L05: Relationships between two categorical variables

Learning objectives for today

Today we will focus on how to visualize and quantify relationships between two categorical variables

- Two way tables
  - marginal vs conditional distributions
- Bar graphs
  - side by side
  - stacked
- Simpson's paradox

Reminder

Categorical variables are just that, categories.

These can be nominal (no underlying order)

or

ordinal (ordered)

Two-way tables

- Two-way, or 2X2 (for a table with two columns and two rows)
  - Used to examine the relationship between 2 categorical variables, originally for those with two levels each
- Foundational to epidemiology, because of the types of variables we are often interested in

Classic 2X2 looks like this:

Exposure group	Disease	No disease	Row total
Exposed	A	B	A+B
Not Exposed	C	D	C+D
Column total	A+C	B+D	Total # observations

Example: Lung cancer and smoking

Group	Lung Cancer	No Lung Cancer	Row total
Smoker	12	238	250
Non-smoker	7	743	750
Column total	19	981	1000

Marginal distributions

- The marginal distribution of a variable is the one that is in the margin of the table (i.e., the Row total or the Column total are the two margins of a two-way table).
- The marginal distribution is the distribution for a single categorical variable
- We learned in Ch. 1 how to plot marginal distributions of categorical variables using `geom_bar()`

Marginal distributions

Group	Lung Cancer	No Lung Cancer	Row total
Smoker	12	238	250
Non-smoker	7	743	750
Column total	19	981	1000

- Overall, what % of the population has lung cancer?
- Overall, what % of the population are smokers?

Marginal distributions

Group	Lung Cancer	No Lung Cancer	Row total
Smoker	12	238	250
Non-smoker	7	743	750
Column total	19	981	1000

- Overall, what % of the population has lung cancer?
  - Answer: $19/1000 = 1.9\%$
- Overall, what % of the population are smokers?
  - Answer: 250/1000 25% smoking
- The marginal distribution of lung cancer is 1.9% lung cancer, 98.1% no lung cancer.

Conditional distributions

Group	Lung Cancer	No Lung Cancer	Row total
Smoker	12	238	250
Non-smoker	7	743	750
Column total	19	981	1000

- The conditional distribution is the distribution of one variable within or conditional on the level of a second variable
- What is the distribution of lung cancer conditional on the individuals being smokers?
- What is the conditional distribution of lung cancer given individuals are non-smoking?

Conditional distributions

Group	Lung Cancer	No Lung Cancer	Row total
Smoker	12	238	250
Non-smoker	7	743	750
Column total	19	981	1000

- The conditional distribution of lung cancer given smoking is: - $12/250 = 4.8\%$ smokers and $238/250 = 95.2\%$
- The conditional distribution of lung cancer given non-smoking is: - $7/750 = 0.9\%$ smokers and $743/750 = 99.1\%$ non-smokers

Visualizing categorical variables in R

Marginal and Conditional distributions in R

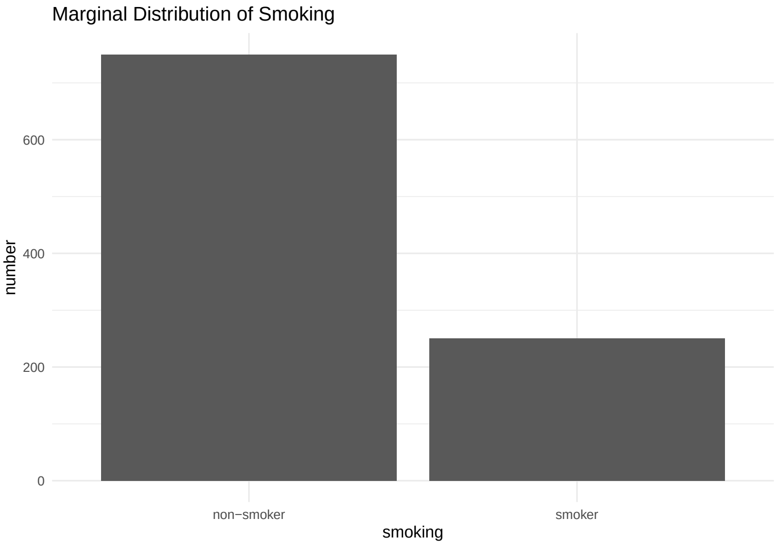
- We learned in Ch.1 how to plot marginal distributions of categorical variables using `geom_bar()`
- Can we generalize our use of `geom_bar()` to allow us to plot multiple conditional distributions? I.e., can we show the conditional distribution of lung cancer for smokers and non-smokers on the same plot?

