CONAXIS – USER MANUAL

1. INTRODUCTION

1.1. Overview

CONAXIS (**con**solidation analysis of **axis**ymmetric system) is an open-source finite element program that is completely free to use, distribute, or modify. The main idea of CONAXIS is to use stress-dependent parameters obtained from the constant rate of strain test (CRS) to analyse consolidation behaviour by prefabricated vertical drains. The axisymmetric finite element model based on the poroelasticity theory is used for CONAXIS.

We hope you enjoy working with CONAXIS.

1.2. About this manual

This manual is used as the supplementary materials for the manuscript *"Fully coupled analysis of consolidation by prefabricated vertical drains with applications of constant strain rate tests: a case study and an open-source program"*.

The manual contains information about:

- The installation
- The consolidation theory (Biot's theory), the constant of strait test, unit cell models
- Main features of CONAXIS

1.3. Contact

If you have any question, I would like to hear it. Please contact me via phamhung.207@gmail.com

2. INSTALLATIONS

2.1. For end-users

CONAXIS is provided on GitHub (https://github.com/pham-hung/Conaxis/tree/master/Releases). The release version is now only available for Windows x64. For Linux users, Mac users, or Windows x32 users, the source code needs to be compiled again.

CONAXIS comes with two files: *CONAXIS.exe* and *libiomp5md.dll*. Users need to download them and store them in the same folder.

2.2. For developers

CONAXIS has been developed by using C++ with the cross-platform framework Qt. Hence, the source code can be compiled on any system with little effort. The following libraries are necessary for further development:

- Qt-Framework (either static or dynamic version): https://www.qt.io/
- Eigen library: http://eigen.tuxfamily.org/index.php?title=Main_Page
- Boost: https://www.boost.org/
- Intel MKL: https://software.intel.com/en-us/mkl
- Compiler: VC++ (on Windows) or g++ (Linux)

2.3. Configuration

Depending on the installed paths of the third party libraries, following lines of *Conaxis.pro* file must be changed accordingly:

```
INCLUDEPATH += "C:/Eigen/Eigen"
INCLUDEPATH +=C:/BoostLib
INCLUDEPATH +=C:/BoostLib/stage/lib

INCLUDEPATH += "C:/Program Files
(x86)/IntelSWTools/compilers_and_libraries_2016.4.246/windows/mkl/include"
INCLUDEPATH += "C:/Program Files
(x86)/IntelSWTools/compilers_and_libraries_2016.4.246/windows/mkl/include/intel6
4"
INCLUDEPATH += "C:/Program Files
(x86)/IntelSWTools/compilers_and_libraries_2016.4.246/windows/mkl/lib/intel64"
INCLUDEPATH += "C:/Program Files
(x86)/IntelSWTools/compilers_and_libraries_2016.4.246/windows/mkl/lib/intel64_wi
n"
LIBS += "C:/Program Files
(x86)/IntelSWTools/compilers_and_libraries_2016.4.246/windows/mkl/lib/intel64_wi
n/mkl_core.lib"
LIBS += "C:/Program Files
(x86)/IntelSWTools/compilers_and_libraries_2016.4.246/windows/mkl/lib/intel64_wi
n/mkl_intel_lp64.lib"
LIBS += "C:/Program Files
(x86)/IntelSWTools/compilers_and_libraries_2016.4.246/windows/mkl/lib/intel64_wi
n/mkl_intel_thread.lib"
LIBS += "C:/Program Files
(x86)/IntelSWTools/compilers_and_libraries_2016.4.246/windows/compiler/lib/intel
64_win/libiomp5md.lib"
```

3. THEORY

3.1. Consolidation theory – Biot's coupled theory

The axisymmetric coordinate is shown in Fig. 3-1. We denote *u, v,* and *p* as the displacements of the x-direction, y-direction, and the excess pore pressure. It is noted that CONAXIS assumes that the soil is always fully saturated. Only the excess pore pressure is considered. The hydrostatic pressure then is ignored in the analysis.

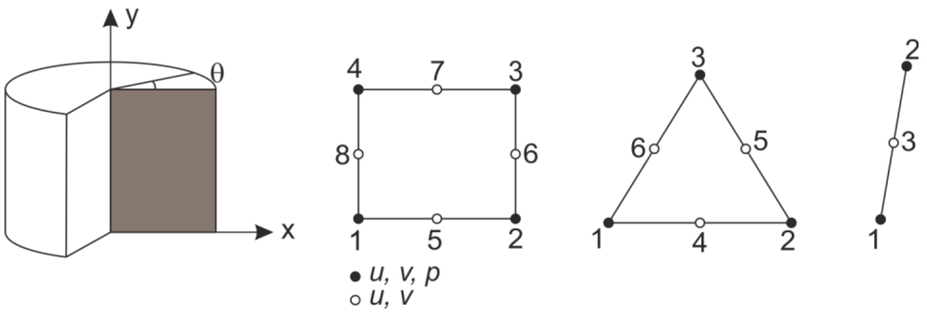


Fig. 3-1: Axisymmetric coordinate system and element types

The system of equations includes the storage equation and the stress equilibrium equations. The storage equation is defined as:

$$\alpha \frac{\partial \varepsilon}{\partial t} + S_s \frac{\partial p}{\partial t} = \frac{\partial}{\partial r} \frac{k_x}{\gamma_f} \frac{\partial p}{\partial x} + \frac{1}{r} \frac{k_x}{\gamma_f} \frac{\partial p}{\partial x} + \frac{\partial}{\partial z} \frac{k_y}{\gamma_f} \frac{\partial p}{\partial y} \tag{3.1}$$

where α is Biot's coefficient; $S_s = nC_f + (\alpha - n)C_s$ is the storativity; *n* is the porosity; C_f is the compressibility of the water; C_s is the compressibility of the solid grain; k_x and k_y are the hydraulic conductivity of the x and y direction; $\varepsilon = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{\theta\theta}$ is the total strain. For the soft soil, $\alpha \sim 1$, and $C_s \sim 0$.

The stress equilibrium equations for x-direction and z-direction are:

$$\begin{aligned} &\frac{\partial \sigma_{xx}}{\partial x} + \frac{\sigma_{xx} - \sigma_{\theta\theta}}{x} + \frac{\partial \sigma_{xy}}{\partial y} - f_x = 0 \\ &\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{xy}}{r} + \frac{\partial \sigma_{yy}}{\partial y} - f_y = 0 \end{aligned} \tag{3.2}$$

where $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, \sigma_{\theta\theta}$ are the total stress components; f_x and f_y are the body force.

The total stress is calculated according to Hook's law and Terzaghi's principle of effective stress with Biot's correction.

$$
\begin{aligned}
\sigma_{xx} &= \sigma'_{xx} + \alpha p = -\left(K - \frac{2}{3}G\right)\varepsilon - 2G\frac{\partial u}{\partial x} + \alpha p \\
\sigma_{yy} &= \sigma'_{yy} + \alpha p = -\left(K - \frac{2}{3}G\right)\varepsilon - 2G\frac{\partial v}{r} + \alpha p \\
\sigma_{\theta\theta} &= \sigma'_{\theta\theta} + \alpha p = -\left(K - \frac{2}{3}G\right)\varepsilon - 2G\frac{u}{r} \\
\sigma_{xy} &= \sigma'_{xy} = -2G\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\
\varepsilon &= \varepsilon_{xx} + \varepsilon_{\theta\theta} + \varepsilon_{yy} = \frac{\partial u}{\partial x} + \frac{u}{r} + \frac{\partial v}{\partial y}
\end{aligned}
\tag{3.3}
$$

Applying Galerkin's approximation method with the backward integration scheme for equations (3.1) and (3.3), the coupled equations are written under the matrix form:

