PRODUCTION AND OPERATIONS MANAGEMENT
Vol. 20, No. 5, September–October 2011, pp. 681–698
ISSN 1059-1478 | EISSN 1937-5956 | 11 | 2005 | 0681
DOI 10.1111/j.1937-5956.2010.01199.x

Optimizing Customer Forecasts for Forecast-Commitment Contracts

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We study a "Forecast-Commitment" contract motivated by a manufacturer's desire to provide good service in the form of delivery commitments in exchange for reasonable forecasts and a purchase commitment from the customer. The customer provides a forecast for a future order and a guarantee to purchase a portion of it. In return, the supplier commits to satisfy some or all of the forecast. The supplier pays penalties for shortfalls of the commitment quantity from the forecast, and for shortfalls of the delivered quantity from the customer's final order (not exceeding the commitment quantity). These penalties allow differential service among customers.

In Durango-Cohen and Yano (2006), we analyzed the supplier's problem for a given customer forecast. In this paper, we analyze the customer's problem under symmetric information, both when the customer is honest and when he strategically orders more than his demand when doing so is advantageous. We show that the customer gains little from lying, so the supplier can use his control over the contract parameters to encourage honesty. When the customer is honest, the contract achieves (near-)coordination of the supply chain in a great majority of instances, and thus provides both excellent performance and flexibility in structuring contracts.

Key words: supply contracts; purchase commitment; supply chain coordination; quantity flexibility; strategic customers
History: Received: June 2009; Accepted: June 2010 by Eric Johnson after 2 revisions.

1. Introduction

Maintaining the loyalty of profitable customers is critical to a firm's long-run success, and firms use a variety of tactics toward that end. Examples include airline frequent flyer programs and discounts for regular customers. The motivation for our work is an application-specific integrated circuit (ASIC) supplier that was seeking to provide its customers "good service" in the form of (soft) delivery commitments during periods of tight capacity, perhaps sacrificing some short-term profits, with the expectation that these customers would be more loyal and thus more inclined to purchase from the firm during periods of cyclically low demand.

In the motivating case, customers were already submitting forecasts. These forecasts were not estimates of future demand, *per se*, but instead were unofficial requests. The term forecast is commonly used to represent such informal communication from a customer to a supplier. This communication was necessary because ASICs are customized and the supplier was unwilling to start production runs without some notion of how much the customer would eventually order. Moreover, the supplier was operating at very high utilization (24 hours a day, 7 days a week). Recognizing that the supplier probably could not satisfy all demand, customers tended to inflate their forecasts, hoping to improve their chances of receiving what they needed. In such an environment, the customers were quite willing to make a firm commitment to purchase a fraction of their forecast in exchange for a reasonably reliable albeit soft delivery commitment. This would be far better than the existing situation in which the supplier did not trust the forecasts, so had difficulty planning and ultimately produced the "wrong" quantities of everything, and thereby inefficiently allocated capacity, so all customers suffered.

With this situation in mind, together with the ASIC supplier, we devised the "Forecast-Commitment" (FC) contract. Under the contract, the customer submits a forecast, $\bar{f}$, for some future time period and is obligated to purchase at least a fraction α of the forecast. The supplier decides on a production quantity, q, and commits to deliver a quantity C (to be selected by the supplier), and pays the customer: (1) a *commitment penalty* per unit (π_1) if the commitment is less than the forecast, and (2) a *delivery penalty* (π_2) per unit if the quantity delivered is less than the committed quantity (or the customer's actual demand, if it is smaller).

With this contract structure, the customer has some incentive to provide reasonable forecasts, and the supplier has some incentive both to make a commitment and to deliver on his promises.

Features of the FC contract appear in a variety of contracts. For example, the template for a long-term supply agreement at ContractStore.com (i) requires the purchaser to give regular forecasts of quantities; (ii) may include minimum purchase quantities and maximum delivery requirements; (iii) requires the supplier to pay a penalty for delivery shortfalls (see http://www.contractstore.com/longtermsupplyagreement). What is most distinctive about the FC contract vis-à-vis other contracts in the research literature (see the next section for a discussion of other contracts) is the inclusion of penalties for shortfalls of the supplier's commitment from the customer's forecast. Many purchase agreements entail a formal or informal delivery quantity promise on the part of the supplier, but we are not aware of any that consider commitment penalties. As we will show later, this aspect of the contract plays a very important role in achieving near-optimal performance.

When our study began, the ASIC supplier was developing a "point system" to prioritize customers (for capacity allocation) on the basis of product profitability, customer's forecast accuracy, etc. The commitment and delivery penalties in our contract represent one mechanism for implementing the ASIC supplier's priority scheme: a customer who has a history of submitting accurate forecasts should command a higher commitment penalty charge on the supplier than a customer who has consistently over-forecasted. Likewise, a high (low) delivery penalty might be associated with high- (low-)margin products. The presence of *two* types of penalties provides the supplier more flexibility to treat customers and products differently, a feature that was quite important to the ASIC supplier. Empirical evidence (e.g., Terwiesch et al. 2005) suggests that manufacturers *do* consider such factors in prioritizing customers. The contract parameters could also be the result of negotiation.

The ASIC supplier viewed the penalties as a way to internalize the point system. However, some supply contracts currently in force require suppliers to pay penalties for failing to meet their obligations. For ease of exposition, we perform our analysis as if the penalties require financial transfers. Without such transfers, the analysis is much messier and some of the detailed insights differ, but even the internalized penalties move both parties' decisions in the same direction as does the contract with financial transfers, although not to the same extent.

Although our research was motivated by long-term supply relationships, the rapid decline in the value of an ASIC over its short life cycle of less than a year (and often only a few months) causes both the supplier and customer to take a relatively short-run view of purchasing and production decisions. Therefore, it is useful to imagine a situation in which the supplier and customer have a long-term relationship, but each transaction is viewed as a "one-off" arrangement, although a few such transactions may occur over time for the same product. With this as a backdrop, we study the supplier's and customer's decisions related to a single demand. Later, we show that, with appropriately chosen contract parameters, typically both parties are better off (in expectation). Thus, even if the (single-period) FC contract were implemented myopically in each period, both parties would likely be better off.

This paper addresses the customer's problem of what forecast to submit to the supplier under an FC contract. We consider two problem variants. In the first, the customer is honest and does not order more than his demand just to extract penalty payments. In Durango-Cohen and Yano (2006), we characterized the jointly optimal commitment and production strategy for the supplier in the honest-customer setting for a given customer forecast. To solve the customer's problem efficiently for this version of the problem, however, we needed to find even stronger characterizations of the supplier's optimal policy; these results are included here, as well.

In the second variant, the customer is strategic and orders more than his demand when it is advantageous to do so. For this scenario, we characterize the supplier's optimal strategy for a given customer forecast and provide a partial characterization of the customer's optimal policy that enables us to compute optimal solutions. We show later that the customer does not gain much from being dishonest, whereas the supply chain may suffer. Consequently, the supplier can use the contract parameters as an enticement or threat to encourage honesty as the relationship develops. For this reason, we concentrate on the honest-customer scenario in this paper and provide a sketch of the results for the strategic-customer case, highlighting differences between the two.

In the next section we review the literature. In section 3, we describe the FC contract formally and briefly summarize results in Durango-Cohen and Yano (2006) on the supplier's problem of choosing commitment and production quantities for a given customer forecast. We also extend this analysis to provide a full characterization of the supplier's dominant strategies for all possible values of the customer's forecast. In section 5 we analyze the customer's problem of determining an optimal forecast to submit. We show that the customer's objective function is usually multi-modal.

In section 6, we provide a full characterization of the supplier's optimal policy when the customer is strategic, and a partial characterization of the customer's optimal policy that enables us to compute optimal solutions, despite the complexity of the policy. In section 7, we report on an extensive numerical study. The results suggest that the customer gains little by being dishonest. When the customer is honest, the structure of the customer's forecast and the supplier's response can be predicted based on relationships among key contract parameters. The results also provide insights into how the contract parameters affect the division of profits between the two parties and characteristics of contract parameters that lead to (near-)coordination of the supply chain. From this, we provide guidelines on how to structure the contract to achieve near-optimal performance. Concluding remarks appear in section 8. Owing to page limitations, the main body of the paper primarily includes statements of the results and key observations. Readers interested in more details should refer to the supporting information Appendices.

2. Related Literature

In recent years, there has been a trend toward reducing the number of suppliers (cf. Kanter 2008) and toward establishing contracts that require customers to provide forecasts or advance orders in exchange for more information about the supplier's anticipated deliveries.

The literature on supply contracts is extensive and growing rapidly. For reviews, see Tsay and Lovejoy (1999), Cachon and Lariviere (1999), and Cachon (2002). For brevity, we do not repeat this material here. Instead, we focus on comparing the FC contract to the most similar contracts in the literature: (i) take-or-pay contracts and capacity reservation contracts with options; (ii) buy-back contracts; (iii) contracts with minimum purchase commitments; and (iv) quantity flexibility (QF) contracts.

Take-or-pay contracts (cf. Grossman et al. 2000, Schultz 1997) require the customer to reserve capacity (or product) in advance. There is a per unit charge for the utilized capacity and a (possibly) reduced per unit charge for any unutilized capacity up to the reservation quantity. The supplier is obliged to provide capacity up to the reserved amount. The FC contract differs in that the customer is required to pay for the lesser of $\alpha\bar{f}$ and the amount committed by the supplier at a constant cost per unit. Thus, the customer must pay for what he takes, and may be required to take more than he needs. The supplier may produce less than the commitment quantity, and may make penalty payments to the customer depending on his commitment and production decisions.

We assume that the two parties have symmetric information. Gan et al. (2009) study a model with asymmetric information in which the customer must purchase his entire forecast and the supplier must produce all of it, so the supplier's decisions are simpler than in our model. In their model, information asymmetry is captured in a stylized manner: the customer's mean demand is high or low. In the concluding section, we discuss the challenges of extending our model to consider information asymmetry.

Buy-back (or returns) contracts (cf. Pasternack 1985), allow the customer to return excess product (possibly with some restrictions) for a full or partial refund. The FC contract could be viewed as a variant of a buy-back contract in which the customer buys whatever the supplier produces (which may be more or less than the customer's forecast) but the supplier offers a buy-back at full price down to the lesser of $\alpha\bar{f}$ and the supplier's commitment. There may also be payments by the supplier that do not occur in a typical buy-back contract, e.g., commitment and delivery penalties.

