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A 2D Engineering Drawing and 3D Model Matching Algorithm for Process Plant

Rui Wen
*School of Computer Science and Engineering,
Nanjing University of Science and Technology,
Nanjing, China;
wenruity@163.com*

Weiqing Tang
*Institute of Computing Technology,
Chinese Academy of Sciences,
Beijing, China*

Zhiyong Su
*School of Automation,
Nanjing University of Science and Technology,
Nanjing, China*

***Abstract*—Process plant Computer-Aided Design (CAD) models can be roughly divided into 3D models and 2D engineering drawings. Matching calculation of these CAD models not only contributes to their coherency verification, but also benefits for the development of model retrieval. However, in process plant area, 3D models and 2D engineering drawings are different in both graphical representations and content structures, which leads to the inapplicability of current shape-feature based 2D & 3D matching approaches. To resolve this problem, a topological structure based algorithm is proposed. Exploiting component as the basic unit, we firstly transform 2D engineering drawings and 3D models into attribute graphs. Then, by introducing related researches of graph similarity, we calculate attribute graph similarity to measure the matching degree between their corresponding CAD models. The proposed algorithm is translation, rotation and similarity transformation consistent. Experimental results demonstrate its effectiveness and feasibility.**

***Keywords*- process plant; 3D model; 2D engineering drawing; matching calculation; graph similarity**

I. INTRODUCTION

Process plant Computer-Aided Design (CAD) models are mainly divided into 3D models and 2D engineering drawings. In engineering design stage, the graphical representations of process plant CAD models are 3D wireframes or entities. Once design is completed, their graphical representations are various 2D engineering drawings such as ISOmetric projection (ISO) drawing, pipeline plane drawing, section drawing, etc., which then serve as a clear and precise guidance for construction [1].

With the expanding investment in process plant, the number of 3D models and 2D engineering drawings is exploding, which brings out difficulties in the retrieval of existing CAD models. A fast and effective model retrieval technique is beneficial to model management and reuse, which is of great significance for improving design efficiency and ensuring design quality. However, due to the difficulty of an accurate model text acquisition and lacking of standardization, a file-name based model retrieval system can no longer meet enterprise-level needs.

Comparatively, through directly exploiting models' internal characteristics, content-based model retrieval technique serves as a useful tool for effectively and precisely locating existing models. On one hand, an accurate matching calculation between 2D engineering drawing and 3D model plays a crucial role in coherency verification. On the other hand, retrieving the relevant 3D model (or 2D engineering drawing) according to 2D engineering drawing (or 3D model) also benefits the management and reuse of CAD models. The key point in tackling these problems lies in 2D engineering drawing and 3D model matching algorithm.

Currently, existing 2D drawing and 3D model matching researches are concentrated on general 3D models [2] and product CAD models [3,4]. These researches focus largely on the shape feature similarity measurement and can be summarized into three steps [5]:

(1) Get a projection drawing of the 3D model under the same viewpoint with the compared 2D drawing;

(2) Extract shape features of the 2D drawing and projection drawing;

(3) Calculate the similarity between their 2D shape features to measure the matching degree between the compared 2D drawing and 3D model.

However, because different kinds of 2D engineering drawings and 3D models derived from the same project have differences in graphical representations and content structures, shape-feature based approaches are inapplicable in process plant CAD models. That is:

- The graphical representations of components are determined by engineering design standards. For instance, in 2D engineering drawings, a component's geometry is not the direct projection of its real 3D shape, but the icon conforming to engineering design standards [6];
- Some kinds of 2D engineering drawings are not drawn proportionally. They only need to keep the original topology intact, such as ISO drawing and Piping & Instrumentation Diagram (P&ID);
- Different kinds of CAD models derived from the same project may also vary in content structures. For

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example, an ISO drawing describes the partial structure of a process plant, but a plane drawing describes its global structure under the certain viewpoint.

Therefore, in process plant area, transforming a 3D model into a 2D engineering drawing requires a series of operations such as replacement and non-scale transformation, rather than a simple projection under a certain viewpoint. This makes the current shape-feature based 2D & 3D matching approaches inapplicable.

Because process plant design concentrates on the accurate topology expression of a process plant, its 2D engineering drawing and 3D model matching algorithm should focus on the similarity measure of topological structures. Process plant CAD models are parametric data structures [7]. Components in 2D engineering drawings and 3D models are all marked with engineering attributes. Because components' attributes and topological relations remain the same in either form, the matching calculation of these heterogeneous CAD models can use the component as a basic unit and base on its topological structures.