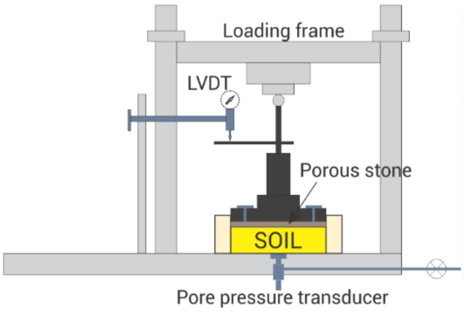
$$
\begin{bmatrix} K & L \\ -L^T & M + \Delta t P \end{bmatrix} \begin{Bmatrix} \Delta d \\ \Delta H \end{Bmatrix} = \begin{Bmatrix} \Delta F \\ -\Delta t P \times H_0 + \Delta t Q \end{Bmatrix} \tag{3.4}
$$

where Δt is the calculation time step; K is the matrix for the displacement field, M and P are the matrices for the transient field; L is the coupled matrix; $\Delta d = \{\Delta u \quad \Delta v\}^T$ is the incremental displacement vector; Δp is the incremental excess pore pressure vector; p_0 is the excess pore pressure of the previous calculation step; ΔF is the incremental load vector; and Q is the extract rate vector.

Detailed information is found in the works of Verruijt [1].

3.2. Constant rate of strain test (CRS)

In the CRST (Fig. 3-2**Error! Reference source not found.**), the soil specimen is axially sandwiched between the porous stone on the top, the rigid Fig. 3-2plate on the bottom, and laterally constrained by O-rings. The load cell is connected to the motor so that the strain rate is constantly kept during the test. The deformation and the pore pressure taken place during the test are measured and recorded through the linear variable differential transformer (LVDT), and the pore pressure transducer, respectively.



Time (min)	Strain (%)	Vertical Stress (kPa)	Pore pressure (kPa)
0.00	0.00	0.01	0.01
1.00	0.01	0.22	0.02
2.00	0.02	0.45	0.04
3.00	0.02	0.67	0.06
4.00	0.03	0.89	0.08
5.00	0.04	1.11	0.09

Fig. 3-2: CRS test scheme and example reading values

The measured data include values recorded by LVDT ΔH (m), the applied load from the loading frame P (kN) or the vertical applied pressure σ (kPa), and from the pore pressure transducer p (kPa). The initial data of a soil sample is the initial height H_0 (m) and the initial void ratio e. According to ASTM-D4186, either the linear theory or the nonlinear theory can be used to interpret data from the CRST. Both theories assume the compressibility m_v (m²/kN) and the consolidation coefficient c_v (m²/s) are constant over the depth of the sample at any time.

At any given time t, in case of the linear theory, equations (23-27) in ASTM-D4186 [2] are used to calculate the average vertical effective stress σ' (kPa), the vertical hydraulic conductivity k_v (m/s), the compressibility m_v and the consolidation coefficient c_v. For the nonlinear theory, equations (X1.1-X1.4) are applied instead.

On the other hand, the compressibility m_v can be expressed as:

$$m_v = \frac{1}{K + 4G/3} = \frac{1+\mu}{3K(1-\mu)} \tag{3.5}$$

where K (kN/m²) is the bulk modulus, G (kN/m²) is the shear modulus and μ is the Poisson's ratio. From equation (3.5), when Poisson's ratio is known, the bulk modulus and the shear modulus are obtained for each time; these parameters are considered to depend on the average vertical effective stress or the void ratio.

3.3. Prefabricated vertical drains – unit cell models

PVDs are band-shaped (typically 100 mm x 4 mm), have channelled plastic cores wrapped with geotextile filters. Water from soft soils passes the geotextile filters and then flows along the channelled cores to the free surfaces. Instead of vertical flow, water flows are radial flows, and the length of the drainage paths is shortened significantly. Thereby, rates of consolidation processes are speeded up. PVDs are often installed in triangular or square patterns with appropriate distances using special machines that have mandrels to push PVDs into the soils. Mandrels can disturb the soils around PVDs and create smear zones.

Each PVD has a specific influence zone that is idealised as a cylinder, which is called a unit cell. The radius of the unit cell r_e (m) is determined based on the field installation pattern, and the distance between PVDs. At the centre of the unit cell, dimensions of the PVD and the smear zone are also converted into equivalent zones, which have radii r_w (m) and r_s (m) respectively.

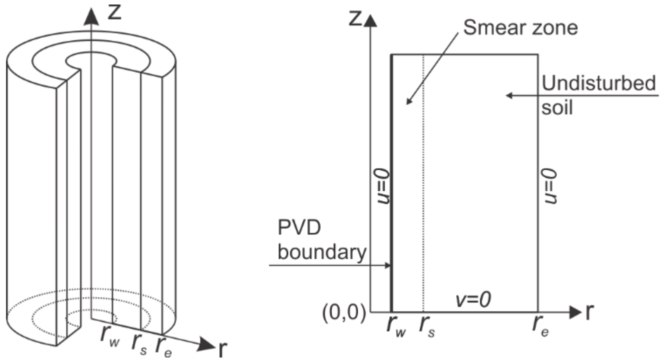


Fig. 3-3: Unit cell models

Consolidation analysis for a unit cell is performed with an axisymmetric model (Fig. 3-3). The left and the right boundaries of the unit cell have *u = 0*; the bottom boundary has *v = 0*. When the well resistance is ignored (i.e. the discharge capacity of PVDs is infinitive), the PVD boundary has zero excess pore pressure (*p = 0*). In contrast, the flow along PVD is considered as the one-dimensional (1D) flow.

4. CONAXIS – MAIN FEATURES

4.1. Graphic user interface (GUI)

The GUI of CONAXIS is shown in Fig. 4-3 that is quite simple and easy to use. The main components are:

- The main toolbar (Fig. 4-3) contains all features of CONAXIS
- The shortcuts of some most used functions
- The main window display the mesh, the result, as well as other information
- The console window for easier debugging