Visualization in R

First, we encode the data to read into R:

```
library(dplyr)
# students, you don't need to know how to do this
two_way_data <- tribble(~ smoking, ~ lung_cancer, ~ percent, ~number,
                     "smoker", "lung cancer", 4.8, 12,
                     "smoker", "no lung cancer", 95.2,238,
                     "non-smoker", "lung cancer", 0.9, 7,
                     "non-smoker", "no lung cancer", 99.1, 743)
```

Bar chart for the visualization of marginal distributions



Conditional distributions

"If there is an explanatory-response relationship, compare the conditional distribution of the response variable for the separate values of the explanatory variable."

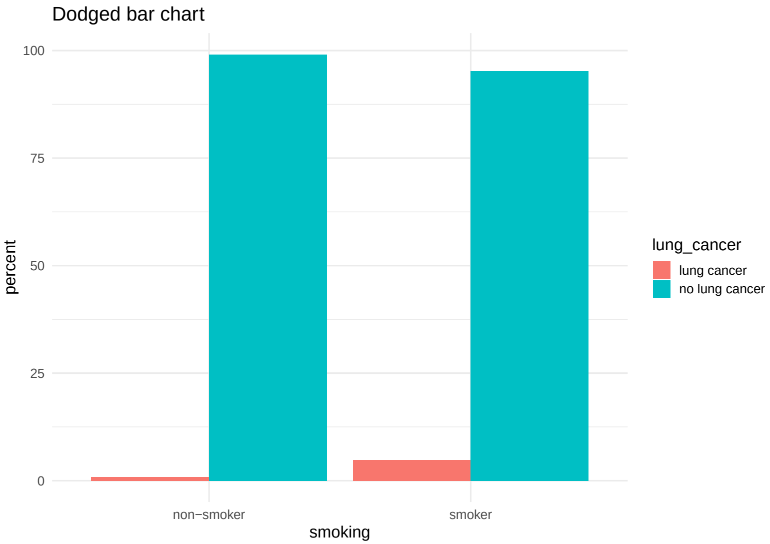
- This allows you to visualize the distribution of the response variable for varying levels of the exposure variable.

Dodged bar chart for the visualization of conditional distributions

Syntax:

```
ggplot(two_way_data, aes(x = smoking, y = percent)) +
geom_bar(aes(fill = lung_cancer), stat = “identity”, position = “dodge”) +
labs(title = “Dodged bar chart”) + theme_minimal(base_size = 15)
```

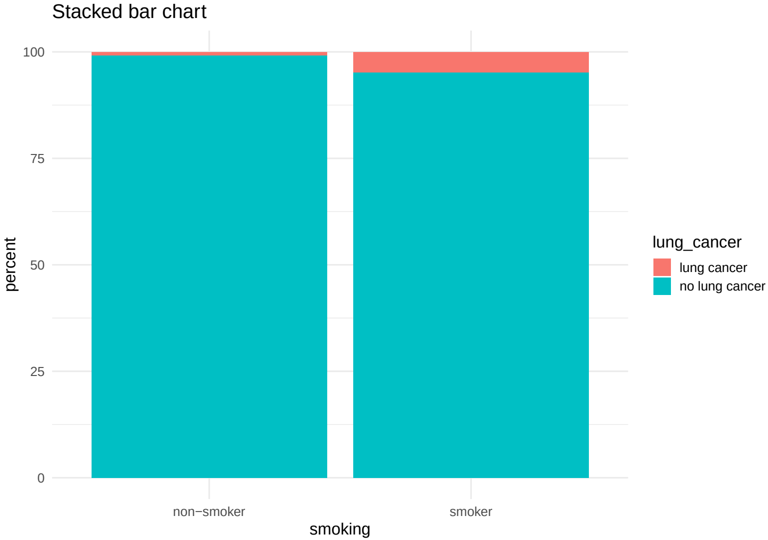
Dodged bar chart for the visualization of conditional distributions



