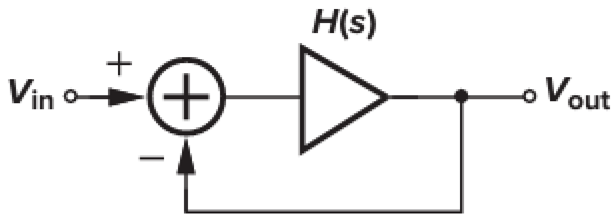
Introduction to oscillator

A. General consideration

- There are many ways to describe how a circuit can oscillate. An oscillator basically has noise as input signal and produces a periodic signal. Consider an unity gain feedback loop, that is:



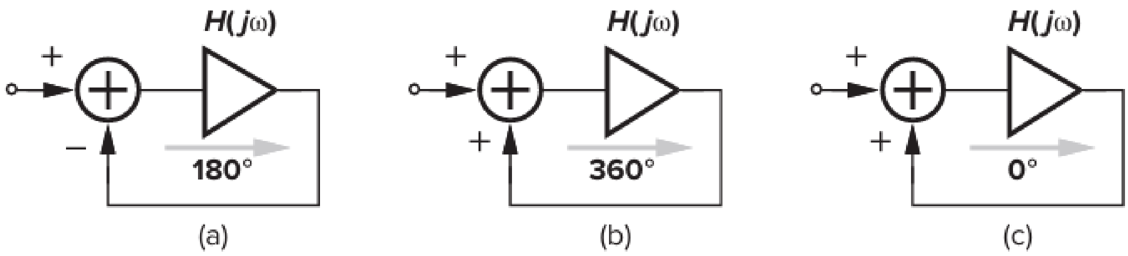
- We can derive the output signal:

$$\frac{V_{out}}{V_{in}}(s) = \frac{H(s)}{1 + H(s)}$$

- How can we get the output signal, whilst the input is noise (that is much small)? The idea is simple: using an infinite-gain amplifier. Look that the equation about, we can observe that to get infinite gain, the condition is:

$$H(s) = -1$$

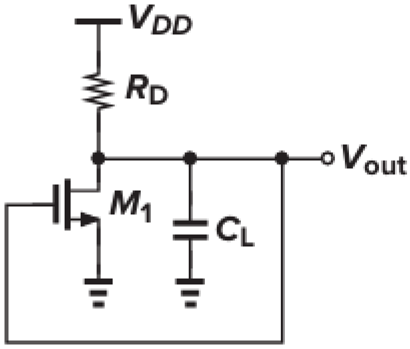
- $H(s)$ is the open loop gain, so the above condition requires the open loop gain has unity amplitude and has the phase shift of 180 degrees (or (2k+1)*pi in general, with k is an integer number). This is called: Bakhausen's criteria. But it is not sufficient. In the real world, there are variety of effects (temperature, non-ideal component, …) that can make the amplitude be lower than unity, so we typically design the loop gain to be at least two or three time than the required value.
- Note that there are various of feedback system's view:



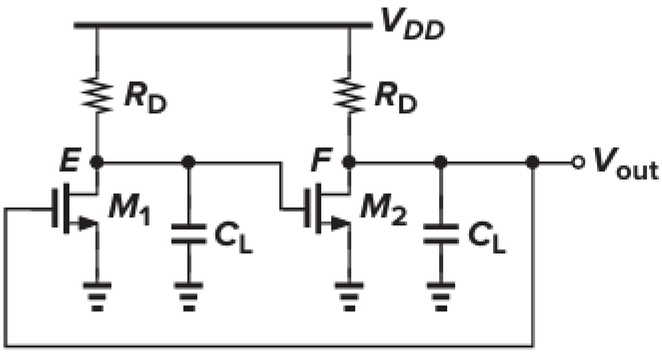
- The foregoing discussion, we used the feedback system with a subtraction to the input signal. What if we add the feedback signal to the input signal? $H(s)$ now should make a phase shift of 2*k*pi (k is integer number). It means if we have 180 degree phase shift, we can get addition 180 degree phase shift by inverse it before applying to the input signal. The key idea here is obtaining 2*k*pi.