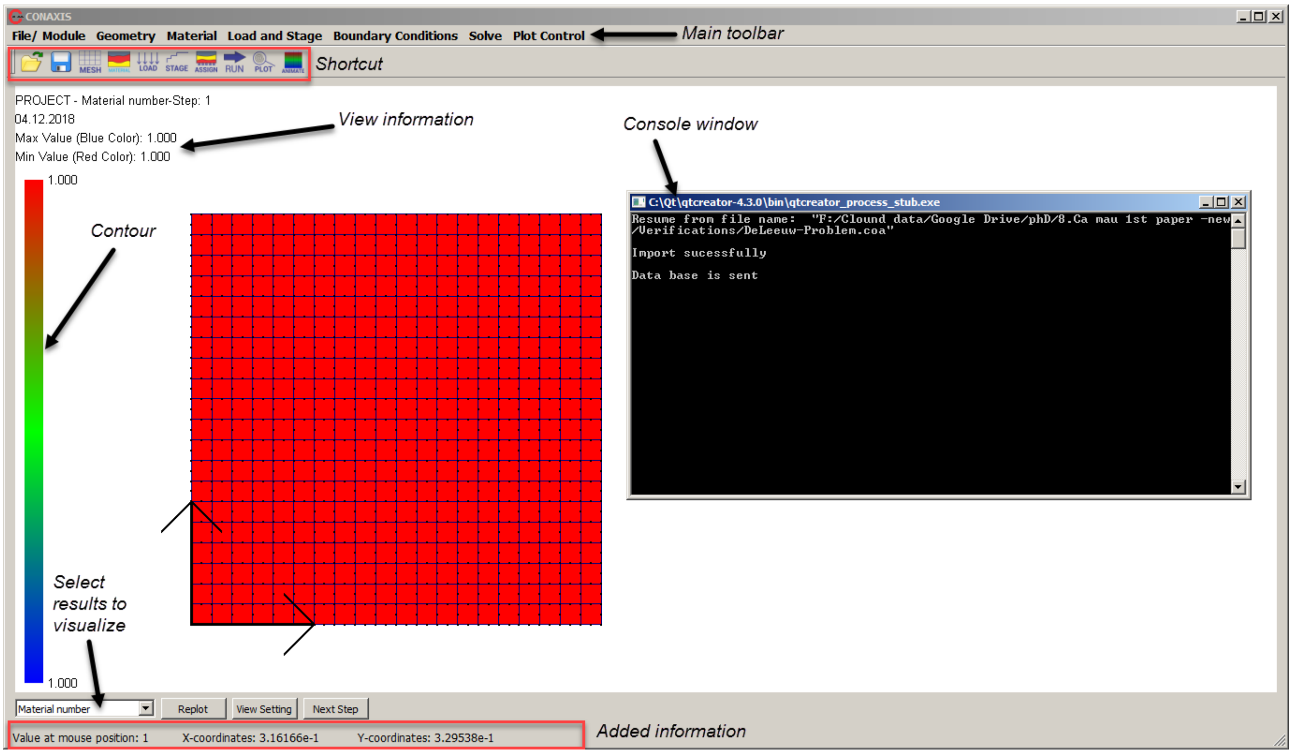


Fig. 4-1: Graphical user interface (GUI) of CONAXIS

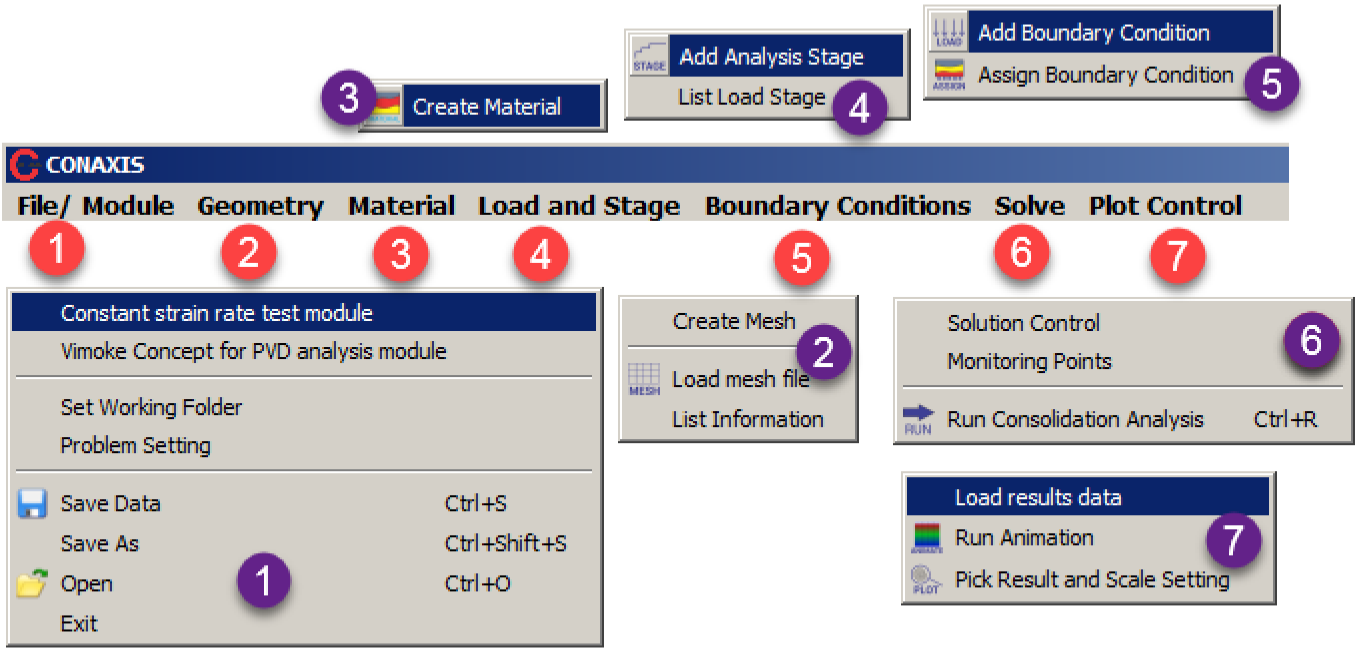


Fig. 4-2: The main functions from the main toolbar

4.2. Unit system

For the current version, CONAXIS uses the unit system as:

- Modulus: kN/m^2
- Length: m
- Hydraulic conductivity: m/s
- Calculation time: day
- Unit weight: kN/m^3

The gravity acceleration is $g = 9.81 m/s^2$ and the unit weight of water is $\gamma_f = 9.81 kN/m^3$.

4.3. Main features

Main features of CONAXIS is presented in Fig. 4-3. Currently, CONAXIS supports:

- ***Mesh***: The simple mesh with rectangular elements can be generated directly. For more complicated geometry, CONAXIS can import mesh data from ASCII file.
- ***Material type***: As stated previously, CONAXIS focuses on stress-dependent parameters related to the CRST. Hence, CONAXIS solves two material laws with constant parameters or with stress-dependent parameters. The source code is open for modification to consider other material laws.
- ***Analysis type***: CONAXIS can be used to analyse the in-situ condition, the undrained behaviour, and the consolidation process. In addition, CONAXIS can simulate the CRST, and perform the back-analysis automatically.

- ***Boundary conditions***: Most of the boundary condition types are supported by CONAXIS, especially the time-dependent boundary conditions.
- ***Element types***: CONAXIS uses the unequal element method approach. The displacement field is approximated with second-order elements as in Fig. 3-1 (8-nodes rectangle, 6-nodes triangle), and the pore pressure field is approximated with first-order elements (4-nodes rectangle, 3-nodes triangle, and 2-nodes line).
- ***Matrix solver***: CONAXIS uses the Intel Pardiso direct solver that is fast and reliable.
- ***Post-processing***: The nodal results, elemental results can be visualised for each time step, or as an animation. Moreover, the coordinate and the deformation can be scaled.

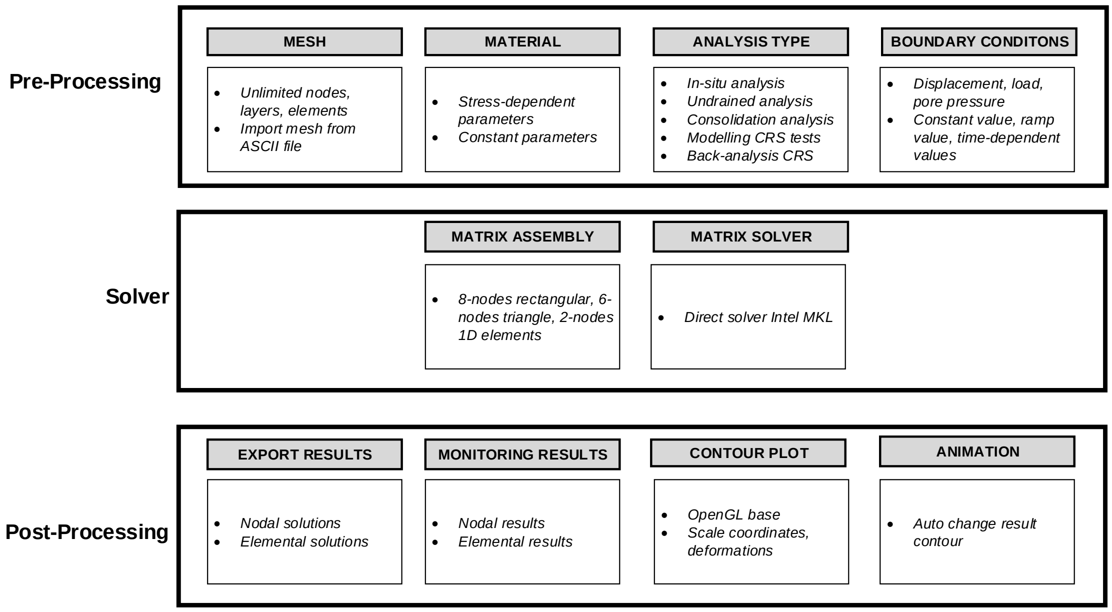


Fig. 4-3: Main features of CONAXIS

4.4. General flow work

Doing an analysis in CONAXIS follows general steps as in Fig. 4-4. The detailed steps are:

- Step1: Creating a mesh
- Step 2: Defining materials
- Step 3: Defining analysis stages
- Step 4: Creating and applying boundary conditions
- Step 5: Defining watch points and run the analysis
- Step 6: Visualising results

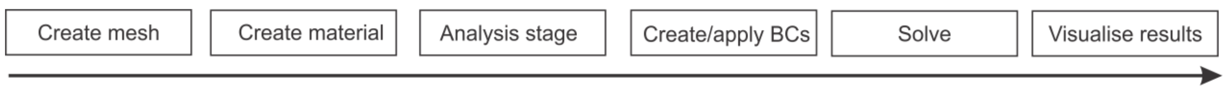


Fig. 4-4: Flow work of CONAXIS

4.5. Step 1 – Meshing

CONAXIS supports two types to create a finite element mesh. The first option is to import ASCII files that contain nodal coordinates and element indices. The second is to create a simple mesh with a built-in function.

4.5.1. *Importing ASCII file*

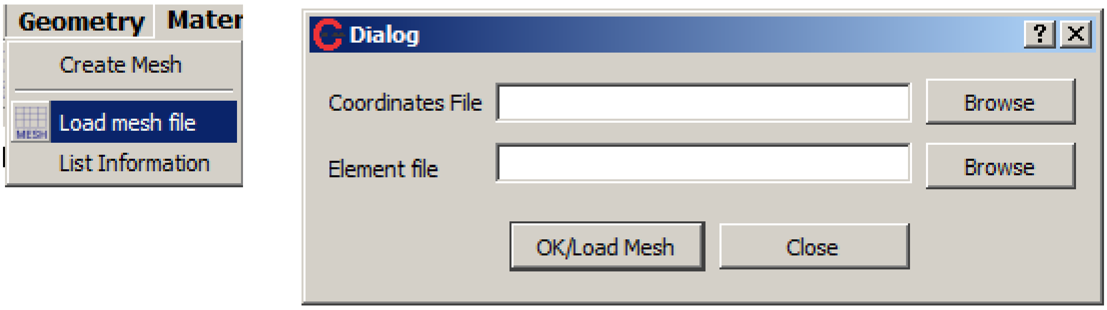


Fig. 4-5: Importing ASCII files

The coordinates file has four columns (Fig. 4-6): (1) – Node index, (2) – X-coordinates; (3) – Y-coordinates, (4) – Z-coordinate. The last column could be ignored.

1	0.10000E+01	0.10000E+02	0.00000E+00
2	0.00000E+00	0.10000E+02	0.00000E+00
3	0.90000E+00	0.10000E+02	0.00000E+00
4	0.80000E+00	0.10000E+02	0.00000E+00
5	0.70000E+00	0.10000E+02	0.00000E+00
6	0.60000E+00	0.10000E+02	0.00000E+00
7	0.50000E+00	0.10000E+02	0.00000E+00
8	0.40000E+00	0.10000E+02	0.00000E+00

Fig. 4-6: Example of coordinates file

The element file has 12 columns (Fig. 4-7). The first column is the element index. The column from 2 to 9 is the node-2 to node-9 of the element. The order of nodes is presented in Fig. 3-1. The 10th column is the number of node per element (8 for a rectangle, 6 for a triangle, and 2 for a line element). The 11th column is the material number of the element. If the element belongs to the smear zone, the 12th column is 2. Otherwise, if the element belongs to the undisturbed zone, the 12th column is 1.

1	1	4	271	219	3	270	221	220	8	1	1
2	4	6	419	271	5	418	369	270	8	1	1
3	6	8	567	419	7	566	517	418	8	1	1
4	8	10	715	567	9	714	665	566	8	1	1
5	10	2	14	715	11	13	813	714	8	1	1
6	219	271	273	217	221	272	222	218	8	1	1
7	271	419	421	273	369	420	370	272	8	1	1
8	419	567	569	421	517	568	518	420	8	1	1
9	567	715	717	569	665	716	666	568	8	1	1
10	715	14	16	717	813	15	814	716	8	1	1

Fig. 4-7: The element file

4.5.2. *Creating mesh with CONAXIS*

CONAXIS only supports to create a simple mesh with all rectangular elements. This feature designs specifically for unit cell models. All the information is given in Fig. 4-8. When the well resistance is considered, CONAXIS creates 1D element along the PVD boundary. Based on the radius of the PVD, and the discharge capacity, CONAXIS calculates the hydraulic conductivity of the 1D element.