Stacked bar chart for the visualization of conditional distributions

```
ggplot(two_way_data, aes(x = smoking, y = percent)) +
geom_bar(aes(fill = lung_cancer), stat = “identity”, position = “stack”) +
labs(title = “Stacked bar chart”) + theme_minimal(base_size = 15)
```

Stacked bar chart for the visualization of conditional distributions



Visualization of conditional distributions: three levels of response variable

- Plots like the one above make less sense when there are only two levels of both of the variables. This is because once you know the percent of lung cancer among smokers, you also know the percent of non-lung cancer among smokers.
- Here is another example with 3 levels: Shoe support by gender (from ch. 5):

Group	Men	Women
Good support	94	137
Average support	1348	581
Poor support	30	1182
Column total	1472	1900

- The question: How does the distribution of support of shoes worn vary between shoes made for men and women?

Visualization of conditional distributions: three levels of response variable

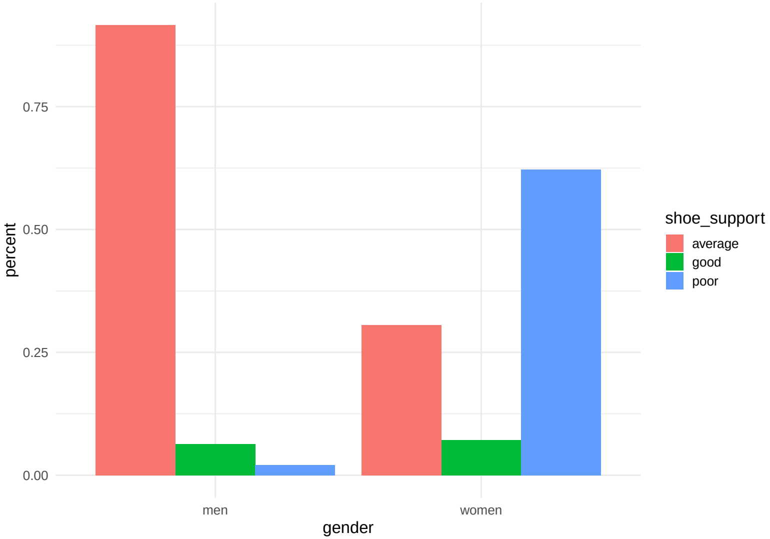
Example using shoe support data from Baldi & Moore page 124 of Ed.4

```
## # A tibble: 6 x 3
##   shoe_support gender percent
##   <chr>        <chr>    <dbl>
## 1 good         men     0.0639
## 2 average      men     0.916
## 3 poor         men     0.0204
## 4 good         women   0.0721
## 5 average      women   0.306
## 6 poor         women   0.622
```

Visualization of conditional distributions: three levels of response variable

```
ggplot(shoe_data, aes(x = gender, y = percent)) +
geom_bar(stat = “identity”, aes(fill = shoe_support), position = “dodge”) +
theme_minimal(base_size = 15)
```

