Lecture9 tree recursion

> ⓘ **recall**
>
> factorial
>
> ```
> def factorial(n):
>     fact = 1
>     i = 1
>     while i <= n:
>         fact *= i # fact = fact * i
>         i += 1    # i = i + 1
>     retrun fact
> ```
>
> ```
> def factorail(n):
>     if n == 0:
>         return 1
>     else:
>         retrun n * factorial(n - 1)
> ```

Orders of Recursive Calls

Cascade Function

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n // 10)
        print(n)


def cascade(n):
    print(n)
    if n >= 10:
        cascade(n // 10)
        print(n)
```

Inverse Cascade

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)

def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)

grow = lambda n: f_then_g(grow, print ,n // 10)
shrink = lambda n: f_then_g(print, f_then_g(print, shrink , n // 10)
```

> well, i have to say that's surprising, cause this function solved this problem elegantly by utilizing the orders of the functions.

> i have to say that this is a helpful way of thinking

Tree Recursion

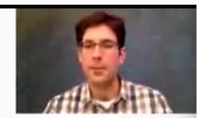
Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one call to that function.

n:	0,	1,	2,	3,	4,	5,	6,	7,	8,	... ,	35
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	9,227,465

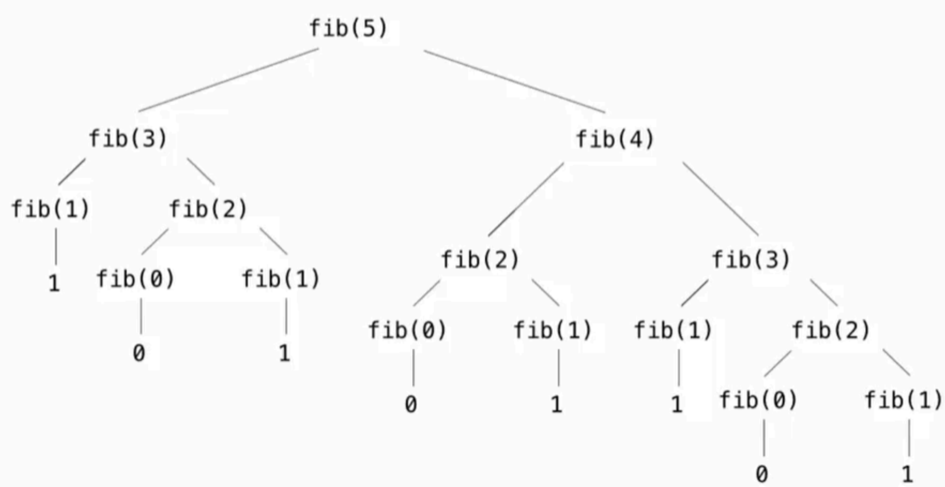
```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

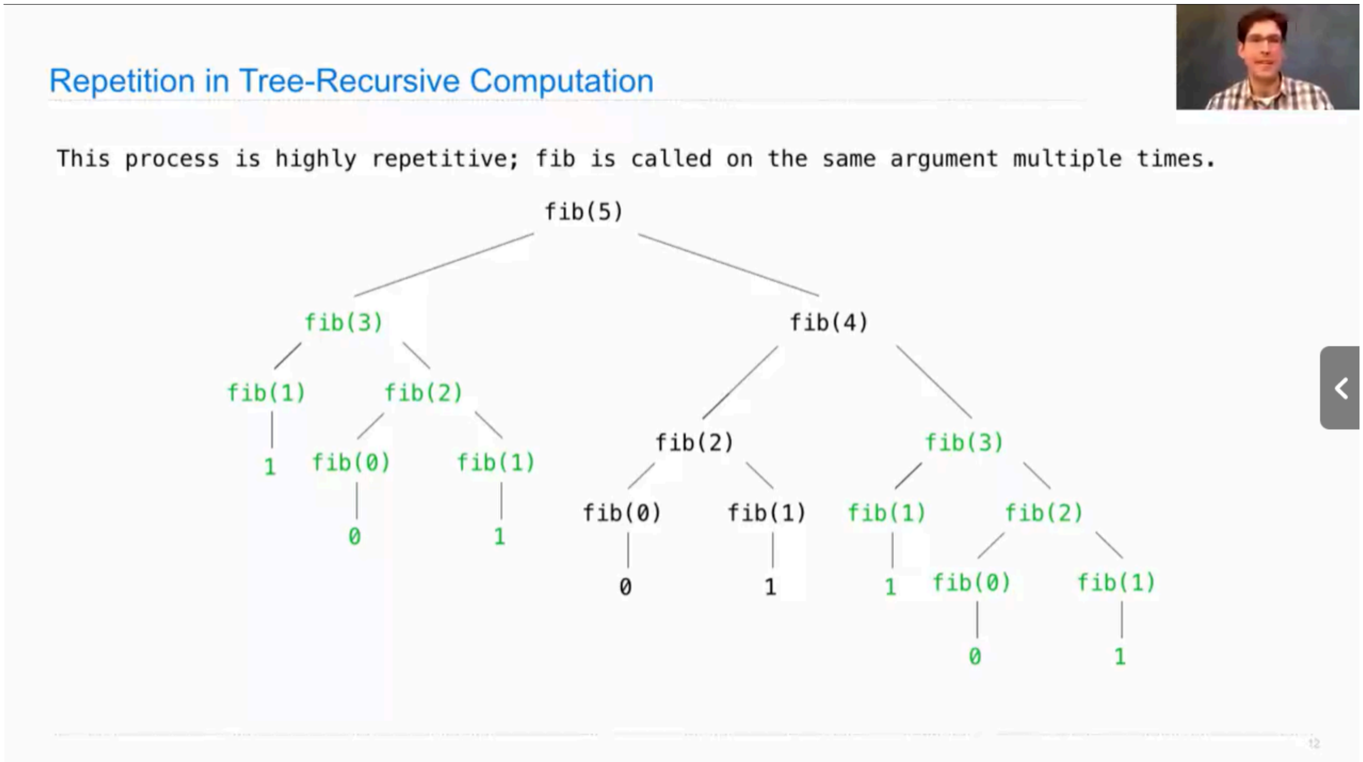
