From Z to T

Roadmap

Roadmap

Part 1 of the course looked at visualizing and describing data

- Continuous(histograms, box plots, mean, median, variance, standard deviation etc.)
- Categorical (bar charts, stacked bars, frequencies/percents marginal and conditional probabilities)

Part II introduced key concepts in probability and distributions

- Probability rules (independence, addition and decomposition, multiplication, Bayes theorem)
- Continuous distribution (Normal)
- Discreet distributions (Binomial and Poisson)
- Sampling variability, central limit theorem, CI and hypothesis testing

Part III will put these together and build your toolkit for statistical testing

Roadmap

In deciding what statistical test to use we will often be thinking about:

- The type of data (continuous vs categorical)
- How many groups we are comparing
- Is there an inherent relationship between the measurements (dependence)?
- Is there a theoretical distribution that is a good fit for the data?

Reduced conditions for inference about a mean

Reduced conditions for inference about a mean

- Data is a SRS form a much larger population (really important)
- Observations follow a Normal distribution (some leeway)

Recap: Z testing

- We have been looking at variables that are continuous in nature
- For the last few lectures we have assumed that the population standard deviation (σ)was known to us
- We conducted the z-test and created CIs using this known σ
- Today we will generalize this framework to a more realistic setting where σ is unknown. We will use s, the sample standard deviation as an estimate of σ

Estimating the standard error based on the sample

- Previously, we knew the standard error of the mean to be

$$\frac{\sigma}{\sqrt{n}}$$

- Now, we don't know σ, so we estimate the standard error by

$$\frac{s}{\sqrt{n}}$$

where s is the sample standard deviation.

standard deviation vs standard error

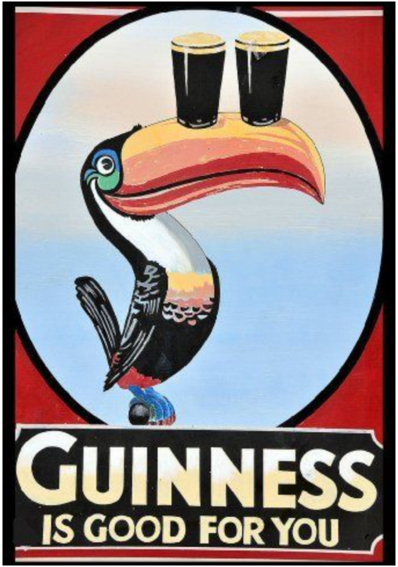
- Variance σ^2: average squared deviation from the mean(absolute value)
- Standard Deviation (σ for a parameter or S for a sample) - square root of the variance of all observations: - on average how far do our values deviate from the samplemean
- Standard Error (SE): - The standard deviation of a statistic estimated from the data is the standard error of the statistic. - The standard error is $se = s/\sqrt{n}$ - The standard error is the sd of all sample means - Tells how close our test statistic is to the true value. - on average how far does our test statistic move from the true population mean?

Recall the z-test!

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

the t-test and t-distribution

Meet the t-test!



T-test

Thanks to William Gosset (published anonymously) and the Guinness company's strategy of hiring statisticians, if we are interested in comparing mean values of a variable to a hypothesized null we can use a t-test.

Meet the t-test!

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

- What is the difference between z and t?
- The t-test is more variable than the z-test statistic because we have to substitute s for σ. Because s is a statistic, it varies across samples.
- Because of this substitution, the t-test will not follow a Normal(0, 1) distribution. It is *more* variable than the standard Normal. Thus, we need a distribution that is like the standard Normal but a little bit wider.

