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Discussion

Mean vs Median: Outliers and sample size, skew,

Measures of spread

Example: Hospital cesarear delivery rates

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Learning objectives for today:

Describing your data:

- 1. Investigate measures of centrality
 - mean and median, and when they're the same vs. different
- 2. Investigate measures of spread
 - ► IQR, standard deviation, and variance
- 3. Create a visualization of the "five number summary"
 - boxplots using ggplot
- 4. Calculate the variance and standard deviation

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Measures of central tendency

Measures of central tendency

► Most common: mean and median

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The arithmetic mean

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

$$\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

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The median

- ▶ Half of the measurements are larger and half are smaller.
 - ▶ What is the median if there is an odd number of observations?
 - ► An even number?

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Bay Area rent

San Francisco

Apartments for rent in San Francisco: What will \$3,400 get you?



From Hoodline.com

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delivery rates

Bay Area rent



Now sitting at \$3,680, average rent in San Francisco has soared 70 percent since 2010 while home prices climbed an eye-popping 95 percent and median income crept up a comparatively modest 61 percent. Across the bay in Oakland, rent climbed even more — 108 percent. Mercury News article

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Discussion

When are these measures approximately equal?

- Answer: When the data has one peak and is roughly symmetric
 - In this case, the mean \approx median, so provide either one in a summary
- Skewed data
 - ▶ mean ≠ median
 - ▶ Right-skewed data will commonly have a _____ mean than median
 - ▶ Left-skewed data will commonly have a _____ mean than median
 - Which statistic should we report?

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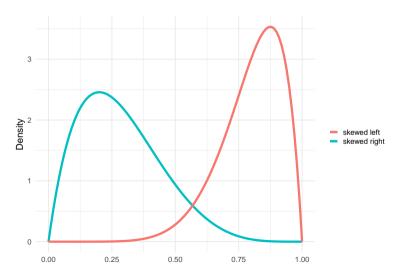
Mean vs Median: Outliers and sample size, skew,

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Example: Hospital cesarear delivery rates

Warning: Using 'size' aesthetic for lines was deprecated in ggplot2 3.4.0.

i Please use 'linewidth' instead.



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Apartment rent in SF

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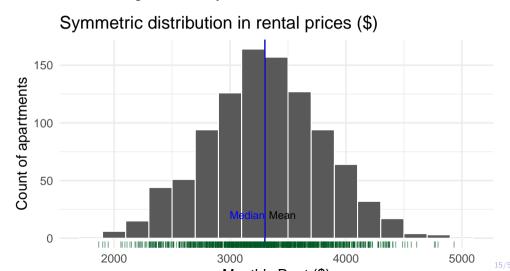
Sample variance and standard deviation

Problem: We want to understand how much it costs for a new resident to rent a 1 bedroom apartment in San Francisco

Plan: Take a sample of 1000 apartment units listed for rent (currently available) and ask the rental price (excluding utilities)

Data: Here I will present data that I simulated in r using a mean value published on rentjungle.com - you will not be expected to do this or be tested on it.

Suppose that the distribution of rent prices looked like this. The green ticks underneath the histograms shows you the exact rent values.



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> ample variance and andard deviation ox plots

From last lecture: We describe this distribution in terms of center, shape and spread:

- ▶ Center: Where is the center of the distribution?
- ► Shape: Is this distribution unimodal or bimodal?
- ➤ Spread: How much variability is there between the lowest and highest rent values?

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Example: Hospital cesarear delivery rates

