Lecture 36:
Non-Parametric
Statistical Tests

Corrine Riddell
(Instructor: Tomer
Altman)

Lecture 36: Non-Parametric Statistical Tests
Chapter 27

Corrine Riddell (Instructor: Tomer Altman)

November 21, 2025
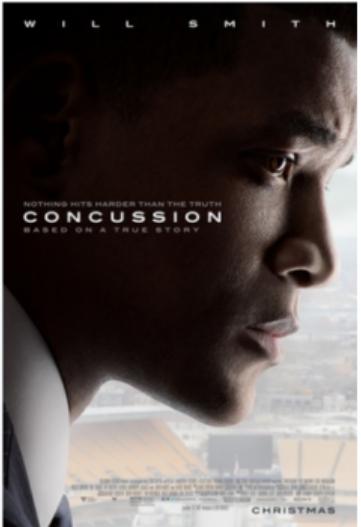
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Concussions

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Non-Parametric Testing
Wilcoxon Two-Sample Tests
Wilcoxon Sign Rank
Non-parametric Test for
Three or More Samples

In Pittsburgh, accomplished pathologist Dr. Bennet Omalu uncovers the truth about brain damage in football players who suffer repeated concussions in the course of normal play.

Concussion (IMDB)
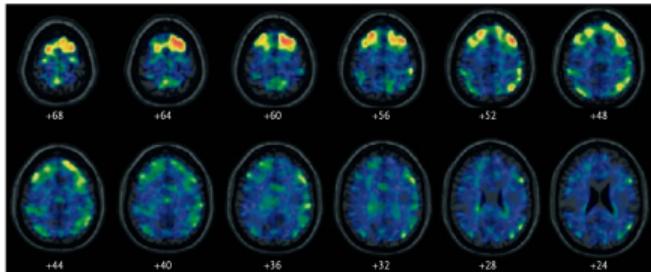
Article on Concussions

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New York Times, April 10, 2019

*Abnormal Levels of a Protein
Linked to C.T.E. Found in N.F.L
Players' Brains, Study Shows*

From the article:

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ORIGINAL ARTICLE

Tau Positron-Emission Tomography in Former National Football League
Players

Robert A. Stern, Ph.D., Charles H. Adler, M.D., Ph.D., Kewei Chen, Ph.D., Michael Navitsky, M.S., Ji Luo, M.S., David W. Dodick, M.D., Michael L. Alosco, Ph.D., Yorghos Tripodis, Ph.D., Dhruman D. Goradia, Ph.D., Brett Martin, M.S., Diego Mastroeni, Ph.D., Nathan G. Fritts, B.A., et al.

▶ The authors of the study and outside experts stressed that such tau imaging is far from a diagnostic test for C.T.E., which is likely years away and could include other markers, from blood and spinal fluid

▶ The results of the study, published in The New England Journal of Medicine are considered preliminary, but constitute a first step toward developing a clinical test to determine the presence of C.T.E. in living players, as well as early signs and potential risk

From the article

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STATISTICAL ANALYSIS
Between-group comparisons of age, years of education, and MMSE
scores were analyzed with Mann–Whitney U tests. Group differences in
race were analyzed with the use of chi-square tests. For between-group
comparisons of amyloid-beta plaque burden, chi-square tests were used
to compare the proportion of participants with a positive florbetapir PET,
and t-tests were used to compare the mean cortical:cerebellar florbetapir
standard uptake value ratio (SUVR, the ratio of radioactivity in a cerebral
region to that in the cerebellum as a reference) between the groups.