Contracts with a minimum purchase commitment require the customer to purchase a certain minimum amount in each period and/or over the life of the contract. These minimum quantities may be specified by the supplier or negotiated, but are assumed to be fixed in advance. These contracts have been analyzed using classical inventory models with constraints on the total or per-period order quantity (cf. Anupindi and Akella 1993, Anupindi and Bassok 1998, Bassok and Anupindi 1997, Cachon and Lariviere 2001, Moinzadeh and Nahmias 2000, Nahmias 1997, Porteus 1990). The FC contract includes a minimum purchase quantity, but this amount is not specified in advance; it is influenced by both the customer's forecast and the supplier's commitment quantity.

QF contracts require that customers provide forecasts and specify a purchase commitment based on the given forecast. The characteristic feature of each variant of this type of contract is the rule used to determine the customer's purchase commitment. The contract structure that is closest to ours is the percentage flexibility contract presented by Tsay (1999). In this contract, the customer commits to purchasing a fraction of the forecast and the supplier agrees to deliver up to a certain fraction above the forecast. Bassok and Anupindi (1997) study the customer's forecasting-ordering policy under a percentage flexibility contract for multiple periods in a rolling horizon framework. Tsay and Lovejoy (1999) extend the rolling horizon contract to a multi-echelon setting. Through simulation experiments, they show that the contract can serve to dampen the "bullwhip effect."

QF also appears in contracts with options. Eppen and Iyer (1997) study agreements that are prevalent in the fashion industry, in which a customer provides a forecast to the supplier before the season, and later purchases a fraction of the forecast at the start of the season. After a short time, the customer has the option to either purchase the remaining units or pay a

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penalty for the units not purchased. Barnes-Schuster et al. (2002) examine the use of options to provide customer flexibility. They consider a two-period setting in which a customer places firm orders for both periods and purchases options in the first period that may be exercised in the second. Martinez-de Albeniz and Simchi-Levi (2005) consider situations with multiple contracts, including those with options.

The aforementioned contract structures emphasize the customer's commitment, and generally assume that the supplier will produce at least as much as the customer's commitment quantity. To the best of our knowledge, our proposed contract is the only one in the literature that entails promises that are decided by both parties. The customer's promise is a binding commitment, but the supplier's promise can be violated at a cost. (When $\alpha < 1$, the customer has some flexibility, although his minimum purchase quantity is fixed.) The supplier, therefore, can make appropriate tradeoffs among customers, leading to more efficient use of his capacity than if he were obliged to make firm commitments for quantities that customers are unlikely to order. In essence, because the supplier can violate his commitments at a cost, he can hedge his risks using financial payments (which are flexible) vs. production commitments for goods that have no secondary market.

Our proposed contract was motivated, in part, by situations in which the supplier needs to allocate limited production capacity among multiple customers or products. We do not treat this problem here, but refer readers to Lee et al. (1999) and Cachon and Lariviere (1999) for papers on allocation issues.

3. The Model

We assume the supplier and buyer have symmetric information about costs and that they share a common distribution, $F(x)$ ($f(x)$ is the probability density function), for demand in a future period. The problem is quite complex even under symmetric information, so generalization to consider asymmetric information remains a topic for future research. We also assume that the supplier has unlimited capacity, although it is possible to account for capacity constraints in the supplier's decision problem for a single customer (see Durango-Cohen and Yano 2006).

The contract gives rise to a Stackelberg game. In the first stage, the customer, as the leader, provides the supplier his forecast, $\bar{f}$. The supplier then makes a commitment to the customer, C, and simultaneously decides a production quantity, q, to be completed in time for the second stage. In the second stage, according to the terms of the contract, the customer must purchase at least $\underline{d} = \min\{\alpha\bar{f}, C\}$, where $0 \le \alpha \le 1$ has been negotiated in advance. Having observed demand, x, at the start of the second stage, the honest customer orders $\min\{C, \max\{x, \underline{d}\}\}$. In simple terms, the customer orders what he needs after observing his demand, except that there is a lower bound of $\alpha\bar{f}$ and an upper bound of C. Because the supplier may produce less than the customer's order, the customer takes delivery of $\min\{q, \max\{x, \underline{d}\}\}$. (We treat the strategic-customer case separately in section 6.) A timeline of events in shown in Figure 1.

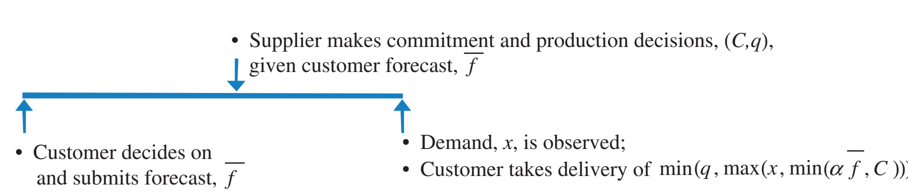
The economic parameters are the retail price, p, wholesale price, w, production cost, c, salvage value, s, commitment penalty, π_1, and the delivery penalty, π_2. Without loss of generality, we assume that prices and costs are normalized so that $s = 0$. Throughout the analysis, we treat α, π_1 and π_2 as fixed parameters (i.e., determined through *a priori* negotiations), but later in the paper, we discuss the joint effect of these parameters on characteristics of contract outcomes.

3.1. The Supplier's Problem in the Honest-Customer Case

In response to the value of $\bar{f}$ chosen by the customer, the supplier chooses commitment and production quantities (C and q) with the goal of maximizing expected profit:

$$\begin{aligned}\phi_S(C,q) = {} & w\left[\int_{x=0}^{\alpha\bar{f}} \alpha\bar{f} f(x)dx + \int_{x=\alpha\bar{f}}^{q} x f(x)dx \right.\\ & \left. + \int_{x=q}^{\infty} q f(x)dx\right] - cq - \pi_1\,[\bar{f} - C]^{+} \\ & - \pi_2\left[\int_{x=q}^{C} (x-q)f(x)dx \right.\\ & \left. + \int_{x=C}^{\infty} (C-q)f(x)dx\right].\end{aligned} \qquad (1)$$

Figure 1 Timeline of Events



The terms represent expected revenue, production cost, and expected commitment and delivery penalties. The objective function is not jointly concave in C and q. However, results in our earlier paper (Durango-Cohen and Yano 2006) and additional results derived later in this paper simplify the effects of the supplier's decision on the customer's problem.

3.2. The Honest Customer's Problem

In this section, we present the customer's problem of choosing a forecast. Define

π_C: shortage penalty (loss of goodwill) per unit incurred by the customer;
$C^*(\bar{f})$: supplier's optimal commitment quantity given the customer forecast $\bar{f}$;
$q^*(\bar{f})$: supplier's optimal production quantity given the customer forecast $\bar{f}$.

The customer seeks a forecast that maximizes his expected profit given the supplier's optimal response. The customer's profit as a function of his decision, $\bar{f}$, given the supplier's optimal commitment $C^*(\bar{f})$ and production quantity $q^*(\bar{f})$, can be written as

$$\begin{aligned}\phi_C(\bar{f}) = p\left[\int_{x=0}^{q^*(\bar{f})} xf(x)dx\right] - w\left[\int_{x=0}^{\alpha\bar{f}} \alpha\bar{f}f(x)dx\right.\\ \left.+ \int_{\alpha\bar{f}}^{q^*(\bar{f})} xf(x)dx + \int_{q^*(\bar{f})}^{\infty} q^*(\bar{f})f(x)dx\right]\\ - \pi_C\left[\int_{q^*(\bar{f})}^{\infty} (x - q^*(\bar{f}))f(x)dx\right]\\ + \pi_1\left[\bar{f} - C^*(\bar{f})\right]^+ + \pi_2\left[\int_q^C (x-q)f(x)dx\right.\\ \left.+ \int_C^{\infty} (C-q)f(x)dx\right].\end{aligned} \quad (2)$$

The first two terms represent the customer's expected revenue and purchase cost. The expected shortage penalty, which considers the quantity produced by the supplier (which is the maximum quantity available to the customer), is given in the third term. The last two terms represent the customer's expected receipts of commitment and delivery penalties.

One important question is the relation of the commitment penalty (π_1) and delivery penalty (π_2) to the direct loss of profit incurred by the customer (i.e., $p-w$) and the loss of goodwill (π_C) when a shortage occurs. Observe that a shortfall of the commitment from the customer's forecast does not necessarily result in shortages. What ultimately affects end-customer shortages is how much the supplier delivers. Thus, π_1 is a penalty that gives the supplier an incentive to commit as much as possible, but it need not be linked to external shortage costs. (Recall that higher values of π_1 typically would be selected for longtime customers and/or those who forecast well.) There is a clearer relation between π_2 and the shortage cost incurred by the customer. In particular, if the full (both direct and loss of goodwill) cost of shortages is passed on to the supplier by the customer via the delivery penalty, then we should have $\pi_2 = p - w + \pi_C$, i.e., the delivery penalty is the sum of the financial loss of profit and the loss of goodwill.

Although an FC contract may be negotiated with any arbitrary values of π_1 and π_2, we proceed under the assumption that $\pi_2 = p - w + \pi_C$. With this assumption, the analysis is cleaner and the insights are more transparent, but the structure of the results is similar even under more general assumptions. Owing to this assumption, the analysis is not completely general. However, as we will see later, the problem is quite complex even with this assumption that dramatically simplifies the algebra. Later in the paper, we explain the primary consequence of this assumption. We often express costs in terms of p and π_C, but sometimes reinterpret the expressions in terms of w and π_2 when it simplifies the analysis. Before considering the customer's problem in detail, we first provide a complete characterization of the supplier's optimal response as a function of the customer's forecast.

4. Supplier's Response as a Function of the Forecast

As mentioned earlier, the supplier's objective function is not jointly concave in C and q for a *fixed customer forecast*, so his optimal response is not a simple function of $\bar{f}$. We have shown in an earlier paper (see Durango-Cohen and Yano 2006) that under the FC contract, the supplier has two dominant commitment strategies: (1) $C^* = \bar{f}$ or (2) $C^* = q^*$. The optimal production quantity, as a function of a given customer forecast, for each dominant commitment strategy, is shown in Table 1, where certain newsvendor-like quantities q_A^*, q_B^* and q_C^* are defined.

For any given forecast, the supplier evaluates both strategies and chooses the better one. Because the two parties share a common demand distribution and know the contract terms, the customer can therefore calculate the supplier's optimal response to any given forecast.