Regarding structured process plant CAD models as attribute graphs, we propose a topology based 2D engineering drawing and 3D model matching algorithm. Firstly, the proposed method introduces a preprocessing procedure to transform 2D engineering drawings and 3D models into attribute graphs; then attribute graph similarity is calculated to measure the matching degree between their corresponding CAD models. Due to the exploitation of topological structures, our proposed algorithm embraces an ideal property of translation, rotation and similarity transformation consistency.

II. Preliminaries

Process plant is the set of reaction vessels, pipelines and supports which are materials for making chemical or physical manufactured products. We will first give a brief introduction to 3D model and 2D engineering drawing for process plant.

A. 3D model

In process plant, 3D models are capable of describing the topological relations between different components, as well as some design constraints. The ultimate goal of constructing 3D models is to generate various 2D engineering drawings for constructional guidance.

Fig.2 gives three 3D model instances. A typical process plant 3D model mainly consists of thousands of basic components, including equipment, pipelines (i.e. pipes and piping components), valves, instruments, etc. Components are comprised of fourteen basic entities as shown in Fig.1, which are cylinder, scylinder, prism, econe, concone, squcir, squcone, box, torus, squtorus, sphere, wedge, saddle and oval.

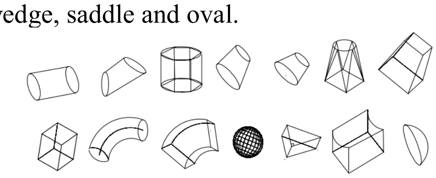
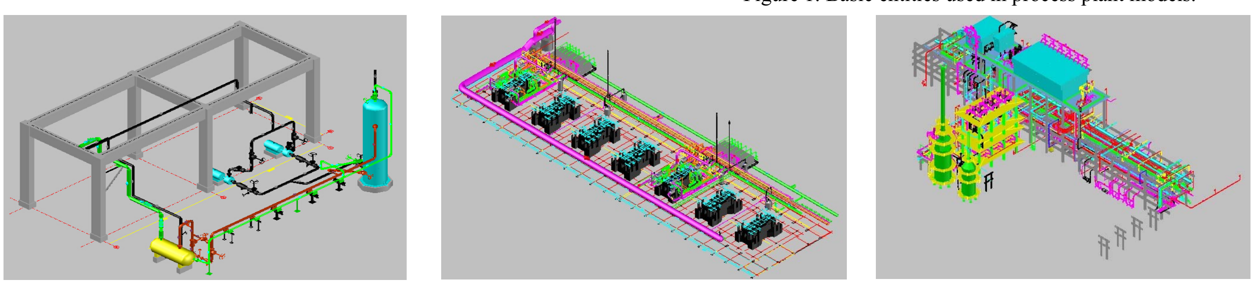


Figure 1. Basic entities used in process plant models.



(a) (b) (c)

Figure 2. 3D process plant models.

Process plant design emphasizes on the structural and topological information of targets, demanding model's capability to precisely describe every topological structure. State-of-the-art topology representations include the dual point and smart line based method [8]. In this method, dual points attempt to describe component-wise topologies; smart lines seek to build up the relational constraints between pipes and pipelines by abstracting them as 3D segments and curves. A simple example is illustrated in Fig.3, wherein the black dots are dual points and dotted lines stand for smart lines.

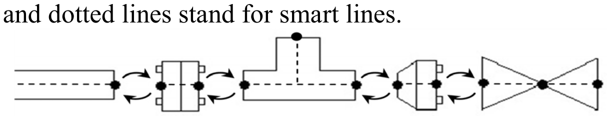
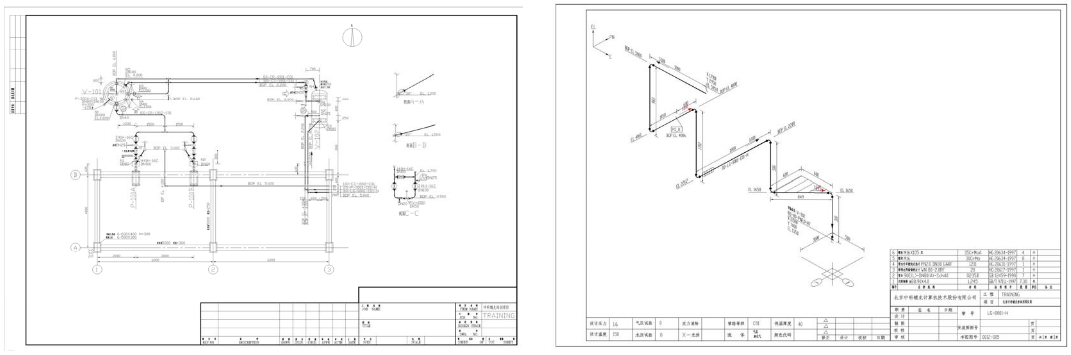


Figure 3. A simple duality point and smart line based pipeline example.