Sample variance and standard deviation Box plots

Summarizing numerically: Center:

```
# in base R
mean(rent data[,"sym"])
## [1] 3301.662
median(rent_data[,"sym"])
## [1] 3298.832
# using the summarize function and a pipe operator
rent_data %>% summarize(
  mean=mean(sym),
  median = median(sym))
```

```
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```

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Mean vs Median: Outliers and sample size, skew, shape

When are the mean and median approximately equal?

- ► If your data has one peak (unimodal), is roughly symmetric, and does not have outliers
 - ightharpoonup mean \approx median, so provide either one in a summary

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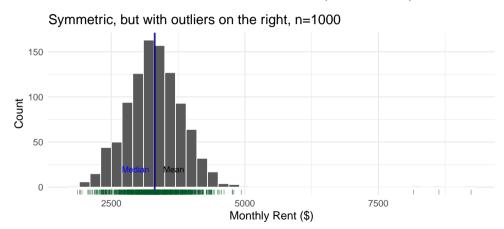
Mean vs Median: Outliers and sample size, skew.

Measures of spread

shane

Example: Hospital cesarean delivery rates

Now suppose that there were three rents within the data set with much larger values than the rest of the distribution. Here is the plot for this updated data.



▶ With 1000 sampled points the outliers do not have a large effect on the mean

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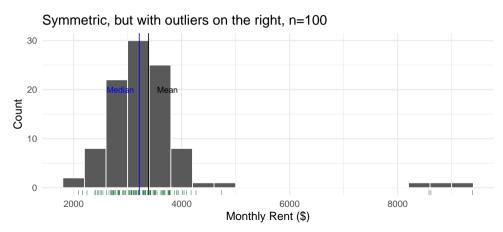
Mean vs Median: Outliers and sample size, skew.

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Example: Hospital cesare delivery rates

Imagine instead, there were only 100 sampled points. Here, the outliers have a larger effect on the mean. The mean is not resistant to outliers.



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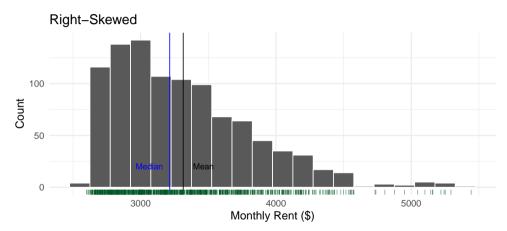
Mean vs Median: Outliers and sample size, skew, shape

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Consider what happens if there are many high-end apartments in the area.



Why is the mean larger than the median in this case?

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Skewed data

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- ▶ mean ≠ median
- ▶ Data with a long right tail will commonly have a _____ mean than median
- ▶ Data with a long left tail will commonly have a _____ mean than median
- ▶ Which statistic should we report?

Measures of central tendency

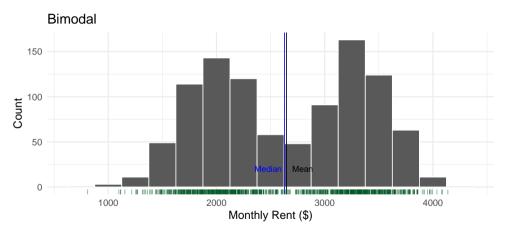
Discussion

Mean vs Median: Outliers and sample size, skew, shape

Measures of spread

Example: Hospital cesarear delivery rates

Now, suppose that the sample of estimates looked like this instead:



Describe the distribution. How does it differ from the first plot? Would you want to provide the mean or median for these data?

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Example: Hospital cesarea delivery rates

standard deviation

Box plots

Summary of measures of central tendency

- ▶ The mean and median are similar when the distribution is symmetric
- Outliers affects the mean and pull it towards their values. But they do not have a large effect on the median.
- Skewed distributions also pull the mean out into the tail.
- Measures of central tendency are not very helpful in multi-modal distributions

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Measures of spread

The inter-quartile range (IQR)

- ▶ Q1 is the 1st quartile/the 25th percentile.
 - ▶ 25% of individuals have measurements below Q1.
- ▶ Q2 is the 2nd quartile/the 50th percentile/the median.
 - ▶ 50% of individuals have measurements below Q2.
- ▶ Q3, the 3rd quartile/the 75th percentile.
 - ▶ 75% of individuals have measurements below Q3.
- ▶ Q1-Q3 is called the inter-quartile range (IQR).
 - What percent of individuals lie in the IQR?

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Example: Hospital cesarea delivery rates

Quantiles using R

```
quantile(variable, 0.25)
```

```
rent_data %>% summarize(
  Q1 = quantile(sym, 0.25),
  median = median(sym),
  Q3 = quantile(sym, 0.75)
)
```

