GEOM20015
Sensing and Measurement

Traversing numerical example

Assignment 2 supplementary material

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Introduction

This is a numerical example of traversing, with planning of measurements, booking, reduction, and adjustment. It serves as a supplementary material to Assignment 2.

The example polygon is given in Figure 1, where horizontal coordinates of control points 3001 and 3002 are known while the coordinates of control points A, B and C are to be determined. The dashed line from 3001 towards 3002 indicates we do not measure the distance between 3001 and 3002, only the direction. As both 3001 and 3002 are permanent CPs of known coordinates, Table 1, the distance between them is known and fixed, as is the bearing.

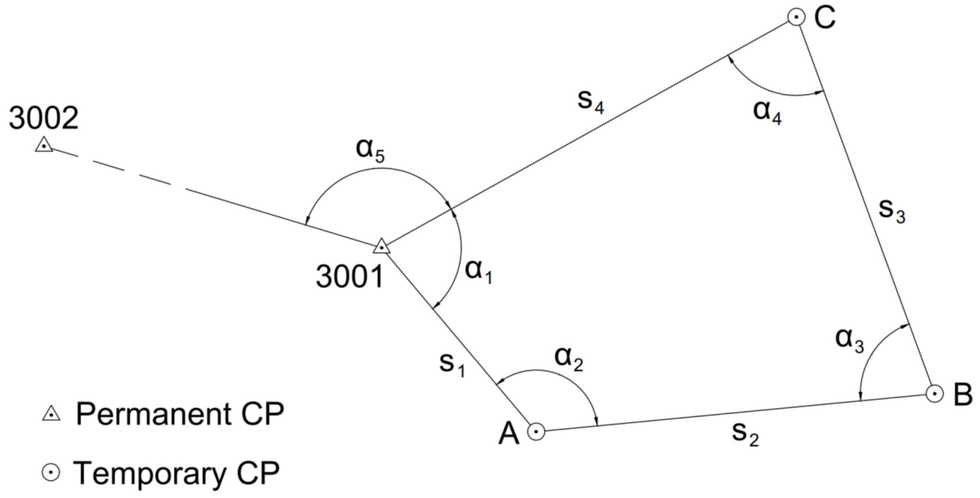

Figure 1. *Closed traverse example*

Planning the field measurements

To perform the traverse along the closed polygon, we must:

- Obtain all internal angles of the polygon ($\alpha_1 - \alpha_4$).
- Obtain the lengths of all sides of the polygon ($s_1 - s_4$). As we are traversing along the polygon, the sides of a polygon are also traverse lines.
- Ensure that we can calculate the bearings for each traverse line by setting the baseline 3001-3002 and obtaining the angle α_5.

We obtain the internal angles and lengths by setting up the total station at the first point of the polygon, measuring the directions and distances towards the adjacent points of the polygon (using face left and face right approach), then moving in the counterclockwise direction, until the total station has been set on all points of the polygon. In our example total station-set ups are:

1. Total station on 3001, backsight prism on C, foresight prism on A.
2. Total station on A, backsight prism on 3001, foresight prism on B.
3. Total station on B, backsight prism on A, foresight prism on C.
4. Total station on C, backsight prism on B, foresight prism on 3001.

However, the polygon contains only one point of known horizontal coordinates. To calculate the bearings, two are needed. Hence, the angle α_5 must be obtained to “connect” the baseline 3001-3002 with the polygon and enable us to calculate the bearings of all traverse lines, so a final total station set-up is:

5. Total station on 3001, backsight prism on 3002, foresight prism on C.

The order of performing the measurements above does not matter, as long as all necessary data has been gathered. Also, although this example traverses in a counterclockwise direction, you are free to traverse in a clockwise direction too.

Table 1. *Permanent control points.*

Point number	Easting [m]	Northing [m]
3001	17042.159	15984.872
3002	16924.184	16020.312

Booking

Reading angles to targets on both faces is a *round* of angles. This may be repeated several times (a *set*) so that a required precision can be achieved. In this example, we use one repetition. For example, readings on point A are:

	Target 3001				Target B			
At A	Direction			Distance	Direction			Distance
FL	0°	0'	0''	83.792 m	124	37'	41''	139.889 m
FR	180°	0'	10''	83.795 m	304	37'	54''	139.887 m

The order of readings is:

	Target 3001	Target B
At A		
FL	Step 1	Step 2
FR	Step 4	Step 3

The reading is set towards the left hand most point and angles are read clockwise. The reading is set to 0°00’00” only once – after switching from FL to FR, the reading is **not** set again to 180°00’00”.

The main purpose of using FL and FR readings is to minimise the collimation error.

There is no standard way of booking directions and distances in traversing. It does not matter which method is used as long as it is clear, rigorous, unambiguous, and simple.

