

= ChatGPT 5.2 Thinking ✓



Generate 5 multiple choice questions based on the attached pictures with increasing difficulty and let students choose the answers and then provide feedback one by one.

## Thought for 14s >

Alright—let's turn those “particle on a ring” slides into a mini boss-fight of understanding. We'll do this **one question at a time**: you answer (A/B/C/D), then I'll give feedback and unlock the next, harder one.

**Q1 (warm-up): Why must  $n$  be an integer?**

For a particle on a ring, the wavefunction is shown as

$$\psi_n(\phi) = \frac{1}{\sqrt{2\pi}} e^{in\phi}, \quad \text{with periodic boundary } \psi(\phi) = \psi(\phi + 2\pi).$$

Which statement best explains why  $n$  must be an integer?

- A) Because energies must be positive.
  - B) Because the wavefunction must be single-valued after going around the ring once.
  - C) Because  $\phi$  is measured in radians.
  - D) Because the potential energy  $V = 0$  everywhere.

**Your move:** Reply with A, B, C, or D.



B

 Q1 Feedback: Correct — B.

**Why:** The boundary condition on a ring is **periodicity**:

$$\psi(\phi + 2\pi) = \psi(\phi).$$

But

$$\psi(\phi + 2\pi) = \frac{1}{\sqrt{2\pi}} e^{in(\phi+2\pi)} = \psi(\phi) e^{i2\pi n}.$$

For this to equal  $\psi(\phi)$  for all  $\phi$ , we must have

$$e^{i2\pi n} = 1 \Rightarrow n \in \mathbb{Z}.$$

So integers come from **single-valuedness + periodic geometry**, not from "energies must be positive" (A), units (C), or  $V = 0$  (D).

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**Q2 (easy → medium): What is the allowed angular momentum?**

Given  $\psi_n(\phi) \propto e^{in\phi}$ , what are the allowed values of the ***z*-component of angular momentum  $L_z$**  for a particle on a ring?

- A)  $L_z = n\hbar$ , where  $n = 0, \pm 1, \pm 2, \dots$
- B)  $L_z = \frac{\hbar}{n}$ , where  $n = 1, 2, 3, \dots$
- C)  $L_z = \left(n + \frac{1}{2}\right)\hbar$ , where  $n = 0, 1, 2, \dots$
- D)  $L_z = \pm\hbar$  only

Reply with A, B, C, or D.



C

Not quite — the correct answer is A.

Why: On a ring, the angular momentum operator is

$$\hat{L}_z = -i\hbar \frac{d}{d\phi}.$$

Acting on  $\psi_n(\phi) \propto e^{in\phi}$ :

$$\hat{L}_z \psi_n = -i\hbar \frac{d}{d\phi}(e^{in\phi}) = -i\hbar(in)e^{in\phi} = n\hbar \psi_n.$$

So the allowed values are

$$L_z = n\hbar, \quad n = 0, \pm 1, \pm 2, \dots$$

Option C ( $n + \frac{1}{2}\hbar$ ) shows up in some *other* quantum situations (e.g., certain boundary-condition shifts or spin-related contexts), but **not** for the standard particle-on-a-ring with  $\psi(\phi) = \psi(\phi + 2\pi)$ .

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### Q3 (medium): Why are $+n$ and $-n$ the same energy?

The energy levels shown are

$$E_n = \frac{n^2 \hbar^2}{8\pi^2 m R^2}.$$

Why do the states  $n$  and  $-n$  have the **same energy**?

- A) Because the particle can't move on a ring, only stand still.
- B) Because the energy depends on  $n^2$ , so changing  $n \rightarrow -n$  doesn't change  $E_n$ .
- C) Because negative  $n$  is forbidden by normalization.
- D) Because the wavefunction must be purely real.

Reply with A, B, C, or D.



B

Q3 Feedback: Correct — B.

**Why:** The energy is proportional to  $n^2$ :

$$E_n \propto n^2 \Rightarrow E_{-n} = E_n.$$

Physically:  $+n$  and  $-n$  are **equal-speed motion in opposite directions** (clockwise vs counterclockwise). Same kinetic energy, opposite angular momentum.

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#### Q4 (harder): Normalization on the ring

The slides state the normalization condition

$$\int_0^{2\pi} |\psi(\phi)|^2 d\phi = 1,$$

and show  $\psi_n(\phi) = \frac{1}{\sqrt{2\pi}} e^{in\phi}$ .

What is the value of  $|\psi_n(\phi)|^2$  (the probability density) for any  $n$ ?