```
## Q1 median Q3
## 1 2981.445 3298.832 3629.012
```

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Example: Hospital cesarear delivery rates

Mean vs Median: Outliers and sample size, skew,

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Example: Hospital cesarea delivery rates

Sample variance and standard deviation

Box plots

- quantile(variable, 0.25) will not always give the exact same answer you calculate by hand
- ► The R function is optimized for its statistical properties and is slightly different than the book's method
- ➤ To get the exact same answer as by hand use quantile(data, 0.25, type = 2)
- ➤ You may use either one in this class. Most commonly, people do not specify type=2

Another measure of spread: The (full) range

▶ The difference between the minimum and maximum value

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Example: Hospital cesarear delivery rates

Concise information about spread and center: The five number summary

- ► The five number summary (min, Q1,median,Q3, max) is a quick way to communicate a distribution's center and spread.
- ▶ Based on the summary you can describe the full range of a dataset, where the middle 50% of the data lie, and the middle value.

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Example: Hospital cesarean delivery rates

dplyr's summarize() to calculate the five number summary

Using our original example of rent data:

```
rent_data %>% summarize(
    min = min(sym),
    Q1 = quantile(sym, 0.25),
    median = median(sym),
    Q3 = quantile(sym, 0.75),
    max = max(sym)
)
```

```
## min Q1 median Q3 max
## 1 1866.829 2981.445 3298.832 3629.012 4932.54
```

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Example: Hospital cesarean delivery rates

Example: Hospital cesarean delivery rates

Lecture 04: Describing data with numbers

These data were provided by the first author (Kozhimannil) of a manuscript published in the journal *Health Affairs*. link

Example: Hospital cesarean delivery rates Sample variance and standard deviation

From the article: Cesarean delivery is the most commonly performed surgical procedure in the United States, and cesarean rates are increasing. In its Healthy People 2020 initiative, the Department of Health and Human Services put forth clear, authoritative public health goals recommending a 10 percent reduction in both primary and repeat cesarean rates, from 26.5 percent to 23.9 percent, and from 90.8 percent to 81.7 percent, respectively.

A targeted approach to achieving such reductions might focus on hospitals with exceptionally high cesarean rates. However, adopting such a strategy requires quantification of hospital-level variation in cesarean delivery rates.

Measures of central tendency

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Measures of spread

Example: Hospital cesarean delivery rates

Problem: To characterize the variation in cesarean rates between Hospitals in the United States

Plan: Collect existing data from a variety of institutions for one year and compare rates of cesarean delivery. They also looked at cesarean rates among only low risk births at each institution. Why might this be important?

Data: For this article, they worked with 2009 data from 593 US hospitals nationwide

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ample variance and tandard deviation tox plots

Example: Hospital cesarean delivery rates

We start by importing the data:

```
library(readx1)
# this library helps with reading xlsx and xls files into R
CS_dat <- read_xlsx("Kozhimannil_Ex_Cesarean.xlsx", sheet = 1)</pre>
```

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Measures of central tendency

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Example: Hospital cesarean delivery rates

head(CS dat)

Example: Hospital cesarean

```
## # A tibble: 6 \times 6
##
     Births HOSP_BEDSIZE cesarean_rate lowrisk_cesarean_rate 'Cesarean rate
##
      <dbl>
                    <dbl>
                                    <dbl>
                                                            <dbl>
        767
                                    0.344
                                                           0.107
## 1
        183
                                    0.454
                                                           0.186
## 2
        668
                                    0.430
                                                           0.195
##
        154
                                    0.279
                                                           0.0844
        327
                                    0.306
                                                           0.119
                                    0.301
       2356
                                                           0.0662
```

... with 1 more variable: 'Low Risk Cearean rate*100' <dbl>

Example: Hospital cesarean delivery rates

