Dynamic Sports Business Decision Model (WNBA): Leverage, Roster Acquisition, Pricing, Expansion, and Injury Response

Example Team: **Indiana Fever** (Gainbridge Fieldhouse, Indianapolis)

Team ________ *(COMAP/ICM Submission Draft)*

Summary Sheet (One Page)

Problem. Professional sports teams must maximize profit and long-term franchise value while maintaining competitive performance. Owners must decide how aggressively to finance operations (leverage), how to acquire players through league mechanisms (draft, free agency, trades, waivers), how to set ticket prices, and how to adapt under league expansion and player injuries.

Our Approach. We build a dynamic *closed-loop decision system*. At each period t, management observes the state

$$X_t = (Q_t, B_t, C_t, D_t, M_t, E_t),$$

where Q_t is team quality, B_t is brand/demand, C_t is cash buffer, D_t is debt, $M_t \in \{0,1\}$ indicates macro stress, and $E_t \in \{0,1\}$ indicates a league-expansion environment. Management then applies decision rules

$$u_t = (\Delta D_t,\ m_t,\ p_t,\ a_t) = \pi(X_t),$$

where ΔD_t is borrowing/paydown, m_t marketing/media investment, p_t game-level ticket pricing, and a_t roster actions (draft/FA/trade/waiver).

Verified WNBA constraints and team constants (Indiana Fever).

- Indiana Fever play at Gainbridge Fieldhouse (Indianapolis), basketball capacity Cap = 17,274. [1]
- WNBA roster size requirement: 11 to 12 players. [2]
- 2024 WNBA salary cap reported as \$1,463,200. [3]

[1] Gainbridge Fieldhouse capacity and tenants: https://en.wikipedia.org/wiki/Gainbridge_Fieldhouse. A venue-industry page also lists total capacity 17,274: https://populous.com/showcases/gainbridge-fieldhouse.

[2] Roster size is widely reported; see salary-cap summary and discussion: https://herhoopstats.com/salary-cap-sheet/wnba/summary/2024/. Also ESPN notes teams can roster 11 or 12 under a hard cap: https://www.espn.com/wnba/story/_/id/45060883/wnba-2025-collective-bargaining-agreement-cba-negotiations-salaries-prioritization-work-stoppage.

[3] Salary cap summary: https://herhoopstats.com/salary-cap-sheet/wnba/summary/2024/. Additional discussion citing the same figure: https://business-law-review.law.miami.edu/basketballs-bargaining-battle-wnba-players-fight-for-new-collective-bargaining-agreement/.

- Expansion example: Toronto awarded a WNBA expansion team to begin play in 2026. [4]

Key Equations (system).

- Profit and state updates:

$$\Pi_t = R_t(Q_t, B_t, p_t) - W_t(a_t) - O_t - m_t - r_t D_t, \quad C_{t+1} = C_t + \Pi_t + \Delta D_t, \quad D_{t+1} = D_t + \Delta D_t.$$

- Demand stock (performance + popularity + marketing):

$$B_{t+1} = (1-\rho)B_t + \alpha\,\mathrm{Wins}_t + \beta \sum_{i \in S_t} s_i(1-\pi_i) + \gamma m_t.$$

- Attendance and revenue:

$$\mathrm{Att}_g = \min(\mathrm{Cap}, A_0 e^{\xi B_t + \zeta Q_t} p_g^{-\epsilon}), \quad R_t = \sum_{g \in t} (p_g + a)\mathrm{Att}_g + \mu B_t.$$

Key Decision Rules (what makes the model dynamic).

- Leverage rule: choose $\Delta D_t = \pi_D(X_t)$ to finance growth when outlook is good and protect the franchise when cash/macro risk is poor.
- Pricing rule: for each game choose $p_g = \pi_p(X_t)$ to maximize per-game revenue given current (Q_t, B_t).
- Roster rule: each offseason (and after major injury shocks) choose $a_t = \pi_a(X_t)$ by solving a constrained acquisition/retention optimization consistent with draft/FA/trade/waiver rules.
- Expansion rule: when $E_t = 1$, modify salaries and demand substitution; then re-optimize π_D, π_p, π_a under the new environment.

