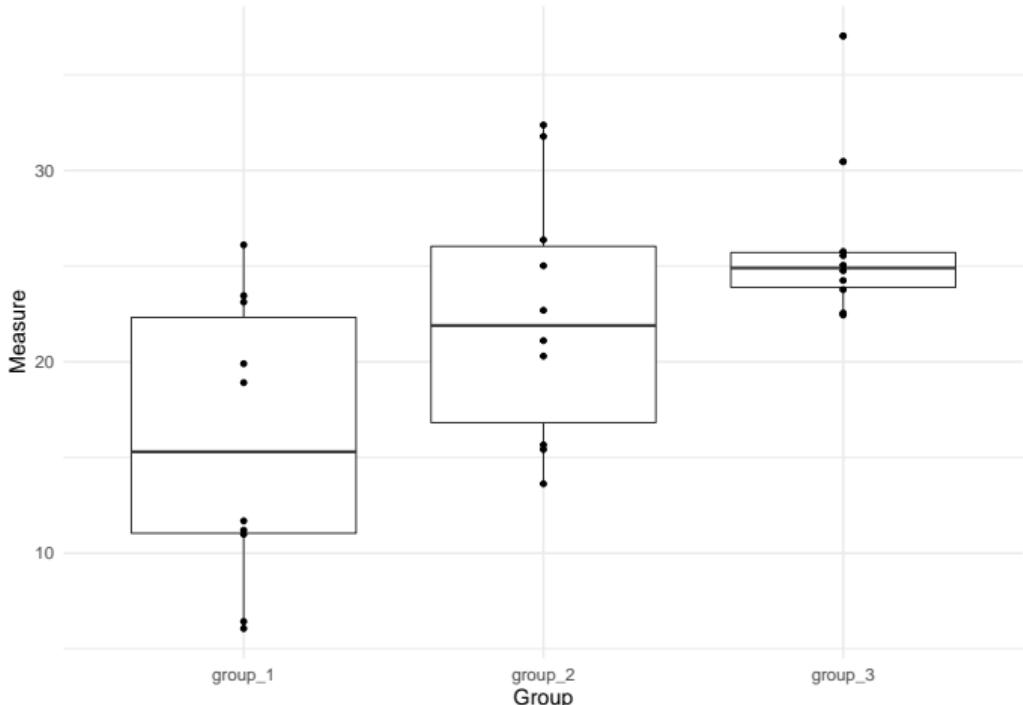
Conditions for using
ANOVAExample: Cannabis to treat
brain cancerAfter a significant
ANOVA...

How would you want to plot these data before you conduct a test?

- ▶ Option 1: Box plot for each level of the grouping variable (with overlaid data points)

```
ggplot(diff_3_narrow, aes(x = Group, y = Measure)) +  
  geom_boxplot() +  
  geom_point() +  
  theme_minimal(base_size = 15)
```

Box plot



ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Descriptive plots

3+ Sample testing
with continuous
outcomes

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

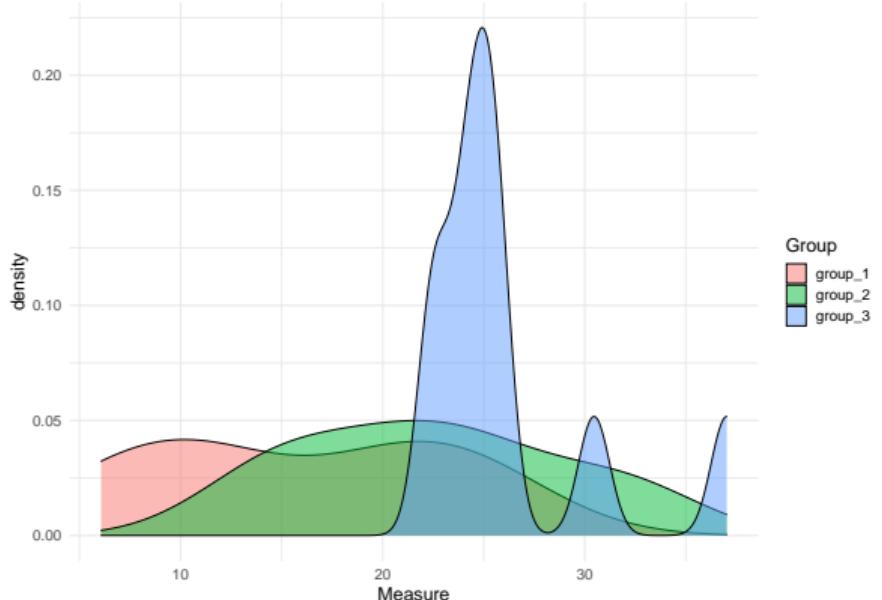
After a significant
ANOVA...

