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Supplier Commitment and Production Decisions Under a Forecast-Commitment Contract

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Manufacturing firms in capital-intensive industries face inherent demand volatility for their products and the inability to change their capacity quickly. To cope with these challenges, manufacturers often enter into contracts with their customers that offer greater certainty of supply in return for more predictable orders. In this paper, we study a "forecast-commitment" contract in which the customer provides a forecast, the supplier makes a production commitment to the customer based on the forecast, and the customer's minimum order quantity is a function of the forecast and committed quantities. We provide a complete analysis of the supplier's decisions when there is a single customer facing uncertain demand. We first show that the supplier has two dominant commitment strategies: committing to the forecast or committing to the production quantity. We then characterize the jointly optimal commitment and production strategy for the supplier and extend the results to consider a capacity constraint. We show that the proposed contract can moderate the supplier's motivation to underproduce, and due to the structure of the contract and the form of the supplier's optimal strategy, also limits the customer's incentive to overforecast. We also provide results for a capacitated two-customer example, which show that the supplier's choice of production quantity for each customer is not necessarily nondecreasing in the total available capacity.

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1. Introduction

As supply chains become more global and decentralized, companies are increasingly adopting mechanisms that facilitate and encourage the sharing of reliable demand and supply information. The need for reliable information is even more amplified in industries where products are highly customized, and significant cyclical variations in demand occur. For example, in the application specific integrated circuits (ASICs) industry, suppliers must effectively manage capacity over the short lifecycle of products, and customers desire to secure consistent supply of high-quality products at competitive prices. Supply contracts have emerged as an important means to facilitate information sharing, and researchers have begun to investigate truth-inducing mechanisms in the context of supply contracts.

But the sharing of reliable information depends, in part, on a mutuality of trust between parties within a supply chain that is often difficult to capture in an analytical model or in the conditions of a formal contract. Our research was initiated at the request of a manufacturer of ASICs whose primary customers produce various finished goods including printers, computers, and medical equipment. When our research began, the ASIC industry was operating near full utilization and many customers were gaming the system by submitting unrealistically high orders, realizing that suppliers would likely fill only a portion of their requests. The management at the company that motivated our research realized that they needed to establish some provisions to limit the extent to which customers would overorder (because ASICs rarely have a profitable secondary market). They also realized that due to the cyclic nature of the industry, it was important to build customer loyalty so that these customers would continue to order in leaner economic times. The management of the company sought to take actions that they hoped would lead to greater trust, and consequently, to more reliable exchange of demand and supply information.

The ASIC manufacturer was already implementing a policy in which it would commit to a delivery quantity in response to each customer forecast of a future order (usually two or three months in advance). The policy of providing commitments was viewed as a long-run strategic weapon to secure customer loyalty by giving customers more accurate information about what would actually be delivered. At the time our research began, the company was also developing a "point system" for prioritizing customers, with the view toward using these points within a decision framework for making commitment and production decisions. The company was already using some notion of priorities in making these decisions (heuristically), but wished to develop more formal agreements with its customers and to better understand the consequences of such agreements.

It was with this environment as a backdrop that we, in discussion with the management of the ASIC manufacturer, developed the concept of a forecast-commitment contract. Under such a contract, the customer provides a forecast and agrees to purchase at least a portion, α, of it. In return for the minimum purchase guarantee, the manufacturer makes a commitment to deliver a specified minimum quantity. The manufacturer would simultaneously make production decisions, taking into consideration the relative priority of the customer and the possibility that the customer's final order might deviate from his initial forecast. What the management of the ASIC manufacturer had in mind was to internalize the cost of securing customer loyalty by "charging" itself a penalty per unit for shortfalls of the commitment from the forecast, as well as a penalty per unit for shortfalls of the delivery quantity from the lesser of the realized demand and the commitment quantity. As an alternative, these penalties could be explicit payments to the customer for the stated shortfalls (we present our analysis in this manner). The former penalty can be viewed as one type of loss-of-goodwill penalty, while the latter would reflect lost sales or backorder penalties, plus any associated loss of goodwill.

The contract has the desirable property that, in addition to price, there are three parameters that can be adjusted depending on the specific characteristics of the customer and the manufacturer's desire to provide good service. Thus, it is somewhat more flexible than most other supply contracts discussed in the literature, and provides explicit methods (via the penalties) for giving preference to "better" customers.

In this paper, we analyze the forecast-commitment contract exclusively from the supplier's perspective. In the concluding section, we briefly discuss the customer's problem of choosing a forecast to submit and other related issues.

The remainder of this paper is organized as follows. In §2, we discuss some relevant aspects of the supply contract literature. We describe our problem framework and introduce the basic model formulation in §3. The structural properties of the optimal policy are discussed in §4. In particular, we show that the supplier has two dominant commitment strategies: to commit to the customer's forecast or to commit to the production quantity. This key result allows us to provide a full characterization of the optimal policy, although the objective function is not jointly concave in the two decisions. We then discuss how the supplier's optimal solution depends on how the customer's forecast influences the supplier's choice of demand distribution. We also extend the results to the capacitated version of the problem. In §5, we present a two-customer example which shows that the supplier's optimal production quantity for each customer may not be nondecreasing in the total capacity. Concluding remarks are presented in §6.

2. Literature Review

The strategic value of supplier-customer relationships has come to the forefront as supply chains have become more decentralized. This realization came as companies struggled to remain competitive and regain customer confidence in the face of fierce global competition. Several studies (Helper 1991, Dyers and Ouchi 1993, Helper and Sako 1995) have compared foreign and domestic business practices, and have found that the most apparent differences are the level of commitment and the amount of information exchange between entities in the supply chain. The level of commitment pertains to the extent to which the entities feel they can rely on each other. Information exchange, on the other hand, includes both the nature and mutuality of the information flow between entities. These studies show that increases in the level of commitment and the sharing of information lead to a reduction in uncertainty levels, and thereby to a reduction in costs for the entire supply chain. To implement these findings into business practices, manufacturers have reduced the number of suppliers (increasing their level of commitment to each supplier) and established supply contracts in which customers give suppliers more information about future demands in exchange for more information about the supplier's ability to fulfill customer requests (increasing the level of information shared).

Supply contracts in the literature that incorporate customer "forecasts" or advance orders are distinguished primarily on the dimensions of what the customer must commit to, or pay for, in exchange for some flexibility or risk sharing on the part of the supplier. For example, quantity flexibility contracts (Tsay 1999, Tsay and Lovejoy 1999) require that customers provide advance forecasts and commit to a minimum purchase quantity. In exchange, the supplier commits to supply at a specified level above the forecast. A special form of a quantity flexibility contract is a back-up agreement (Eppen and Iyer 1997) in which the customer pays for a specified fraction of his order up front and has the option to purchase the remainder at a later date. Variants of this contract form allow customers to update their forecasts over time (Bassok et al. 1997; Bassok and Anupindi 1997a, b; Anupindi and Bassok 1998). Other contract forms distinguish between advance reservations for capacity and the use of that capacity. Brown and Lee (1997) study a "pay to delay" contract in which the customer pays a fixed up-front fee for the option to place an order at a later date. Other researchers (e.g., Barnes-Schuster et al. 2002) have analyzed capacity reservation contracts in which customers pay a per-unit fee in advance for the capacity reservation and a separate per-unit fee if the option to utilize the capacity is exercised. In all of these papers, the authors study the customer's optimal ordering (and where relevant, forecasting) policy and generally conclude that the customers overforecast while the suppliers tend to underproduce in response to the customer's exaggerated forecast. These tendencies are mitigated in the expected direction by terms of the contract such as purchase or payment obligations that depend on the forecast.