Data/Simulation (planned). Player inputs (b_i, s_i, π_i, c_i) come from public performance stats, popularity proxies, injury history, and salary data. Unknown team-level financials (debt, cash, sponsor/media revenue) are treated as parameters to be estimated or scenario-tested. *(Implementation details left for Data/CS teammates.)*

Key Words: dynamic decision-making; leverage policy; roster acquisition; ticket pricing; expansion; injury response.

[4] WNBA official release: https://www.wnba.com/news/expansion-toronto-2026-announcement.

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1 Introduction and Problem Restatement

Sports teams are entertainment enterprises. Owners seek to maximize profit and franchise value, while team operations seek to maintain on-court performance. The prompt asks for a dynamic model that (i) adjusts leverage to changing performance and economic conditions, (ii) develops a player acquisition strategy via draft/free agency/trade/waiver mechanisms, (iii) accounts for league rules and expansion, (iv) optimizes an additional business decision (ticket pricing), and (v) adapts when a key player is injured. Our model is explicitly *decision-based*: it defines policies mapping state $\to$ action and updates state over time.

2 Modeling Assumptions (Minimal, Defensible)

2.1 Business and Finance

- (A1) The owner chooses leverage adjustments ΔD_t each period and sets marketing spend m_t subject to feasibility.
- (A2) Financing cost increases during macro stress and when leverage rises.
- (A3) The owner maintains a minimum cash buffer $C_{\min}$ to avoid distress (forced asset sale or emergency dilution).

2.2 Sport Performance, Popularity, and Demand

- (A4) Team quality depends on expected available player performance and a roster balance penalty.
- (A5) Demand depends on brand stock B_t, performance Q_t, and price via a standard elasticity form.
- (A6) Popularity/personality/clutch are represented by a single proxy s_i (to be operationalized with measurable indicators).

2.3 League Mechanisms and Expansion

- (A7) Roster changes follow standard league mechanisms: draft, free agency, trades, waivers, all enforced via constraints.
- (A8) Expansion changes the environment through salary inflation (more teams competing for talent) and demand substitution (market overlap).

3 Variables and Notation

3.1 Indices and Sets

- $t = 1, \ldots, T$: decision periods (e.g., weeks)

- $g \in \mathcal{G}(t)$: games in period t
- S_t: active roster set in period t

3.2 State Variables

$$X_t = (Q_t, B_t, C_t, D_t, M_t, E_t)$$

3.3 Decision Variables (Policies)

$$u_t = (\Delta D_t, m_t, p_t, a_t) = \pi(X_t)$$

3.4 Player Inputs

For each player i:

$$(b_i, s_i, \pi_i, c_i)$$

4 Decision Loop: What Makes the Model Dynamic

At each period t:

1. Observe state X_t (performance, demand, cash, debt, macro/expansion environment).
2. Apply policies (decisions) $u_t = \pi(X_t)$.
3. Realize outcomes (wins, attendance, profit) and update state X_{t+1}.
4. If shocks occur (key injury, macro stress, expansion), re-optimize policies under the new state.

This directly matches the prompt: decisions made over time, dependent on current state, with re-optimization after shocks.

5 Dynamic Model: Performance → Demand → Profit → Leverage

5.1 Team Quality from Roster (Performance + Injury + Fit)

$$Q_t = \sum_{i \in S_t} b_i(1 - \pi_i) - \eta \, \text{Imbalance}(S_t). \tag{1}$$

What the variables mean. b_i is on-court contribution, $(1-\pi_i)$ is availability, and $\text{Imbalance}(S_t)$ penalizes missing roles or redundancy.

Why this formula works (assumptions).

- Expected contribution is additive across players at the season-planning level (A4).
- Injury risk reduces expected availability.
- Team fit matters, but we keep it implementable using a simple penalty rather than an unobservable chemistry model.

Decision link. Roster decision a_t determines S_t; injury shocks change π_i; both immediately change Q_t.