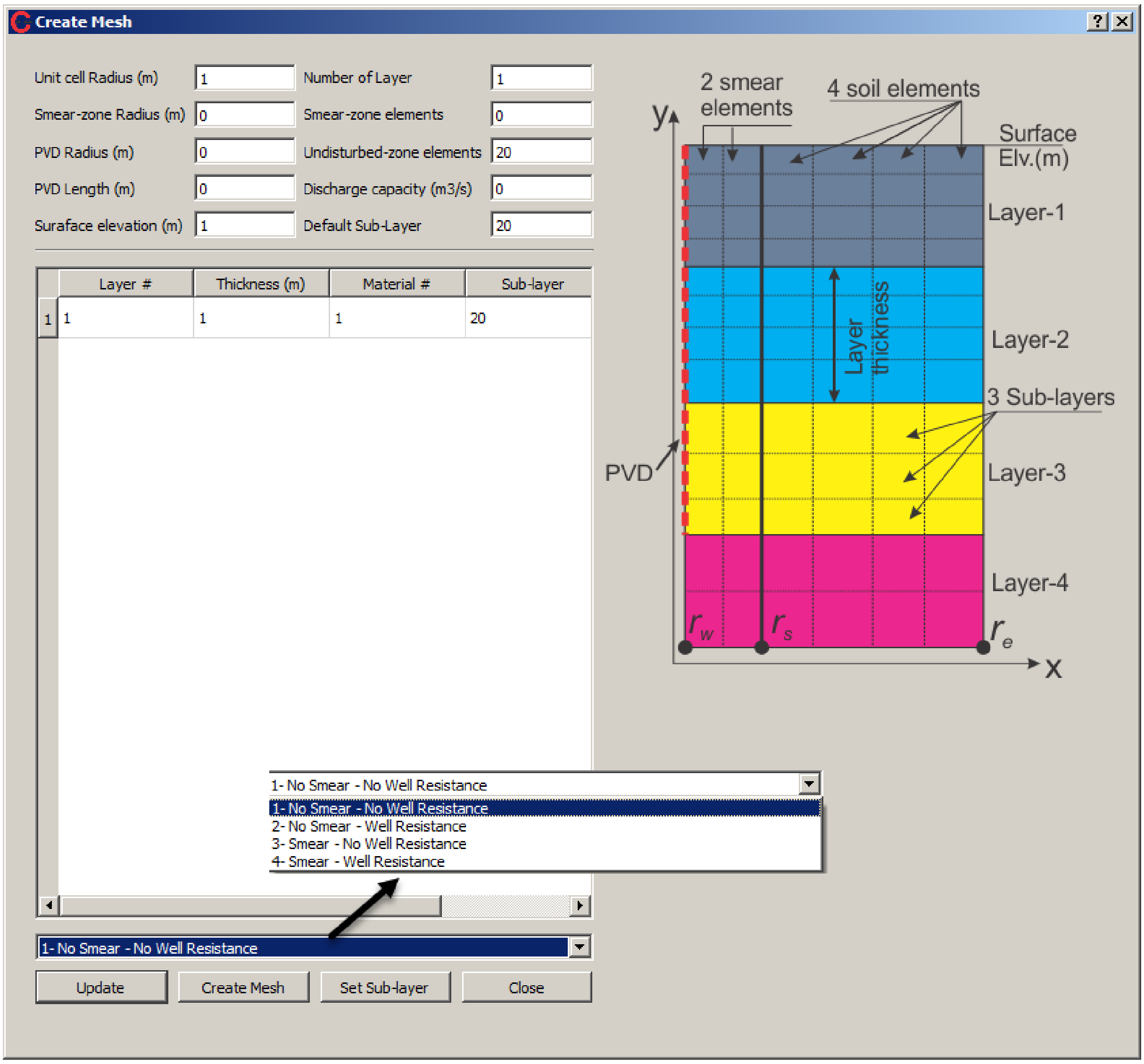


Fig. 4-8: Creating mesh with CONAXIS

4.6. Step 2 – Defining materials

Materials can be defined as in Fig. 4-9. Currently, CONAXIS has two material types.

- The first type is the linear elastic material that has all constant parameters. This type of material is suitable for verification purposes.
- The second type that have parameters depend on the vertical effective stress. These dependences are obtained from the CRS test.

In Fig. 4-9, the k_h/k_v ratio is the ratio between the horizontal hydraulic conductivity and the vertical hydraulic conductivity. The k_s/k_h ratio (or *Cd*) is the ratio between hydraulic conductivity of the smear zone and the undisturbed zone.

In the smear zone, the horizontal hydraulic conductivity is calculated as:

$$k_{hs} = C_d \times \left(\frac{k_h}{k_v}\right) \times k_v \qquad \textbf{(3.6)}$$

For example, if the vertical hydraulic conductivity is $k_v = 10^{-9}(m/s)$, the ratio *$k_h/k_v=3$,* and the *Cd=0.4,* the horizontal hydraulic conductivity of the smear zone is:

$$k_{hs} = 0.4 \times 3 \times 10^{-9}(m/s) \qquad \textbf{(3.7)}$$

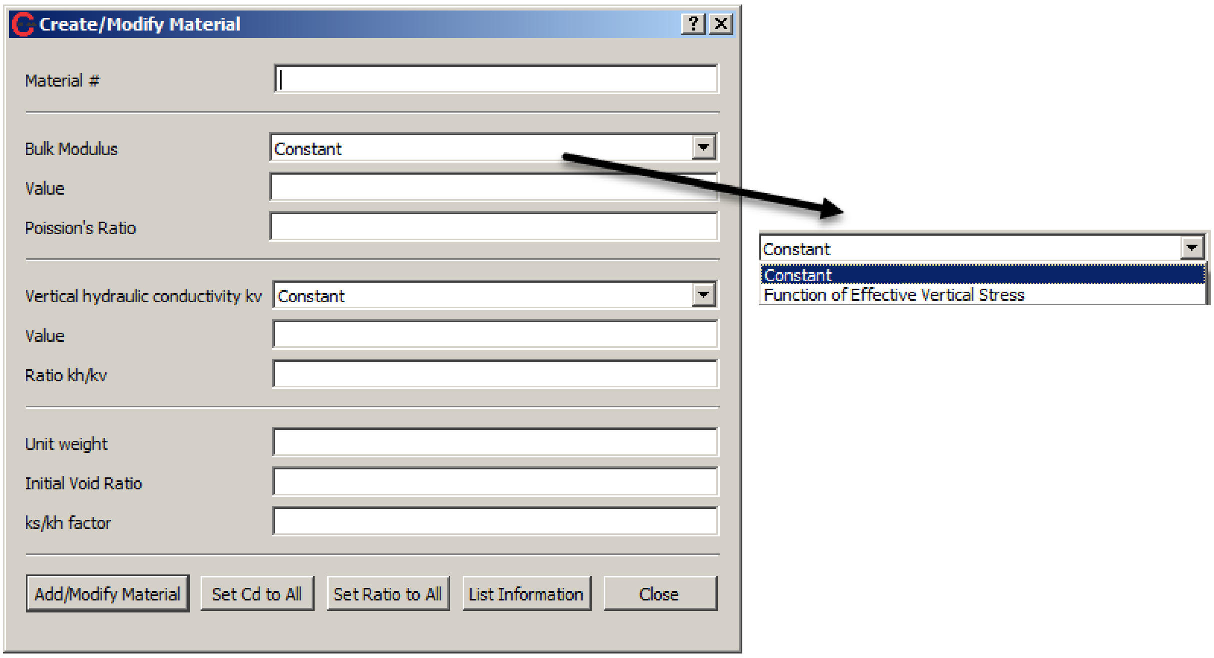


Fig. 4-9: Defining materials

4.7. Step 3 - Analysis stages

CONAXIS has three types of analysis: in-situ, undrained, and consolidation (Fig. 4-10).

- **In-situ analysis**: the excess pore pressure is not considered. This is used to generate the field effective stress before applying load. The gravity load (based on the density of materials) is used. Hence, the checkbox “include gravity load” is checked.
- **Undrained analysis** has calculation time $\Delta t = 0$, or it means that water cannot dissipate. This analysis is used to generate the initial excess pore pressure before the consolidation process.
- **Consolidation analysis**: The excess pore pressure dissipates during the process.

In Fig. 4-10, the time step can be calculated linear based on the total consolidation time (duration) and the number of sub-step. Alternatively, the time step can be imported via ASCII file. In this case, the number of sub-step is taken as the number of rows of ASCII file.

For example, if the consolidation time is 100 day and the number of sub-step is 200, the time step is:

$$\Delta t = \frac{T}{ns} = \frac{100}{200} = 0.5(day) \quad \textbf{(3.8)}$$

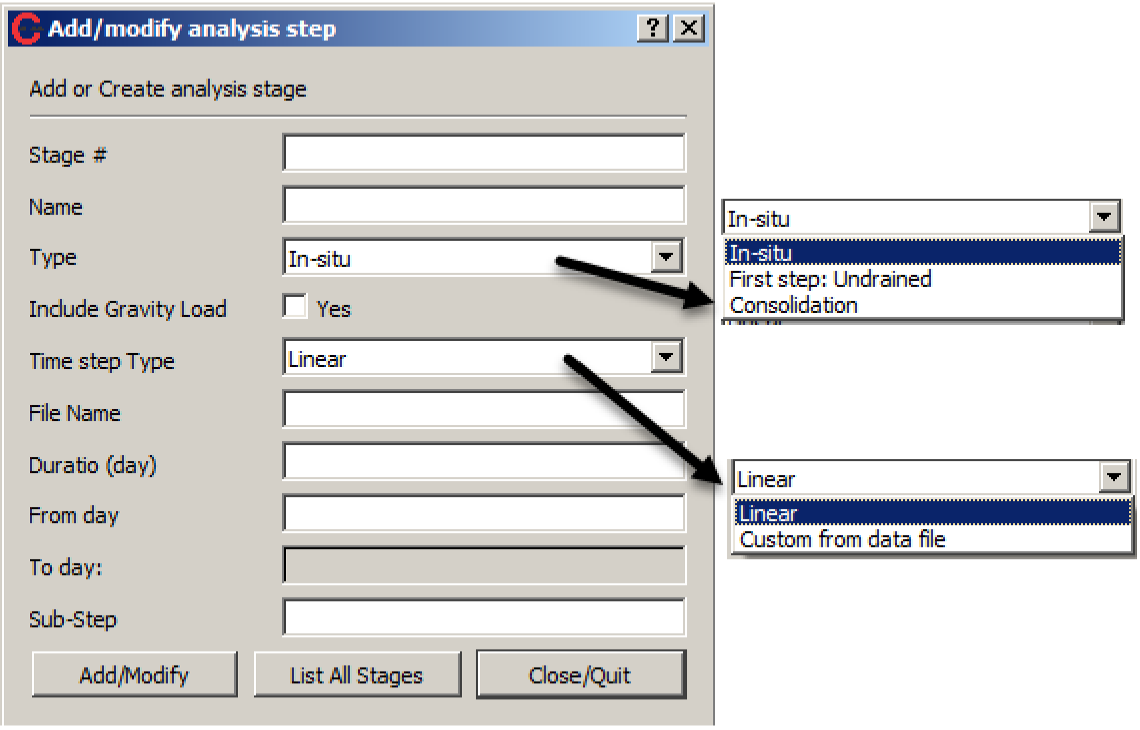


Fig. 4-10: Defining analysis stages

4.8. Step 4 – Creating/applying boundary conditions (BCs)

4.8.1. *Creating BCs*

CONAXIS supports all type of boundary conditions (Fig. 4-11). Each boundary condition can have three options for the value:

- **Constant**: The value does not change overtime
- **Ramp**: The value depend on the start value, end value and the calculation time. For instance, if the start value is 0, the end value is 100, the analysis lasts 10 days with 20 sub-steps. At the day 3 (or the calculation step 10), the value of the boundary condition will be 30.
- **Time dependence**: An data file (Fig. 4-12) can be imported. The first column is the time, and the second column is the boundary condition value. CONAXIS uses this file to interpolate the value corresponding to the calculation time.

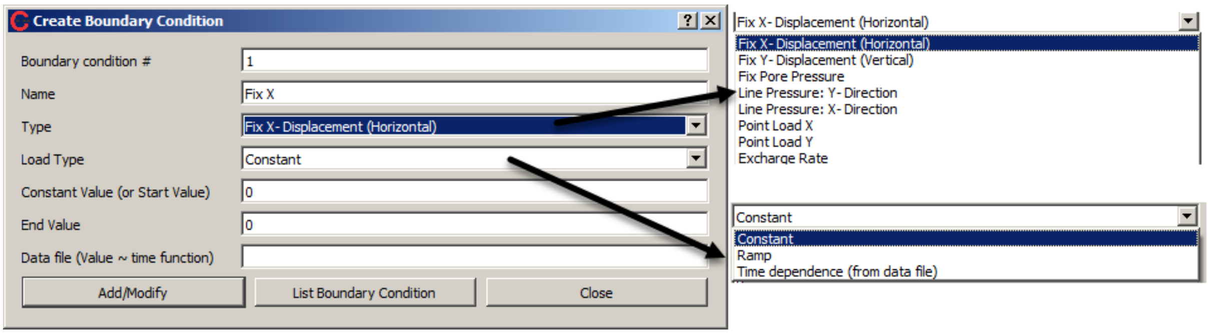


Fig. 4-11: Defining boundary conditions

0	0.00
1	0.30
2	0.61
3	0.91
4	1.22
5	1.52
6	1.82

Fig. 4-12: Time dependent boundary condition table

4.8.2. *Applying BCs*

Boundary conditions from Step-4 need to be assigned for each analysis stage (Fig. 4-13). The boundary conditions are assigned to nodes. Hence, to find nodes, it is necessary to provide ranges of coordinates. Alternatively, a file contains a list of nodes can be imported.

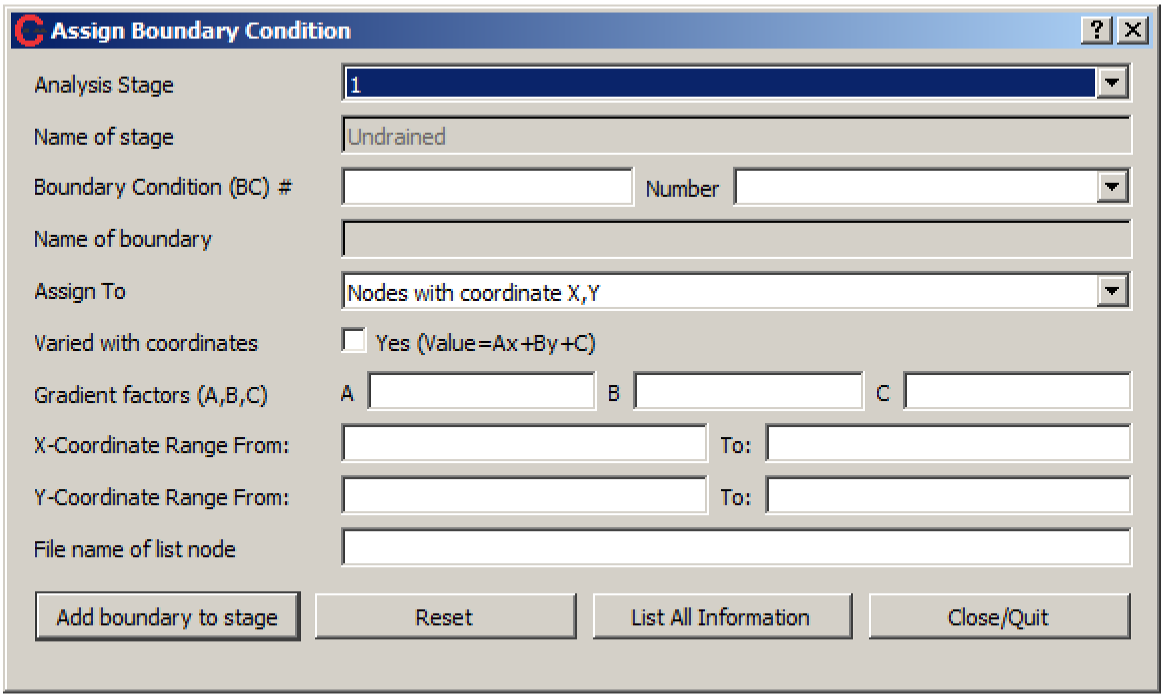


Fig. 4-13: Applying boundary condition to each analysis stage

4.9. Step 5 – Run analysis

4.9.1. *Controlling output results*

CONAXIS has four options for output results that are saved as ASCII files.

- **Nodal solution**: The displacement and the excess pore pressure of all nodes.
- **Element stress** (Discontinue): The stress components of all elements are calculated and saved. The element stress is not continuous.
- **Average element stress:** For instance, in Fig. 4-14, there are four elements that have stress S1, S2 S3, S4. All four elements share one common node – node 3. The average stress of node 3 is calculates as:

$$S_{n-3} = \frac{S1 + S2 + S3 + S4}{4} \quad \textbf{(3.9)}$$

- **Material parameters:** The bulk modulus and the vertical hydraulic conductivity of each element are saved.

