Homework 2

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Problem 4.1

It is ***true*** that an FID signal lasts as long as the transverse magnetization.

Problem 4.2

For a spin system with N spectral components at their corresponding individual frequencies, the spectral density function $\rho(\omega)$ is a combination of N delta functions of frequencies:

$$\rho(\omega) = \sum_{k=1}^{N} M_{z,k}^{0}\delta(\omega - \omega_k)$$

In addition, an FID signal resulting from an α pulse is mathematically presented by:

$$S(t) = \sin\alpha \int_{-\infty}^{\infty} \rho(\omega) e^{-t/T_2(\omega)} e^{-j\omega t} d\omega$$

Thus, an FID signal with N spectral components can be derived as follows:

$$S(t) = \sin\alpha \int_{-\infty}^{\infty} \sum_{k=1}^{N} M_{z,k}^{0}\delta(\omega - \omega_k) e^{-t/T_2(\omega)} e^{-j\omega t} d\omega$$

$$\Rightarrow S(t) = \sin\alpha \sum_{k=1}^{N} \left(M_{z,k}^{0} \int_{-\infty}^{\infty} \delta(\omega - \omega_k) e^{-t/T_2(\omega)} e^{-j\omega t} d\omega \right)$$

For any function $f(x)$ defined within $[a, b]$, we consider the following characteristic

$$\int_{a}^{b} \delta(x - x_0) f(x) dx = f(x_0), \ \forall x_0 \in [a, b]$$

Hence, with any $k \in [1, N]$, we easily see that

$$\int_{-\infty}^{\infty} \delta(\omega - \omega_k) e^{-t/T_2(\omega)} e^{-j\omega t} d\omega = e^{-t/T_2(\omega_k)} e^{-j\omega_k t}$$

Therefore, we finally obtain the expression of a FID with N spectral components,

$$S(t) = \sin\alpha \sum_{k=1}^{N} M_{z,k}^{0} e^{-t/T_2(\omega_k)} e^{-j\omega_k t}$$

Problem 4.11

a Let's denote M_z^0 as the bulk magnetization at thermal equilibrium. So, we have,

$$\begin{cases} \text{the process } \vec{M}_{rot}(0_-) \xrightarrow{90^o_{x'}} \vec{M}_{rot}(0_+) \xrightarrow{90^o_{y'}} \vec{M}_{rot}(0_{++}) \\ \text{the pre-pulse condition } \vec{M}_{rot}(0_-) = [0, 0, M_z^0]^T \end{cases}$$

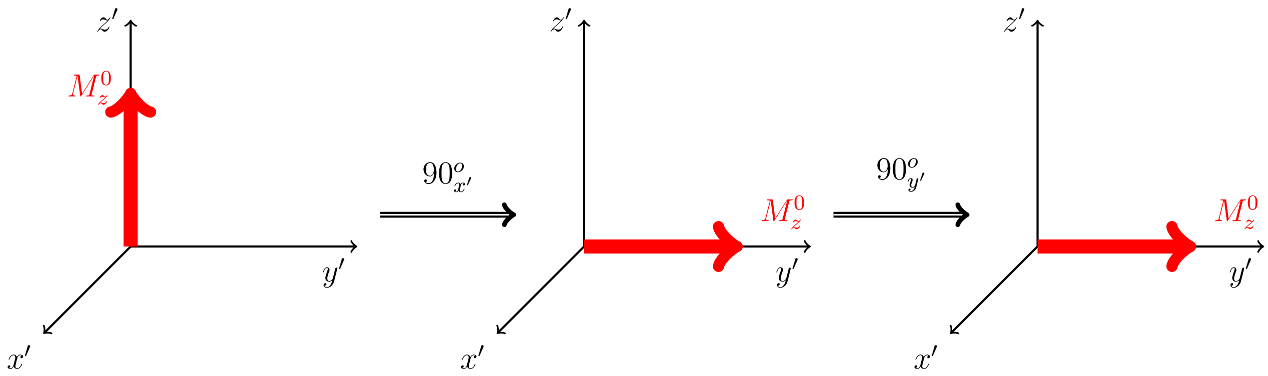
•Calculation of $\vec{M}_{rot}$ after $90^o_{x'}$-pulse: $\vec{M}_{rot}(0_+) = R_{x'}(90^o)\vec{M}_{rot}(0_-)$

$$\Rightarrow \vec{M}_{rot}(0_+) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 90^o & \sin 90^o \\ 0 & -\sin 90^o & \cos 90^o \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ M_z^0 \end{bmatrix} = \begin{bmatrix} 0 \\ M_z^0 \\ 0 \end{bmatrix}$$

