L10: More Probability

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Learning Objectives

- General addition rule for probability
- Conditional probability
- Determine whether two events are independent or dependent
- General multiplication rule for probability
- Introducing tree diagrams and frequencies (tables) as tools
- The prosecutor's fallacy
- Bayes Theorum
- Recap of key probability rules

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

General addition rule

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

For any two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

- Why do we subtract off $P(A \cap B)$?
- This formula simplifies to $P(A \cup B) = P(A) + P(B)$ when A and B are disjoint.
- What does the Venn diagram look like for disjoint events?

Decomposition of a probability

For any two events A and B, $P(A) = P(A \cap B) + P(A \cap \bar{B})$

Which pieces of our diagram does this indicate?

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Conditional probability

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Statistics is everywhere

You may have heard people talk about the probability of dying from a bee sting being greater than the probability of being killed by a shark. As in this recent article, published during "shark week"

ENTERTAINMENT

Shark week: You're way more likely to die from these than a shark attack

Here are seven things more likely to kill you than sharks.

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Statistics is everywhere

From the article:

Your odds of a dog killing you are 1 in 112,400, Your odds of dying from one of these insects touching you is 1 in 63,225 Your odds of dying from a fired gun are 1 in 6,905 Your odds of dying from lightning are 1 in 161,856 Your odds of dying while in a car are 1 in 114

compared to about 1 in 3,748,067 for a shark attack.

It's worth noting that these are probabilities(risks) NOT odds in the statistical sense

People use these probabilities to argue that swimming in the ocean is safe. Why do conditional probabilities make this argument questionable?

Conditional probability

Conditional Probability

When $P(A) > 0$, the conditional probability of B, given A is:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

- Rearrange this formula for $P(A \cap B)$

$$P(A|B) = \frac{P(B \cap A)}{P(B)}$$

- What pieces of a Venn Diagram would these correspond to?

Conditional probability JUUL and vaping example

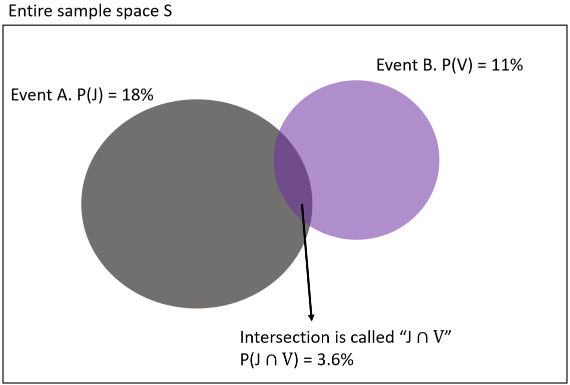
Among those who have seen an ad for JUUL, what percent vaped in the past month?

$$P(Vape|JUUL)$$

$$P(V|J) = \frac{P(J \cap V)}{P(J)}$$

Conditional probability

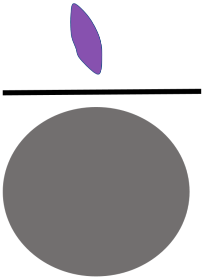
A basic Venn diagram



Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Conditional probability

What percent of individuals vape given they have seen an ad for JUUL?



L10: More Probability

Conditional probability

Independent vs. dependent events

Multiplication rules

Conditional probability and diagnostic testing

Tools for complex proability calculations: Trees and Absolutes

Tree diagrams

Absolute frequencies

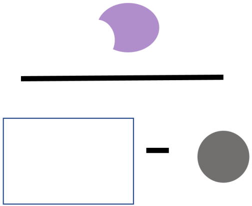
The prosecutor's fallacy

Bayes' theorem

Conditional probability

$$P(V|J') = \frac{P(J' \cap V)}{P(J')}$$

What percent of individuals vape given they have NOT seen an ad for JUUL?



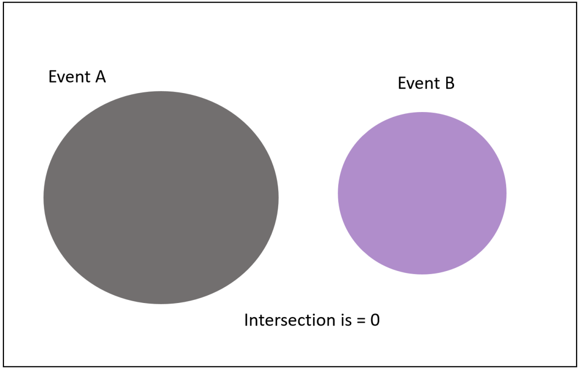
Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Recall the definition for disjoint events

- What did it mean for two events to be disjoint?
- Another term for disjoint is mutually exclusive
- How would we draw disjoint events in a Venn diagram?