How would you want to plot these data before you conduct a test?

- ▶ Option 2: Density plot for each level of the grouping variable

```
ggplot(diff_3_narrow, aes(x = Measure)) +  
  geom_density(aes(fill = Group), alpha = 0.5) +  
  theme_minimal(base_size = 15)
```

Density plot



ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

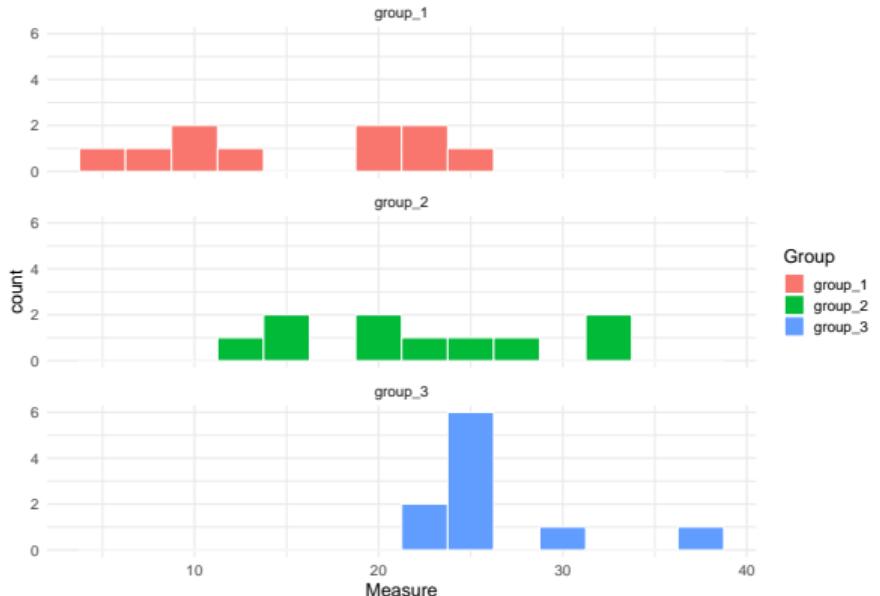
After a significant
ANOVA...

How would you want to plot these data before you conduct a test?

- ▶ Option 3: Histogram for each level of the grouping variable

```
ggplot(diff_3_narrow, aes(x = Measure)) +  
  geom_histogram(aes(fill = Group), col = "white", binwidth = 2.5) +  
  theme_minimal(base_size = 15) +  
  facet_wrap(~ Group, nrow = 3)
```

Histograms with facet wrap



ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Testing with ANOVA

The hypotheses

Null hypothesis

$H_0 : \mu_1 = \mu_2 = \dots = \mu_K$, where K is the number of levels of the grouping variable

- ▶ Can you also state the null hypothesis in words?

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

The hypotheses

Alternative hypothesis

H_a : not all $\mu_1, \mu_2, \dots, \mu_K$ are equal

- ▶ In words: Not all means are the same. Or, **at least one of the means** differs from the others.



ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

The test statistic (ANOVA F Statistic)

$$F = \frac{\text{variation among group means}}{\text{variation among individuals in the same group}}$$

- ▶ Numerator is, fundamentally, the variance of the sample means
- ▶ Denominator is, fundamentally, an average of the group variances.
- ▶ The F statistic follows an F distribution
- ▶ Computation details are at the end of the book chapter (these computation details will not be tested)

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVAExample: Cannabis to treat
brain cancerAfter a significant
ANOVA...

The F distribution

- ▶ Skewed right
- ▶ Take only positive values
- ▶ The F distribution depends on the number of means being compared and the sample size for each of the groups
- ▶ Let k be the number of groups being compared and $N_{Total} = n_1 + n_2 + \dots + n_k$ (the total sample size across all the groups)
- ▶ Then the F statistic follows an F distribution with $k - 1$ degrees of freedom in the numerator and $N_{Total} - k$ degrees of freedom in the denominator
- ▶ The p-value of the ANOVA F statistic is always the area to the right of the test statistic

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVAExample: Cannabis to treat
brain cancerAfter a significant
ANOVA...