To solve the customer's problem of choosing a forecast, however, we need to know how the supplier's optimal response changes *as a function of the forecast*. In Durango-Cohen and Yano (2006), we showed that for $\bar{f} \in [0, \min(q_B^*, q_C^*)]$, the supplier's optimal policy is the same under the two commitment strategies, namely $C^* = \bar{f}$ or $C^* = q_A^*$ and $q^* = q_A^*$ for $\bar{f} \in [0, q_A^*]$ and $C^* = \bar{f}$ and $q^* = \bar{f}$ for $\bar{f} \in [q_A^*, \min(q_B^*, q_C^*)]$, so we focus here on $\bar{f} \geq \min(q_B^*, q_C^*)$. We also found that the structure of the supplier's optimal response depends heavily on the

Table 1 Optimal Supplier Response for Different Customer Forecasts

Commitment strategy, $C = q$		Commitment strategy, $C = \bar{f}$	
Forecast quantity	Optimal supplier response	Forecast quantity	Optimal supplier response
$\bar{f} \le q_A^*$	$C^* = q^* = q_A^*$	$\bar{f} \le q_A^*$	$C^* = \bar{f}, q^* = q_A^*$
$q_A^* \le \bar{f} \le q_C^*$	$C^* = q^* = \bar{f}$	$q_A^* \le \bar{f} \le q_B^*$	$C^* = \bar{f}, q^* = \bar{f}$
$q_C^* \le \bar{f} \le \frac{q_C^*}{\alpha}$	$C^* = q^* = q_C^*$	$q_B^* \le \bar{f} \le \frac{q_B^*}{\alpha}$	$C^* = \bar{f}, q^* = q_B^*$
$\frac{q_C^*}{\alpha} \le \bar{f}$	$C^* = q^* = \alpha\bar{f}$	$\frac{q_B^*}{\alpha} \le \bar{f}$	$C^* = \bar{f}, q^* = \alpha\bar{f}$

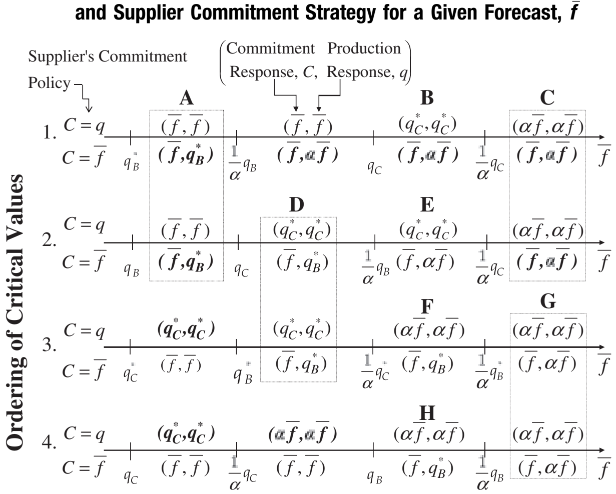
Here $q_A^* = F^{-1}\left(\frac{w-c}{w}\right), q_B^* = F^{-1}\left(\frac{w+\pi_2-c}{w+\pi_2}\right)$, and $q_C^* = F^{-1}\left(\frac{w+\pi_1-c}{w}\right)$.

ordering of four critical values: q_B^*, q_C^*, $\frac{q_B^*}{\alpha}$, and $\frac{q_C^*}{\alpha}$, and we utilize this fact in our analysis. Our earlier results establish that, for each of the four orderings of critical values, one of the supplier's strategies is dominant for certain ranges of forecast values. These dominant policies are shown, without special labeling, in boldface in Figure 2.

Each row in the figure represents one ordering of the critical values. For brevity, we use the term ''interval'' to refer to an interval between two consecutive critical values. The supplier's optimal response to a forecast in an interval when the supplier commits to q (respectively, $\bar{f}$) is shown above (respectively, below) the axis.

The remaining (pairs of) cases are labeled A through H and we investigate them next. In supporting information Appendix S1, we show that for the two cases labeled A, the case labeled B, and the two cases labeled C, the strategy shown in boldface dominates the other. Thus, in all of the unlabeled cases as well as in Cases A, B, and C, one of the supplier's policies is dominant, so the customer can easily anticipate what the supplier will do. In the other cases (i.e., cases labeled D through H), the customer needs to consider both supplier policies in general, although one may be dominant for certain combinations of parameter values. The customer's problem would be extremely complicated if the supplier's policy were to change many times as the customer's forecast changes. Fortunately, we are able to show that the supplier's objectives under the two commitment policies may cross at most once in the G and H cases, and at most twice in the D, E, and F cases. The proof of Theorem 1 appears in supporting information Appendix S2.

Figure 2 Dominant Supplier Strategies for Each Ordering of Critical Values and Supplier Commitment Strategy for a Given Forecast, $\bar{f}$



THEOREM 1. *Within each of the intervals shown in Figure 2, either (i) one of the supplier's two commitment policies is dominant for the entire interval or (ii) the supplier's objective functions under the two commitment strategies cross at most twice.*

When solving his problem, the customer must anticipate which strategy the supplier will choose, and the crossing (or switching) points are critical in defining ranges of forecast values for which each supplier strategy is relevant. In supporting information Appendix S2, we provide an analysis of the number of possible switches in the dominant strategy (from $C = \bar{f}$ to $C = q$, or vice versa) in the supplier's commitment strategy for each ordering of the critical values.

5. The Honest Customer's Choice of a Forecast

Before considering the effects of the supplier's switching points on the customer's choice of $\bar{f}$, we first characterize the customer's objective function for each dominant supplier response. In Table 2, we list the relevant forecast intervals along with the dominant supplier solution(s). For each case, we list either the unconstrained stationary point of the customer's objective or the first-order condition that defines the optimal forecast. For each case, we show (details follow) that the customer's objective function *for each dominant supplier commitment strategy* is concave in the customer's forecast. Thus, for a given forecast interval and supplier commitment strategy, the optimal customer forecast is either the *unconstrained* stationary point if it falls within the interval, or an interval boundary.

Table 2 Customer's Optimal Forecast Given Supplier's Commitment Strategy

			Optimal supplier response given $\bar{f}$		
	Case	Forecast range	$C^*(\bar{f})$	q^*	Customer optimal forecast given C^* and q^*
Commitment strategy $C = q$	1.	$0 \le \bar{f} \le q_A^*$	$C^* = q_A^*$	$q^* = q_A^*$	$\bar{f}^* = 0$
	2.	$q_A^* \le \bar{f} \le q_C^*$	$C^* = \bar{f}$	$q^* = \bar{f}$	Solution to first-order condition (#1): $(p + \pi_C - w)[1 - F(\bar{f})] - \alpha wF(\alpha\bar{f}) = 0$
	3.	$q_C^* \le \bar{f} \le \frac{q_C^*}{\alpha}$	$C^* = q_C^*$	$q^* = q_C^*$	$\bar{f}^* = \frac{1}{\alpha}F^{-1}\left(\frac{\pi_1}{\alpha w}\right) \equiv q_D^*$
	4.	$\frac{q_C^*}{\alpha} \le \bar{f}$	$C^* = \alpha\bar{f}$	$q^* = \alpha\bar{f}$	$\bar{f}^* = \frac{1}{\alpha}F^{-1}\left(\frac{\alpha(p+\pi_C-w)+\pi_1(1-\alpha)}{\alpha(p+\pi_C)}\right) \equiv q_E^*$
Commitment strategy $C = \bar{f}$	5.	$0 \le \bar{f} \le q_A^*$	$C^* = \bar{f}$	$q^* = q_A^*$	$\bar{f}^* = 0$
	6.	$q_A^* \le \bar{f} \le q_B^*$	$C^* = \bar{f}$	$q^* = \bar{f}$	Solution to first-order condition (#1): $(p + \pi_C - w)[1 - F(\bar{f})] - \alpha wF(\alpha\bar{f}) = 0$
	7.	$q_B^* \le \bar{f} \le \frac{q_B^*}{\alpha}$	$C^* = \bar{f}$	$q^* = q_B^*$	Solution to first-order condition (#2) $-\alpha wF(\alpha\bar{f}) + \pi_2[1 - F(\bar{f})] = 0$
	8.	$\frac{q_B^*}{\alpha} \le \bar{f}$	$C^* = \bar{f}$	$q^* = \alpha\bar{f}$	Solution to first-order condition (#3): $\alpha(p + \pi_C - (w + \pi_2))[1 - F(\alpha\bar{f})] - \alpha wF(\alpha\bar{f}) + \pi_2[1 - F(\bar{f})] = 0$

THEOREM 2. *For each dominant supplier response, the customer's objective function is concave in $\bar{f}$ and Table 2 characterizes the customer's dominant unconstrained solutions.*

See supporting information Appendix S3 for a proof of Theorem 2. We note that for Cases 1 and 5, the customer's objective function is concave decreasing in $\bar{f}$ so $\bar{f}^* = 0$. One way to interpret a forecast of zero is to view it as though the customer does not submit a forecast at all. In other words, in response to a forecast $\bar{f}^* = 0$, the supplier produces his "unconstrained" economic newsvendor quantity, $q_A^* = F^{-1}\left(\frac{w-c}{w}\right)$. Thus, such a forecast does not change the supplier's production quantity relative to a no-contract setting.

We make a few other observations about the results in Table 2. For Case 3, incremental changes to the forecast within the interval do not affect the supplier's optimal commitment and production quantities, so the customer chooses the forecast that optimally trades off his costs of under- and over-forecasting (π_1 and $\alpha w - \pi_1$, respectively), and then inflates this quantity by a multiplicative factor of $1/\alpha$ (because he is obliged to take delivery of only a fraction α of his forecast). Case 4 provides interesting insights. The first portion of the forecast, $F^{-1}\left(\frac{p+\pi_C-w}{p+\pi_C}\right)$, is the customer's newsvendor quantity if he were producing the product himself at a cost of w per unit. In addition to inflating the forecast by a factor of $1/\alpha$, there is an increase due to the term $\frac{\pi_1(1-\alpha)}{\alpha(p+\pi_C)} > 0$. This term arises because the supplier-paid commitment penalty plays the role of a subsidy, allowing the customer to submit a forecast for, and ultimately also purchase, a higher fractile of his demand distribution. In Cases 7 and 8, the optimal forecast trades off the benefits of receiving the delivery penalty but possibly having to purchase (and salvage) more units, with the possibility of selling more units, as increases in the forecast induce the supplier to produce more. A similar rationale applies to Cases 2 and 6.

Thus far we have presented results showing that, within each forecast interval in Table 2, the customer's value function is concave for each relevant supplier commitment policy, and have characterized stationary points that define the optimal solution within each interval for a given supplier commitment strategy. This, however, is not a full characterization of the customer's optimal policy because it does not account for the supplier's choice of commitment strategy. We next present a numerical example to illustrate how the supplier's strategy choice complicates the customer's decision.