B. Ring oscillator

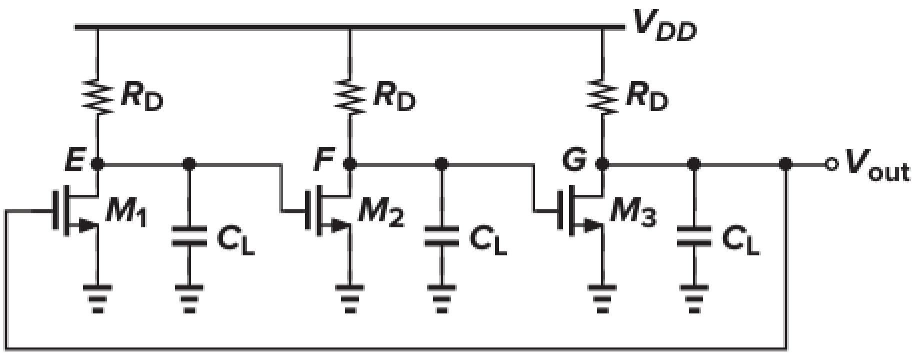
- Let look at an common source stage with unity feedback loop:



- Can it oscillate ? We can get the maximum total phase shift of 270 degree(180 degree of inversion property of common source stage, and 90 degree at infinite frequency due to one pole). So the answer is no.
- What about this circuit:



- Now there are two poles, so we hope it will operate. Unfortunately, M1 and M2 introduce two times of signal inversion, so the maximum total frequency phase shift is (2+1)*pi at infinite frequency. So it cannot oscillate at all.
- Let look at three stage configuration:



- Here it contains 3 poles, and 3 times of inversion(that means 3*pi). So each pole must exhibit pi/3 phase shift. The loop gain can be written as:

$$H(s) = -\frac{A_0^3}{\left(1+\frac{s}{\omega_0}\right)^3}$$

- Because of 60 degree phase shift of each pole, we have:

$$\tan^{-1}\frac{\omega_{osc}}{\omega_0} = 60^\circ$$

- And hence:

$$\omega_{osc} = \sqrt{3}\omega_0$$

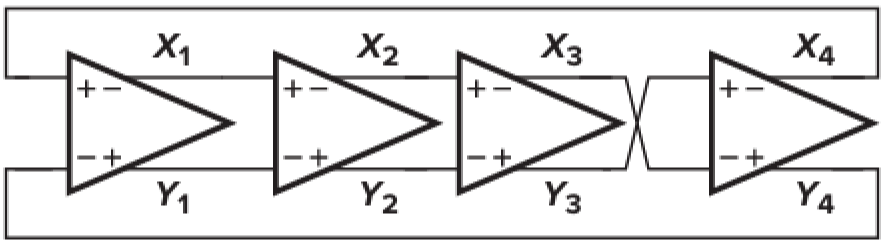
- The loop gain should have the gain at least:

$$\frac{A_0^3}{\left[\sqrt{1+\left(\frac{\omega_{osc}}{\omega_0}\right)^2}\right]^3} = 1$$