In all of the aforementioned papers on supply contracts, it is assumed that the supplier will honor the agreement (and thus assume a portion of the demand risk). However, the supplier may not voluntarily honor the terms of the agreement, and even if he does, the contract terms themselves can induce gaming behavior. Moreover, the incentive to do so increases if the customer must compete for the supplier's capacity. Lee et al. (1999) show that mechanisms in which capacity is allocated in proportion to order quantities induce gaming behavior that results in inflated orders. Cachon and Lariviere (2001) study arrangements with both forced and voluntary compliance for the supplier and show that the entire class of mechanisms that are "individually responsive"—i.e., an increase in any customer's order results in a strict increase in his allocation—induce competitive behavior among customers which makes it impossible to construct a mechanism that induces the distributed system to perform like the centrally-planned system. Cachon and Lariviere (1999) study a "turn-and-earn" mechanism that gives a higher allocation in the second period to the customer who sells more in the first period and compare it to an equal allocation scheme. Under the "turn-and-earn" mechanism, the supplier's profits are higher but the retailers' profits may not be because retailers choose to sell a greater volume at lower price to protect their allocation.

The contract that we analyze has the features of (i) minimum purchase quantity that depends on the customer's forecast and the possible availability of supply beyond the minimum purchase quantity if the supplier chooses to produce it; and (ii) voluntary compliance on the part of the seller. What distinguishes it from other contracts, however, are two provisions: (i) the need for the supplier to declare a commitment quantity; and (ii) financial payments from the supplier to the customer for deviations of the commitment quantity from the customer's forecast and deviations of the delivered quantity from the commitment quantity. In essence, our contract is a type of quantity flexibility contract that includes additional economic incentives for supplier compliance.

An extensive overview of the supply contract literature can be found in Cachon (2002); see also earlier review papers by Anupindi and Bassok (1999) and Tsay et al. (1999). With the exception of Tsay and Lovejoy (1999), who consider replenishment decisions a multistage supply chain, focusing on the role of flexibility, and a recent paper by Kamrad and Siddique (2004) that analyzes both the customer's and supplier's decisions under a fairly complicated contract (with an emphasis on risk reduction), all of the literature is concerned with the customer's decisions, whereas our focus is on the supplier's decisions. In the next section, the supplier-customer relationship under a forecast-commitment contract is described and the model formulation is presented.

3. Model Formulation

We consider a two-stage problem in which the supplier and a single customer interact as follows. In the first stage, the customer provides the supplier its order forecast, $\bar{f}$. Based on his distribution of demand that will occur in the second stage, the supplier then makes a commitment to the customer, C, and decides on a production quantity, q, to be completed in time for the second stage. In the second stage, according to the terms of the contract, the customer must order at least $\alpha\bar{f}$, $0 \leq \alpha \leq 1$, unless the supplier has committed to a smaller amount, i.e., if $C \leq \alpha\bar{f}$. That is, the customer is not forced to order more than the supplier is willing to commit and is not required to order more than C even if his original forecast exceeded C. Thus, the minimum required order is $\underline{d} = \min\{\alpha\bar{f}, C\}$. Having observed demand, x, at the start of the second stage, the customer orders $\max\{x, \underline{d}\}$. The supplier delivers $\max\{x, \underline{d}\}$ if $q \geq \max\{x, \underline{d}\}$, and q otherwise.

In exchange for the fractional-purchase clause of the contract, the supplier agrees to pay a linear penalty for any shortfall of its commitment from the forecast (Type 1 penalty), as well as a linear penalty for the lesser of the shortfall of the delivery quantity from the commitment quantity or the shortfall of the delivery quantity from the realized demand (Type 2 penalty). The supplier "charges" himself the Type 1 penalty to reflect the loss of goodwill he incurs if his commitment is less than the forecast, and so the Type 1 penalty can be viewed as a commitment penalty. The Type 2 penalty is incurred if the supplier cannot meet his commitment and, as a result, can be viewed as a delivery penalty. We assume that the supplier and buyer have symmetric information about other costs. The model parameters are

w wholesale price
c per-unit production cost
π_1 customer commitment (Type 1) penalty
π_2 delivery (Type 2) penalty

The supplier's goal is to choose a commitment amount and production quantity so as to maximize expected profits. Thus, the supplier's profit for a given C, q, and $\bar{f}$ is

$$\begin{aligned}\phi_s(C,q,\bar{f}) &= wE[\min\{\max\{x,\underline{d}\},q\}] - cq - \pi_1\max\{\bar{f}-C,0\}\\ &\quad - \pi_2E[\max\{\min\{x,C\}-q,0\}]. \end{aligned} \tag{1}$$

The first term represents the expected sales revenue. The production cost is expressed in the second term. The penalty incurred if the supplier fails to commit up to the forecasted amount is captured by the third term. The final term represents the supplier's expected liability if he fails to deliver the committed amount.

The supplier's beliefs about demand are based on the forecast he receives from the customer. Let $f_s(x\mid\bar{f})$ and $F_s(x\mid\bar{f})$ be the supplier's conditional density of demand and cumulative distribution function, respectively, given the customer's forecast. Because $\bar{f}$ is a constant in the model, for expositional convenience, the dependence of $f_s(x)$, $F_s(x)$, and $\phi_s(C,q)$ on $\bar{f}$ will be omitted. For a given $\bar{f}$, we wish to maximize $\phi_s(C,q)$ with respect to C and q. This objective function is not jointly concave, so we employ a nested solution approach. We analyze the "inner problem" of finding the optimal commitment quantity for a fixed production quantity, then utilize the characterization of the optimal policy for the inner problem in solving the "outer problem" of optimizing the production quantity (with the corresponding optimal commitment quantity implicitly defined).

3.1. The Optimal Commitment Response

Theorem 1. *For a given production quantity, q, and the customer's forecast, $\bar{f}$, an optimal commitment response for the supplier, $C^*(q)$, is*

$$C^*(q) = q \text{ or } \bar{f},$$

where $C^*(q) = \arg\max_C[\phi_s(C,q)]$. That is, the supplier either commits to the amount to be produced or commits to the amount forecasted by the customer.

Proof. In the following, we characterize the supplier's expected profit as a function of the commitment response, C, where C depends on q. Clearly, it is always optimal to choose $q \geq \alpha\bar{f}$. Furthermore, if $q > \bar{f}$, the supplier pays no order delivery penalties. Therefore, we separate our analysis of the optimal commitment response into two cases: $\alpha\bar{f} \leq q \leq \bar{f}$ and $q > \bar{f}$.

Case 1. $\alpha\bar{f} \leq q \leq \bar{f}$. For q in this interval, the supplier's objective function has a different form in each of three subintervals: (a) $\alpha\bar{f} \leq C \leq q$; (b) $q \leq C \leq \bar{f}$; and (c) $C \geq \bar{f}$. We characterize the dominant value(s) of C for each.

Subcase (a). For $\alpha\bar{f} \leq C \leq q$, profit increases linearly at a rate of π_1 for each unit increase in the commitment amount, so the maximum occurs at $C = q$.

Subcase (b). For $q \leq C \leq \bar{f}$, the supplier's profit function can be written as

$$\begin{aligned}\phi_s(C,q) &= w\left[\int_{x=0}^{\alpha\bar{f}} \alpha\bar{f} f_s(x)\,dx + \int_{x=\alpha\bar{f}}^{q} x f_s(x)\,dx + \int_{x=q}^{\infty} q f_s(x)\,dx\right]\\ &\quad - cq - \pi_1[\bar{f}-C] - \pi_2\left[\int_{x=q}^{C}(x-q)f_s(x)\,dx\right.\\ &\quad \left. + \int_{x=C}^{\infty}(C-q)f_s(x)\,dx\right]. \end{aligned} \tag{2}$$

It is easy to show that $\phi_s(C,q)$ is convex in C, so the profit-maximizing commitment is one of the boundary points.