From the article

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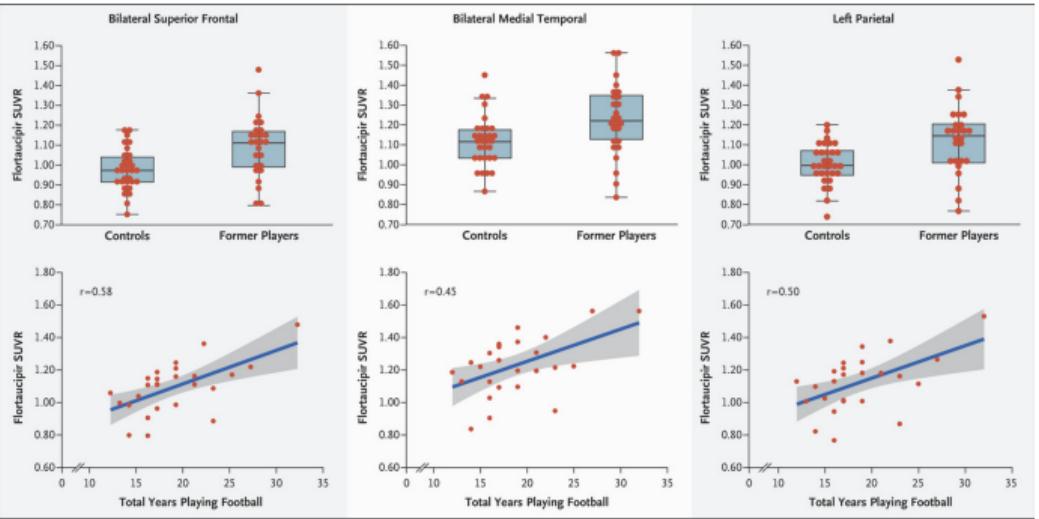
Roadmap

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Parametric Testing:

▶ One sample comparison to a mean (one sample t-test)
▶ Two independent samples (two sample t-test)
▶ Two non-independent samples (paired t-test)
▶ Multiple samples/groups (ANOVA)
  ▶ Bonferroni
  ▶ Tukey's HSD

Roadmap

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But all of the methods we have looked at so far depend on some assumptions about the underlying distribution.

What have we assumed?

What do we do if our assumptions are violated?

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Non-Parametric Testing

Non-Parametric Testing

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From http://biostatisticsryangoslingre
turns.tumblr.com/

Non-Parametric Testing

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PROS: Non-parametric methods make very few assumptions about the variable(s) we samples or their distribution and thus rely less on "parameters".

▶ Use a ranking of the data instead of actual values
▶ Do not assume a normal distribution of the data
▶ Less sensitive to outliers and skewed data
▶ Do not need a large sample size (sometimes)

CONS: Non-parametric methods use less of the information offered in the data

▶ If the assumptions for a parametric test are met and a non-parametric test is used, it will have lower power (probability of detecting a false null hypothesis)
▶ They are less specific in what they test (e.g., independence)

Non-Parametric Testing

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We will discuss non-parametric equivalents for:

Two sample t : Wilcoxon Rank-Sum

Paired t : Wilcoxon sign-rank

ANOVA: Kruskal-Wallis
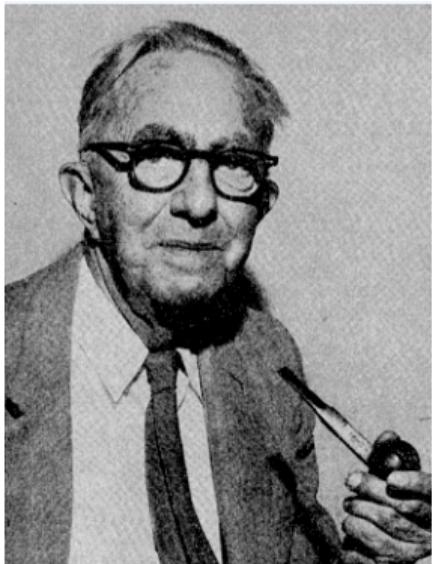
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Wilcoxon Two-Sample Tests

Frank Wilcoxon

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In one paper in 1945 he proposed both
the Wilcoxon rank-sum test and the
Wilcoxon signed-rank test.