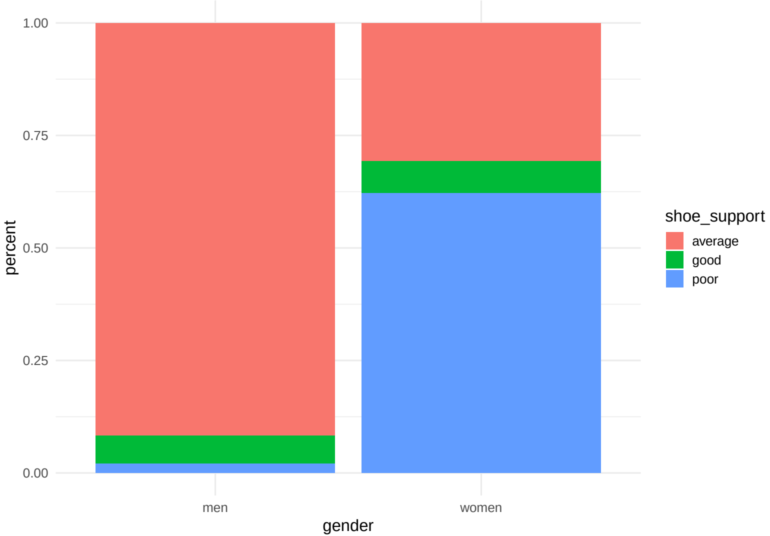
Visualization of conditional distributions: three levels of response variable



Visualization of conditional distributions: three levels of response variable

```
ggplot(shoe_data, aes(x = gender, y = percent)) +
geom_bar(stat = “identity”, aes(fill = shoe_support), position = “stack”) +
theme_minimal(base_size = 15)
```

Visualization of conditional distributions: three levels of response variable



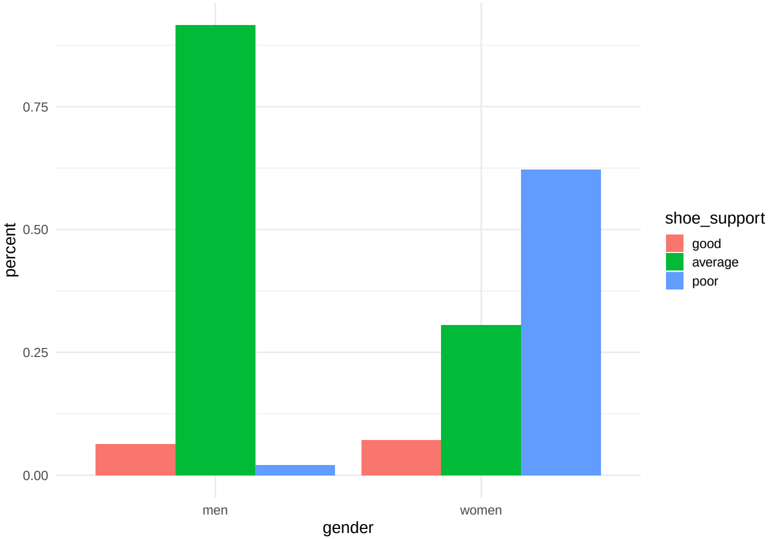
Visualization of conditional distributions: three levels of response variable

Recall from last class we learned how to reorder factor variables that affect the look of the plot:

```
shoe_data <- shoe_data %>% mutate(shoe_support =
fct_relevel(shoe_support, “good”, “average”, “poor”))
```

```
ggplot(shoe_data, aes(x = gender, y = percent)) + geom_bar(stat =
“identity”, aes(fill = shoe_support), position = “dodge”) +
theme_minimal(base_size = 15)
```

Visualization of conditional distributions: three levels of response variable



Visualization of conditional distributions: three levels of response variable

Why might we prefer dodged plots to stacked plots?