Subcase (c). For $C \geq \bar{f}$, the supplier incurs only the Type 2 penalty and his profit function is convex nonincreasing in this range, so the maximum occurs at $C = \bar{f}$.

For the subcases above, it follows that $C^*(q) = q$ or $\bar{f}$ for $\alpha\bar{f} \leq q \leq \bar{f}$.

Case 2. $q > \bar{f}$. The supplier will commit to at least the forecast, and may commit up to the amount to be produced without incurring any penalties, because the amount to be produced exceeds the forecasted amount, i.e., the objective function is constant for $C \in [\bar{f}, q]$. It is easy to show that if the supplier commits to anything greater than the amount to be produced, $\phi_s(C,q)$ is convex nonincreasing in C. Therefore, the supplier can optimally commit to any amount between $\bar{f}$ and q. Thus, $C^*(q) = q$ or $\bar{f}$ is a dominant policy. In most settings, for reasons not modeled here, one could argue that the supplier would have no incentive to disclose to the customer that he has produced more than the forecasted quantity. □

Theorem 1 states that the supplier will either choose to incur the commitment penalty with certainty, or he will commit to the forecast and possibly incur the delivery penalty, but never both. For the remainder of this paper, we will restrict our attention to $q \in [\alpha\bar{f}, \infty)$, as it is never optimal to produce $q < \alpha\bar{f}$, given that the customer must order at least $\alpha\bar{f}$.

3.2. Related Problems

Before presenting the structure of the optimal solution, we note that there are closely related problems that provide the motivation for the solution to the supplier's problem.

Problem A. The supplier must decide on a production amount that maximizes expected profit, without being required to make any commitment to its customer or receiving any minimum purchase commitment in return. This is analogous to the supplier solving the "unconstrained" newsvendor problem. The customer orders the observed demand, and can only influence the supplier's behavior through the forecast quantity, insofar as the forecast affects the demand distribution used by the supplier. The supplier's problem in this case is

$$PA: \quad \max_q \left\{ -cq + w\left[\int_{x=0}^{q} x f_s(x)\,dx + \int_{x=q}^{\infty} q f_s(x)\,dx\right]\right\}.$$

The optimal production quantity is $F_s^{-1}((w-c)/w)$.

Problem B. The supplier must decide on a production amount that maximizes expected profit when he is fully liable for any shortages. That is, the supplier pays π_2 for each unit short of the observed demand. The customer agrees to purchase at least $\alpha\bar{f}$. The supplier's problem in this case is

$$PB: \quad \max_q \left\{ -cq + w\left[\int_{x=0}^{\alpha\bar{f}} (\alpha\bar{f}) f_s(x)\,dx + \int_{x=\alpha\bar{f}}^{q} x f_s(x)\,dx + \int_{x=q}^{\infty} q f_s(x)\,dx\right] - \pi_2\left[\int_{x=q}^{\infty} (x-q) f_s(x)\,dx\right]\right\}.$$

The optimal production quantity is

$$\begin{cases} F_s^{-1}\left(\dfrac{w+\pi_2-c}{w+\pi_2}\right) & \text{if } F_s^{-1}\left(\dfrac{w+\pi_2-c}{w+\pi_2}\right) \geq \alpha\bar{f}, \\ \alpha\bar{f} & \text{otherwise.} \end{cases}$$

Another key newsvendor quantity arises from the following scenario.

Problem C. The supplier must decide on a production amount that maximizes expected profit when he commits to a production quantity $q < \bar{f}$, and the customer agrees to purchase at least $\alpha\bar{f}$. The supplier's problem in this setting is

$$PC: \quad \max_q \left\{ -cq + w\left[\int_{x=0}^{\alpha\bar{f}} (\alpha\bar{f}) f_s(x)\,dx + \int_{x=\alpha\bar{f}}^{q} x f_s(x)\,dx + \int_{x=q}^{\infty} q f_s(x)\,dx\right] - \pi_1[\bar{f}-q]^+.\right.$$

The optimal production quantity is

$$\begin{cases} F_s^{-1}\left(\dfrac{w-c}{w}\right) & \text{if } \bar{f} \leq F_s^{-1}\left(\dfrac{w-c}{w}\right), \\ \bar{f} & \text{if } F_s^{-1}\left(\dfrac{w-c}{w}\right) \leq \bar{f} \leq F_s^{-1}\left(\dfrac{w+\pi_1-c}{w}\right), \\ F_s^{-1}\left(\dfrac{w+\pi_1-c}{w}\right) & \text{if } F_s^{-1}\left(\dfrac{w+\pi_1-c}{w}\right) \leq \bar{f} \leq \alpha^{-1}F_s^{-1}\left(\dfrac{w+\pi_1-c}{w}\right), \\ \alpha\bar{f} & \text{if } \bar{f} \geq \alpha^{-1}F_s^{-1}\left(\dfrac{w+\pi_1-c}{w}\right). \end{cases}$$

We refer to the unconstrained stationary points that arise in Problems A and B as q_A^* and q_B^*, respectively, and to the new stationary point that arises in Problem C as q_C^*. That is,

$$q_A^* = F_s^{-1}\left(\frac{w-c}{w}\right), \quad q_B^* = F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right), \quad \text{and} \quad q_C^* = F_s^{-1}\left(\frac{w+\pi_1-c}{w}\right).$$

It is clear that the supplier's problem is closely connected to the problems presented above. Based on the forecasted quantity ($\bar{f}$), the supplier is required to make some commitment to its customer, but is only liable for Type 2 penalties up to the committed amount. This leads us to three useful observations: (1) The optimal production quantity for the supplier's problem will be at least as large as that of the optimal production quantity for the supplier with no contractual constraints, i.e., $q^* \geq q_A^*$. This observation highlights a benefit of forecast-commitment contracts: such a contract curbs the supplier's motivation to underproduce. (2) The actual delivery penalty paid by the supplier is less than that included in Problem B. (The supplier is not obligated to pay delivery penalties on demand exceeding $\bar{f}$.) Consequently, the supplier may wish to produce less than q_B^* (unless the commitment penalties have countervailing effects). On the other hand, the customer can induce the supplier to produce at least q_B^* by submitting $\bar{f} \geq q_B^*/\alpha$. (3) In Problem C, the customer can induce the supplier to produce at least q_C^* by submitting $\bar{f} \geq q_C^*/\alpha$. The last two observations highlight another benefit of forecast-commitment contracts—the contract provides a means for the customer to induce the supplier to produce more than he might otherwise by guaranteeing the purchase of a fraction α of the forecast.

3.3. The Optimal Production Quantity

In §3.1, we showed that the supplier has two dominant commitment strategies: to commit to the forecast or to commit to the amount to be produced. We next characterize the structure of the optimal production policy for each commitment strategy. As we will show, for each commitment strategy, the optimal production quantity depends on the ordering of five critical values:

$$\alpha\bar{f},\quad \bar{f},\quad q_A^* = F_s^{-1}\left(\frac{w-c}{w}\right),$$

$$q_B^* = F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right),\quad \text{and}\quad q_C^* = F_s^{-1}\left(\frac{w+\pi_1-c}{w}\right).$$

The critical value

$$q_C^* = F_s^{-1}\left(\frac{w+\pi_1-c}{w}\right)$$

is the optimal solution to the problem the supplier solves when committing to a production level q, $\alpha\bar{f} \le q \le \bar{f}$.

