SUPELEC

Lecture by M. Debbah

Digital Signal Processing

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I.1.1.1. Notations

Ω : sample space

A : $\{\omega,\ fulfill\ a\ given\ property\}$ – A forms a family of events of F.

$\forall A \in F, P(A) \in [0,1]$ (Probability of the event)

The triplet $\{\omega, F, P\}$ forms a probability space.

$(\Omega, F, P) \to (\mathbb{R}, B, P_X)$

X : $P[X(\omega) \leq x] = F_X(x) \longleftarrow$ Cumulative distribution fonction of the variable X.

I.1.1.2. Discret Case

$X(\omega) \in \{x_1, ..., x_n\}$

Let us call $p_i = P[X(\omega) = x_i]$

$$\begin{cases} \sum_i p_i & = & 1 \\ F_X(x) & = & \sum_{i, x_i \leq x} p_i \end{cases}$$

I.1.1.3. Probability density

$P_X(x) = \sum_i p_i \delta(x - x_i)$

I.1.1.4. Continuous case

$X(\omega)$ continuous ensemble of values and $F_X(x)$ is continuous and derivable.

$\frac{dF_X(x)}{dx} = p_X(x)$: probability density of X.

$$\begin{aligned} \int_{\mathbb{R}} p_X(x)dx &= 1 \\ \int_{-\infty}^{x} p_X(u)du &= F_x(x) \\ dF_X(x) &= \underbrace{p_X(x)dx}_{\text{elementary probability}} = P[x \leq X \leq x + dx] \end{aligned}$$

$F_X(-\infty) = 0$ and $F_X(+\infty) = 1$

I.1.1.5. Moments of X

$m_n = E[x^n] = \int_{\mathbb{R}} x^n dF_X(x)$

$m_1 = E[X]$: statistical mean

$m_2 = E[X^2]$: moment of second order

$Var(X) = \sigma_X^2 = E[|X - E[X]|^2]$

$E[f(X)] = \int_R f(x) dF_X(x)$

If $f(X) = e^{iuX}$, $E[f(X)] = \int_{\mathbb{R}} e^{iux} dF_X(x) = \Phi_X(u)$

Φ_X is the caracteristic function of first order.

I.1.1.6. X is continuous

$\Phi_X(u) = \int_{\mathbb{R}} e^{iuX} p_X(x) dx$

$p_X(x) = \frac{1}{2\pi} \int_{\mathbb{R}} e^{-iuX} \Phi_X(u) du$

I.1.1.7. Characteristic function of second order

$\Psi_X(u) = \ln \Phi_X(u)$

$\Psi_X(u) = \sum_{k=0}^{+\infty} \frac{(iu)^k}{k!} C_k$, C_k is a cumulant.

$C_1 = m_1$

$C_2 = m_2 - m_1^2$ (variance)

I.1.1.8. Couple of random real vectors (X, Y)

$*$ **In $\mathbb{R}^2$:**

- $E[X, Y^T] = 0$, if $X \perp Y$

- Let us call $X_c = X - E[X]$:

  $E[X_C, Y_c^T] = Cov(X, Y) = E[(X - E[X])(Y - E[Y])] = E[XY^T] - E[X]E[Y^T]$

  If $Cov(X, Y) = 0$ then X and Y are uncorelated.