

≡ ChatGPT 5.2 Thinking ▾

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Generate 5 multiple choice questions based on the attached pictures with increasing difficulty and let students choose the answers and then provide feedback one by one.

Thought for 14s >

Alright—let's turn those "particle on a ring" slides into a mini boss-fight of understanding. We'll do this **one question at a time**: you answer (A/B/C/D), then I'll give feedback and unlock the next, harder one.

Q1 (warm-up): Why must n be an integer?

For a particle on a ring, the wavefunction is shown as

$$\psi_n(\phi) = \frac{1}{\sqrt{2\pi}} e^{in\phi}, \quad \text{with periodic boundary } \psi(\phi) = \psi(\phi + 2\pi).$$

Which statement best explains why n must be an integer?

- A) Because energies must be positive.
- B) Because the wavefunction must be single-valued after going around the ring once.
- C) Because ϕ is measured in radians.
- D) Because the potential energy $V = 0$ everywhere.

Your move: Reply with **A, B, C, or D**.

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B

✅ Q1 Feedback: Correct — B.

Why: The boundary condition on a ring is **periodicity**:

$$\psi(\phi + 2\pi) = \psi(\phi).$$

But

$$\psi(\phi + 2\pi) = \frac{1}{\sqrt{2\pi}} e^{in(\phi+2\pi)} = \psi(\phi) e^{i2\pi n}.$$

For this to equal $\psi(\phi)$ for all ϕ , we must have

$$e^{i2\pi n} = 1 \Rightarrow n \in \mathbb{Z}.$$

So integers come from **single-valuedness + periodic geometry**, not from "energies must be positive" (A), units (C), or $V = 0$ (D).

Q2 (easy → medium): What is the allowed angular momentum?

Given $\psi_n(\phi) \propto e^{in\phi}$, what are the allowed values of the z -component of angular momentum L_z for a particle on a ring?

- A) $L_z = n\hbar$, where $n = 0, \pm 1, \pm 2, \dots$
- B) $L_z = \frac{\hbar}{n}$, where $n = 1, 2, 3, \dots$
- C) $L_z = \left(n + \frac{1}{2}\right) \hbar$, where $n = 0, 1, 2, \dots$
- D) $L_z = \pm\hbar$ only

Reply with A, B, C, or D.

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C

Not quite — **the correct answer is A.**

Why: On a ring, the angular momentum operator is

$$\hat{L}_z = -i\hbar \frac{d}{d\phi}.$$

Acting on $\psi_n(\phi) \propto e^{in\phi}$:

$$\hat{L}_z \psi_n = -i\hbar \frac{d}{d\phi}(e^{in\phi}) = -i\hbar(in)e^{in\phi} = n\hbar \psi_n.$$

So the allowed values are

$$L_z = n\hbar, \quad n = 0, \pm 1, \pm 2, \dots$$

Option **C** $(n + \frac{1}{2})\hbar$ shows up in some *other* quantum situations (e.g., certain boundary-condition shifts or spin-related contexts), but **not** for the standard particle-on-a-ring with $\psi(\phi) = \psi(\phi + 2\pi)$.

Q3 (medium): Why are $+n$ and $-n$ the same energy?

The energy levels shown are

$$E_n = \frac{n^2 \hbar^2}{8\pi^2 m R^2}.$$

Why do the states n and $-n$ have the **same energy**?

- A) Because the particle can't move on a ring, only stand still.
- B) Because the energy depends on n^2 , so changing $n \rightarrow -n$ doesn't change E_n .
- C) Because negative n is forbidden by normalization.
- D) Because the wavefunction must be purely real.

Reply with A, B, C, or D.



B

✓ Q3 Feedback: Correct — B.

Why: The energy is proportional to n^2 :

$$E_n \propto n^2 \Rightarrow E_{-n} = E_n.$$

Physically: $+n$ and $-n$ are **equal-speed motion in opposite directions** (clockwise vs counterclockwise). Same kinetic energy, opposite angular momentum.

Q4 (harder): Normalization on the ring

The slides state the normalization condition

$$\int_0^{2\pi} |\psi(\phi)|^2 d\phi = 1,$$

and show $\psi_n(\phi) = \frac{1}{\sqrt{2\pi}} e^{in\phi}$.

What is the value of $|\psi_n(\phi)|^2$ (the probability density) for any n ?