ANOVA in R: use `aov()`, then `tidy()` it up!

3+ Sample testing
with continuous
outcomes

- ▶ `aov()` stands for analysis of variance

The general syntax for the ANOVA is:

```
aov(outcomevariable ~ groupvariable, data=dataset)
```

We will save the output of this as an object and then use `tidy(object)` to get the output we want.

reference: https://broom.tidyverse.org/reference/anova_tidiers.html

ANOVA
Descriptive plots
Testing with ANOVA
Conditions for using
ANOVA
Example: Cannabis to treat
brain cancer
After a significant
ANOVA...

ANOVA in R: use `aov()`, then `tidy()` it up!

3+ Sample testing
with continuous
outcomes

We will focus on two parts of the output from this package:

- ▶ `statistic` is the F test statistic, the ratio of the variation between means vs. the variation within groups.
- ▶ `p.value` is the p-value for the test.

ANOVA
Descriptive plots
Testing with ANOVA
Conditions for using
ANOVA
Example: Cannabis to treat
brain cancer
After a significant
ANOVA...

p of an f statistic in R

You can check that you can calculate the p-value from the F distribution.

Remember, that you need to specify a degrees of freedom for the numerator and for the denominator:

```
pf(value, df1= numerator degrees of freedom, df2= denominator degrees of  
freedom, lower.tail=F)
```

This general pattern of syntax should look familiar by now....

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

ANOVA

Descriptive plots

Testing with ANOVA

**Conditions for using
ANOVA**

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Conditions for using ANOVA

Conditions for ANOVA

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Condition 1: k independent SRSs, one from each of k populations.

- ▶ The most important assumption, because this method, like the others in Part III of the course, depends on the premise of having taken a random sample.

Conditions for ANOVA

Condition 2: Each of the k populations has a Normal distribution with an unknown mean μ_i .

- ▶ This assumption is less necessary
- ▶ The ANOVA test is **robust** to non-Normality.
- ▶ Remember that the ANOVA is based on comparing the differences of sample means
- ▶ What did the CLT tell us about variability of sample means when the samples are not normally distributed?

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVAExample: Cannabis to treat
brain cancerAfter a significant
ANOVA...

Conditions for ANOVA

Condition 3: All the populations have the same standard deviation σ , whose value is unknown.

- ▶ Hardest condition to satisfy and check
- ▶ If this condition is not satisfied ANOVA is often okay if the sample sizes are large enough and if they are similar across the groups
- ▶ Can use `group_by()` and `summarize()` to calculate the sample SDs to see if they're similar and indicative that the population parameters are too
- ▶ General rule: we want the largest sample standard deviation to be less than twice as large as the smallest one. I.e., $s_{max}/s_{min} < 2$