5.1. An Illustrative Example

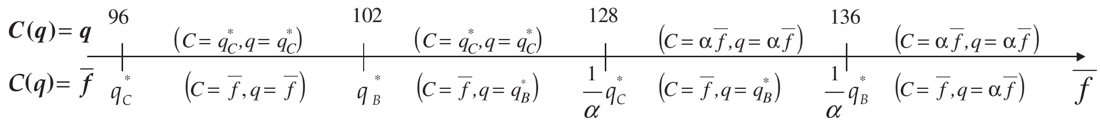
This example illustrates how the supplier's choice of commitment strategy determines the customer's expected profit function and how this, in turn, leads to the choice of the optimal forecast quantity.

For this example, the ordering of the critical values is q_A^*, q_C^*, q_B^*, $\frac{q_C^*}{\alpha}$, and $\frac{q_B^*}{\alpha}$. The supplier's optimal responses under the two commitment strategies for a range of customer forecasts spanning from zero to greater than the largest critical value are shown in Figure 3.

We note that the two strategies are identical up to q_C^*, which is less than q_B^*. If the customer submits a forecast between $q_C^* = 96$ and $q_B^* = 102$, the supplier's optimal response when committing to the production quantity is to commit to and produce $q_C^* = 96$, and when committing to the forecast, the optimal production quantity is to produce the forecast, $q = \bar{f}$. The responses under the two commitment strategies for the remaining intervals can be interpreted similarly.

The left pane of Figure 4 shows the supplier's expected profit under the two commitment strategies given his optimal response in each interval, as a function of the customer's forecast. The supplier's optimal production quantity for forecasts $\bar{f} \in [0, q_A^*]$ is $q^* = q_A^* = 83$, while for $\bar{f} \in [q_A^*, q_C^*]$, we have $C^* = q^* = \bar{f}$. The supplier's objective functions for the two strategies cross at $\bar{f} = 124$. (Values are rounded for convenience.)

Figure 3 Optimal Supplier Response under Each Commitment Strategy. The Problem Parameters are: Demand $\sim N(\mu = 100,\ \sigma = 20)$; p = US\$1.55; π_C = US\$0.65; $\alpha = 0.75$; w = US\$1.25; c = US\$1.00; π_1 = US\$0.275; and $\pi_2 = 0.95$



For customer forecasts between $q_C^* = 96$ and $\bar{f} = 124$ (the portion of the subinterval to the left of the crossing point), the supplier commits to and produces $q_C^* = 96$. For $\bar{f} \in [124, q_B^*/\alpha = 136]$, the supplier's optimal strategy is to commit to the forecast and produce $q_B^* = 102$. For forecasts larger than q_B^*/α, the supplier's optimal strategy is to commit to the forecast and produce $q^* = \alpha\bar{f}$.

The right pane of Figure 4 shows the customer's expected profit, which is calculated taking into account the supplier's optimal response to forecasts in the different intervals. The discontinuity in the customer's objective function results from the supplier's switch in his preferred strategy. In this example, the optimal forecast is $\bar{f} = 124$, and the optimum is at a point where the supplier switches strategies. The supplier's optimal response is to commit to the forecast and produce $q_B^* = 102$. The optimal expected profits for the customer and supplier are US\$22.80 and US\$14.80, respectively.

The key observation from this example is that identification of the customer's optimal forecast must account for switches in the supplier's commitment strategy. In particular, crossing points of the supplier's objective functions for the two strategies are candidates for the customer's optimal forecast.

5.2. Finding the Customer's Best Forecast

For each of the four orderings of the critical values q_B^*, q_C^*, $\frac{q_B^*}{\alpha}$, and $\frac{q_C^*}{\alpha}$ and for each interval in which the customer forecast may lie, Figure 5 provides a "shorthand" representation of how to find the best solution within each interval. Shown above or below the axis is the relevant stationary point (i.e., $\bar{f}_{\mathrm{FOC}}$, q_D^* or q_E^*) for each supplier commitment policy, where $q_D^* = \frac{1}{\alpha}F^{-1}\left(\frac{\pi_1}{\alpha w}\right)$ and $q_E^* = \alpha^{-1}F^{-1}\left(\frac{\alpha\pi_2+(1-\alpha)\pi_1}{\alpha(w+\pi_2)}\right)$, and $\bar{f}_{FOC}$ denotes the solution to the first order necessary condition shown in Table 2. (The three first-order conditions in Table 2 are equivalent under the assumption that $\pi_2 + w = p + \pi_C$, so the customer needs to find only one stationary point. Otherwise, the customer would need to compute three stationary points, but the solution process remains fundamentally the same.) If the stationary points q_D^* and q_E^* and the solution(s) to the FOC(s) do not lie in the specified interval, it is necessary to evaluate the boundaries of

Figure 4 Supplier Expected Profit under Each Commitment Strategy (Left), and Customer's Expected Profit Given Supplier's Optimal Commitment Choice (Right)

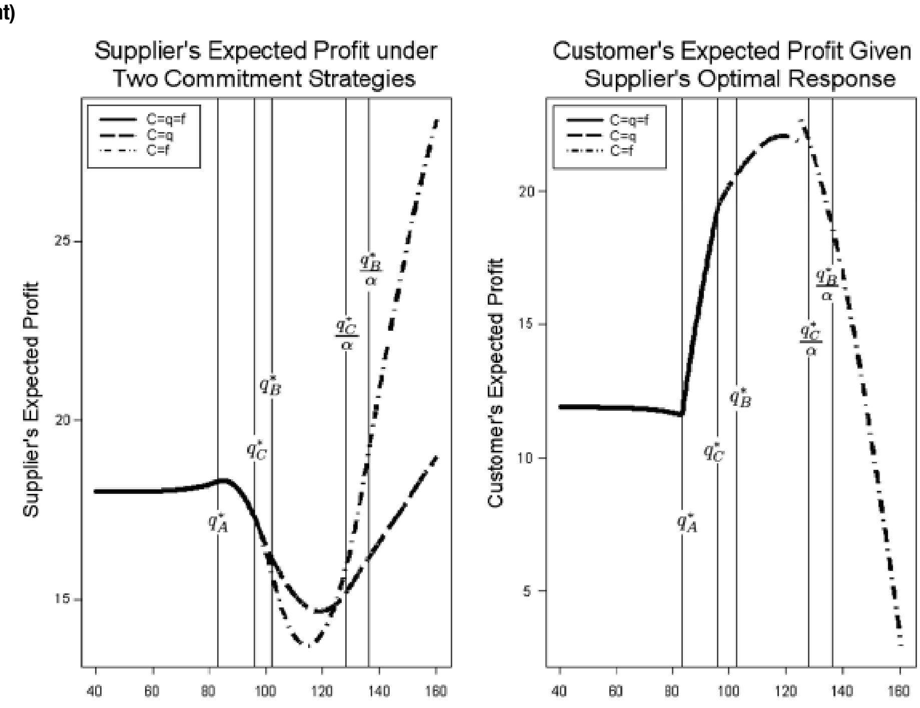
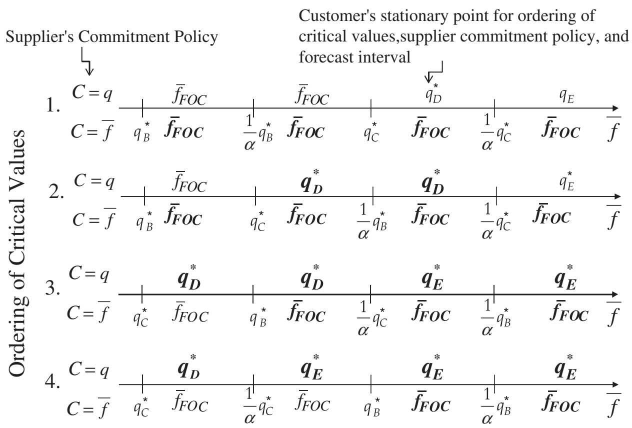


Figure 5 Customer's Stationary Points for Each Ordering of Critical Values, Supplier Commitment Strategy, and Forecast Interval



the interval as candidate solutions. The FOCs for which solutions must be determined are shown in boldface. The other options can be eliminated because the corresponding commitment strategy would not be selected by the supplier. If two options are shown in boldface within the same interval, then the supplier's objectives functions for his two commitment strategies may cross and the customer has to anticipate the switch in the supplier's strategy at the (potential) crossing point(s).

Finding the optimal solution requires computing q_D^*, q_E^*, and $\bar{f}_{\mathrm{FOC}}$, as well as the crossing points of the supplier's objective functions under the two commitment strategies for Cases D through H (cf. Figure 2). Then, each unconstrained candidate solution must be adjusted, if necessary, to account for the relevant constraints imposed by the interval boundaries and crossing points of the supplier's objective function. Finally, because the customer's objective function may be multi-modal, the objective function values must be computed for the candidate solutions and best alternative selected. The most complicated part of the process is determining the crossing points, but these steps can be performed using line searches. The fact that the number of crossings under each ordering of the critical values, as well as the intervals in which crossings may occur, are known (Corollary 1 in supporting information Appendix S2) streamlines the process. In supporting information Appendix S5, we also show how to find the optimal forecast for the various orderings of the critical values.

6. The Strategic-Customer Case

6.1. The Supplier's Problem Under Strategic Customer Behavior

One can easily show that it is never optimal for the supplier to commit to less than what he has produced, as such a strategy would cause the supplier to incur commitment penalties unnecessarily. Thus, the supplier's commitment strategy is such that $C \geq q$. The customer, on the other hand, can only benefit from strategic behavior if the supplier commits to more than he produces. If the supplier produces $q \leq C$, we note the following: (i) If the observed demand, x, exceeds the production quantity, q, the customer can order C, take delivery of q, and receive a penalty payment $\pi_2(C-q)$. (ii) If $x \leq q \leq C$, i.e., demand is less than production quantity, the customer needs to decide whether to over-order, and if so, how much. If the customer orders y, $q < y \leq C$, he receives a penalty payment $\pi_2(y-q)$ and pays $w(q-x)$ for the excess units. Because the penalty payment is increasing in y but his payment for excess units remains constant, it is optimal for him to order C if he chooses to over-order. The customer acts strategically if $\pi_2(C-q) > w(q-x)$. Thus, if $x \geq \frac{(w+\pi_2)q-\pi_2 C}{w} = T(C,q)$, the customer orders C and takes delivery of q. Otherwise, he orders x.