- A) $|\psi_n|^2 = \frac{1}{2\pi}$ (constant)
- B) $|\psi_n|^2 = \frac{1}{2\pi} \cos^2(n\phi)$
- C) $|\psi_n|^2 = \frac{n^2}{2\pi}$
- D) $|\psi_n|^2$ depends on ϕ and is larger near $\phi = 0$

Reply with A, B, C, or D.



A

☒ **Q4 Feedback: Correct — A.**

Why:

$$|\psi_n(\phi)|^2 = \left| \frac{1}{\sqrt{2\pi}} e^{in\phi} \right|^2 = \frac{1}{2\pi} |e^{in\phi}|^2 = \frac{1}{2\pi} \cdot 1 = \frac{1}{2\pi}.$$

So the probability is **uniform around the ring** for a single eigenstate $e^{in\phi}$. (The “wavy” real-part plots in the energy diagram are showing the **real part**, not the probability density.)

Q5 (boss level): Energy spacing and scaling with radius

Using

$$E_n = \frac{n^2 \hbar^2}{2mR^2} \quad \left(\text{equivalent to } \frac{n^2 h^2}{8\pi^2 m R^2} \right),$$

suppose the ring radius is doubled: $R \rightarrow 2R$. What happens to **all** energy levels E_n ?

- A) They double.
- B) They become 4 times larger.
- C) They become 4 times smaller.
- D) They are unchanged.

Reply with A, B, C, or D.



C

✅ Q5 Feedback: Correct — C.

Why: The energy scales like $1/R^2$:

$$E_n \propto \frac{1}{R^2}.$$

So if $R \rightarrow 2R$,

$$E'_n = \frac{n^2 \hbar^2}{2m(2R)^2} = \frac{n^2 \hbar^2}{2m \cdot 4R^2} = \frac{E_n}{4}.$$

So every level becomes **four times smaller**, and the spectrum gets “squeezed” closer together.

If you want an extra challenge variant, the next natural step is to ask about **superpositions** (e.g., $\psi \propto e^{in\phi} + e^{-in\phi}$) and how that changes the **probability density** into a standing-wave pattern—same energies available, different spatial distribution.



yes

Nice 😊 Here's the **extra challenge (superposition/standing wave)**. Same rule: answer A/B/C/D and I'll give feedback.

Q6 (extra boss): What happens to the probability density?

Consider the normalized superposition

$$\psi(\phi) = \frac{1}{\sqrt{2}}(\psi_n(\phi) + \psi_{-n}(\phi)), \quad \text{where } \psi_n(\phi) = \frac{1}{\sqrt{2\pi}}e^{in\phi}.$$

What is the probability density $|\psi(\phi)|^2$?

- A) $|\psi(\phi)|^2 = \frac{1}{2\pi}$ (still uniform)
- B) $|\psi(\phi)|^2 = \frac{1}{\pi} \cos^2(n\phi)$
- C) $|\psi(\phi)|^2 = \frac{1}{\pi} \sin^2(n\phi)$
- D) $|\psi(\phi)|^2 = \frac{n^2}{2\pi}$

Reply with **A, B, C, or D**.



B

✅ **Q6 Feedback: Correct — B.**

Let's do it cleanly and mechanically (because the universe rewards algebra nerds):

$$\psi(\phi) = \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2\pi}}e^{in\phi} + \frac{1}{\sqrt{2\pi}}e^{-in\phi} \right) = \frac{1}{\sqrt{4\pi}} (e^{in\phi} + e^{-in\phi}).$$

Use the identity $e^{ix} + e^{-ix} = 2 \cos x$:

$$\psi(\phi) = \frac{1}{\sqrt{4\pi}} (2 \cos(n\phi)) = \frac{1}{\sqrt{\pi}} \cos(n\phi).$$

So

$$|\psi(\phi)|^2 = \left(\frac{1}{\sqrt{\pi}} \cos(n\phi) \right)^2 = \frac{1}{\pi} \cos^2(n\phi).$$

Interpretation: this superposition makes a **standing wave** around the ring. The particle is no longer equally likely to be found everywhere—there are **nodes** where probability goes to zero, and **antinodes** where it peaks.

If you want to go full “final form”: the other superposition $\frac{1}{\sqrt{2}}(\psi_n - \psi_{-n})$ gives a $\sin(n\phi)$ standing wave instead, shifting where the nodes are.