B. 2D engineering drawing

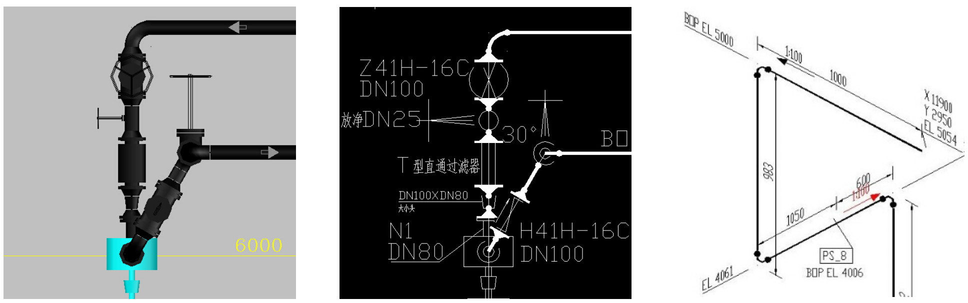
In the present state of engineering CAD development, design results still largely remain in the form of 2D engineering drawings, including P&ID, ISO drawing, section drawings, etc. Engineering drawings exploit 2D graphs as well as annotations and illustrations to describe the relevant engineering attributes, such as structures, geometrical features, material attributes, etc. Fig.4 gives two instances derived from 3D model Fig.2(a), as are a pipeline plane drawing and an ISO drawing respectively.

A typical engineering drawing generally consists of drawing field, material block, title block and sign block. Contents in drawing field are the set of component shapes, dimensions and projecting direction indicators. Material block is responsible for storing the quantities of various materials as well as specifications in the drawing field. Title block is the symbol of engineering design company. Sign block contains authors' signatures after the engineering drawing is revised.



(a) A pipeline plane drawing (b) An ISO drawing

Figure 4. 2D engineering drawings of 3D model Fig.2(a).



(a) 3D model Fig.2(a) (b) Pipeline plane drawing Fig.4(a) (c) ISO drawing Fig.4(b)

Figure 5. The zoomed-in views of 3D model Fig.2(a) and its 2D engineering drawings.

For 2D engineering drawings, due to the constraints of engineering design standards, a component's geometry can be different from the direct projection of its real 3D shape. For example, Fig.5(a) and Fig.5(b) are the zoomed-in views of 3D model Fig.2(a) and its engineering drawing Fig.4(a) respectively. We can see that various related annotations are added in engineering drawings and several components' 2D shapes differ from their 3D direct projections.

Another example is Fig.5(c). An ISO drawing is the non-scale projection of a pipeline under a special view. The purpose of ISO drawing is to thoroughly depict the relative relations and every detail among pipes. Thus, its drawing process requires certain transformations and projections in conformity with standardized icon, instead of directly projecting proportionally to the real pipe sizes. This kind of engineering drawing only keeps topology consistent with its corresponding 3D model. Therefore, 2D engineering drawings and 3D models derived from the same project may differ in graphical representations.

In order to guide construction, 2D engineering drawings and 3D models may also vary in content structures. For example, the ISO drawing Fig.4(b) describes a partial structure of 3D model Fig.2(a). But the pipeline plane drawing Fig.4(a) describes its global structure.

In conclusion, because 2D engineering drawings and 3D models derived from the same project have differences in graphical representations and content structures, the current shape-feature based 2D & 3D matching methods are not applicable for process plant.

III. ALGORITHM DETAIL

The kernel of process plant lies in its topological structure. For any 3D model and 2D engineering drawing, their topologies will remain invariant if they are derived from the same process plant. Intuitively, the matching algorithm between 3D model and 2D engineering drawing should regard topology as an essential fundamental.