Lemma 1. *When the supplier's commitment strategy is to commit to the forecast, i.e., $C(q) = \bar{f}$, the optimal quantity to be produced is*

$$q^* = \begin{cases} q_A^* & \text{if } \alpha\bar{f} \le \bar{f} \le F_s^{-1}\left(\dfrac{w-c}{w}\right), \\ \bar{f} & \text{if } F_s^{-1}\left(\dfrac{w-c}{w}\right) \le \bar{f} \le F_s^{-1}\left(\dfrac{w+\pi_2-c}{w+\pi_2}\right), \\ q_B^* & \text{if } F_s^{-1}\left(\dfrac{w+\pi_2-c}{w+\pi_2}\right) \le \bar{f} \\ & \qquad \le \alpha^{-1}F_s^{-1}\left(\dfrac{w+\pi_2-c}{w+\pi_2}\right), \\ \alpha\bar{f} & \text{if } \alpha^{-1}F_s^{-1}\left(\dfrac{w+\pi_2-c}{w+\pi_2}\right) \le \bar{f}. \end{cases}$$

Proof. Recall that when the commitment strategy is to commit to the forecast, $C(q) = \bar{f}$, the supplier never incurs the commitment (Type 1) penalty. Therefore, the optimal production quantity depends only on the ordering of $\alpha\bar{f}$, $\bar{f}$, q_A^*, and q_B^*. The optimal production quantity for each of the possible orderings is obtained by evaluating the necessary and sufficient conditions for profit maximization when the production quantity is restricted to either $q \in [\alpha\bar{f}, \bar{f}]$ or $q \in [\bar{f}, \infty)$.

When $\alpha\bar{f} \le q \le \bar{f}$,

$$\begin{aligned}\phi_s(C,q) &= w\left[\int_{x=0}^{\alpha\bar{f}} \alpha\bar{f} f_s(x)\,dx + \int_{x=\alpha\bar{f}}^{q} x f_s(x)\,dx + \int_{x=q}^{\infty} q f_s(x)\,dx\right] - cq \\ &\quad - \pi_2\left[\int_{x=q}^{\bar{f}} (x-q) f_s(x)\,dx + \int_{x=\bar{f}}^{\infty} (\bar{f}-q) f_s(x)\,dx\right]. \qquad (3)\end{aligned}$$

Because the profit function is concave in q ($\partial^2\phi_s(C,q)/\partial q^2 = -(w+\pi_2)f_s(q) \le 0$), the profit-maximizing production quantity will either be an interior stationary point or a boundary point. Setting the first derivative to zero, we get

$$q_B^* = F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right)$$

as the optimal production quantity if it falls in the required range.

Next, we consider the case in which $q \ge \bar{f}$. In this case, the supplier's profit is

$$\phi_s(C,q) = w\left[\int_{x=0}^{\alpha\bar{f}} \alpha\bar{f} f_s(x)\,dx + \int_{x=\alpha\bar{f}}^{q} x f_s(x)\,dx + \int_{x=q}^{\infty} q f_s(x)\,dx\right] - cq. \qquad (4)$$

Equation (4) is concave in q, and $q_A^* = F_s^{-1}((w-c)/w)$ is the optimal production quantity if $q_A^* \ge \bar{f}$.

Because we do not know in advance which intervals

$$q_A^* = F_s^{-1}\left(\frac{w-c}{w}\right) \quad \text{and} \quad q_B^* = F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right)$$

will fall into, we have to analyze the various possibilities to determine the structure of the supplier's objective function, given by (3) for $q \in [\alpha\bar{f}, \bar{f})$ and (4) for $q \in [\bar{f}, \infty)$, which then determines whether a boundary or an interior point (q_A^* or q_B^*) is optimal. Note that (3) and (4) define a single objective function that is continuous, so we do not need to distinguish the cases of $q \le \bar{f}$ and $q \ge \bar{f}$ in the analysis of the objective function structure. However, given the forecast and the demand distribution used by the supplier, $F_s(\cdot)$, the specific form of the objective depends on the ordering of relevant critical values. Observe that only two of the stationary points,

$$q_A^* = F_s^{-1}\left(\frac{w-c}{w}\right) \quad \text{and} \quad q_B^* = F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right),$$

are applicable here. Thus, because $q_A^* \le q_B^*$ and $\alpha\bar{f} \le \bar{f}$, there are four mutually exclusive and collectively exhaustive orderings of the relevant critical values. We analyze each in turn.

$$(1)\quad \alpha\bar{f} \le \bar{f} \le F_s^{-1}\left(\frac{w-c}{w}\right) < F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right).$$

Given this ordering, the objective function is concave and $q^* = F_s^{-1}((w-c)/w)$ is the profit-maximizing point. Thus, the supplier's optimal production quantity equals the "unconstrained" newsvendor quantity, q_A^*.

$$(2)\quad F_s^{-1}\left(\frac{w-c}{w}\right) \le \bar{f} \le F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right).$$

The objective function is concave increasing in q for $q \leq \bar{f}$, and concave decreasing for $q > \bar{f}$. Therefore, the optimal production quantity for the supplier is $\bar{f}$.

(3) $\alpha\bar{f} \leq F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right) \leq \bar{f}$.

For this ordering, the objective function is also concave and

$$q^* = F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right)$$

is the profit-maximizing point. That is, in this case it is optimal for the supplier to hedge against the order delivery (Type 2) penalty by producing $q_B^* > q_A^*$.

(4) $F_s^{-1}\left(\frac{w-c}{w}\right) < F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right) \leq \alpha\bar{f} \leq \bar{f}$.

The objective function is concave decreasing in q, and so the supplier's optimal production quantity is $\alpha\bar{f}$. □

Lemma 2. *When the supplier's commitment strategy is to commit to the amount to be produced, i.e., $C(q) = q$, the optimal quantity to be produced is*

$$q^* = \begin{cases} q_A^* & \text{if } \bar{f} \leq F_s^{-1}\left(\frac{w-c}{w}\right), \\ \bar{f} & \text{if } F_s^{-1}\left(\frac{w-c}{w}\right) \leq \bar{f} \leq F_s^{-1}\left(\frac{w+\pi_1-c}{w}\right), \\ q_C^* & \text{if } F_s^{-1}\left(\frac{w+\pi_1-c}{w}\right) \leq \bar{f} \\ & \quad \leq \alpha^{-1}F_s^{-1}\left(\frac{w+\pi_1-c}{w}\right), \\ \alpha\bar{f} & \text{if } \bar{f} \geq \alpha^{-1}F_s^{-1}\left(\frac{w+\pi_1-c}{w}\right). \end{cases}$$

Proof. Omitted. The proof is similar to that of Lemma 1. Note that the result presented in Lemma 2 is the solution to Problem C. □

When following a strategy to commit to the amount to be produced, the supplier's production strategy is similar to that when the commitment strategy is to commit to the forecast, except that the supplier may choose to produce $q_C^* = F_s^{-1}((w+\pi_1-c)/w)$ as a way to reduce his commitment penalty liability when the customer's forecast is sufficiently high, i.e., $\bar{f} \geq F_s^{-1}((w+\pi_1-c)/w)$. The optimal policy for the supplier's problem of choosing commitment and production amounts given a customer forecast, which follows from Lemmas 1 and 2, is presented next.

Theorem 2. *The optimal commitment and production quantity pair for the supplier is*

$$(C^*, q^*) = \underset{\substack{(\bar{f}, q_A^*), (\bar{f}, q_B^*), (q_C^*, q_C^*), \\ (\bar{f}, \bar{f}), (\bar{f}, \alpha\bar{f}), (\alpha\bar{f}, \alpha\bar{f})}}{\arg\max} \phi_s(C, q).$$

Evaluation of the above expression provides the supplier's optimal policy. It is not, however, necessary to evaluate all of these candidate strategies to find the optimal policy. For each commitment policy, a subset of candidate strategies can be eliminated from consideration based on the ordering of critical values that arise. The feasible orderings under each of the commitment strategies are presented in Table 1, with orderings under $C(q) = \bar{f}$ and $C(q) = q$ labeled with the numbers 1 through 4 and the letters A through D, respectively.