Reduction

To reduce the direction observations to angles we first calculate the angle for each set, then take the average of all sets. For one set of observations above:

FL	124°	37’	41”	-	0°	0’	0”	=	124°	37’	41”
FR	304°	37’	54”	-	180°	0’	10”	=	124°	37’	44”
Average of FL and FR									124°	37’	42.5”

The distances are simply the average of all observations:

Distance A-3001 = (83.792 m + 83.795 m) / 2 = 83.7935 m

Distance A-B = 139.888 m

During processing, we usually end up with more decimal places then we started with, as is the case in the example above. In surveying, the decimal places are important, as they imply the accuracy of measurements, coordinates, and all other spatial data. So, it is necessary to round the processed numbers back to the number of decimal places of our measurements, which gives us the final reduced measurements for total station set-up on point A:

Angle α_2:	124°37’43”
Distance A-3001:	83.794 m
Distance A-B:	139.889 m

Note, you will obtain the length of each polygon side twice, from two total station set-ups at each end. The final length is the average of these two. For example, if we measure the A-B length from point A to be 139.888 m, and the same length from point B to be 139.890 m, the final length of s_2 is 139.889 m.

The process is the same for all other total station set-ups. All reduced angles and distances of this example are shown in Figure 2.

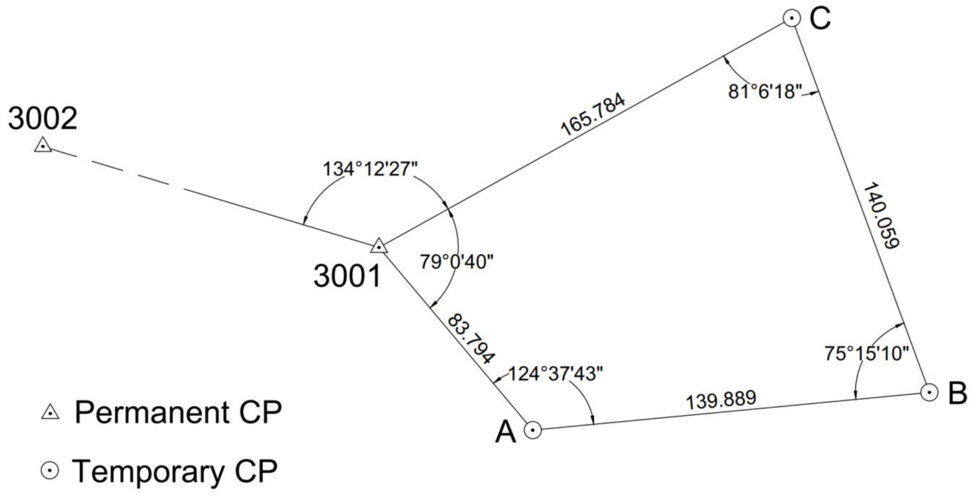

Figure 2. *All reduced angles and distances*

Adjustment

The traverse adjustment in this example is split into three sections, which calculate:

1. Angular misclosure and angle adjustment,
2. Bearing calculation and initial coordinates,
3. Linear misclosure and Bowditch adjustment.

(1) Angular misclosure and angle adjustment

This correct sum of internal angles in a polygon is:

$$(n-2)\pi,$$

where n is the number of angles in the polygon (in this example n = 4), as well as the number of measured angles we want to adjust.

Traversing along a polygon allows us to compare the sum of our measured angles with the correct sum and perform the adjustment. The sum of measured angles is:

$$\sum_{i=1}^{n} \alpha_i = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 359°59'51",$$

which gives the angular misclosure of:

$$\text{angular misclosure} = 360°00'00 - 359°59'51"=9".$$

Before continuing, we must first check if the misclosure is in the acceptable range. The maximum allowed misclosure is:

$$12.0\sqrt{\text{n}}=24.0",$$

which means the calculates misclosure is acceptable.

To adjust our n measured angles we calculate the misclosure per angle:

$$\text{misclosure per angle} = 9"/\text{n}=2.25".$$

As in the reduction to angles, we want our adjusted angles to be rounded to whole seconds. This means we need integer corrections, but our misclosure per angle is not an integer. We simply correct three angles with 2”, and one randomly chosen angle with 3”, which adds up to the total misclosure of 9”. This might seem confusing at first but is a common practice in surveying, as it avoids calculating and working with unnecessary decimal places, and in the end has no effect on the results as long as the adjustment is carried out correctly.