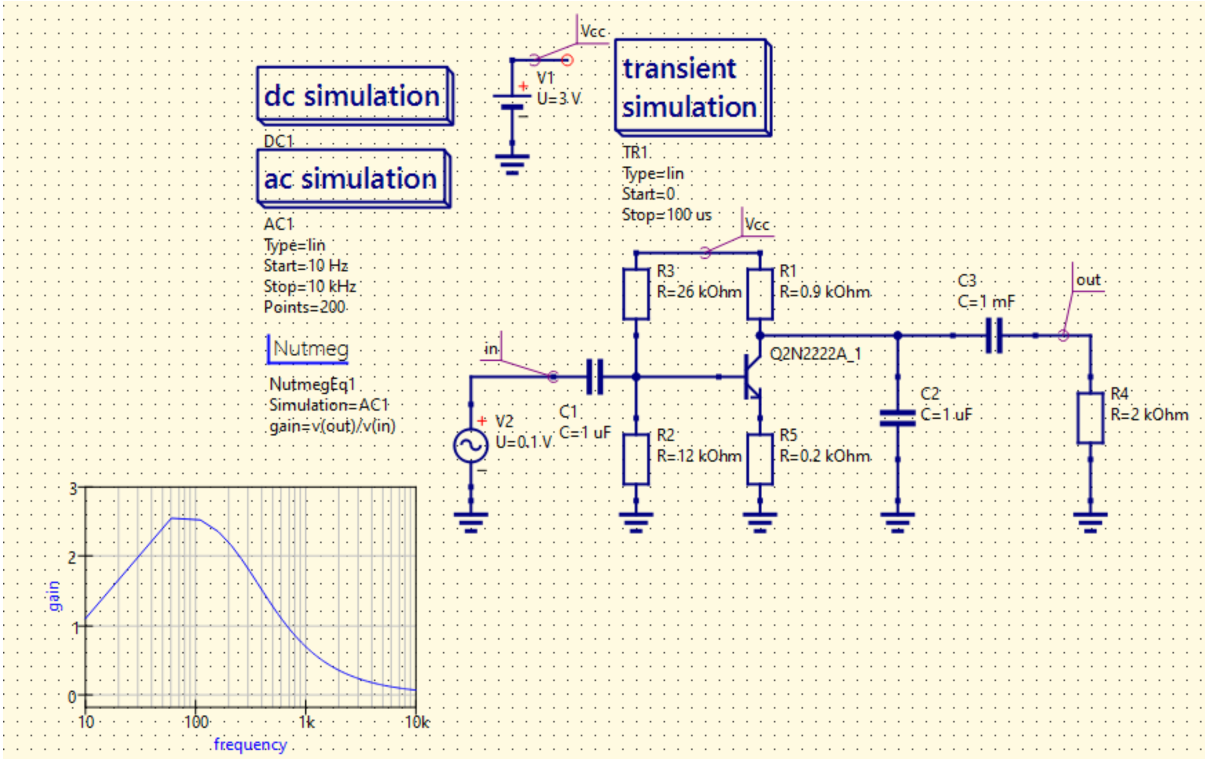
- Or in other word:

$$A_0 = 2$$

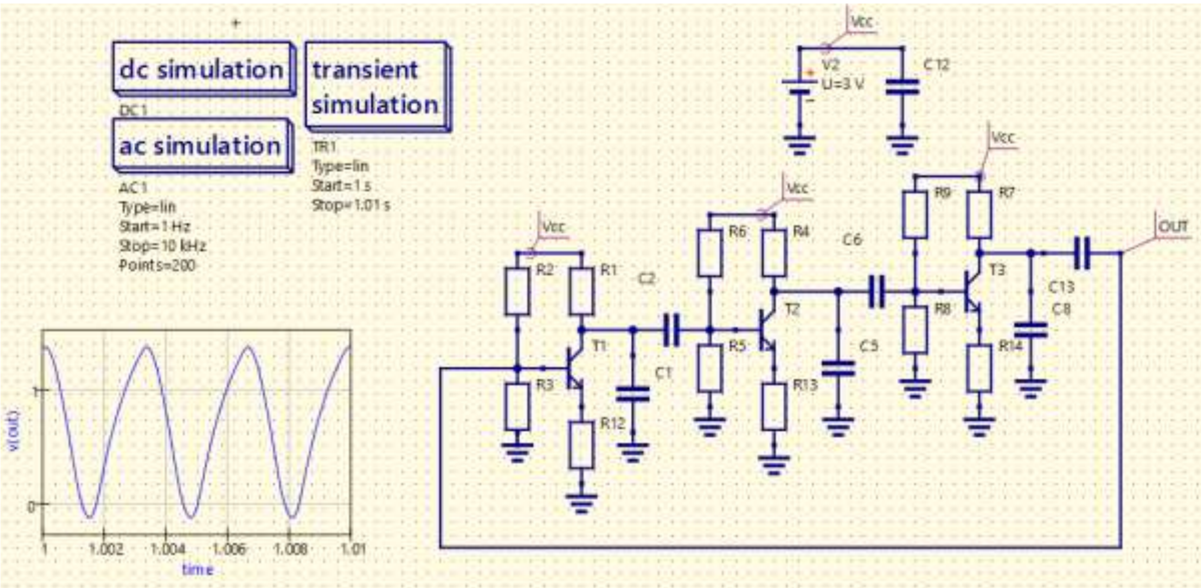
- This is called "ring oscillator". A ring oscillator typically has odd stages. The procedure above can be used to analysed for another ring configuration (5 stages, 7 stages,…).
- I have said that the ring oscillator typically has odd stage. And it also can have even stages, if you configure each stage as differential circuit:



- Notice this that we have 4 stages. But the output of the third stage is intentionally inversed before applying it to the fourth stage. Then we can say: there are 5 time of inversions, or 5*pi. And finally we just make 45 degree phase shift for each poles(because we have 4 capacitors now).
- So it is all about the "analog" ring oscillator. We also have digital ring oscillator, that consists digital inverting circuit (the NOT gate). But I don't mention it here because I focus to the analog type.
- In conclusion, let construct a three stages ring oscillator. I use QUCS-S here for simulation, Notice that it is just an quick example. Firstly, I try to get an element with 2 or higher in gain:



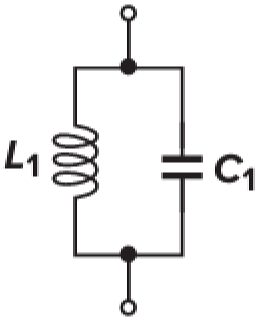
- The equivalent output resistor of this stage is about 620 Ohm. The capacitor in used is 1e-6Nf. We then can find out the frequency is about $\frac{1}{2*pi*900*1e-6} = 256.8\ Hz$
- Then I place it into the main circuit:



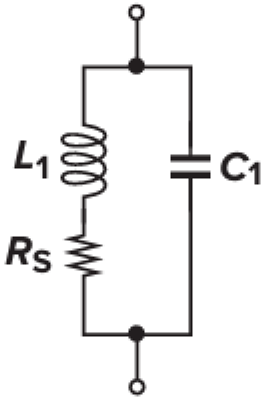
- You can see the result is in the low left hand side. The frequency is about 300Hz, with the peak to peak amplitude is about 1.5V. The frequency is about 300 Mhz (larger than our calculation).
- I don't like this type of oscillator at all (complexity, low frequency operation, require a lot of stage). Instead of that, the digital ring oscillator is more useful. Now let look at another type of oscillator.

C. LC oscillator

- Now let look at a LC resonator:

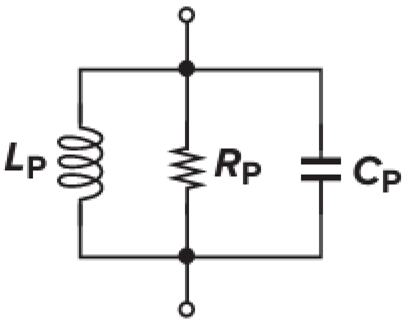


- It is quite interesting. The components are ideal, so that the Q factor is infinite. If we apply an amount of energy to this circuit, it will oscillate at resonance frequency $f = \frac{1}{2 * pi * \sqrt{(L_1 C_1)}}$
- Unfortunately, the realistic inductor has series parasitic resister Rs:



$$|Z_{eq}(s = j\omega)|^2 = \frac{R_S^2 + L_1^2\omega^2}{(1 - L_1C_1\omega^2)^2 + R_S^2C_1^2\omega^2}$$

- Due to the Rs, the resonance frequency is in vicinity of $f = \frac{1}{2 * pi * \sqrt{(L_1 C_1)}}$.
- We can transfer this circuit into an easy configuration to analyse:

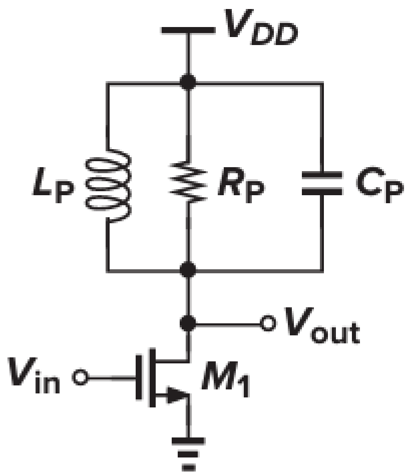


- Cp has the same value as C1. You can derive Lp and Rp by:

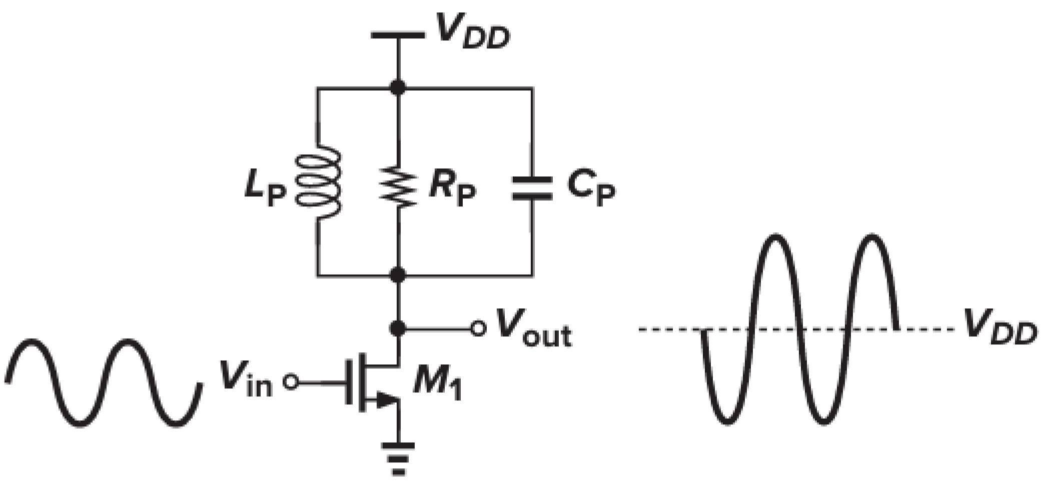
$$L_1R_P + L_PR_S = R_PL_P$$

$$R_SR_P - L_1L_P\omega^2 = 0$$

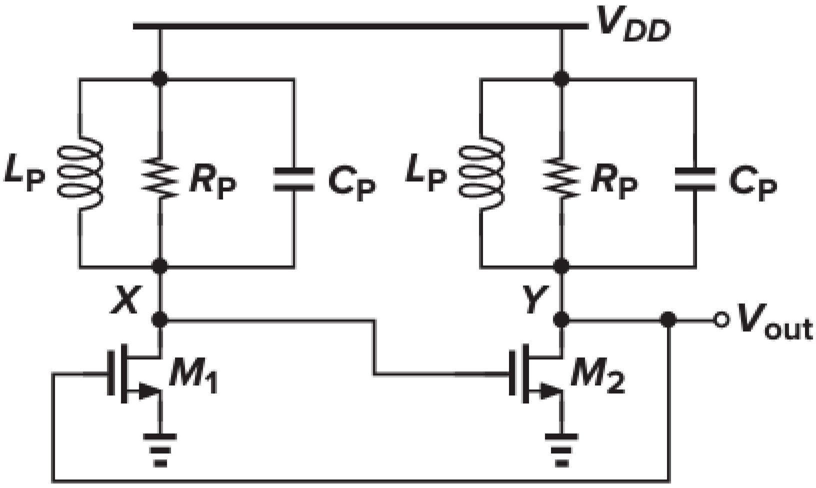
- So now, the resonance frequency is: $f = \frac{1}{2*pi*\sqrt{(L_P C_P)}}$.
- We have observed that the above circuit has frequency-depended impedance. What happen if we apply it to an amplifier:



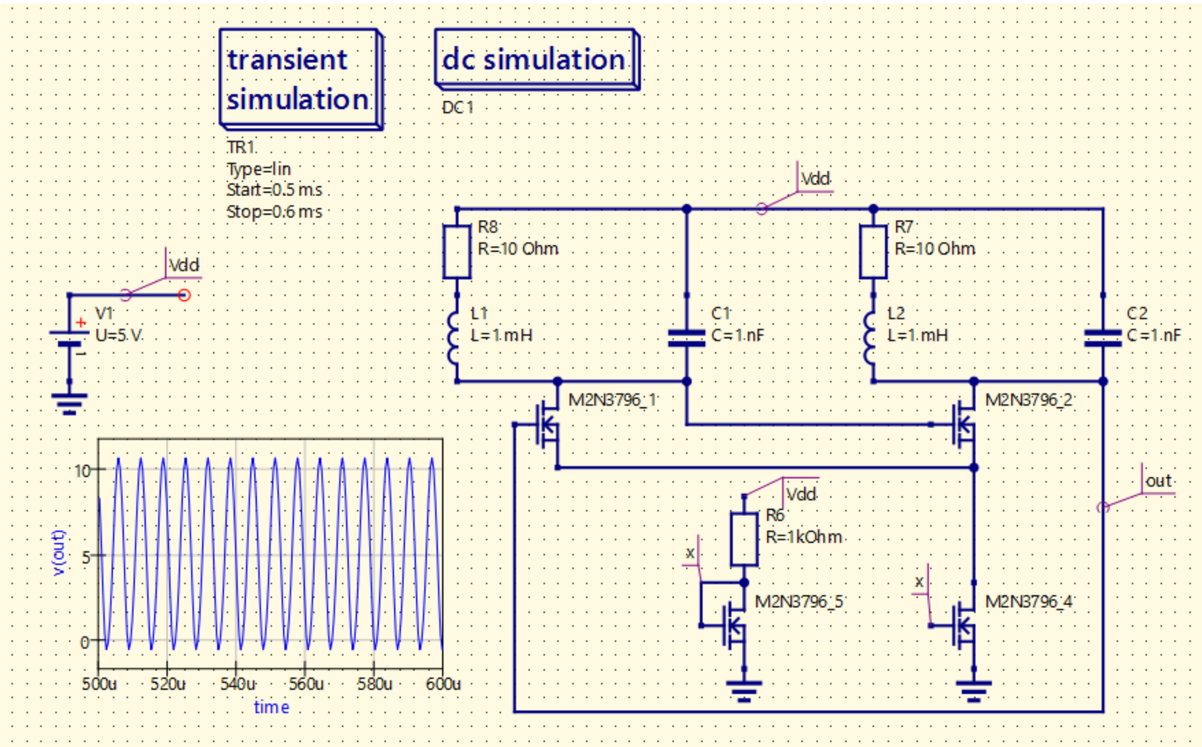
- It is called: the tuned gain oscillator, because the gain is changed while the frequency changes. And the gain is highest at resonance frequency. Let look at the fact of this configuration. If we apply an input that have average value near VDD, the output voltage at Vout varies, and hence the current. But the inductor tends to keep the current changes slowly. So if the current is reduced, the inductor will forced the voltage at Vout higher than VDD (to resist the transistor current changes). So that the output voltage can be higher than the DC power source:



- Let look at two tuned-gain stages in feedback loop:



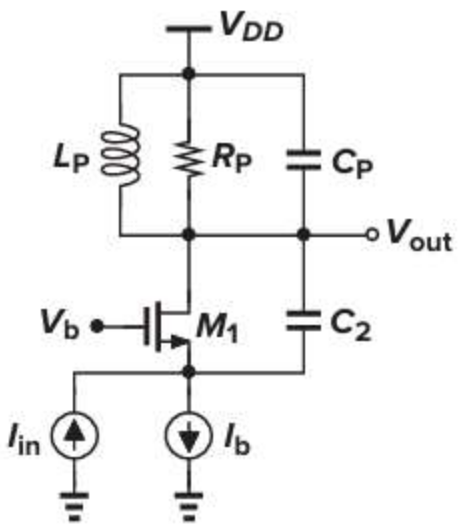
- The condition to oscillate is the gain of the overall circuit is higher than 1:
  $$(g_{m1,2}R_P)^2 \geq 1$$
- This circuit faces a problem: the frequency resonance is supply sensitivity. That is because the parasitic capacitor at the drain is changed by the supply capacitor changes.
- Finally, we construct an example. The software in used is QUCS-S:



- The frequency is about 154kHz. The theoretical frequency is about 159,2 kHz. You can see the maximum output voltage is larger than 5V (about 10 V), that is the characteristic of using LC circuit as load. The bias current is formed in term of current mirror. Now let look at the next type of oscillator.

3. Colpitts oscillator

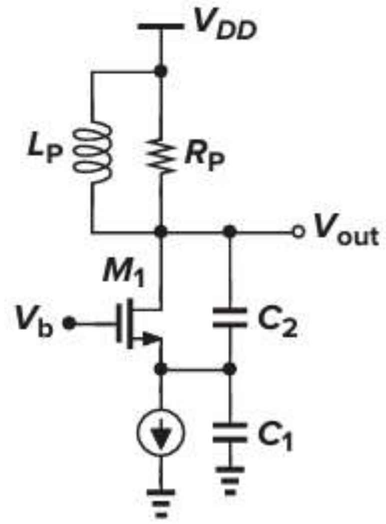
- Now, turn back to the tuned-gain oscillator. It cannot oscillate, because the phase shift is just 180 degrees. But note that the drain and the source are in phase. So we surmise that if we connect the drain and the source together, it will work:



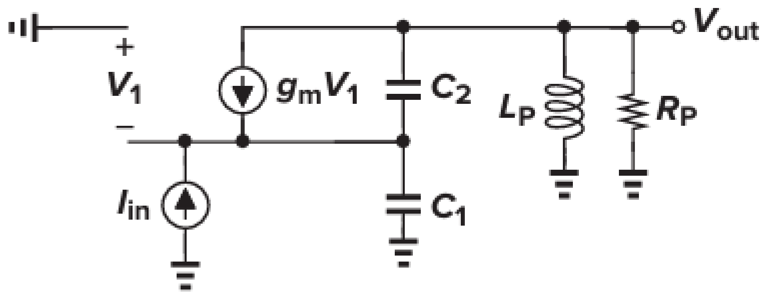
- Why do we apply a current source as input? If we apply voltage source, due to the ideal current biasing Ib, hence Vout doesn't change with voltage source. Now we has the closed-loop gain:

$$\frac{V_{out}}{I_{in}} = L_P s || \frac{1}{C_P s} || R_P$$

- We need infinite closed-loop gain, and it isn't sastified.
- Due to the infinite impedance that look from source of M1 to ground, the opened-loop gain is zero. So let connect a capacitor from the source to the ground to seek the condition od oscillation.



- Here, C1 is connected from the source to the ground, let find the gain of this voltage. The capacitor of the load is removed, we will consider it later. Now we need the equivalent small signal model:



- The gain can be derived:

$$\frac{V_{out}}{I_{in}} = \frac{R_P L_P s(g_m + C_2 s)}{R_P C_1 C_2 L_P s^3 + (C_1 + C_2) L_P s^2 + [g_m L_P + R_P(C_1 + C_2)]s + g_m R_P}$$