Wilcoxon Rank-Sum

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▶ Sometimes also called the Mann-Whitney U test
▶ Non-parametric test for comparing two independent samples with a continuous outcome
▶ This is the non-parametric counterpart of the two sample t-test
▶ Assumes that the distributions have the same general shape but assumes nothing about that shape
▶ Evaluates the null hypothesis that the two population distributions are identical

Wilcoxon Rank-Sum

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To calculate a rank sum test:

1. The observations from both groups are ordered from lowest to highest and assigned the rank of their order

2. If there are "tied" values, these are assigned the average of the ranks

    ▶ E.g., if two observations have the same value and the next lower value has a rank of three, then the two observations are both given the rank of 4.5 (because they would have been ranks 4 and 5)

3. Then the sum of ranks belonging to Group 1 are compared to the sum of ranks belonging to Group 2

Wilcoxon Rank-Sum

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Values in group 1: 4, 3, 5, 2, 6

Values in group 2: 6, 5, 7, 4, 8

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Wilcoxon Rank-Sum: Ranking

Number	2	3	4	4	5	5	6	6	7	8
Index	1	2	3	4	5	6	7	8	9	10
Rank	1	2	3.5	3.5	5.5	5.5	7.5	7.5	9	10

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Wilcoxon Rank-Sum: Summation

Group 1	rank	Group 2	rank
4	3.5	6	7.5
3	2	5	5.5
5	5.5	7	9
2	1	4	3.5
6	7.5	8	10
sum	19.5	sum	35.5

Wilcoxon Rank-Sum

The smaller of the two sums is called W, with size of n_S, and the larger with size n_L. This is then used in the following equation to generate a Z statistic.

$$Z_w = \frac{W - \mu_w}{\sigma_w}$$

where

$$\mu_w = \frac{n_S(n_S + n_L + 1)}{2}$$

and

$$\sigma_w = \sqrt{\frac{n_S n_L(n_S + n_L + 1)}{12}}$$

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Wilcoxon Rank-Sum

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So from our example where group 1 had a rank sum of 19.5 and group 2 had a rank sum of 35.5

$$\mu_w = \frac{n_S(n_S + n_L + 1)}{2} = \frac{5(5 + 5 + 1)}{2} = 27.5$$

and

$$\sigma_w = \sqrt{\frac{n_S n_L(n_S + n_L + 1)}{12}} = \sqrt{\frac{5 \times 5(5 + 5 + 1)}{12}} = 4.8$$

$$Z_w = \frac{W - \mu_w}{\sigma_w} = \frac{19.5 - 27.5}{4.8} = -1.67$$

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Wilcoxon Rank-Sum

The Z_w we generate follows an approximate standard Normal distribution. So we can use our Z score to get a p-value in R

```
2*pnorm(-1.67)
```

```
## [1] 0.09491936
```

Wilcoxon Rank-Sum in R

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The general syntax will be:

`wilcox.test(group1, group2, paired=F)`

or

`wilcox.test(outcome ~ group)`

Remember that you can always type `help(wilcox.test)` in your console to get the full details.

Wilcoxon Rank-Sum example: Phenylketonuria

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Normalized mental age scores for children with phenylketonuria (PKU):

Group 1: "low exposure" < 10.0 mg/dl

Group 2: "high exposure" $>= 10.0$ mg/dl

Wilcoxon Rank-Sum: Phenylketonuria

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```
##   Group  nMA
## 1   low  34.5
## 2   low  37.5
## 3   low  39.5
## 4   low  40.0
## 5   low  45.5
## 6   low  47.0
```

Wilcoxon Rank-Sum: Phenylketonuria

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In this example there 18 High and 21 Low exposure individuals.