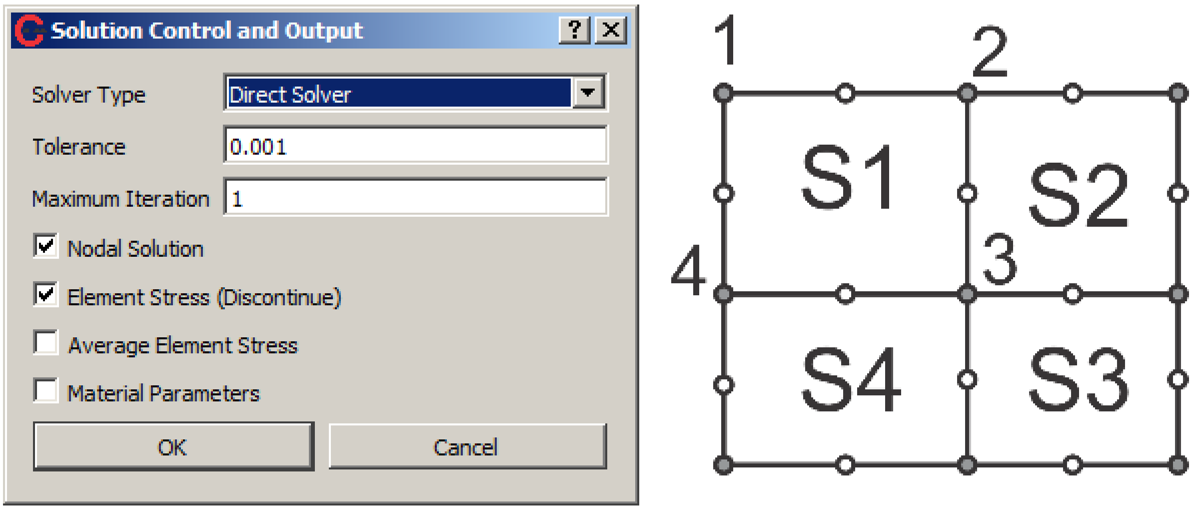


Fig. 4-14: Controlling output results

4.9.2. *Watch lists – Monitoring results*

Sometimes, it is more convenient to save directly results of specific nodes for plotting graph, or processing data on Excel, or comparing model results with field data. We call these are “watch lists” (Fig. 4-15).

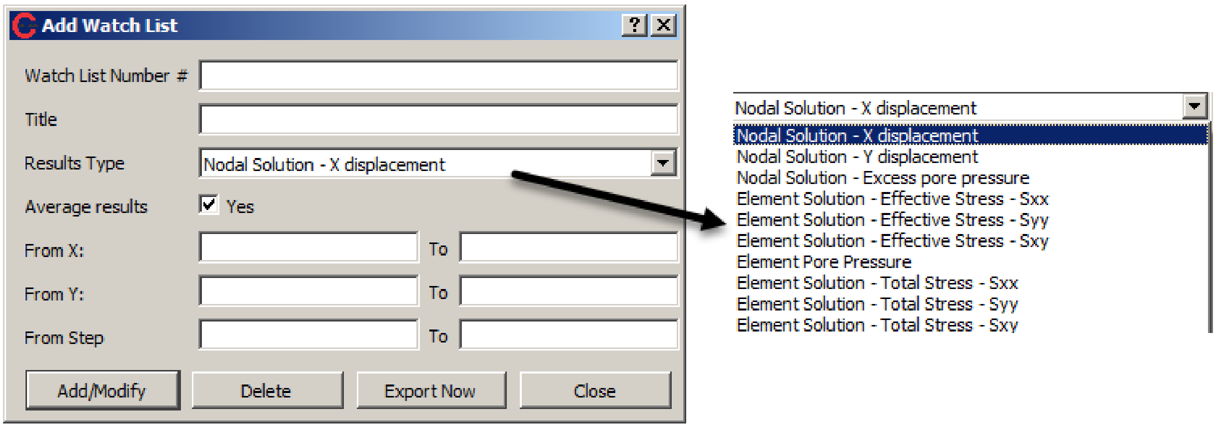


Fig. 4-15: Defining watch lists

A watch list is defined by:

- **Title:** This is used as the name of the output file.
- **Result types:** It can be nodal solution, or elemental solution.
- **Check box Average results:** If this is check, results of found nodes are averaged. If not, results of each node are saved.
- **From X to ..:** Range of X-coordinate to find the nodes or elements.
- **From Y to..:** Range of Y-coordinate.
- **From Step..To..:** Range of calculation steps.

4.10. Visualising results

4.10.1. *Setting contour bar and plotted information*

The contour bar can be placed on the left, the right, or the bottom of the screen (Fig. 4-17). The number of contour and the font size can be adjusted to have a better view. For small number, the numeric type should be set to scientific type (1.264e-2). Otherwise, it can be set to the float type (6.123) or the integer type.

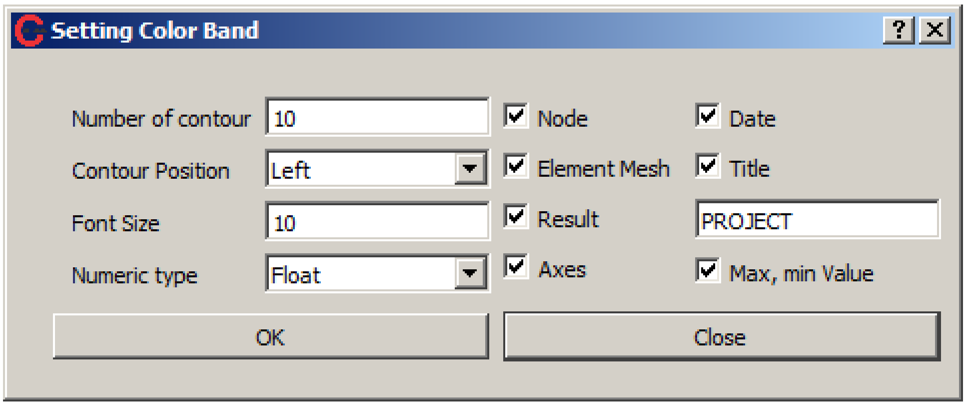


Fig. 4-16: Options for visualisation

4.10.2. *Plotting result for a calculation step*

The following results can be chosen to be plotted (Fig. 4-17):

- The horizontal displacement U

- The vertical displacement V
- The excess pore pressure P
- The material number
- The element type (type 2 for the smear zone and type 1 for the undisturbed zone)
- Imported nodal solution or elemental solution can be plotted also via *Custom Plot* and *Element Result Plot* option.

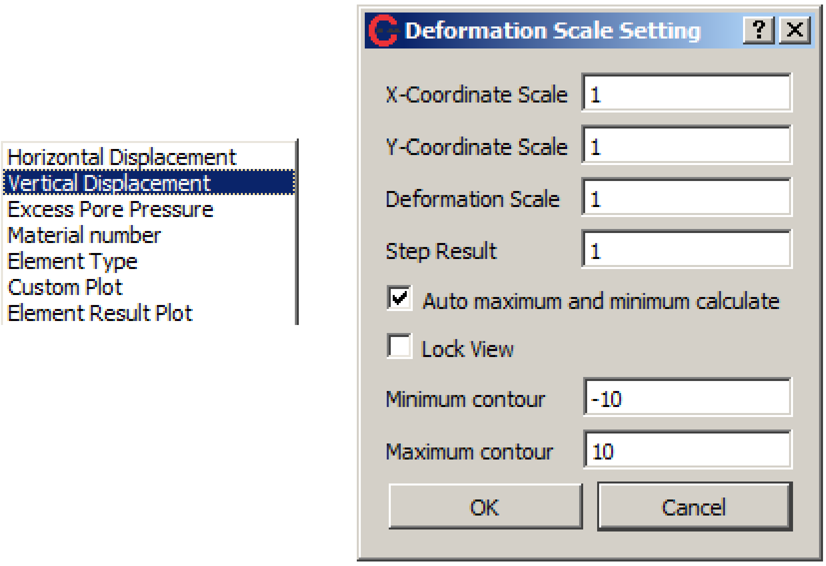


Fig. 4-17: Plotting result for one calculation step

In some cases, when one direction is much larger than the other, it is easier if the coordinate can be scaled to have a better view (Fig. 4-18). The deformation also can be exaggerated.