Under each commitment policy, the corresponding conditions are exhaustive and mutually exclusive. Conditions are also ordered monotonically with respect to the ranges into which the value of $\bar{f}$ may fall. This implies that for any given forecast, at most two distinct orderings hold, one for each commitment policy. Table 2 provides a list of candidate strategies that must be compared when each possible pair of conditions is satisfied. In the table, column labels represent the orderings under $C(q) = \bar{f}$, while row labels represent orderings under $C(q) = q$. A missing entry indicates that the pair of orderings is inconsistent. Finally, because some orderings lead to the same solution, the supplier needs to compare the objective function value for at most two candidate strategies.

Table 1 Ordering of Critical Values Under Commitment Policies

	Commitment strategy	
	$C(q) = \bar{f}$	$C(q) = q$
Ordering of critical values	1. $\alpha\bar{f} \leq \bar{f} \leq F_s^{-1}\left(\frac{w-c}{w}\right)$	A. $\alpha\bar{f} \leq \bar{f} \leq F_s^{-1}\left(\frac{w-c}{w}\right)$
	2. $F_s^{-1}\left(\frac{w-c}{w}\right) \leq \bar{f} \leq F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right)$	B. $F_s^{-1}\left(\frac{w-c}{w}\right) \leq \bar{f} \leq F_s^{-1}\left(\frac{w+\pi_1-c}{w}\right)$
	3. $\alpha\bar{f} \leq F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right) \leq \bar{f}$	C. $\alpha\bar{f} \leq F_s^{-1}\left(\frac{w+\pi_1-c}{w}\right) \leq \bar{f}$
	4. $F_s^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right) \leq \alpha\bar{f} \leq \bar{f}$	D. $F_s^{-1}\left(\frac{w+\pi_1-c}{w}\right) \leq \alpha\bar{f} \leq \bar{f}$

Table 2 Set of Consistent Strategies for Different Orderings of Critical Values Under the Two Commitment Policies

Orderings under commitment policy	$C(q)=\bar{f}$ 1.	2.	3.	4.
$C(q)=q$				
A.	$(\bar{f}, q_A^*)$	—	—	—
B.	—	$(\bar{f}, \bar{f})$	$(\bar{f}, \bar{f})$, $(\bar{f}, q_B^*)$	$(\bar{f}, \bar{f})$, $(\bar{f}, \alpha\bar{f})$
C.	—	(q_C^*, q_C^*), $(\bar{f}, \bar{f})$	(q_C^*, q_C^*), $(\bar{f}, q_B^*)$	(q_C^*, q_C^*), $(\bar{f}, \alpha\bar{f})$
D.	—	$(\alpha\bar{f}, \alpha\bar{f})$, $(\bar{f}, \bar{f})$	$(\alpha\bar{f}, \alpha\bar{f})$, $(\bar{f}, q_B^*)$	$(\alpha\bar{f}, \alpha\bar{f})$, $(\bar{f}, \alpha\bar{f})$

Note that orderings 1 and A apply when $\bar{f} < q_A^*$. Under this condition, the supplier produces q_A^*, so the strategy (q_A^*, q_A^*) is equivalent to $(\bar{f}, q_A^*)$, so the former is not listed in the table.

To illustrate the use of Table 2, suppose that Condition 4 and Condition D are satisfied. Because the optimal production quantity is the same under the two candidate strategies, the production cost and expected revenue can be ignored. Therefore, the supplier must weigh whether incurring the commitment penalty is less costly than incurring the expected order delivery penalty, in choosing the optimal policy. Similar comparisons can be performed when other pairs of conditions hold. Thus, given a forecast, the supplier can compare the objective function value under the two commitment strategies for the prescribed production quantities to select the optimal strategy.

4. Structural Properties of the Optimal Policy

In this section, we discuss the structure of the supplier's overall commitment strategy. We then present conditions in which one commitment strategy is dominant, followed by conditions that guarantee unimodality of the value function when a single commitment strategy is not dominant.

4.1. Structure of the Optimal Commitment Strategy

Theorem 3. *The supplier's optimal commitment, $C^*(q)$, is nondecreasing in the amount to be produced, q. Furthermore, the optimal commitment policy is a threshold policy. That is, for q less than the threshold (which may be as small as $\alpha\bar{f}$), it is optimal to commit to the production quantity, and for q greater than the threshold (which may be as large as $\bar{f}$), it is optimal to commit to the forecast.*

Proof. For $q \in [\alpha\bar{f}, \bar{f}]$, let

$$A(q) = \pi_1[\bar{f} - q], \tag{5}$$

$$B(q) = \pi_2\left[\int_{x=q}^{x=\bar{f}} (x-q) f_s(x)\,dx + \int_{x=\bar{f}}^{x=\infty} (\bar{f}-q) f_s(x)\,dx\right]. \tag{6}$$

Then, $A(q)$ and $B(q)$ are the expected penalty costs associated with a production quantity of q under different commitment strategies. $A(q)$ corresponds to the strategy of committing to the production quantity and $B(q)$ corresponds to committing to the forecast. We restrict our attention to $q \in [\alpha\bar{f}, \bar{f}]$ because for $q \geq \bar{f}$, the two commitment strategies are equivalent, as the supplier pays no penalties under either. Note that when comparing the profit functions under the two commitment strategies for a given production level q, all terms are the same except for the penalties. Therefore, if $A(q) < B(q)$, then it is better to commit to q than to commit to the forecast. Equations (5) and (6) imply that $A(q)$ is linearly decreasing in q, $B(q)$ is convex decreasing in q ($\partial B(q)/\partial q = -\pi_2(1 - F_s(q))$ and $\partial^2 B(q)/\partial q^2 = \pi_2 f_s(q)$), and $A(\bar{f}) = B(\bar{f}) = 0$. Thus, if $B(q) > A(q)$ for $q \in [\alpha\bar{f}, \bar{f})$, then it is optimal to commit to the amount to be produced for all q, and so $C^*(q)$ is nondecreasing in q. Otherwise, we have

(1) $B(q) \leq A(q)$ for all $q \in [\alpha\bar{f}, \bar{f}]$, or

(2) There exists some threshold q_T such that for $q \leq q_T$, $B(q) \geq A(q)$ and for $q > q_T$, $B(q) < A(q)$.

In the first case, it is optimal to commit to the forecast for all q, so $C^*(q)$ is nondecreasing in q. In the second case, it is optimal to commit to the production quantity for $q \leq q_T$ so $C^*(q)$ is increasing in this region, and it is optimal to commit to the forecast for $q > q_T$. Consequently, the optimal policy is a threshold policy and the optimal commitment quantity is nondecreasing for $q \in [\alpha\bar{f}, \bar{f})$. This completes the proof. □

A corollary of the second part of Theorem 3 is the following:

Corollary 1. *$\phi_s(q, q)$ and $\phi_s(\bar{f}, q)$ intersect at most once in $q \in [\alpha\bar{f}, \bar{f})$. Consequently, because both functions are concave, the upper envelope has at most two modes.*

4.2. Refining the Supplier's Optimal Strategy

The results above allow us to further reduce the number of candidate strategies that must be compared.