```
group_by(pku,Group) %>%
  summarise(
    count = n(),
    median = median(nMA, na.rm = TRUE),
    IQR = IQR(nMA, na.rm = TRUE)
  )
```
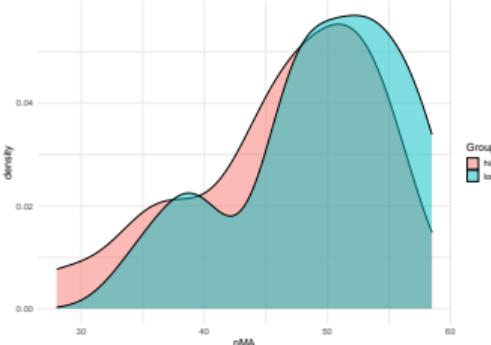
```
## # A tibble: 2 x 4
##    Group count median   IQR
##    <chr> <int>  <dbl> <dbl>
## 1 high      18   48.2  9.12
## 2 low       21   51    7
```

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Wilcoxon Rank-Sum: Phenylketonuria

If we graph the distributions with a
density plot what do we notice?

```
ggplot(pku, aes(x = nMA)) +
  geom_density(aes(fill = Group),
  theme_minimal(base_size = 15)
```

Wilcoxon Rank-Sum: Phenylketonuria

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```
wilcox.test(nMA ~ Group, data=pku)
```

```
##
##   Wilcoxon rank sum test with continuity correction
##
## data:  nMA by Group
## W = 142, p-value = 0.1896
## alternative hypothesis: true location shift is not equal to 0
```

Check your understanding!

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Wilcoxon Rank-Sum vs T: NHANES example

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Here I will again use the NHANES data as an example, looking at height by gender:

```
# Read CSV into R
nhanes <- read.csv(file="./data/nhanes.csv", header=TRUE, sep=",")
names(nhanes)
```

```
##  [1] "ridageyr"  "agegroup"  "gender"   "military"  "born"     "citizen"
##  [7] "drinks"    "drinkscat" "bmxwt"    "bmxht"     "bmxbmi"   "bmicat"
## [13] "bpxpls"    "bpxsy1"    "bpxsy2"   "sys1d"     "sys2d"    "bpxdi1"
## [19] "bpxdi2"    "dias1d"    "dias2d"   "bpcat"     "chest"    "fs1"
## [25] "fs2"       "fs3"       "lbdhdd"   "hdlcat"    "highhdl"  "hi"
## [31] "asthma"    "vwa"       "vra"      "va"        "aspirin"  "sleep"
## [37] "is"        "hs"        "lbdldl"   "highldl"
```
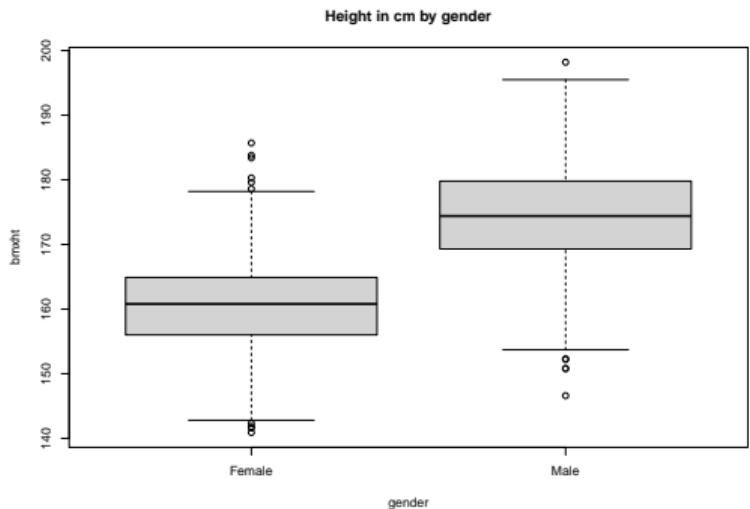
Wilcoxon Rank-Sum vs T: NHANES example

