# Digital Image & Video Processing

## Lecture 6

Image Pre-processing

(Image Local Pre-processing: Edge Detection)

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# 3. Image Pre-processing

**3.1**. Color Transformations

**3.2**. Geometric Transformations

**3.3**. Local Pre-processing

# 3.3. Local Pre-processing

**3.3.1**. Image Smoothing

**3.3.2**. Edge Detection

# 3.3.2. Edge Detection

**3.3.2.1.** Gradient operator

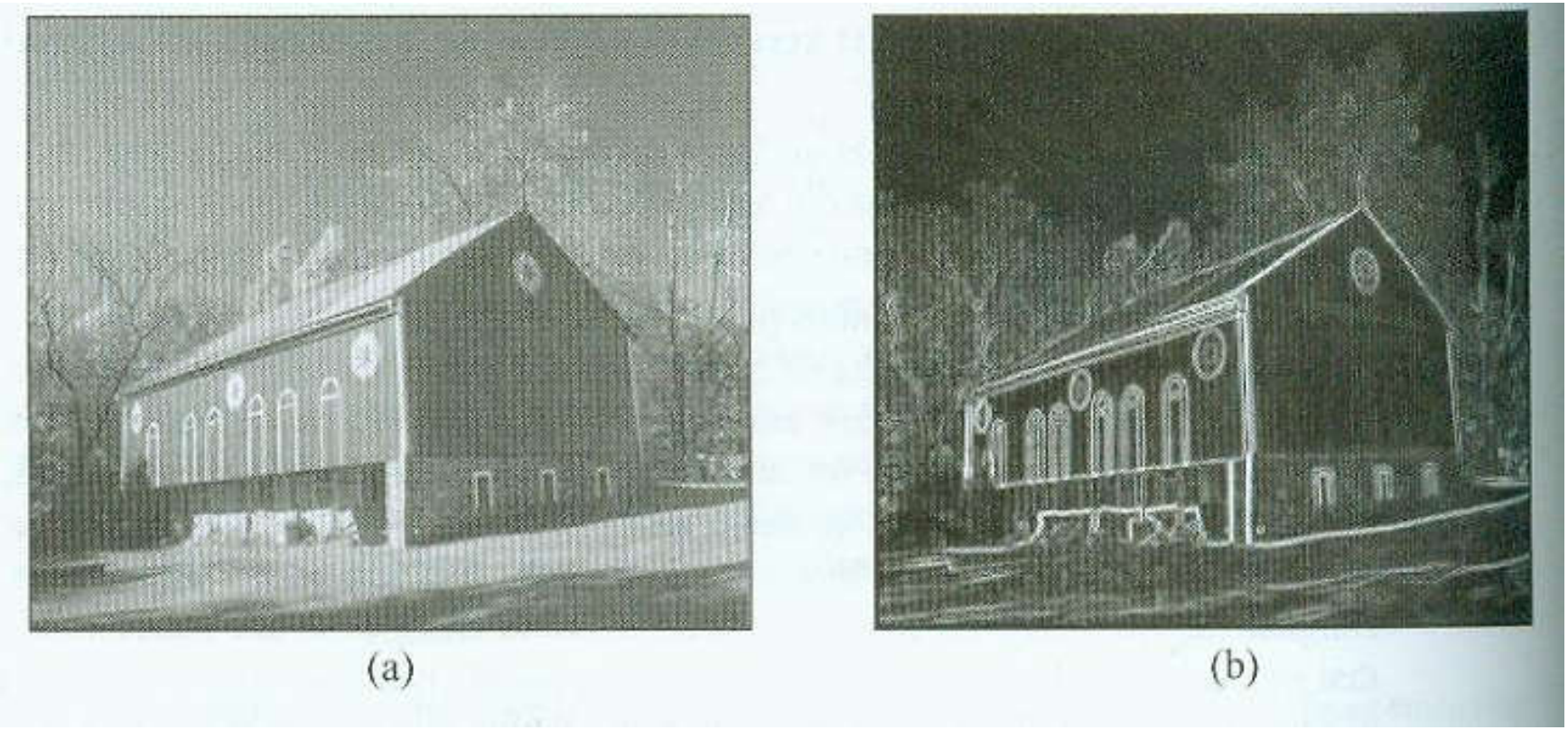
**3.3.1.2.** Laplace operator

**3.3.1.3.** Laplace of Gaussian

**3.3.1.4.** Canny method

# 3.3.2. Edge Detection

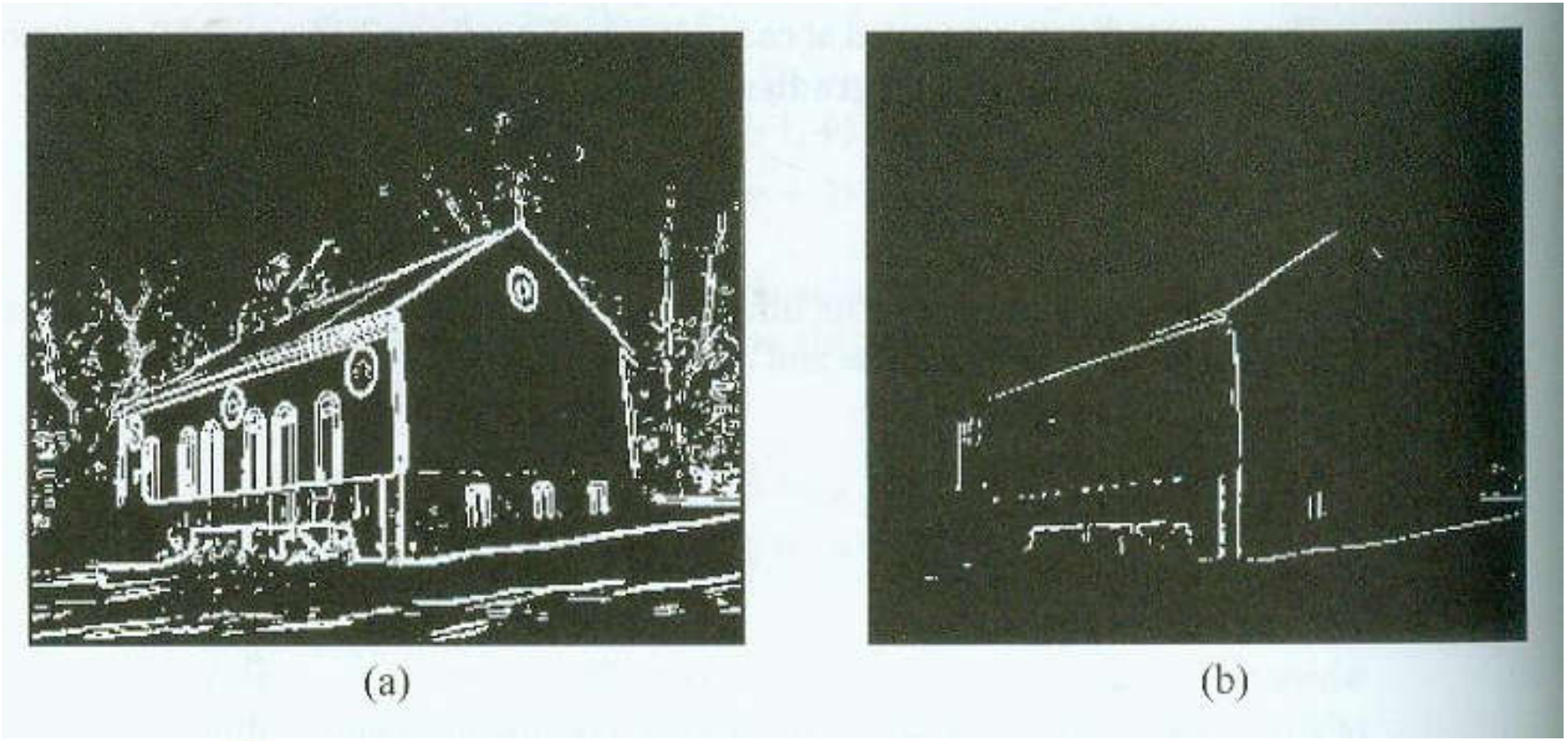
## 3.3.2.1. Gradient operator



(a) (b)