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Disjoint events



Independent vs. dependent events

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Independent vs. dependent events

What sorts of events are independent?

Two events are independent if knowing that one event occurred does not change the probability that the other occurred

Independent vs dependent: example 1. Down syndrome

Down syndrome is a genetic disorder caused when abnormal cell division results in an extra full or partial copy of chromosome 21.[1] The largest risk factors for having a child with Down syndrome are advanced maternal age.[1] Suppose that Martha is 40 and her baby has been diagnosed with Down syndrome. Martha's best friend Jane, also 40, is hoping to conceive. Is her baby's risk of Down syndrome independent of Martha's baby's risk?

1. https://www.mayoclinic.org/diseases-conditions/down-syndrome/symptoms-causes/syc-20355977

Independent vs. dependent events

Written out in probability notation, for any two events A and B, the events are independent if:

$$P(A|B) = P(A)$$

or

$$P(B|A) = P(B)$$

or

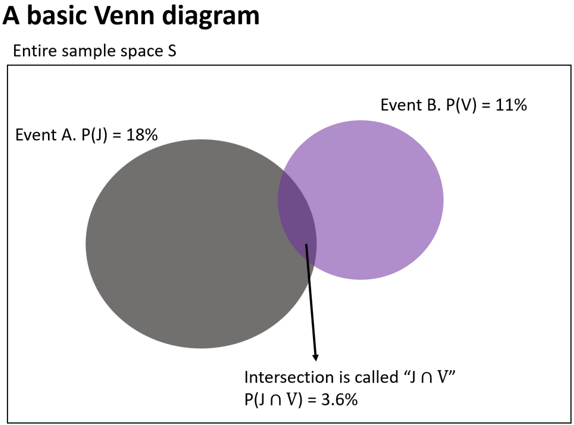
$$P(A \cap B) = P(A) * P(B)$$

The "|" is read as "given" or "conditional on"

If one of these is true, they are all true.

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Independent vs. dependent



In our example, are the probability of seeing and ad for JUUL and vaping in the past month independent?

Independent vs. dependent events

From our JUUL and vaping example:

$$P(V|J) = \frac{P(V \cap J)}{P(J)} = \frac{.036}{.18} = .20$$

and

$$P(V) = 0.11$$

so

$$P(V|J) \neq P(V)$$

proving that we have events that are NOT independent, knowing someones exposure to JUUL ads gives us information about their probability of having vaped.

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Multiplication rules

Multiplication rule

For any two events, the probability that both events occur is given by:

$$P(A \cap B) = P(B|A) \times P(A)$$

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Multiplication rule for independent events

Two events A and B are independent if knowing that one occurs does not change the probability that the other occurs.

If events are independent, $P(B|A) = P(B)$ so the general multiplication rule:

$$P(A \cap B) = P(B|A) * P(A)$$

Simplifies to:

$$P(A \cap B) = P(A) \times P(B)$$

Conditional Probability using tables

Ad exposure	Vape	No Vape	Total
seen an ad for JUUL	0.036	0.144	0.18
did not see ad for JUUL	0.074	0.746	0.82
Total	0.11	0.89	1

What are the conditional probabilities of vaping by Ad exposure?

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Conditional probability and diagnostic testing

Conditional probability and diagnostic testing

Key terms:

- Sensitivity: P(test positive | truly have disease)
- Specificity: P(test negative | truly do not have disease)
- Positive predictive value: P(truly have disease | test positive)
- Negative predictive value: P(truly do not have disease | test negative)

Conditional probability and diagnostic testing

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

More key terms:

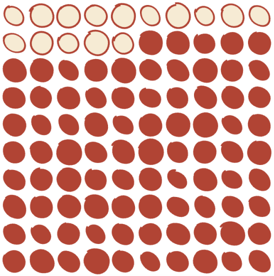
	Have the disease	Do not have the disease
Test positive	True positive	False positive
Test negative	False negative	True negative

Statistics is everywhere: prenatal testing

The New York Times

When They Warn of Rare Disorders, These Prenatal Tests Are Usually Wrong

Some of the tests look for missing snippets of chromosomes. For every 15 times they correctly find a problem ...



Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Statistics is everywhere: prenatal testing

"The tests initially looked for Down syndrome and worked very well. But as manufacturers tried to outsell each other, they began offering additional screenings for increasingly rare conditions... Nonetheless, on product brochures and test result sheets, companies describe the tests to pregnant women and their doctors as near certain. They advertise their findings as "reliable" and "highly accurate," offering "total confidence" and "peace of mind" for patients who want to know as much as possible."

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Statistics is everywhere: prenatal testing

THE UPSHOT | When They Warn of Rare Disorders, These Prenatal Tests Are Usually Wrong

As prenatal tests have expanded to more rare conditions, a larger share of their positive results are incorrect. Some of the worst-performing tests look for microdeletions, which are small missing snippets of chromosomes.

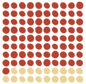
AIBE-D-22-00448.pdf

Chance positive results are wrong

DiGeorge syndrome

Affects 1 in 4,000 births

Can cause heart defects and delayed language acquisition. (May appear on lab reports as "22q.")

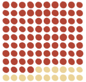


81% wrong

1p36 deletion

1 in 5,000 births

Can cause seizures, low muscle tone and intellectual disability.

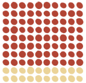


84% wrong

Cri-du-chat syndrome

1 in 15,000 births

Can cause difficulty walking and delayed speech development.

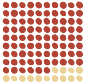


80% wrong

Wolf-Hirschhorn syndrome

1 in 20,000 births

Can cause seizures, growth delays and intellectual disability.



86% wrong

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

How to calculate the chance of having condition if you test positive?

- The article gives the probability of NOT having the disease given that the test is positive, or the false positive proportion - for DiGeorge syndrome this is given as 81%.
- We don't have all of the information needed in the article to calculate all of the conditional probabilities, but if we imagine that the test has a 96% sensitivity and a 99.9% specificity we can look at how we could get to such a high proportion of false positives

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Example: How to calculate the chance of having DiGeorge syndrome if you test positive?

	Have Cancer	Do not have Cancer	Total
Test positive	True positive	False positive	
Test negative	False negative	True negative	
Total			

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Example: How to calculate the chance of having DiGeorge syndrome if you test positive?

96% chance of testing positive when the individuals it 99.9% chance of testing negative when the patient does not have it. 1 in 4000 births have this syndrome, or 0.025% this is one of the least rare conditions mentioned in the article.

	Have DiGeor	Do not have DiGeorge	Total
Test positive	24	100	124
Test negative	1	99875	99876
Total	25	99975	100000

*note that I have rounded the numbers is the non-disease column slightly

Conditional probability and diagnostic testing

What is the chance that a patient has cancer given that they test positive?

- Positive predictive value: P(truly have disease | test positive)
- P(truly have disease | test positive)$= 24/124 = 19.4\%$
- P(do not have disease | test positive) $= 100/124 = 80.6\%$

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Tools for complex proability calculations: Trees and Absolutes

Example: Unintended pregnancies by maternal age group (pg 257)

The question: Birth certificates show that approximately 9% of all births in the US are to teen mothers (aged 15-19), 24% to young-adult mothers (ages 20-24) and the remaining 67% to adult mothers (aged 25-44). A survey found that only 23% of births to teen mothers are intended. Among births to young adult women, 50% are intended, and among women aged 25-44 75% are intended

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Define events using probability notation

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

The first step in probability questions is to translate the written text into probability statements.
Define our notation: Let M denote the age of the mother and B denote whether the birth was intended.

- $P(M = \text{teen}) = 0.09$
- $P(M = \text{young adult}) = 0.24$
- $P(M = \text{older adult}) = 0.67$
- $P(B = \text{intended} \mid M = \text{teen}) = 0.23$
- $P(B = \text{intended} \mid M= \text{young adult}) = 0.5$
- $P(B = \text{intended} \mid M = \text{older adult}) = 0.75$

Example: Unintended pregnancies by maternal age group (pg 257)

What if we want to know the probability that any given live birth in the U.S. is unintended?

- rewrite this question as a probability statment

We will cover two strategies for answering this question: - Using tree diagrams - Using absolute frequencies (not covered in your book)

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Tree diagrams

Tree diagrams

- Tree diagrams can be used to perform complex probability calculations
- Tree diagrams place conditional probabilities down the branch of the tree and multiply them to obtain the probability of two (or more) events occurring.