As in the honest-customer setting, we model the problem as a Stackelberg game with the customer as the leader. For any customer forecast, $\bar{f}$, we must consider two cases: (i) $q \geq \bar{f}$ or (ii) $q \leq \bar{f}$. In Case (i), the supplier's optimization problem for a given customer forecast $\bar{f}$ is

$$\begin{aligned} \max \phi_S(C,q) = {} & w\int_0^{\min(\alpha\bar{f},T(C,q))} \alpha\bar{f}f(x)dx \\ & + w\int_{\min(\alpha\bar{f},T(C,q))}^{T(C,q)} xf(x)dx \\ & + wq[1-F(T(C,q))] \\ & - \pi_2(C-q)[1-F(T(C,q))] - cq \end{aligned} \tag{3}$$

$$\text{s.t.} \quad \bar{f} \leq q \leq C \tag{4}$$

The first three terms in (3) collectively represent expected revenue. The fourth term is the delivery penalty and the last term captures the variable production costs. As in the honest-customer case, it is never optimal for the supplier to commit to less than q. For $C \geq q$, the profit function also depends on whether the customer's demand threshold, $T(C,q) \equiv \frac{(w+\pi_2)q-\pi_2 C}{w}$, is less or greater than $\alpha\bar{f}$. Note that if $C = q$, then $T(C,q) = q$, which is necessarily greater than or equal to $\alpha\bar{f}$, so we cannot have $T(C,q) < \alpha\bar{f}$ when $C = q$. It is straightforward to show that the objective function is convex decreasing in C in Case (i), and so the optimal commitment is $C^* = q$. If we now substitute q for C in the objective function, and then take the first derivative with respect to q, we obtain

$$q^* = \begin{cases} F^{-1}\left(\frac{w-c}{w}\right) \equiv q_A^* & \text{if } \bar{f} \le F^{-1}\left(\frac{w-c}{w}\right) \\ \bar{f} & F^{-1}\left(\frac{w-c}{w}\right) \le \bar{f} \end{cases}.$$

For Case (ii), we also need to consider whether $\alpha\bar{f} \le T(C,q)$ or $\alpha\bar{f} \ge T(C,q)$, as the supplier may commit to more than the amount to be produced. Note that these conditions place *implicit* constraints on $\bar{f}$ because C and q are the supplier's choices *for the given* $\bar{f}$. The supplier's problem for a given customer forecast $\bar{f}$ is

$$\begin{aligned} \max \phi_S(C,q) = {} & w\int_0^{\min(\alpha\bar{f},T(C,q))} \alpha\bar{f}f(x)dx \\ & + w\int_{\min(\alpha\bar{f},T(C,q))}^{T(C,q)} xf(x)dx \\ & + wq[1-F(T(C,q))] - \pi_1(\bar{f}-C)^+ \\ & - \pi_2(C-q)[1-F(T(C,q))] - cq, \end{aligned} \tag{5}$$

$$\text{s.t.} \quad q \ge \alpha\bar{f}, \tag{6}$$

$$q \le C \le \bar{f}. \tag{7}$$

The first three terms in the objective function collectively represent expected revenue. The other terms are the commitment penalty, delivery penalty, and variable production costs, respectively. We consider the cases of $\alpha\bar{f} \le\ge T(C,q)$. If $\alpha\bar{f} \le T(C,q)$, then

$$\frac{\partial\phi_S(C,q)}{\partial q} = (w+\pi_2)\{1-F(T(C,q))\} - c, \tag{8}$$

$$\frac{\partial\phi_S(C,q)}{\partial C} = -\pi_2\{1-F(T(C,q))\} + \pi_1. \tag{9}$$

When $\alpha\bar{f} \ge T(C,q)$, then the first-order conditions with respect to q and C are

$$\begin{aligned} \frac{\partial\phi_S(C,q)}{\partial q} = {} & (w+\pi_2)\{[\alpha\bar{f}-T(C,q)]f(T(C,q)) \\ & +[1-F(T(C,q))]\} - c, \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{\partial\phi_S(C,q)}{\partial C} = {} & -\pi_2\{[\alpha\bar{f}-T(C,q)]f(T(C,q)) \\ & +[1-F(T(C,q))]\} + \pi_1. \end{aligned} \tag{11}$$

Equations (8)–(11) are first derivatives, but we will use the same equation numbers when referring to the corresponding FOCs. For example, FOC (8) means the FOC that arises when (8) is set equal to zero.

For both $\alpha\bar{f} \le\ge T(C,q)$, the determinant of the Hessian of ϕ_S is zero. However, we can take advantage of the structure of the FOCs and certain monotonicity properties of the first derivatives to characterize the supplier's dominant solutions. In particular, the structure of the FOCs is such that for each pair ([8] and [9], or [10] and [11]), at most one of the FOCs can be satisfied (unless $\pi_1 = \frac{\pi_2 c}{w+\pi_2}$, in which case both FOCs can be satisfied but the solution is not unique). Thus, the relationships between π_1 and $\frac{\pi_2 c}{w+\pi_2}$, and between $\alpha\bar{f}$ and $T(C,q)$ (as noted earlier), are critical, and they define the four cases that must be considered.

A summary of the supplier's dominant policies appears in Table 3; detailed derivations appear in supporting information Appendix S6. Interestingly, for Cases A and C, where $\pi_1 \le \frac{\pi_2 c}{w+\pi_2}$, in nearly all the policies that emerge, the supplier commits to q. As such, the customer *cannot* benefit from being dishonest, so it is not surprising that the policies are the same as in the honest-customer model. Intuitively, the commitment penalty is low relative to the delivery penalty, and so the supplier prefers to pay the commitment penalty rather than opening himself up to strategic behavior by the customer. The only policy that differs qualitatively is $q = \alpha\bar{f}$ and C from (11) in Case C. Here, the forecast is high and the supplier uses the interesting hedging strategy of producing $\alpha\bar{f}$ but choosing an intermediate commitment quantity that balances the commitment penalty against the delivery penalty. For the customer, this means that he must provide a large forecast, which, in turn, may require him to purchase a significant amount. Because C is defined implicitly by a messy FOC (11), it is not possible to prove concavity of the supplier's objective for this policy. Nevertheless, it is easy to determine the supplier's optimal policy for any given $\bar{f}$ when $\pi_1 \le \frac{\pi_2 c}{w+\pi_2}$. We have the easy-to-express ranges of $\bar{f}$ values (up to $\frac{q_C^*}{\alpha}$) for which specific supplier policies are dominant. And when $\bar{f} \ge \frac{q_C^*}{\alpha}$, the supplier only needs to compare the profits from the two policies listed in Case C.

For Case B the solution is more complex. In addition to policies in which the supplier commits to q, we observe policies that are qualitatively different than

Table 3 Supplier's Dominant Policies

	$\pi_1 \le \frac{\pi_2 C}{w+\pi_2}$	$\pi_1 \ge \frac{\pi_2 C}{w+\pi_2}$
	Case A	Case B
$\alpha\bar{f} \le T(C,q)$	$C = q = q_A^*$, if $\bar{f} \le q_A^*$ $C = q = \bar{f}$, if $q_A^* \le \bar{f} \le q_C^*$ $C = q = q_C^*$, if $q_C^* \le \bar{f} \le \frac{q_C^*}{\alpha}$ $C = q = \alpha\bar{f}$, if $\frac{q_C^*}{\alpha} \le \bar{f}$	All the solutions in Case A, and $C = q = \bar{f}$, if $q_A^* \le \bar{f} \le q_B^*$ $C = \bar{f}, q = q_G^*$, if $q_B^* \le \bar{f} \le \frac{q_B^*}{\alpha}$ where $q_G^* \equiv \frac{wq_B^* + \pi_2\bar{f}}{w+\pi_2}$
	Case C	Case D
$\alpha\bar{f} \ge T(C,q)$	$C = q$, $q = \alpha\bar{f}$, or C from (11), $q = \alpha\bar{f}$, if $\frac{q_C^*}{\alpha} \le \bar{f}$	$C = \bar{f}$, $q = \alpha\bar{f}$, if $\frac{q_B^*}{\alpha} \le \bar{f}$

we have seen in the honest-customer case. Because the commitment penalty, π_1, is relatively large (i.e., $\geq \pi_2 c/(w+\pi_2)$) the supplier may prefer to commit to $\bar{f}$, but the production quantities generally do not take on newsvendor forms as they did in the honest-customer model, where typically all forecasts in an interval (defined by certain thresholds) would lead to the same choice of q by the supplier. Instead, when the customer is strategic, q may be more sensitive to the customer's forecast.

When $\pi_1 \geq \frac{\pi_2 c}{w+\pi_2}$, the problem is further complicated because we cannot establish ranges of $\bar{f}$ for which one of the supplier's strategies is dominant. In the honest-customer setting, we characterized how the supplier's optimal response changed as a function of the forecast. For example, we defined intervals in which one of the supplier's two commitment policies was dominant, or showed that at most two switches (where the dominant strategy changes from $C = \bar{f}$ to $C = q$, or vice versa) in the optimal commitment strategy could occur. In the strategic-customer setting, we cannot prove that the supplier's optimal policy exhibits a "clean" structure; this is what makes the strategic-customer version of the problem so difficult. However, for any given $\bar{f}$, it is straightforward (but tedious) to determine the supplier's optimal policy.

Finally, the results in the table also provide qualitative insights into how the supplier may adjust his solution in response to strategic customers. Recall that the customer can benefit from strategic behavior only when the supplier commits to more than he produces. The supplier is economically motivated to do so when $\pi_1 \geq \pi_2 c/(w + \pi_2)$ (i.e., the marginal commitment penalty exceeds the marginal expected payment of delivery penalties). A comparison of the results in the second column of Table 3 with results in the honest-customer case (shown in the "Optimal Supplier Response" column in Table 2) leads to the conclusion that q may not be much larger than in the comparable case where the supplier anticipates that the customer will be honest. For example (and this is the primary difference that we observed in our numerical results), in cases where the supplier would have produced q_B^* in the honest-customer case (see Case 7 in Table 2), he might produce $q_G^* = \frac{wq_B^* + \pi_2 \bar{f}}{w+\pi_2}$ when the customer is strategic. But the latter value exceeds the former by a sizable margin only if $\bar{f} \gg q_B^*$, and the customer might not be happy about the additional purchase obligation that a large forecast would imply. The customer's potential for inducing the supplier to produce more increases as π_2 increases, but if $\frac{\pi_2 c}{w+\pi_2}$ is larger than π_1, the supplier will likely switch to a policy with $C = q$ and thereby thwart the customer's attempt to gain from being dishonest. (This follows from the fact that five of the six dominant solutions—all but the solution involving the FOC—in the middle column of the table—are such that $C^* = q$, and, as we show later, the solution involving the FOC is unlikely to arise.) We discuss these issues in more detail in section 7.