ANOVA

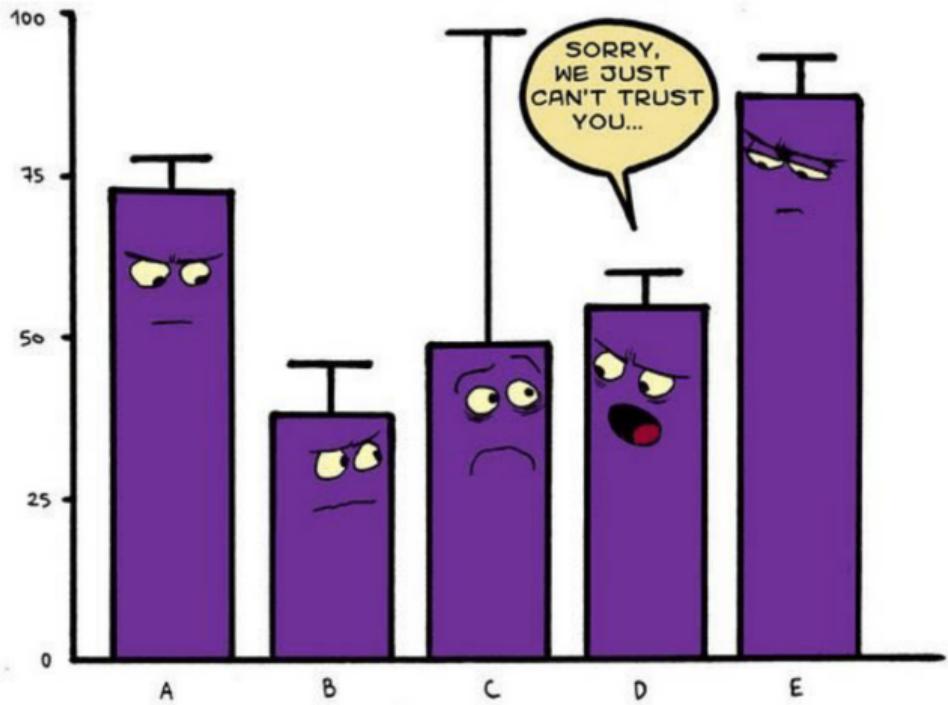
Descriptive plots

Testing with ANOVA

Conditions for using
ANOVAExample: Cannabis to treat
brain cancerAfter a significant
ANOVA...

Conditions for ANOVA

3+ Sample testing
with continuous
outcomes



ANOVA
Descriptive plots
Testing with ANOVA
Conditions for using ANOVA
Example: Cannabis to treat brain cancer
After a significant ANOVA...

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Example: Cannabis to treat brain cancer

Example: Cannabis to treat brain cancer (in mice)

High-grade glioma is an aggressive type of brain cancer with a low long-term survival rate. Cannabinoids, which are chemical compounds found in cannabis, are thought to inhibit glioma cell growth. Researchers transplanted glioma cells in otherwise-healthy mice, and then randomly assigned these mice to 4 cancer treatments: irradiation alone, cannabinoids, alone, irradiation combined with cannabinoids, or no cancer treatment. The treatments were administered for 21 days, after which the glioma tumor volume (in cubic millimeters) was assessed in each mouse using brain imaging.

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

The data

```
head(cancer_data)
```

```
##          treatment tumor_volume
## 1   Irradiation         30
## 2   Irradiation         46
## 3   Irradiation         46
## 4   Irradiation        95
## 5 Cannabinoids          12
## 6 Cannabinoids          14
```

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Organize the data

- ▶ Think about how you want the data to look.
- ▶ I want to plot the raw data points and display the mean for each treatment group
- ▶ I also want to specify the order that the treatment groups show up in the graph

```
# specify the order of the treatment groups for plotting
library(forcats)
cancer_data <- cancer_data %>%
  mutate(trt_order = fct_relevel(treatment, c("Neither", "Irradiation",
                                             "Cannabinoids", "Both")))
```

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVAExample: Cannabis to treat
brain cancerAfter a significant
ANOVA...

Look at summary statistics

```
# calculate the means and SD for each group
summary_stats <- cancer_data %>%
  group_by(trt_order) %>%
  summarise(mean_vol = mean(tumor_volume),
            sd_vol = sd(tumor_volume),
            samp_size = n())
summary_stats
```

```
## # A tibble: 4 x 4
##   trt_order   mean_vol   sd_vol samp_size
##   <fct>        <dbl>     <dbl>     <int>
## 1 Neither      48.3      24.8       7
## 2 Irradiation  54.2      28.2       4
## 3 Cannabinoids 26.0      16.6       5
## 4 Both         6.0       2.76       6
```

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVAExample: Cannabis to treat
brain cancerAfter a significant
ANOVA...

Graph the data

```
ggplot(cancer_data, aes(x = trt_order, y = tumor_volume)) +  
  geom_jitter(pch = 4, width = 0.1) +  
  geom_point(data = summary_stats, aes(y = mean_vol, col = "Mean"), pch = 19) +  
  labs(y = "Tumor volume (mm cubed)", x = "Treatment") +  
  theme_minimal(base_size = 15)
```

note: `geom_jitter()` with `width = 0.1` randomly “jitters” the location of the points along the x axis so that we can see each of them since some have the exact same values.