Our proposed method has two primary steps: (1) preprocessing is applied to 3D model M_{3D} and 2D engineering drawing M_{2D} as to transform them into attribute graphs G_{3D} and G_{2D}; (2) related graph similarity methodologies are introduced to calculate the similarity $sim(G_{3D}, G_{2D})$. The result of $sim(G_{3D}, G_{2D})$ is then used to measure the matching degree between M_{3D} and M_{2D}, denoting as $sim(M_{3D}, M_{2D})$.

A. *Preprocessing detail*

A process plant model is designed to be parametric. All components basically follow the same serialized standard which records components' information such as geometric parameters, types and other engineering attributes, rather than raw meshes as in general 3D models.

In process plant design, type attribute plays a critical role for designers to distinguish among different components. In addition, insertion position attributes of components are equivalent in either form. Guided by these factors, we propose to exploit both of type and insertion coordinate as the main identifications of a component.

During the preprocessing from heterogeneous CAD models to attribute graphs, we setup the graph as follows: components are described as nodes, type and insertion coordinate as nodes' attributes and topological relations as edges. We use $G_M = (\mathbf{C}, \Sigma_{\mathbf{C}})$ as the symbolic representation of heterogeneous CAD models, where **C** is the set of components and $\Sigma_{\mathbf{C}}$ the set of topological relations.

Preprocessing of 2D engineering drawings and 3D models should obey the following principles:

(1) Each component type in both models is uniquely identified;

(2) Components of each model are uniquely identified;

(3) Each component should be labeled by one type uniquely;

(4) Each component has at most one relation set. A component has none relation set if it is isolated or does not have any linking relationship.

Moreover, the preprocessing of 2D engineering drawings should extract relevant elements in drawing fields such as types and topological relations of components by filtering annotations and illustrations, as shown in Fig.6.

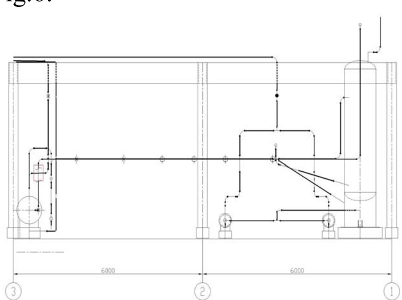


Figure 6. 2D engineering drawings without annotations and illustrations.

B. Matching calculation

After the preprocessing, 3D models and 2D engineering drawings are all in attribute graph structures. The 2D & 3D matching problem is now boiled down to calculating the similarity of attribute graphs.

1) Graph similarity. Graph similarity can be roughly divided into precise measurement and error-tolerant measurement [9], each with a representative solution of Max-Common Sub-graph [10] method and Min-Edit Distance method [11,12]. As Edit Distance is defined within a common theoretical framework, it is actually a quite flexible method and has been successfully applied to many fields [13]. So as a natural choice, we try to tackle the similarity problem using Min-Edit Distance in our proposed method.

Definition 1. Min-Edit Distance. The min-edit distance between graph G_1 and G_2, denoting as $d(G_1,G_2)$, is the minimum operations to transform G_1 into G_1' such that $G_1' = G_2$.

Definition 2. Graph Similarity. Given two graphs G_1 and G_2, and edit distance threshold τ. G_1 and G_2 are similar if $d(G_1, G_2) \leq \tau$.

Notably, Edit Distance method becomes equivalent to Max-Common Sub-graph if τ is set to zero.

2) Matching calculation detail. Combining with the related graph similarity researches introduced in Section III-B1, we propose a 2D engineering drawing and 3D model matching algorithm for process plant. Denote τ ($\tau \geq 0$) as the predefined edit distance threshold and $d(G_p, G_q)$ (initialized to 0) as the edit distance.

Step 1: Preprocessing 3D model M_{3D} and 2D engineering drawing M_{2D} by transforming them into attribute graphs $G_{3D} = (\mathbf{C}_{3D}, \Sigma_{\mathbf{C}_{3D}})$ and $G_{3D} = (\mathbf{C}_{2D}, \Sigma_{\mathbf{C}_{2D}})$ respectively;

Step 2: Compare the quantities of topological relations of G_{3D} and G_{2D}. Let the smaller one be the reference graph G_q and the larger one be the comparison graph G_p, which means that if $e_{3D} > e_{3D}$, then $G_p = G_{3D}$ and $G_q = G_{2D}$; otherwise, $G_p = G_{2D}$ and $G_q = G_{3D}$;

Step 3: If the traversal of G_q is finished, go to Step 6; otherwise, for each to-be-traversed node C, randomly choose a node C' in G_p who has the same type and coordinate with C, then go to Step 4;