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Wilcoxon Rank-Sum vs T

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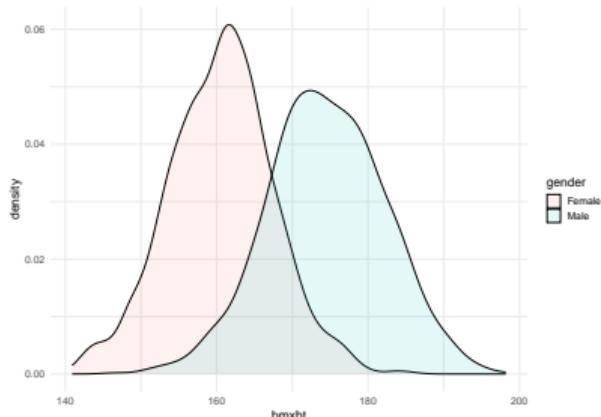
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```
ggplot(nhanes, aes(x = bmxht)) +
  geom_density(aes(fill=gender), alpha=0.1) +
  theme_minimal(base_size = 15)
```

Wilcoxon Rank-Sum vs T

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```
t.test(malesht, femalesht, paired=F)
```

```
##
##  Welch Two Sample t-test
##
## data:  malesht and femalesht
## t = 47.285, df = 2384, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  13.37441 14.53172
## sample estimates:
## mean of x mean of y
##  174.4717  160.5186
```

Wilcoxon Rank-Sum vs T

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```
wilcox.test(malesht,femalesht)

##
##   Wilcoxon rank sum test with continuity correction
##
## data:  malesht and femalesht
## W = 1402065, p-value < 2.2e-16
## alternative hypothesis: true location shift is not equal to 0
```

Wilcoxon Rank-Sum vs T

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▶ When the sample size is quite large (as with these NHANES data) the assumption of approximate normality (due to CLT) is reasonable one and the results of the hypothesis testing will generally not be different using a parametric or non-parametric approach

▶ In smaller sample sizes, with potential outliers, can get more reliable results using Wilcoxon (exact version) than equivalent t-test

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Wilcoxon Sign Rank

Wilcoxon Sign Rank

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Non-Parametric Testing
Wilcoxon Two-Sample Tests
Wilcoxon Sign Rank
Non-parametric Test for Three or More Samples

▶ Non-parametric test for comparing two non-independent (paired) sample means

▶ This is the non-parametric counterpart of the paired t-test

▶ Assumes that the distributions have the same general shape but assumes nothing about that shape

▶ Evaluates the null hypothesis that the difference between the paired values is 0

Wilcoxon Sign Rank

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Steps:

1) Calculate the difference between each pair of observations

2) Rank the difference by absolute value from smallest to largest (again, tied values get the average of the ranks). Any pair where there is no difference is thrown out.

3) Record the "sign" of the difference (i.e., positive or negative)

4) Take the sum of the positive ranks and the sum of the negative ranks (the smaller sum is denoted with a T)

Wilcoxon Sign Rank