- For infinite closed-loop gain, we need the denominator becomes zero, hence:

$$-R_P C_1 C_2 L_P \omega_R^3 + [g_m L_P + R_P(C_1 + C_2)]\omega_R = 0$$

$$-(C_1 + C_2) L_P \omega_R^2 + g_m R_P = 0$$

- (This is cruatial, I think that this came from the experiences). The author assumes that:

$$g_m L_P \ll R_P(C_1 + C_2)$$

- The resonance frequency can be derived:

$$\omega_R^2 = \frac{1}{L_P \dfrac{C_1 C_2}{C_1 + C_2}}$$

- And the ratio of C1 and C2 also can be derived:

$$g_m R_P = \frac{(C_1 + C_2)^2}{C_1 C_2} = \frac{C_1}{C_2}(1 + \frac{C_2}{C_1})^2$$

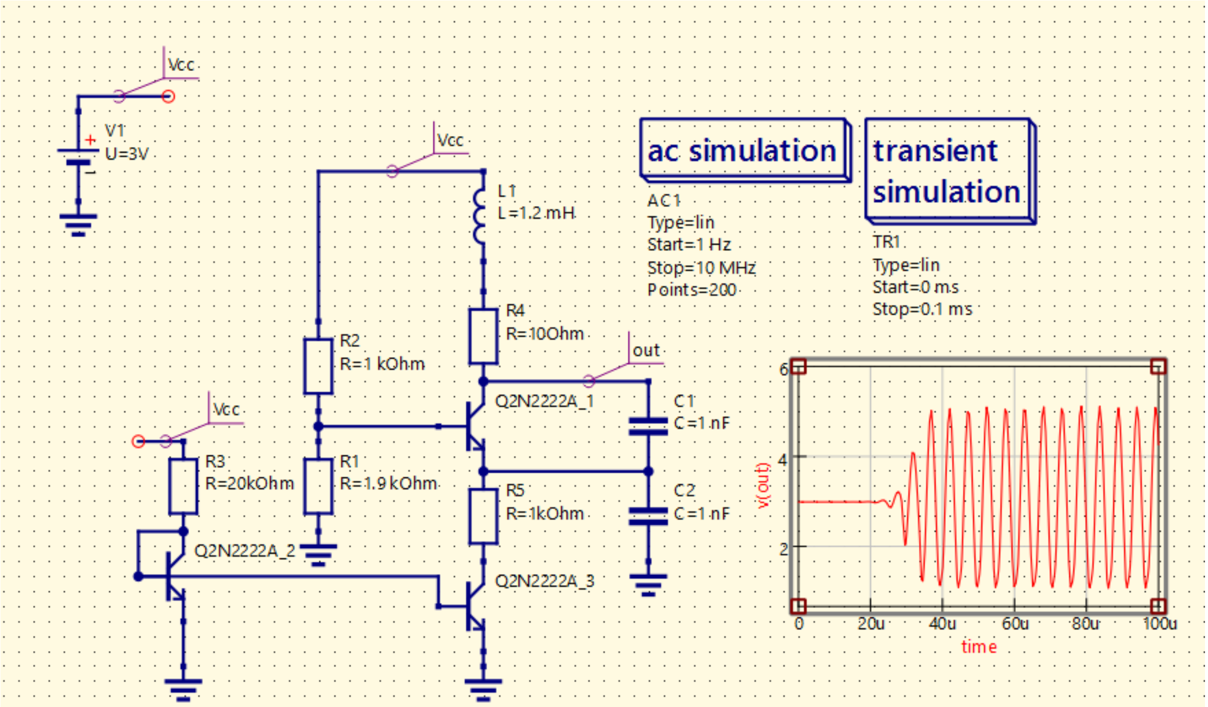
- You can prove that:

$$g_m R_P \geq 4$$

- It occurs at C1/C2 = 1. The higher Rp means higher Q-factor of inductor, so it is the limitation of this configuration. In CMOS tech, inductor is usually low in Q-factor, so the cross-coupled oscillator is preferred.
- What happen if we keep the capacitor at the load? The frequency will be changed:

$$\omega_R^2 = \frac{1}{L_P(C_P + \frac{C_1 C_2}{C_1 + C_2})}$$

- Now let construct an Colpitts oscillator. I use QUCS-S to simulate:



- The theoretical frequency is about 205.572 kHz. The simulation frequency is about 195 kHz.