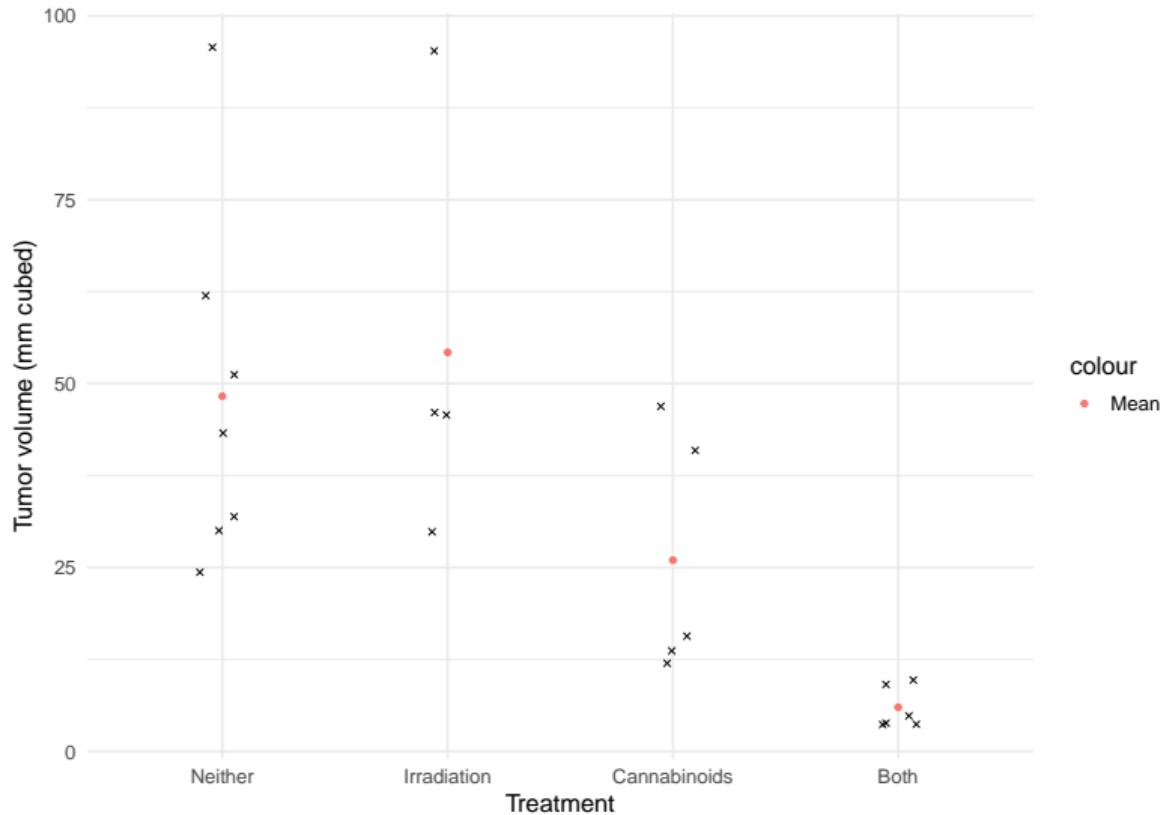
ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVAExample: Cannabis to treat
brain cancerAfter a significant
ANOVA...

Graph the data



ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

ANOVA in R: use `aov()`, then `tidy()` it up!

```
library(broom)
```

```
## Warning: package 'broom' was built under R version 4.0.4
```

```
cancer_anova <- aov(formula = tumor_volume ~ treatment, data = cancer_data)  
tidy(cancer_anova)
```

```
## # A tibble: 2 x 6
```

term	df	sumsq	meansq	statistic	p.value
<chr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
treatment	3	8060.	2687.	6.70	0.00313
Residuals	18	7218.	401.	NA	NA

This F says that the variation between the means is nearly 7 times as large as the variation within the groups.

This p-value is equal to 0.3%. There is a 0.3% chance of observing the F statistic we observed (or more extreme) under the null hypothesis that all the means are the same.

P value of the F test

Remember

- ▶ k is the number of groups being compared and $N_{Total} = n_1 + n_2 + \dots + n_k$ is the total sample size across all the groups.
- ▶ The F statistic follows an F distribution with $k - 1$ degrees of freedom in the numerator and $N_{Total} - k$ degrees of freedom in the denominator
- ▶ The p-value of the ANOVA F statistic is always the area to the right of the test statistic

```
pf(6.699489, df1 = 3, df2 = 22 - 4, lower.tail = F)
```

```
## [1] 0.003131703
```

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA ...

What now?

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

- ▶ The p-value equaled 0.003, indicating a difference. But what groups are actually different?

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

After a significant ANOVA...

next steps...

3+ Sample testing with continuous outcomes

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

- ▶ You could look at all pairwise differences (i.e., comparing each combination of treatments), but we have to be careful because we will find differences "just by chance" if we compare enough groups.

A key example of what not to do?

3+ Sample testing
with continuous
outcomes

FOOD FOR THOUGHT

Cornell Food Researcher's Downfall Raises Larger Questions For Science

September 26, 2018 - 3:07 PM ET

BRETT DAHLBERG

FROM **WXXI**



ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

A brief reminder about p-hacking

Remember, one of the issues with multiple comparisons is that when you repeatedly question the same dataset, you can end up finding “significant” results by chance alone.

