L12: More Probability

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Learning Objectives

- Making and using Venn diagrams for probabilities
- General addition rule for probability
- Conditional probability
- Determine whether two events are independent or dependent
- General multiplication rule for probability

Venn diagrams

Venn diagrams

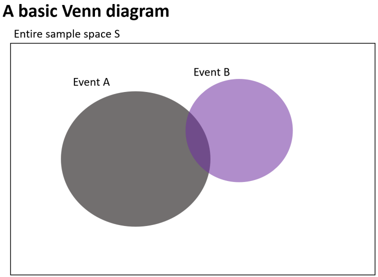
Popularized in a paper in 1880

John Venn

Born 4 August 1834

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Venn diagrams

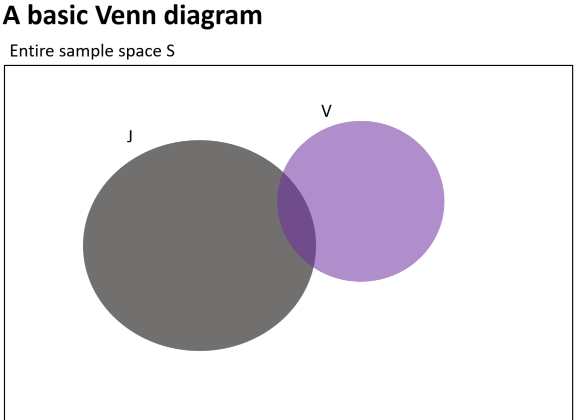


- Venn diagrams generally consist of at least two events
- When we draw a rectangle around the diagram this is the sample space. All the probabilities inside must sum to 1.
- When you make a Venn diagram label the events using capital letters to represent the random variables

Venn diagrams

Suppose you had access to survey data about vaping and JUUL-related advertisements.

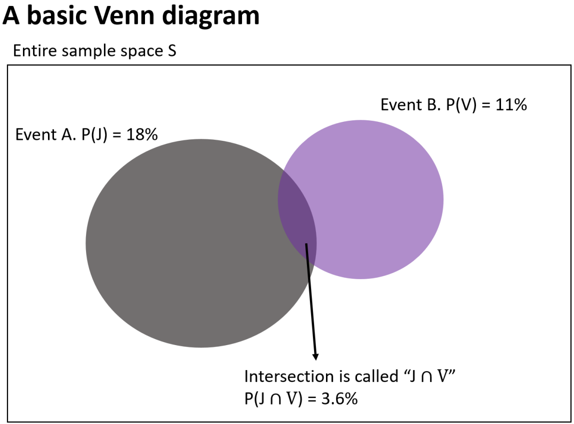
You could define the following random variables: J is the event "seen ad for JUUL" V is the event "vaped in the last 30 days"



Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Venn diagrams

Now let's say we do a survey - we can add the percents to the diagram:



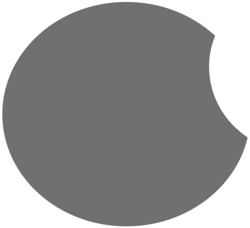
what percent of the individuals saw an ad for JUUL and do not vape?

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Venn diagram

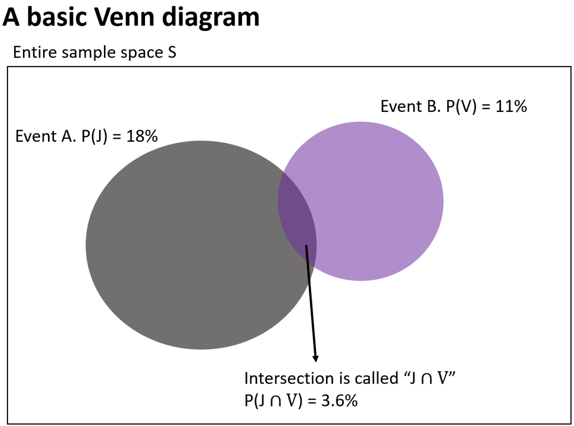
What percent of individuals saw an ad for JUUL and do not vape?