# 3.3.2. Edge Detection

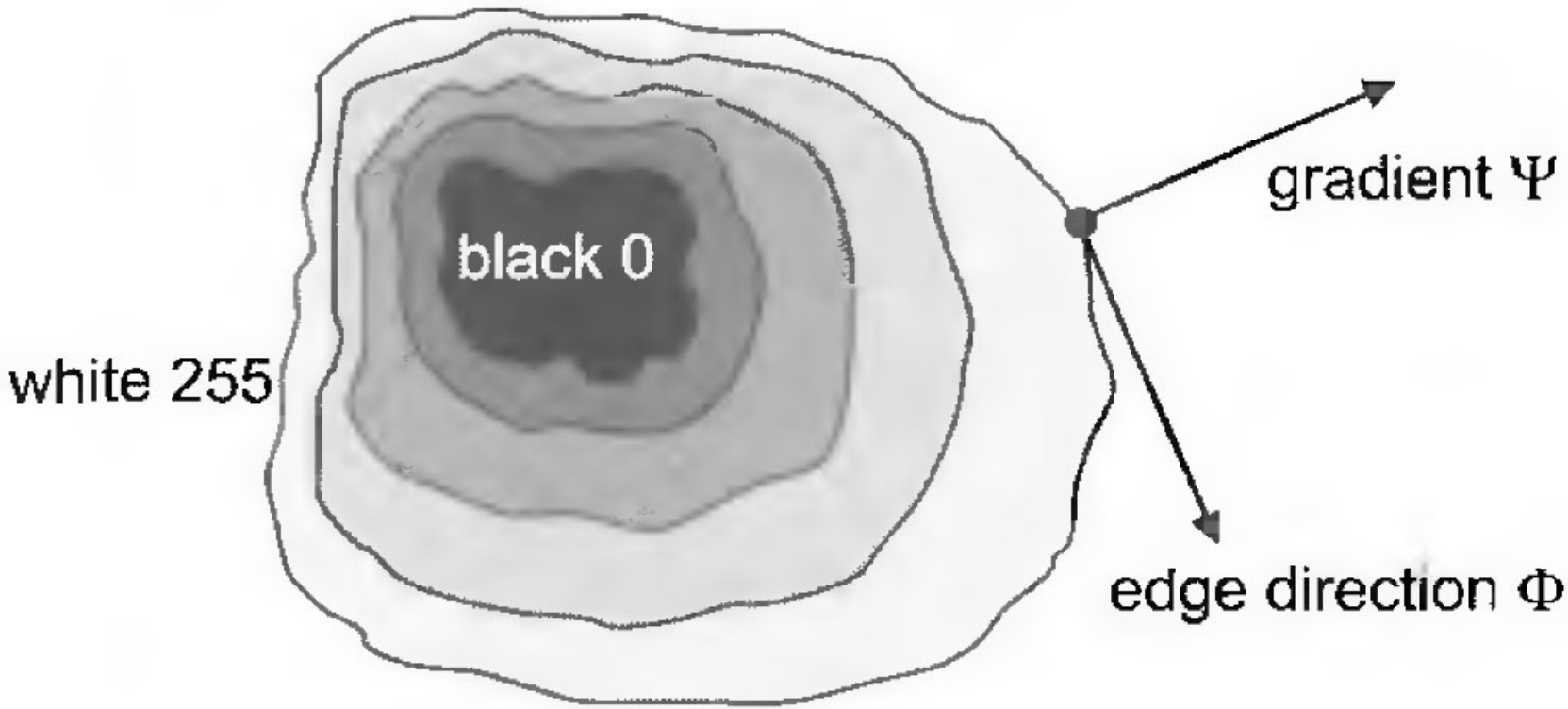
## 3.3.2.1. Gradient operator



(a) (b)

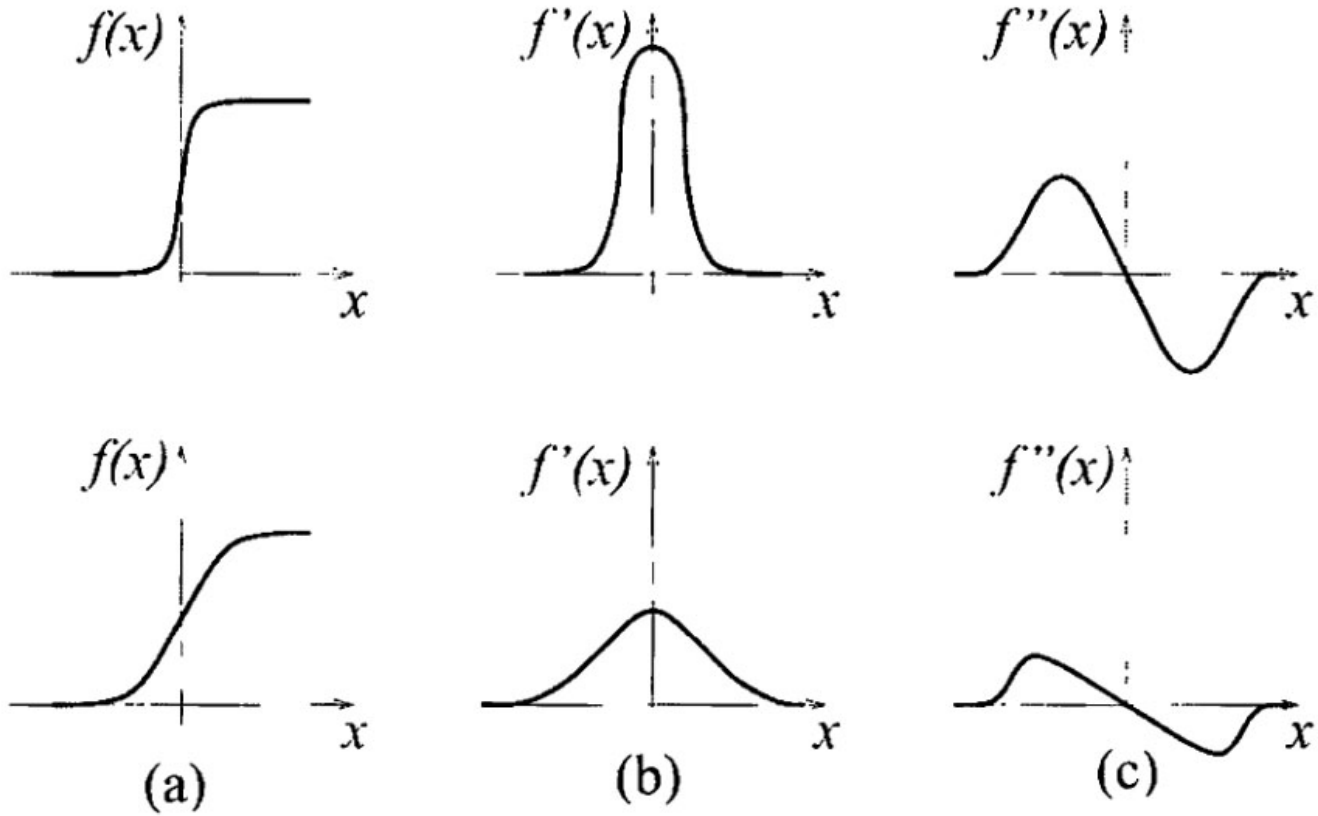
# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator



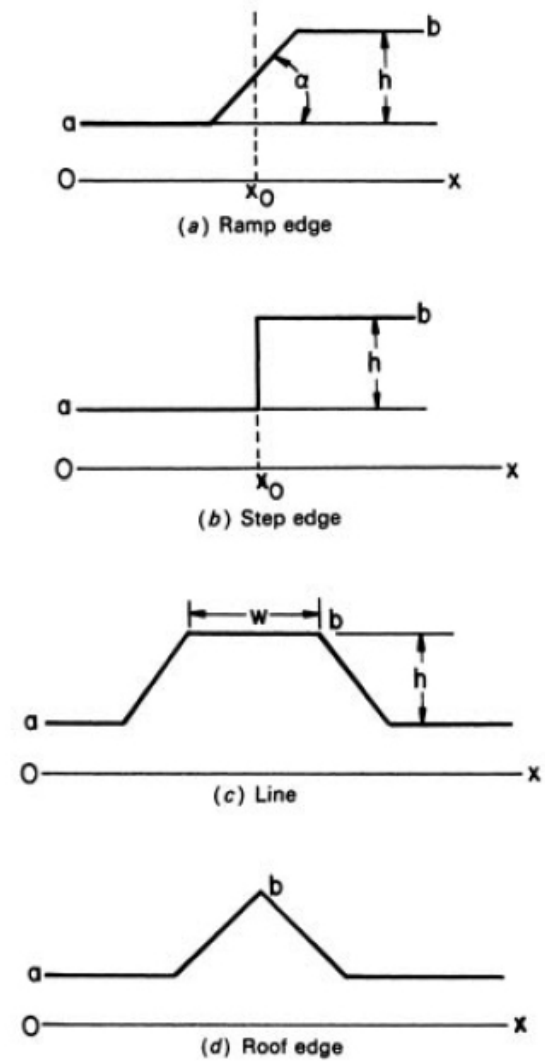
# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator



# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator



# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

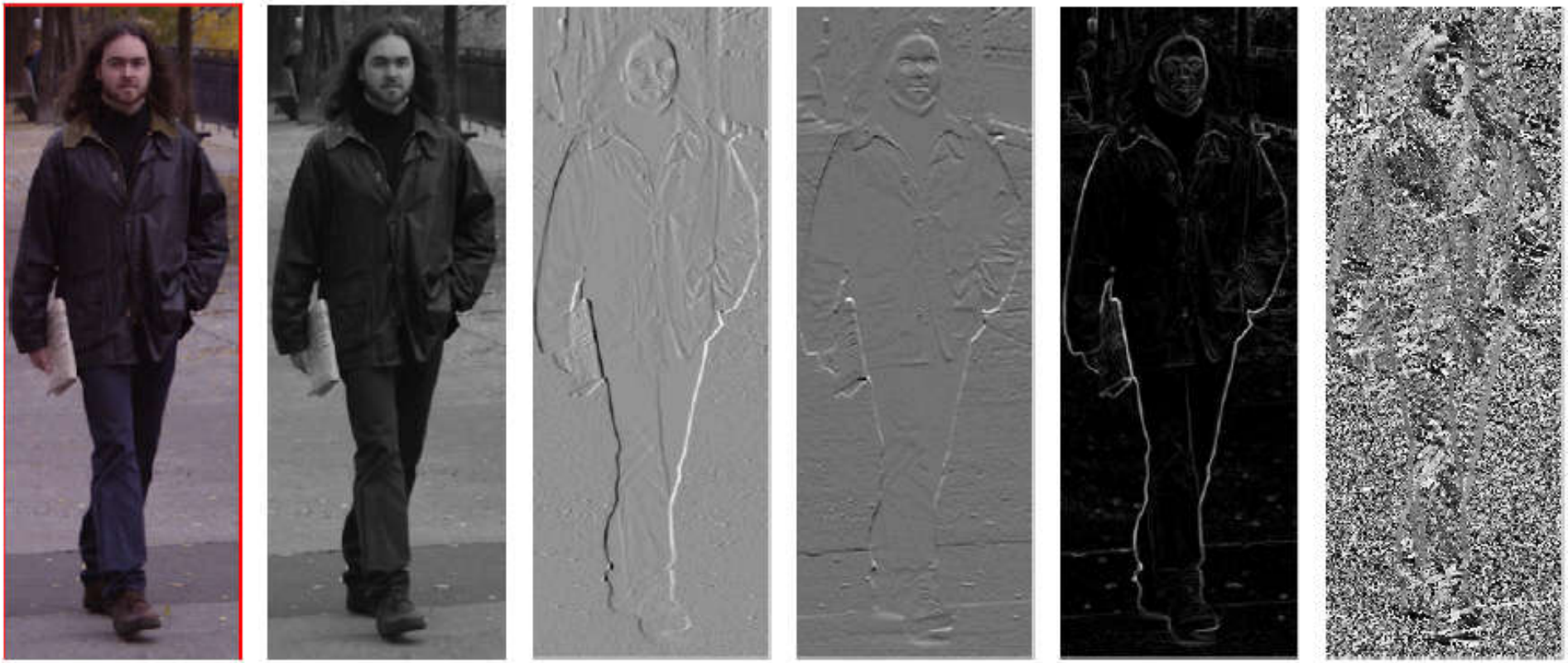
$$(\nabla f)(x,y) = [\partial f / \partial x \;\; \partial f / \partial y]^T = [f_x \;\; f_y]^T$$

$$e(x,y) = (f_x^{\,2}(x,y) + f_y^{\,2}(x,y))^{1/2}$$

$$\phi(x,y) = \arctan(f_x \;\; f_y)$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator



(a) Initial Image (b) $I$ (c) $I_x$ (d) $I_y$ (e) $\sqrt{I_x^2 + I_y^2}$ (f) $\arctan(I_y/I_x)$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | a | *b* | b | b | b | b |
| a | a | a | a | a | *b* | b | b | b | b |
| a | a | a | a | a | *b* | b | b | b | b |
| a | a | a | a | a | *b* | b | b | b | b |
| a | a | a | a | a | *b* | b | b | b | b |

Vertical step edge

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | *b* | b | b | b | b | b | b |
| a | a | a | a | *b* | b | b | b | b | b |
| a | a | a | a | a | *b* | b | b | b | b |
| a | a | a | a | a | a | *b* | b | b | b |
| a | a | a | a | a | a | a | *b* | b | b |

Diagonal step edge

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | a | a | a | a | a | a |
| a | a | a | a | a | a | a | a | a | a |
| a | a | a | a | a | *b* | *b* | *b* | *b* | *b* |
| a | a | a | a | a | *b* | b | b | b | b |
| a | a | a | a | a | *b* | b | b | b | b |

Corner step edge

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | *c* | b | b | b | b | b |
| a | a | a | a | *c* | b | b | b | b | b |
| a | a | a | a | *c* | b | b | b | b | b |
| a | a | a | a | *c* | b | b | b | b | b |
| a | a | a | a | *c* | b | b | b | b | b |

Vertical ramp edge

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | *c* | b | b | b | b | b | b | b |
| a | a | a | *c* | b | b | b | b | b | b |
| a | a | a | a | *c* | b | b | b | b | b |
| a | a | a | a | a | *c* | b | b | b | b |
| a | a | a | a | a | a | *c* | b | b | b |

Diagonal ramp edge

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | a | a | a | a | a | a |
| a | a | a | a | *c* | *c* | *c* | *c* | *c* | *c* |
| a | a | a | a | *c* | b | b | b | b | b |
| a | a | a | a | *c* | b | b | b | b | b |
| a | a | a | a | *c* | b | b | b | b | b |

Corner ramp edge

Single pixel transition

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |

Vertical ramp edge

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | c | b | b | b | b | b | b | b |
| a | a | a | c | b | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | a | c | b | b | b | b |
| a | a | a | a | a | a | c | b | b | b |

Diagonal ramp edge

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | a | a | a | a | a | a |
| a | a | a | a | c | c | c | c | c | c |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |

Corner ramp edge

Single pixel transition

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |

Vertical ramp edge

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | d | e | b | b | b | b | b | b |
| a | a | a | d | e | b | b | b | b | b |
| a | a | a | a | d | e | b | b | b | b |
| a | a | a | a | a | d | e | b | b | b |
| a | a | a | a | a | a | d | e | b | b |

Diagonal ramp edge

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | a | a | a | a | a | a |
| a | a | a | a | d | c | c | c | c | c |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |

Corner ramp edge

Smoothed transition

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

$c=(a+b)/2,$

$d= (3a+b)/4$

$e=(a+3b)/4$

$(a<d<c<e<b)$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

**Differencing**

$$f_x(x,y) \approx f(x,y) - f(x+1,y)$$

$$f_y(x,y) \approx f(x,y) - f(x,y-1)$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

**Differencing**

h=b-a

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | a | b | b | b | b | b |
| a | a | a | a | a | b | b | b | b | b |
| a | a | a | a | a | b | b | b | b | b |
| a | a | a | a | a | b | b | b | b | b |
| a | a | a | a | a | b | b | b | b | b |

Vertical step edge

0 0 0 0 h 0 0 0 0

# 3.3.2. Edge Detection

## 3.3.2.1. Gradie

**Differencing**

h=b-a

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |

Vertical ramp edge

0 0 0 0 h/2 h/2 0 0 0

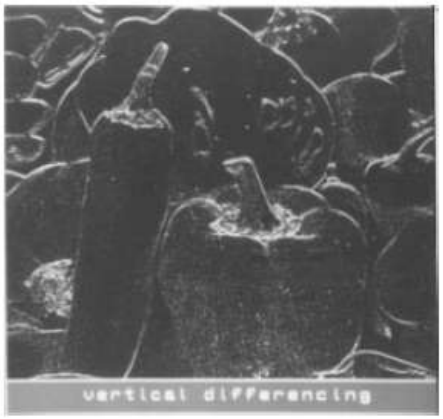
# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

