Problem 1:

a, Marginal distribution:

Using the formular of marginal distributions:

$$P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$$

From the table that:

$$R_X = \{1,2,3,4,5\}, R_Y = \{1,2,3\}$$

We have:

+ P(x):

$$P(X = x_1) = 0.01 + 0.05 + 0.1 = 0.16$$

$$P(X = x_2) = 0.02 + 0.1 + 0.05 = 0.17$$

$$P(X = x_3) = 0.03 + 0.05 + 0.03 = 0.11$$

$$P(X = x_4) = 0.1 + 0.07 + 0.05 = 0.22$$

$$P(X = x_5) = 0.1 + 0.2 + 0.04 = 0.34$$

+ P(y):

$$P(Y = y_1) = 0.01 + 0.02 + 0.03 + 0.1 + 0.1 = 0.26$$

$$P(Y = y_2) = 0.05 + 0.1 + 0.05 + 0.07 + 0.2 = 0.47$$

$$P(Y = y_3) = 0.1 + 0.05 + 0.03 + 0.05 + 0.04 = 0.27$$

B, Conditional distribution

Using formula:

$$P(x|Y = y_j) = \frac{P(x, y_i)}{P(Y = y_j)}$$

We calculate the conditional distribution of $P(x|Y = y_1)$:

$$P(X = x_1|Y = y_1) = \frac{0.01}{0.26} = \frac{1}{26}$$

$$P(X = x_2|Y = y_1) = \frac{0.02}{0.26} = \frac{1}{13}$$

$$P(X = x_3|Y = y_1) = \frac{0.03}{0.26} = \frac{3}{26}$$

$$P(X = x_4|Y = y_1) = \frac{0.1}{0.26} = \frac{5}{13}$$

$$P(X = x_5|Y = y_1) = \frac{0.1}{0.26} = \frac{5}{13}$$

Similarly, we can calculate $P(x|Y = y_3)$:

$$P(X = x_1|Y = y_3) = \frac{0.01}{0.27} = \frac{1}{27}$$

$$P(X = x_2|Y = y_2) = \frac{0.02}{0.27} = \frac{2}{27}$$

$$P(X = x_3|Y = y_3) = \frac{0.03}{0.27} = \frac{1}{9}$$

$$P(X = x_4|Y = y_4) = \frac{0.1}{0.27} = \frac{10}{27}$$

$$P(X = x_5|Y = y_5) = \frac{0.1}{0.27} = \frac{10}{27}$$

Problem 3:

We have:

$$P(X) = 0.207$$

$$P(Y) = 0.5$$

$$P(X|Y) = 0.365$$

A, Probability of user use both X and Y:

$$P(X, Y) = P(X|Y) \times P(Y) = 0.365 \times 0.5 = 0.1825$$

B, We have:

$$+ P(\bar{X}|Y) = 1 - P(X|Y) = 0.635$$

$$+ P(\bar{X}) = 1 - P(X) = 0.793$$

Using Baye formula:

$$P(Y|\bar{X}) = \frac{P(\bar{X}|Y) \times P(Y)}{P(\bar{X})}$$

$$= \frac{(0.635 \times 0.5)}{0.793}$$

$$= 0.4$$

Problem 4:

By the definition of variance, we have

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\begin{aligned}
Var[X] &= E[(X - E[X])^2] \\
&= E[X^2 - 2X \times E[X] + (E[X])^2] \\
&= E[X^2] + E\left[-2X \times E[X]\right] + E[E[X]]^2 \\
&= E[X^2] - 2E[X]E[X] + E[X]^2 E[1] \\
&= E[X^2] - 2E[X]^2 + E[X]^2 \\
&= E[X^2] - E[X]^2
\end{aligned}
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