5.2 Wins (Optional Simulation Layer)

$$\Pr(\mathrm{Win}_g = 1) = \sigma\big(\lambda(Q_t - Q_g^{opp}) + h\big), \quad \sigma(z) = \frac{1}{1+e^{-z}}. \tag{2}$$

Why this formula works. A logistic form is a standard way to map a performance advantage into win probability while keeping probabilities in $[0, 1]$ and allowing diminishing returns at extremes.

5.3 Brand/Demand Stock (Winning + Popularity + Marketing)

$$B_{t+1} = (1-\rho)B_t + \alpha\,\mathrm{Wins}_t + \beta \sum_{i \in S_t} s_i(1-\pi_i) + \gamma m_t. \tag{3}$$

Why this formula works.

- Brand behaves like a *stock*: it persists but decays without continued success/attention.
- Winning increases attention; star popularity increases attention even without winning (prompt requirement).
- Marketing is a controllable business lever that increases attention.

5.4 Attendance and Ticket Pricing (Business Decision)

For each home game g:

$$\mathrm{Att}_g = \min\Big(\mathrm{Cap}, A_0 e^{\xi B_t + \zeta Q_t} p_g^{-\epsilon}\Big). \tag{4}$$

Why this formula works.

- $p^{-\epsilon}$ is a standard constant-elasticity demand model (interpretable and calibratable).
- $e^{\xi B + \zeta Q}$ scales demand smoothly with brand and performance.
- $\min(\mathrm{Cap}, \cdot)$ enforces arena capacity (real constraint).

Indiana Fever capacity (verified). For our chosen team, Cap = 17,274 seats at Gainbridge Fieldhouse.

Revenue:

$$R_t = \sum_{g \in t} (p_g + a)\mathrm{Att}_g + \mu B_t. \tag{5}$$

Pricing decision rule (explicit).

$$p_g^\star(X_t) \in \arg\max_{p_{\min} \le p \le p_{\max}} (p+a) \cdot \mathrm{Att}_g(p; Q_t, B_t). \tag{6}$$

Why this is a decision model. The model chooses $p_g^\star$ *each game* based on current state (Q_t, B_t); if injuries reduce Q_t, optimal price shifts downward automatically.

5.5 Profit, Cash, Debt, and Financing Cost

$$\Pi_t = R_t - W_t - O_t - m_t - r_t D_t. \tag{7}$$

$$C_{t+1} = C_t + \Pi_t + \Delta D_t, \qquad D_{t+1} = D_t + \Delta D_t. \tag{8}$$

$$r_t = r_0 + r_M M_t + r_L \frac{D_t}{V_t}. \tag{9}$$

Why this links leverage to profit (your key concern).

- Borrowing increases cash now (C_{t+1}) and can fund roster/marketing to increase future revenue.
- But debt increases financing cost ($r_t D_t$) and raises future expense, especially in macro stress ($M_t = 1$).
- Thus leverage is an explicit trade-off between growth investment and risk/cost.

Feasibility:

$$W_t + m_t + \text{Fees}_t \le C_t + \text{CreditLimit}(D_t). \tag{10}$$

Cash safety:

$$C_t \ge C_{\min}. \tag{11}$$

5.6 Owner Leverage Policy (State → Action)

Policy definition.

$$\Delta D_t(X_t) = \begin{cases} +\Delta^+ & \text{if } C_t \ge C_{\text{target}},\ \widehat{\Pi}_{t:t+H} > 0,\ M_t = 0 \\ 0 & \text{if } C_{\min} \le C_t < C_{\text{target}} \\ -\Delta^- & \text{if } C_t < C_{\min} \text{ or } M_t = 1 \end{cases} \tag{12}$$

Why this policy works. It operationalizes a conservative finance principle:

- Borrow only when liquidity is healthy and the next-H-period profit outlook is positive.
- Stop borrowing when buffers shrink.
- Deleverage when distress risk or macro stress rises (borrowing becomes costly and risky).

This directly answers: *"adjust leverage in response to changing team performance and economic conditions."*

6 Player Acquisition Strategy (Draft / Free Agency / Trades / Waivers)

6.1 Player "Owner Value" Score

$$\text{Score}_i = \frac{\omega_b b_i + \omega_s s_i - \omega_r \pi_i}{c_i}. \tag{13}$$

Why this formula works (prompt alignment).