We finally correct our internal angles to obtain final adjusted angles:

$$\alpha_1 = 79°00'40" + 2" = 79°00'42",$$

$$\alpha_2 = 124°37'43" + 2" = 124°37'45",$$

$$\alpha_3 = 75°15'10" + 2" = 75°15'12",$$

$$\alpha_4 = 81°06'18" + 3" = 81°06'21".$$

For control, we check if these angles add up to $(n-2)\pi = 360°00'00"$:

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 360°00'00".$$

(2) Bearing calculation and initial coordinates

We need bearings of traverse lines to calculate the Easting and Northing coordinate differences, ΔE and ΔN, between subsequent points of the traverse. In our example, we must first calculate the bearing θ_{3001}^{3002} (from 3001 to 3002) from their known coordinates (Table 1):

$$\theta_{3001}^{3002} = tan^{-1}\frac{\Delta E_{3001}^{3002}}{\Delta N_{3001}^{3002}} = tan^{-1}\frac{(E_{3002} - E_{3001})}{(N_{3002} - N_{3001})} = tan^{-1}\frac{-117.975}{35.440} = -73°16'47''$$

From coordinate differences $(\frac{-}{+})$ we can see that the quadrant is between North and West, which means we add 360° to the angle. The bearing is thus:

$$\theta_{3001}^{3002} = -73°16'47'' + 360° = 286°43'13''.$$

Bearings are covered in more detail in the lectures, but in general your case might be different based on the quadrant of your bearing. From $\frac{\Delta E}{\Delta N}$ coordinate differences, different cases are:

$$\frac{+}{+} \rightarrow \frac{East}{North} \rightarrow +0°$$

$$\frac{+}{-} \rightarrow \frac{East}{South} \rightarrow +180°$$

$$\frac{-}{-} \rightarrow \frac{West}{South} \rightarrow +180°$$

$$\frac{-}{+} \rightarrow \frac{West}{North} \rightarrow +360°$$

The bearing of the baseline is $\theta_{3001}^{3002} = 286°43'13''$. The bearing of the first traverse line is obtained through basic trigonometry as (see figures 2 and 3):

$$\theta_{3001}^{A} = \theta_{3001}^{3002} + (\alpha_1 + \alpha_5) = 139°56'22''.$$

Now the bearing of the first traverse line can be used to compute bearings of all subsequent traverse lines, using the equations for computing bearings from internal angles of a polygon:

$$\theta_n = \theta_{n-1} + 180° - \alpha \qquad \text{(clockwise direction)},$$

$$\theta_n = \theta_{n-1} - 180° + \alpha \qquad \text{(counterclockwise direction)}.$$

For our example bearings are:

$$\theta_A^B = \theta_{3001}^A - 180° + \alpha_2 = 84°34'07",$$

$$\theta_B^C = \theta_A^B - 180° + \alpha_3 = 339°49'19",$$

$$\theta_C^{3001} = \theta_B^C - 180° + \alpha_4 = 240°55'40".$$

It is a good practice to check the plan to see if the calculated bearings look correct. For example, if the line looks like it should have a bearing of roughly 190-210°, and your calculated bearing is 20°, there is an error somewhere – either in the calculated bearing or in the plan.

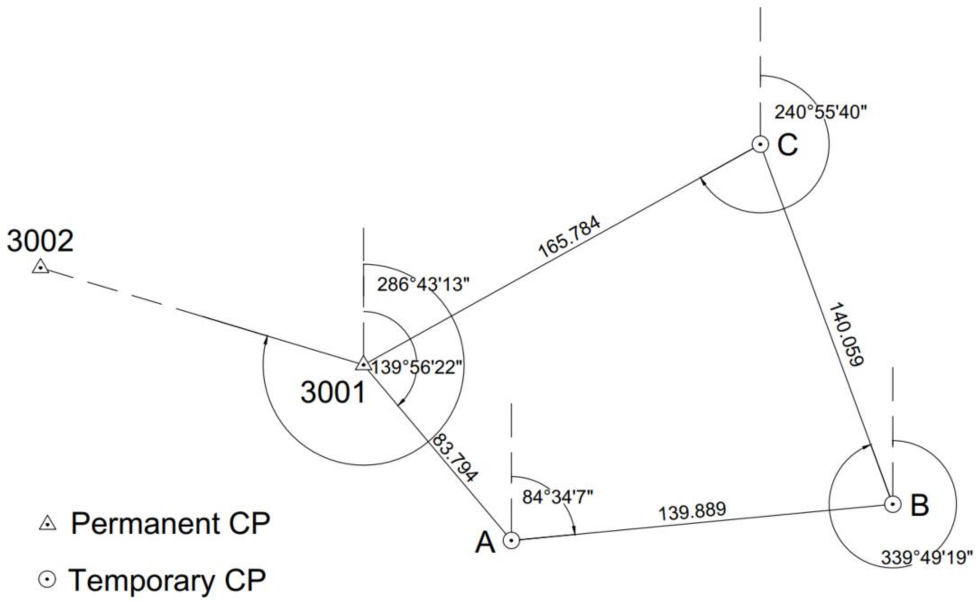

Figure 3. *Bearings and distances*

We now have all information necessary to calculate the coordinate differences between the points of our polygon using the equations:

$$\Delta E = D\sin(\theta),$$

$$\Delta N = D\cos(\theta).$$

The coordinate differences can be immediately used to calculate the initial coordinates of unknown points. Using the known and fixed coordinates of the permanent CP 3001 (Table 1) for our example we get:

Line	Bearing θ			Distance D [m]	ΔE [m]	ΔN [m]	E [m]	N [m]
3001-A	139°	93'	94''	83.794	53.930	-64.133	17042.159	15984.872
A-B	84°	56'	86''	139.889	139.261	13.241	17096.089	15920.739
B-C	339°	82'	19''	140.059	-48.312	131.463	17235.349	15933.980
C-3001	240°	92'	77''	165.784	-144.897	-80.556	17187.038	16065.443
							17042.141	15984.887

(3) Linear misclosure and Bowditch adjustment

Again, as with the adjustment of angles, we must first calculate the linear misclosure and check if it is in the acceptable range. Beginning and finishing a traverse on points of known horizontal coordinates allows us to compare the obtained ΔE_i and ΔN_i with the correct ones and calculate the misclosures. Because we traverse along a closed polygon and return to the starting point, the correct ΔE_i and ΔN_i are zero, and the misclosures are simply the sums of obtained ΔE_i and ΔN_i:

$$\Delta E = \sum_{i=1}^{n} \Delta E_i = -\,0.018\ m,$$

$$\Delta N = \sum_{i=1}^{n} \Delta N_i = 0.015\ m.$$

This gives us the misclosure along the E axis and along the N axis, which can be used to calculate the total linear misclosure as the length of the vector formed by ΔE and ΔN:

$$\text{linear misclosure} = \sqrt{\Delta E^2 + \Delta N^2} = 0.0230\ m.$$

Note that an additional decimal place is used. We use additional decimal places when calculating the misclosure and the adjustment, and round the final result back to the initial number of decimal places, same as with angles.

Now, we can calculate the accuracy from the total length of the traverse (perimeter) and the misclosure as:

$$\text{accuracy} = \frac{\text{perimeter}}{\text{linear misclosure}} = \frac{529.526}{0.0230} = 1{:}\,23022.87.$$

The linear misclosure accuracy is better (bigger) than 1:8000 which confirms our linear misclosure is acceptable, and we can continue and perform the adjustment.

The Bowditch adjustment linearly distributes misclosures ΔE and ΔN through the traverse based on the lengths of traverse lines. The correction applied to each traverse line is proportional to the length of that side as a ration of the perimeter:

$$\delta\Delta\mathrm{E_i} = \Delta\mathrm{E}\frac{\mathrm{D_i}}{\sum_{i=1}^{n}\mathrm{D_i}},$$

$$\delta\Delta\mathrm{N_i} = \Delta\mathrm{N}\frac{\mathrm{D_i}}{\sum_{i=1}^{n}\mathrm{D_i}},$$

where D_i is the length of a traverse line, and $\delta\Delta E_i$ and $\delta\Delta N_i$ are corrections to ΔE_i and ΔN_i. This means that longer traverse lines will end up having larger corrections, as they are more likely to contain a random error, and vice versa.

Corrections $\delta\Delta E_i$ and $\delta\Delta N_i$ are used to compute the adjusted ΔE_i and ΔN_i:

$$\mathrm{Adj}\ \Delta\mathrm{E_i} = \Delta E_i - \delta\Delta E_i,$$

$$\mathrm{Adj}\ \Delta\mathrm{N_i} = \Delta N_i - \delta\Delta N_i,$$

which are in turn used to compute the final adjusted coordinates of unknown points.

For our example we get:

Line	ΔE [m]	ΔN [m]	δΔE [m]	δΔN [m]	Adj ΔE [m]	Adj ΔN [m]	Adj E [m]	Adj N [m]
3001-A	53.930	-64.133	-0.0028	0.0023	53.9328	-64.1353	17042.159	15984.872
A-B	139.261	13.241	-0.0047	0.0038	139.2657	13.2372	17096.092	15920.737
B-C	-48.312	131.463	-0.0047	0.0039	-48.3073	131.4591	17235.358	15933.974
C-3001	-144.897	-80.556	-0.0056	0.0046	-144.8914	-80.5606	17187.050	16065.433
							17042.159	15984.872

For control, the adjusted coordinates of the point the traverse begins and finishes on, CP 3001, must be equal.

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