Lemma 3. *When Cases 2C and 2D hold, the candidate strategies (q_C^*, q_C^*) and $(\alpha\bar{f}, \alpha\bar{f})$, respectively, dominate the strategy $(\bar{f}, \bar{f})$ for all $q \in [\alpha\bar{f}, \bar{f}]$. When Case 4B holds, the strategy $(\bar{f}, \alpha\bar{f})$ dominates the strategy $(\bar{f}, \bar{f})$ for all $q \in [\alpha\bar{f}, \bar{f}]$.*

Proof. To establish the results for Cases 2C and 2D, we use the facts that $\phi_s(\bar{f}, q) = \phi_s(q, q)$ when $q = \bar{f}$, and $C^*(q)$ is nondecreasing in q (Theorem 3). In Case 2C, $\phi_s(\bar{f}, q)$ is concave increasing and $\phi_s(q, q)$ is concave for $q \in [\alpha\bar{f}, \bar{f}]$ with an interior stationary point at q_C^*. This means that $\phi_s(\bar{f}, q)$ cannot be dominant for all q (otherwise the two functions would not be equal at $\bar{f}$), nor can a threshold policy hold (because the optimal commitment would not be nonincreasing in q). As a result, committing to the amount to be produced must be dominant for all q and we therefore only need to consider the candidate strategy (q_C^*, q_C^*) in Case 2C. A similar argument holds for Case 2D. When Case 4B holds, the value function under the strategy to commit to the forecast is concave decreasing for $q \in [\alpha\bar{f}, \bar{f}]$. Therefore, strategy $(\bar{f}, \alpha\bar{f})$ dominates $(\bar{f}, \bar{f})$. □

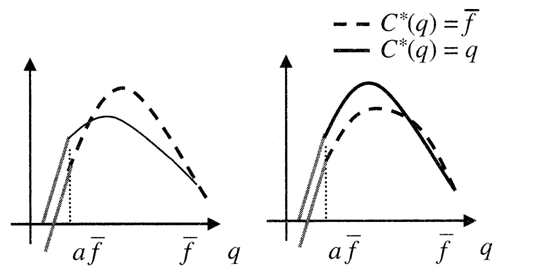
The reduced sets of candidate strategies that must compared are shown in Table 3.

The table is arranged so that as the customer's forecast, $\bar{f}$, increases, one moves (weakly) south, or east or southeast in the table. Because a single commitment policy is not dominant for many of the cells in Table 3, we cannot make broad statements about how the customer's forecast affects the supplier's response. From the table, we can infer, however, that for $\bar{f} \le \min(q_B^*, q_C^*)$ (i.e., Cases 1A and 2B), it is optimal to commit to the forecast. Only if the forecast exceeds this threshold would the supplier commit to a larger value. Also, as the forecast grows larger, it becomes more likely that a policy of producing $\alpha\bar{f}$ is the only, or one of the candidate, optimal production quantities.

4.3. Unimodality Results

As mentioned in the previous subsection, in five cases (1A, 2B, 2C, 2D, and 4B) there is a single consistent strategy or a single dominant strategy, so the optimal value function is concave and therefore also unimodal (by Lemmas 1 and 2). We next show that the function is always unimodal also in three of the remaining five cases presented in Table 3. (In the other two cases, we can derive sufficient conditions for unimodality,

Figure 1 Structural Characteristics Needed for Unimodality of the Optimal Value Function When the Objective Functions Under the Two Commitment Strategies Intersect



but they are complicated and consequently, are not of practical benefit in solving the problem.)

The optimal value function is unimodal when either: (1) there is a single dominant commitment strategy for all q, or (2) the objective functions under the two commitment strategies intersect when both functions are increasing or both functions are decreasing as shown in Figure 1.

Lemma 4. *When Case* 3B, 4C, *or* 4D *holds, the optimal value function is unimodal.*

Proof. For each of the three cases, we know that if a single commitment policy does not dominate for all q, the value functions under the two commitment strategies intersect once, and up to the intersection point, the value function under the strategy to commit to the amount to be produced is dominant (by Theorem 3).

For Case 3B, $\bar{f} \le F_s^{-1}((w + \pi_1 - c)/w)$ holds, and hence $\phi_s(C(q) = q, q)$ is concave increasing in q, $q \in [\alpha\bar{f}, \bar{f})$. If an intersection occurs, the value function under the strategy to commit to the forecast becomes dominant. Thus, because the functions intersect when $\phi_s(C(q) = q, q)$ is increasing and because $\phi_s(C(q) = \bar{f}, q)$ is concave in q and both functions must be equal at $q = \bar{f}$, the optimal value function is unimodal in q (the upper envelope is unimodal in q).

In Cases 4C and 4D, $\phi_s(C(q) = \bar{f}, q)$ is concave decreasing. Thus, if the value functions under each of the commitment strategies intersect, they must do so

Table 3 Reduced Set of Consistent Strategies for Different Orderings of Critical Values Under the Two Commitment Policies

Orderings under commitment policy	$C(q) = \bar{f}$ 1.	2.	3.	4.
$C(q) = q$				
A.	$(\bar{f}, q_A^*)$	—	—	—
B.	—	$(\bar{f}, \bar{f})$	$(\bar{f}, \bar{f})$, $(\bar{f}, q_B^*)$	$(\bar{f}, \alpha\bar{f})$
C.	—	(q_C^*, q_C^*)	(q_C^*, q_C^*), $(\bar{f}, q_B^*)$	(q_C^*, q_C^*), $(\bar{f}, \alpha\bar{f})$
D.	—	$(\alpha\bar{f}, \alpha\bar{f})$	$(\alpha\bar{f}, \alpha\bar{f})$, $(\bar{f}, q_B^*)$	$(\alpha\bar{f}, \alpha\bar{f})$, $(\bar{f}, \alpha\bar{f})$

while both are decreasing. This follows from Corollary 1, which states that $\phi_s(q, q)$ and $\phi_s(\bar{f}, q)$ intersect at most once, and Theorem 3, which states that the latter becomes dominant if the functions cross. For Case 4C, $\phi_s(q, q)$ is concave in q, $q \in [\alpha\bar{f}, \bar{f})$, so the upper envelope of the two functions is unimodal in q. For Case 4D, the upper envelope is unimodal because both functions are concave decreasing in q, $q \in [\alpha\bar{f}, \bar{f})$. □

4.4. Sensitivity of the Solution to the Customer Forecast and the Supplier's Selected Distribution

In the previous sections, we have fully characterized the supplier's optimal policy given the supplier's distribution of demand based on the customer's forecast. It is clear that the selected distribution and its parameters will be major determinants of the optimal commitment and production quantities. In Table 4, we present numerical results that illustrate the sensitivity of the optimal policy to the demand distribution selected by the supplier. The parameters of the demand distributions were selected to illustrate what can happen and why. The other relevant parameters are $w = 1.25$, $c = 1$, $\alpha = 0.85$, $\pi_1 = 0.3$, and $\pi_2 = 0.75$.

Example 1 illustrates a situation in which the supplier believes that the mean demand is 50, and reacts to a larger forecast by increasing the standard deviation (we call this a *variance-only response*). If the customer submits a forecast of 50, the supplier uses his (baseline) prior distribution with a standard deviation of 5, and the optimal solution is to commit to and produce the forecasted quantity ($C^* = q^* = \bar{f} = 50$). Under the increased variance response (to the larger forecast of 55), the supplier increases the standard deviation to 7.5, and consequently switches strategy by committing to and producing $q_C^* = 48.87$, a smaller quantity. For a forecast of 60, which causes the supplier to increase the standard deviation to 10, the optimal strategy is to commit and produce $\alpha\bar{f} = 51$. In this last case, the optimal production quantity is larger than that for the original solution (for the demand distribution $N(50, 5)$). Thus, we cannot say in general how a variance-only change will affect the supplier's optimal policy, because increases in the variance may lead to a different ordering of the forecast relative to the critical values (q_A^*, q_B^*, etc.). The revised ordering may lead to a change in the supplier's optimal commitment strategy and therefore also to a (possibly) nonmonotonic change in the optimal production and commitment quantities.

The second example represents a situation in which the supplier does not update his beliefs about the demand distribution when the customer submits higher forecasts, i.e., the supplier does not believe that the customer's forecast provides any useful information (we call this a *no-change response*). Again, we see that higher forecasts do not necessarily induce the supplier to increase the production and/or commitment quantities, and may actually cause a decrease.