Simpson's Paradox

Simpson's Paradox: Example from Baldi and Moore

- Here is the data presented in your book to illustrate Simpson's paradox.
- It looks at mortality rates by community and age group for two communities

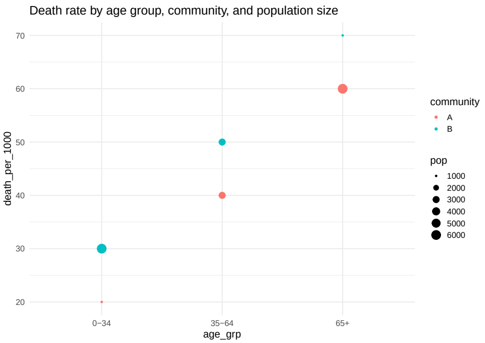
```
simp_data <- tribble(~ age_grp, ~ community, ~ deaths, ~ pop,
                     "0-34", "A", 20, 1000,
                     "35-64", "A", 120, 3000,
                     "65+", "A", 360, 6000,
                     "all", "A", 500, 10000,
                     "0-34", "B", 180, 6000,
                     "35-64", "B", 150, 3000,
                     "65+", "B", 70, 1000,
                     "all", "B", 400, 10000)
simp_data <- simp_data %>%
  mutate(death_per_1000 = (deaths/pop) * 1000)
simp_data_no_all <- simp_data %>% filter(age_grp != "all")
```

Simpson’s Paradox Example: Only Conditional data

Plot the mortality rates according to age group and community, linking size of dot to population size

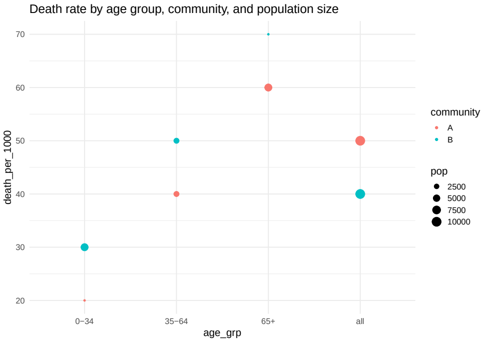
```
ggplot(simp_data_no_all, aes(x = age_grp, y = death_per_1000)) +
geom_point(aes(col = community, size = pop)) +
labs(title = “Death rate by age group, community, and population size”) +
theme_minimal(base_size = 15)
```

Simpson's Paradox Example: Only Conditional data



- What do we notice about mortality by age groups?
- Which community is larger?
- If someone ask you which community has higher mortality, what would you say?

Simpson’s Paradox Example: with marginal data



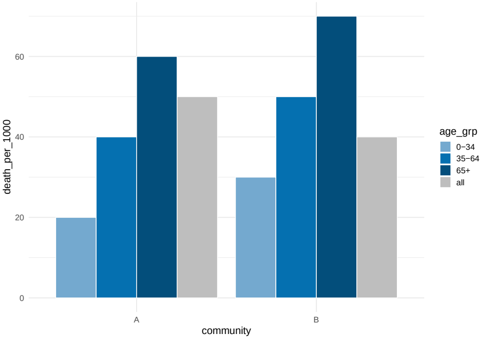
- Notice that the mortality rates for the communities overall show community A having a higher rate than community B. Why is that?

Simpson's Paradox

“An association or comparison that holds for all of several groups can reverse direction when the data are combined to form a single group. This reversal is called Simpson's Paradox”

Simpson's Paradox

- Here are the same data shown using a bar chart



Which visualization gives you more information?

Simpson's Paradox Berkeley example

A famous example of Simpson's paradox related to admissions to Berkeley by gender:

Watch: https://www.youtube.com/watch?v=E_ME4P9fQbo

Recap: Code and concepts

1. `geom_bar(aes(col = var), stat = "identity", position = "dodge")`
2. `geom_bar(aes(col = var), stat = "identity", position = "stack")`
3. Marginal vs conditional distributions
4. Simpson's Paradox

Comic Relief

First, I looked in the back of the book, but it wasn't an odd-numbered problem.
Then I asked my little brother, but he wanted me to pay him $5.
Finally, I found it on the Internet with Google.
MY MATH TEACHER WANTS US TO SHOW HOW WE GET OUR ANSWERS.
AH.
2-10