6.2. The Strategic Customer's Problem

The customer's problem for a given supplier policy can be formulated as

$$\begin{aligned}\phi_C(\bar{f}) = {} & p\left[\int_0^q xf(x)dx + q[1 - F(q)]\right] \\ & - \pi_C \int_q^\infty (x-q)f(x)dx \\ & - w\left[\int_0^{\min(\alpha\bar{f}, T(C,q))} \alpha\bar{f}f(x)dx \right. \\ & \left. + \int_{\min(\alpha\bar{f}, T(C,q))}^{T(C,q)} xf(x)dx + q[1 - F(T(C,q))]\right] \\ & + \pi_1(\bar{f} - C)^+ + \pi_2(C-q)[1 - F(T(C,q))],\end{aligned}$$

where q and C are the supplier's optimal choices that depend on $\bar{f}$. (We have suppressed the dependence on $\bar{f}$ for expositional simplicity.) The sum of the first two terms represents expected revenue. The third term is the expected shortage cost (above and beyond lost profits). The sum of the next three terms represents expected purchase costs, and the final two terms represent expected receipts of penalty payments.

The customer cannot benefit from strategic behavior when the supplier commits to q (including cases where $C = q = \bar{f}$), so the customer's optimal forecast remains the same as in Table 2 except in two cases, which are shown in Table 4 (see supporting information Appendix S4 for derivations).

Because the supplier's dominant strategies cannot be expressed in closed form in all cases (cf. bottom row in Table 4), finding the customer's optimal forecast requires an enumerative search: *for each candidate forecast*, it is necessary to compute the supplier's solution for *each* dominant strategy and then choose the best one. Moreover, unlike in the honest-customer setting where we could identify intervals (for $\bar{f}$) in which a specific supplier policy is dominant, the more complicated structure of the supplier's objective function prevents derivation of such well-defined specifications. Thus, only after enumerating the supplier's various undominated policies, can we evaluate the resultant customer objective for a given forecast. This process, in principle, needs to be repeated for each viable forecast.

For the reasons discussed above, finding the customer's optimal forecast is significantly more difficult in the strategic-customer case than in the honest-customer case. (For Ordering #3, i.e., $q_C^* \leq q_B^* \leq \frac{q_C^*}{\alpha} \leq \frac{q_B^*}{\alpha}$, however,

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Table 4 Added Characterization of Customer's Optimal Forecast under Strategic Behavior

		Optimal supplier response given $\bar{f}$		
Commitment strategy	Forecast range	C^*	q^*	Customer's unconstrained optimal forecast given C^* and q^*
$C=\bar{f}$	$q_B^* \le \bar{f} \le \frac{q_B^*}{\alpha}$	$C^*=\bar{f}$	$q^*=\frac{wq_B^*+\pi_2\bar{f}}{w+\pi_2}$	Solution to: $-\alpha wF(\alpha\bar{f})+\pi_2\left[1-F\left(\frac{wq_B^*+\pi_2\bar{f}}{w+\pi_2}\right)\right]=0$
$q^*<C\le\bar{f}$	$\frac{q_C^*}{\alpha}\le\bar{f}$	C^* from (11)	$q^*=\alpha\bar{f}$	$\bar{f}^*$ from enumerative search

we are able to further characterize the customer's optimal policy; see supporting information Appendix S7.) The numerical study presented next suggests that the considerable additional effort required of the customer to solve the strategic-behavior problem may have only minimal benefits.

7. Numerical Study

We present results from a numerical study that allow us to characterize the performance of the contract with respect to the no-contract setting. We also analyze how strategic behavior affects both supply chain coordination and the division of profits between the parties. We then report on typical contract outcomes as a function of the parameter values. This is followed by a discussion of the implications of the results for selection of contract parameters and a comparison of the performance of the FC contract with QF contracts.

Throughout the numerical study, we assume that demand is normally distributed with a mean demand of 100; the coefficient of variation of demand is set to 0.1, 0.15, 0.20, 0.25, or 0.30; the production cost per unit, c, is normalized to US\$1.00; the wholesale price per unit is defined as a function of the production cost, i.e., $w=\$(1+\delta)c$ with $\delta=0.10$, 0.25, 0.40, 0.6, or 0.75; the retail price per unit is $p=\$(1+\gamma)w$ with $\gamma=0.20$, 0.40, 0.75, 0.80, or 1.25; the customer shortage penalty per unit is $\pi_C=\$\kappa p$ with $\kappa=0.20$, 0.50, 0.75, 1.00, or 1.50; the fractional purchase requirement α is 0.5, 0.75, 0.85, 0.9, or 0.95; finally, the commitment penalty per unit π_1 is US\$0.40, US\$0.50, US\$0.75, US\$0.85, or US\$0.90. The delivery penalty per unit incurred by the supplier, π_2, is calculated based on the relation $\pi_2=p-w+\pi_C$. We calculated equilibrium outcomes under the contract for all 15,625 combinations of the parameters listed above. We note that some combinations of the above parameter values may not be reasonable to use in practice, but our intent is to find the two parties' solutions for a wide variety of parameter settings so that we can better assess how the parameters affect the qualitative and quantitative features of the outcomes.

7.1. Performance of the FC Contract

In this section, we discuss how the FC contract performs in comparison with the no-contract situation, and how strategic customer behavior affects the performance of the FC contract (relative to the honest-customer case). This is followed by a comparison of system-wide profits under the FC contract with that of the coordinated (centrally planned) solution, for both the honest-customer and strategic-customer settings.

In the discussion to follow, we use shorthand characterizations of the parameter values: π_1 is high if $\pi_1 \ge \frac{\pi_2 c}{\pi_2+w}$ and low otherwise. (The condition for π_1 to be high (low) is equivalent to $q_B^* \le (>) q_C^*$.) We call α high for Orderings 1 and 2 if $\alpha \ge \frac{q_B^*}{q_C^*}$, and low otherwise. Similarly, α is high for Orderings 3 and 4 if $\alpha \ge \frac{q_C^*}{q_B^*}$. A mapping of these conditions to the different orderings of critical values appears in Table 5.

Table 6 displays how the two parties individually and the supply chain as a whole fare under the two FC contract settings vis-à-vis the no-contract setting (in columns 2, 4, and 6). It also shows the incremental impact on profits/losses of strategic customer behavior in comparison with the honest-customer setting (in columns 3, 5, and 7). Finally, it shows the performance gap (percentage loss of profit) of the chain under the FC contract (honest- and strategic-customer versions, respectively) relative to that of the centralized system (in columns 8 and 9). Under the headings "Customer," "Supplier," and "Chain," the subheading "Hon. vs. NC" refers to a comparison of profits under the FC contract vs. those in the no-contract setting when the customer is honest; the column labeled "Strat. vs. Hon." shows the added gains/losses of the parties under the contract assuming strategic customer behavior vs. the parties' profits when the customer is honest. For example, the columns labeled "Strat. vs. Hon." displays $\frac{\phi^{\text{Strat.}}-\phi^{\text{Hon.}}}{\phi^{\text{Hon.}}}\times 100\%$ for the relevant parties in the supply chain. The values in columns labeled "Hon. vs. NC" are similarly computed.

Below we provide a more detailed discussion of these results.

Table 5 Mapping of Parameter Magnitudes to Orderings of Critical Values

	π_1 high	π_1 low
α high	Ordering #1	Ordering #4
α low	Ordering #2	Ordering #3

Table 6 Percentage Change in Profits: Forecast-Commitment (FC) Contract with Honest Customer vs. No Contract; FC Contract with Honest Customer vs. FC Contract with Strategic Customer; Performance Gap between FC Contract with Honest and Strategic-Customer Relative to Centrally Planned System

	Percentage profit gains or losses							
	Customer		Supplier		Chain		Gap vs. centralized	
Ordering	Hon. vs. NC (%)	Strat. vs. Hon. (%)	Hon. vs. NC (%)	Strat. vs. Hon. (%)	Hon. vs. NC (%)	Strat. vs. Hon. (%)	Hon. (%)	Strat. (%)
#1	184	0.30	12	− 11.75	95	− 0.32	− 1.93	− 2.24
#2	227	2.40	− 14	− 145	75	− 6.36	− 0.21	− 8.21
#3	237	0.74	− 9	− 80.81	86	− 1.42	− 0.85	− 2.41
#4	176	0.0001	33	− 0.0154	157	− 0.0008	− 2.27	− 2.28

Hon., FC honest setting; NC, no-contract setting; Strat., FC strategic setting.

7.1.1. FC Contract (Honest and Strategic) vs. No-Contract and Centralized Settings. Here, we discuss the results in columns 2 through 7 of Table 6. We know that *the honest customer always fares at least as well under the FC contract as under the no-contract scenario* because, if he does not like the terms of the contract, he can simply submit $\bar{f} = 0$ and the supplier will behave the same as in the no-contract case. The detailed numerical results are consistent with this; averages are shown in the second column. Notably, the average increase in the honest customer's profit under the FC contract is approximately 200%. Detailed results (not reported here) also show that the FC contract improves supply chain profits consistently, often providing very substantial improvements over the no-contract setting (averages are shown in the sixth column). The key question, then, is how the FC contract affects the supplier's profit. The results (fourth column) suggest that this is sensitive to the contract parameters.

For Ordering #1, the supplier's profit increases an average of 12%. For Ordering #4, the supplier sees an average profit improvement of 33%. Interestingly, under Ordering #4, the supplier gains substantially although he rarely produces the coordinating quantity, as we discuss in more detail later in this section. Both Orderings #1 and #4 have high values of α, so the customer must purchase a significant fraction of his forecast. This suggests that *the supplier benefits from a high α even if π_1 is also high*. The story is quite different for Orderings #2 and #3, where profits are, respectively, 14% and 9% lower, on average. The main reason is that these orderings correspond to low α values, so the customer has a limited obligation to purchase and the supplier, nevertheless, must pay commitment and/or delivery penalties.