http://en.wikipedia.org/wiki/File:Fibonacci.jpg



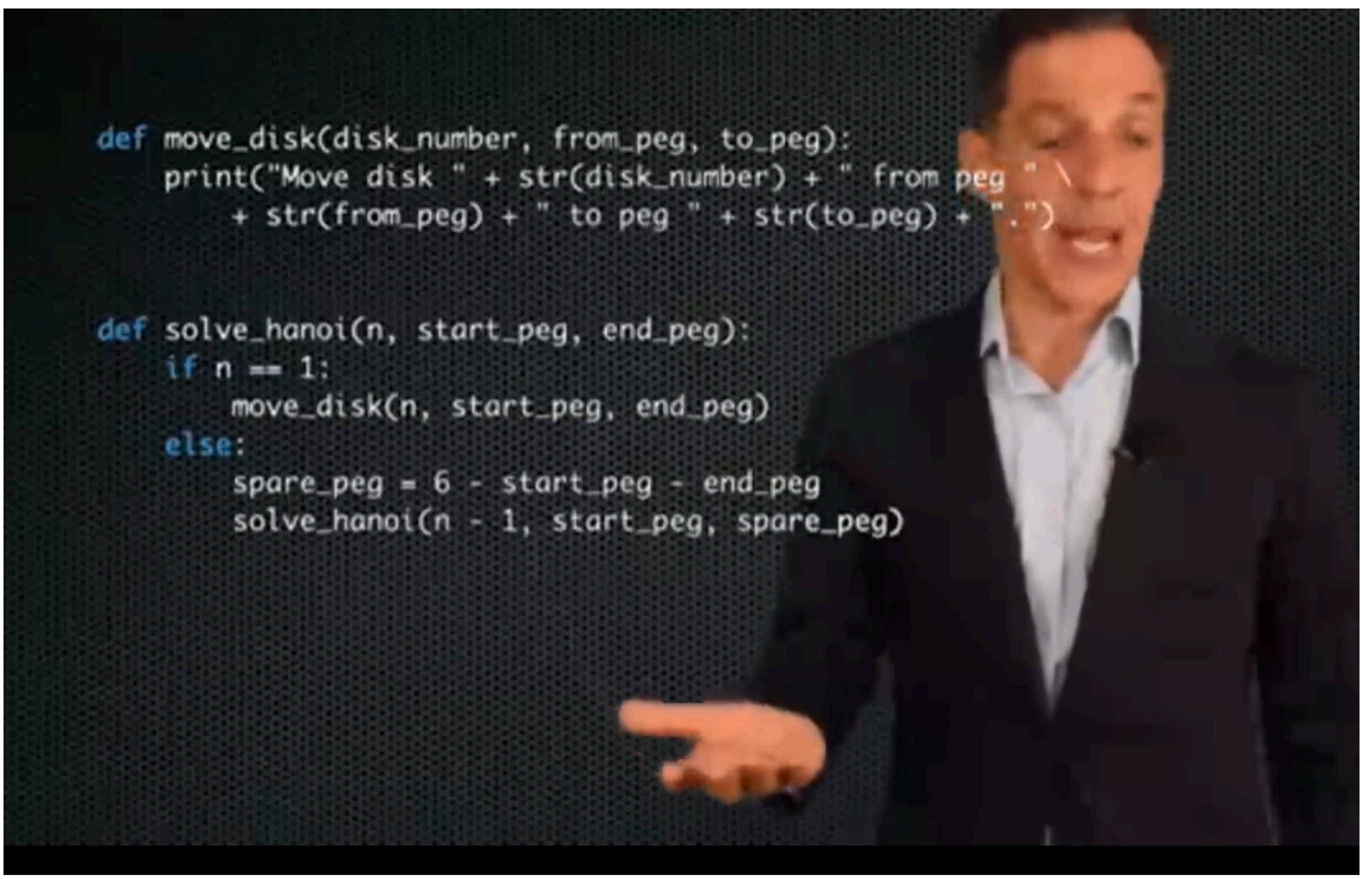
A Tree-Recursive Process

The computational process of fib evolves into a tree structure

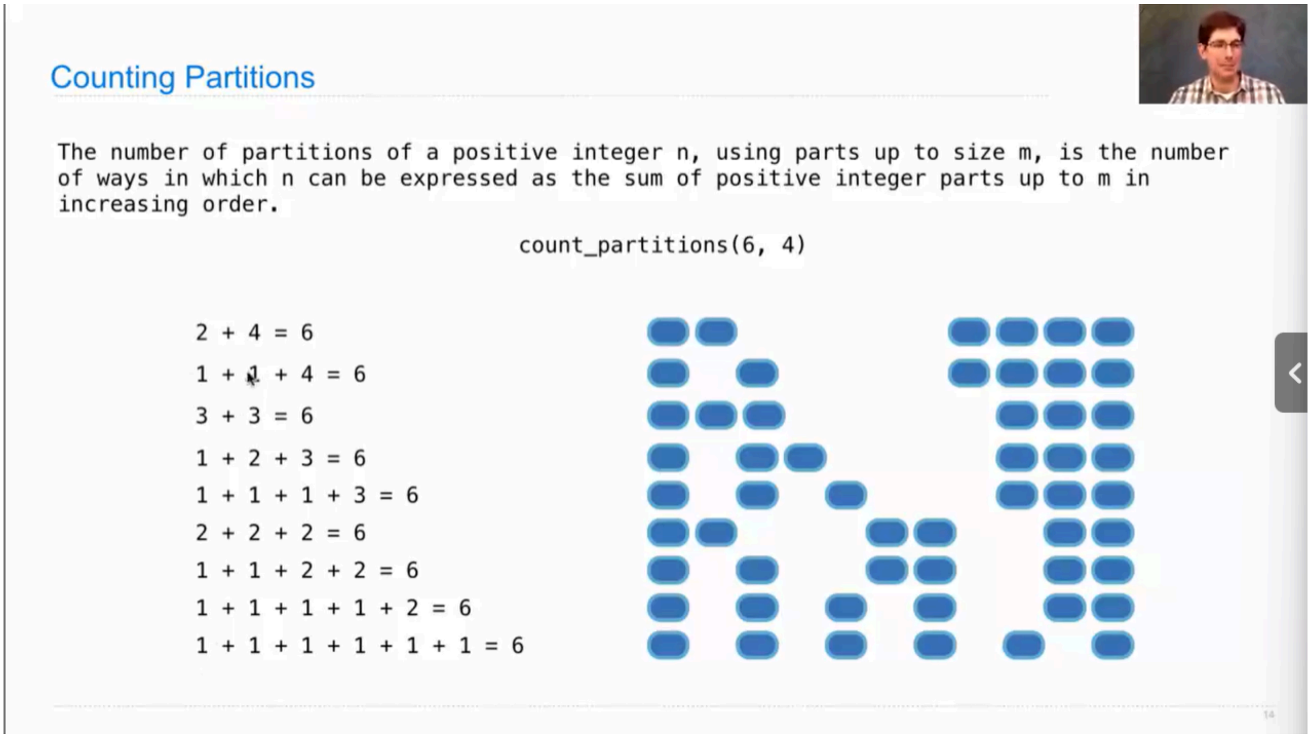




Hanoi Tower



Example: Counting Partitions

> a good example for tree recursion.
>
> 
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strategies to deal with such problems

- recursive decomposition: finding simpler instances of the problem
- explore two possibilities:
  - use at least one 4
  - don't use any 4
- solve tow simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices

reflect on recursion homework

Q2. pingpong