Variability and sample size

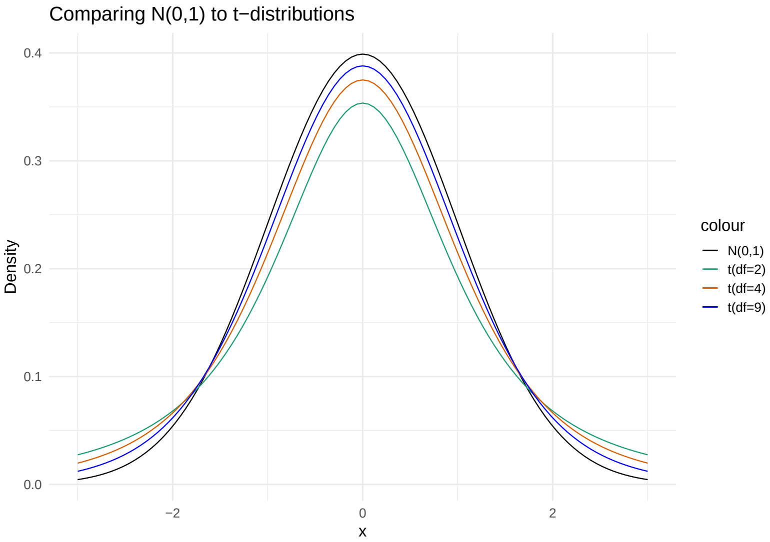
What do we know about our estimate of mean and variability as the sample size grows?

Introducing the t distribution

- Like the standard Normal distribution, but wider.
- It's width depends on n, the sample size which determines the degrees of freedom

This is because as n increases, our estimate s gets better and better, and approaches σ. Thus, as n increases the t-distribution approaches a Normal(0, 1) distribution.

Introducing the t distribution



Meet the t-test!

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

The one-sample t statistic has a t distribution with $n - 1$ degrees of freedom

You calculate the t statistic using $\bar{x}$ and s estimated from your sample, and n which is also a property of your sample, and μ_0 from the null hypothesis. Then compute the probability of observing a value of t or more extreme.

Assumptions of the t-test

- The dependent variable must be continuous (interval/ratio).
- The observations are independent of one another.
- Data come from a random sample of the underlying population.
- The dependent variable should be approximately normally distributed.
- The dependent variable should not contain any outliers.

Guess the R functions

```
pt(q = , df = , lower.tail = )
qt(p = , df = , lower.tail = )
```

Which one would we use to calculate the p-value for a hypothesis test after we calculated the t-test statistic? `pt` or `qt`?

Confidence intervals based on t

Calculating a confidence interval for the t-test

Draw an SRS of size n from a large population having unknown mean μ and unknown standard deviation σ. A level C confidence interval for μ is:

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

where t^* is the critical value for the $t(n-1)$ density curve with area C between $-t*$ and $t*$.

Supposing we had $n = 100$, what is t^* for a 95% confidence interval?

```
qt(p = 0.975, df = 99)
```

Example: Testosterone and obesity in adolescent males (pg 422 B&M Ed 4)

Here are the data for $n = 25$ adolescent males between the ages of 14 and 20:

```
library(dplyr)
testosterone <- c(0.30, 0.24, 0.19, 0.17, 0.18, 0.23, 0.24, 0.06, 0.15,
                  0.17, 0.18, 0.17, 0.15, 0.12, 0.25, 0.25, 0.25, 0.32,
                  0.35, 0.37, 0.39, 0.46, 0.49, 0.42, 0.36)
dat_test <- data.frame(testosterone)
```

Example: Testosterone and obesity in adolescent males (pg 422 B&M Ed 4)

Use R to calculate a 95% confidence interval for testosterone. We can do this using `summarize`

```
dat_test %>% summarize(sample_mean = mean(testosterone),
                       sample_sd = sd(testosterone),
                       sample_size = length(testosterone),
                       sample_se = sample_sd/sqrt(sample_size))
```

```
##   sample_mean sample_sd sample_size  sample_se
## 1      0.2584 0.1115303          25 0.02230605
```

Example: Testosterone and obesity in adolescent males (pg 422 B&M Ed 4)

We still need the t^* value:

```
t_star <- qt(p = 0.975, df = 24)
t_star
```

```
## [1] 2.063899
```

Example: Testosterone and obesity in adolescent males (pg 422 B&M Ed 4)