- A)  $|\psi_n|^2 = \frac{1}{2\pi}$  (constant)
- B)  $|\psi_n|^2 = \frac{1}{2\pi} \cos^2(n\phi)$
- C)  $|\psi_n|^2 = \frac{n^2}{2\pi}$
- D)  $|\psi_n|^2$  depends on  $\phi$  and is larger near  $\phi = 0$

Reply with A, B, C, or D.



A

✓ Q4 Feedback: Correct — A.

**Why:**

$$|\psi_n(\phi)|^2 = \left| \frac{1}{\sqrt{2\pi}} e^{in\phi} \right|^2 = \frac{1}{2\pi} |e^{in\phi}|^2 = \frac{1}{2\pi} \cdot 1 = \frac{1}{2\pi}.$$

So the probability is **uniform around the ring** for a single eigenstate  $e^{in\phi}$ . (The “wavy” real-part plots in the energy diagram are showing the **real part**, not the probability density.)

## Q5 (boss level): Energy spacing and scaling with radius

Using

$$E_n = \frac{n^2 \hbar^2}{2mR^2} \quad \left( \text{equivalent to } \frac{n^2 \hbar^2}{8\pi^2 m R^2} \right),$$

suppose the ring radius is doubled:  $R \rightarrow 2R$ . What happens to **all** energy levels  $E_n$ ?

- A) They double.
- B) They become 4 times larger.
- C) They become 4 times smaller.
- D) They are unchanged.

Reply with A, B, C, or D.



C

Q5 Feedback: Correct — C.

Why: The energy scales like  $1/R^2$ :

$$E_n \propto \frac{1}{R^2}.$$

So if  $R \rightarrow 2R$ ,

$$E'_n = \frac{n^2 \hbar^2}{2m(2R)^2} = \frac{n^2 \hbar^2}{2m \cdot 4R^2} = \frac{E_n}{4}.$$

So every level becomes **four times smaller**, and the spectrum gets "squeezed" closer together.

If you want an extra challenge variant, the next natural step is to ask about **superpositions** (e.g.,  $\psi \propto e^{in\phi} + e^{-in\phi}$ ) and how that changes the **probability density** into a standing-wave pattern—same energies available, different spatial distribution.

□ ◀ ▶ ↑ ⟳ ...

yes

Nice 😊 Here's the extra challenge (superposition/standing wave). Same rule: answer A/B/C/D and I'll give feedback.

## Q6 (extra boss): What happens to the probability density?

Consider the normalized superposition

$$\psi(\phi) = \frac{1}{\sqrt{2}} (\psi_n(\phi) + \psi_{-n}(\phi)), \quad \text{where } \psi_n(\phi) = \frac{1}{\sqrt{2\pi}} e^{in\phi}.$$

What is the probability density  $|\psi(\phi)|^2$ ?

- A)  $|\psi(\phi)|^2 = \frac{1}{2\pi}$  (still uniform)
- B)  $|\psi(\phi)|^2 = \frac{1}{\pi} \cos^2(n\phi)$
- C)  $|\psi(\phi)|^2 = \frac{1}{\pi} \sin^2(n\phi)$
- D)  $|\psi(\phi)|^2 = \frac{n^2}{2\pi}$

Reply with A, B, C, or D.

□ ◀ ▶ ↑ ⟳ ...

B

✓ Q6 Feedback: Correct — B.

Let's do it cleanly and mechanically (because the universe rewards algebra nerds):

$$\psi(\phi) = \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2\pi}} e^{in\phi} + \frac{1}{\sqrt{2\pi}} e^{-in\phi} \right) = \frac{1}{\sqrt{4\pi}} (e^{in\phi} + e^{-in\phi}).$$

Use the identity  $e^{ix} + e^{-ix} = 2 \cos x$ :

$$\psi(\phi) = \frac{1}{\sqrt{4\pi}} (2 \cos(n\phi)) = \frac{1}{\sqrt{\pi}} \cos(n\phi).$$

So

$$|\psi(\phi)|^2 = \left( \frac{1}{\sqrt{\pi}} \cos(n\phi) \right)^2 = \frac{1}{\pi} \cos^2(n\phi).$$

**Interpretation:** this superposition makes a **standing wave** around the ring. The particle is no longer equally likely to be found everywhere—there are **nodes** where probability goes to zero, and **antinodes** where it peaks.

If you want to go full “final form”: the other superposition  $\frac{1}{\sqrt{2}}(\psi_n - \psi_{-n})$  gives a  $\sin(n\phi)$  standing wave instead, shifting where the nodes are.

