

MapReduce and PageRank

Question 1:

Suppose our input data to a map-reduce operation consists of integer values (the keys are not important). The map function takes an integer i and produces the list of pairs (p,i) such that p is a prime divisor of i . For example, $\text{map}(12) = [(2,12),(3,12)]$.

The reduce function is addition. That is, $\text{reduce}(p,[i_1,i_2,\dots,i_k])$ is $(p,i_1+i_2+\dots+i_k)$.

Compute the output, if the input is the set of integers 15, 21, 24, 30, 49.

Answer: The output of map function is

$\text{map}(15) = [(3,15),(5,15)]$

$\text{map}(21) = [(3,21),(7,21)]$

$\text{map}(24) = [(2,24),(3,24)]$

$\text{map}(30) = [(2,30),(3,30),(5,30)]$

$\text{map}(49) = [(7,49)]$

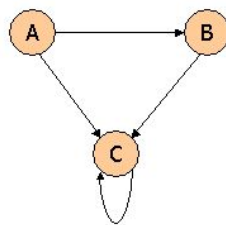
There are the respective prime divisors of inputs

The output reduction function is

$\text{reduce}(2,54)$, $\text{reduce}(3,90)$, $\text{reduce}(5,45)$ and $\text{reduce}(7,70)$.

Question 2:

Consider three Web pages with the following links:



Suppose we compute PageRank with a β of 0.7, and we introduce the additional constraint that the sum of the PageRanks of the three pages must be 3, to handle the problem that otherwise any multiple of a solution will also be a solution. Compute the PageRanks a , b , and c of the three pages A, B, and C, respectively.

$$2) \text{ for } a = \beta(a) + (1-\beta)$$

$$b = \beta(a/2) + (1-\beta)$$

$$c = \beta(a/2 + b + c) + (1-\beta)$$

$$\text{Here } \beta = 0.7, a+b+c = 3$$

$$a = 0.7(0) + (1-0.7) = 0.3$$

$$b = 0.7(0.3/2) + (1-0.7) = 0.405$$

$$\begin{aligned} c &= 0.7(0.3/2 + 0.405 + c) + 1.07 \\ &= 0.405 + 0.7(0.405) + (0.7)(c) \end{aligned}$$

$$c = 1.7(0.405) + 0.7(c)$$

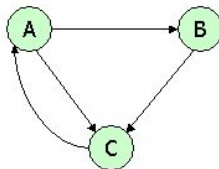
$$\Rightarrow (1.07)c = 1.7(0.405)$$

$$0.3c = 0.6885$$

$$c = \frac{0.6885}{0.3}$$

$$\boxed{c = 2.295}$$

Question 3:



Suppose we compute PageRank with $\beta=0.85$. Write the equations for the PageRanks a , b , and c of the three pages A, B, and C, respectively.

3) Here $\beta = 0.85$

$$a = 0.85 + c + (1 - 0.85) \frac{1}{3}$$

$$a = 0.85c + 0.05$$

$$b = 0.85 \times 0.5 \times a + 0.05$$

$$b = 0.425a + 0.05$$

$$c = 0.85 \times (0.5 \times (a + b)) + 0.05$$

$$= 0.425a + 0.85 + 0.05$$

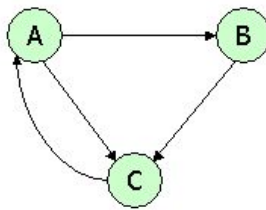
Formulae :-

$$a = \beta \times (c + (1 - \beta) \frac{1}{3})$$

$$b = \beta \times (a/2 + (1 - \beta) \frac{1}{3})$$

$$c = \beta \times (a/2 + b) + (1 - \beta) \frac{1}{3}$$

Question 4:



Assuming no "taxation," compute the PageRanks a , b , and c of the three pages A, B, and C, using iteration, starting with the "0th" iteration where all three pages have rank $a = b = c = 1$. Compute as far as the 5th iteration, and also determine what the PageRanks are in the limit.

4) formula :

$$a=c \quad b=a/2 \quad c=a/2+b$$

0th iteration :

$$a=1 \quad b=1 \quad c=1$$

1st iteration :

$$a=1 \quad b=1/2 \quad c=1/2+1=3/2$$

2nd iteration :

$$a=c=3/2 \quad ; \quad b=a/2=1/2 \quad c=1/2+3/2=2$$

3rd iteration :

$$a=c=2 \quad ; \quad b=a/2=1 \quad c=1+2=3$$

4th iteration :

$$a=c=3 \quad b=a/2=3/2 \quad c=3/2+3=9/2$$