Expand the previous code chunk to calculate the margin of error (which uses the critical t^* value), and then calculate the lower and upper CI

```
dat_test %>% summarize(sample_mean = mean(testosterone),
                       sample_sd = sd(testosterone),
                       sample_size = length(testosterone),
                       sample_se = sample_sd/sqrt(sample_size),
                       margin_of_error = sample_se*t_star,
                       lower_CI = sample_mean - margin_of_error,
                       upper_CI = sample_mean + margin_of_error)
```

```
##   sample_mean sample_sd sample_size  sample_se margin_of_error  lower_CI
## 1      0.2584 0.1115303          25 0.02230605      0.04603743 0.2123626
##    upper_CI
## 1 0.3044374
```

Hypothesis testing with unknown σ using the t-test

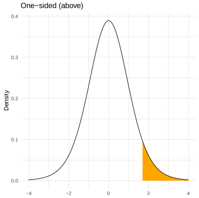
Draw an SRS of size n from a large population having unknown mean μ and unknown standard deviation σ. To test the hypothesis $H_0 : \mu = \mu_0$, calculate the t statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Hypothesis testing with unknown σ using the t-test

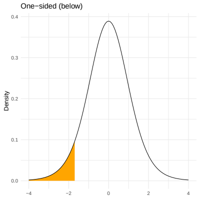
In terms of a variable T having the $t(n-1)$ distribution, the p-value for a test of H_0 against

H_a: $\mu > \mu_0$ is $P(T \geq t)$



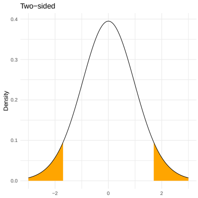
Hypothesis testing with unknown σ using the t-test

H_a: $\mu < \mu_0$ is $P(T \leq t)$



Hypothesis testing with unknown σ using the t-test

H_a: $\mu \neq \mu_0$ is $2 \times P(T \geq |t|)$



Example and t-testing in R

Example of a t-test (pg 426 B&M Ed 4)

Here are 18 measures of pulse wave velocity (PWV) from a sample of children diagnosed with progeria, a genetic disorder that produces rapid aging.

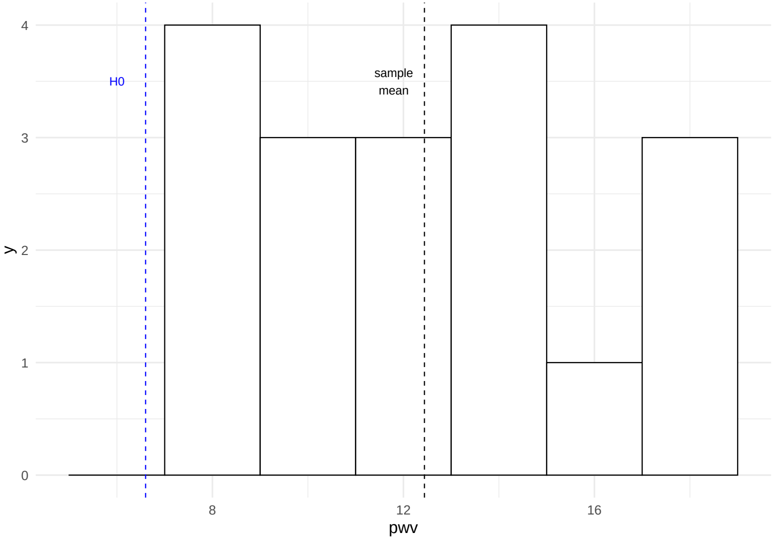
```
pwv <- c(18.8, 17.6, 17.5, 16.0, 14.8, 14.1, 13.7, 13.1, 12.9,
         12.9, 12.4, 10.1, 9.3,  9.1,  8.3,  8.3,  7.9,  7.2)

pwv_dat <- data.frame(pwv)
```

Example of a t-test (pg 426 B&M Ed 4)

pwv measures greater than 6.6 are considered abnormally high. We would like to test the hypothesis that the mean for this subset of children is abnormally high.