This percent is represented by this area:



How would we write this as a probability statement? What percentage is this?

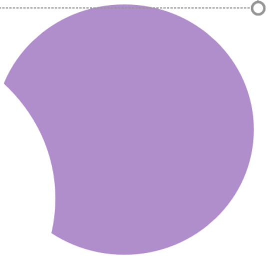
Venn diagram



What percent of individuals vape but have not seen an ad for JUUL?

Venn diagram

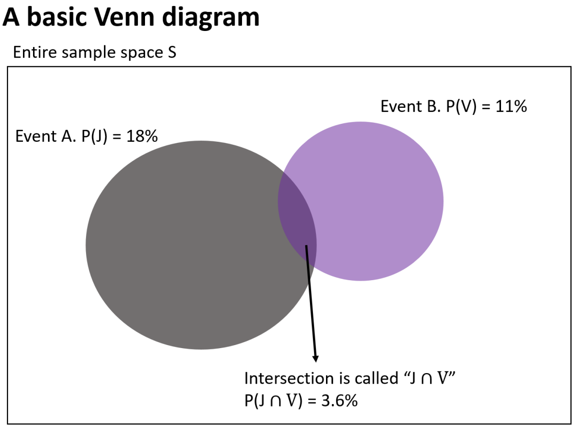
What percent of individuals vape but have not seen an ad for JUUL?



How would we write this as a probability statment? What percentage is this?

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

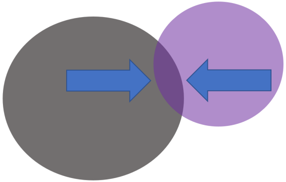
Venn diagram



What percent of individuals vape AND have seen an ad for JUUL? What is this called?

Venn diagram

What percent of individuals have seen an ad for JUUL and vape?

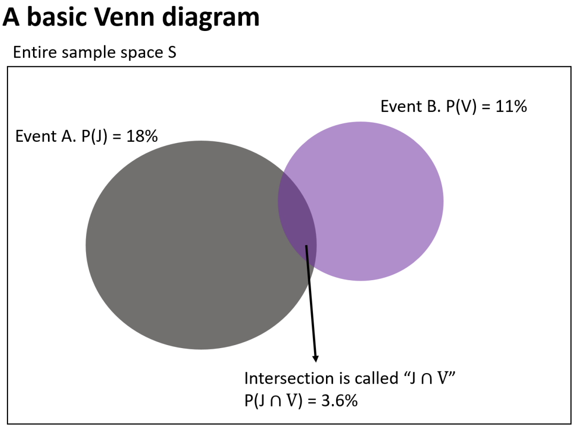


The "and" represents the intersection, the area where both events have occurred:

$P(J \cap V) = 3.6\%$

3.6% of individuals have seen an ad for JUUL and have vaped in past 30 days

Venn diagram



What percent of individuals vape OR seen an ad for JUUL? What is this called?

Venn diagram

The union is all events that have either of the outcomes - all the shaded parts of this diagram are part of the union.

P(J or V)
$P(J \cup V)$
$= P(J) + P(V) - P(J \cap V)$ this is the general rule for addition
$= 0.18 + 0.11 - 0.036$
$= 0.256 = 25.6\%$

General addition rule

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

For any two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

- Why do we subtract off $P(A \cap B)$?
- This formula simplifies to $P(A \cup B) = P(A) + P(B)$ when A and B are disjoint.
- What does the Venn diagram look like for disjoint events?

Decomposition of a probability

For any two events A and B, $P(A) = P(A \cap B) + P(A \cap \bar{B})$

Which pieces of our diagram does this indicate?

Compliments

- What is the compliment of seeing an ad for JUUL?
- What is the compliment of the intersect?
- What is the compliment of the union?