•Calculation of $\vec{M}_{rot}$ after $90^o_{y'}$-pulse: $\vec{M}_{rot}(0_{++}) = R_{y'}(90^o)\vec{M}_{rot}(0_+)$

$$\Rightarrow \vec{M}_{rot}(0_{++}) = \begin{bmatrix} \cos 90^o & 0 & -\sin 90^o \\ 0 & 1 & 0 \\ \sin 90^o & 0 & \cos 90^o \end{bmatrix} \begin{bmatrix} 0 \\ M_z^0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ M_z^0 \\ 0 \end{bmatrix}$$

•Sketch the bulk magnetization $\vec{M}_{rot}(t)$ after the two pulses



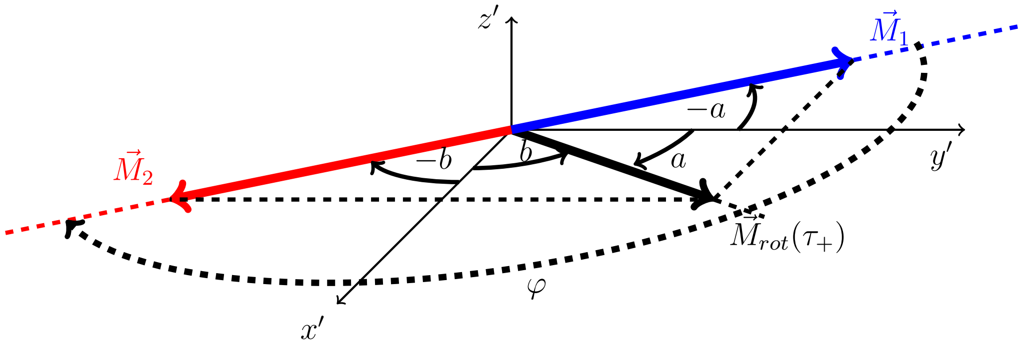
(b) From the sketch, because the signal has only one single spectral component, it is obvious that the FID signals $S_1(t)$ and $S_2(t)$ generated by the two pulses are ***the same***.

Problem 4.12 a

Let's denote the two given processes as follows:

$$\begin{cases} \text{Process 1: } \vec{M}_{rot}(0_-) \xrightarrow{90^o_{x'}} \vec{M}_{rot}(\tau_-) \xrightarrow{\tau} \vec{M}_{rot}(\tau_+) \xrightarrow{180^o_{y'}} \vec{M}_1 \\ \text{Process 2: } \vec{M}_{rot}(0_-) \xrightarrow{90^o_{x'}} \vec{M}_{rot}(\tau_-) \xrightarrow{\tau} \vec{M}_{rot}(\tau_+) \xrightarrow{180^o_{x'}} \vec{M}_2 \end{cases}$$

The sketch of the bulk magnetization vector from the time point τ_+ onwards,



It can be seen that the two echo signals resulted from $\vec{M}_1$ and $\vec{M}_2$ should be equal in their magnitude, and have 180^o phase difference. To illustrate, lets consider some angle coordination as in the sketch, then $\varphi = -(-a) + a - b + (-b) = 2(a-b) = 2(-\pi/2) = -\pi$ or 180^o.

Problem 4.19

(a) $B_z(x,y,z) = 3 - 2x \Rightarrow \vec{\nabla} B_z = \dfrac{\partial(3-2x)}{\partial x}\vec{i} + \dfrac{\partial(3-2x)}{\partial y}\vec{j} + \dfrac{\partial(3-2x)}{\partial z}\vec{z} = -2\vec{i}$
$\Rightarrow$ A linear x-gradient field with $G_x = -2$.

(b) $B_z(x,y,z) = 3-2x+x^2 \Rightarrow \vec{\nabla} B_z = \dfrac{\partial(3-2x+x^2)}{\partial x}\vec{i} + \dfrac{\partial(3-2x+x^2)}{\partial y}\vec{j} + \dfrac{\partial(3-2x+x^2)}{\partial z}\vec{z}$
$\Rightarrow \vec{\nabla} B_z = (-2+2x)\vec{i} \Rightarrow$ Not a linear gradient field.

(c) $B_z(x,y,z) = 5 - x - y - z \Rightarrow \vec{\nabla} B_z = -\vec{i} - \vec{j} - \vec{k} \Rightarrow$ A linear gradient field.

Problem 5.1

It is ***false*** that selection of an envelope function for an RF pulse has nothing to do with the $\vec{B}_0$ field strength. Because the sharp of RF envelope function together with the inhomogeneity of the $\vec{B}_0$ field will affect the number of selective spectral components (or the scaling interval of equally two-sided rectangular function in frequency domain) in the transverse magnetization vector (the solution of Bloch equation in which both $\vec{B}_0$ and $\vec{B}_1$ participate) as well as in the selection accuracy of expected parts of an object.

Problem 5.4

In the slice-selective excitation, the ***cross-talk*** artifacts means the unexpected excitation of spins in the neighboring slices to some varying degrees, which is due to the truncation of RF-pulse (spatially support-limited time-domain signal impacts on entire frequency-domain).