- $\omega_b b_i$ = value through wins/performance.
- $\omega_s s_i$ = value through popularity-driven revenue.
- $-\omega_r \pi_i$ = expected loss from missed games and reduced demand.
- dividing by c_i yields a *value-per-dollar* measure (fits salary-cap reality).

6.2 Roster Decision Optimization (GM Decision)

Let $y_i \in \{0, 1\}$ indicate whether player i is acquired/kept.

$$y^\star \in \arg\max \sum_i y_i(\omega_b b_i + \omega_s s_i - \omega_r \pi_i) \tag{14}$$

subject to:

$$\sum_i y_i c_i \leq \text{Cap} \quad \text{(salary cap)} \tag{15}$$

$$11 \leq \sum_i y_i \leq 12 \quad \text{(WNBA roster requirement)} \tag{16}$$

$$\sum_{i \in \mathcal{D}} y_i \leq \#\text{DraftPicks} \tag{17}$$

$$\sum_{i \in \mathcal{T}} y_i \cdot \text{AssetCost}_i \leq \text{AssetsAvailable}. \tag{18}$$

Why this answers the acquisition question. This is a concrete decision mechanism that uses standard practices: draft limitation (picks), free agency (cap), trades (assets), waivers (in-season re-run with smaller pool).

What we still need for the chosen team. For Indiana Fever and the chosen season, we still need the *exact* cap year used, draft picks owned, and trade asset budget definition (how you measure assets). These are league/team-specific and will be filled by the Data teammate.

6.3 Policy Summary (Readable Strategy)

- **Draft policy:** choose high Score_i prospects (cheap upside).
- **Free agency policy:** only sign expensive players if leverage policy (12) allows growth financing and feasibility (10) holds.

- **Trade policy:** trade only if objective gain exceeds asset cost and does not violate cash buffer (11).
- **Waiver policy:** after injuries, re-solve (14) on waiver pool to replace missing performance cheaply.

7 League Expansion and Location Effects (with WNBA Example)

7.1 Expansion Environment Variables

Let $E_t \in \{0, 1\}$ indicate the presence of expansion (new franchises entering).

Example expansion event (verified). The WNBA awarded Toronto an expansion team to begin play in 2026. This provides a concrete expansion scenario.

7.2 Expansion Effects (Formulas)

We model three effects:

(1) Salary inflation (competition for talent).

$$c_i \leftarrow c_i(1 + \delta_c E_t). \tag{19}$$

Why: More teams compete for the same talent pool, increasing bargaining power and expected salaries.

(2) League-wide attention boost.

$$B_{t+1} \leftarrow B_{t+1} + \delta_{\text{league}} E_t. \tag{20}$$

Why: Expansion can increase total media coverage and fan interest in the league overall.

(3) Local market substitution (harm depends on location overlap). Define distance d between our team market and the new franchise market, and overlap:

$$\text{Overlap}(d) = e^{-d/\tau}. \tag{21}$$

Then apply substitution:

$$B_{t+1} \leftarrow B_{t+1} - \delta_{\text{sub}} \text{Overlap}(d) E_t. \tag{22}$$

Why: Nearby franchises compete for attention, sponsorship, and occasional travel fans more than distant ones.

7.3 Worked Example: Indiana Fever vs Toronto Expansion

Indiana Fever are located in Indianapolis. Toronto expansion is in Toronto, Ontario (Canada). The distance d is large, so

$$\text{Overlap}(d) \approx e^{-d/\tau} \text{ is small,}$$

meaning substitution harm is limited. Under this scenario, our model predicts:

- The Fever benefit more from the league-wide boost (20) than they lose from substitution (22).
- The main negative pressure is salary inflation (19), which makes value-per-dollar roster strategy more important.

Decision rule under expansion. When $E_t = 1$, tighten financial posture by increasing the buffer target:

$$C_{\text{target}} \leftarrow C_{\text{target}}(1 + \kappa E_t),$$

and re-solve roster optimization (14) using inflated salaries c_i.