**Differencing**

(*a*) Original

(*b*) Horizontal magnitude




(*c*) Vertical magnitude

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

**Robert operator**

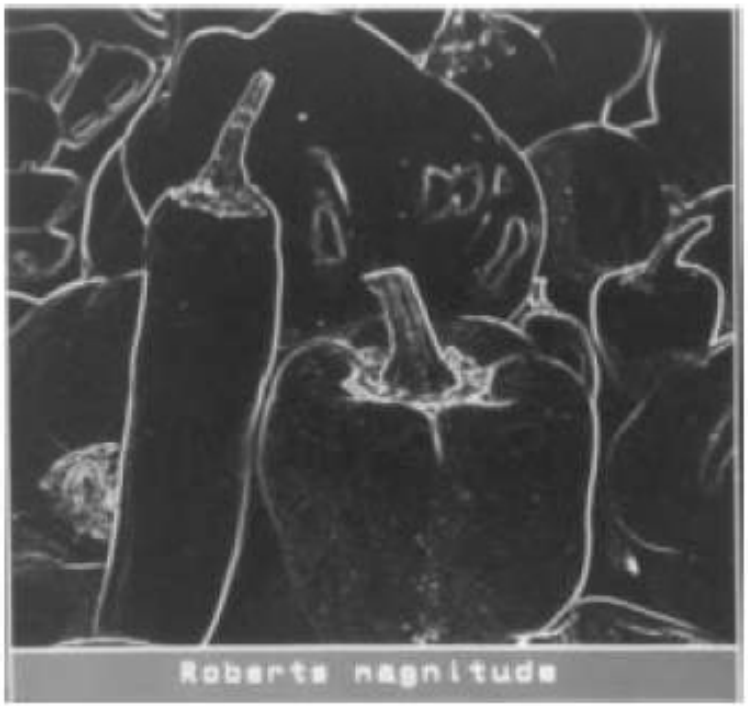
$$f_x(x,y) \approx f(x,y) - f(x+1,y+1)$$

$$f_y(x,y) \approx f(x,y+1) - f(x+1,y)$$

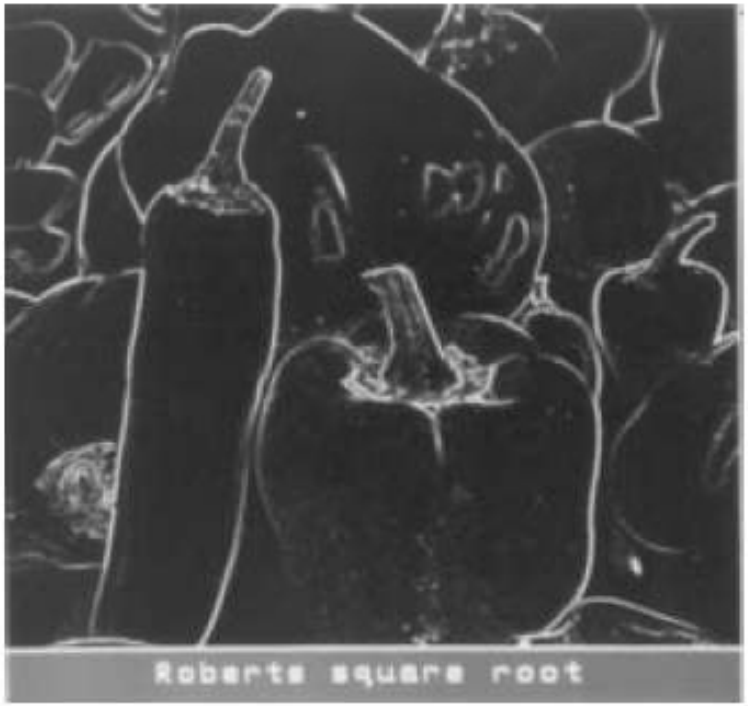
# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

### Robert operator




(*a*) Magnitude




(*b*) Square root

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

**Differencing (localize edge center of ramp edge)**

$$f_x(x,y) \approx f(x-1,y) - f(x+1,y)$$

$$f_y(x,y) \approx f(x,y+1) - f(x,y-1)$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

**Differencing (localize edge center of ramp edge)**

h=b-a

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |

Vertical ramp edge

0 0 h/2 h h/2 0 0

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

| | | |
|---|---|---|
| $A_0$ | $A_1$ | $A_2$ |
| $A_7$ | $F(j,k)$ | $A_3$ |
| $A_6$ | $A_5$ | $A_4$ |

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

**Prewitt operator (k=1)**

$$f_x \approx \frac{1}{k+2}[(A_2 + kA_3 + A_4) - (A_0 + kA_7 + A_6)]$$

$$f_y \approx \frac{1}{k+2}[(A_0 + kA_1 + A_2) - (A_6 + kA_5 + A_4)]$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

**Sobel operator (k=2)**

$$f_x \approx \frac{1}{k+2}[(A_2 + kA_3 + A_4) - (A_0 + kA_7 + A_6)]$$

$$f_y \approx \frac{1}{k+2}[(A_0 + kA_1 + A_2) - (A_6 + kA_5 + A_4)]$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

**Frei-Chen ($k=2^{1/2}$)**

$$f_x \approx \frac{1}{k+2}[(A_2 + kA_3 + A_4) - (A_0 + kA_7 + A_6)]$$

$$f_y \approx \frac{1}{k+2}[(A_0 + kA_1 + A_2) - (A_6 + kA_5 + A_4)]$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

**Prewitt**

**Sobel**

**Frei-Chen**

h=b-a

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |

Vertical ramp edge

0 0 h/2 h h/2 0 0

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

**Prewitt**

**Sobel**

**Frei-Chen**

h=b-a

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | c | b | b | b | b | b | b | b |
| a | a | a | c | b | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | a | c | b | b | b | b |
| a | a | a | a | a | a | c | b | b | b |

Diagonal ramp edge

$$0 \quad \frac{h}{\sqrt{2}(2+k)} \quad \frac{h}{\sqrt{2}} \quad \frac{\sqrt{2}(1+k)h}{(2+k)} \quad \frac{h}{\sqrt{2}} \quad \frac{h}{\sqrt{2}(2+k)} \quad 0$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

$$\text{Frei - Chen}: k = \sqrt{2} \Rightarrow \frac{\sqrt{2}(1+k)h}{(2+k)} = h$$

$$\text{Prewitt}: \quad k = 1 \Rightarrow \frac{\sqrt{2}(2)h}{(3)} = 0.94 \times h$$

$$\text{Sobel}: \quad k = 2 \Rightarrow \frac{\sqrt{2}(3)h}{(4)} = 1.06 \times h$$

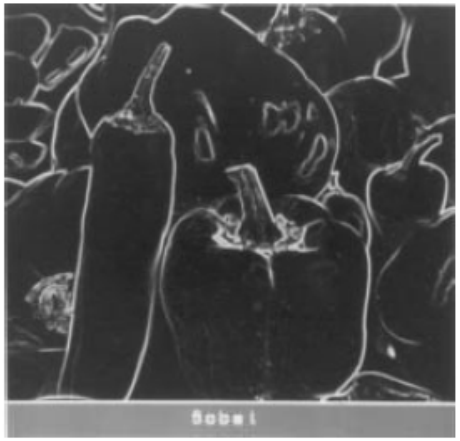
$$h = b - a$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

**Prewitt-Sobel-FreiChen**

(*a*) Prewitt




(*b*) Sobel

(*c*) Frei–Chen

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

Convolution operator

$$g = f * h$$

$$g(x,y) = \sum_i \sum_j f(x-i, y-j).h(i,j),$$

$$(i,j) \in O$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

Comsider image f and filter h as follows:

| | -1 | 0 | +1 |
|---|---|---|---|
| -1 | -1 | 0 | 1 |
| 0 | -2 | 0 | 2 |
| +1 | -1 | 0 | 1 |

| | | | | |
|---|---|---|---|---|
| | | x | | |
| | | | | |
| | 72 | 53 | 60 | |
| y | 76 | 56 | 65 | |
| | 88 | 78 | 82 | |
| | | | | |

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

Ouput image g computed at (x,y):

$$g(x,y) = h(-1,-1)f(x+1,y+1) + h(0,-1)f(x,y+1) +$$
$$h(1,-1)f(x-1,y+1) + h(-1,0)f(x+1,y) +$$
$$h(0,0)f(x,y) + h(1,0)f(x-1,y) +$$
$$h(-1,1)f(x+1,y-1) + h(0,1)f(x,y-1) +$$
$$h(1,1)f(x-1,y-1)$$

$$g(x,y) = 40$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

Pixel difference

$$f_x \approx \hat{f}_x = f * W_x$$

$$f_y \approx \hat{f}_y = f * W_y$$

$$W_x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \quad W_y = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

Separated Pixel difference

$$f_x \approx \hat{f}_x = f * W_x$$

$$f_y \approx \hat{f}_y = f * W_y$$

$$W_x = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \quad W_y = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

Robert operator

$$f_x \approx \hat{f}_x = f * W_x$$

$$f_y \approx \hat{f}_y = f * W_y$$

$$W_x = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad W_y = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

Prewitt operator

$$f_x \approx \hat{f}_x = f * W_x$$

$$f_y \approx \hat{f}_y = f * W_y$$

$$W_x = \frac{1}{3}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}, \quad W_y = \frac{1}{3}\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

Sobel operator

$$f_x \approx \hat{f}_x = f * W_x$$

$$f_y \approx \hat{f}_y = f * W_y$$

$$W_x = \frac{1}{4}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}, \quad W_y = \frac{1}{4}\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

# 3.3.2. Edge Detection

## 3.3.2.1. Gradient operator

Frei-chen operator

$$f_x \approx \hat{f}_x = f * W_x \,, f_y \approx \hat{f}_y = f * W_y \,,$$

$$W_x = \frac{1}{2+\sqrt{2}} \begin{bmatrix} 1 & 0 & -1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & 0 & -1 \end{bmatrix},$$

$$W_y = \frac{1}{2+\sqrt{2}} \begin{bmatrix} -1 & -\sqrt{2} & -1 \\ 0 & 0 & 0 \\ 1 & \sqrt{2} & 1 \end{bmatrix}$$