```
names(CS_dat)
```

let's take a moment to discuss variable names containing spaces

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Example: Hospital cesarean delivery rates

Sample variance and standard deviation

Box plots

- ► Two variables in CS dat contain spaces.
- ▶ We generally want to remove spaces from variable names.
- ▶ Question: Which dplyr function can we use to change the variable names?
- Answer: rename(new_name = old_name) can be used. When the old variable name contains spaces, you need to place back ticks around it like this:

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Sample variance and standard deviation
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See this paper for tips on storing data in Excel for later analysis.

Mean vs Median: Outlier

snape

Example: Hospital cesarean delivery rates

Sample variance and standard deviation

```
CS_dat <- CS_dat %>%
  select(Births, cs_rate,low_risk_cs_rate) %>%
  rename(num_births = Births)
```

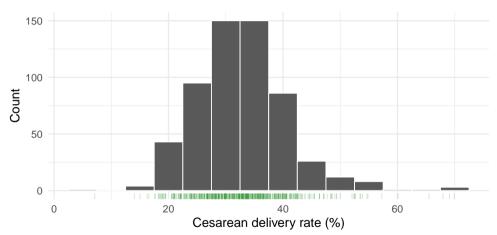
For our example, we are only interested in each hospital's cesarean delivery rate, the rate for lower risk pregnancies, and the number of births at the hospital.

ggplot(CS dat, aes(x = cs rate)) +

Example: Hospital cesarean delivery rates

```
geom histogram(col = "white", binwidth = 5) +
labs(x = "Cesarean delivery rate (%)", y = "Count",
caption = "Data from: Kozhimannil, Law, and Virnig. Health Affairs. 2013;32(3
geom rug(alpha = 0.2, col = "forest green") + \#alpha controls transparency
theme_minimal(base_size = 15)
```

Histogram of cesarean delivery rates across US hospitals



Data from: Kozhimannil, Law, and Virnig. Health Affairs. 2013;32(3):527–35.

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Measures of spread

Example: Hospital cesarean delivery rates

standard deviation

Spread of cesarean delivery rates across US hospitals

- ► What can you say about this distribution? Would you expect so much variation across hospitals in their rates of cesarean delivery?
- Let's describe the spread of these data using the methods from Chapter 2.

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```

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snape

Example: Hospital cesarean delivery rates

andard deviation

```
CS_dat %>% summarize(
  Q1 = quantile(cs_rate, 0.25),
  median = median(cs_rate),
  Q3 = quantile(cs_rate, 0.75)
)
```

```
## # A tibble: 1 x 3
## Q1 median Q3
## <dbl> <dbl> <dbl> 4bl> 37.6
```

and sample size, skew,

Measures of spread

Example: Hospital cesarean delivery rates

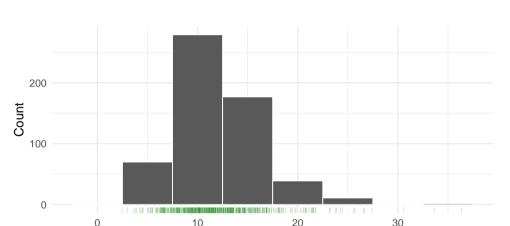
andard deviation

```
CS dat %>% summarize(
  min = min(cs rate),
  Q1 = quantile(cs_rate, 0.25),
  median = median(cs rate),
  Q3 = \text{quantile}(\text{cs rate}, 0.75),
  max = max(cs rate)
## # A tibble: 1 \times 5
##
       min
               Q1 median
                              Q3
                                    max
```

<dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <0.9</pre>

##

Histogram of low risk cesarean delivery rates across US hospitals



Low risk Cesarean delivery rate (%)

Data from: Kozhimannil, Law, and Virnig. Health Affairs. 2013;32(3):527–35.

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Example: Hospital cesarean delivery rates

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Example: Hospital cesarean delivery rates

tandard deviation

```
CS_dat %>% summarize(
    min = min(low_risk_cs_rate),
    Q1 = quantile(low_risk_cs_rate, 0.25),
    median = median(low_risk_cs_rate),
    Q3 = quantile(low_risk_cs_rate, 0.75),
    max = max(low_risk_cs_rate)
)
### # A tibble: 1 x 5
```