8 Key Player Injury Adjustment (Decision Response)

If a key player k is injured, then π_k increases, reducing Q_t in (1). This lowers demand (4) and revenue (5), reducing cash in (8). The model responds through decisions:

- **Roster:** re-run (14) on waiver/free-agent pool to restore Q_t.
- **Pricing:** recompute $p_g^\star$ by (6) under new (Q_t, B_t).
- **Leverage:** policy (12) shifts toward $\Delta D_t \leq 0$ if C_t approaches $C_{\min}$ or macro stress rises.

9 Model Strengths and Weaknesses

9.1 Strengths

- **Truly dynamic:** explicit policies map state $\to$ action, with re-optimization after shocks.
- **Prompt-aligned:** separates (and links) performance value and popularity value through b_i and s_i.
- **Implementable:** requires only 4 player-level inputs plus a few team-level parameters.
- **League-aware:** roster size and salary cap constraints are explicit; draft/trade assets appear as constraints.
- **Transparent:** each formula has an economic interpretation and a managerial meaning.

9.2 Weaknesses / Risks

- **Brand index is latent:** B_t must be proxied (search trends, followers, sales, etc.), creating measurement noise.
- **Simplified chemistry:** Imbalance(S_t) is a crude representation of fit and strategy.
- **Team financials often private:** true C_t, D_t, μ (media/sponsor scaling), and a (ancillary spend) may be hard to observe, requiring estimation or scenario analysis.
- **Expansion mechanisms can be specific:** if the league uses an expansion draft or special rules, those constraints must be added explicitly (Data teammate).

10 Planned Simulation Framework (Placeholder for Data/CS)

10.1 Simulation Goals

Compare strategies by evaluating profit and franchise value under uncertainty: injuries (π_i), opponent variation (Q_g^{opp}), macro regimes (M_t), and expansion scenarios (E_t and distance d).

10.2 Implementation Placeholders

- **Data teammate:** estimate b_i, s_i, π_i, c_i for candidate players and calibrate elasticity ϵ.
- **CS teammate:** implement the loop: observe X_t, compute $p_g^\star$, apply ΔD_t, solve roster decisions when triggered, update state, repeat; run Monte Carlo.

Letter to Owner and General Manager (1–2 Pages)

To: Team Owner and General Manager
From: Modeling Group
Subject: Decision System for Leverage, Roster Acquisition, Pricing, Expansion, and Injury Response
Date: February 2, 2026

Dear Owner and General Manager,

We developed a dynamic decision system that updates weekly. Each period, we observe the team state $(Q_t, B_t, C_t, D_t, M_t, E_t)$ and choose actions $(\Delta D_t, m_t, p_t, a_t)$ using explicit policies. This closed-loop structure links performance and popularity to demand and revenue, which then determine feasible spending and leverage choices.

Leverage. We recommend a state-based leverage rule: borrow only when liquidity is strong, near-term profit outlook is positive, and macro conditions are stable; deleverage when cash approaches a minimum threshold or macro stress rises. This balances growth investments with protection against distress risk.

Roster acquisition. We evaluate players by a value-per-dollar metric combining performance, popularity, and injury risk. Offseason and injury-triggered roster decisions are solved as a constrained optimization consistent with salary cap, roster size, draft pick limits, and trade asset constraints.

Pricing. For each home game, we select a ticket price that maximizes expected per-game revenue given current demand conditions. Prices rise with strong brand/performance and fall after negative shocks (injury, losing streak).

Expansion. Expansion increases league-wide attention but can inflate salaries and create market substitution depending on location overlap. Under a Toronto-style expansion, Indianapolis has limited substitution exposure (large distance), so we expect net demand upside but increased payroll pressure; thus we recommend emphasizing value-per-dollar acquisitions and tightening liquidity buffers during expansion periods.

Injury response. When a key player is injured, we immediately update expected quality, recompute optimal pricing, and (if feasible) re-optimize roster decisions through waivers/low-cost signings. Financially, leverage becomes more conservative until the cash buffer stabilizes.

Sincerely,
Modeling Group

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