That is: $H_0 : \mu = 6.6$ and $H_a : \mu > 6.6$

Look at the data and see if there is evidence against the null hypothesis



Example of a one-sided t-test (pg 426 B&M Ed 4)

```r
pwv_dat %>%
  summarize(sample_mean = mean(pwv),
            sample_sd = sd(pwv),
            sample_size = length(pwv),
            sample_se = sample_sd/sqrt(sample_size),
            t_test = (sample_mean - 6.6)/sample_se,
            p_value = 1 - pt(t_test, df = sample_size - 1))
```

```
##   sample_mean sample_sd sample_size sample_se   t_test      p_value
## 1    12.44444  3.637747          18 0.8574252 6.816273 1.501248e-06
```

There's a function for that...

Rather than doing the test using summarize, we could have R do it for us using `t.test`:

```r
t.test(x = pwv_dat %>% pull(pwv), alternative = "greater", mu = 6.6)
```

```
##
##  One Sample t-test
##
## data:  pwv_dat %>% pull(pwv)
## t = 6.8163, df = 17, p-value = 1.501e-06
## alternative hypothesis: true mean is greater than 6.6
## 95 percent confidence interval:
##  10.95286      Inf
## sample estimates:
## mean of x
##  12.44444
```

Robustness of t procedures

- A confidence interval or hypothesis test is called robust if the confidence level or P-value does not change very much when the conditions for use of the procedure are violated.
- In particular, how robust are the procedures against non-Normality?
- The t procedures are quite robust against non-Normality of the population except when outliers or strong skewness are present.
- The t procedures are not robust against outliers unless the sample size is sufficiently large.

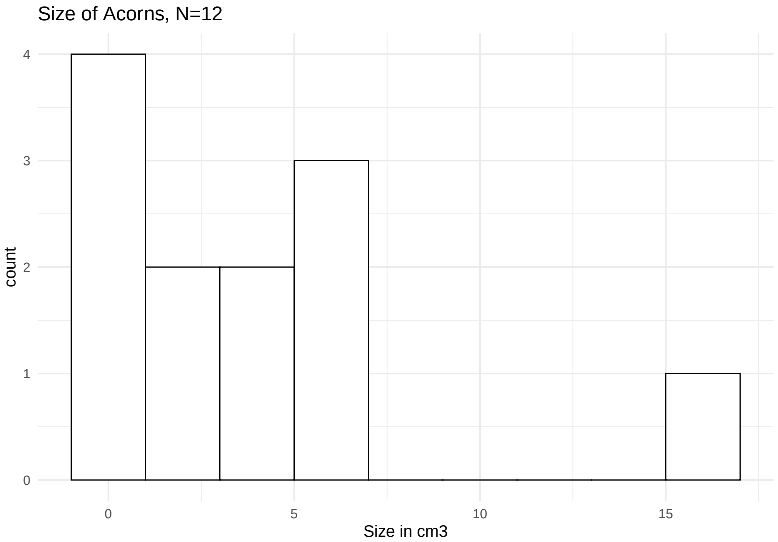
Checking assumptions

- Always plot your data first:
  - Are there any outliers
  - Is the distribution of the data skewed?

