VERIFICATIONS OF A FINITE ELEMENT CODE BASED ON THE POROELASTICITY THEORY

1. MANDEL'S PROBLEM

1.1. Problem description

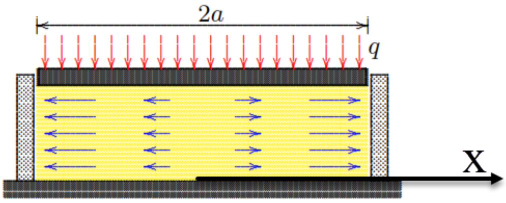


Figure 1: Mandel's problem – 2D section

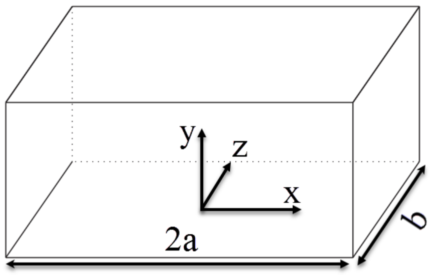


Figure 2: Mandel's problem – 3D geometry

A rectangular soil sample, which has $2a \times b$ dimension, is constrained by two rigid plates on the top and on the bottom. Two sides of x-direction are free and drained. The z-direction is also constrained.

A constant force 2F causes a uniform pressure q (*kN/m²*) on the top boundary.

$$q = \frac{2F}{2a \times b}$$

When $b = 1\,(m)$, $q = \dfrac{F}{a}$

1.2. ANALYTICAL SOLUTION

The soil has properties:

- k : *Hydraulic conductivity* (m/s)
- C_s, C_m, C_f : Compressibility of solid grains, porous material, and water (*m²/kN*)
- *n*: Porosity
- γ_f : Unit weight of water (*9.81 kN/m³*)
- ν : Poisson's ratio
- *K, G*: Bulk modulus and shear modulus (*kN/m²*). $G = \dfrac{3K(1-2\nu)}{2(1+\nu)}$ and $K = \dfrac{1}{C_m}$
- c_v : Consolidation coefficient (*m²/s*). $c_v = \dfrac{k}{\gamma_f\left(\alpha^2 m_v + S\right)}$
- $S = nC_f + (\alpha - n)C_s$: Specific storage

- $m_v = \dfrac{1}{(K+4G/3)}$: Compressibility

When the soil is isotropic, the analytical solutions of the excess pore pressure *p* at the position *x (m)* and at the time *t (s)* is calculated (Abousleiman et al. 1996):

$$p = \frac{2F}{aA_1}\sum_{i=1}^{\infty}\frac{\sin\beta_i}{\beta_i - \sin\beta_i\cos\beta_i}\times\left(\cos\frac{\beta_i x}{a} - \cos\beta_i\right)\times\exp\left(-\frac{\beta_i^2 c_v t}{a^2}\right)$$

where:

- *p*: Pore pressure (kN/m²)
- *q*: Top pressure (kN/m²)
- $A_1 = \dfrac{3}{B(1+\nu_u)}$ where B is Skempton's coefficient, and ν_u is undrained Poisson's ratio. For soft soil, $\nu_u = 0.5$
- $A_2 = \dfrac{\alpha(1-2\nu)}{1-\nu}$
- β_i with i=1, 2, 3… are the roots of function: $\tan\beta_i - \beta_i\dfrac{A_1}{A_2} = 0$

1.3. Numerical model verification

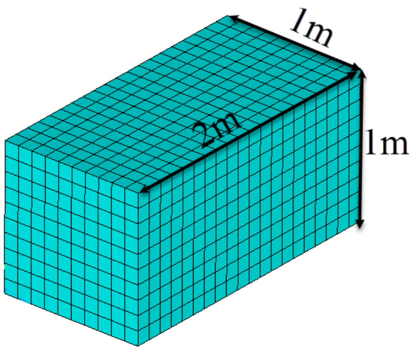
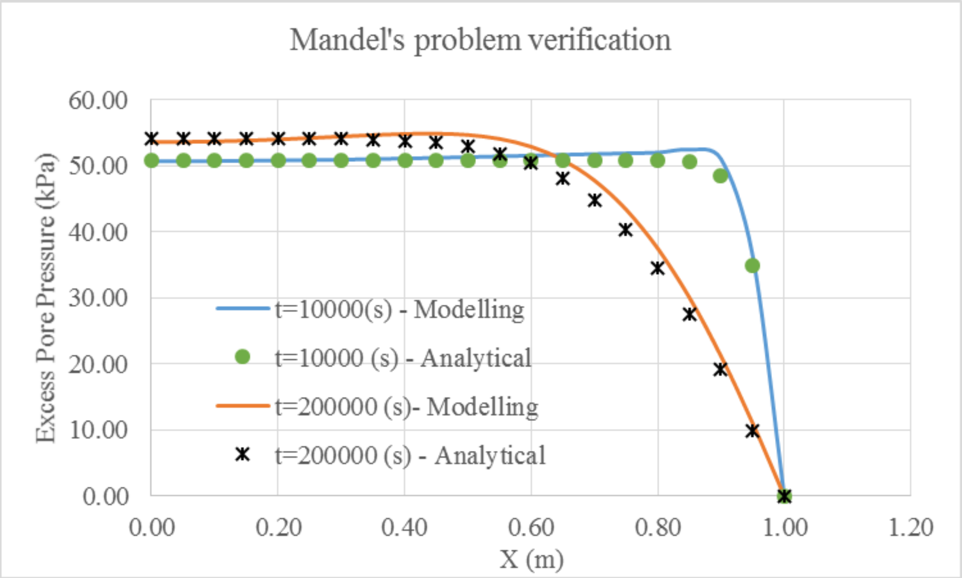