Conditional probability

Statistics is everywhere

You may have heard people talk about the probability of dying from a bee sting being greater than the probability of being killed by a shark. As in this recent article, published during "shark week"

ENTERTAINMENT

Shark week: You're way more likely to die from these than a shark attack

Here are seven things more likely to kill you than sharks.

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Statistics is everywhere

From the article:

Your odds of a dog killing you are 1 in 112,400, Your odds of dying from one of these insects touching you is 1 in 63,225 Your odds of dying from a fired gun are 1 in 6,905 Your odds of dying from lightning are 1 in 161,856 Your odds of dying while in a car are 1 in 114

compared to about 1 in 3,748,067 for a shark attack.

It's worth noting that these are probabilities(risks) NOT odds in the statistical sense

People use these probabilities to argue that swimming in the ocean is safe. Why do conditional probabilities make this argument questionable?

Conditional Probability

When $P(A) > 0$, the conditional probability of B, given A is:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

- Rearrange this formula for $P(A \cap B)$

$$P(A|B) = \frac{P(B \cap A)}{P(B)}$$

- What pieces of a Venn Diagram would these correspond to?

Conditional probability

Among those who have seen an ad for JUUL, what percent vaped in the past month?

$$P(Vape|JUUL)$$

$$P(V|J) = \frac{P(J \cap V)}{P(J)}$$

L12: More Probability

Venn diagrams

Conditional probability

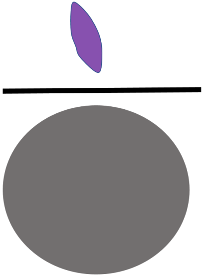
Independent vs. dependent events

Multiplication rules

Conditional probability and diagnostic testing

Conditional probability

What percent of individuals vape given they have seen an ad for JUUL?

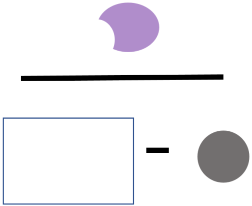


Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Conditional probability

$$P(V|J') = \frac{P(J' \cap V)}{P(J')}$$

What percent of individuals vape given they have NOT seen an ad for JUUL?



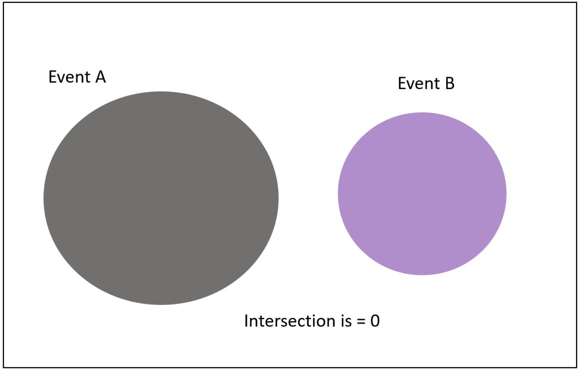
Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Recall the definition for disjoint events

- What did it mean for two events to be disjoint?
- Another term for disjoint is mutually exclusive
- How would we draw disjoint events in a Venn diagram?

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Disjoint events



Independent vs. dependent events

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Independent vs. dependent events

What sorts of events are independent?

Two events are independent if knowing that one event occurred does not change the probability that the other occurred

Independent vs dependent: example 1. Down syndrome

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Down syndrome is a genetic disorder caused when abnormal cell division results in an extra full or partial copy of chromosome 21.[1] The largest risk factors for having a child with Down syndrome are advanced maternal age.[1] Suppose that Martha is 40 and her baby has been diagnosed with Down syndrome. Martha's best friend Jane, also 40, is hoping to conceive. Is her baby's risk of Down syndrome independent of Martha's baby's risk?

1. https://www.mayoclinic.org/diseases-conditions/down-syndrome/symptoms-causes/syc-20355977

Independent vs. dependent events

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Question: The risk of having a baby with Down syndrome is $1/100$ among 40 year olds. Suppose that Jenny and Samantha are two 40-year old women. What is the probability that they both have babies with Down syndrome?

