Regularitet og Automater: Assignment 5

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Introduction

This assignment is about pumping $L \subseteq \Sigma^*$ of a given FA for regular expressions, and showing that only finite automata accepting regular languages can be pumped.

Consider following language, use the pumping lemma to show that it cannot be accepted by an FA:

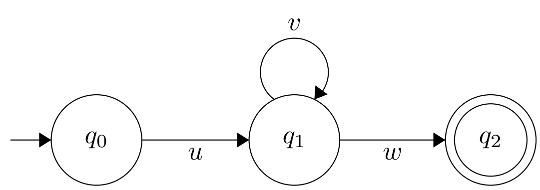
$L = \{a^i b^j a^k | k > i + j\}$

Pumping and the lemma

Now, before going any further, let's have a look at what pumping is, how it operates and what we may use it for. The formal definition is that for any given $M = (Q, \Sigma, q_0, A, \delta)$ is an FA accepting $L \subseteq \Sigma^*$ where Q has n elements, and if string $x \in L$ with $|x| < n$, then x has n distinct prefixes and M may be in a different state for every processed $\sigma \in \Sigma$. However, if $|x| \geq n$, then M must have entered a state twice; there must be two different prefixes u and uv such that

$$\delta^*(q_0, u) = \delta^*(q_0, uv)$$

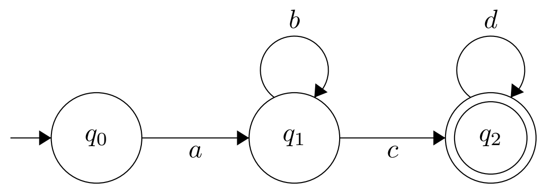
We may now extrapolate that if $x \in L$ and w is a string satisfying $x = uvw$, then we have



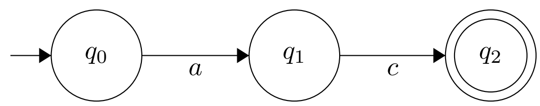
and this allows for "pumping" up a string with additional v values in the form of v^i, such that $x = uv^iw, i \geq 0$. Knowing that any x pumped by this procedure will still be accepted by M, we have pumped this particular language with no side effects. This isn't always the case, though, for the pumping lemma is usually used to show that some languages can be pumped into irregularity.

Usage example

Let's look at an example based on $L(ab^*cd^*)$



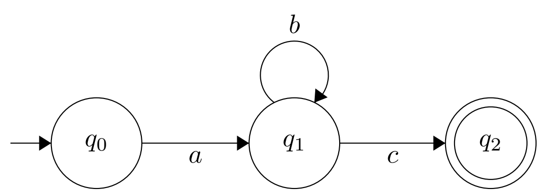
Let $M = (Q, \Sigma, q_0, A, \delta)$ where $Q = \{q_0, q_1, q_2\}$, $\Sigma = \{a, b, c, d\}$, $|Q| = n = 3$ and $A = \{q_2\}$.
Considering $x = ac$, then $|x| = 2 < n$ which means that M can reach $q_2 \in A$ without visiting any state twice and therefore isn't pumpable with the chosen n, like this:



If we were to pick an x such that $|x| \geq n$ where $x \in L \subseteq \Sigma^*$, then M would have to

$$\delta(q_0, a) \to \delta(q_1, b) \to \delta(q_1, c) \to q_2 \in A$$

visting q_1 more than once, using these paths:

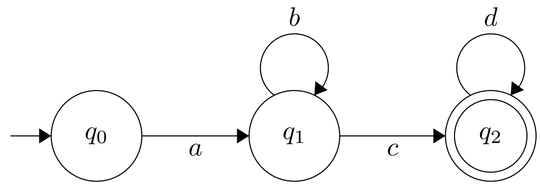


Showing that if $|x| \geq |Q|$ then we must use any $q \in Q$ more than once, and this exact property opens up for potential pumping. To show the whole $x = uv^iw$ part, let's pick a word way longer than just 2 or 3: $x = abbcdd \in L \Rightarrow |x| = 6 \geq n$.

There must exist a seperation such that $x = uvw$, and $|uv|$ – according to the lemma – must be $\leq n$, and in this case $|v| \geq 1$ due to it being the pumpable variable and therefore it must exist.

Also, if $|x| < n$ then it will suffice to only consider $\sigma_{1 \to n-1} \in x$, because any string longer than that will fall under the constraint of having to use a state more than once.
For the separation of *abbcdd* into $x = uvw$, in this example $u = ab$ and $v = b$ will do. $u = a$ and $v = bb$ would also do, but just any one seperation is necessary for pumping. This leaves us with $w = cdd$.

Glancing at the lemma yet again, we see that $uv^iw \in L$ must be upheld. $\forall i : i \geq 0 \Rightarrow uv^iw \in L$, for example if $i = n = 3$ then $v^i = b^3$ yielding the result of $u = ab$, $v = bbb$, $w = cdd$ which gives us $abbbbcdd \in L$ still contained by our FA, and therefore without side effects,



with the following transitions

$$\delta(q_0, a) \to i \cdot \delta(q_1, b) \to \delta(q_1, c) \to 2 \cdot \delta(q_2, d)$$

Main problem

Considering $L = \{a^i b^j a^k | k > i + j\}$, use the pumping lemma to show that it cannot be accepted by an FA.

Picking a random case within the constraints of $k > i+j$, $|v| \geq 1$ and $|uv| \leq |Q|$:

$$i = n$$

$$j = 1$$

$$k = i + j + 1 = n + 2$$

Using these values we get a string $x \in L \subseteq \Sigma^*$

$$x = a^n b a^{n+2}$$

Considering $|x| \geq n$ and therefore x must visit some $q \in Q$ twice or more, revealing a pumpable loop.

Using the lemma, we may split $x = uvw$ whilst obeying the lemma's decrete of $|v| \geq 1$ and $|uv| \leq n$. Knowing that $|uv| \leq n$ and the prefix of x is a^n, it's given that $|uv|$ consists only of a's.

Pumping v^i where $i = n - n + 2 \to i \geq 2$, yields a v greater than allowed in L where the constraint of $k > i + j$ no longer holds. The pattern is mismatched by pumping and therefore the pumped L is not accepted by the FA.