We talked about this before as p-hacking or p-fishing or data dredging

This along with other issues that are sometimes unconscious can lead to bias in what is found and what is published.

Ioannidis, John P.A. (August 30, 2005). “Why Most Published Research Findings Are False”. PLoS Medicine. <https://journals.plos.org/plosmedicine/article?id=10.1371/journal.pmed.0020124>

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using ANOVA

Example: Cannabis to treat brain cancer

After a significant ANOVA...

Adjustment for multiple comparisons

We could do a series, comparing each combination of groups in pairs or $\binom{k}{2}$ comparisons.

To compensate for making multiple comparisons and set the overall probability of making a type I error at 0.05, we can adjust our α to α^* for each comparison by dividing by the number of comparisons we are making.

$$\alpha^* = \left(\frac{0.05}{\binom{k}{2}} \right)$$

We then use α^* as the significance level for each individual comparison. So for a comparison of 3 groups we would use an α of 0.0167 as the significance level for each comparison.

This modification is known as the Bonferroni correction. Bonferroni is fairly basic and can become unwieldy - what happens if you have a lot of groups?

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVAExample: Cannabis to treat
brain cancerAfter a significant
ANOVA...

Tukey's honest significant differences (Tukey's HSD)



John Wilder Tukey

ANOVA
Descriptive plots
Testing with ANOVA
Conditions for using ANOVA
Example: Cannabis to treat brain cancer
After a significant ANOVA...

Tukey's honestly significant differences (Tukey's HSD)

- ▶ Tukey's test maintains a 5% experimentwise or “family” error rate.
- ▶ Even if you make many pairwise comparisons, the overall error rate is 5% (at most)
- ▶ Overcomes the issue of multiple testing. Recall: If you conducted 100 tests with a 5% error rate (i.e., $\alpha = 0.05$) AND the H_0 was always true, how many p-values would you expect to be < 0.05 ?
- ▶ The Tukey's error rate is 5% overall, no matter how many tests you do. Thus it overcomes the problem of multiple testing

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVAExample: Cannabis to treat
brain cancerAfter a significant
ANOVA...

TukeyHSD() to calculate the differences in R

You can think of the TukeyHSD() as a wrap-around for the anova, you can either nest the statements like this:

```
TukeyHSD(aov(outcome ~ group))
```

or save the ANOVA as an object and use that in the statement:

```
modelresult<-aov(outcome ~ group)
```

```
TukeyHSD(modelresult, overall_alpha)
```

ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

TukeyHSD() to calculate the differences in R

Here is the R code and output for the cancer example:

```
diffs <- TukeyHSD(cancer_anova, conf.level = 0.05) %>% tidy()  
diffs
```

```
## # A tibble: 6 x 7  
##   term      contrast    null.value estimate conf.low conf.high adj.p  
##   <chr>     <chr>          <dbl>     <dbl>     <dbl>     <dbl>  
## 1 treatme~ Cannabinoids-Both        0     20.      13.5      26.5  0  
## 2 treatme~ Irradiation-Both        0     48.3      41.4      55.1  0  
## 3 treatme~ Neither-Both           0     42.3      36.4      48.2  0  
## 4 treatme~ Irradiation-Canna~      0     28.2      21.1      35.4  0  
## 5 treatme~ Neither-Cannabino~     0     22.3      16.0      28.5  0  
## 6 treatme~ Neither-Irradiati~     0    -5.96     -12.6      0.720 0
```

TukeyHSD() to calculate the differences in R

Each row in the table corresponds to a pairwise test. So the first row is looking at the difference between Cannabinoids vs. Both treatments. The estimated difference in means is 20 and the 95% CI is 13.54 to 26.45. The adjusted p-value is 0.38.

- ▶ “Adjusted” means that it is adjusted for conducting multiple tests. The unadjusted p-value would be smaller. You can tell the unadjusted p-value would be < 0.05 because the 95% CI doesn’t include 0.
- ▶ Thus, when you have an adjusted test you can’t use the CI to infer the value of the p-value!