# 3.3.2. Edge Detection

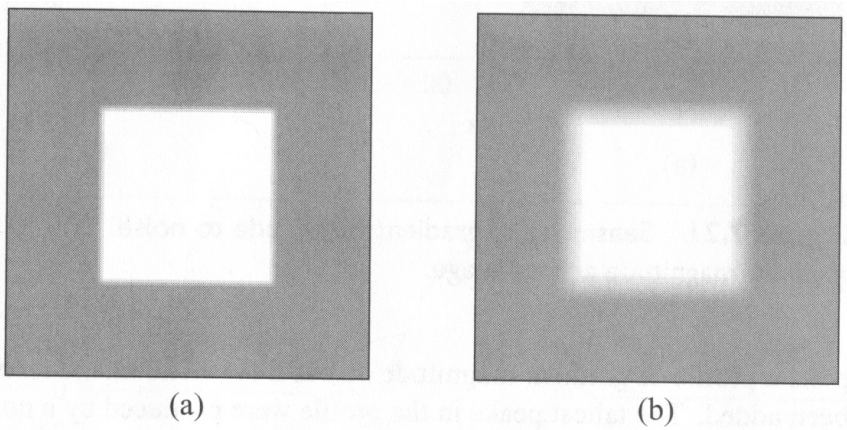
## 3.3.2.2. Laplace operator

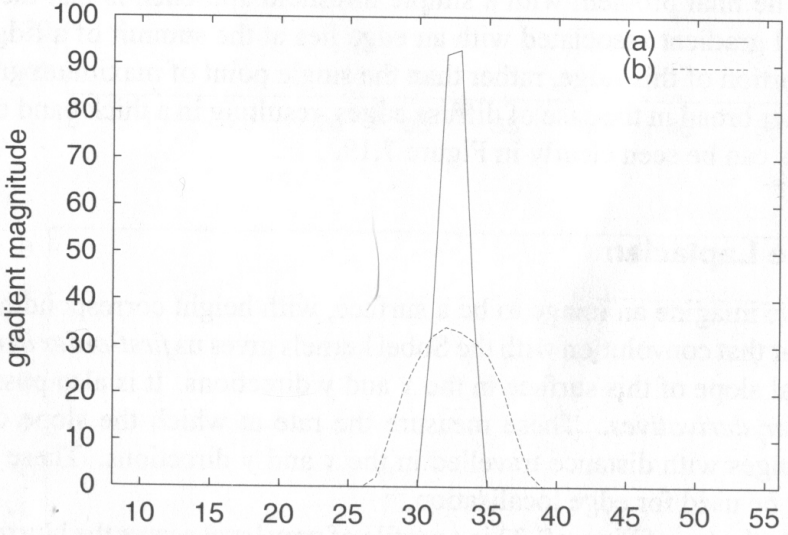
$$(\nabla^2 f)(x,y) = \partial^2 f / \partial x^2 + \partial^2 f / \partial y^2$$

$$(\nabla^2 f)(x,y) \approx f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

# 3.3.2. Edge Detection

## 3.3.2.2. Laplace operator



# 3.3.2. Edge Detection

## 3.3.2.2. Laplace operator

$$\nabla^2 f \approx f * Laplace$$

$$Laplace = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

# 3.3.2. Edge Detection

## 3.3.2.2. Laplace operator

$$\nabla^2 f \approx f * Laplace$$

$$Laplace = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

# 3.3.2. Edge Detection

## 3.3.2.2. Laplace operator

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | a | b | b | b | b | b |
| a | a | a | a | a | b | b | b | b | b |
| a | a | a | a | a | b | b | b | b | b |
| a | a | a | a | a | b | b | b | b | b |
| a | a | a | a | a | b | b | b | b | b |

Vertical step edge

$$0 \quad -3h/8 \quad 3h/8 \quad 0$$

# 3.3.2. Edge Detection

## 3.3.2.2. Laplace operator

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |

Vertical ramp edge

0 -3h/16 0 3h/16 0

# 3.3.2. Edge Detection

## 3.3.2.2. Laplace operator

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | c | b | b | b | b | b | b | b |
| a | a | a | c | b | b | b | b | b | b |
| a | a | a | a | c | b | b | b | b | b |
| a | a | a | a | a | c | b | b | b | b |
| a | a | a | a | a | a | c | b | b | b |

Diagonal ramp edge

$$0 \quad -h/8 \quad -h/8 \quad 0 \quad h/8 \quad h/8 \quad 0$$

# 3.3.2. Edge Detection

## 3.3.2.2. Laplace operator

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a | a | d | e | b | b | b | b | b | b |
| a | a | a | d | e | b | b | b | b | b |
| a | a | a | a | d | e | b | b | b | b |
| a | a | a | a | a | d | e | b | b | b |
| a | a | a | a | a | a | d | e | b | b |

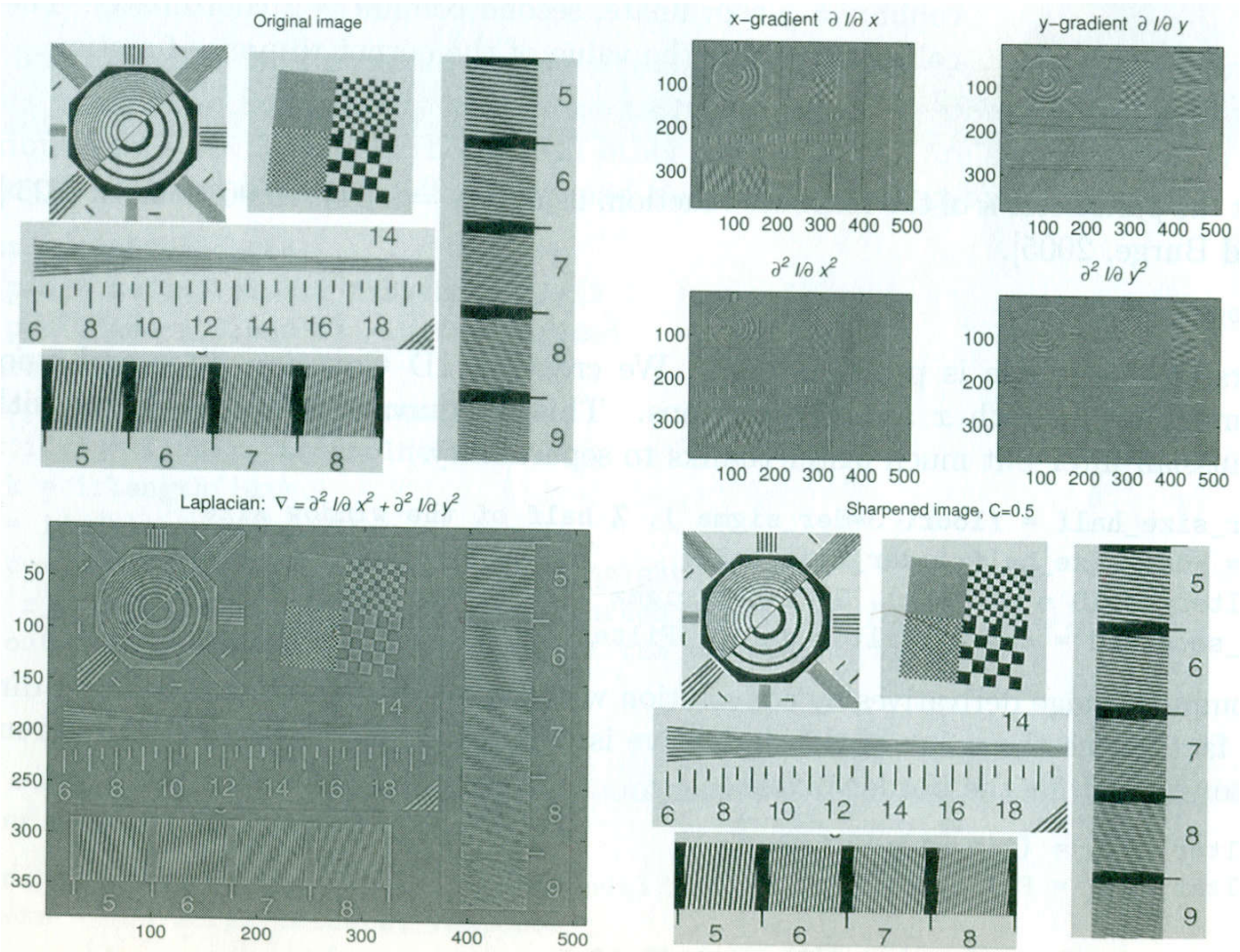
Diagonal ramp edge

Smoothed transition

$$0 \quad -h/16 \quad -h/8 \quad -h/16 \quad 0 \quad h/16 \quad h/8 \quad h/16 \quad 0$$