Step 4: If the neighbor traversal of C is finished, go to Step 3; otherwise, for each to-be-traversed neighbor N, if there exists a neighbor N' of C' who has the same type and coordinate with N, repeat Step 4 to continue this traversal; otherwise, increment edit distance d by 1, then go to Step 5;

Step 5: If $d(G_p, G_q) \leq \tau$, meaning the current edit distance is within an acceptable range, go to Step 4 to continue the neighbor traversal of C; otherwise, stop the comparison and mark all traversed nodes in G_q and G_p as to-be-traversed, then go to Step 3;

Step 6: The traversal of G_q is finished now and $d(G_p, G_q) \leq \tau$. It indicates that in G_p, there exists a topological structure similar with G_q. Thus, we regard M_{3D} and M_{2D} as a match.

As described above, in our method, the matching degree between model M_{3D} and M_{2D} is measured by the topology similarity between G_{3D} and G_{2D}. We use the smaller scale model as reference and the other as target. Here the model scale is determined by its number of topological relations. Comparing with the reference model, if the target model has some local structure whose min-edit distance is no more than threshold τ, we regard M_{3D} is matching with M_{2D}.

IV. EXPERIMENT RESULTS

Details about our experimental platform are an Intel dual core 2.1 GHz CPU and 3G memory laptop. All experiments are strictly conducted as described in Section III-A and Section III-B1.

A. Preprocessing results

As a process plant model is designed to be parametric, we use PDSOFT® 3DPiping and Visual Studio 2008 as the development environment to preprocess process plant models automatically. The preprocessing results of 3D models in Fig.2 and its 2D engineering drawings in Fig.4 are presented in TABLE I.

TABLE I. Preprocessing results

Model	Components | C |	Topological relations $|\sum_C|$
Fig.2(a)	395	785
Fig.2(b)	9,795	19,044
Fig.2(c)	22,596	45,373
Fig.4(a)	395	785
Fig.4(b)	35	66

2D engineering drawing Fig.4(a) is the global structure of 3D model Fig.2(a). From TABLE I we can see, by filtering annotations and illustrations as shown as Fig.2(a), their quantities of components and topological relations are both the same after preprocessing.

B. Matching results

Since a precise matching between 3D model and 2D engineering drawing is needed in real engineering applications, we set edit distance threshold τ to zero throughout experiments. TABLE II gives the matching results between 3D models and various 2D engineering drawings, we can see that:

(1) Our experimental results are in accordance with the actual fact. As described in Section III-B, if $d(G_{3D}, G_{2D}) \leq \tau$, we consider that the attribute graph G_{2D} is matching with G_{3D}. From TABLE II we can see, because their min-edit distances are both zero, we regard that 3D model Fig.2(a) is matching with 2D engineering drawings Fig.4(a) and Fig.4(b). As Fig.4(a) and Fig.4(b) are both derived from Fig.2(a), our matching results meet the fact.

(2) In this experiment, we denote $\tau = 0$ which means that if $d(M_{3D}, M_{2D}) > 0$, M_{3D} and M_{2D} are regarded as a mismatch. As shown in TABLE II, because the source model of 2D engineering drawing Fig.4(b) is not 3D model Fig.2(b), their min-edit distance is more than zero, resulting in a mismatch result.

(3) Most calculation times in our experiment lie in a proper range. As shown in TABLE II, acceptable calculation times are observed in almost all experiments, except for two large scale groups (large in both 3D model and 2D drawing) whose matching calculation times reach a length of 532,993ms and 1,496,800ms respectively. With the scales of the smaller models under comparison shrinking, their measure time is reducing.

TABLE II. Matching results ($\tau = 0$)