Independent vs. dependent events

Written out in probability notation, for any two events A and B, the events are independent if:

$$P(A|B) = P(A)$$

or

$$P(B|A) = P(B)$$

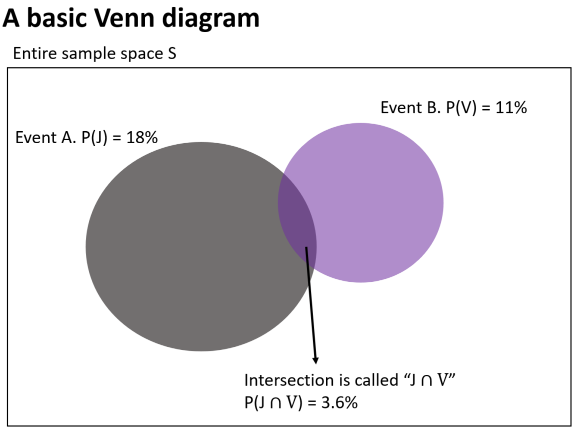
or

$$P(A \cap B) = P(A) * P(B)$$

The "|" is read as "given" or "conditional on"

If one of these is true, they are all true.

Independent vs. dependent



In our example, are the probability of seeing and ad for JUUL and vaping in the past month independent?

Independent vs. dependent events

From our JUUL and vaping example:

$$P(V|J) = \frac{P(V \cap J)}{P(J)} = \frac{.036}{.18} = .20$$

and

$$P(V) = 0.11$$

so

$$P(V|J) \neq P(V)$$

proving that we have events that are NOT independent, knowing someones exposure to JUUL ads gives us information about their probability of having vaped.

Multiplication rules

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Multiplication rule

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

For any two events, the probability that both events occur is given by:

$$P(A \cap B) = P(B|A) \times P(A)$$

Multiplication rule for independent events

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Two events A and B are independent if knowing that one occurs does not change the probability that the other occurs.

If events are independent, $P(B|A) = P(B)$ so the general multiplication rule:

$$P(A \cap B) = P(B|A) * P(A)$$

Simplifies to:

$$P(A \cap B) = P(A) \times P(B)$$

Conditional Probability using tables

Ad exposure	Vape	No Vape	Total
seen an ad for JUUL	0.036	0.144	0.18
did not see ad for JUUL	0.074	0.746	0.82
Total	0.11	0.89	1

What are the conditinal probabilies of vaping by Ad exposure?

Venn diagrams
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing

Conditional probability and diagnostic testing

Conditional probability and diagnostic testing

Key terms:

- Sensitivity: P(test positive | truly have disease)
- Specificity: P(test negative | truly do not have disease)
- Positive predictive value: P(truly have disease | test positive)
- Negative predictive value: P(truly do not have disease | test negative)

Conditional probability and diagnostic testing

More key terms:

	Have the disease	Do not have the disease
Test positive	True positive	False positive
Test negative	False negative	True negative

How to calculate the chance of having cancer if you test positive?

- Suppose that there is test for breast or prostate cancer that has a 95% chance of testing positive for cancer when the individuals it and a 98% chance of testing negative for cancer when the patient does not have it. 3% of patients in a population have the cancer being tested for.

From last lecture: How to calculate the chance of having cancer if you test positive?

	Have Cancer	Do not have Cancer	Total
Test positive	True positive	False positive	
Test negative	False negative	True negative	
Total			

From last lecture: How to calculate the chance of having cancer if you test positive?

95% chance of testing positive for cancer when the individuals it 98% chance of testing negative for cancer when the patient does not have it. 3% of patients in a population have the cancer being tested for.

	Have Cancer	Do not have Cancer	Total
Test positive	57	39	96
Test negative	3	1901	1904
Total	60	1940	2000

Conditional probability and diagnostic testing

What is the chance that a patient has cancer given that they test positive?

- Positive predictive value: P(truly have disease | test positive)
- P(truly have disease | test positive)$= 57/96 = 59\%$

Parting humor

from XKCD