data

$$\int_0^2 \frac{x}{x+1}\,dx = 0{,}90139$$

$f(x) = \frac{x}{x+1}$

$f''(x) = \frac{2x}{(x+1)^3} - \frac{2}{(x+1)^2}$

$M_2 = \max_{a \leq \xi \leq b} |f''(x)|$

$f''(2) = \frac{4}{3^3} - \frac{2}{3^2} = \frac{4}{27} - \frac{2}{9}$

$f''(0) = \frac{0}{1^3} - \frac{2}{1} = -2$

$f''(1) = \frac{2}{2^3} - \frac{2}{2^2}$

$h = \frac{b-a}{n} = \frac{2-0}{9} = 0{,}22$

x	$y = f(x)$
0	0
0,22	0,1803
0,44	0,3056
0,66	0,3976
0,88	0,4680
1,00	0,5
1,22	0,5495
1,44	0,5902
1,66	0,6241
1,88	0,6527
2,00	0,6667

Regra do trapézio

$T_{10}(f) = \frac{h}{2}\left(f(x_0) + 2(f(x_1) + \dots + f(x_{n-1})) + f(x_n)\right)$

$T_{10}(f) = \frac{0{,}22}{2}\left(0 + 2(4{,}268) + 0{,}6667\right)$

$= 0{,}11(8{,}536 + 0{,}6667) = 0{,}11(9{,}2027)$

$= 1{,}012297$

$|I(f) - T_{10}(f)| = 0{,}90139 - 1{,}012297 = |-0{,}110907|$

$= 0{,}110907$

$n > \frac{(b-a)^2 M_2}{\sqrt{12 \cdot 10^{-7}}} = \frac{4 \cdot M_2}{\sqrt{12 \cdot 10^{-7}}} = \frac{8}{1{,}0954 \times 10^{-3}} \rightarrow n > 7302{,}9674$

Regra de $\frac{1}{3}$ de Simpson

$S_n(f) = \frac{h}{3}\left(f(x_0) + 4(f(x_1) + f(x_3) + \dots + f(x_{n-1})) + 2(f(x_2) + \dots f(x_{n-2})\right)$

$= \frac{h}{3}\left(0 + 4(0{,}1803 + 0{,}3976 + 0{,}5 + 0{,}5902 + 0{,}6527) + 2(0{,}3056 + 0{,}4680 + 0{,}5495 + 0{,}6241) + 0{,}6667\right)$

$= \frac{h}{3}\left(4(2{,}3208) + 2(1{,}9472) + 0{,}6667\right)$

$= \frac{0{,}22}{3}(9{,}2832 + 3{,}8944 + 0{,}6667)$

$= \frac{0{,}22}{3}(13{,}8443) = \frac{3{,}0457}{3} = 1{,}0152$

$|I(f) - S_{10}(f)| = 0{,}90139 - 1{,}0152 = |-0{,}11381| = 0{,}11381$

$M_4 = \max_{a \leq \xi \leq b} |f^{4}(\xi)| \Rightarrow f^{(4)}(x) = \frac{24x}{(x+1)^5} - \frac{24}{(x+1)^4}$

$f^{(4)}(0) = 0 - \frac{24}{1} = -24$

$f^{(4)}(1) = \frac{24}{32} - \frac{24}{16}$

$f^{(4)}(2) = \frac{48}{343} -$

$\frac{b-a}{180} \cdot h^4 M_4 \leq 10^{-7}$

$\frac{2}{180}\left(\frac{2-0}{n}\right)^4 24 \leq 10^{-7}$

$\frac{48 \cdot 16}{180\, n^4} \leq 10^{-7}$

$\frac{768}{180 \cdot 10^{-7}} \leq n^4$

$42666666{,}67 \leq n^4$

$n \geq \sqrt[4]{42666666{,}67}$

$n \geq 80{,}8206$