```
def pingpong(n):
    """Return the nth element of the ping-pong sequence.
    >>> pingpong(8)
    8
    >>> pingpong(10)
    6
    >>> pingpong(15)
    1
    >>> pingpong(21)
    -1
```

```
    >>> pingpong(22)
    -2
    >>> pingpong(30)
    -2
    >>> pingpong(68)
    0
    >>> pingpong(69)
    -1
    >>> pingpong(80)
    0
    >>> pingpong(81)
    1
    >>> pingpong(82)
    0
    >>> pingpong(100)
    -6
    >>> from construct_check import check
    >>> # ban assignment statements
    >>> check(HW_SOURCE_FILE, 'pingpong', ['Assign', 'AugAssign'])
    True
    """
    "*** YOUR CODE HERE ***"
    def helper(val, dir, idx):
        if idx > n: return 0
        if num_eights(idx - 1) > 0 or (idx - 1) % 8 == 0: return helper(val, -
dir, idx + 1) - dir
        return helper(val, dir, idx + 1) + dir
    return helper(1, -1, 1)
```

> why recalling on this
> we know that recursion use parameters as the record of the status, however, what should we do if we only have a single parameter but several status to be passed?
> the answer is, use a helper function, which is naturally a gread state machine
> moreover, this recursive function, infact, this is logically equivalent to iteration; the difference is purely in form.

another good example from hw02

```
def count_coins(total):
    """Return the number of ways to make change for total using coins of value
of 1, 5, 10, 25.
    >>> count_coins(15)
    6
    >>> count_coins(10)
    4
```

```
    >>> count_coins(20)
    9
    >>> count_coins(100) # How many ways to make change for a dollar?
    242
    >>> from construct_check import check
    >>> # ban iteration
    >>> check(HW_SOURCE_FILE, 'count_coins', ['While', 'For'])

    True
    """
    "*** YOUR CODE HERE ***"
    def helper(total, coin):
        if total <= 0: return 0
        elif total == coin: return 1
        elif coin == 25: return helper(total - coin, coin)
        return helper(total - coin, coin) + helper(total,
next_largest_coin(coin)
    return helper(total, 1)
```

```
def make_anonymous_factorial():
    return (lambda n:
        (lambda f: f(f))
        (lambda f: lambda n: 1 if n == 1 else mul(n, f(f)(sub(n, 1))))
        (n)
    )
```

> this one is impressive too, though cs61a don't require us to handle this but it is still helpful for us to understand what happen's here

> the reason why i didn't solve this at the first time is that i was struggling about how to call a function in its body, but in stead of considering creating a function that calls itself. a solution to this might be thinking of things step by step, and consider them with their functions.