In the special case where the supplier believes that the customer is submitting the mean demand as his forecast, and does not change the standard deviation in response to the modified forecast (i.e., a *mean-only response*), the supplier's optimal solution changes monotonically with the forecast, with a one unit change in the solution for each unit change in the forecast. This occurs because the ordering of the forecast in relation to the critical values does not change.

If the supplier responds to a revised customer forecast by changing both the mean and standard deviation of the demand distribution, then we cannot say definitively what will happen. If we extrapolate from the above examples, however, there is reason to believe that less predictable behavior will be observed when there is a switch in the supplier's optimal commitment policy due to a change in the ordering of the forecast relative to the critical values.

These examples, which are based upon not entirely unreasonable choices of "updated" demand distributions by the supplier, show that, except in special cases, there is little that can be said about how the supplier's choice of conditional demand distribution and of its moments will affect the supplier's optimal decisions and resulting profits. Yet, the examples also provide useful insights into why unexpected changes in the supplier's solution may occur as the customer changes his forecast.

Table 4 Sensitivity of the Optimal Policy to Customer Forecast—Two Counterintuitive Examples

Example	Forecast	Demand distribution	Optimal policy
1. Variance-only response	50	$N(50, 5)$	$C^* = \bar{f} = 50$, $q^* = \bar{f} = 50$
	55	$N(50, 7.5)$	$C^* = q_C^* = 48.87$, $q^* = q_C^* = 48.87$
	60	$N(50, 10)$	$C^* = \alpha\bar{f} = 51$, $q^* = \alpha\bar{f} = 51$
2. No-change response	75	$N(75, 20)$	$C^* = \bar{f} = 75$, $q^* = \bar{f} = 75$
	80	$N(75, 20)$	$C^* = q_C^* = 71.98$, $q^* = q_C^* = 71.98$
	90	$N(75, 20)$	$C^* = \alpha\bar{f} = 76.5$, $q^* = \alpha\bar{f} = 76.5$

5. The Capacitated Supplier

In this section, we briefly discuss the impact of a capacity constraint in the single-customer problem. We then turn to the multiproduct scenario with capacity constraints. When the optimal value function is unimodal in q, the uncapacitated solution is useful in solving the capacitated problem. If the unconstrained solution is feasible, it is optimal. If capacity is binding, the facts that at most one switch occurs in the supplier's commitment policy (Theorem 3) and that the optimal production quantity is nondecreasing in capacity, means that the supplier need only compare $\phi_s(\text{Cap}, \text{Cap})$ and $\phi_s(\bar{f}, \text{Cap})$.

If the optimal value function is not unimodal, the supplier must first check for feasibility of the optimal unconstrained production level under each commitment strategy. If either solution is not feasible, it must be adjusted downward to account for the capacity constraint. Then, the solutions, after any necessary adjustments, for the two commitment policies must be compared.

We now generalize the model to consider multiple products, each ordered by a different customer. We assume that products are not interchangeable and that forecasts and orders are independent across customers. In particular, we assume that that customers do not have information about other customers' demands or forecasts and do not use any gaming behavior to their advantage. In other words, each customer behaves as if he were the only customer. Situations in which the same product is ordered by multiple customers, or in which products may be substituted for one another, are more complicated because they require additional allocation decisions. These generalizations represent interesting areas for future research. We show here that even when customers' products are distinct, the capacitated problem is difficult.

5.1. Multiproduct Problem with a Capacity Constraint

The supplier's goal is to choose commitment and production amounts for each customer so as to maximize the sum of its total expected profits from all customers, given their respective forecasts:

$$\max_{C_i, q_i} \sum_{i=1}^{N} \phi_s(C_i, q_i, \bar{f}_i) \tag{7}$$

$$\text{subject to } \sum_{i=1}^{N} \mu_i q_i \le \text{Cap}, \tag{8}$$

$$q_i \ge 0, \quad i = 1, 2, \ldots, N.$$

The production requirements are given on the left-hand side of inequality (8), where μ_i represents the capacity utilization per unit of product i. The final constraint ensures nonnegativity of production quantities.

We use the following two-product example to illustrate how the structure of the supplier's objective function, in particular, the possibility that the objective function may be bimodal, complicates allocation decisions when capacity is binding. The data used in the example are presented in Table 5. Note that in this example, the supplier uses the same a priori distribution for both customers, although the two customers have submitted quite different forecasts. This would reflect a scenario in which Customer 1 has a history of submitting relatively unbiased forecasts on the average, so the supplier has a good reason to center his a priori distribution at the customer-supplied forecast. On the other hand, Customer 2 may have a history of consistently submitting forecasts that are larger than the demand that is eventually observed.

Table 5 **Data for the Multiple-Product Problem**

	Customer 1	Customer 2
Forecast, $\bar{f}$	60	75
Alpha	0.75	0.75
Demand distribution	$N(60, 13)$	$N(60, 13)$
Wholesale price, w	1.50	1.50
Production cost, c	1.00	1.00
Commitment penalty, π_1	0.55	0.30
Delivery penalty, π_2	0.80	0.80
Capacity utilization (per unit produced), μ	1	1

The optimal production quantities for the unconstrained, individual customer problems are 60 and 62 for Customer 1 and Customer 2, respectively. Thus, if the total available capacity is at least 122, the supplier will use the solution to the unconstrained problem. The optimal quantity produced for each customer, as a function of the total available capacity, is shown in Figure 2.

Figure 2 shows that when capacity increases from 100 to 101, the optimal allocation to Customer 1 decreases from 50 to 45, while Customer 2's allocation increases from 50 to 56. Hence, while the total production in this example is nondecreasing in the total available capacity, the optimal allocation of capacity to each customer is not necessarily nondecreasing in capacity.

This phenomenon occurs because the objective function in the capacitated, multiproduct problem is not jointly concave in the capacity allocations to

Figure 2 **Optimal Capacity Allocations to Customers as a Function of Total Available Capacity**

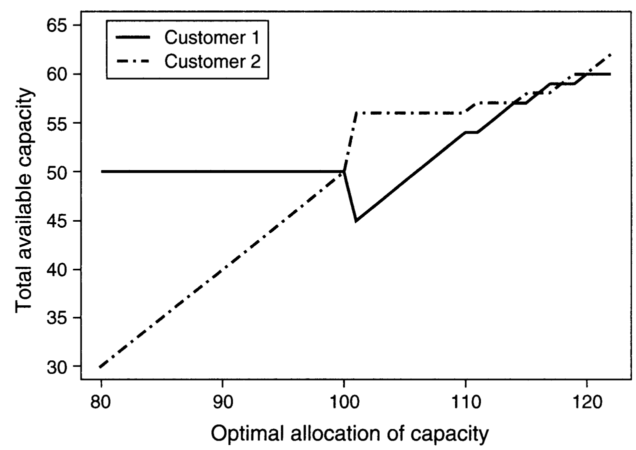


Figure 3 Supplier's Objective Function Value for Different Capacity Allocations at Two Capacity Levels

