Quarto Example

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1 Andy's Quarto Example

This is a Quarto website.

To learn more about Quarto websites visit https://quarto.org/docs/websites.

This is an example showing how I use Quarto for my lectures notes. I wrote a blog post about it here.

2 Parallel-transport and geodesics

Here is an excerpt from the GR lecture notes.

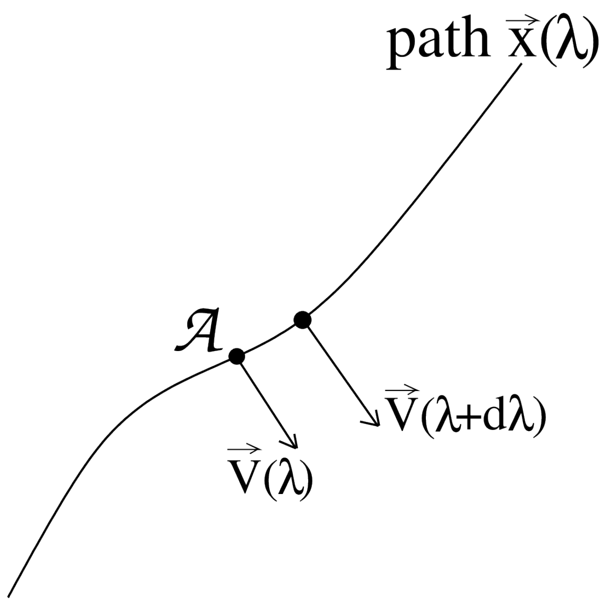

Figure 2.1: Parallel transport of a vector $\vec{v}(\lambda)$ where λ is an affine parameter. For geodesics the vector $\vec{v}$ is a tangent vector, $\vec{u}(\lambda)$.

We can use the idea of parallel transport to construct *geodesics*, defined as curves that parallel-transport their own tangent vectors. That is, for a geodesic

$$\begin{aligned} \boldsymbol{\nabla}_{\vec{u}}\vec{u} &= 0 \\ \text{i.e.} \qquad u^\beta u^\alpha{}_{;\beta} &= 0 \\ \text{i.e.} \qquad u^\beta u^\alpha{}_{,\beta} + \Gamma^\alpha{}_{\beta\gamma} u^\beta u^\gamma &= 0 \\ \text{or} \qquad \frac{d}{d\lambda}\left(\frac{dx^\alpha}{d\lambda}\right) + \Gamma^\alpha{}_{\beta\gamma}\frac{dx^\beta}{d\lambda}\frac{dx^\gamma}{d\lambda} &= 0 \end{aligned}$$

where in the last of these expressions (often called the *geodesic equation*, though the first is also the geodesic equation), λ is a continuous parameter along the curve.

We have some freedom to choose λ – if we choose it to be the proper time of a particle with the curve as its world line, then $\vec{u}$ is the velocity of the particle. However, λ is a more general quantity and can be used also for light rays with no proper time. Any linear transformation of λ, such as $\phi = a\lambda + b$ with a, b constants, has $\vec{x}(\phi)$ a valid solution of the geodesic equation (try the transformation $\lambda \to \phi$) – we refer to λ (or ϕ) as an **affine parameter**.

In a locally-flat region, where the Christoffel symbols vanish, clearly the geodesic equation reduces to

$$\frac{d^2 x^\alpha}{d\lambda^2} = 0$$

which solves to the straight-line solution

$$x^\alpha = A^\alpha \lambda + B^\alpha.$$

In fact we can say, in a very real sense, that all geodesics are **straight**. This definition about "parallel transport of the tangent vector" is the only sensible definition of a straight line – it means that the curve at each point keeps moving in the direction of its local tangent vector. No other frame-independent definition of "straight" makes sense.

3 About

About this site