# 3.3.2. Edge Detection

## 3.3.2.2. Laplace operator



# 3.3.2. Edge Detection

## 3.3.2.2. Laplace of Gaussian operator

$$\nabla^2[G(x,y,\sigma) * f(x,y)] = [\nabla^2 G(x,y,\sigma)] * f(x,y)$$

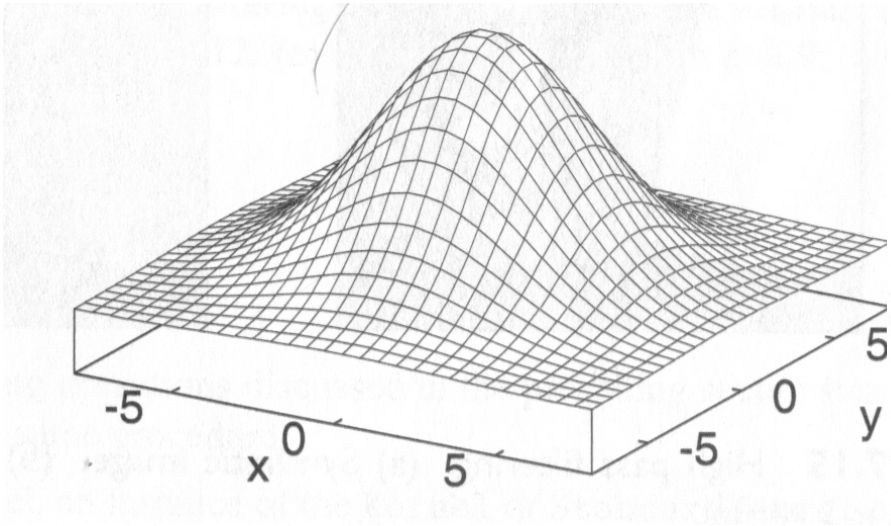
$$G(x,y,\sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$

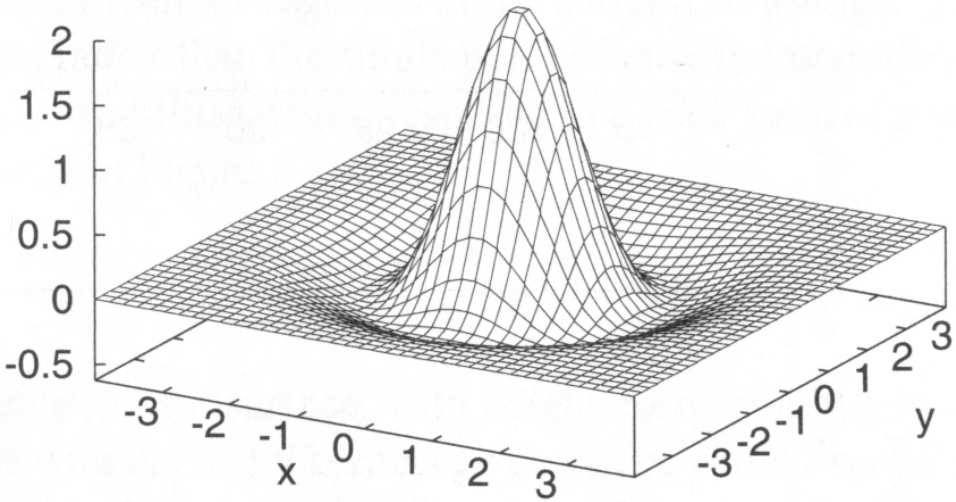
$$\nabla^2 G(x,y,\sigma) = \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}$$

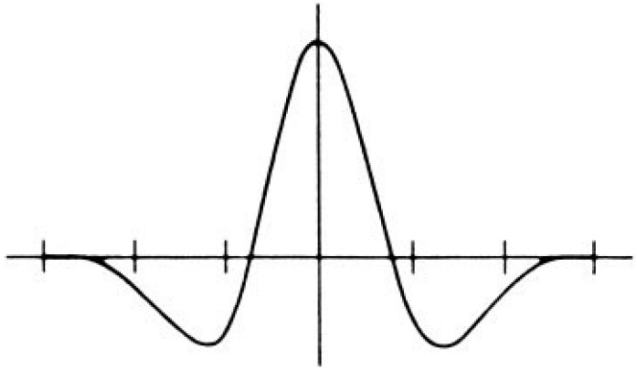
$$= \frac{1}{\pi\sigma^4}\left[\frac{x^2+y^2}{2\sigma^2} - 1\right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$