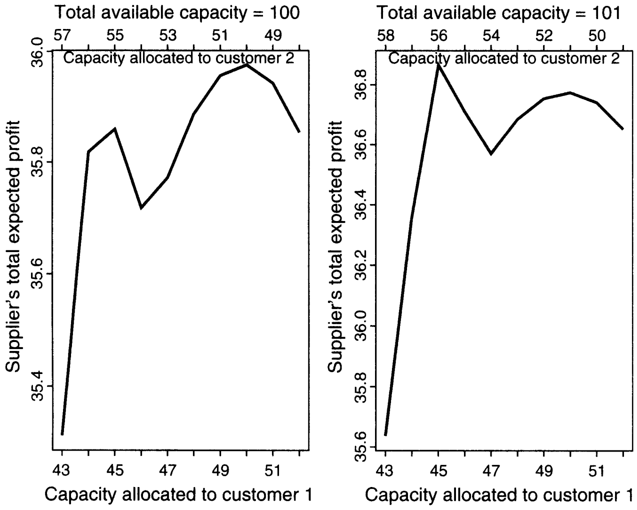
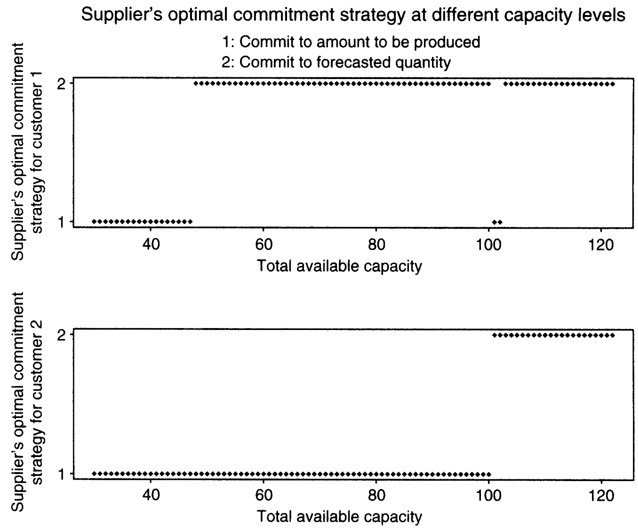


Figure 4 Supplier's Optimal Commitment Strategy at Different Capacity Levels



the two customers for a given capacity level, as shown in Figure 3. The graph on the left shows the total expected profit for the supplier for allocation pairs that add up to 100, the available capacity. The optimal commitment strategy corresponding to the maximum capacity utilization is $(q_1^*, q_2^*) = (50, 50)$, with $(C_1^*, C_2^*) = (\bar{f}, q_2^*)$. When the supplier has an additional unit of capacity available, the supplier switches commitment strategies to $(C_1^*, C_2^*) = (q_1^*, \bar{f})$, and produces $(q_1^*, q_2^*) = (45, 56)$, as shown in the graph on the right. A further implication is that unlike in the single-customer problem, where at most one switch in the commitment strategy occurs as q increases (Theorem 3), the supplier may switch from committing from the production quantity to the forecasted amount and back more than once, as shown in Figure 4. The figure shows the supplier's optimal commitment strategy for each customer as a function of the total available capacity.

It is important to emphasize that these seemingly counterintuitive results are not for contrived examples, but instead, should be expected in view of the (usually) bi-modal nature of the supplier's objective function. Small changes in the problem parameters may cause the supplier to choose a solution on the other "hump," thereby leading to a substantial change in the production (and commitment) quantity for that customer.

Finally, we note that another implication of the lack of monotonicity of the optimal capacity allocations, which is a consequence of the fact that the multicustomer objective is not jointly concave in the q values, is that the problem cannot be solved by marginal analysis. To solve the capacitated version of the problem for N customers, the supplier must solve 2^N constrained (with respect to the allocation to each customer), concave maximization problems, one problem for each partition of customers between the two commitment strategies, with appropriate constraints on the value of q for each customer.

6. Conclusions and Future Research Directions

In this paper, we have considered a contract in which a supplier makes production commitments to its customer, and in exchange, the customer commits to provide a forecast and to purchase a fraction of his forecast. The contract also stipulates penalties the supplier must pay if he fails to commit to the customer forecast, as well a penalty if he cannot deliver on his commitment.

We have provided a full characterization of the supplier's optimal commitment and production policy. In particular, we have shown that the supplier has two dominant commitment policies: commit to the amount to be produced or commit to the forecasted amount. We have also shown that either one of these commitment strategies is optimal for all production levels, or that a threshold policy is optimal, i.e., the supplier commits to the production quantity up to some production threshold, and then commits to the forecasted quantity for larger production levels. We showed that the supplier's optimal value function has at most two modes as a function of the amount produced, and provided sufficient conditions for unimodality.

We then analyzed the multicustomer, multiproduct problem. In the presence of a capacity constraint, we showed that capacity allocation to individual customers may decrease as the total capacity increases because the objective function may be highly irregular. The highly irregular structure of the supplier's value function means that small changes in a customer's forecast may lead to a significant shift in his own allocation or those of other customers. A managerial implication of this observation is that customers may experience additional volatility in the quantities they receive from period to period as a result of the contract structure and the supplier's (optimum) response to it.

The inherent volatility of the ASIC industry necessitates contractual agreements. As stated in the literature, contracts provide a means for parties to share private information credibly to guarantee reliable delivery of services and goods. Contracts such as the forecast-commitment contract presented above or the quantity flexibility contract presented by Tsay (1999) serve to mitigate problems, such as customers overforecasting and suppliers underproducing, that pervade the ASIC industry. Such problems would be even more magnified in the absence of these contracts. For example, without the forecast-commitment contract, the supplier would produce

$$q^* = F_s^{-1}\left(\frac{w-c}{w}\right),$$

which was shown (see §3.2) to be less than the optimal production amount(s) in the presence of the contract, except when the customer's forecast is low enough that producing the "unconstrained" newsvendor quantity is economically optimal for the supplier. Therefore, the proposed contract serves to curb the supplier's motivation to underproduce.

In a follow-up paper (Durango-Cohen and Yano 2003), we have analyzed the customer's problem of choosing a forecast to submit. There, we show that the contract limits the customer's incentive to overforecast and provides a means for the customer to induce the supplier to produce more than he would in the absence of the contract. This is achieved through the fractional-purchase clause of the contract. This result can also be inferred from the results presented here. The results in Table 3 show that for sufficiently large values of $\bar{f}$, the supplier will simply produce $\alpha\bar{f}$, which the customer is then obliged to purchase. Thus, although the customer may still overforecast, the fact that the customer is contractually committed to purchase at least $\alpha\bar{f}$ constrains his behavior and induces him to provide more accurate information.

Another benefit of the forecast-commitment contract is the contract's penalty structure. The "commitment" penalty captures the loss of goodwill incurred by the supplier when he does not commit to a customer's forecast, while the "delivery" penalty reflects a standard lost sales/backorder penalty. The penalties serve to align the relative value of customers and short-term profit considerations, and thus provide the supplier with an explicit mechanism for giving preference to "better" customers. The contract also benefits customers as they may be better able to plan their downstream production based on the advance-commitment information provided by the supplier.

The customer's problem is quite complex (cf. Durango-Cohen and Yano 2003) because he must consider how the supplier will respond to the submitted forecast. This effect results in a piecewise concave objective function for the customer, with potentially many modes. We have found that some of the potentially optimal solutions will lead to coordination of the supply chain (same total system profit as in the centralized solution). Indeed, in our numerical study, the customer's optimal forecast and the consequent optimal response of the supplier lead to a coordinated solution in the vast majority of problem instances. We consider this to be remarkable considering that the contract was designed with a different intent in mind, and considering that the supplier's and customer's decision problems are not concave in the respective decision variables. Thus, the forecast-commitment contract may provide some flexibility not afforded by other simpler contracts without much loss in system profit.

It is important to reiterate that our analysis examines the forecast-commitment contract with the firm requirement for the customer to purchase at least a fraction α of his forecast. In practice, if a customer has a history of greatly overforecasting, the supplier might choose to take the risk of producing less than α of the customer's forecast, if the capacity could be used more profitably to satisfy other demands.

Future research directions include extending the model to multiple periods, possibly with an increased commitment/forecast window and/or rolling forecasts, and considering the contract under generalized penalty functions. In addition, studying the multi-customer, single-product problem, where the optimal allocation is likely to be greedy and risk pooling plays a significant role, may provide interesting insights.

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