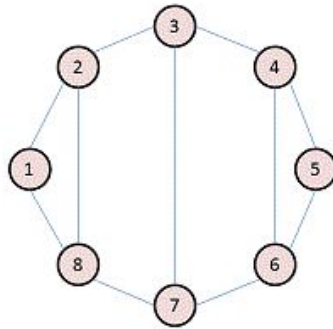


# Communities

## Question 1:

For the following graph:



Write the adjacency matrix  $A$ , the degree matrix  $D$ , and the Laplacian matrix  $L$ . For each, find the sum of all entries and the number of nonzero entries.

i) Adjacency matrix:-

	1	2	3	4	5	6	7	8
1	0	1	0	0	0	0	0	1
2	1	0	1	0	0	0	0	1
3	0	1	0	1	0	0	1	0
4	0	0	1	0	1	1	0	0
5	0	0	0	1	1	0	0	0
6	0	0	0	1	1	0	1	0
7	0	0	1	0	0	1	0	1
8	1	1	0	0	0	0	1	0

no. of non zero entries = 22

sum of all entries = 22.

Degree matrix:-

	1	2	3	4	5	6	7	8
1	2	0	0	0	0	0	0	0
2	0	3	0	0	0	0	0	0
3	0	0	3	0	0	0	0	0
4	0	0	0	3	0	0	0	0
5	0	0	0	0	2	0	0	0
6	0	0	0	0	0	3	0	0
7	0	0	0	0	0	0	3	0
8	0	0	0	0	0	0	0	3

no of non-zero entries = 8

sum of all entries = 8

Laplacian matrix  $\therefore (L = D - A)$

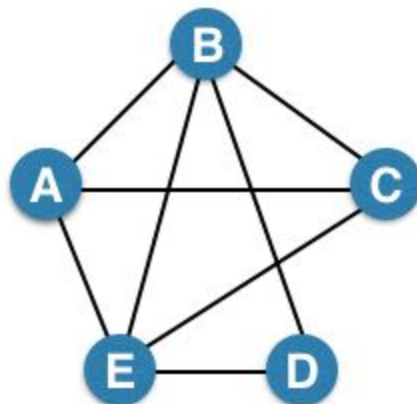
	1	2	3	4	5	6	7	8
1	2	-1	0	0	0	0	0	-1
2	-1	3	-1	0	0	0	0	-1
3	0	-1	3	-1	0	0	-1	0
4	0	0	-1	3	-1	-1	0	0
5	0	0	0	-1	2	-1	0	0
6	0	0	0	-1	-1	3	-1	0
7	0	0	-1	0	0	-1	3	-1
8	-1	-1	0	0	0	0	-1	3

no. of non-zero entries = 30

sum of all entries = 0.

**Question 2:**

Consider the following undirected graph (i.e., edges may be considered bidirectional):



Run the "trawling" algorithm for finding dense communities on this graph and find all complete bipartite subgraphs of types  $K_{3,2}$  and  $K_{2,2}$ . Note: In the case of  $K_{2,2}$ , we consider  $\{\{W, X\}, \{Y, Z\}\}$  and  $\{\{Y, Z\}, \{W, X\}\}$  to be identical.

2) from the given graph :-

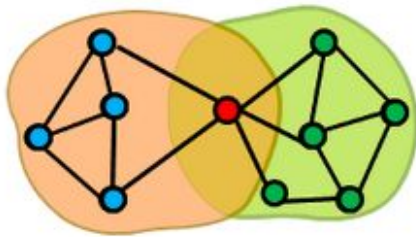
$A = \{B, C, E\}$      $B = \{A, C, D, E\}$      $C = \{A, B, E\}$   
 $D = \{B, E\}$      $E = \{A, B, C, D\}$

So B and E are having support more than 3.  
 $\therefore$  Bipartite subgraph of  $K_{3,2}$

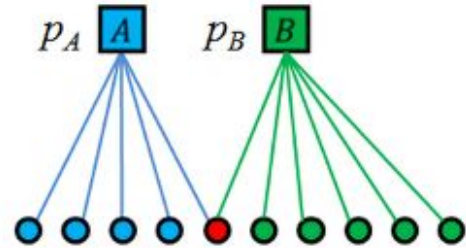
Bipartite subgraph of  $K_{2,2}$  :

**Question 3:**

We fit AGM to the network on the left, and found the parameters on the right:



Network



Learned AGM parameters

Find the optimal values for  $p_A$  and  $p_B$ .

$$\begin{aligned} \text{a) } p_A &= \frac{\text{no. of edges in the network}}{\text{Total possible no. of edges}} \\ &= \frac{7}{10} = 0.7 \end{aligned}$$

$$\begin{aligned} \text{b) } p_B &= \frac{\text{no. of edges in the network}}{\text{Total possible no. of edges}} \\ &= \frac{9}{15} \\ &= 0.6 \end{aligned}$$