Figure 3: Mandel's problem – Finite element mesh

Input parameters	Value
Model width *a*	1.0 *m*
Model thickness *b*	1.0 *m*
Bulk modulus	500 *kN/m²*
Poisson's ratio (drained/undrained)	0.1/0.5
Hydraulic conductivity	1e-9 *(m/s)*
Top pressure	100 *(kN/m²)*
C_s, C_f	0 and 1e-7 *(m²/kN)* (solid grain is incompressible)
Porosity	0.64

Time step	1000 *s*
Calculation step	200



2. CRYER'S PROBLEM

2.1. PROBLEM DESCRIPTION

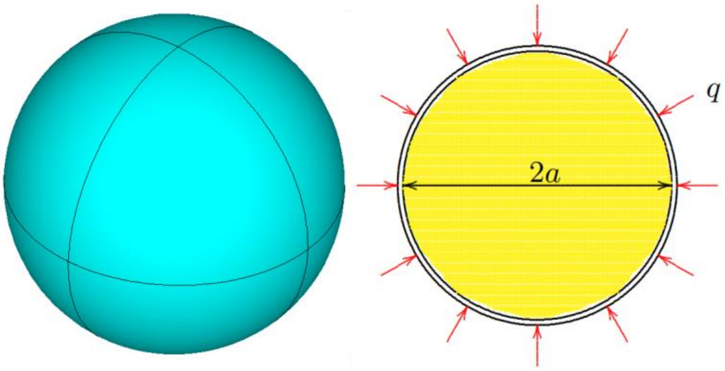


Figure 4: Cryer's problem

A sphere soil sample, which has the radius *a*, is compressed by a uniform load q (kN/m^2) via the outer boundary. The outer boundary is drained.

2.2. ANALYTICAL SOLUTION

The pore pressure at the center of the soil sample at the time *t* (s) is calculated by (Verruijt 2016):

$$\frac{p_c}{p_0} = \eta \sum_{j=1}^{\infty} \frac{\sin\xi_j - \xi_j}{(\eta - 1)\sin\xi_j + \eta\xi_j \cos\xi_j / 2} \exp\left(-\xi_j^2 c_v t / a^2\right)$$

where

- ξ_j are the positive roots of the equation: $\left(1 - \eta\xi_j^2\right)\tan\xi_j - \xi_j = 0$

- $\eta = \dfrac{K + 4G/3}{2G}\left(1 + KS/\alpha^2\right)$
- $p_0 = \dfrac{q}{\alpha\left(1 + KS/\alpha^2\right)}$: The initial pore pressure (kN/m^2)

The other parameters are same as the Mandel's problem.

2.3. Numerical model verification

Table. 1. Input parameters for the verification

Input parameters	**Value**
Model radius	1.0 m
Bulk modulus	500 kN/m^2
Poisson's ratio	0.1
Hydraulic conductivity	1e-9 m/s
Pressure *q*	100 kN/m^2
C_s, C_f	0 and 1e-7 m^2/kN (solid grain is incompressible)
Porosity	0.64
Time step	86400 s
Calculation step	100

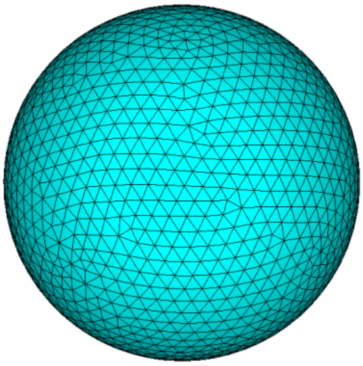


Figure 5: 3D FEM model for Cryer's problem

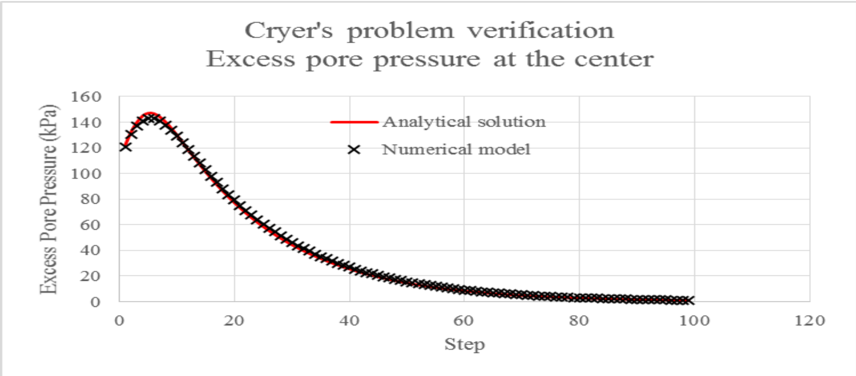


Figure 6: Numerical results vs analytical solutions

3. CONCLUSION

The numerical results for both Mandel and Cryer problems fit well with the analytical solutions. The finite element code is therefore considered to be reliable.

References

Abousleiman, Y., Cheng, A. H.-D., Cui, L., Detournay, E., and Roegiers, J.-C. (1996). "Mandel's problem revisited." *Géotechnique*, 46(2), 187-195.

Verruijt, A. (2016). *PoroElasticity*.