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Under the null hypothesis that the difference is zero, we would expect the sample to have equal numbers of positive and negative ranks with equivalent sums. This expectation is tested against the statistic

$$Z_T = \frac{T - \mu_T}{\sigma_T}$$

Where

$$\mu_T = \frac{n(n+1)}{4}$$
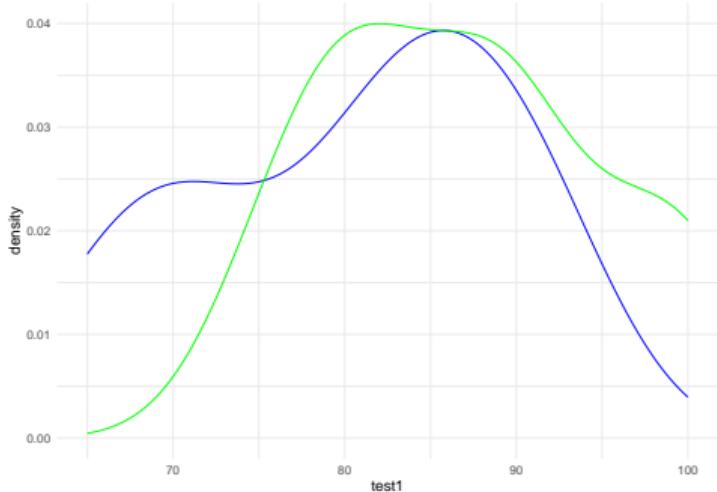
and

$$\sigma_T = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

Wilcoxon Sign Rank: Example Pre- and Post-Test

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Time 1	Time 2
65	77
87	100
77	75
90	89
70	80
84	81
92	91
83	96
85	84
91	89
68	88
72	100
81	81
———	——

Sign Rank Example

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Note the central peak in the data.

Sign Rank Example: Calculate Difference and Sign

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Time 1	Time 2	Difference	sign
65	77	12	+
87	100	13	+
77	75	-2	-
90	89	-1	-
70	80	10	+
84	81	-3	-
92	91	-1	-
83	96	13	+
85	84	-1	-
91	89	-2	-
68	88	20	+
72	100	18	+
81	81	0	?

Sign Rank Example: Sort by Absolute Value and Assign Rank

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Time 1	Time 2	Difference	sign	rank
90	89	-1	-	2
92	91	-1	-	2
85	84	-1	-	2
77	75	-2	-	4.5
91	89	-2	-	4.5
84	81	-3	-	6
70	80	10	+	7
65	77	12	+	8
87	100	13	+	9.5
83	96	13	+	9.5
72	100	18	+	11
68	88	20	+	12
81	81	0	?	drop

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Sign Rank Example: Sum the Positive and Negative Ranks

Negative signs

Time 1	Time 2	Difference	sign	rank
90	89	-1	-	2
92	91	-1	-	2
85	84	-1	-	2
77	75	-2	-	4.5
91	89	-2	-	4.5
84	81	-3	-	6
_____	____	__-	____	

Sum of negative sign ranks is 21.

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Sign Rank Example: Sum the Positive and Negative Ranks

Time 1	Time 2	Difference	sign	rank
70	80	10	$+$	7
65	77	12	$+$	8
87	100	13	$+$	9.5
83	96	13	$+$	9.5
72	100	18	$+$	11
68	88	20	$+$	12
_____	____	__	___	

Sum of the positive sign ranks is 57.

Wilcoxon Sign Rank: Example

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Our expectation would be:

$$\mu_T = \frac{n(n+1)}{4} = \frac{12(12+1)}{4} = 39$$

Remember that we had 13 observations, but we dropped one because the values at times 1 and 2 were the same and

$$\sigma_T = \sqrt{\frac{n(n+1)(2n+1)}{24}} = \sqrt{\frac{12(12+1)(2 \times 12 + 1)}{24}} = 12.75$$

Wilcoxon Sign Rank: Example

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Altman)

Non-Parametric Testing
Wilcoxon Two-Sample Tests
Wilcoxon Sign Rank
Non-parametric Test for
Three or More Samples

And we compare our expectation to the smaller rank value (Sum of negative ranks was 21, sum of positive ranks was 57):

$$Z_T = \frac{T - \mu_T}{\sigma_T} = \frac{21 - 39}{12.75} = -1.412$$