3D model M_{3D}		2D engineering drawing M_{2D}			Matching calculation		
Model	*Relations* e_{3D}	*Drawing*	*Relations* e_{2D}	*Source model*	*Edit distance* $d(M_{3D}, M_{2D})$	*Result*	*Time*
Fig.2(a)	785	Plane drawing (Fig.4(a))	785	Fig.2(a)	0 ($\leq \tau$)	Matching	1,981ms
		Back-side section (Fig.6)	785	Fig.2(a)	0 ($\leq \tau$)	Matching	1,545ms
		ISO drawing (Fig.4(b))	66	Fig.2(a)	0 ($\leq \tau$)	Matching	93ms
		ISO drawing	213	Fig.2(a)	0 ($\leq \tau$)	Matching	313ms
		Plane drawing	19,044	Fig.2(b)	785 ($> \tau$)	NOT Matching	13ms
		ISO drawing	133	Fig.2(b)	133 ($> \tau$)	NOT Matching	16ms
Fig.2(b)	19,044	Plane drawing (Fig.4(a))	785	Fig.2(a)	785 ($> \tau$)	NOT Matching	13ms
		ISO drawing (Fig.4(b))	66	Fig.2(a)	66 ($> \tau$)	NOT Matching	239ms
		Left-side section	19,044	Fig.2(b)	0 ($\leq \tau$)	Matching	532,993ms
		ISO drawing	64	Fig.2(b)	0 ($\leq \tau$)	Matching	156ms
		ISO drawing	133	Fig.2(b)	0 ($\leq \tau$)	Matching	249ms
Fig.2(c)	45,373	Plane drawing (Fig.4(a))	785	Fig.2(a)	785 ($> \tau$)	NOT Matching	13ms
		ISO drawing (Fig.4(b))	66	Fig.2(a)	66 ($> \tau$)	NOT Matching	16ms
		Plane drawing	45,373	Fig.2(c)	0 ($\leq \tau$)	Matching	1,496,800ms
		ISO drawing	40	Fig.2(c)	0 ($\leq \tau$)	Matching	93ms
		ISO drawing	170	Fig.2(c)	0 ($\leq \tau$)	Matching	432ms

V. ALGORITHM DISCUSSION

A. Robustness discussion

The proposed algorithm is similarity transformation consistent. It is proven in Fig.5 that, the direct projections of some components (in Fig.2(a)) may have different geometrical shapes with their shapes in 2D engineering drawings (as shown in Fig.4). Nonetheless, results in TABLE II indicate a nice match between 3D model Fig.2(a) and its 2D drawings Fig.4(a) and Fig.4(b). It convincingly demonstrates the great property of similarity transformation consistency of our proposed method.

The proposed algorithm is also translation and rotation consistent. Plane drawing, back-side section drawing and left-side section drawing are all 2D engineering drawings derived from 3D models under different views. If we use the shape-feature based methods to measure their similarities with 3D model, an initial projection under the same view will be needed which requires several operations such as rotation and translation for 3D models. However, the topological relations in these models are reserved consistently during these transformations. Thus, our method seeks to solve the problem in a topology perspective, effectively bypassing the problems of numerical error, low efficiency and expensive running cost brought by traditional 3D coordinate standardization.

B. Complexity discussion

Comparing to the geometry based matching methods, our proposed algorithm has a lower computation complexity as several trivial operations like rotations and translations are discarded. Let n_{3D} and n_{2D} be the quantities of components for M_{3D} and M_{2D}, e_{3D} and e_{2D} are the quantities of topological relations respectively. Our proposed method will result in a $O(\min((n_{3D} + e_{3D}), (n_{2D} + e_{2D})))$ complexity. Because we use the smaller one as a reference model when M_{3D} and M_{2D} are of different scales, the choice of reference model has a direct effect on computation efficiency. As indicated in TABLE II, Fig.2(a) has an equal scale with Fig.6 while differing significantly from Fig.4(b), resulting in a shorter calculation time in the latter case.

Because the time complexity of our matching method is proportional to the scale of the smaller model, matching calculation time would become longer with the increase of this smaller scale. As shown in TABLE II, for 3D model Fig.2(b) and 2D left-side section, number of relations of the smaller model is 19,044, with a corresponding calculation time of 532,993ms; for 3D model Fig.2(c) and 2D plane drawing, number of relations is up to 45,373, resulting in a longer calculation time of 1,496,800ms.

VI. CONCLUSIONS

In engineering design, the matching calculation between 2D engineering drawings and 3D models is of great significance in the management and reuse of existing CAD models. Due to some constraints of process plant design standards, various 2D engineering drawings and 3D models are different in graphical representations and content structures, which leads to the inapplicability of existing shape-feature based 2D & 3D matching methods. In order to solve this problem, we propose an algorithm based on components' topological relations. This algorithm firstly introduces a preprocessing procedure to transform the compared 2D engineering drawings and 3D models into attribute graphs. Attribute graphs similarity is then calculated to measure the matching degree between two corresponding CAD models. Experiment demonstrates that our proposed method yields realistic results within a proper time cost and meets real engineering application demands. Furthermore, the proposed method embraces a good property of translation, rotation and similarity transformation consistency, making it even more scalable toward other engineering models.

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