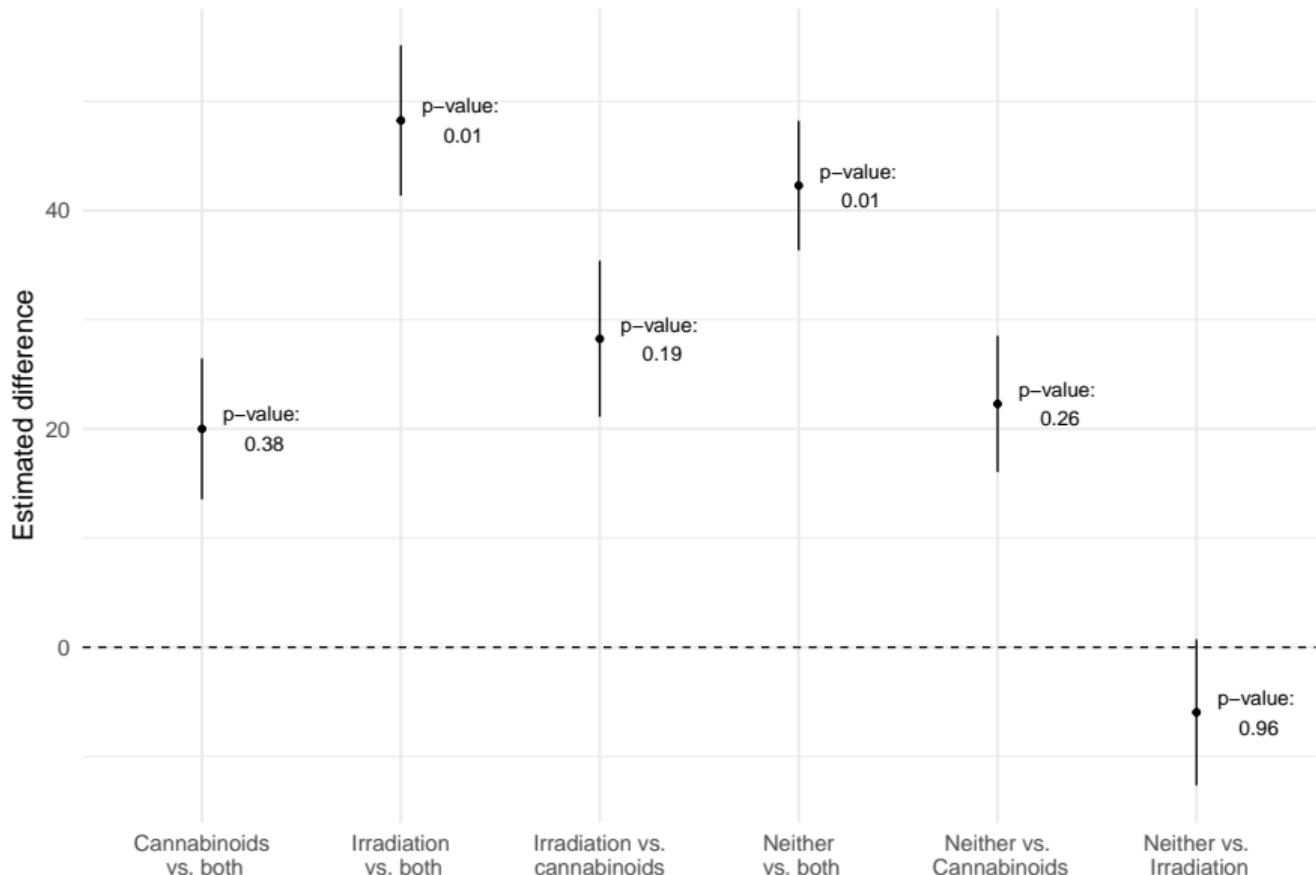
ANOVA

Descriptive plots

Testing with ANOVA

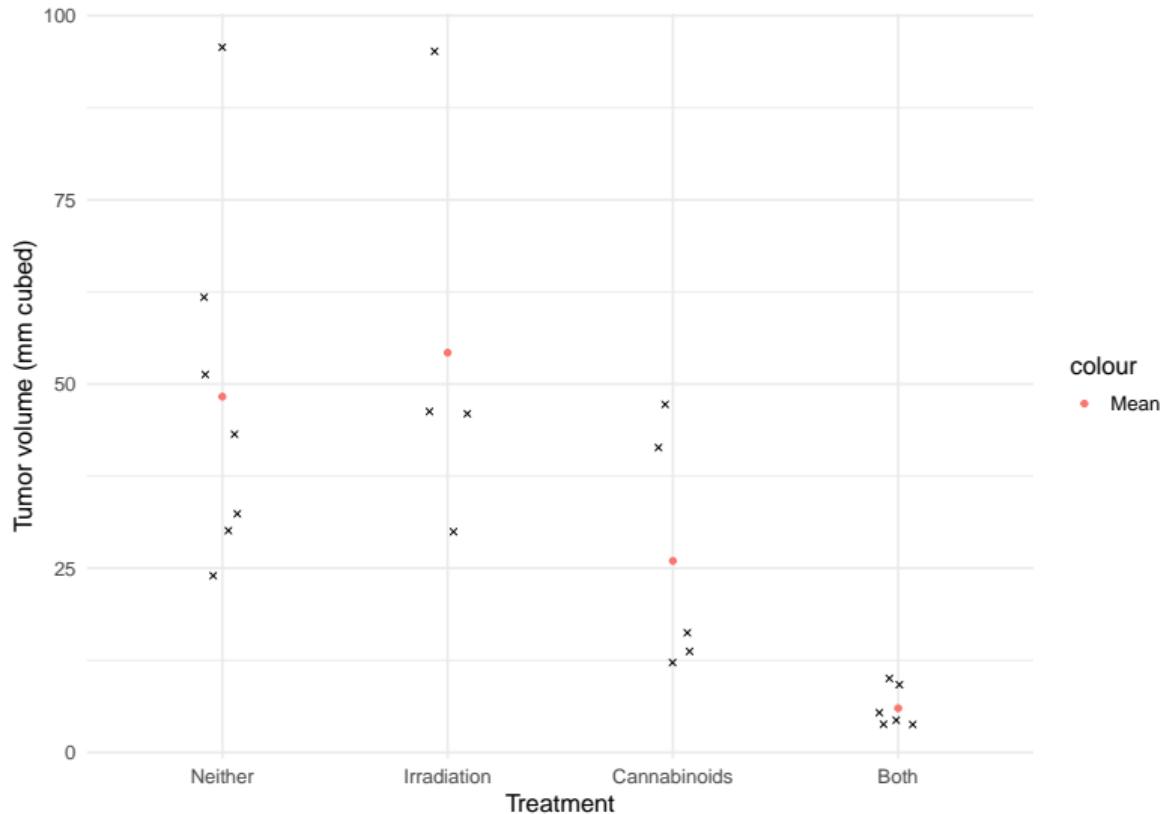
Conditions for using
ANOVAExample: Cannabis to treat
brain cancerAfter a significant
ANOVA...

Visualize the pairwise differences



ANOVA
Descriptive plots
Testing with ANOVA
Conditions for using ANOVA
Example: Cannabis to treat brain cancer
After a significant ANOVA...