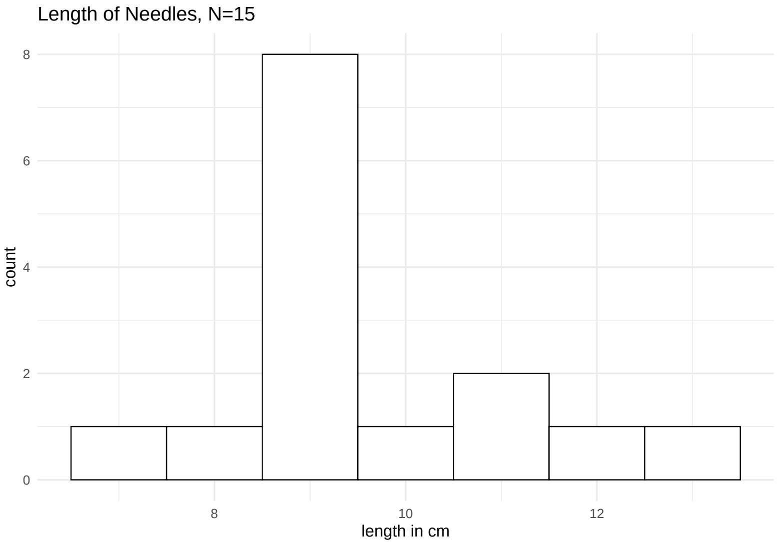
Guidelines for using the t procedures

- The SRS condition is more important that the Normality condition
- If n $<$ 15: Use t procedures if the data appear close to Normal (at least roughly symmetric, single peak, no outliers). If the data are skewed or there are outliers, don't use t.
- Moderate sample size $>$ 15: The t procedures can be used except in the presence of outliers or strong skewness
- Large sample size, roughly $n \geq 40$: The t procedures can be used even for strongly skewed distributions when the sample is large, roughly $n \geq 40$

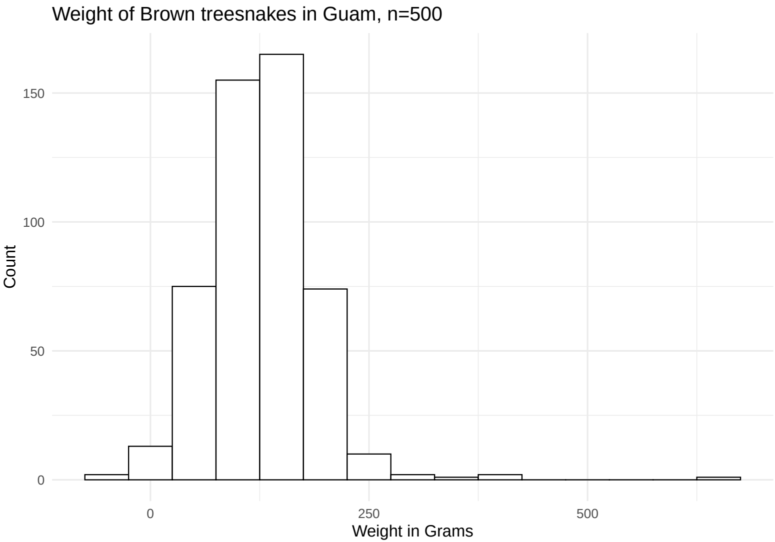
Example 17.5: Can we use t?



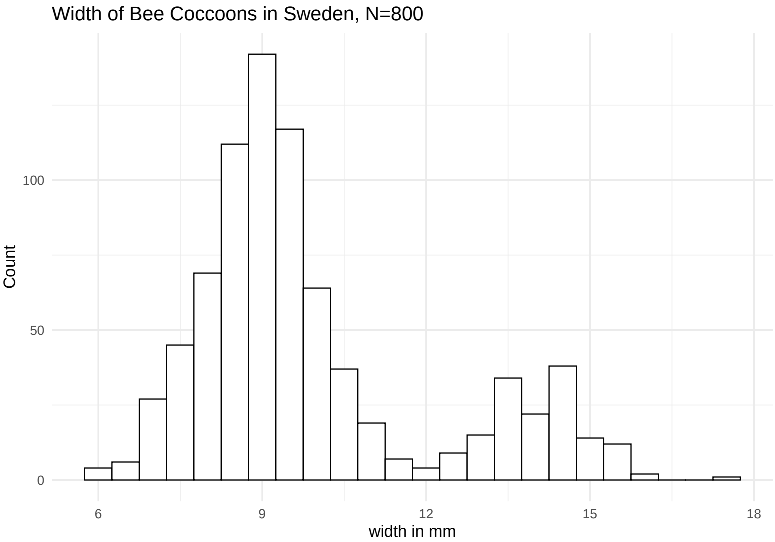
Example 17.5: Can we use t?



Example 17.5: Can we use t?



Example 17.5: Can we use t?



Parting Humor