The highest customer gains from strategic behavior occur under Ordering #3 (on average, 2.4%), with smaller gains occurring under the other orderings. On the whole, we see that *the customer gains little by lying, while the supplier and the supply chain, to a lesser extent, may lose appreciably*. Although some of the supplier profit reductions due to strategic customer behavior are quite large (cf. fifth column), the largest deviations for individual problem instances uniformly apply to cases where the original profits were quite small (e.g., less than a dollar). Thus, in a practical sense, the supplier's profit reductions are not as significant as what the percentages convey. It is evident that the customer's gains (supplier's losses) are more pronounced when the high commitment penalty values (of Ordering #2) induce the supplier to commit to the forecast, which then allows the customer to take advantage of the supplier. Low α values compound the problem for the supplier (in Ordering #2) and also affect the profits significantly under Ordering #3. And, although the customer is generally better off, it is unclear that there exists a significant impetus for him to behave strategically. In addition, strategic customer behavior leads to a diminution of the chain's profits relative to the honest-customer setting. This is due to the reductions in the supplier's profits and small gains in the customer's profits.

7.1.2. Comparison of FC Contract to the Centrally Planned System. Here, we discuss columns 8 and 9 of Table 6. The decentralized solution under the honest-customer FC contract produces the same system-wide expected profits as in the centralized (*coordinated*) case if the supplier chooses $q^* = q_B^*$. Because the supplier may also produce q_A^*, q_C^*, $\bar{f}$, or $\alpha\bar{f}$, depending on the customer forecast, coordination cannot be guaranteed. We therefore seek to answer two key questions: (i) How often does the contract coordinate the system? and (ii) How far from optimal are the profits when the supplier produces other than the system-optimal quantity? *For the problem instances in our numerical study, supply chain coordination is achieved in 71% of the cases in the honest-customer setting, and, even when coordination is not achieved, the performance loss is quite small*, as shown in the next-to-last column of Table 6. The gap is only 0.88% across all instances, on average, and exceeds 10% in only about 2% of the instances. Nearly all of the latter instances had very low supply chain profit—e.g., a few dollars—so small deviations from the optimum can lead to a large percentage change.

The overall performance of the chain suffers across all orderings in the strategic-customer setting. The main reason for this is that *all of the coordinated equilibria in the honest-customer setting shift to non-coordinated equilibria in the strategic-customer setting*. (We discuss changes in the observed equilibria in the next section.) On average, the performance gap between the FC contract with a strategic customer and the centrally planned system increases to 4.81% (details not shown), and the performance gap relative to the centralized system for each of the orderings also increases, particularly for Orderings #2 and #3 because these orderings have a larger proportion of coordinating equilibria in the honest-customer setting. Overall, the supply chain is better off with strategic customer behavior as compared with honest customer behavior in only 18 (0.115%) of the 15,625 instances considered. The supply chain performance remains unchanged in 5796 instances (37.09%). In the remaining 9811 (62.79%) instances the chain's performance is worse.

7.2. Typical Outcomes and Changes in the Equilibria

The distribution of the types of supplier solutions induced by the customer's optimal forecast for both the honest-customer and strategic-customer cases is presented in Table 7 for each of the orderings. Recall that if the supplier commits to his production quantity, the customer cannot benefit from strategic behavior. Thus, certain cells in the table have no associated problem instances. The breakdown is tabulated for each ordering, so the column totals sum to 100% (subject to rounding). (The selected parameter combinations define the number of instances for each ordering of critical values and affect the distribution of solutions for each ordering of critical values, so one should not attach much importance to the exact percentages. What is of significance is that some types of outcomes occur with some regularity but others rarely or never appear.) For a given set of problem parameters, the customer's optimal forecast sometimes differs depending on whether he is honest or strategic; further details can be found in supporting information Appendix S7.

Table 7 also shows that *in all instances in which the supplier's optimal policy in the honest-customer case was to commit to the production quantity (i.e., $C^* = q$), the solutions remained (qualitatively) unchanged in the strategic-customer setting* (first, second, third, and last rows (5a and 5b) of the table). These results represent a situation where the supplier prefers $C^* = q$ even when the customer is guaranteed to be honest. For these cases, it is not surprising that customer strategic behavior causes the supplier to continue to take a defensive posture against paying delivery penalties, so he only commits to q. Similarly, when the supplier's optimal policy in the honest-customer setting is $q^* = C^* = q_A^*$ as a response to $\bar{f}^* = 0$, the supplier produces his decentralized newsvendor quantity. Evidently, the opportunity to be strategic does not cause the customer to change his mind because he would have to submit a relatively high forecast (at least q_B^*) to induce the supplier to choose a strategy in which $C^* > q$, which are the only situations in which the customer may benefit from strategic behavior.

The equilibria change in instances where the supplier's optimal policy in the honest-customer setting is ($q^* = q_B^*$,

Table 7 Breakdown of Outcomes in Honest (Hon.)-Customer and Strategic (Strat.)-Customer Models

<table>
<tr><td></td><td colspan="10">Proportion of solutions for each ordering</td></tr>
<tr><td></td><td colspan="2">Optimal supplier solution</td><td colspan="2">#1</td><td colspan="2">#2</td><td colspan="2">#3</td><td colspan="2">#4</td></tr>
<tr><td>Row</td><td>Hon.</td><td>Strat.</td><td>Hon.</td><td>Strat.</td><td>Hon.</td><td>Strat.</td><td>Hon.</td><td>Strat.</td><td>Hon.</td><td>Strat.</td></tr>
<tr><td>1.</td><td colspan="2">$C^* = q^* = q_A^*$</td><td colspan="2">18%</td><td colspan="2">3%</td><td colspan="2">4%</td><td colspan="2">3%</td></tr>
<tr><td>2.</td><td colspan="2">$C^* = q^* = q_C^*$</td><td colspan="2">NA</td><td colspan="2">NA</td><td colspan="2">10%</td><td colspan="2">5%</td></tr>
<tr><td>3.</td><td colspan="2">$C^* = q^* = \bar{f}$</td><td colspan="2">44%</td><td colspan="2">15%</td><td colspan="2">16%</td><td colspan="2">30%</td></tr>
<tr><td rowspan="4">4.</td><td rowspan="3">$q^* = q_B^*$</td><td>$C^* = q^* = \bar{f}$</td><td rowspan="4">38%</td><td>–</td><td rowspan="4">82%</td><td>–</td><td rowspan="4">66%</td><td>–</td><td rowspan="4">1%</td><td></td></tr>
<tr><td>$C^* = q^* = q_C^*$</td><td>–</td><td>–</td><td>47%</td><td>–</td></tr>
<tr><td>$C^* = q^* = \alpha\bar{f}$</td><td>–</td><td>0.22%</td><td>19%</td><td>1%</td></tr>
<tr><td>$C^* = \bar{f}$</td><td>$C^* = \bar{f}, q^* = q_G^*$</td><td>38%</td><td>81.88%</td><td>–</td><td>–</td></tr>
<tr><td>5a.</td><td>$q^* = \alpha\bar{f}$</td><td>Unchanged</td><td colspan="2">NA</td><td colspan="2">NA</td><td rowspan="2">4%</td><td>3.75%</td><td rowspan="2">60%</td><td>59.5%</td></tr>
<tr><td>5b.</td><td>$C^* = \alpha\bar{f}$</td><td>Unchanged*</td><td colspan="2">NA</td><td colspan="2">NA</td><td><0.25%</td><td><0.5%</td></tr>
</table>

*The outcome type is the same, i.e., $q^* = \alpha\bar{f}, C^* = \alpha\bar{f}$, but the resulting forecast differs, so the production and commitment quantities as well as the profits differ.

$C^* = \bar{f}$) *(fourth row). When the customer is strategic, the supplier either commits to* $\bar{f}$ *and hedges by producing a larger quantity, or switches strategies entirely (to* $C = q^*$*) and pays the commitment penalty instead*. We elaborate on this point for the various orderings of critical values.

- Under Ordering #1, $q_C^* \geq q_B^*/\alpha$, which roughly implies that $\pi_1 \gg \pi_2 c/(w + \pi_2)$. Hence, the supplier commits to $\bar{f}$ because the commitment penalty is very high, but he hedges by producing a larger quantity, $q_G^* = \frac{wq_B^* + \pi_2 \bar{f}}{w + \pi_2}$. This quantity optimizes the tradeoff between additional production costs and the reduction in delivery penalties incurred when the customer orders $C = \bar{f}$ (exceeding his actual demand). A similar rationale holds for Ordering #2. Furthermore, under Orderings #1 and #2, the customer is never worse off, while the supplier is never better off.
- Under Ordering #3, in the honest-customer case, the supplier commits to $\bar{f}$, but with the introduction of a strategic customer, the delivery penalty is high enough (i.e., $\frac{\pi_2 c}{w + \pi_2} \geq \pi_1$) that the supplier prefers to pay the commitment penalty rather than to pay delivery penalties. Therefore, the supplier switches strategies entirely (to $C^* = q$) and his production quantity depends on the customer's forecast. Under Ordering #3, both the customer and the supplier's performance are the same in the honest- and strategic-customer cases 33.81% of the time. The customer's profits improved in 36.97% of instances, and declined in 29.16% of the instances. The supplier's profits, on the other hand, improved in 10.14% of instances under Ordering #3, but declined in 56.05% of instances.
- Under Ordering #4, the equilibria are the same under the honest- and strategic-customer settings in virtually all instances. Notice that under Ordering #4 the supplier rarely produces the coordinating quantity q_B^*. The $C = \bar{f}$, $q^* = q_B^*$ solution occurs in only 1% of the instances and $q^* = q_B^*$ can arise elsewhere only if the customer chooses $\bar{f} = q_B^*/\alpha$ (which is an extremely high forecast because it would require the customer to purchase the system-coordinating quantity q_B^*) and the supplier chooses $q^* = \alpha\bar{f}$. The main reasons why equilibria with $q^* = q_B^*$ are rare under Ordering #4 are as follows. The value of α is high, so if the supplier commits to the forecast, the customer is obligated to purchase a substantial portion of it. This, and the fact that he cannot collect much in the way of commitment penalty payments (if the supplier commits to the forecast) because π_1 is low, deters the customer from submitting a high enough forecast to induce the supplier to produce q_B^*.

Also noteworthy is the absence of equilibria involving FOC (11). Although these solutions cannot be ruled out as candidates for supplier's solutions, it appears that the customer does not choose forecasts that induce the supplier to choose these solutions. One important reason is that the associated forecasts need to be extremely high, forcing the customer to make a large minimum purchase with small expected penalty payment gains. Likewise, we did not observe any equilibria in which the supplier chooses $(C^* = \bar{f}, q^* = \alpha\bar{f})$. As noted in section 2, this supplier policy leads to thresholds of customer demand (above which he orders C) that are less than $\alpha\bar{f}$, so the customer would always extract delivery penalties. Such a policy is undesirable to the supplier, so instead he commits to q.