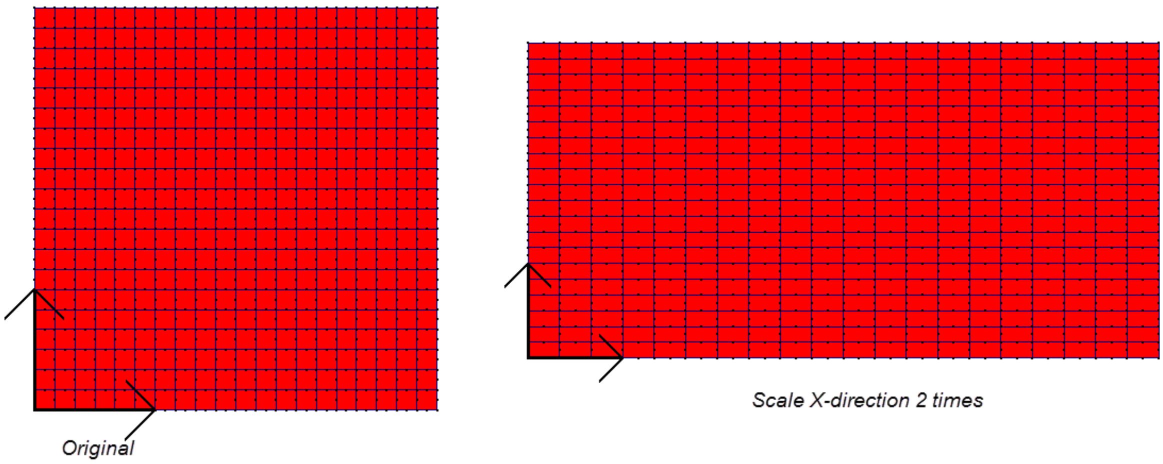


Fig. 4-18: Scale coordinates

The maximum and minimum contour values can be automatically calculated from results or can be fixed. In Fig. 4-17, when the *Lock view* is checked, the mouse functions (scroll, drag) is disabled.

4.10.3. *Animation*

To observe a continuous change of the deformation or the excess pore pressure, an animation is a useful option (Fig. 4-19).

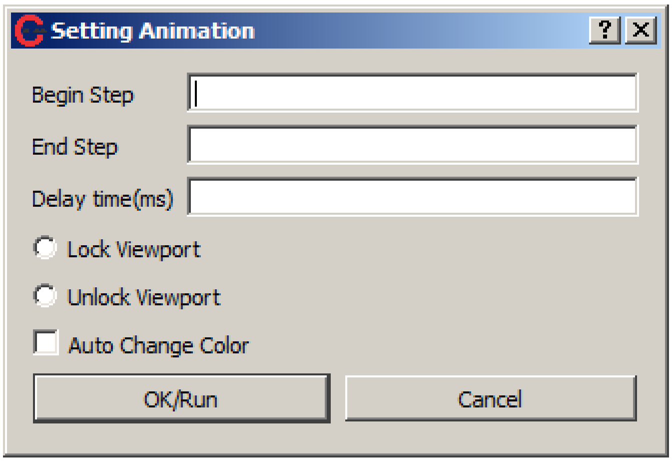


Fig. 4-19: Plotting results as an animation

4.11. Modelling CRS tests

The CRS test can be modelled automatically with CONAXIS using either load control procedure or strain control procedure (Fig. 4-20). The input is the test data file which contains four columns (Fig. 4-21): (1) – Testing time - minutes, (2) – Strain %, (3) – Total applied stress – kPa, (4) – Excess pores pressure – kPa.

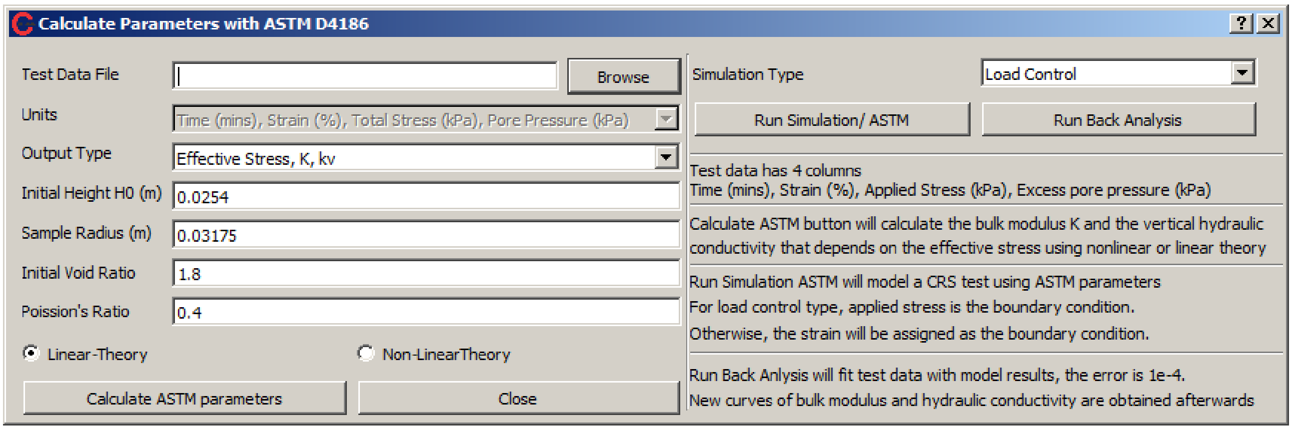
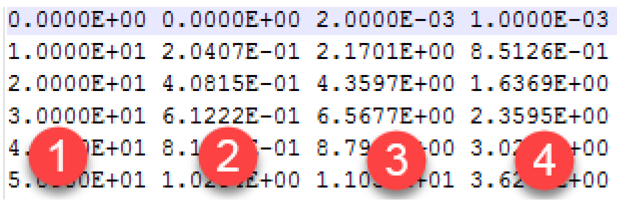


Fig. 4-20: Modelling CRS test



0.0000E+00	0.0000E+00	2.0000E-03	1.0000E-03
1.0000E+01	2.0407E-01	2.1701E+00	8.5126E-01
2.0000E+01	4.0815E-01	4.3597E+00	1.6369E+00
3.0000E+01	6.1222E-01	6.5677E+00	2.3595E+00
4.[illegible]E+01	8.1[illegible]-01	8.79[illegible]00	3.0[illegible]+00
5.[illegible]E+01	1.0[illegible]E+00	1.10[illegible]+01	3.62[illegible]+00

Fig. 4-21: CRS test data

Values of the bulk modulus and the vertical hydraulic conductivity that depend on the effective stress can be obtained with three different ways:

- Using linear theory from ASTM D4186 [2]
- Using nonlinear theory from ASTM D4186
- Using back analysis method

5. FUTURE WORKS

CONAXIS is still under developing. Those following features are considered:

- Add more material laws such as the modified Cam-Clay model, the Mohr Coulomb model.
- Automatic time step

REFERENCES

1. Verruijt, A., *PoroElasticity*. 2016: http://geo.verruijt.net/.
2. ASTM/D4186M-12e1, *Standard Test Method for One-Dimensional Consolidation Properties of Saturated Cohesive Soils Using Controlled-Strain Loading*. 2014, ASTM International: West Conshohocken, PA, 2012.