# 3.3.2. Edge Detection

## 3.3.2.2. Laplace of Gaussian operator

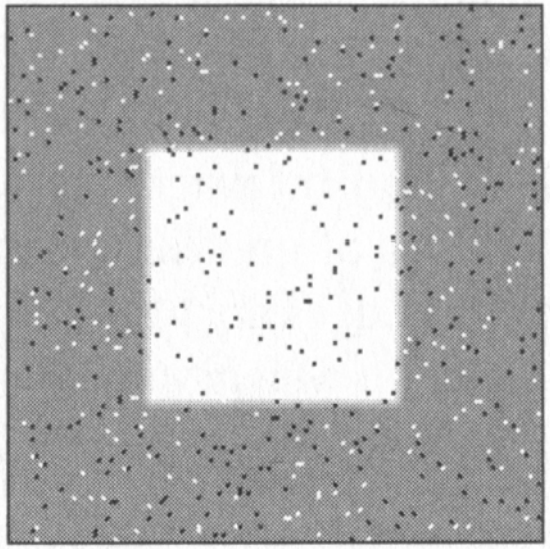


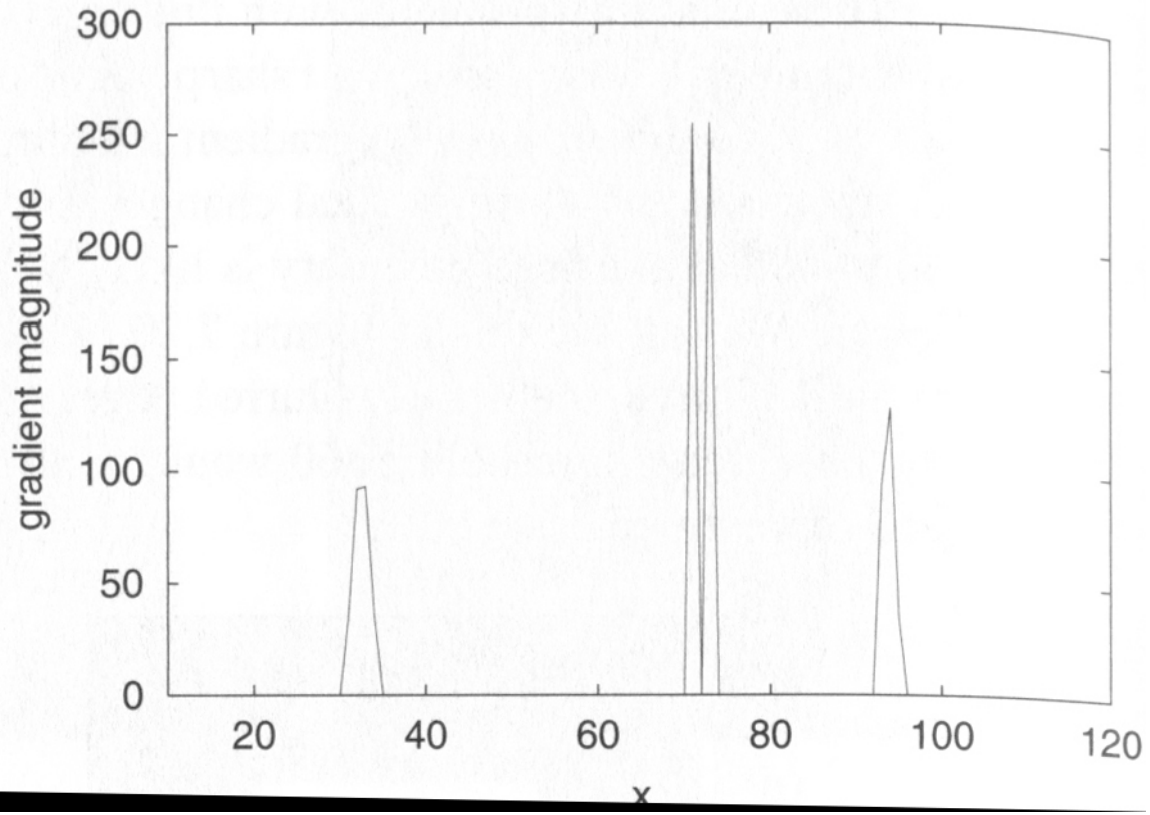



# 3.3.2. Edge Detection

## 3.3.2.2. Laplace of Gaussian operator





# 3.3.2. Edge Detection

## 3.3.2.2. Canny edge detection method

**1.** $G(x, y, \sigma) * f(x, y)$

**2.** $n = \nabla(G * f)$

**3.** Find the location of the edges using non-maximal suppression approach.

**4.** Compute the magnitude of the edge is measured as

$$|\nabla(G * f)|$$

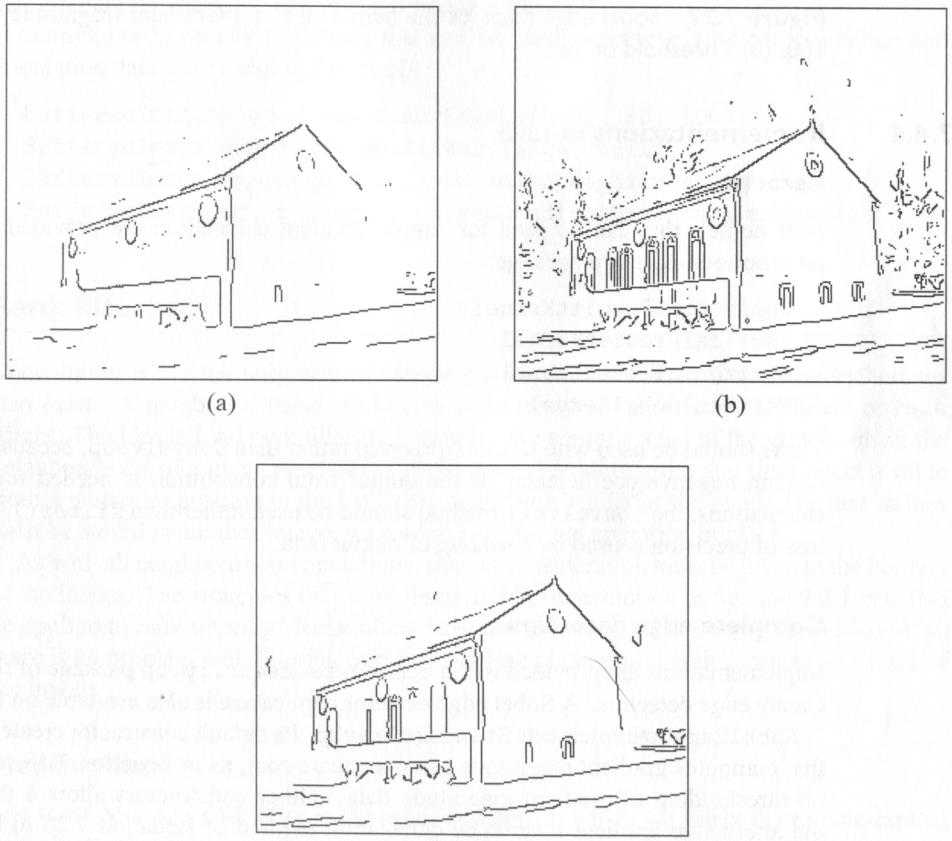
**5.** Threshold edges in the image with hysteresis to eliminate spurious responses.

**6.** Repeat steps 1-5 for ascending values of the standard deviation $\sigma$ .

**7**. Aggregate the final information about edges at multiple scale using 'feature systhesis' approach

# 3.3.2. Edge Detection

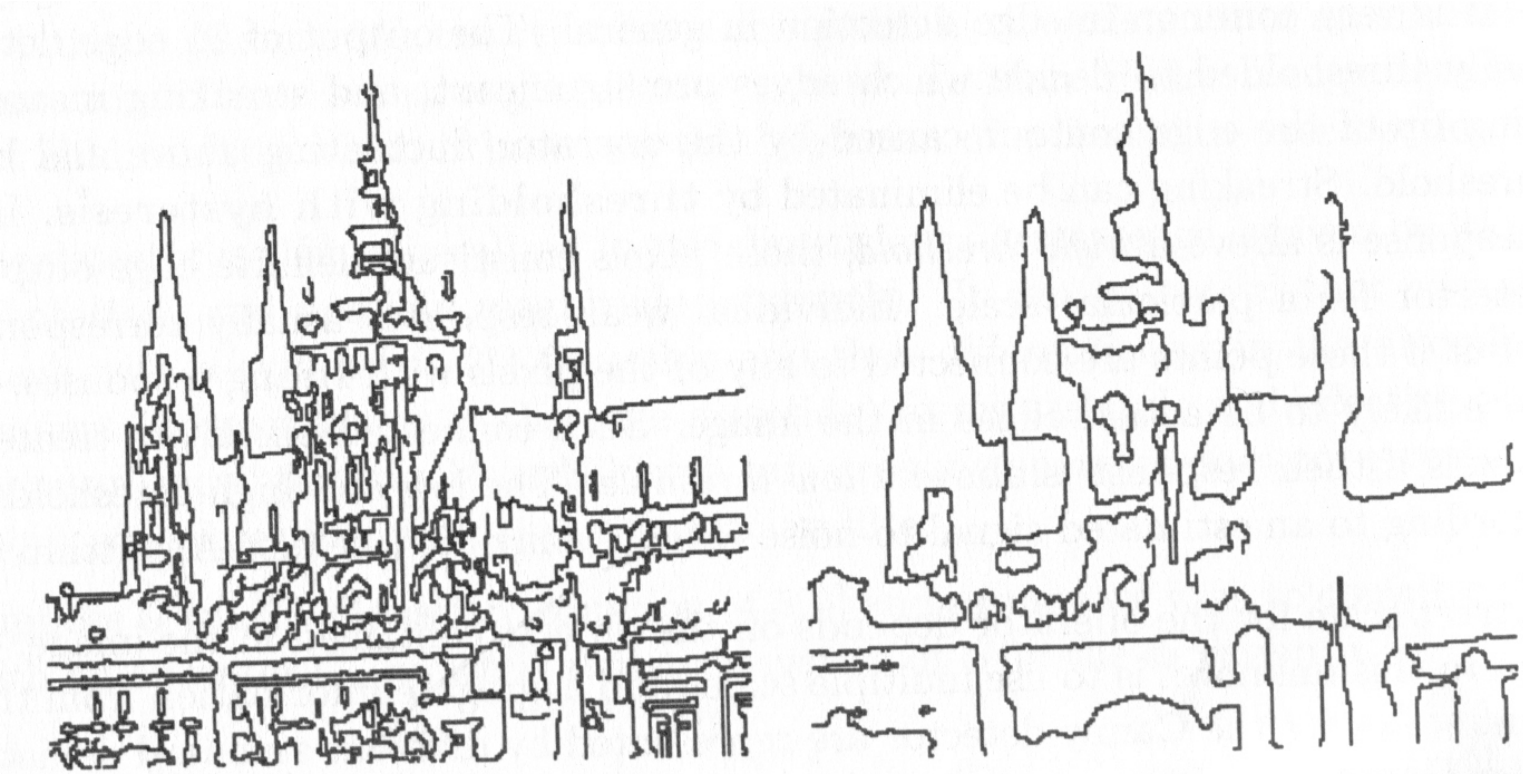
## 3.3.2.2. Canny edge detection method



(a) (b) (c)

# 3.3.2. Edge Detection

## 3.3.2.2. Canny edge detection method



(a) (b)

# 3.3.2. Edge Detection

## 3.3.2.2. Canny edge detection method

**Non-maximal suppression of directional edge data**

**3.** Find the location of the edges using non-maximal suppression approach.

**3.1**. For each pixel with non-zero edge magnitude, inspect two adjacent pixels indicated the direction of its edge.

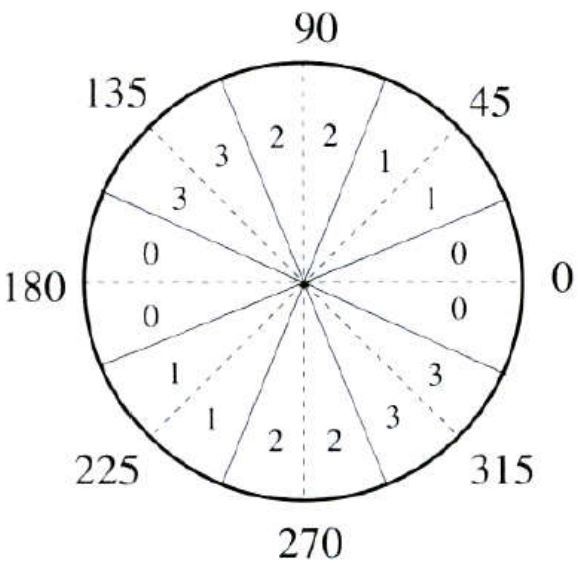
**3.2**. If the edge magnitude of either of these two exceeds that of the pixel under inspection, mark it for deletion.