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

The tree diagram for these calculations

Getting to the answer

What if we want to know the probability that any given live birth in the U.S. is unintended?
We add up all of the “branches” that contain Unintended pregnancy:

$P(B{=}\text{unintended}) = P(B = \text{unintended} \cap M = \text{teen}) +$
$P(B{=}\text{unintended} \cap M{=} \text{younger adult}) +$
$P(B = \text{unintended} \cap M = \text{older adult}) = 35.7\%$

$P(B{=}\text{unintended}) = 0.0693 + 0.12 + 0.1675 = 35.7\%$

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor’s fallacy
Bayes’ theorem

Absolute frequencies

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Absolute frequencies

- Another method for thinking about these kinds of complex probabilities is to use absolute frequencies
- We make a table of the probabilities in an imaginary population
- this may be more intuitive for most people.

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Calculations using absolute frequencies

Pretend there are 1000 women. Given that 9%, 24%, and 67% of the mothers are teens, younger, and older mothers (respectively) this means that out of the 1000:

- 90 are teens,
- 240 are younger mothers, and,
- 670 are older mothers.

These are the marginal values of your table

Pregnancy	Teens	Younger	Older	Total
Intended				
Unintended				
Total	90	240	670	1000

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Calculations using absolute frequencies

Now, *conditional* on being a teen, 23% of the pregnancies are intended. This means that 90*23% $=$ 20.7 teen mothers had intended pregnancies. We can calculate these joint probabilities for each age group

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Calculations using absolute frequencies

- 90 are teens, $90 \times 23\% = 20.7$ teens with intended pregnancies (69.3 teens with unintended pregnancies).
- 240 are younger mothers, $240 \times 50\% = 120$ younger mothers with intended pregnancies.
- 670 are older mothers, $670 \times 75\% = 502.5$ older mothers with intended pregnancies.

Pregnancy	Teens	Younger	Older	Total
Intended	20.7	120	502.5	
Unintended				
Total	90	240	670	1000

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Calculations using absolute frequencies

Now since we know the total and we know how many in each group had intended pregnancies, we can find the marginal total of Intended pregnancies.

Pregnancy	Teens	Younger	Older	Total
Intended	20.7	120	502.5	
Unintended				
Total	90	240	670	1000

Thus the number of total intended pregnancies is $20.7 + 120 + 502.5 = 643.2$. Therefore, approximately 64% of all pregnancies are intended. Subtracting from 1000 we now know that 356.8 pregnancies were unintended - so 35.7% of pregnancies were unintended.

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

The prosecutor's fallacy

Statistics is Everywhere

from - Slate.com article, August 2018:

Consider this case from 1964: In the wake of a robbery in Los Angeles, someone reported seeing a blond woman with a ponytail commit the deed before jumping into a yellow getaway car driven by a bearded and mustachioed black man.

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Statistics is Everywhere

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Malcolm Collins and his wife Janet Collins, who matched the descriptions but could not be identified by the eyewitnesses, were initially convicted of the crime. The conviction hinged largely on the testimony of a mathematics instructor at a local state college who made the damning assertion that the probability of the Collins' innocence was 1 in 12 million. He came to that number by simply multiplying together some rough guesses for the probabilities of each of the separate attributes: About 1 in 4 men had a mustache. About 1 in 10 women had a ponytail. And so forth.

Prosecutors' fallacy

but:

- Even if a concordance of all those attributes (blond hair, ponytail, yellow car, etc.) is unlikely, innocence isn't necessarily equally unlikely.

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Prosecutors' fallacy and conditional probability

consider the following:

- P(evidence|innocence)
- P(innocence|evidence)

Prosecutors often point to low values of P(evidence|innocence) for example, "it would be very unlikely for us to have this evidence if the defendant were innocent"

But as we saw in the Collins case, a low value of P(evidence|innocence) does not necessarily imply a low value of P(innocence|evidence)

Prosecutors' fallacy

A more revealing statistic was computed in the appeal: The probability that there were two couples in the Los Angeles area that both matched the description was 40 percent.

Fortunately:

The Collins case had a relatively happy ending. Their case was appealed, the system corrected its statistical misstep, and the case became a standard example in legal pedagogy for the misuse of statistical evidence.

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Bayes' theorem

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Bayes' theorem

If we know the conditional probability for B given A: $P(B|A)$ and we want to know the opposite - the conditional probability of A given B: $P(A \mid B)$ we can use Bayes' theorem to get the probability we want.

