Dimensionality Reduction

Question 1: Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is [2/7,3/7,6/7], and another is [6/7, 2/7, -3/7]. Let the third column be [x,y,z]. Since the length of the vector [x,y,z] must be 1, there is a constraint that $x^2+y^2+z^2 = 1$. However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x, y, and z. Compute these ratios.

1) let c_1 be $[2/7, 3/7, 6/7]$ c_2 be $[6/7, 2/3, -3/7]$ and c_3 be $[x, y, z]$

The dot product of any two columns must be zero

$c_1 c_2 = (2/7 * 6/7) + (3/7 * 2/7) + (6/7 * -3/7) = 0$

$c_2 . c_3 = (6/7 * x) + (2/7 * y) + (-3/7 * z) = 0 \Rightarrow 6x+2y-3z=0 \rightarrow (1)$

$c_3 . c_1 = (x * 2/7) + (y * 3/7) + (z * 6/7) = 0 \Rightarrow 2x+3y+6z=0 \rightarrow (2)$

(1) + (2) $\Rightarrow 12x+4y-6z+2x+3y+6z=0 \Rightarrow 14x+7y=0 \Rightarrow y=2x.$

(2) − (1) $\Rightarrow 6x+9y+18z-6x-2y+3z=0 \Rightarrow 7y+21z=0$

$\Rightarrow y=-3z$

$\boxed{x:y:z = -2:1:-3}$

Question 2: Find the eigenvalues and eigenvectors of the following matrix:

2	3
3	10

You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue and One eigenvector.

2) let the given matrix be $A = \begin{matrix} 2 & 3 \\ 3 & 10 \end{matrix}$ and the eigen vector be of the form $\frac{1}{e}$

$Ax = \lambda x \rightarrow \begin{matrix} 2 & 3 \\ 3 & 10 \end{matrix} = \frac{1}{e} = \lambda \times \frac{1}{e} \rightarrow 2+3e = \lambda$ and

$3+10e = \lambda e \rightarrow 3+10e = (2+3e)e$

$3e^2 - 8e + 3 = 0 \Rightarrow e = 3, -1/3$

The eigen vector are $1/3$ and -3

The eigen values are $2+3e = \lambda \rightarrow \lambda = 2+3*3 = 11$ and

$\lambda = 2+3(-\frac{1}{3}) = 1.$

Question 3: Suppose [1,3,4,5,7] is an eigenvector of some matrix. What is the unit eigenvector in the same direction? Find out the components of the unit eigenvector.

3) Given the eigen vector g some matrix be

$$M = [1,3,4,5,7]$$

To get the unit eigen vector of given matrix, we need to divide each component by square root of sum of squares in the same direction

sum of squares $= 1^2+3^2+4^2+5^2+7^2 = 100$ and its square root is 10.

unit eigen vector $= [1/10, 3/10, 4/10, 5/10, 7/10]$

Question 4: Suppose we have three points in a two dimensional space: (1,1), (2,2), and (3,4). We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N, whose eigenvectors are the directions that best represent these three points. Construct the matrix N and identify, its elements.

4) The given three points in a 2-D space are

(1,1) (2,2) (3,4)

we should construct matrix whole rows correspond to. Points and columns correspond to dimensions of the

Then the given matrix will be,

$$a = \begin{matrix} 1 & 1 \\ 2 & 2 \\ 3 & 4 \end{matrix} \qquad a^T \cdot a = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 17 \\ 17 & 21 \end{bmatrix}$$

Question 5: Consider the diagonal matrix M =

1	0	0
0	2	0
0	0	0

Compute its Moore-Penrose pseudoinverse.

5) moore-penrose pseudo inverse means the matrix having diagonal elements replaced by 1 and divided by corresponding elements of given matrix and the other elements will be zero.

moore - penrose pseudo inverse of given matrix.

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 0 \end{matrix}$$

Question 6: When we perform a CUR dcomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose:

1	2	3
4	5	6
7	8	9
10	11	12

Calculate the probability distribution for the rows.

$0 \quad 0 \quad 0$

6) Probability with which we choose row

$$= \frac{\text{sum of square of element in row}}{\text{sum of square of elements in the matrix}}$$

sum of square of elements in the matrix $= 12 * 13 * 25/6 = 3900/6$

$= 650$

$$P(R_1) = \frac{1^2+2^2+3^2}{650} = 14/650 = 0.02$$

$$P(R_2) = \frac{4^2+5^2+6^2}{650} = 77/650 = 0.12$$

$$P(R_3) = \frac{7^2+8^2+9^2}{650} = 194/650 = 0.298$$

$$P(R_4) = \frac{10^2+11^2+12^2}{650} = \frac{365}{650} = 0.56.$$