A key insight from this section is that *although the customer's objective function may be multi-modal, in nearly all instances, the structure of the optimal forecast and the supplier's response are well characterized and fairly intuitive*. Our analysis played an essential role in defining parameter thresholds (in terms of other parameters) that separate qualitatively distinct policy regions. Because these parameters are interlinked (e.g., threshold on π_1 depends on π_2, threshold on α depends on both π_1 and π_2, and the customer's choice of $\bar{f}$ depends on both α and π_2), the structure of the policy regions would have been difficult to conjecture in advance.

7.3. Structuring the Contract

It is clear from the discussion in section 7.1 that certain combinations of contract parameters are less advantageous than others. In particular, if π_1, π_2, and α are all low, then the contract requires minimal commitment from either party and does not serve its intended purpose.

The commitment penalty, π_1, serves two purposes. First, if the commitment penalty is not high and the supplier elects not to commit to the forecast, it provides the customer a subsidy that then permits the customer to forecast more aggressively, which induces the supplier to produce more. Second, if it is high, it tends to induce the supplier to commit to the forecast, and because there are also delivery penalties, it indirectly increases the supplier's production quantity, as well. Our numerical results suggest that coordination is achieved more often when π_1 is high; if it is too small, its role as a subsidy is diminished.

From the (equivalent) first-order conditions in Table 2, recalling that $p - \pi_C - w = \pi_2$, we can also infer that the *ratio* of π_2 to αw is important, as it defines the customer's stationary point. For fixed w, pairing higher (lower) values of α with higher (lower) value of π_2 discourages the customer from attempting to game the system by over-forecasting, which tends to be beneficial for the entire system.

With these insights and a few simple calculations using the customer's first-order condition, the supplier can fairly easily determine contract parameters that prevent him from sacrificing more profits than he is willing (or to strictly improve his expected profit, if he insists) while simultaneously considering the customer's risk preferences with respect to α.

One might also ask whether similar benefits can be achieved using a simpler contract form. In the next subsection we compare an FC contract with a QF contract.

7.4. Comparison with the QF Contract

The FC contract can be viewed as a more flexible form of a QF contract. Under a typical QF contract, the customer submits a forecast, say ψ and is required to purchase a fraction (say β) of it, and the supplier guarantees availability of a fraction (say γ) greater than ψ. This corresponds to an FC contract with $\bar{f} = (1+\gamma)\psi/\beta$, $\alpha = \beta/(1+\gamma)$ and $\pi_2 = \infty$.

We now examine whether this combination of parameters is likely to achieve supply chain coordination based on the patterns observed in our numerical study. First, we observed that "high" values of π_1, i.e., $\pi_1 \geq \pi_2 c/(\pi_2+w)$, contribute to achieving coordination. Now, because $\pi_2 = \infty$, the condition on π_1 is essentially equivalent to $\pi_1 \geq c$ (commitment penalty exceeds the unit production cost). The supplier would be disinclined to agree to so large a commitment penalty, but if he does agree, the combination of $\pi_2 = \infty$ and $\pi_1 \geq c$ leads to q_C^* being at the upper limit of the support of the demand distribution and q_B^* being essentially at the upper limit, as well. Then, we would need to have $\alpha = 1$ to satisfy the condition $\alpha \geq q_B^*/q_C^*$. Thus, from the patterns observed in our numerical study, we would predict that supply chain coordination would be difficult to achieve with a QF contract for any values of β and γ when the wholesale price is fixed. Tsay (1999) showed that supply chain coordination can be achieved if the wholesale price can be adjusted, and our results provide further support for the fact that more degrees of freedom are needed to achieve coordination.

Whether the supplier prefers the FC contract to a QF contract depends on the contract parameters. However, if we take the parameters of a QF contract and set the parameters of a FC contract as described at the beginning of this subsection, there are reasons that a supplier will prefer the FC contract. Unlike the QF contract, under the FC contract, coordination can often be achieved without the need to adjust the wholesale price, providing the supplier the possibility of extracting additional profits. Further, viewing this issue more broadly, the FC contract has more parameters than the QF contract, and additional parameters almost always give the party who offers the contract more control over his share of the profits.

The FC contract can be viewed as a generalization of a QF contract in which the parties make decisions based on economic tradeoffs rather than imposing many hard constraints. The customer may choose a forecast that communicates more than the minimum or maximum purchase quantity while considering his economic tradeoffs and maintaining the same flexibility to purchase less than the upper bound on supply. The use of penalties in place of hard constraints allows the supplier to choose a production quantity that considers economic tradeoffs and is more likely to coordinate the supply chain. Although it is more difficult for the customer to choose his forecast under the FC contract, he can utilize the distinctive patterns in our numerical study to narrow down his choices to a few good alternatives to be evaluated. The FC contract also allows the supplier to maintain the current wholesale price and add contract parameters (α for the customer and π_1 and π_2 for the supplier) that increase the required "commitment" on the part of both parties while achieving (near-) coordination in the vast majority of situations.

8. Summary and Conclusions

We have analyzed the customer's problem of choosing a forecast to submit in the context of an FC contract for a capacity-unconstrained setting under symmetric information. We modeled the problem as a Stackelberg game, with the customer submitting a forecast and the supplier subsequently deciding commitment and production quantities. We have analyzed both the case in which the customer honestly reports his demand and the case in which he strategically orders more than his demand when it is advantageous to do so.

To characterize the customer's optimal policy, we first derived stronger characterizations of the *supplier's* dominant policies for the honest-customer case (supplementing those in Durango-Cohen and Yano 2006) and fully characterized the supplier's dominant policies for the strategic-customer case. In particular, we show that, in addition to the two dominant commitment policies found in the honest-customer case (i.e., commit to the production quantity or to the forecast), two hedging strategies arise: (i) the supplier produces $\alpha\bar{f}$ and commits to an intermediate amount that trades off the commitment and delivery penalties; and (ii) the supplier commits to the forecast and increases the production quantity by a fraction $\pi_2/(\pi_2+w)$ for each unit by which the forecast exceeds q_B^*. In addition, when the delivery penalty is high, the supplier commits to the production quantity so the customer cannot gain anything from lying.

We have provided a full characterization of the honest customer's optimal policy. For any ordering of critical values determined from newsvendor-type tradeoffs, the customer's objective function is piecewise concave. The number of potentially optimal solutions is small, and, although some of them cannot be expressed in closed-form, their values are straightforward to compute. Although our analysis suggests that the customer's problem is not simple, our numerical study indicates that both parties' solutions are well structured and the policy regions can be characterized by thresholds that are functions of key contract parameters.

We have also shown that in the honest-customer setting, the FC contract may devolve to a wholesale price contract for some contract parameter combinations, but in a large majority of cases, achieves (near-) coordination of the supply chain. These performance gains arise because penalties paid by the supplier—primarily commitment penalties—indirectly provide an economic incentive for the customer to submit a higher forecast than he would otherwise, which then induces the supplier to produce more than he would otherwise, quite often equal or similar to the system-coordinating quantity. Recall that the FC contract was not designed with coordination as the key objective, but the structure provides incentives for greater truth-telling and greater purchase and delivery commitments on the parts of both parties, so it might be expected to improve the degree of coordination.

Because the supplier's dominant strategies in the strategic-customer case cannot always be expressed in closed-form, solving the customer's problem is more difficult, but we have partially characterized the customer's optimal policy, which eases the solution process.

The results of an extensive numerical study indicate that the customer gains very little by lying but the cost to the supply chain may be significant because his dishonesty usually causes the supply chain to shift from a coordinated solution when he is honest to a non-coordinated one when he is not. This sometimes arises because the supplier chooses to commit to the production quantity so as to thwart the customer's dishonest behavior altogether.

Our results also provide clear guidelines for choosing contract parameters that are more likely to achieve Pareto improvement in the honest-customer case. In particular, the value of α should be relatively high if one wishes to ensure that the supplier is better off. Also, pairing higher values of α with higher values of π_2 (rather than, e.g., a high π_2 with a low α) leads the customer to choose reasonable forecasts (i.e., those that do not excessively overstate the amount the customer would like to have available for purchase), which tends to benefit the supply chain.

The FC contract has other attractive features. For the ASIC manufacturer that motivated this work, as discussed in the Introduction, having a common framework for providing differential service to its customers was very important, and few supply contracts in the literature have enough parameters to provide this flexibility. Another attractive feature is voluntary supplier compliance (but he is penalized for not complying). Moreover, no monitoring is required: the supplier either commits to the forecast or he does not, and he either delivers according to his commitment or the shortfall is evident. The customer may choose not to participate by submitting a forecast of zero, but, if he does participate, a little analysis will show that the supplier will produce more than in the absence of a contract and quite often the system-coordinating quantity. Our numerical study indicates that the contract also achieves (near-)coordination without the need to change the wholesale price.

Variants of our problem with information asymmetry are quite difficult because small changes in the forecast can lead to shifts in the supplier's optimal strategy. As mentioned in the literature review, Gan et al. (2009) treat a restricted case in which $\alpha = 1$ and $\pi_1 = \pi_2 = \infty$ and information asymmetry pertains to whether the mean demand is either low or high. Although costs and prices may be common knowledge at different divisions within a firm, this will not be true for parties in arms-length transactions. Relaxing the assumption of a common demand distribution will be especially challenging. In particular, it is not well understood how customer forecasts influence a supplier's beliefs about future demand, and one would expect the introduction of an FC contract to change how the customer chooses forecasts. These issues go beyond the usual effects of information asymmetry and extend to consideration of what each party believes about the other party's beliefs.

More broadly, the FC contract works best in a supplier–buyer relationship where the customer is not intentionally deceitful. Yet, with a sensible choice of contract parameters by the supplier, the two parties can initially act in their own self-interest and the customer can be dishonest about his demand, but, because the structure of the contract requires mutual commitment and the customer does not benefit much from being dishonest, it can serve as a vehicle for engendering mutual trust and honesty as the supply relationship matures.

Acknowledgments

The authors thank the management and staff of an unnamed ASIC manufacturing firm for introducing us to the issues that motivated this research. We also thank the Senior Editor and referees for constructive comments on earlier versions of this paper.

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Supporting Information

Additional supporting information may be found in the online version of this article:

Appendix S1. Proof of Dominance for Cases Labeled A, B and C in Table 1.
Appendix S2. Proof of Theorem 1.
Appendix S3. Proof of Theorem 2 and Results in Table 2.
Appendix S4. Proof of Results in Table 4.
Appendix S5. Finding Candidate Forecasts.
Appendix S6. Derivation of Results for the Supplier's Problem in the Strategic-Customer Case.
Appendix S7. Changes in the Equilibria.