**3.3**. When all pixels have been inspected, re-scan the image and erase to zero all edge data marked for deletion.

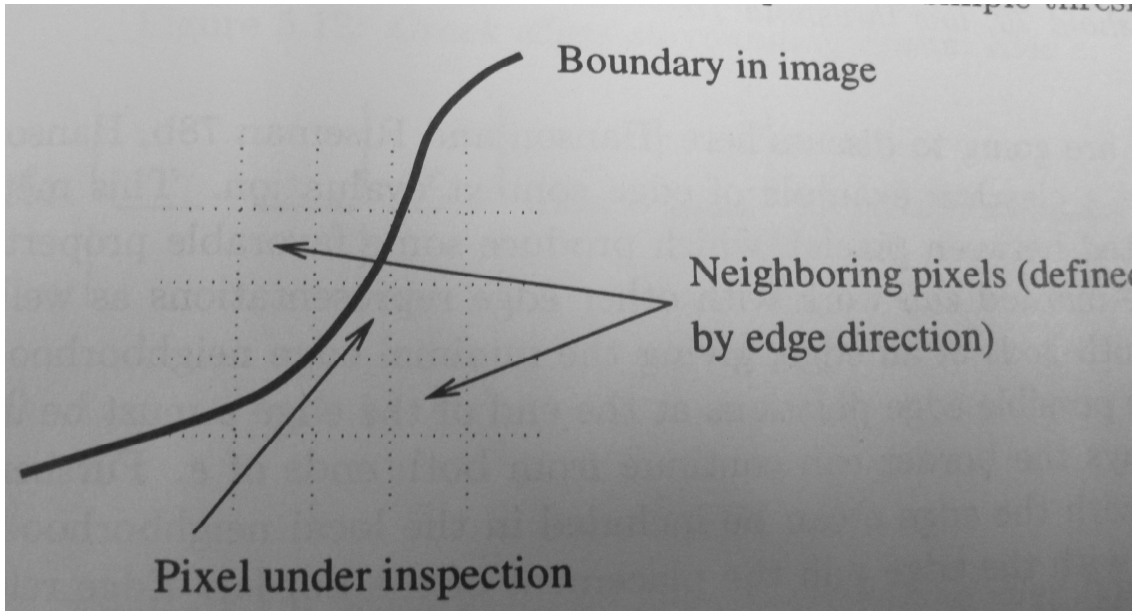
# 3.3.2. Edge Detection

## 3.3.2.2. Canny edge detection method

## (Non-maximal suppression of directional edge data)



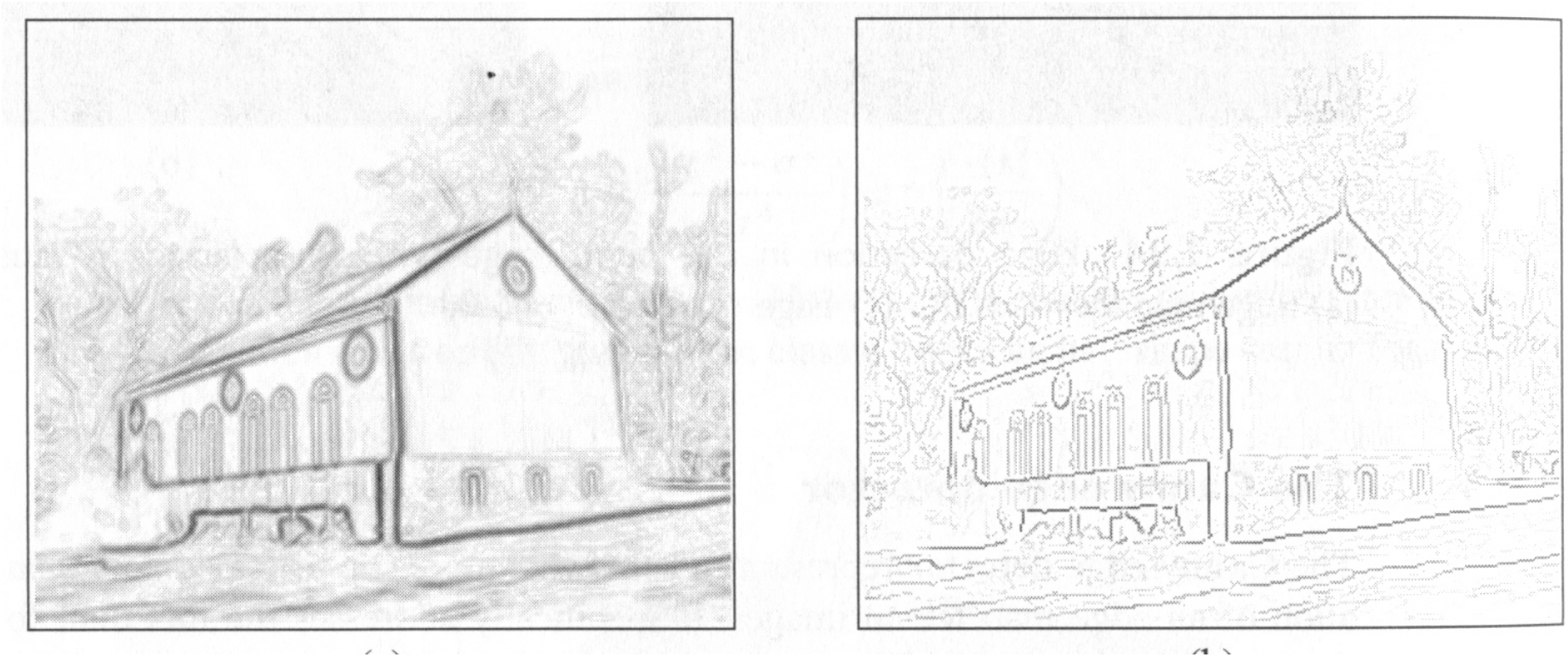

Canny Edge Detector



# 3.3.2. Edge Detection

## 3.3.2.2. Canny edge detection method

## (Non-maximal suppression of directional edge data)



## 3.3.2.2. Canny edge detection method

**Hysteresis to filter output of an edge detector**

**5.** Threshold edges in the image with hysteresis to eliminate spurious responses.

**5.1**. Mark all edges with magnitude greater than $\mathbf{t_1}$.

**5.2**. Scall all pixels with edge magnitude in the range $\mathbf{[t_0\ t_1]}$.

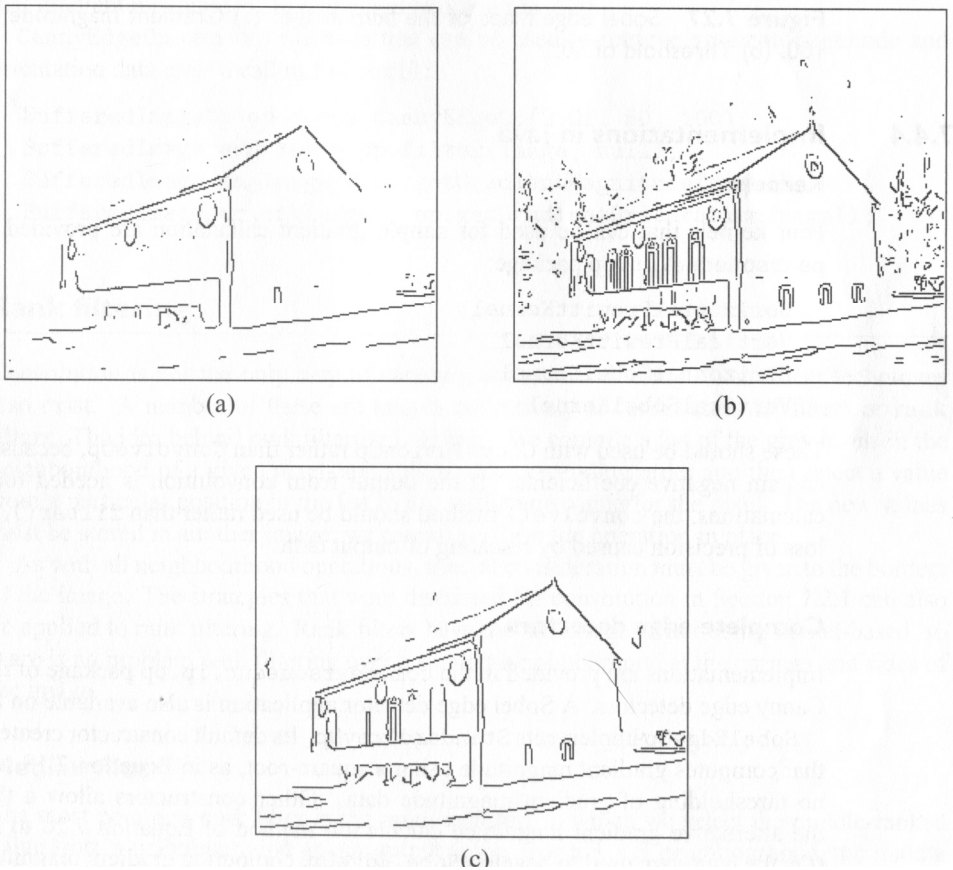
**5.3**. If such a pixel borders another marked as an edge, the mark it too.

**5.4**. Repeat from step 5.2 until stability.

# 3.3.2. Edge Detection

## 3.3.2.2. Canny edge detection method

**Hysteresis to filter output of an edge detector**



(a) (b) (c)