```
## # A tibble: 1 x 5
## min Q1 median Q3 max
## <dbl> <dbl> <dbl> <dbl> <dbl> 36.4
```

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Mean vs Median: Outlier and sample size, skew.

.....

Measures of spread

delivery rates

Sample variance and standard deviation

Box pic

Sample variance and standard deviation

Sample variance and standard deviation

Let s^2 represent the variance of a sample. Then,

$$s^{2} = \frac{(x_{1} - \bar{x})^{2} + (x_{2} - \bar{x})^{2} + \dots + (x_{n} - \bar{x})^{2}}{n - 1}$$

$$s^2 = \frac{1}{n-1}((x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + ... + (x_n - \bar{x})^2)$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

Let s represent the standard deviation of a sample. Then,

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

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Example: Hospital cesarear delivery rates

Sample variance and standard deviation

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Sample variance and standard deviation

► Some intuition on why we divide by n-1: link

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Measures of central tendency

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Mean vs Median: Outlier and sample size, skew,

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Example: Hospital cesarean delivery rates

Sample variance and standard deviation

```
CS_dat %>% summarize(
    cs_sd = sd(cs_rate),
    cs_var = var(cs_rate)
)

## # A tibble: 1 x 2
## cs sd cs var
```

<dbl> <dbl>

64.5

8.03

##

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Measures of spread

Example: Hospital cesarean delivery rates

Sample variance and standard deviation

Example: Hospital cesarean delivery rates

What might we conclude from these data?

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Evample: Hospital cesarea

Example: Hospital cesarear delivery rates

Sample variance and standard deviation

Example: Hospital cesarean delivery rates

From the article:

"we found that cesarean rates varied tenfold across hospitals, from 7.1 percent to 69.9 percent. Even for women with lower-risk pregnancies, in which more limited variation might be expected, cesarean rates varied fifteenfold, from 2.4 percent to 36.5 percent. Thus, vast differences in practice patterns are likely to be driving the costly overuse of cesarean delivery in many US hospitals."

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Box plots

Box plots provide a nice visual summary of the center and spread

Lecture 04: Describing data with numbers

Also called box and whisker plots

The box:

- ▶ The centre line is the median
- ► The top of the box is the Q3
- ▶ The bottom of the box is the Q1

The whiskers - depends:

- The top of the top whisker is either the max value, or equal to the highest point that is below Q3 + 1.5*IQR
- The bottom of the bottom whisker is either min value, or equal to the lowest point that is above Q1 - 1.5*IQR
- In plots where the whiskers are not the min and max, the data points above and below the whiskers are the outliers

Box plots in R

```
ggplot(CS_dat, aes(y = cs_rate)) +
geom boxplot() +
ylab("Cesarean delivery rate (%)") +
labs(title = "Box plot of the CS rates across US hospitals",
   caption = "Data from: Kozhimannil et al. 2013.") +
theme minimal(base size = 15) +
scale x continuous(labels = NULL) \# removes the labels from the x axis
```

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Measures of central tendency

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Mean vs Median: Outlier

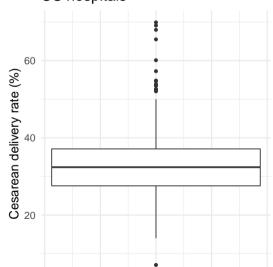
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measures or spread

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Box plots provide a nice visual summary of the center and spread

Box plot of the CS rates across US hospitals



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Example: Hospital cesarear delivery rates

R Recap: What new functions did we use?

- quantile(data, 0.25), quantile(data, 0.75) for Q1 and Q3, respectively
- 2. min() and max() for the full range of the data
- 3. sd() and var() for sample standard deviation and variance
- 4. Used the above within summarize() to easily output these measures
- 5. ggplot's geom_boxplot

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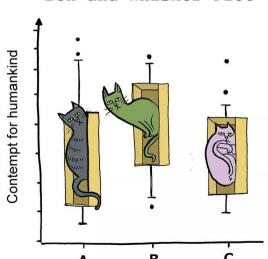
Measures of spread

Example: Hospital cesarear delivery rates

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Box-and-Whisker Plot



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> ample variance and andard deviation