```
2*pnorm(-1.412)
```

```
## [1] 0.15795
```

Wilcoxon Rank-Sum in R

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Statistical Tests

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The general syntax will be:

```
wilcox.test(group1, group2, paired=T)
```

or

```
wilcox.test(Pair(group,outcome) ~ 1)
```

Wilcoxon Sign Rank: Example

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```
wilcox.test(test1,test2,paired=T, correct=FALSE)

## Warning in wilcox.test.default(test1, test2, paired = T, correct = FALSE):
## cannot compute exact p-value with ties

## Warning in wilcox.test.default(test1, test2, paired = T, correct = FALSE):
## cannot compute exact p-value with zeroes

##
##   Wilcoxon signed rank test
##
## data:  test1 and test2
## V = 21, p-value = 0.157
## alternative hypothesis: true location shift is not equal to 0
```

Wilcox Sign Rank: Compare to T

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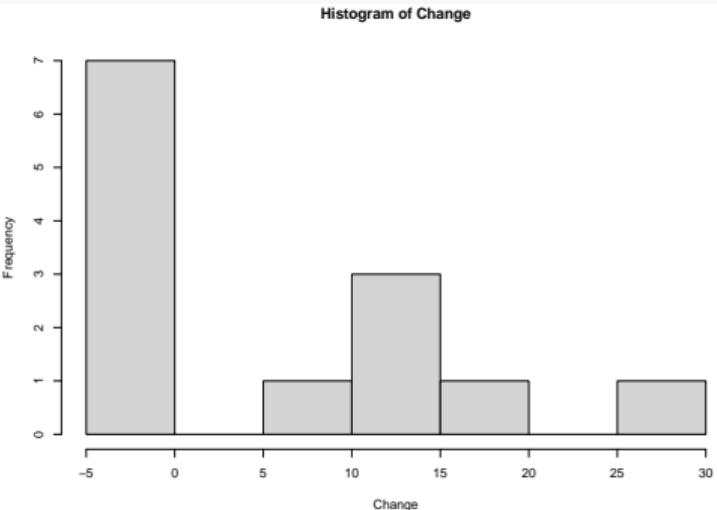
```
t.test(test1,test2,paired=TRUE)
```

```
##
##  Paired t-test
##
## data:  test1 and test2
## t = -2.3684, df = 12, p-value = 0.0355
## alternative hypothesis: true mean difference is not equal to 0
## 95 percent confidence interval:
##  -12.7011701  -0.5295991
## sample estimates:
## mean difference
##       -6.615385
```

Wilcox Sign Rank: Compare to T

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▶ Our sample size is 13 (one pair is thrown out for having a difference of zero)
▶ Note the t-test had more power due to the data having a symmetric peak
▶ The distribution of change is plotted as a histogram

```
hist(Change)
```


Histogram of Change

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Non-parametric Test for Three or More Samples

Kruskal-Wallis

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▶ The Kruskal-Wallis test is a non-parametric alternative to the ANOVA test
▶ Kruskal-Wallis looks at the medians of the groups, not the means, and tests
  if one or more are significantly different from the rest (but does not report
  which one):
  ▶ H_0: There is no difference between the group medians
  ▶ H_1: There is a statistically-significant difference in the group medians

Kruskal-Wallis

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▶ This test can be thought of as an extension of the Wilcoxon Rank-Sum test
▶ We will not do this one by hand
▶ In R the syntax is generally: `kruskal.test(outcome ~ group, dataset)`

Kruskal-Wallis

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```
## 
##  Kruskal-Wallis rank sum test
## 
## data:  outcome by treatment
## Kruskal-Wallis chi-squared = 13.096, df = 3, p-value = 0.004434
```

Non-Parametric Summary

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Non-Parametric
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Non-Parametric Testing
Wilcoxon Two-Sample Tests
Wilcoxon Sign Rank
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▶ Most parametric tests have an analogous non-parametric test
▶ We have covered the following:

Samples	Parametric	Non Parametric
Two independent samples	two sample *t*-test	Wilcoxon rank sum
Two paired samples	paired *t*-test	Wilcoxon sign rank
Three or more samples	ANOVA	Kruskal-Wallis

Non parametrics in R

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Samples	test name	R function
Two independent samples	Wilcoxon rank sum	`wilcox.test(group1,group2,paired=F)`
Two paired samples	Wilcoxon sign rank	`wilcox.test(Pair(group1,group2) ~ 1)`
Three or more samples	Kruskal-Wallis	`kruskal.test(outcome ~ group)`

Parting humor

Lecture 36:
Non-Parametric
Statistical Tests
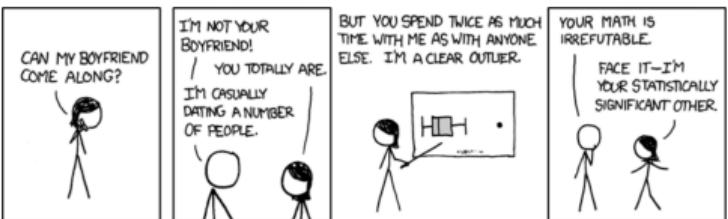
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Check your understanding!

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