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Bayes' theorem (simple version)

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Suppose that A and A^c are disjoint events whose probabilities are not 0 and add exactly to 1. That is, any outcome has to be exactly in one of these events. Then if B is any other event whose probability is not 0 or 1,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

6 min video about Baye's theorem

Bayes' theorem (simple version)

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

How did we end up with the formula?

1. First recall that $P(A|B) = \frac{P(A\&B)}{P(B)}$ by conditional probability rule
2. Also: $P(B|A) = P(A\&B)/P(A)$, which implies $P(A\&B) = P(B|A) \times P(A)$
3. Plugging (2) into (1): $\mathrm{P(A|B)} =$

Bayes' theorem (simple version)

3. $P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$
4. Now think about P(B): $P(B) = P(B\&A) + P(B\&A^c) = P(B|A) \times P(A) + P(B|A^c) \times P(A^c)$
5. Plug in (4) into (3):

Bayes' theorem (simple version)

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B|A) \times P(A) + P(B|A^c) \times P(A^c)}$$

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Bayes' theorem example: HIV testing

suppose we randomly select an individual living in South Africa

- we test the individual for HIV
- the prevalence of HIV in South Africa is 20%
- the sensitivity of our testing method is 85%, ie $P(T+|HIV+) = 0.85$
- the specificity of our testing method is 60%, ie $P(T-|HIV-) = 0.60$
- what is the positive predictive value of this testing method?

PPV as a conditional probability

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Let's look at this as a tree

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Now let's look at this in a table format

Bayes' theorem (generalized)

- Rather than only having A and A^c, suppose you had the events A_1, and $A_2, \ldots,$ through to A_k as disjoint events whose probabilities are not 0 or 1 That is, any outcome has to be exactly in one of these events. Then if B is any other event whose probability is not 0 or 1,

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + ... + P(B|A_k)P(A_k)}$$

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Review of probability rules

Probabilities are numbers between 0 and 1.

$0 \leq P(A) \leq 1$

The probabilities in the probability space must sum to 1.

The probabilities of an event and it's complement must sum to 1

$P(A) + P(\bar{A}) = 1$

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Adding and decomposing probability

For any two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

For any two events A and B, $P(A) = P(A \cap B) + P(A \cap \bar{B})$

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Rules for independence

Written out in probability notation, for any two events A and B, the events are independent if:

$$P(A|B) = P(A)$$

or

$$P(B|A) = P(B)$$

or

$$P(A \cap B) = P(A) * P(B)$$

Multiplication rule and conditional probability

For any two events, the probability that both events occur is given by:

$$P(A \cap B) = P(B|A) \times P(A)$$

When $P(A) > 0$, the conditional probability of B, given A is:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Peter Backus application of Drake's equation to dating

"Why I don't have a Girlfriend"

Number of potential girlfriends=Population of UK X P(Woman) X P(London) X P(age appropriate) X P(University education) *P(Subjectively attractive to Peter)

Or 60,975,000 X (.51) X (.13) X (.20) X (.26) X (0.05)

= 10,510

Further estimating P(woman finds peter subjectively attractive) X P(Single) X P(get along)

His conclusion is: "there are 26 women in London with whom I might have a wonderful relationship"

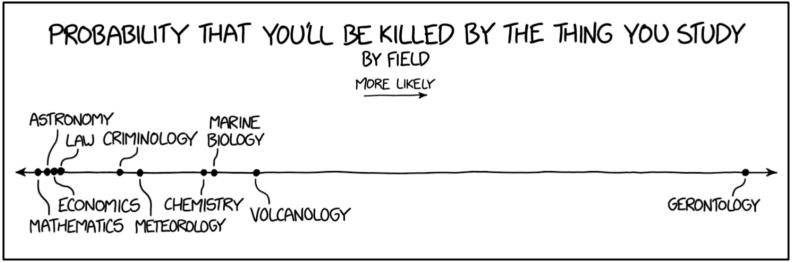
What assumption is he making in order to come to this conclusion?

full article here:

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem

Parting humor

from XKCD



L10: More Probability

Conditional probability
Independent vs. dependent events
Multiplication rules
Conditional probability and diagnostic testing
Tools for complex proability calculations: Trees and Absolutes
Tree diagrams
Absolute frequencies
The prosecutor's fallacy
Bayes' theorem