## **Dimensionality Reduction**

**Question 1**: Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is [2/7,3/7,6/7], and another is [6/7,2/7,-3/7]. Let the third column be [x,y,z]. Since the length of the vector [x,y,z] must be 1, there is a constraint that  $x^2+y^2+z^2=1$ . However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x, y, and z. Compute these ratios.

1) Oct c, be 
$$(\frac{2}{7}, \frac{3}{7}, \frac{6}{7})$$
 cz be  $(\frac{6}{7}, \frac{2}{3}, -\frac{3}{7})$  and c3 be(x.y.3)

The dot product of any two columns must be zero

 $c_{1}(2) \cdot (\frac{3}{7} + \frac{6}{7}) + (\frac{3}{7} \times \frac{1}{7}) + (\frac{6}{7} \times -\frac{3}{7}) = 0$ 
 $c_{2} \cdot 6_{3} = (\frac{6}{7} \times 1) + (\frac{1}{7} \times 1) + (\frac{3}{7} \times 2) = 0 \Rightarrow 6x + 2y - 32 = 0 \rightarrow 0$ 
 $c_{3} \cdot c_{1} = (x \times \frac{1}{7}) + (y \times \frac{3}{7}) + (3 \times \frac{4}{7}) = 0 \Rightarrow 2x + 3y + 63 = 0 \rightarrow 0$ 
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 $c_{3} \cdot c_{1} = (x \times \frac{1}{7}) + (x$ 

**Question 2**: Find the eigenvalues and eigenvectors of the following matrix:

2	2	3
3	3	10

You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue

and One eigenvector.

2) let the given matrix be 
$$A = \frac{2}{3}$$
 and the eigenvector. be  $g$  the form  $\frac{1}{e^{-}}$ 

$$Ax = \lambda \pi + \frac{2}{3} + \frac{3}{3} = \frac{1}{e^{-}} = \lambda \times \frac{1}{e^{-}} \rightarrow 2+3e^{-}\lambda \text{ and}$$

$$3+3e^{-} = \lambda = \frac{1}{3} + 10e = \frac{1}{3} + \frac{1}{3} = \frac{1}{3} + \frac{1}{3} = \frac{1}{3}$$

**Question 3**: Suppose [1,3,4,5,7] is an eigenvector of some matrix. What is the unit eigenvector in the same direction? Find out the components of the unit eigenvector.

3) Given the eigen vector of some matrix be M = [1,3,4,5,7]To get the unit eigen vector of given matrix. we need to divide each component by square most of sum of squares in the same direction

Sum of squares =  $1^2+3^2+4^2+5^2+9^2=100$  and its square most is so unit (igen vector = [1/10,3/10,4/10,5/10,7/10]

**Question 4**: Suppose we have three points in a two dimensional space: (1,1), (2,2), and (3,4). We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N, whose eigenvectors are the directions that best represent these three points. Construct the matrix N and identify, its elements.

H) The given three points in a 2-D space are

(1,1) (2.2) (3.34)

We should construct matrix whole nows convergend to.

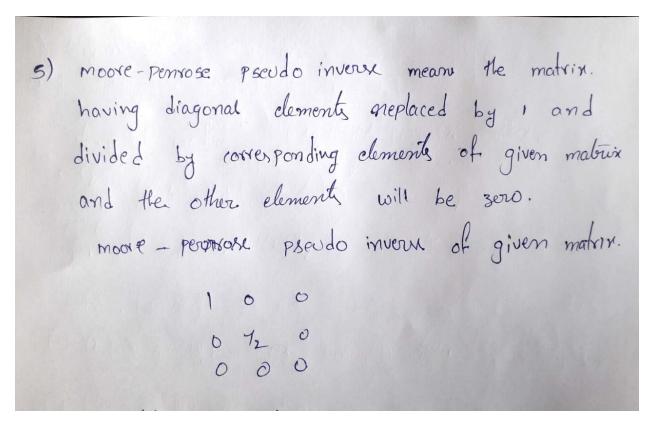
Ports and column correspond to dimensions of the then the given matrix will be.

$$a = \frac{1}{2} \frac{1}{2}$$
 $a = \frac{12}{3} \frac{3}{4}$ 
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Question 5: Consider the diagonal matrix M =

1	0	0
0	2	0
0	0	0

Compute its Moore-Penrose pseudoinverse.



**Question 6**: When we perform a CUR dcomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose:

1	2	3
4	5	6
7	8	9
10	11	12

Calculate the probability distribution for the rows.

6) Probability with which we choose vow . som of square of element in rou

50m of 3quare of element in you som of square of elements in the matrix = 12 \* B \* 25/6 = 3900/6

$$P(R_1) = \frac{1^2 + 2^2 + 3^2}{650} = \frac{14}{650} = 0.62$$

$$P(R_3) = \frac{9^2 + 8^2 + 9^2}{650} = \frac{194}{650} = 0.298$$

$$P(R_{H}) = \frac{10^{2} + 11^{2} + 12^{2}}{650} = 0.56.$$