Review raw data for comparison



ANOVA

Descriptive plots

Testing with ANOVA

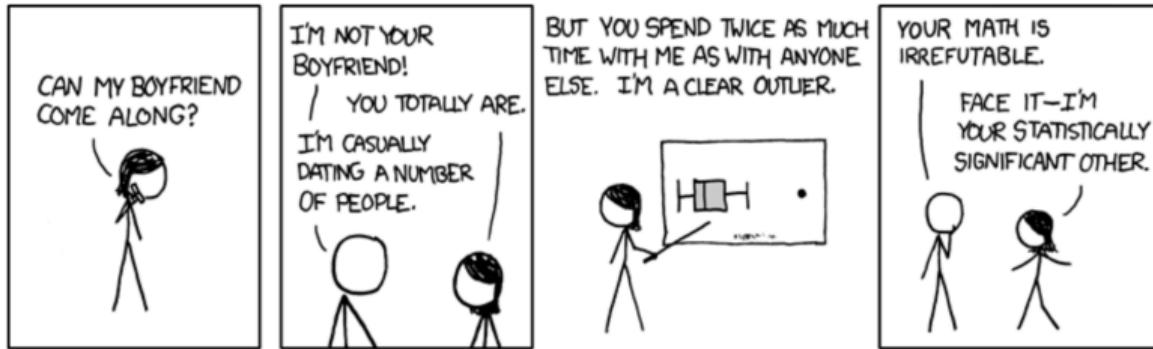
Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...

Parting humor XKCD.com

3+ Sample testing
with continuous
outcomes



ANOVA

Descriptive plots

Testing with ANOVA

Conditions for using
ANOVA

Example: Cannabis to treat
brain cancer

After a significant
ANOVA...